18.3. The pivot mechanism

In the literature are a number of applications of the general ideas just given to the design of optimal mechanisms that deserve your attention. In particular, the analysis of optimal auction design is especially rich and well developed; references will be supplied at the end of this chapter.

Rather than pursue one of these, we look at a different sort of problem in mechanism design that uses some of the notions of the previous section. Instead of looking for an "optimal" mechanism, according to the interests of some individual who (presumably) is designing the mechanism with her own interests in mind, we will look at a classic example of mechanism design where the objective is to satisfy a set of criteria.

Consider the following story. Several farmers live on the banks of a stream. (Some live on one side, some on the other.) The number of farmers is I, and we index them by $i = 1, 2, \ldots, I$. They consider building a bridge over the stream that will permit them all to cross back and forth. There is only one place where the bridge can be constructed, and the bridge will cost K to construct. Each farmer attaches some value to having the bridge constructed, but none is quite sure what value the others attach to it. If they construct the bridge, the funds to do so will have to come from their own pockets.

These farmers must decide whether or not to build this bridge. We take it as given that if the bridge is built, each farmer will pay K/I, his pro rata share of the costs of building the bridge. But in addition to this, the farmers are willing to consider transfers among themselves; we write t^i for the amount of money *taken* from farmer i, above and beyond the K/I that is collected if the bridge is to be built. We do not preclude $t^i < 0$, which means that a subsidy is paid to farmer i.

Farmer i's utility depends on whether the bridge is built and on any monetary transfer t^i that is made to or from him. We suppose that each farmer i attaches some monetary value to the bridge, u^i, and that farmer i's utility is linear in money and in this valuation. That is, farmer i's utility depends on (a) whether the bridge is built, (b) the building tax K/I that he pays if it is built, and (c) any further tax or subsidy, t^i; this utility is given by

$$\text{Farmer } i\text{'s utility} = \begin{cases} u^i - K/I - t^i & \text{if the bridge is built, and} \\ -t^i & \text{if the bridge isn't built.} \end{cases}$$

Note that we permit transfers even if the bridge isn't built. We assume that farmers may dislike having this bridge built; i.e., $u^i < 0$ is possible.

In fact, we assume that u^i could be any real number whatsoever; no value is precluded.

It will be expositionally convenient to work with the farmer's valuation of the bridge, *net* of his contribution for building it, or $v^i = u^i - K/I$. Note that with this substitution, farmer i's utility is $v^i - t^i$ if the bridge is built.

The farmers meet together one evening to try to decide whether to build the bridge and what taxes and subsidies to enforce beyond the pro rata contribution to building it. One farmer suggests majority rule and no transfers: Put building the bridge to a vote, and if a majority vote to build the bridge, then everyone pays only his contribution K/I; if a majority vote against, no transfers are made at all. If this plan is adopted, farmers will vote for the bridge if and only if their net personal valuation v^i exceeds 0. But there are several objections to this. One farmer observes that some farmers may really want this bridge, and if most farmers don't care that much (i.e., if $u^i < K/I$ for many farmers), then the bridge won't be built, even though general social welfare would be improved if it were built. Another objects that he doesn't want this bridge much at all, in fact he positively dislikes it, and yet he may be assessed with a building fee of K/I. What justice is there in that?

So someone suggests that farmers "subscribe" to the bridge. Everyone will write his name and a pledge (an amount of money) on a piece of paper and toss it into a hat. After all the pledges are collected, they will be added together. Negative pledges are allowed; farmers in this way demand compensation for having the bridge built. If the sum of the pledges exceeds 0, the bridge will be built, and every farmer must contribute the amount of his pledge (plus his share K/I). If the pledges amount to less than 0, then the bridge isn't built and no transfers will be made. What to do with the surplus, if more than zero is pledged? One farmer suggests that this surplus be divided equally among all the farmers. Another suggests that it be divided proportionally (in proportion to original pledges) among those farmers who pledged positive amounts. Still another suggests that it might be a good thing to collect the excess and burn it. (This last suggestion is met with some incredulity.)

Thinking about this mechanism, the farmers recognize some difficulties. Imagine a farmer who values the bridge at a net amount v^i. How much should he pledge? If he believes that it will take a pledge of v^i or more to get the bridge built, then he is willing to pledge precisely v^i. He certainly would never wish to pledge more than this, since he will be stuck with a pledge that is greater than the net value of the bridge to him. On the other hand, if he believes that a pledge of less than v^i will do, then he wishes to shade his pledge down from v^i and "free-ride" on the pledges of his fellow farmers. In general, he won't be sure which of these two circumstances pertains, but as long as he assesses positive probability for the second, his optimal bid will be something less than v^i. Hence the total pledges will be less than $\sum_i v^i$, and the bridge may not be built when it should be. In any case, another farmer objects that figuring out how to behave in this mechanism is too hard for simple country bumpkins, and all sorts of bad outcomes may result.

After much discussion, the farmers decide that they wish to find some mechanism for deciding whether to build the bridge and what taxes to assess that has the following properties:

(a) The bridge will be built if and only if it is socially efficient to do so; that is, if and only if $\sum_i v^i \geq 0$.

(b) No farmer should have to waste his time doing complicated analysis of how to behave in the mechanism. In particular, no farmer should have to spend time trying to assess what his fellow farmers will do. Put another way, the optimal actions of each farmer in the mechanism (as a function of the farmer's private valuation v^i) should dominate any other actions the farmer might take, no matter what his fellow farmers do.

(c) The mechanism, if played optimally by a farmer, should never be so injurious to the welfare of the farmer that he would prefer that the decision to build or not to build is taken by decree. That is, farmer i should not wind up with utility less than $\min\{v^i, 0\}$.

(d) Since no outsider is willing to put up money to permit the mechanism to function, the taxes collected (less any subsidies, and not including the building tax K/I per farmer, if the bridge is built) must be nonnegative.

One can certainly quibble (a), (b), and (c) (and later we will have a very large quibble with [a], in particular), but let us accept them and see where they lead.

We translate (b) as saying that the farmers want a dominant strategy mechanism. They wish to design a general mechanism in which each farmer, as a function of his personal valuation v^i, has a dominant strategy to play. With this, we appeal to the revelation principle for dominant strategy mechanisms: Anything we can do with a general dominant strategy mechanism can be done with a direct revelation mechanism in which truth-telling is a dominant strategy. So we can restrict attention to direct revelation mechanisms.

In this context a direct revelation mechanism takes the form: Each farmer is asked to reveal his personal valuation v^i. We put hats on variables to distinguish what farmers reveal from their true valuations; that is, $\hat{v}^i$ is what farmer i reveals. As a function of the vector of revealed valuations $\hat{v} = (\hat{v}^1, \ldots, \hat{v}^I)$, a decision is made whether to build the bridge or not, and taxes on each farmer are determined. We write $\alpha(\hat{v})$ for the decision whether to build the bridge, where $\alpha(\hat{v}) = 1$ means that the bridge is built and $\alpha(\hat{v}) = 0$ means it is not. (In a more general treatment of these issues, we might let $\alpha(\hat{v})$ take on any value in the interval $[0, 1]$, interpreting this as the probability that the bridge will be built. But given the farmers' requirement [a], we know what α must be, and this doesn't entail randomized decisions whether to build the bridge.) And we write $t^i(\hat{v})$ for the tax imposed on farmer i. (More generally, we could permit random taxes, but as farmers are risk neutral, this would add nothing to the story except horrid notation.)

It will be handy to have the following pieces of notation. We let $\hat{v}^{-i} = (\hat{v}_1, \ldots, \hat{v}_{i-1}, \hat{v}_{i+1}, \ldots, \hat{v}_I)$. That is, $\hat{v}^{-i}$ is the vector of reported valuations for all the farmers except for i. We sometimes write $t^i(\hat{v})$ as $t^i(\hat{v}^i, \hat{v}^{-i})$; i.e., we put i's valuation as the first argument. Finally, we write $\hat{\Sigma}$ for $\sum_{j=1}^{I} \hat{v}^j$ and $\hat{\Sigma}^{-i}$ for $\sum_{j \neq i} \hat{v}^j$. That is, $\hat{\Sigma}$ is the sum of all the announced valuations, and $\hat{\Sigma}^{-i}$ is the sum of all the announced valuations save i's.

We are looking for a direct revelation mechanism in which truth-telling is a dominant strategy and in which the bridge is built if and only if $\sum_i v^i \geq 0$. The latter restriction, combined with the notion that truth-telling will be dominant tells us what the function α must be:

$$\alpha(\hat{v}) = \begin{cases} 1, & \text{if } \hat{\Sigma} \geq 0, \text{ and} \\ 0, & \text{if } \hat{\Sigma} < 0. \end{cases} \tag{1}$$

Furthermore, we have the following results:

Lemma 1. *The taxes paid by farmer i must take the form*

$$t^i(\hat{v}^i, \hat{v}^{-i}) = \begin{cases} \bar{t}^i(\hat{v}^{-i}), & \text{if } \hat{\Sigma} \geq 0, \text{ and} \\ \underline{t}^i(\hat{v}^{-i}), & \text{if } \hat{\Sigma} < 0, \end{cases} \tag{2}$$

for $\bar{t}^i$ and $\underline{t}^i$ functions of $\hat{v}^{-i}$.

The idea is that what a farmer pays in taxes cannot depend on what he himself reveals as his valuation, except insofar as this revelation changes the decision whether or not to build the bridge. To see why this is so, suppose v^i and w^i, two valuations for i, and $\hat{v}^{-i}$ are such that $v^i + \hat{\Sigma}^{-i} > 0$, $w^i + \hat{\Sigma}^{-i} > 0$, and $t^i(v^i, \hat{v}^{-i}) > t^i(w^i, \hat{v}^{-i})$. In words, if the other farmers are announcing $\hat{v}^{-i}$, then whether i announces v^i or w^i, the bridge will be built. But announcing v^i (when the others are announcing $\hat{v}^{-i}$) results in higher taxes than does announcing w^i. In such circumstances, the mechanism doesn't have truth-telling as a dominant strategy; farmer i, if his valuation is v^i, would prefer to misrepresent his valuation as w^i (if his fellow farmers announce $\hat{v}^{-i}$). This establishes the first part of the proposition: For $\hat{v}$ such that $\hat{\Sigma} \geq 0$, $t^i(\hat{v})$ depends only on $\hat{v}^{-i}$. A similar argument gives the other half.

***Lemma* 2.**

$$\bar{t}^i(\hat{v}^{-i}) - \underline{t}^i(\hat{v}^{-i}) = -\hat{\Sigma}^{-i}. \tag{3}$$

To prove this, fix $\hat{v}^{-i}$ and consider the case where $v^i = -\hat{\Sigma}^{-i}$. If farmer i truthfully reveals v^i when the others reveal $\hat{v}^{-i}$, then farmer i must prefer revealing v^i to revealing $v^i - \epsilon$ for $\epsilon > 0$. But, in these circumstances, revealing v^i means the bridge will be built and revealing $v^i - \epsilon$ means that the bridge won't be built. So by revealing v^i, farmer i nets $v^i - \bar{t}^i(\hat{v}^{-i})$, while revealing $v^i - \epsilon$ nets $-\underline{t}^i(\hat{v}^{-i})$. The former must be at least as large as the latter, so

$$v^i = -\hat{\Sigma}^{-i} \geq \bar{t}^i(\hat{v}^{-i}) - \underline{t}^i(\hat{v}^{-i}).$$

Now consider the case where $v^i = -\epsilon - \hat{\Sigma}^{-i}$ for $\epsilon > 0$. Truthful revelation of v^i causes the bridge not to be built, giving farmer i utility $-\underline{t}^i(\hat{v}^{-i})$. Falsely revealing $v^i + \epsilon$ causes the bridge to be built, and farmer i has utility $v^i - \bar{t}^i(\hat{v}^{-i})$. Since the former must be at least as large as the latter (to support truth-telling),

$$v^i = -\epsilon - \hat{\Sigma}^{-i} \leq \bar{t}^i(\hat{v}^{-i}) - \underline{t}^i(\hat{v}^{-i}).$$

This is true for arbitrary $\epsilon > 0$, so by letting $\epsilon \to 0$,

$$-\hat{\Sigma}^{-i} \leq \bar{t}^i(\hat{v}^{-i}) - \underline{t}^i(\hat{v}^{-i}).$$

Together with the next to last inequality, this gives the desired result.

Lemma 3.

$$\underline{t}^i(\hat{v}^{-i}) \leq \begin{cases} \hat{\Sigma}^{-i}, & \text{if } \hat{\Sigma}^{-i} \geq 0, \text{ and} \\ 0, & \text{if } \hat{\Sigma}^{-i} < 0. \end{cases} \tag{4}$$

Consider first the case where $\hat{\Sigma}^{-i} \geq 0$. Suppose $v^i = 0$. By truthfully reporting v^i, farmer i causes the bridge to be built. His utility in this case is $v^i - \bar{t}^i(\hat{v}^{-i}) = 0 - [\underline{t}^i(\hat{v}^{-i}) - \hat{\Sigma}^{-i}]$. Since (c)[9] entails that farmer i is no worse off for having revealed the truth than 0 (in utility), we have $-[\underline{t}^i(\hat{v}^{-i}) - \hat{\Sigma}^{-i}] \geq 0$ or $\underline{t}^i(\hat{v}^{-i}) \leq \hat{\Sigma}^{-i}$. Consider next the case where $\hat{\Sigma}^{-i} < 0$. Again suppose $v^i = 0$. By truthfully revealing v^i, farmer i causes the bridge not to be built. Hence he nets $-\underline{t}^i(\hat{v}^{-i})$. Since he must be no worse off than 0 by so doing, we have $-\underline{t}^i(\hat{v}^{-i}) \geq 0$, or $\underline{t}^i(\hat{v}^{-i}) \leq 0$.

Lemma 4.

$$\underline{t}^i(\hat{v}^{-i}) = \begin{cases} \hat{\Sigma}^{-i}, & \text{if } \hat{\Sigma}^{-i} \geq 0, \text{ and} \\ 0, & \text{if } \hat{\Sigma}^{-i} < 0. \end{cases} \tag{5}$$

In other words, inequality (4) must be an equation.

To see this, we note first that from lemmas 1 and 2, (d) rendered in symbols is

$$\textstyle\sum_i \underline{t}^i(\hat{v}^{-i}) \geq 0 \text{ if } \hat{\Sigma} < 0, \text{ and} \tag{$d1$}$$

$$\textstyle\sum_{n=1}^{N} \bar{t}^i(\hat{v}^{-i}) = \sum_i [\underline{t}^i(\hat{v}^{-i}) - \hat{\Sigma}^{-i}] \geq 0 \text{ if } \hat{\Sigma} \geq 0. \tag{$d2$}$$

Suppose, then, that for some i and $\hat{v}^{-i}$ such that $\hat{\Sigma}^{-i} \geq 0$ we had a strict inequality in (4). That is, $\underline{t}^i(\hat{v}^{-i}) < \hat{\Sigma}^{-i}$. Choose v^i to be sufficiently large so that $\hat{\Sigma}^{-i'} > 0$ for all i' and so that $\hat{\Sigma} > 0$. The bridge will be built, and by lemmas 1 and 2, farmer i' pays

$$\bar{t}^{i'}(\hat{v}^{-i'}) = \underline{t}^{i'}(\hat{v}^{-i'}) - \hat{\Sigma}^{-i'}$$

By lemma 3, each such term is nonpositive and, by the supposition, the term for i is strictly negative. Hence the sum of all the payments is strictly negative, contradicting ($d2$).

Similarly, suppose that for i and $\hat{v}^{-i}$ such that $\hat{\Sigma}^{-i} < 0$, we had $\underline{t}^i(\hat{v}^{-i}) < 0$. We can choose v^i sufficiently small so that $\hat{\Sigma}^{-i'} < 0$ for all i'

[9] In case you forgot, (c) says that, in general, a farmer should do no worse than $\min\{v^i, 0\}$ by reporting the truth.

and $\hat{\Sigma} < 0$. Arguing along the lines just given, we obtain a violation of ($d1$).

We now put all the pieces together and provide a converse.

Proposition 2. *There is only one direct revelation mechanism for which (a) through (d) hold, namely the direct revelation mechanism defined by (1), (2), (3), and (5). This mechanism is alternatively defined by (1) and*

$$t^i(\hat{v}) = \begin{cases} 0, & \text{if } \hat{\Sigma} \geq 0 \text{ and } \hat{\Sigma}^{-i} \geq 0, \\ 0, & \text{if } \hat{\Sigma} < 0 \text{ and } \hat{\Sigma}^{-i} < 0, \\ \hat{\Sigma}^{-i}, & \text{if } \hat{\Sigma} < 0 \text{ and } \hat{\Sigma}^{-i} \geq 0, \text{ and} \\ -\hat{\Sigma}^{-i}, & \text{if } \hat{\Sigma} \geq 0 \text{ and } \hat{\Sigma}^{-i} < 0. \end{cases} \tag{6}$$

Proof. The lemmas establish that this is the only possible candidate. Showing that (6) is equivalent to (2), (3), and (5) is a matter of simple bookkeeping. We see from (6) that $t^i(v) \geq 0$ in all cases, so (d) is obvious. So all that remains is to show that the mechanism defined by (1) and (6) satisfies (b) and (c). We will do half of this, leaving the other half for homework.

Suppose that, for some i and $\hat{v}^{-i}$, $\hat{\Sigma}^{-i} \geq 0$ and $v^i + \hat{\Sigma}^{-i} \geq 0$. If i tells the truth, the mechanism calls for the bridge to be built and for i to pay no tax. The only way that i can change the outcome is to report (falsely) a valuation $\hat{v}^i$ less than $-\hat{\Sigma}^{-i}$. Then the bridge won't be built, and i will pay a tax of $\hat{\Sigma}^{-i}$. Reporting truthfully, then, leaves i with utility v^i, and a misrepresentation sufficient to change the outcome leaves i with utility equal to $-\hat{\Sigma}^{-i}$. Since we assume in this case that $v^i + \hat{\Sigma}^{-i} \geq 0$, $v^i \geq -\hat{\Sigma}^{-i}$ Thus, telling the truth is a best reply; also, i ends up with utility v^i, so (c) holds in this case.

Suppose that, for some i and $\hat{v}^{-i}$, $\hat{\Sigma}^{-i} < 0$ and $v^i + \hat{\Sigma}^{-i} \geq 0$. The bridge will be built if i reports truthfully, and he will pay a tax of $-\hat{\Sigma}^{-i}$, which gives i utility $v^i + \hat{\Sigma}^{-i} \geq 0$. So (c) holds. Moreover, the only way that i can change the outcome (and his utility) is by reporting a valuation $\hat{v}^i < -\hat{\Sigma}^{-i}$. But if he does this, $\hat{\Sigma}^{-i} < 0$ and $\hat{\Sigma} < 0$, which means that there is no transfer, and i winds up with utility 0. So misrepresenting is not beneficial.

(The other two cases are left for you.)

This particular mechanism is called the *pivot mechanism* because a tax is paid by farmer i only if his valuation v^i is pivotal, that is, only if his valuation changes the decision from what it would be if he reported zero.

Moreover, when i's valuation is pivotal, i is taxed an amount that is just equal to the "social distress" his pivotal valuation causes; i.e., if he causes the bridge to be built when it otherwise wouldn't be, he pays $-\hat{\Sigma}^{-i}$, which is the monetary "cost" to the rest of the farmers of the bridge; and if he causes the bridge not to be built when it otherwise would be, he pays $\hat{\Sigma}^{-i}$, the monetary benefit to the other farmers of the bridge.

This mechanism satisfies our four criteria, and in fact it is the only mechanism that does this, so if we accept the four as desirable, we have come upon quite a nice result. Condition (a), however, seems especially suspect. The justification for it is that we wish to achieve a "social optimum," which in this society where utility is linear in money means maximizing the sum of individual utilities. But the mechanism doesn't achieve a social optimum if there are any pivotal individuals, because it produces a positive net collection of taxes. Note well: The transfers are all nonnegative, and they are all zero only if there are no pivotal individuals.

This raises the question, If there are pivotal individuals, so a net surplus of funds is collected, what happens with that surplus? Unthoughtful answers are to give it back to the farmers, or to hold it for the next project to come along, or to use it to throw a dance. If the surplus is used in any fashion that gives utility to the farmers, and if the farmers anticipate this, then the direct revelation mechanism isn't what we described. We would have to include in the farmer's utilities the value they attach to the uses to which the surplus is put. But our uniqueness result is that the only mechanism that will achieve the four required properties is the one described. This means that if we wish to achieve (a) through (d), we must find a use for the surplus that is of no benefit (or detriment) to our farmers. The surplus must be burned, or mailed off to aid some other lucky group of individuals about whom none of our farmers cares in the least. (And our farmers can't anticipate that any such largesse may result in a reciprocal gift coming to them. It is probably safest to burn any surplus.)

Condition (a), then, can be attacked on the grounds that it doesn't guarantee that a social optimum is reached. The decision whether to build the bridge will be done "optimally," but at a waste of other social resources, if there are any pivotal individuals.

We can put this another way. Condition (a) would imply that a social optimum is reached *if* it were joined to the following modification of (d): The sum of taxes and transfers, not including the taxes collected to build the bridge if the bridge is built, totals zero precisely. This condition is known in the literature as the *balanced budget condition*, and what we know from our analysis is that it is impossible to satisfy this together with (a), (b), and (c). In fact, one can show that the balanced budget condition is inconsistent with (a) and (b) alone. It is impossible to achieve a Pareto optimal outcome in this closed society with a dominant strategy mechanism.

18.4. The Gibbard-Satterthwaite theorem

One way to view the results of the previous section is that asking for a dominant strategy mechanism may be asking too much. We close by alerting you to another result that reinforces this message (or which, at least, implies that asking for a dominant strategy mechanism is asking for rather a lot).

Recall Arrow's possibility theorem from chapter 5. The setting is one with a finite number I of individuals, and a finite number of possible social outcomes. We let X be the set of social outcomes. Each individual has preferences $\succ_i$ defined on X that are asymmetric and negatively transitive.

In Arrow's theorem, we were concerned with aggregation of the array of society's preferences $(\succ_1, \ldots, \succ_I)$ into a social ordering $\succ^*$ on X. The story before was that members of society attempted to design some social choice rule that would take the array of preferences $(\succ_1, \ldots, \succ_I)$ and transform them in desirable fashion into society's preferences. We saw that a few, seemingly quite desirable properties for the social choice rule, forces social choice to be dictatorial, which doesn't seem very desirable.

One thing unexplained was how, even if we had found a nice social choice rule, we were going to work out what the preferences of each member of society were. Perhaps the preferences of individual i would be obvious. But it seems more likely that at some point individual i would have to volunteer what she liked and didn't like. And even if we had a nice social choice rule, we might worry that individual i would misrepresent her preferences in an attempt to get social preferences more to her liking.

So we can approach the matter as one of mechanism design. Suppose we had a desirable social choice rule. In fact, we'll make things easier on ourselves: Suppose we had a desirable social choice *function* ϕ that associates with every array of individual preferences $(\succ_1, \ldots, \succ_I)$ some outcome $\phi(\succ_1, \ldots, \succ_I) = x \in X$, which is the outcome that is implemented. All we want to know is whether the given ϕ can be implemented "reliably" by some mechanism, where what we mean by reliable implementation is that if individual preferences are given by $(\succ_1, \ldots, \succ_I)$, then the mechanism (which involves actions of some sort by the members of society, each of whom knows her own preferences) has *dominant strategies* for members of society that lead to the outcome $\phi(\succ_1, \ldots, \succ_I)$. By the revelation principle for dominant strategy mechanisms, we can turn this into the following question:

Given ϕ, is there a direct revelation mechanism, where each individual is called upon to reveal her preferences over X, which has truth-telling as a dominant strategy and which results in the outcome $\phi(\succ_1, \ldots, \succ_I)$ when preferences are $(\succ_1, \ldots, \succ_I)$?

The Gibbard-Satterthwaite theorem. *If the domain of ϕ is the space of all I-tuples of preferences over X, if the range of ϕ has at least three elements, and if ϕ can be implemented by a dominant strategy mechanism, then ϕ must be dictatorial; there is some i such that $\phi(\succ_1, \ldots, \succ_I)$ is one of the $\succ_i$-most preferred elements of X.*

For elegant proofs, see Schmeidler and Sonnenschein (1978) and Barbera (1983).

You may find the conjunction of the Gibbard-Satterthwaite theorem and proposition 2 (on the pivot mechanism) a bit jarring. The pivot mechanism implements a particular social choice function with dominant strategy mechanism, yet that social choice function is hardly dictatorial. As we noted in the previous section, the social choice function of the pivot mechanism is not efficient. But the Gibbard-Satterthwaite theorem says nothing about efficiency of the social choice function. So how are these reconciled?

They reconcile because the Gibbard-Satterthwaite theorem requires that domain of the social choice function is the set of *all* possible I-tuples of preferences over the social outcome. In the farmers and bridge story, preferences over social outcomes come from a very restrictive domain. Preferences are "quasi-linear" in the decision whether to build the bridge and the transfer received by a farmer, and farmers don't care at all about the transfers their neighbors receive. If we allowed farmers to have all manner of preferences over the full social outcome, proposition 2 would crumble, as the Gibbard-Satterthwaite theorem says it must.

18.5. Bibliographic notes

The general subject of optimal contract and mechanism design covers many different categories in the literature. In this chapter, we have limited attention to contract design when adverse selection (or hidden information) is the issue; compare with chapter 16, where we studied some basic results in optimal contract design in situations of moral hazard. (Although we didn't discuss it except in passing in chapter 16, issues related to implementation of particular sets of actions for interacting agents arises there, just as in this chapter. If you did problem 3 in chapter 16, you at least got a taste of this.) And while we have looked at problems that involve only moral hazard or only adverse selection, problems that mix the two can be tackled by a mixture of the methods we have explored.

At the same time, the techniques discussed in this chapter are used to study optimal contracts and problems in social choice, where the objective is to see which social choice rules can be implemented in particular environments. (Sections 18.3 and 18.4 gave two examples of the latter sort of activity.) Finally, there is a wide range of applications of these techniques.