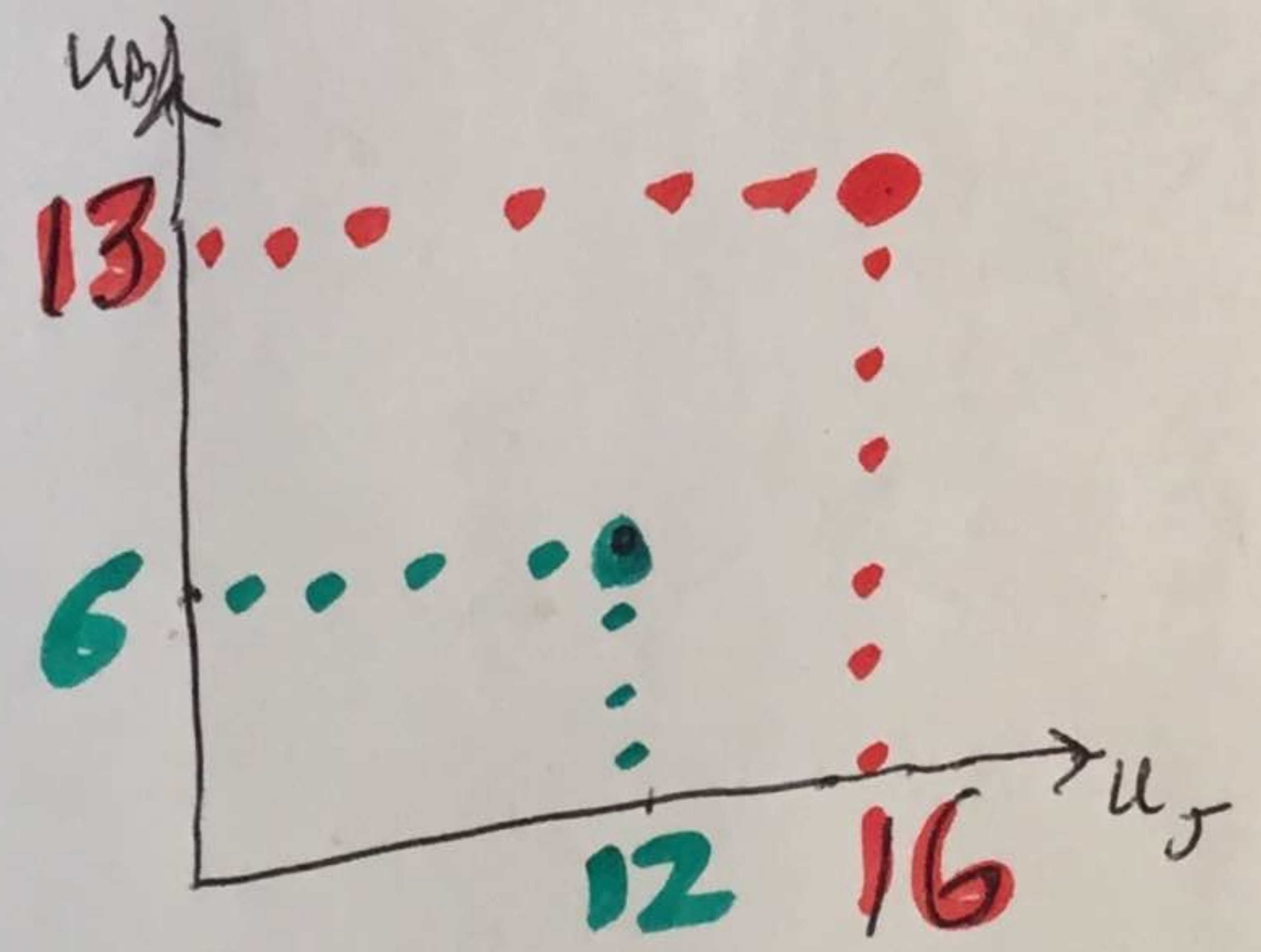
Bill's goods	Utility to Bill	Utility to Jack
book	2	4
whip	2	2
ball	2	1
bat	2	2
box	4	1

Jack's goods	Ut to Bill	Ut to Jack
pen	10	1
toy	4	1
knife	6	2
hat	2	2

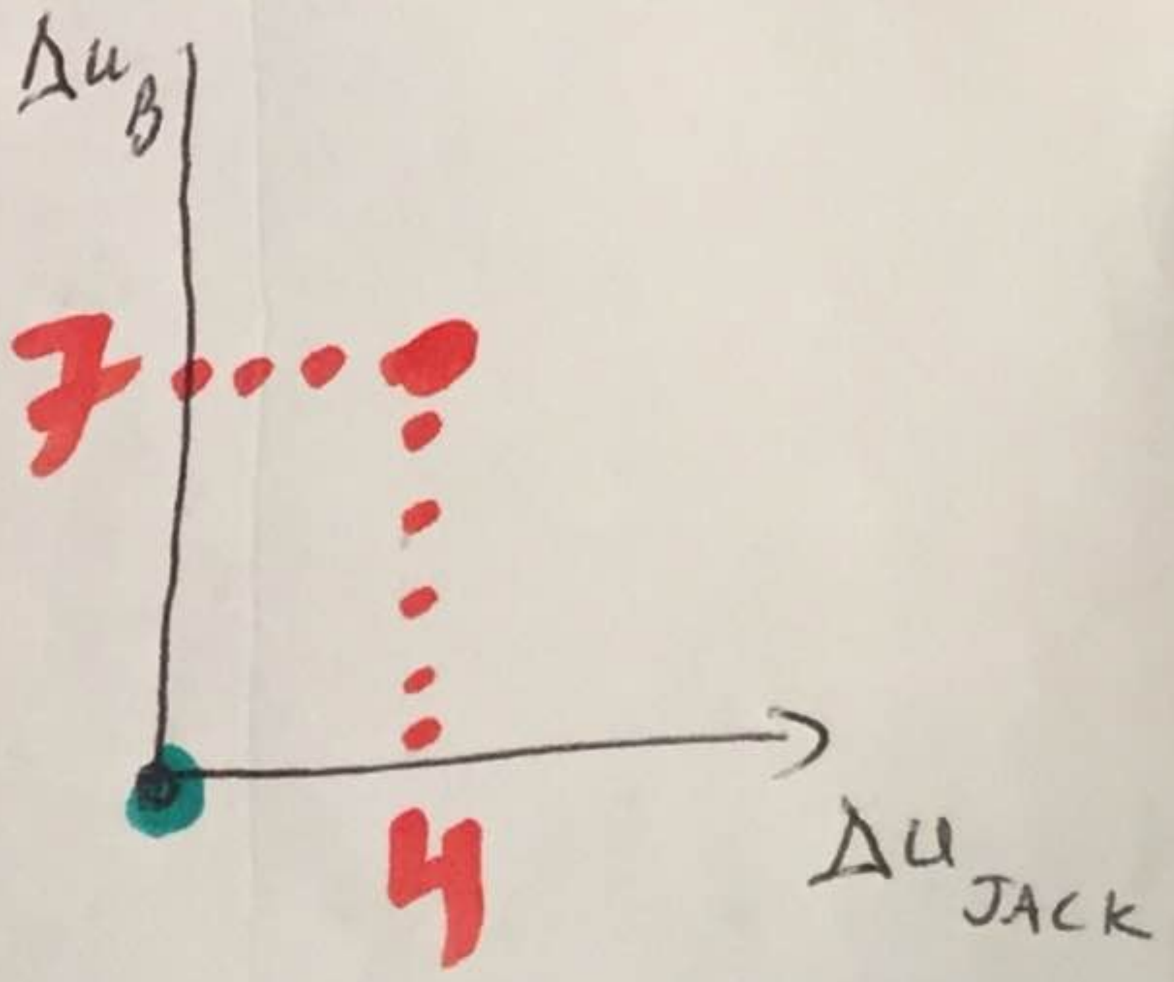
$\left\{ \underbrace{\text{pen, knife}}_{\text{BILL}} \quad \underbrace{\text{. . . . . .}}_{\text{Jack}} \right\} \rightarrow (16, 13)$

$\rightarrow (12, 6)$

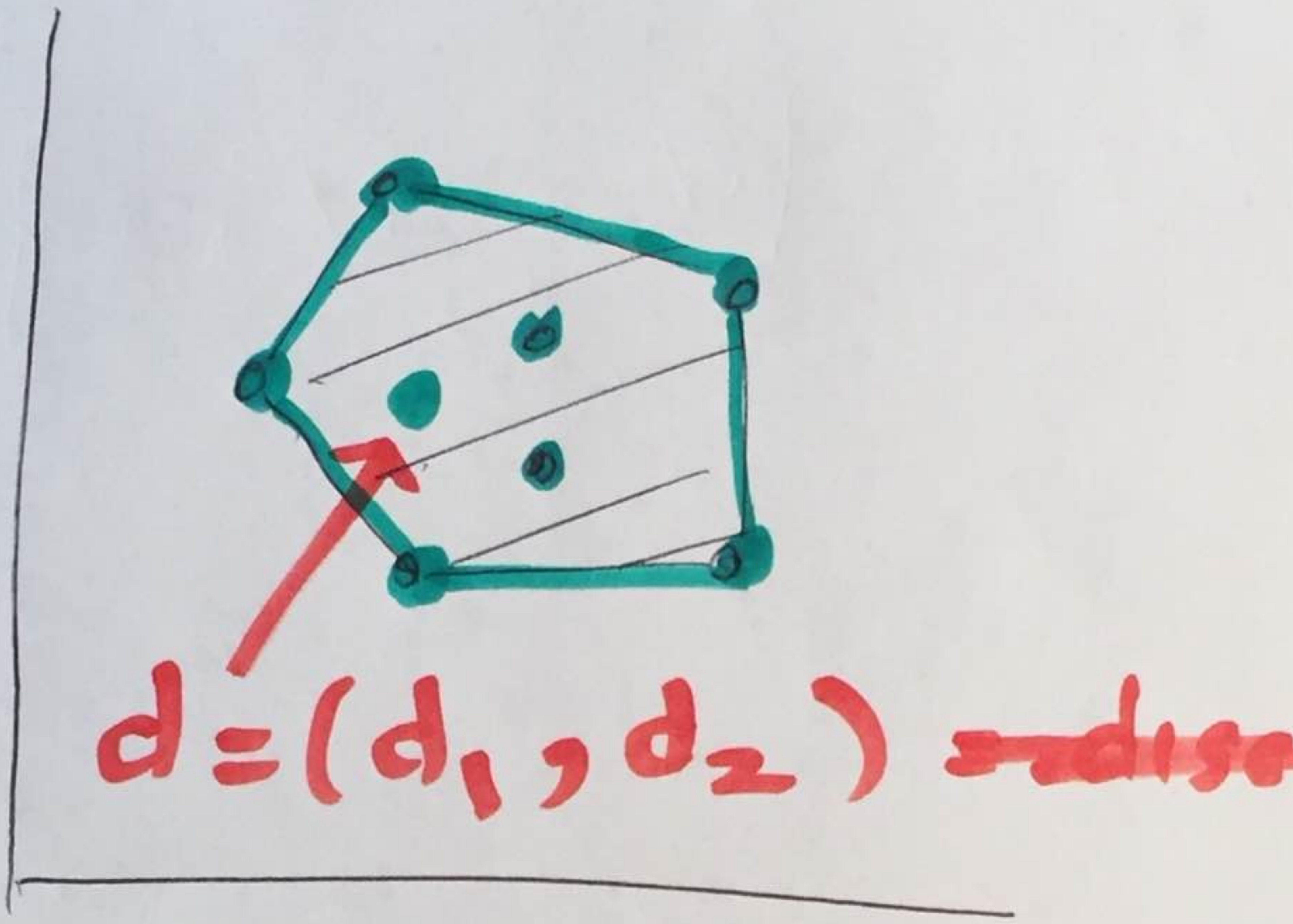
No trade :



or



LOTTERIES



d = disagreement point
(or, status quo)

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

a_i real number $\qquad$ λ_i <u>positive</u>

<u>DEF</u>^{n} (1) $\quad a + \lambda x = \begin{pmatrix} a_1 + \lambda_1 x_1 \\ a_2 + \lambda_2 x_2 \end{pmatrix}$ for $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$

(2) For any set $S \subset \mathbb{R}^2$,

$$a + \lambda S = \{a + \lambda x : x \in S\}$$

a_i = change of origin
λ_i = change of scale
} measuring utility of i = utility scale of i

Take two lotteries

p	$1-p$
10	7

q	$1-q$
11	5.

$p10 + (1-p)7$

$>$

$q \cdot 11 + (1-q)5.$

Replace these numbers 10, 7, 11, 5 by $a_i + \lambda_i 10$, $a_i + \lambda_i 7$, etc.

$\Rightarrow$ LOTTERIES RANKED THE SAME WAY

$$\left.\begin{array}{lccc} \$ & x_1 & \cdots & x_n \\ \text{prob} & p_1 & & p_n \end{array}\right\} \mathcal{L}$$

$$\left.\begin{array}{ccc} y_1 & \cdots & y_k \\ q_1 & & q_k \end{array}\right\} \tilde{\mathcal{L}}$$

$\mathcal{L} \succ \tilde{\mathcal{L}}$ if

$$\underbrace{\sum_{i=1}^{n} p_i x_i}_{\text{Exp } \mathcal{L}} > \underbrace{\sum_{j=1}^{k} q_j y_j}_{\text{Exp } \tilde{\mathcal{L}}}$$

~~does not~~ invariant under change of scale

$$x_i \longrightarrow \tilde{x}_i = -5 + 7x_i$$

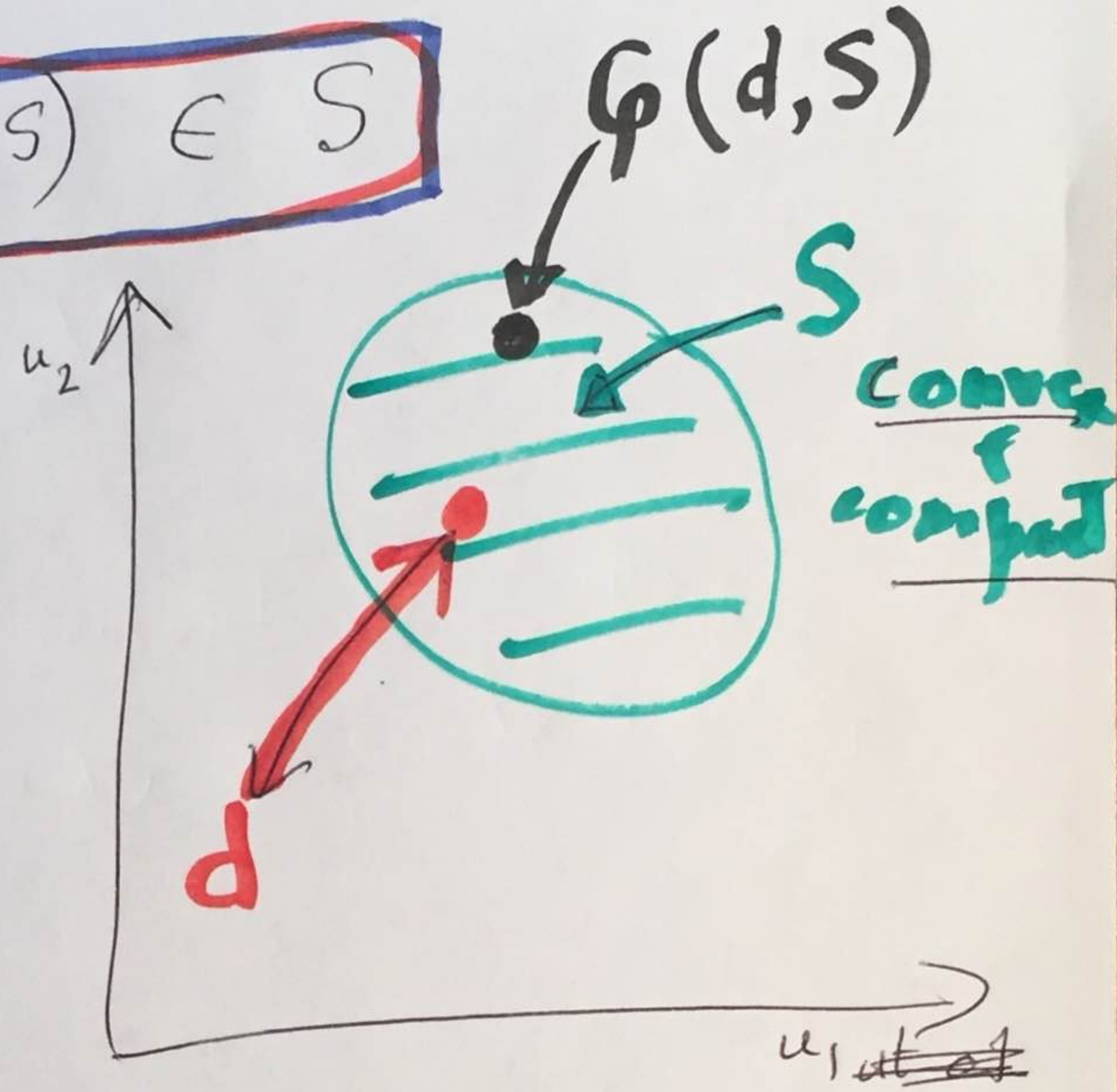
$$y_j \longrightarrow \tilde{y}_j = -5 + 7y_j$$

$$\sum p_i \tilde{x}_i > \sum q_j \tilde{y}_j$$

$$> \Longleftrightarrow >$$

BARGAINING PROBLEM

$$\boxed{\varphi(d,S) \in S}$$



$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Assume $\exists \ x \in S$

such that

$x_1 > d_1$

$x_2 > d_2$

AXIOMS ON φ

$\varphi(d,S)$ $\qquad\qquad$ $\varphi(a+\lambda d,\ a+\lambda S)$

$$(d,S) \sim (a+\lambda d,\ a+\lambda S)$$

SO SAME SOL^{n}.

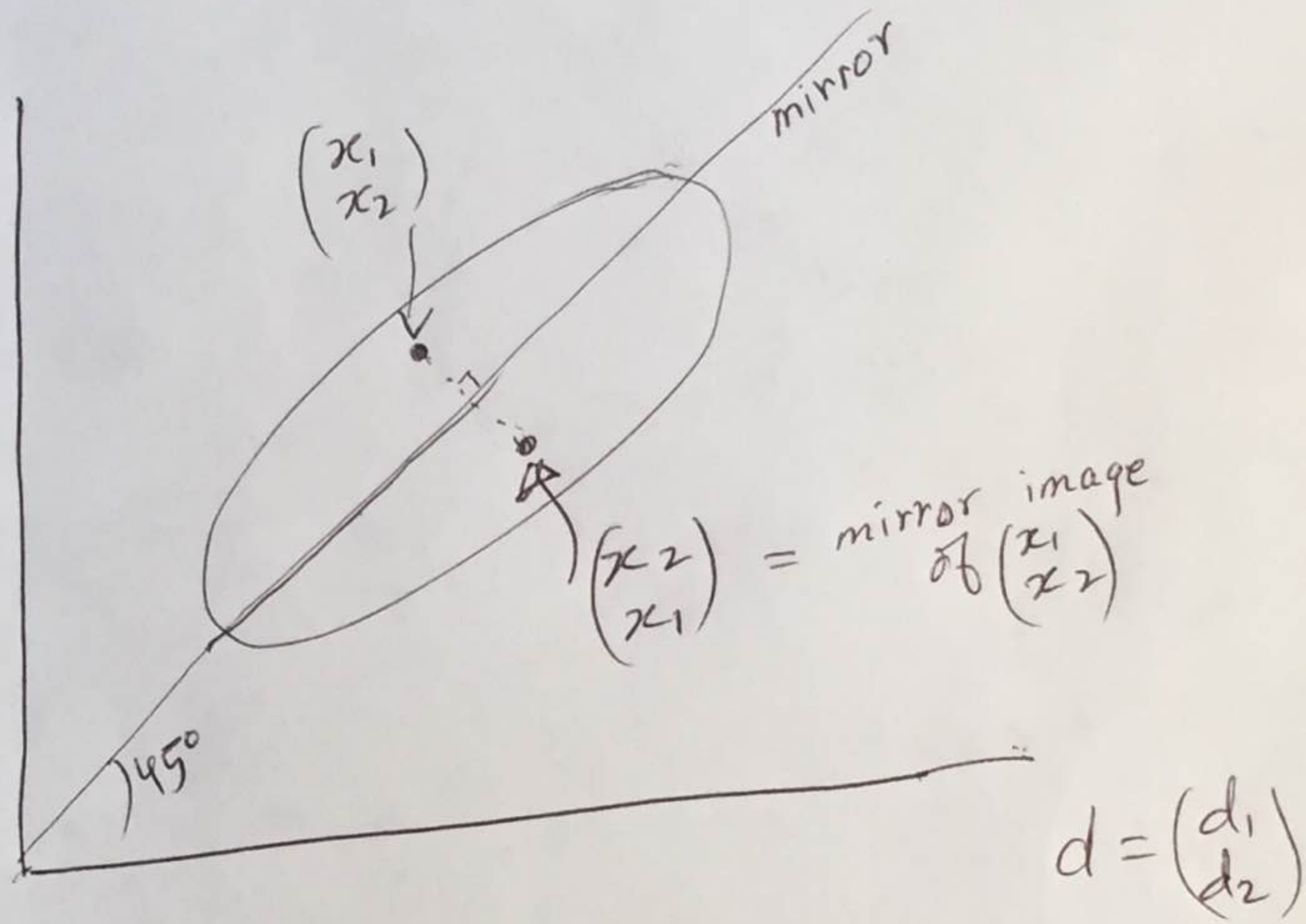
AX I $\boxed{\varphi(a+\lambda d,\ a+\lambda S) = a + \lambda\varphi(d,S)}$

SCALE INVARIANCE

(

<u>DEF^n</u> S is **symmetric** if

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in S \implies \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} \in S$$



AX II

$$\boxed{\text{If } d_1 = d_2 \text{ and } S \text{ is symmetric then } \varphi_1(d,S) = \varphi_2(d,S)}$$

SYMMETRY

PARETO - OPTIMALITY

AX III

$$\boxed{\begin{gathered}\nexists \quad x = \binom{x_1}{x_2} \in S \quad \text{such that} \\ x_1 > \varphi_1(d,S) \\ x_2 > \varphi_2(d,S\end{gathered}}$$

PARETO-OPTIMALITY

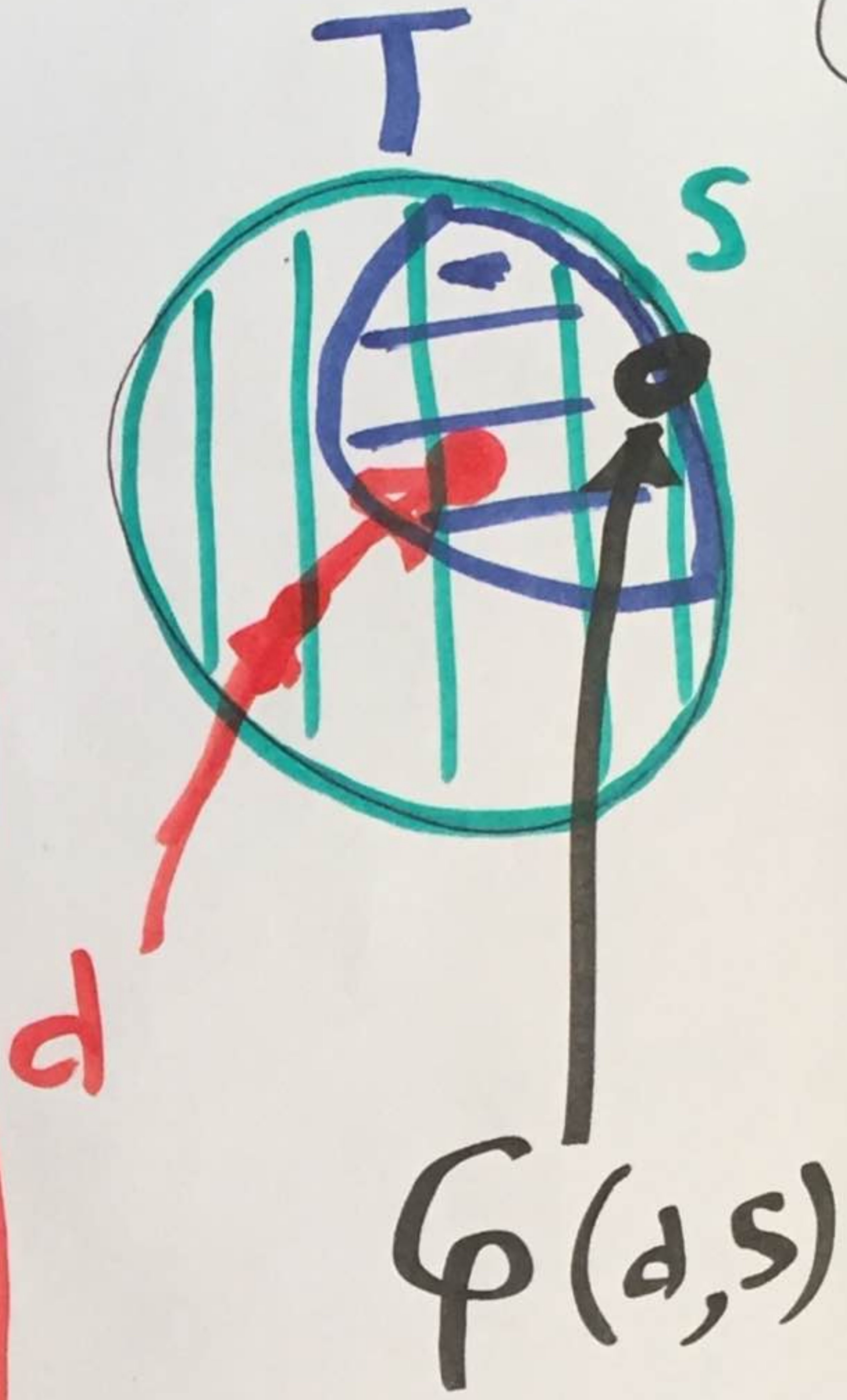
$T \subset S$,

AND $\varphi(d,S) \in T$

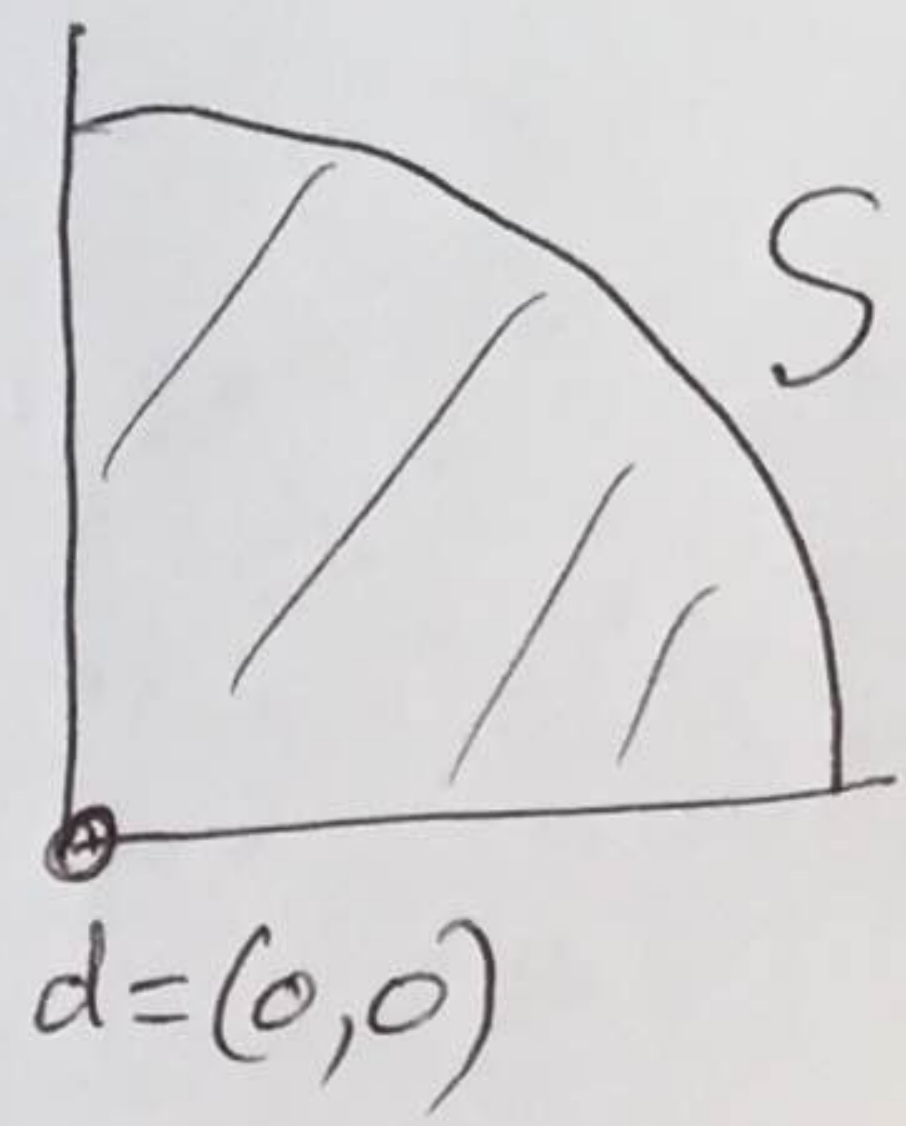
$$\Downarrow$$

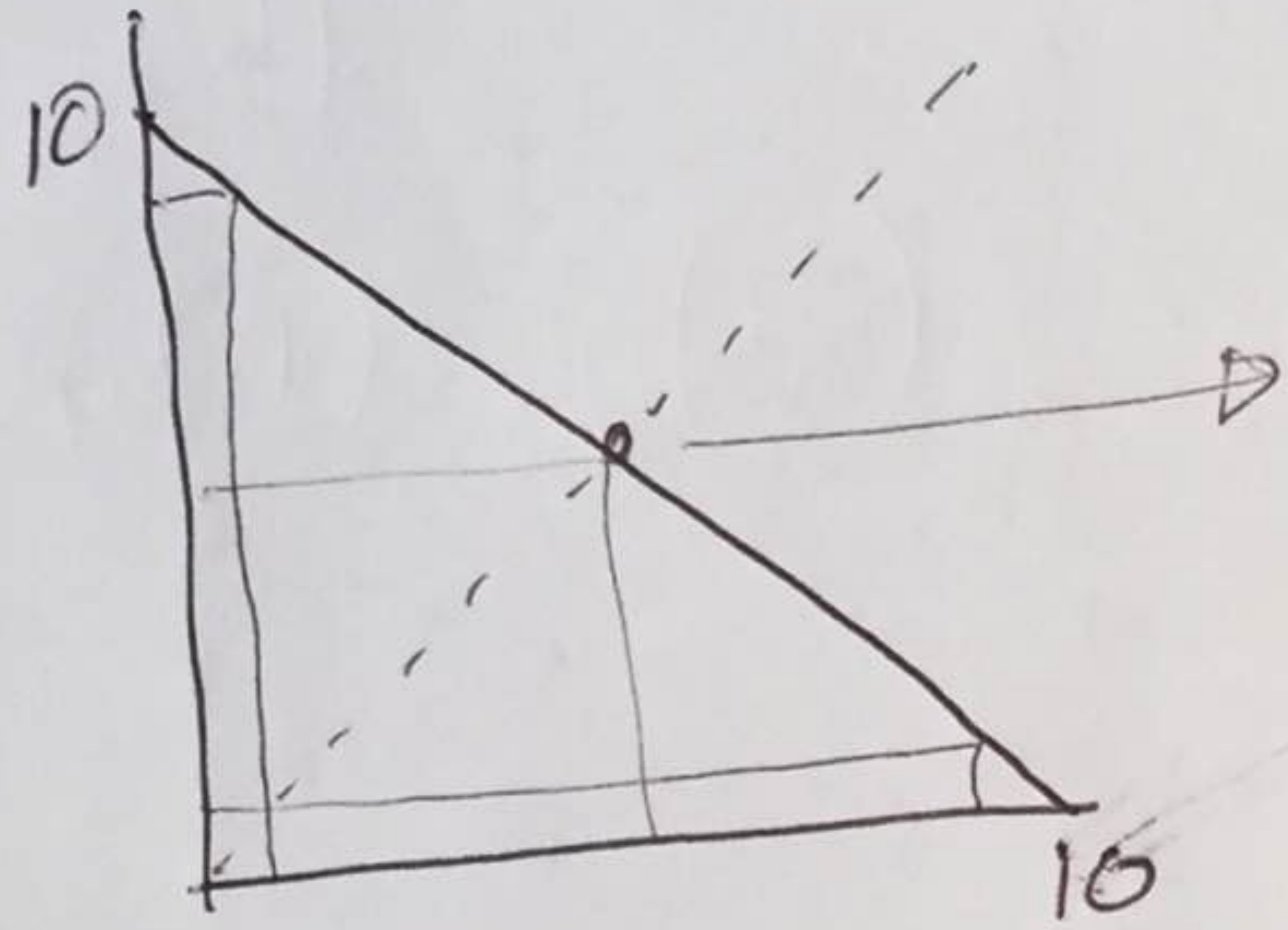
$$\varphi(d,T) = \varphi(d,S)$$

11A



Proof Restrict to



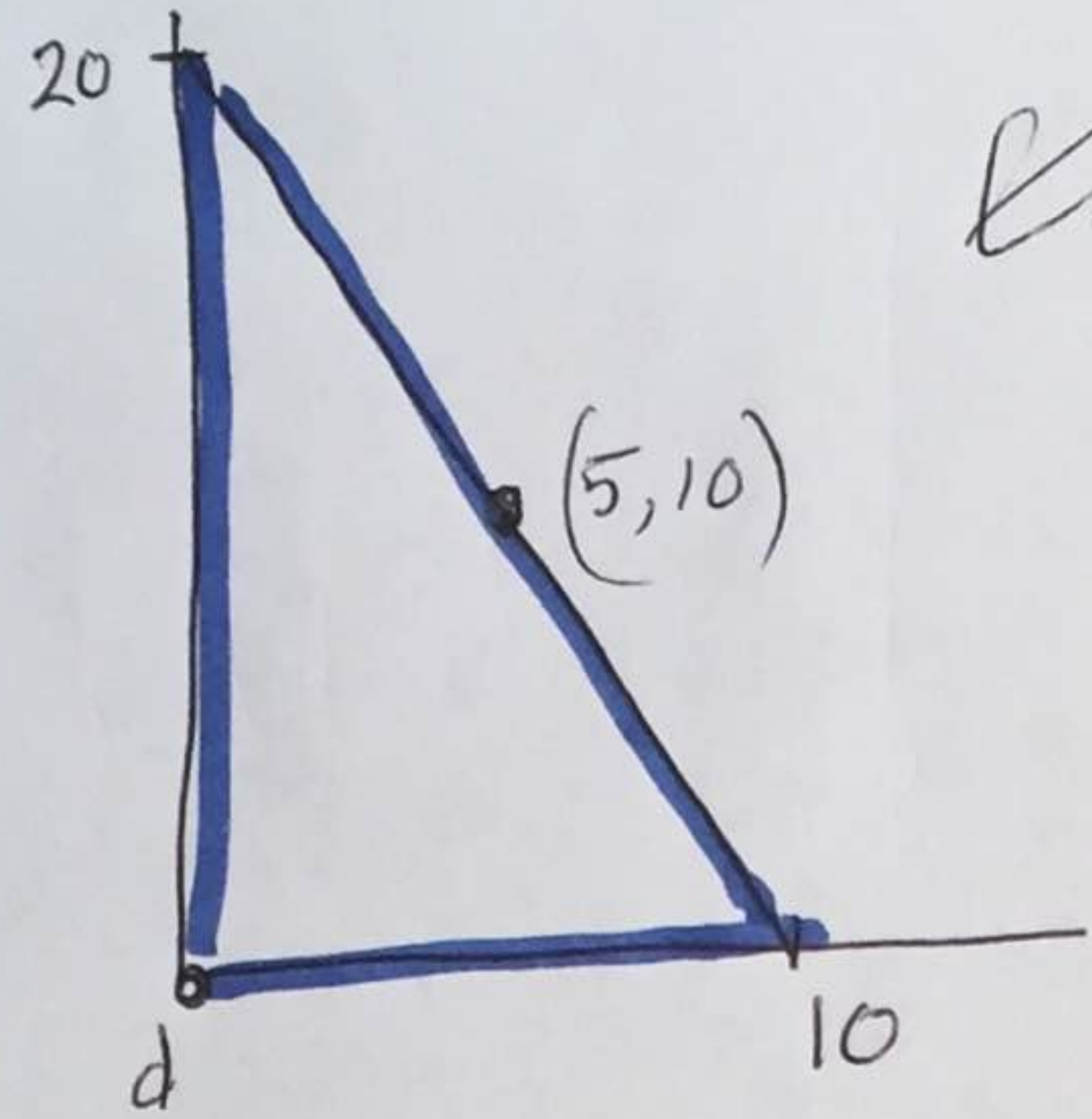


maximizes
$x_1 x_2$
for $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \triangle$

$\max \; x(10-x)$

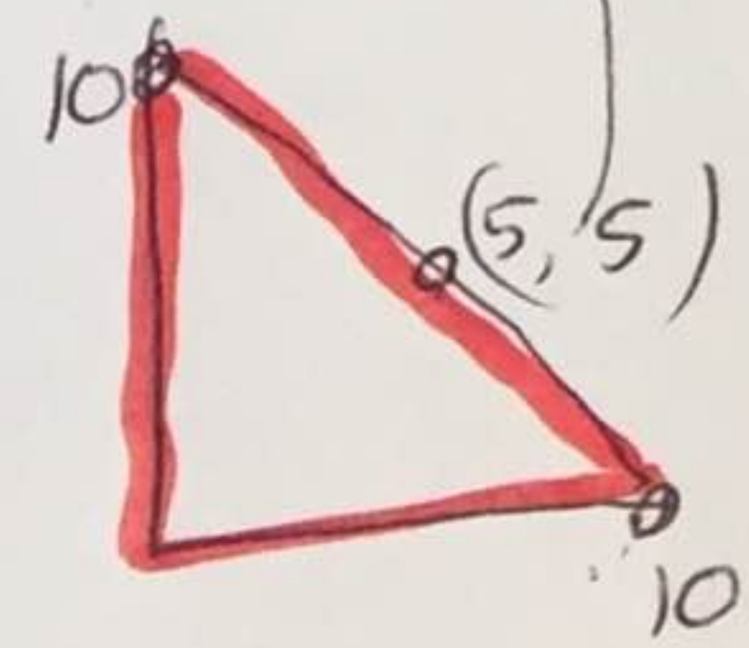
$\max\limits_{0 \le x \le 1} \; x(10-x)$

So $\frac{d}{dx}\left[x(10-x)\right]$
$= (10-x) + x \cdot -1 = 0$
$\Rightarrow x = 5.$



Scale of u_2 has changed

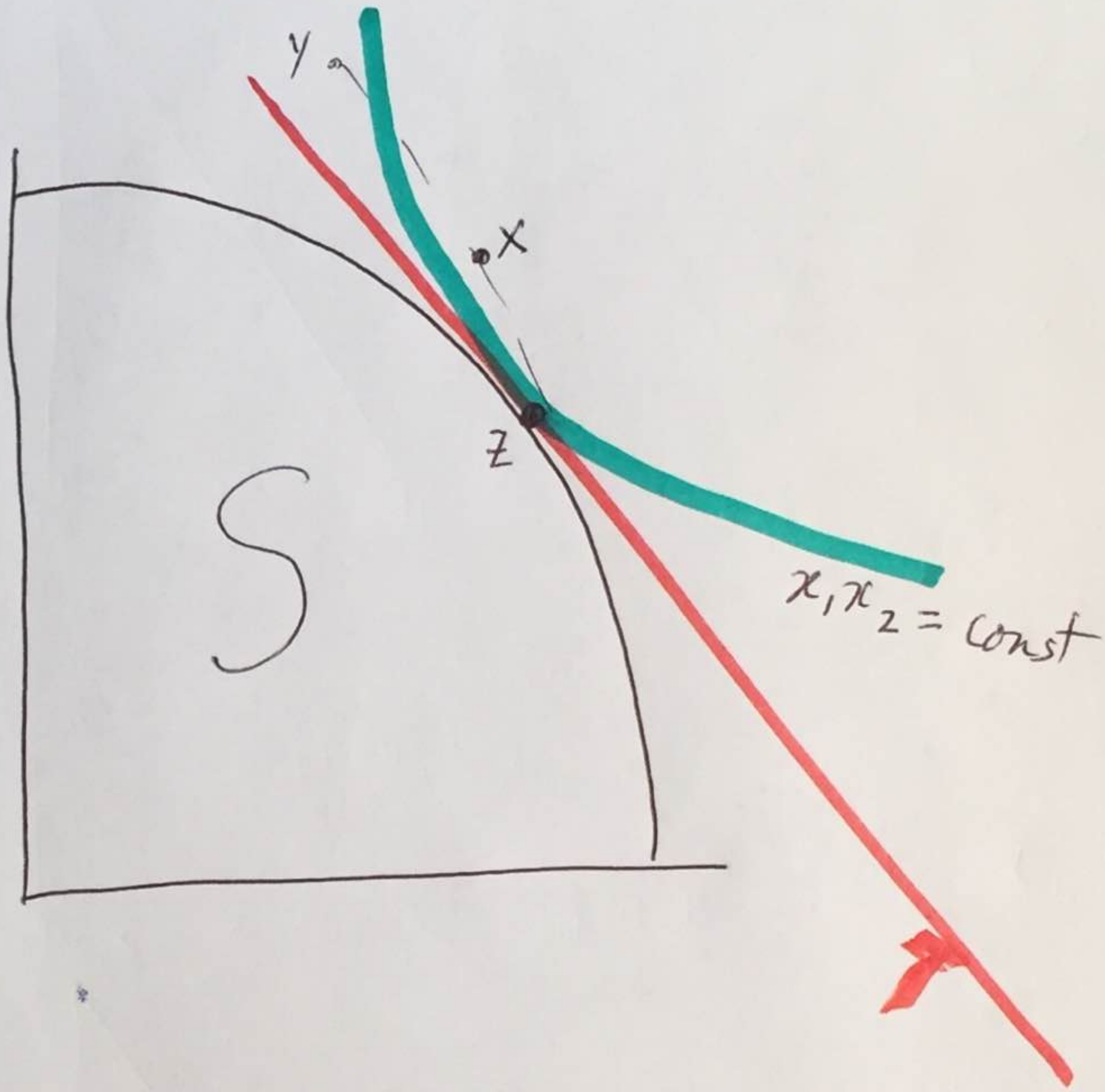
$\lambda = 2$

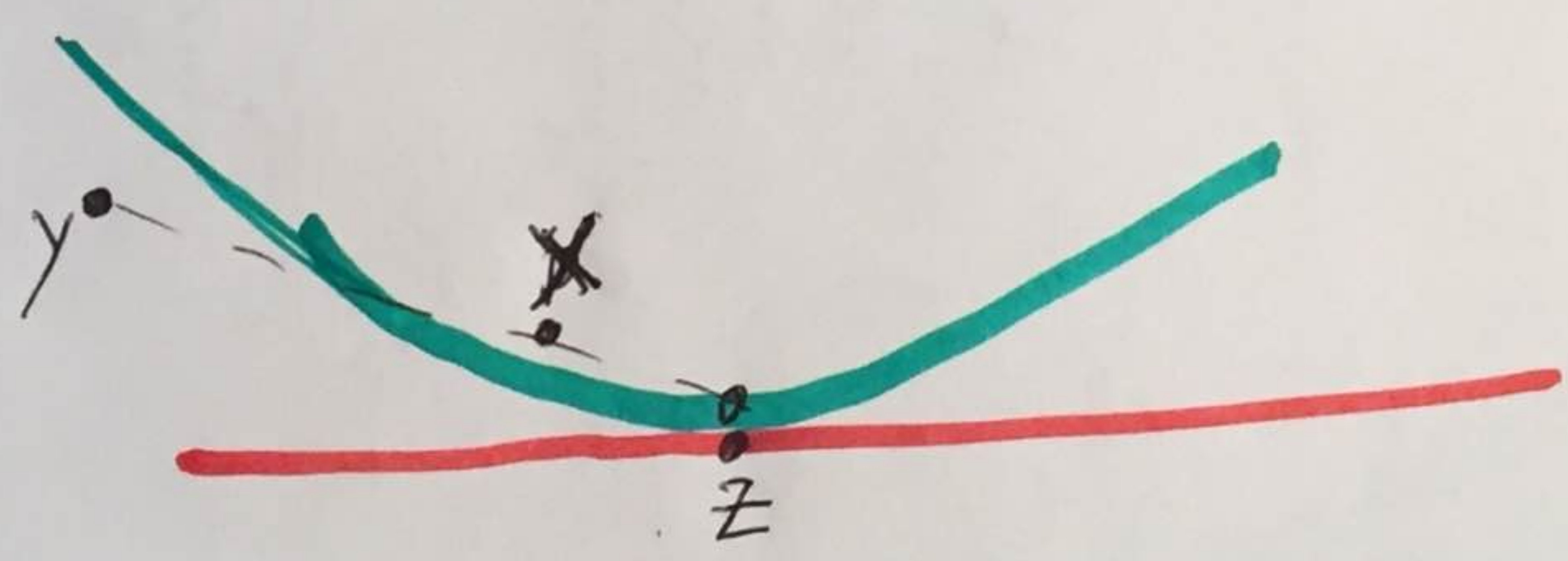


Again $x_1 x_2$ is maximited at $(5, 10)$

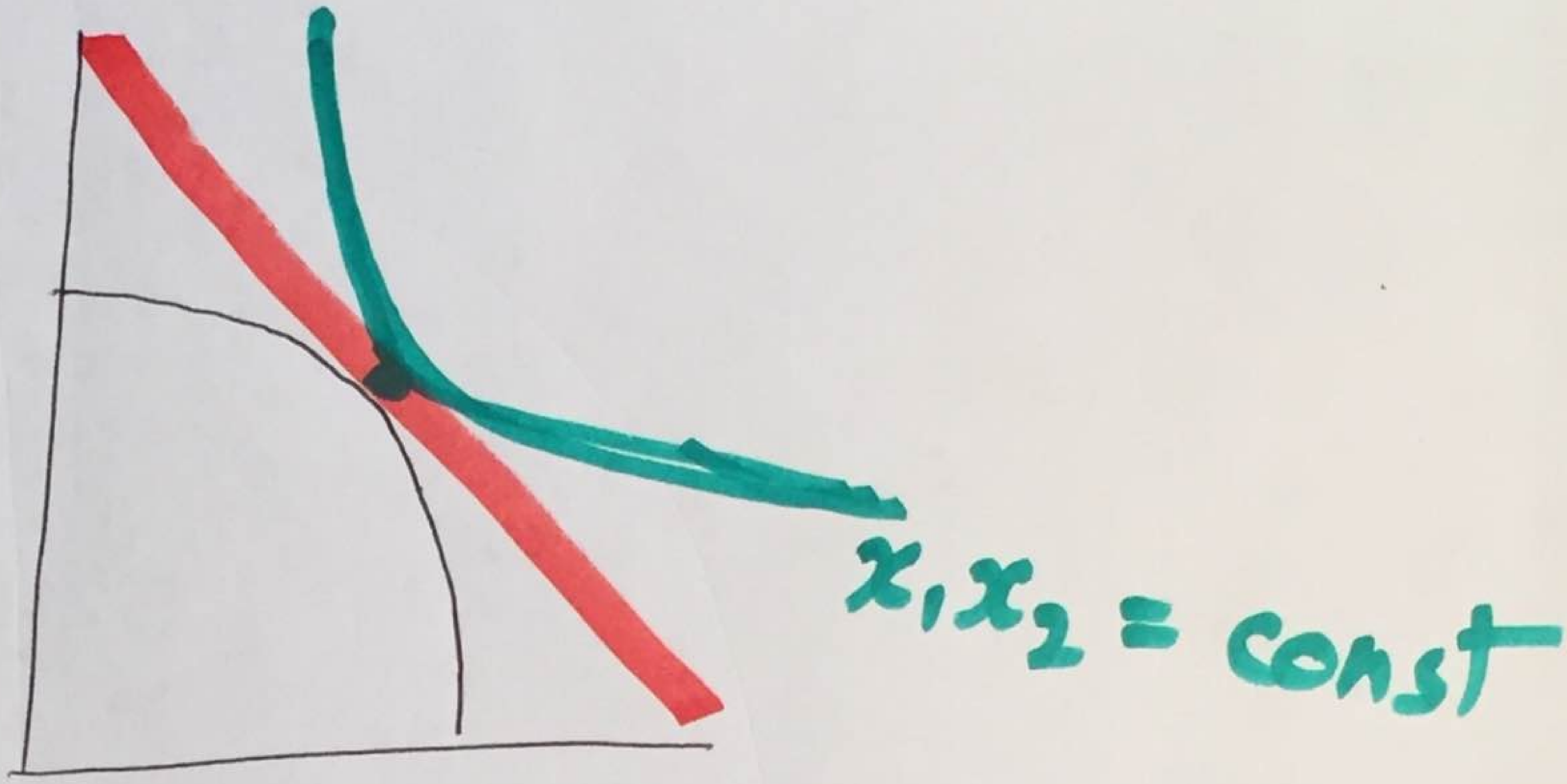
$\text{Max}_{x \in \Delta}\ x_1 x_2$ SAME AS $\text{Max}_{x \in \Delta}\ x_1 2x_2$

$= 2\,\text{Max}\ x_1 x_2$


y
x
z
S
$x_1 x_2 = \text{const}$


y
x
z

So must have



$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$ A

More generally,

if $\quad \max x_1 x_2 \quad$ is achieved at $x^* = \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}$

(1) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in S$

THEN

$\quad \max y_1 y_2 \quad$ is achieved at $y^* = \lambda x^* = \begin{pmatrix} \lambda_1 x_1^* \\ \lambda_2 x^2_* \end{pmatrix}$

(2) $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \lambda S$

<u>Proof</u>

$$y_1 y_2 = \lambda_1 x_1 \lambda_2 x_2$$
$$= (\lambda_1 \lambda_2)\, x_1 x_2$$

So ~~solve~~ solve (2) by

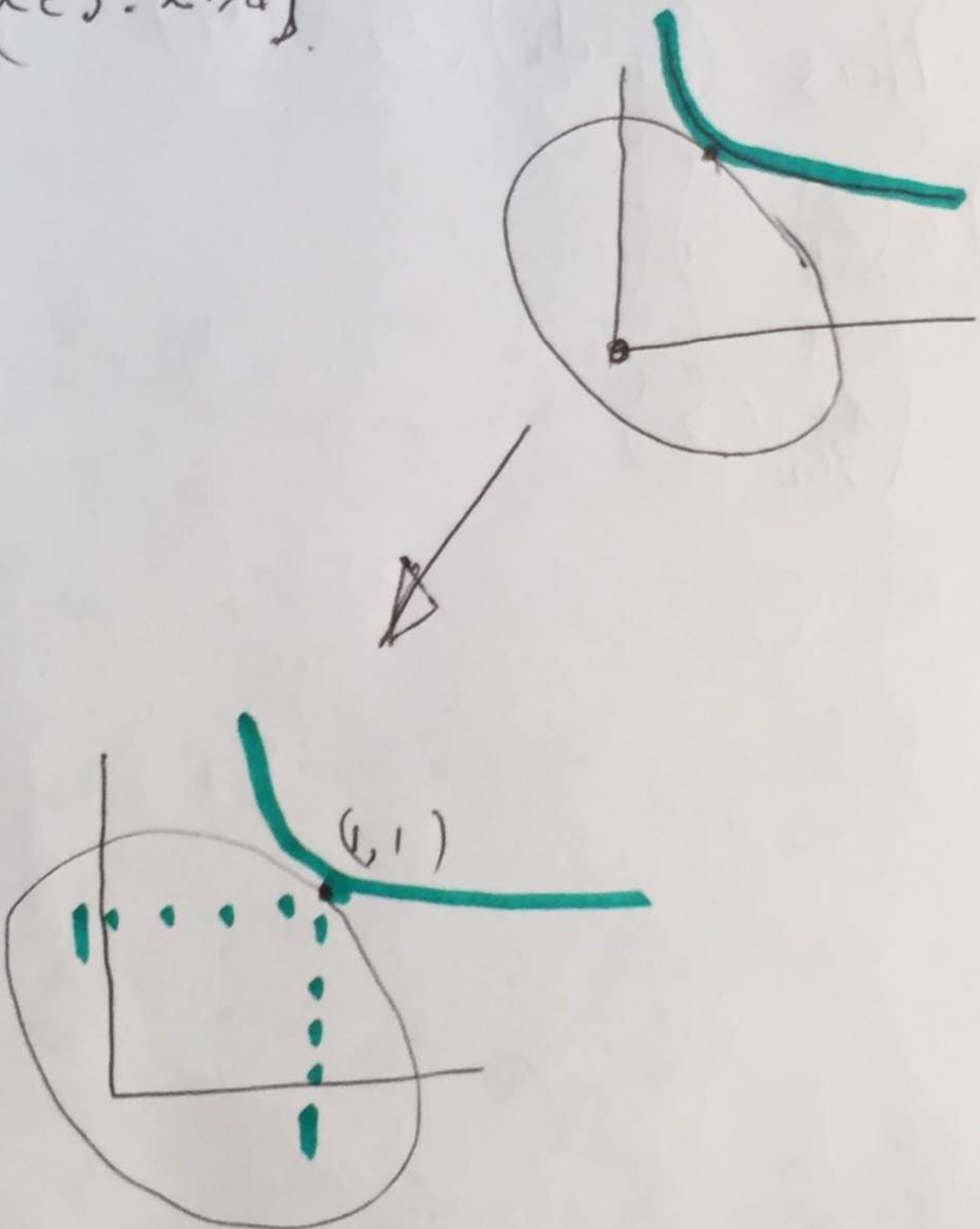
$$\max y_1 y_2$$
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \lambda S$$

$$\sim \quad \max (\lambda_1 \lambda_2)\, x_1 x_2$$
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in S$$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$,

General sol^n is.

$$\underset{\{x \in S : x \geq d\}}{\text{Max}} (x_1 - d_1)(x_2 - d_2)$$



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