Lecture Note 2: Uncertainty

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1 Decision Making under Uncertainty

The analysis of consumer behavior up to this point has assumed lack of *uncertainty* in the problem. The consumers operated their choices with all relevant data (preferences, prices, wealth) exactly known. Needless to say, there are many situations of interest in which there is uncertainty over some of the relevant variables. How should individuals make investment plan when they face uncertainty about their future income? How should firms make production decisions when they face uncertainy of demand? What are the appropriate fiscial policies when governments face uncertainty about future productivity, growth and inflation? In this section, we extend the consumer theory to take into account uncertainty and address the following issues

1. How to model uncertainty?
2. How to measure risk aversion?
3. How to compare risky prospects?
4. Comparative statics of choice under uncertainty
5. limitation of the model

1.1 Modeling uncertainty

In standard consumer theory under certainty, the decision maker is given a preference relation on a set of possible alternatives X. This means that the consumer is able to rank any two alternatives x and y in X. Consider now the following problem.

- Imagine that there are three alternatives, x, y and z, ranked by the consumer $x \succ y \succ z$.

- Now the decision maker is offered a choice between the two following alternatives: having y for sure; a coin will be tossed; if 'head' comes up, the decision maker will obtain x. If 'tail' comes up, the decision maker will obtain z.

- Which of the two should the decision maker choose? Simply knowing that $x \succ y \succ z$ is not sufficient to provide an answer. Factors like the intensity of the preferences or the risk aversion of the decision maker will have to play a role. Our task is to develop a framework in which these factors can be made precise.

1.1.1 Lotteries

- Assume that the set of alternatives X is **finite** (an important assumption used in various proofs), and let $X \equiv \{x_1, ..., x_n\}$. The set X could be a set of consumption goods or a set of occupation choices like {doctor, lawyer, enginer, artist...}. More generally, X is a set of prizes or consequences. For example, if we are considering who will win a soccor game between Germany and Italy. The set of consequences is {Germany wins, Italy wins, Tie}.

- A *simple lottery over* X is a vector $L = (p_1, ...p_n)$, with $p_i \geq 0$ for each i and $\sum_{i=1}^{n} p_i = 1$. Therefore, L is a probability distribution over the elements of X, and p_i is the probability that the lottery assigns to alternative x_i. **Example** of soccor game, the lottery $\{2/5, 2/5, 1/5\}$ means Germany and Italy will win with the same probability $2/5$ and the two teams will tie with probability $1/5$. The case that Germany will win for sure can be represented by a lottery $\{1, 0, 0\}$. Similarly, $\{0, 1, 0\}$ means Italy will win for sure.

- We denote the set of lotteries over X as

$$P(X) \equiv \left\{ (p_1, ...p_n) \middle| p_i \geq 0 \text{ and} \sum_{i=1}^{n} p_i = 1 \right\}.$$

Insert Figure 6.B.1

- Imagine now that we offer to the decision maker *lotteries over lotteries.* We might offer a lottery giving a certain simple lottery $L^a = (p_1^a, p_2^a, \dots, p_n^a)$ with probability α, and another lottery $L^b = \left(p_1^b, p_2^b, \dots, p_n^b\right)$ with probability $1-\alpha$. For example, imagine that you are playing a slot machine with two arms. Each arm is a lottery that determins a probability distribution of your prizes. Before you decide which arm to pull, you first toss a coin. If the coin turns out to be head, you play the left arm. If it is tail, you play the right arm.

- This *lottery over lotteries* will be called a *compound lottery.* How do we deal with such an alternative? We will **assume** that **the decision maker only cares about the final distribution of the outcomes**. The compound lottery has a corresponding simple lottery $\alpha L^a + (1-\alpha) L^b$.

Insert Figure MWG: 6.B.3

- Recall that L^a and L^b are two vectors. Take the slot machine example, suppose the left arm will yield \$10 with probability 0.1 while the right arm will yeild \$10 with probability 0.2.Given a fair coin, the compound lottery will yeild \$10 with probability $0.5 \times 0.1 + 0.5 \times 0.2 = 0.15$. We can actually compound any finite number of simple lotteries. Let $\left(L^{(1)}, \dots L^{(k)}\right)$ be k lotteries, and $(\alpha_1, \dots \alpha_k)$ be k positive numbers summing to 1. Then we can build the lottery $\sum_{j=1}^k \alpha_j L^{(j)}$. In this lottery, the probability of outcome x_i is given by $\sum_{j=1}^k \alpha_j p_i^{(j)}$.

- Mathematically, this means the set $P(X)$ is convex. To see this, let $\widetilde{L} = \alpha L^a + (1-\alpha) L^b$ and $\widetilde{p}_i = \alpha p_i^a + (1-\alpha) p_i^b \geq 0$, $\forall i$. In addition,

$$\begin{aligned}
\sum_{i=1}^{n} \widetilde{p}_i &= \sum_{i=1}^{n} (\alpha p_i^a + (1-\alpha) p_i^b) \\
&= \alpha \sum_{i=1}^{n} p_i^a + (1-\alpha) \sum_{i=1}^{n} p_i^b \\
&= 1.
\end{aligned}$$

  So, $\widetilde{L} \in P(X)$.

1.1.2 Preference Axioms

- When we study consumer theory under certainty, preference $\succsim$ is defined on the set of alternative X.

- When individuals make decisions under uncertainty, we assume that the decision maker is endowed with a preference relation *over the set of all possible lotteries*. In other words, $\succsim$ is now defined on the set $P(X)$.

- We will maintain the rational assumption that the preference relation $\succsim$ is complete and transitive. **Note** that when $\succsim$ is defined over $P(X)$ the rationality assumption is much stronger than when $\succsim$ is defined over X. For example, consumers often have trouble ranking different investment portofolios, especially when the portofolios become more complex.

- We will add two other axioms: *continuity* and *independence*. The former assumption ensures the existence of a utility function representing $\succsim$ and the latter imposes some structure on $U(.)$

Our first assumption is relatively technical, and it is the counterpart of the similar assumption we have seen for the certainty case.

Continuity Consider 3 lotteries p, q, $r \in P(x)$ with $p \succeq q \succeq r$. There exists some $\alpha \in [0,1]$ such that

$$\alpha p + (1-\alpha) r \sim q.$$

- An implication of the continuity axiom is that if p is preferred to q, then a lottery "close" to p is still preferred to q.
- There are situations where continuity might be doubtful. Example: Suppose you the the following three gambles

$$p: \text{ you get \$10 for sure}$$

$$q: \text{ nothing for sure}$$

$$r: \text{ car crash and killed for sure}$$

Naturally, your preference is

$$p \succ q \succ r$$

.Continuity implies that there is some $\alpha \in (0,1)$ such that

$$\alpha p + (1-\alpha) r \sim q$$

you would be indifferent between getting nothing and getting \$10 with probability α and getting killed with probability $1-\alpha$.

- Given that preferences are complete, transitive and continuous, they can be represented by a utility function $U(L)$ which is continuous in L and represents preferences over lotteries; this result closely parallels the one we had for the certainty case.
- Our next axiom, which is more controversial, will allow us to say a great deal about the structure of U. This axiom is the key to expected utility theory and it has *no counterpart* in the certainty case.

Independence The preference relation $\succeq$ on the space of simple lotteries $P(X)$ satisfies the *independent axiom* if for **all simple** lotteries p, q and r and $\alpha \in (0,1)$ we have

$$p \succeq q$$

if and only if

$$\alpha p + (1 - \alpha) r \succeq \alpha q + (1 - \alpha) r.$$

The assumption says the following. Take two lotteries, p and q, with p preferred to q. Now assume that you have to choose between two compound lotteries. The first gives p with probability α and another lottery r with probability $(1 - \alpha)$. The second gives q with probability α and another lottery r with probability $(1 - \alpha)$. The assumption says that if you prefer p to q, then you must also prefer $\alpha p + (1 - \alpha) r$ to $\alpha q + (1 - \alpha) r$.

- The name 'independence' comes from the fact that your preference of p over q is independent of any mix that you can make with another lottery.

- While the assumption may look uncontroversial, there are well known examples in which reasonable decision makers exhibit preferences which do not satisfy independence. **Note that this axiom has no counterpart in the standard consumer theory**. To see this, consider three bundles

$$X_1 = \{0 \text{ cups of coffee,0 coffee cakes,2 cups of tea}\}$$

$$X_2 = \{2 \text{ cups of coffee,0 coffee cakes,0 cups of tea}\}$$

$$X' = \{0 \text{ cups of coffees,2 coffee cakes,0 cups of tea}\}$$

Let

$$\begin{aligned} X_3 &= 1/2X_1 + 1/2X' \\ &= \{0 \text{ cups of coffee,1coffee cake,1cup of tea}\} \end{aligned}$$

$$\begin{aligned} X_4 &= 1/2X_2 + 1/2X' \\ &= \{1 \text{ cup of coffee,1 coffee cake,0 cups of tea}\} \end{aligned}$$

Suppose $X_1 \succeq X_2$. This does not imply $X_3 \succeq X_4$.

1.1.3 Expected Utility

We now introduce the idea of an expected utility function.

Definition A utility function $U : P(X) \rightarrow R$ has an expected utility form (or is a von Neumann-Morgenstern utility function) if there are numbers $(u_1, ..., u_n)$ for each of the N outcomes $(x_1, ..., x_n)$ such that for **every** $L \in P(X), U(L) = \sum_{i=1}^{n} p_i u_i$.

In other words, U has an expected utility form if it is linear in probabilities.

Proposition A utility function $U : P(X) \rightarrow R$ has an expected utility form **if and only if** it satisfies the property that

$$U(\sum_{k=1}^{K} \alpha_k L^k) = \sum_{k=1}^{K} \alpha_k U(L^k)$$

for any K lotteries $L^k \in P(X), k = 1, ...K$, and any probability $(\alpha_1, ..., \alpha_K) \geq 0, \sum\limits_{k=1}^{K} \alpha_k = 1$.

Proof : Necessary (only if): Suppose U has the expected utility form.

$$U(L) = \sum_{i=1}^{n} p_i u_i$$

for each simple lottery $L \in P(X)$. The compound lottery $(L^1, ..., L^K; \alpha_1, ..., \alpha_K)$ can be reduced to a simple lottery

$$L^{'} = \sum_{k=1}^{K} \alpha_k L^k = \begin{matrix} \sum\limits_{k=1}^{K} \alpha_k p_1^k \\ \cdot \\ \cdot \\ \sum\limits_{k=1}^{K} \alpha_k p_n^k \end{matrix}$$

, where the probability of consequence x_i is $\sum_{k=1}^{K} \alpha_k p_i^k$. So, we can write

$$\begin{aligned} U(\sum_{k=1}^{K} \alpha_k L^k) &= U(L^{'}) \\ &= \sum_{i=1}^{n} (\sum_{k=1}^{K} \alpha_k p_i^k) u_i \\ &= \sum_{k=1}^{K} \alpha_k \sum_{i=1}^{n} p_i^k u_i \\ &= \sum_{k=1}^{K} \alpha_k U(p^k). \end{aligned}$$

Sufficient (If): simple lottery $L = (p_i, ..., p_n)$ can be written as a compound lotteries ($\underbrace{r^1, r^2, ..., r^n}_{\text{degenerated lottery}}$; $\underbrace{p_1, ..., p_n}_{\text{weights on each dengerated lottery}}$), where

$$r^i = \begin{array}{ll} 0 & \\ . & \\ 1 & \text{ith} \\ 0 & \end{array}$$

is a degenerated lottery in which prize x_i occurs with probability one. We can write

$$\begin{aligned} U(L) &= U(\sum_{k=1}^{n} p_k r^k) \\ &= \sum_{k=1}^{n} p_k U(r^k) \\ &= \sum_{k=1}^{n} p_k u_k. \end{aligned}$$

Hence, $U(.)$ has the expected utility form. Q.E.D.

Proposition 1 *Suppose that* $U : P(X) \to R$ *is an expected utility representation of the preference relation* $\succeq$ *on* $P(X)$*. Then* $V : P(X) \to R$ *is an* **expected** *utility representation of* $\succeq$ **if and only if** *there are scalars* γ *and* $\beta > 0$ *such that* $V(L) = \gamma + \beta U(L)$ *for all* $L \in P(X)$*. (preference is preserved up to increasing linear transformation)*

Proof: It's presumed that U is a v.N-M expected function throughout.

(1) Sufficiency (if): Suppose $\widetilde{U}(L) = \beta U(L) + \gamma$. Show that $\widetilde{U}(\sum_{k=1}^{K} \alpha_k L^k) = \sum_{k=1}^{K} \alpha_k \widetilde{U}(L^k)$.(Linearity)

$$
\begin{aligned}
& \widetilde{U}(\sum_{k=1}^{K} \alpha_k L^k) \\
= \; & \beta U(\sum_{k=1}^{K} \alpha_k L^k) + \gamma \\
= \; & \beta \sum_{k=1}^{K} \alpha_k U(L^k) + \gamma \text{ ($\because U$ is expected utility)} \\
= \; & \sum_{k=1}^{K} \alpha_k \beta U(L^k) + \sum_{k=1}^{K} \alpha_k \gamma \text{ (} \because \sum_{k=1}^{K} \alpha_k = 1) \\
= \; & \sum_{k=1}^{K} \alpha_k [\beta U(L^k) + \gamma] \\
= \; & \sum_{k=1}^{K} \alpha_k \widetilde{U}(L^k) \text{ (} \because \widetilde{U}(L) = \beta U(L) + \gamma)
\end{aligned}
$$

Easy to see that $\widetilde{U}(L)$ and $U(L)$ represent the same preference.

(2) Necessity (only if): Suppose $\widetilde{U}$ is v.N-M for $\succsim$. Show $\exists \beta > 0$, γ s.t. $\widetilde{U}(L) = \beta U(L) + \gamma$ for all $L \in P(X)$.

Let $\overline{L}$ (best lottery) and $\underline{L}$ (worst lottery) be lotteries such that $\overline{L} \succsim L \succsim \underline{L}$ for all $L \in P(X)$.

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First, establish the existence of $\overline{L}$ & $\underline{L}$.

Consider lotteries $\widetilde{L}_n = e_n = (0, ..., 0, 1, 0, ..., 0)$ (1 in the n^{th} position) which is essentially a deterministic outcome C_n.

$\exists \widetilde{L}_n$ such that $\widetilde{L}_n \succsim \widetilde{L}_m$, $\forall m = 1, ..., N$.

Let L be $(p_1, ..., p_N)$

$$\begin{aligned}
\widetilde{L}_n &\sim p_1\widetilde{L}_n + p_2\widetilde{L}_n + ... + p_N\widetilde{L}_n \\
&\succsim p_1\widetilde{L}_1 + p_2\widetilde{L}_n + ... + p_N\widetilde{L}_n \\
&\quad \vdots \\
&\succsim p_1\widetilde{L}_1 + p_2\widetilde{L}_2 + ... + p_N\widetilde{L}_N \sim L \text{ (repeated application of the independence axiom)}
\end{aligned}$$

Existence of $\underline{L}$ is similarly established.

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Uninteresting case: $\underline{L} \sim \overline{L}$. In this case, $U(L) = c$ and $\widetilde{U}(L) = \widetilde{c}$, where c and $\tilde{c}$ are constants, and a continuum of β, γ exsit.

So, we focus on the case of $\overline{L} \succ \underline{L}$.

- Since $\overline{L} \succ \underline{L}$, we have $U(L) \in [U(\underline{L}), U(\overline{L})]$, $\forall L$. Consider any $L \in P(X)$ and define $\lambda_L \in [0, 1]$ by

$$U(L) = \lambda_L U(\overline{L}) + (1 - \lambda_L)U(\underline{L}).$$

  This can be rewritten as

$$\lambda_L = \frac{U(L) - U(\underline{L})}{U(\overline{L}) - U(\underline{L})}$$

  .

- Since $U(L)$ have the expected form, we have

$$\begin{aligned}
U(L) &= \lambda_L U(\overline{L}) + (1 - \lambda_L)U(\underline{L}) \\
&= U(\lambda_L \overline{L} + (1 - \lambda_L)\underline{L}),
\end{aligned}$$

  which implies

$$L \sim \lambda_L \overline{L} + (1 - \lambda_L)\underline{L}.$$

- Because $\widetilde{U}$ represents the same preference as U. It should also follow that

$$\begin{aligned}
\widetilde{U}(L) &= \widetilde{U}(\lambda_L \overline{L} + (1 - \lambda_L)\underline{L}) \\
&= \lambda_L \widetilde{U}(\overline{L}) + (1 - \lambda_L)\widetilde{U}(\underline{L}) \quad \widetilde{U} \text{ has the expected utility form.}
\end{aligned}$$

Hence,

$$\lambda_L = \frac{\widetilde{U}(L) - \widetilde{U}(\underline{L})}{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})}.$$

- Therefore,

$$\begin{aligned}
\frac{U(L) - U(\underline{L})}{U(\overline{L}) - U(\underline{L})} &= \frac{\widetilde{U}(L) - \widetilde{U}(\underline{L})}{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})} \\
\widetilde{U}(L) &= \left(\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})\right) \frac{U(L) - U(\underline{L})}{U(\overline{L}) - U(\underline{L})} + \widetilde{U}(\underline{L}) \\
&= \left(\frac{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})}{U(\overline{L}) - U(\underline{L})}\right) U(L) + \widetilde{U}(\underline{L}) - \left(\frac{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})}{U(\overline{L}) - U(\underline{L})}\right) U(\underline{L}) \\
&= \beta U(L) + \gamma.
\end{aligned}$$

where $\beta = \frac{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})}{U(\overline{L}) - U(\underline{L})}, \gamma = \widetilde{U}(\underline{L}) - \frac{\widetilde{U}(\overline{L}) - \widetilde{U}(\underline{L})}{U(\overline{L}) - U(\underline{L})} U(\underline{L})$. Q.E.D.

Theorem (von Neumann and Morgenstern, 1947) A complete and transitive preference relation $\succeq$ on $P(X)$ satisfies continuity and independence if and only if it admits a **expected utility representation** $U : P(X) \rightarrow R$.That is, we can assign a number u_n to each outcome $n = 1, ..., N$ in such a manner that for **any** two lotteries p and q, we have

$$p \succeq q$$

if and only if

$$\sum_{i=1}^{N} p_i u_i \geq \sum_{i=1}^{N} q_i u_i$$

Let's first provide some intution for the theorem. We continue with the three dimensional simplex example.

- Expected Utility form implies that indifference curves are straight and paralell.

straight consider $L \sim L'$ and $\alpha \in [0,1]$. Expected utility representation implies

$$\begin{aligned} U(\alpha L + (1-\alpha)L') &= \alpha U(L) + (1-\alpha)U(L') \\ &= u \ (\because U(L) = U(L') = u). \end{aligned}$$

Hence, $L \sim L' \sim \alpha L + (1-\alpha)L'$, $\forall \alpha \in [0,1]$. Figure

Parallel If indifference curves are not parallel, there must exist two indifference curves which cross each other at L^*. Then, we can find a small triangle such that all the lotteries in this triangle yield the same utility. But the utility must increase in one dimension, and hence we can find in this small triangle two lotteries which yield different utilities. Figure

- Independent theorem implies indifferent curves are straight and parallel.

straight Consider $L \sim L'$ and $\alpha \in [0,1]$. Independence implies $L \sim \alpha L + (1-\alpha)L' \sim L'$, $\forall \alpha \in [0,1]$.

parallel Levin's Figure 3

Proof : The proof has several steps. The idea is to first construct a utlity function over $P(X)$ and then show tha the Utilility function has the expected utility form Let $\succeq$ be given and assume it satisifes continuity and independence. Let $\overline{L}$ and $\underline{L}$ be the best and the worst lotteries in $P(X)$. So, $\overline{L} \succeq L \succeq \underline{L}$ for all $L \in P(X)$. (the case of infinite outcome is more invovled and is discussed in Kreps)

Step 1 Because of continuity of $\succeq$, for each L, $\exists \lambda_L$ such that $L \sim \lambda_L\ \overline{L} + (1-\lambda_L)\ \underline{L}$. (we wish to claim λ_L is $U(L)$.)

Step 2 (This step shows λ_L is unique) We show that

$$\overline{L} \succ \beta\overline{L} + (1-\beta)\underline{L} \succ \alpha\overline{L} + (1-\alpha)\underline{L} \succ \underline{L}$$

if and only if $1 > \beta > \alpha > 0$.

If: It is easy to see that the first and the last preference is dervied from independence axiom. We need to show the preference in the middle. Define $\gamma = \frac{\beta-\alpha}{1-\alpha} \in (0,1]$. We can rewrite $\beta\overline{L} + (1-\beta)\underline{L}$ as

$$\gamma\overline{L} + (1-\gamma)[\alpha\overline{L} + (1-\alpha)\underline{L}].$$

Since $\overline{L} \succ \alpha\overline{L} + (1-\alpha)\underline{L}$, by independence

$$\begin{aligned} \gamma\overline{L} + (1-\gamma)[\alpha\overline{L} + (1-\alpha)\underline{L}] &\succ \alpha\overline{L} + (1-\alpha)\underline{L} \Leftrightarrow \\ \beta\overline{L} + (1-\beta)\underline{L} &\succ \alpha\overline{L} + (1-\alpha)\underline{L}. \end{aligned}$$

Define

$$\begin{aligned} L_\beta &= \beta\overline{L} + (1-\beta)\underline{L} \\ L_\alpha &= \alpha\overline{L} + (1-\alpha)\underline{L}. \end{aligned}$$

This step shows that $L_\beta \succ L_\alpha$ if and only if $\beta > \alpha$. This shows that λ_L is unique. To see this, suppose

$$L \sim \lambda_1\overline{L} + (1-\lambda_1)\underline{L} \sim \lambda_2\overline{L} + (1-\lambda_2)\underline{L}$$

and $\lambda_1 > \lambda_2$ (WLOG). Then, step 2 implies that

$$\lambda_1\overline{L} + (1-\lambda_1)\underline{L} \succ \lambda_2\overline{L} + (1-\lambda_2)\underline{L}.$$

A contradiction.

Only if: this part can be easily shown by contradiction and is ignored.

Let

$$U(L) = \lambda_L.$$

Step 3 (This step shows λ_L represents $\succeq$).

$$\begin{aligned} L_1 &\succeq L_2 \Leftrightarrow \\ \lambda_1\overline{L} + (1-\lambda_1)\underline{L} &\succeq \lambda_2\overline{L} + (1-\lambda_2)\underline{L} \Leftrightarrow \\ \lambda_1 &\geq \lambda_2 \text{ (by step 2)} \end{aligned}$$

Step 4 (This step shows U has the expected form) Consider any L_1, $L_2 \in P(X)$, $\alpha \in [0,1]$..

$$\begin{aligned}
L_1 &\sim \lambda_1 \overline{L} + (1-\lambda_1)\underline{L} \Leftrightarrow U(L_1) = \lambda_1 && (1)\\
L_2 &\sim \lambda_2 \overline{L} + (1-\lambda_2)\underline{L} \Leftrightarrow U(L_2) = \lambda_2 && (2)\\
\alpha L_1 + (1-\alpha)L_2 &\sim \lambda_a \overline{L} + (1-\lambda_a)\underline{L} \Leftrightarrow U(\alpha L_1 + (1-\alpha)L_2) = \lambda_a && (3)
\end{aligned}$$

We want to show

$$U(\alpha L_1 + (1-\alpha)L_2) = \alpha U(L_1) + (1-\alpha)U(L_2)$$

By independence

$$\begin{aligned}
\alpha L_1 + (1-\alpha)L_2 &\sim \alpha[\lambda_1 \overline{L} + (1-\lambda_1)\underline{L}] + (1-\alpha)L_2 \text{ (apply independent axiom to 1)}\\
&\sim \alpha[\lambda_1 \overline{L} + (1-\lambda_1)\underline{L}] + (1-\alpha)[\lambda_2 \overline{L} + (1-\lambda_2)\underline{L}]\\
&\quad \text{(apply independent axiom to 2)}\\
&= [\alpha\lambda_1 + (1-\alpha)\lambda_2]\overline{L} + [1 - \alpha\lambda_1 - (1-\alpha)\lambda_2]\underline{L}
\end{aligned}$$

Then by the definition of U, we have

$$\begin{aligned}
U(\alpha L_1 + (1-\alpha)L_2) &= \alpha\lambda_1 + (1-\alpha)\lambda_2\\
&= \alpha U(L_1) + (1-\alpha)U(L_2)
\end{aligned}$$

Q.E.D.

- U defined over the lottery space $P(X)$ is called the von-Neumann-Morgenstern utility function. The utility function u defined over the outcome space X is called the Bernoulli function.

- The theorem says that when preferences $\succsim$ satisfy both continuity and independence then it is possible to find a utility function $U : P(X) \to R$ representing $\succsim$ which is **linear** in q, that is, $U(q) = \sum_i u_i q_i$. Here u_i is the coefficient of q_i, and it can be interpreted as the utility that the decision maker assigns to outcome x_i.

- We can restate the theorem as the following: When preferences over lotteries satisfy continuity and independence then it is possible to find a utility function $u : X \to R$ (that is, the utility function u is defined over X, the set of 'pure' alternatives) such that the utility of a lottery q can be computed simply as $U(q) = \sum_{i=1}^{n} q_i u(x_i)$, and a lottery q is better than a lottery r if and only if $\sum_{i=1}^{n} q_i u(x_i) \geq \sum_{i=1}^{n} r_i u(x_i)$.

- If a decision maker's preferences can be represented by an expected utility function, all we need to know to pin down her preferences over uncertain outcomes are her payoffs from the certain outcomes x_1, ..., x_n.

- In the standard consumer theory without uncertainty, $\succeq$ is preserved when we have a monotone transformation of the utility function. When we have uncertainty, The function $U(p)$ is unique up to positive affine transformations. (notice that now general monotone transformations *of the Bernoulli function* u are not allowed.)

From now on, we will focus on preferences which can be represented in expected utility form.

1.2 Risk Aversion

Up to now we have taken X to be a generic choice set. We now specialize the analysis, and assume that X is an interval of real numbers, interpreted as quantity of money received by the decision maker.

- $X = [0, +\infty)$

- A monetery lottery is described by a cumulative distribution function $F : R \to [0, 1]$. Let $f(x)$ be the density function.

- A compound lottery $(q^1, ..., q^K; \alpha_1, ..., \alpha_K)$ can be reduced to a simple lottery with distribution funciton $F(x) = \sum_{i=1}^{k} \alpha_i F_i(x)$, where $F_i(.)$ is the distribution of the payoff under lottery q^i.

- The lottery space $P(X)$ is the set of all distribution functions over interval $[0, +\infty)$.

The expected utility theorem allows us to provide a neat characterization of risk aversion. Let us start with some intuition. Suppose that the decision maker is offered a choice between two lotteries:

- p : \$50 with probability 1.
- q : \$100 with probability $\frac{1}{2}$, \$0 with probability $\frac{1}{2}$.

Notice that the two lotteries have the same expected value, since $\frac{1}{2}100+\frac{1}{2}0 = 50$. This does not mean that the two lotteries generate the same expected utility. If u is the Bernoulli utility function, then the first lottery gives an expected utility of $U(p) = u(50)$, while the second gives an expected utility of $U(q) = \frac{1}{2}u(0)+\frac{1}{2}u(100)$. When is a decision maker risk averse? An intuitive answer is 'when a sure outcome is preferred to an uncertain outcome'. The two lotteries give the same expected value, but the second is clearly riskier than the first. Thus, in this case we expect that the risk-averse decision maker will choose the first lottery, that is $\frac{1}{2}u(0) + \frac{1}{2}u(100) < u(50)$.

Example. Suppose the **Bernoully** utility function is $u(x) = \sqrt{x}$. In this case, the utility of having \$50 for sure is $u(50) = \sqrt{50} = 7.0711$. The expected utility of the lottery giving \$0 or \$100, each with probability $\frac{1}{2}$, is $\frac{1}{2}\sqrt{0}+\frac{1}{2}\sqrt{100} = 5$. Therefore, in this case the decision maker prefers the sure outcome.

The Example shows that when the **Bernoully** utility function is concave, the decision maker is risk averse. We can now generalize this intuition.

Definition A decision maker is a risk averter if for any lottery $F(.)$, we have

$$U(F) = \int u(x)dF(x) \leq u\left(\int xdF(x)\right). \tag{4}$$

That is, the degenerated lottery that yields the amount $\int xdF(x)$ with certainty is at least as good as the lottery $F(.)$ itself. The decision maker is strictly risk averse if indifference holds only when the two lotteries are the same [i.e., when $F(x)$ is degenerated]. If the decision maker is always indifferent between the two lotteries, we say that he is risk neutral.Finally, if the decision maker prefers the lottery $F(.)$ to the degenerated lottery, he is a risk lover.

- From the mathematical point of view, inequality (4) is called *Jensen's inequality*. It is the property of a *concavity* function. Therefore, we will say that a decision maker is risk averse if preferences can be represented by means of a **concave Bernoulli** utility function. Strict risk aversion is equivalent to strict concavity.

- The consumer is risk neutral if and only if Bernoulli utility function is *linear*, that is $u(x) = a + bx$ with $b > 0$.

- Figure 6.C.2 MWG.

Another way to characterize risk attitudes is the following.

Definition Suppose that the decision maker is facing a lottery $F(.)$. Certainty equivalent x^{ce} is defined as the sure amount of money which makes the decision maker indifferent between the lottery $F(.)$ and x^{ce}. Therefore,

$$u(x^{ce}) = \int u(x)dF(x).$$

Figure representation...

We can restate our characterization of risk attitudes as follows:

Proposition

- A decision maker is risk-averse if $x^{ce} \leq \int xdF(x)$ for all $F(.)$, and strictly risk averse if $x^{ce} < \int xdF(x)$ whenever $F(x)$ does not put probability 1 on $E(x)$.

- A decision maker is risk-neutral if $x^{ce} = \int xdF(x)$ for each lottery $F(x)$.

- A decision maker is risk-loving if $x^{ce} \geq \int xdF(x)$ for each lottery $F(x)$, and strictly risk-loving if $x^{ce} > \int xdF(x)$ whenever $F(x)$ does not put probability 1 on $E(x)$.

Example. Consider again the case $u(x) = \sqrt{x}$. This is a concave function, so we can conclude that our decision maker is risk-averse. We want to find the certainty equivalent of the lottery giving \$0 or \$100, each with probability $\frac{1}{2}$. In order to do this, we solve:

$$\sqrt{x^{ce}} = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{100}.$$

The solution is $x^{ce} = 25$. As expected, this is less than the expected value of the lottery.

Suppose now that the utility function is $u(x) = x^2$. This function is convex, and therefore the decision maker is a risk lover. The certainty equivalent in this case is obtained solving:

$$(x^{ce})^2 = \frac{1}{2}(0)^2 + \frac{1}{2}(100)^2$$

The solution is $x^{ce} = 70.711$, which is greater than the expected value of the lottery. Again, this was expected given the convexity of the utility function.

1.2.1 Measuring Risk Aversion

We have provided a general characterization of risk aversion in terms of properties of the utility function $u(x)$. We now want to find a way to measure **quantitatively** risk aversion. For example, we may want to compare the risk aversion of two different individuals. Or we may want to know whether a decision maker becomes more or less risk averse when the level of wealth increases.

Definition A common measure adopted in decision theory is the *Arrow-Pratt coefficient of absolute risk aversion*:

$$r_A(x) = -\frac{u''(x)}{u'(x)}.$$

The value of $r_A(x)$ indicates the degree of risk aversion of a decision maker having a wealth of x.

- Since degree of risk aversion is related to the curvature of the Bernouli uitlity function u, it is natural to consider using $u''(x)$ to measure risk aversion. However, this is not an adequate measure because it is not invariant to positive linear transformation. Consider $v(x) = a + bu(x)$, $b > 0$. Utility function $v(x)$ represents the same preference as $u(x)$ but $v''(x) = bu''(x)$. So, we use $-\frac{u''(x)}{u'(x)}$ to take care of it.

- . Notice that, since for a concave function we have $u''(x) \leq 0$, we have $r_A(x) \geq 0$ when the decision maker is risk averse; higher values of $r_A(x)$ indicate more risk aversion. In the case of a risk-neutral decision maker, that is $u(x) = x$, we have $u''(x) = 0$, and therefore $r_A(x) = 0$. At any point at which the function is strictly concave we have $u''(0) < 0$, and therefore $r_A(x) > 0$.

Another common measure used in economics is the *coefficient of relative risk aversion*. This is given by:

$$r_R(x) = -\frac{xu''(x)}{u'(x)} = xr_A(x).$$

The coefficient of relative risk aversion is useful when we want some measure of risk attitudes normalized with respect to the level of wealth.

Example. Consider first the utility function $u^*(x) = \sqrt{x}$, defined on $x \geq 0$ and differentiable on $x > 0$. In this case $u'(x) = \frac{1}{2\sqrt{x}}$ and $u''(x) = -\frac{1}{4x^{\frac{3}{2}}}$, so that

$$r_A^*(x) = \frac{1}{2x}.$$

Consider now the utility function $\overline{u}(x) = \sqrt[4]{x}$. Intuitively, this should denote a higher risk aversion, since the utility function is 'more concave' (more precisely, we have that $\overline{u}(x) = \sqrt{u^*(x)}$, so that $\overline{u}(x)$ is a concave transformation of $u^*(x)$). When we compute the coefficient of absolute risk aversion in this case we have:

$$\overline{r}_A(x) = \frac{3}{4x}.$$

For each $x > 0$ we have $\overline{r}_A(x) > r_A^*(x)$, confirming our intuition that $\overline{u}(x)$ is more risk averse than u^*.

Coming now to the coefficient of relative risk aversion, we observe that:

$$r_R^*(x) = \frac{1}{2} \quad \text{and} \quad \overline{r}_R(x) = \frac{3}{4},$$

so that in both cases the relative risk aversion is constant.

Example. Continuing the previous example, we may ask which utility functions yield constant absolute risk aversion or constant relative risk aversion. If $r_A(x) = r$ for each x then we have the following differential equation:

$$-r = \frac{u''(x)}{u'(x)}$$

This can be written as:

$$-r = \frac{d \ln(u'(x))}{dx}$$

and integrating both sides:

$$\begin{aligned} -rx &= \ln(u'(x)) + A \\ e^{-rx} &= e^{A} e^{\ln(u'(x))} \\ e^{-A} e^{-rx} &= u'(x) \end{aligned}$$

integrating again on both sides we obtain:

$$u(x) = -\frac{1}{r} e^{-A} e^{-rx} + B$$

Since multiplicative and additive constant are irrelevant, we conclude that, up to affine transformations, the only function exhibiting constant absolute risk aversion is $u(x) = -e^{-rx}$.

For the case of relative risk aversion we proceed similarly. We can start from the equality:

$$-\frac{r}{x} = \frac{d \ln(u'(x))}{dx}$$

Integrating on both sides now yields:

$$\begin{aligned} \ln x^{-r} &= \ln(u'(x)) + A \\ e^{-A} x^{-r} &= u'(x). \end{aligned}$$

Assume $r \neq 1$; integrating again we obtain

$$u(x) = e^{-A}\frac{x^{1-r}}{1-r} + B.$$

Again, since we can normalize constants we conclude that, up to affine transformations, the only function exhibiting constant relative risk aversion is $u(x) = \frac{x^{1-r}}{1-r}$ when $r \neq 1$ (notice that $1 - r$ can be positive or negative, but the function is always increasing).If $r = 1$ then the equation becomes

$$u'(x) = \frac{1}{x}e^{-A}.$$

Integrating we have $u(x) = e^{-A}\ln x + B$. Thus, up to affine transformations, the only Bernoulli utility function with relative risk aversion always equal to 1 is $u(x) = \ln x$.

1.2.2 Application: The Insurance Problem

- A risk averse person has assets (e.g. a house and money) for a total value of W. With probability α the house may suffer damage (e.g. a fire), which reduces its value by L. In other words, absent any further action, the final wealth of the person is a random variable $\widetilde{W}$ which takes value W with probability $1-\alpha$, and value $W-L$ with probability α.

- The expected utility of the owner is

$$U(F) = \alpha u(W - L) + (1 - \alpha) u(W).$$

- An insurance company offers a policy which promises to pay \$1 in case of accident for each p dollars of premium paid by the insured (that is, p is the premium paid for each unit of insurance).

- The owner has to decide the optimal level of coverage Q, where Q is the amount of dollars received in case of damage,

- We can write the problem as follows:

$$\max_{Q} \ \alpha\, u(W - L - pQ + Q) + (1 - \alpha)\, u(W - p\,Q)$$

The FOC is:
$$\frac{\alpha\, u'(W-L+(1-p)Q^*)}{(1-\alpha)\, u'(W-p\,Q^*)}=\frac{p}{1-p}$$
This can be interpreted in the same way we did in the standard consumer problem. On the left hand side we have the marginal rate of substitution between wealth when the accident occurs and wealth when the accident does not occur. On the right hand side we have a price ratio: p is the unit price of insurance, and $1-p$ is the net amount of dollars (per unit of insurance) received in case of accident.

- The condition can be rewritten as:
$$\frac{u'(W-L+(1-p)Q^*)}{u'(W-p\,Q^*)}=\frac{\frac{p}{1-p}}{\frac{\alpha}{1-\alpha}} \tag{5}$$

- Suppose first that $p=\alpha$ (this is sometimes called 'actuarially fair premium', since an insurance company charging this price would break even on average) the first order condition becomes
$$u'(W-L+(1-p)Q^*)=u'(W-p\,Q^*). \tag{6}$$
This implies $Q^*=L$, or full coverage.

- In this case the decision maker bears no risk, and only pays the premium.

- The insurance company has an expected profit of zero The expected profit is $E(\pi)=pQ^*-\alpha Q^*$, equal to 0 when $\alpha=p$.

- In the case of $p>\alpha$, so that $\dfrac{p}{1-p}>\dfrac{\alpha}{1-\alpha}$. In this case, the optimal amount of insurance Q^* is such that
$$u'(W-L+(1-p)Q^*)>u'(W-p\,Q^*). \tag{7}$$

- If u is a strictly concave function then u' is a strictly decreasing function, and therefore $W-L+(1-p)Q^*<W-p\,Q^*$, or $Q^*<L$. This implies that insurance is not full. This happens because in this case insurance is relatively expensive, since the price of a unit of insurance p is higher than the probability α of accident.

- It is useful to check what amount of insurance would be bought under constant absolute or relative risk aversion. Assume first that $u(x) = -e^{-rx}$. Then, setting $K = \dfrac{p(1-\alpha)}{(1-p)\alpha}$, equation (5) becomes:

$$\begin{aligned} \frac{e^{-r(W-L+(1-p)Q^*)}}{e^{-r(W-pQ^*)}} &= K \\ e^{r(L-Q^*)} &= K \\ Q^* &= L - \frac{1}{r}\ln K. \end{aligned}$$

- Notice that if $K = 1$ (actuarially fair insurance) then we have full insurance, $Q^* = L$, consistent with the previous result.

- When $K > 1$ (that is, $p > \alpha$) the coverage is less than full.

- The level of coverage decreases with K and increases with r. In the first case we have a simple price effect at work: K increases when p increases, that is insurance is more expensive. In the second case we see the effect of risk aversion. Since r is the coefficient of absolute risk aversion, an higher r corresponds to a more risk averse decision maker. It is natural that a more risk averse decision maker is willing to buy more insurance.

- Consider now the case of constant relative risk aversion. Now equation (5) becomes:

  $$(W - L + (1-p)Q^*)^{-r} = K\,(W - p\,Q^*)^{-r}$$

  and after manipulations we obtain:

  $$Q^* = \frac{L + \left(K^{-\frac{1}{r}} - 1\right)W}{1 + \left(K^{-\frac{1}{r}} - 1\right)p}.$$

  Again, notice that if $K = 1$ we have full insurance, $Q^* = L$. When $K > 1$ then $Q^* < L$; as before, when r increases, the amount of insurance bought increases.

1.3 Ranking of risky prospects

In the previous section, we compared utility functions (risk aversion). Now, we compare payoff distributions. There are two natural ways that random outcomes can be compared: 1) according to the level of returns and 2) according to the dispersion of returns.

A natural question is when one lottery could be said to pay more than another This leads to the idea of first order stochastic dominance.

Definition The distribution F_1 *first order stochastically dominates* F_2 if for *every* nondecreasing function $u : R \to R$,

$$\int u(x)dF_1(x) \geq \int u(x)dF_2(x). \tag{8}$$

So, if F_1 *first order stochastically dominates* F_2, every decision maker who prefers more to less will necessarily prefer $F_1(x)$ to $F_2(x)$. Note that this is true regardless of the decision maker's risk atitude.

Proposition The distribution F_1 *first order stochastically dominates* F_2 if and only if $F_1(x) \leq F_2(x)$ for each x.

Insert Figure 1

Insert Figure 2

Sktech of the proof: →The only if part is actually very easy to prove; just consider utility functions of the form

$$u(x) = \begin{cases} 0 & \text{if} \quad x < x^* \\ 1 & \text{if} \quad x \geq x^* \end{cases}$$

for each possible x^*. The constructed utility function is nondecreasing. F_1 FSD F_2 implies that

$$\begin{aligned} \int_{x^*}^{+\infty} dF_1(x) &\geq \int_{x^*}^{+\infty} dF_2(x) \\ 1 - F_1(x^*) &\geq 1 - F_2(x^*) \\ F_1(x^*) &\leq F_2(x^*). \end{aligned}$$

←The if part is more invovled. Let $H(x) \equiv F_1(x) - F_2(x)$. Integrating by parts, we have

$$\int u(x) dH(x) = u(x) H(x) |_0^{+\infty} - \int u'(x) H(x) \, dx.$$

Given $H(0) = H(+\infty) = 0$, it follows that

$$\int u(x) dH(x) = -\int u'(x) H(x) \, dx.$$

Because $u(x)$ is an increasing function, so $u'(x) \geq 0$. If $H(x) \leq 0$ for each x, it must be true that

$$\begin{aligned} -\int u'(x) H(x) \, dx &\geq 0 \\ \int u(x) dH(x) &\geq 0 \\ \int u(x) dF_1(x) - \int u(x) dF_2(x) &\geq 0. \end{aligned}$$

Q.E.D.

Intuitively, $F_1(x) \leq F_2(x)$ for all x means that $F_1(x)$ puts a higher weight on high value return than $F_2(x)$. See Figure 1.

Consider an example,

	1	2	3	4	5
lottery 2	1/2	0	0	1/2	0
lottery 1	0	1/4	1/4	0	1/2

Clearly, lottery 1 will give the decision maker a better return than lottery 2. Let's compute $F_i(x)$, $i = 1, 2$.

.Figure 2 shows the distribution of the returns from the two lotteries. We have $F_1(x) \leq F_2(x)$ for all x.

Remarks

- Note that *first order stochastically dominance* does not mean that every possible return of the suprior lottery is larger than every possible return of the inferior one. ...

- F_1 *first order stochastically dominates* F_2 implies that the mean under $F_1(x)$ is greater than the mean under $F_1(x)$. This is straightforward by setting $u(x) = x$.

- The reverse is not necessarily true. Having a higher mean does not imply first order stochastic dominance. For example consider two lotteries. Lottery 1 will pay \$100 or 0 with equal probabilities. Lottery 2 will pay \$36 for sure. Clearly, lottery 1 has a higher mean. Consider an increasing utility function $\sqrt{x}$. The expected utility from Lottery 1 is \$5 where the expected utility from Lottery 2 is \$6. Figure 3 shows the c.d.f. of the two lotteries.

First order stochastic dominance is based on the level of return. We'd like to introduce another ranking based on *riskness* or *dispersion.*

- Example: Two lotteries

	-1	0	1
lottery 1	1/3	1/3	1/3
lottery 2	1/2	0	1/2

The two lotteries have the same expected value of zero; can we say that lottery 2 is riskier than lottery 1? In other words, is it true that any risk-averse decision maker would prefer lottery 1 to lottery 2?

Definition For any two distributions F_1 and F_2 with the same mean, F_1 *second order stochastically dominates* F_2 if for *every* nondecreasing *concave* function $u : R \rightarrow R$,

$$\int u(x)\, dF_1(x) \geq \int u(x)\, dF_2(x).$$

Insert Figure 3

Proposition The distribution F_1 *second order stochastically dominates* F_2 if and only if $\int\limits_{-\infty}^{x} F_1(y) \leq \int\limits_{-\infty}^{x} F_2(y)$ for each x.

The proof is ignored. In other words, F_1 second order stochastically dominates F_2 if every risk–averse decision maker prefers F_1 to F_2. In this case, we will say that F_2 is riskier than F_1. Let's get back to the example. Figure 4 shows the c.d.f. of $F_i(x)$, $i = 1, 2$. Note that the area below the two curves are the same. For simplicity, assume that the two lotteries have the highest price $\overline{x}$. Given that the two lotteries have the same mean, we have

$$\int_0^{\overline{x}} x dF_1(x) = \int_0^{\overline{x}} x dF_2(x).$$

By integration by parts, we have

$$\begin{aligned} xF_1(x)\,|_0^{\overline{x}} - \int F_1(x)\,dx &= xF_2(x)\,|_0^{\overline{x}} - \int F_2(x)\,dx \\ x[F_1(x) - F_2(x)]|_0^{\overline{x}} &= \int F_1(x)\,dx - \int F_2(x)\,dx \\ 0 &= \int F_1(x)\,dx - \int F_2(x)\,dx. \end{aligned}$$

- Second order stochastic dominance is linked to the idea of a *mean-preserving spread*. The idea is the following. Pick any random variable $\widetilde{x}_1$, with distribution F_1. We construct a new random variable as follows: when $\widetilde{x}_1$ takes a value x_1, then we randomize further according to the random variable ε_{x_1}, with $E(\varepsilon_{x_1}) = 0$ for each possible x_1. Let F_2 be the distribution resulting from this additional randomization. The variable $\widetilde{x}_2$ is called a mean-preserving spread of $\widetilde{x}_1$. Observe that $\widetilde{x}_2$ has the same mean as $\widetilde{x}_1$. It is intuitive that $\widetilde{x}_2$ is riskier than $\widetilde{x}_1$, since after determining the value x_1, we make an additional roulette spin.

- Note that in the previous example, F_2 is a mean preserving spread of F_1. A figure illustration is Figure 5

- The result that we state without proof is the following: a distribution F_1 second-order stochastically dominates F_2 if and only if F_2 is obtained as a mean-preserving spread of F_1.

- Note that FSD$\nRightarrow$SSD and SSD$\nRightarrow$FSD.

Insert Figure 4

Insert Figure 5

1.3.1 Criticisms of the expected utility theory

Expected utility theory predicts how rational people should behave under uncertainty. However, may experiments demonstrate that the theory does not describe how people actually behave in reality. Below are some criticisms of the expected utility theory.

Allais Paradox Here is an experiment conducted by Kahneman and Tversky (1979).

Problem 1 Choose between two lotteries. The first pays $\$55,000$ with probability 0.33, $\$48,000$ with probability 0.66, and 0 with probability 0.01. The second pays $\$48,000$ for sure.

	$\$55,000$	$\$48,000$	0
lottery 1	0.33	0.66	0.01
lottery 2	0	1	0

Problem 2 Choose between two lotteries. The first pays $\$55,000$ with probability 0.33 and nothing with probability 0.67. The second pays $\$48,000$ with probability 0.34 and nothing with probability 0.66.

	$\$55,000$	$\$48,000$	0
lottery 3	0.33	0	0.67
lottery 4	0	0.34	0.66

A typical response to these two problems is Lottery 2 $\succ$Lottery 1 and Lottery 3 $\succ$Lottery 4. But this preference contradicts the expected utility theorm. Lottery 2 $\succ$Lottery 1 implies

$$\begin{aligned}
u(48,000) &> 0.33u(55,000) + 0.66u(48,000) + 0.01u(0) \\
0.34u(48,000) &> 0.33u(55,000) + 0.01u(0) \\
0.34u(48,000) + 0.66u(0) &> 0.33u(55,000) + 0.67u(0) \\
\text{Lottery 4} &\succ \text{Lottery 3}
\end{aligned}$$

Kahneman and Tversky explain the experimental result as a certainty effect: people tend to overvalue a sure thing. Their paper has many other examples that also lead experimental subjects to violate the independence axiom.

Paradox Suppose that a decision maker prefers a trip to Prague to a movie about Prague and go back home. We can think that the decision maker is facing three lotterires

$$\begin{aligned} L_1 &= (1,0,0) \\ L_2 &= (0,1,0) \\ L_3 &= (0,0,1), \end{aligned}$$

and his preference is

$$L_1 \succ L_2 \succ L_3.$$

By Independent theorem, we should have

$$0.01L_2 + 0.99L_1 \succ 0.01L_3 + 0.99L_1.$$

But in reality, many people prefer

$$0.01L_2 + 0.99L_1 \prec 0.01L_3 + 0.99L_1$$

because they anticipate that if they lose the trip to Prague they would be upset and don't want to be reminded of what they have missed!

Risk Aversion ? As we saw before, in the expected utility model, risk aversion is synonymous with concavity of the utility function, or decreasing marginal utility. Many researchers, however, have argued that this does not provide a reasonable way to think about aversion to medium sized risks. The reason is that turning down medium sized risks implies a high degree of curvature to the utility function, so much so that aversion to medium risks must imply implausible aversion to larger risks. Rabin (2000) gives some numerical examples that bring this point home.

Example 1 Suppose that from any initial wealth, an expected utility maximizer would turn down a 50/50 chance to loses \$1000 or gains \$1050. Then this person must also always turn down a 50/50 bet of losing \$20,000 and gaining any sum.

Example2 Suppose that from any initial wealth, an expected utility maximizer would turn down a 50/50 lose \$100, gain \$110 gamble. She must also turn down a 50/50 lose \$1000, gain \$10,000,000,000 gamble.

Framing Effect The following experiment is due to Kahneman and Tversky, and is a classic.

Decision 1 The U.S. is preparing for an outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. The exact scientific estimates of the consequences of these programs are as follows:

If program A is adopted, 200 people will be saved.

If program B is adopted, there is a 2/3 probability that no one will be saved, and

a 1/3 probability that 600 people will be saved.

Decision 2 The U.S. is preparing for an outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. The exact scientific estimates of the consequences of these programs are as follows:

If program C is adopted, 400 people will die with certainty.

If program D is adopted, there is a 2/3 probability that 600 people will die, and a 1/3 probability that no one will die.

Kahneman and Tversky found that 72% of subjects chose A over B, while 78% chose D over C. Therefore a majority of subjects preferred A to B and D to C. But note that A and C are identical, as are B and D so these are really the same chice problem restated in different way. One way to explain this is via a reference point theory. The first question is asked so the reference point is that everyone will die. The second question is asked so the reference point is that no one will die. Of course, this is not the only way to think about this experiment (and there are many others of its ilk). Another possibility is just that the way a question is asked triggers subjects to start thinking about the answer in a different way – for instance, perhaps the second question makes one focus on trying to save everyone.