

## Homework 3

1. Consider a production and exchange economy with two commodities, two households, labeled 1 and 2, and one firm with production set  $Y = \{(x, y) | x \leq 0, y \leq \sqrt{-x}\}$ . Household 1 has endowment  $e^1 = (2, 1)$ , utility  $u^1(x_1, y_1) = \frac{1}{2} \ln x_1 + \ln y_1$  and  $\frac{1}{3}$  share of the firm. Household 2 has endowment  $e^2 = (6, 1)$ , utility  $u^2(x_2, y_2) = \ln x_2 + \frac{1}{2} \ln y_2$ , and  $\frac{2}{3}$  share of the firm. Compute a competitive equilibrium.

2. Consider a production and exchange economy with two commodities  $x$  and  $y$ , two households 1 and 2 and one firm with production set

$$Y = \left\{ (x, y) \in \mathbb{R}^2 \mid x \leq 0, y \leq -\frac{1}{2}x \right\}.$$

Household 1 has endowment  $e^1 = (4, 1)$ , utility function  $u^1(x, y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$  and  $\frac{1}{3}$  share of the firm. Household 2 has an endowment  $e^2 = (2, 0)$ , utility function  $u^2(x, y) = \sqrt{xy}$  and  $\frac{2}{3}$  share of the firm.

- (a) Compute a competitive equilibrium of this economy. Is it unique?
- (b) If firm shares are changed from  $(\frac{1}{3}, \frac{2}{3})$  to  $(\frac{1}{2}, \frac{1}{2})$ , how will the competitive equilibrium change?

3. Suppose that we have  $n \geq 2$  individuals and  $k > 2$  alternatives. Assume for simplicity that individuals' rankings of the alternatives are strict. Consider the following social welfare functions (SWF):

- (a) Each individual,  $i = 1, \dots, n$  gives  $k$  points to the alternative he likes most,  $k - 1$  to the alternative he likes second most, etc. The social ranking is according to the total points received from individuals to alternatives.
- (b) There is an individual  $i$  so that  $x$  is socially preferred to  $y$  if and only if  $y \succ_i x$ .

For (a) and (b), check whether *transitivity*, *IIA* and *unanimity* hold. If a property holds, provide a proof, otherwise provide a counterexample.