Entry games

2 firms

period 1: Each firm decides whether to enter (simultaneously and independently)

entry cost is $U \sim [0,50]$. Each firm only knows its own entry cost.

period 2: B competition with demand function $q = 10 - p$

(If only one firm enters – revenue 25
If both firms enter – revenue 0)

	Enter	NE
Enter		
NE		

Battle of the sexes/
Chicken

Asymmetric information / incomplete information

A pure strategy: $f: [0,50] \to \{\text{Enter}, \text{NE}\}$
If my cost is c I will do $f(c)$

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entering when your cost is more than 25 is (weakly) dominated.

exercise The strategy
* enter if my cost is between 3 and 20
is also weakly dominated

Marsam doesn't enter

(Kanidia plays against Marsam, she enters if her

Kadidia ...
cost is ≤ 25

Suppose James is playing against Kadidia.
If James' cost is 5

If James enter

prob 1/2 — If Kadidia cost is ≤ 25 his revenue will be 0
prob 1/2 — " " " > 25 " " " " 25

James' expected from entering is $\underbrace{\frac{1}{2} * 25 + \frac{1}{2} * 0}_{12.5}$ − James' cost

James' best response to Kadidia is the strategy
enter if my cost is ≤ 12.5

What is the best-response to James strategy?

my revenue from entering

prob $\frac{1}{4} = \frac{12.5}{50}$ 0 if James enters

prob. $\frac{3}{4}$ 25 " James does not enter

best response: enter if my cost is $\le \underbrace{\frac{3}{4} * 25}$ = 18.7

(Bayes-Nash Equilibrium) A NE A pair of strategies such that each player's strategy is the best response to opponent's strategy.

If my opponent enters when his cost is $\le C$

what is my best response?

If I enter my revenue

prob. $C/50$ 0 if opponent enters
25 " " does not enter

prob. $1-c/50$ ...

Best response: enter if my cost $\leq (1-c/50)*25$

$= 25 - c/2$

$C_2 = 25 - C_1/2$ $\qquad$ $C_1 = 25 - C_2/2$

player 1 : enter is cost $\leq C_1$

player 2 : " " " $\leq C_2$

$$C_1 = C_2 = 50/3$$

If my opponent enters if his cost $\leq \frac{50}{3}$

" " " w.p $1/3$

my expected revenue from entering is $\frac{2}{3} * 25 = \frac{50}{3}$

I should enter if my cost $\leq \frac{50}{3}$

Homework: Assume to players:

player 1's entry cost is $U[0, 40]$

" 2's " " " $U[0, 80]$

find the Bayes Nash Equilibrium

Auction games:

2 bidders, one painting for sale.

Each player has a private value $\sim U[0,100]$

Each player submit a sealed bid b

The winner is the player who submitted the highest b

The payoff to the winner is my value − my bid

The payoff to the loser is 0

A pure strategy: A function $\mathbb{R}_+ \to \mathbb{R}_+$

if my value is v I bid $f(v)$

- It's a dominated strategy to bid above your value
- Try to find the equilibrium strategy:

Suppose my opponent bid his own value: what is my best response?

Repeated games

A small town with two pizza shops

Each shop can charge high (\$17) or low (\$15) per pie

Each store has loyal customers who buy 3000 pie from that store

There are 4000 pies that will be bought from store with lowest price

One shot game

	H	L
H	85, 85	51, 105
L	105, 51	75, 75

prisoner's Dilemma

* Five shot games

Sequential elimination of dominated strategies; play the one-shot equilibrium every period.

let's assume discount factor β

which means that if my payoffs are

$a_0, a_1, a_2, a_3, a_4, \ldots$

then I aggregate this sequence of payoffs to a single number

$a_0 + a_1\beta + a_2 \cdot \beta^2 + a_3\beta^3 + \ldots\ldots$

~~one~~ Two interpretations

- interest rate: \$1 tomorrow equals $\beta^{=1-r}$ today

* - After every period there is a probability β that we live another day, and prob. $1-\beta$ the world ends

Grim trigger strategy

- At day 1 set a high price
- Continue to have a high price as long as opponent set a high price. If at some poin opponent defect (low price) switch to low price forever

At home: Grim trigger is not an equilibrium in the 5 shot game.

Claim: GT is an equilibrium in the discounted game.

Suppose my opponent play GT.

If I play GT I get

$$85 + 85 \cdot \beta + 85 \cdot \beta^2 + \ldots \simeq 85/1-\beta$$

what if I decide to deviate and play L today

$$105 + 75 \cdot \beta + 75 \cdot \beta^2 + \ldots = 105 + 75 \cdot \beta/1-\beta$$

This is an equilibrium if

$$85/1-\beta > 105 + 75 \cdot \beta/1-\beta$$

$$\beta > \frac{2}{3}$$