Eco 500 Fall 2020 final exam

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Instructions

Each solution should **start with the immediate answer** (One sentence in question 1, the list of strategies in question 2, the votes and outcome in question 3, the value of p in question 4, etc.) and then in case you did some calculation or require an argument a **very short, no more than a few sentences** explanation, just to show your reasoning.

All payoffs in the questions are Von-Neumann Morgenstern utilities. In addition, all payoffs are additives, so if I say that Pitagoras got a utility 7 and paid a cost 3 then his total payoff is 4.

Consider the following normal-form game

	L	C	R
T	5,7	1,6	9,4
M	5,9	7,3	1,7
B	20,5	8,6	10,5

1. Explain, in one sentence, why the strategy C of the column player is not dominant.

2. Which strategies survive sequential elimination of strictly dominated strategies ?

Gandalf, Radagast, and Saruman are three legislators who have to vote on a bill that, if passes, will give each of them a pay raise of 20. In addition, each legislator who votes for the bill incurs a voter resentment cost of 10. The bill passes if at least two members vote for the bill.

3. Assume that the vote is sequential and public: First Gandalf votes, then Radagast, then Saruman. How will they vote, and what will be the outcome?

4. Assume now that the vote is simultaneous and independent. Find a mixed strategy equilibrium in which all legislators vote in favor of the bill with the same probability p.

Two tooth fairies each has to decide whether to fly to little Dorothy's bedroom and replace the baby teeth she has placed under her pillow with a brand new iPhone 12. Each fairy will get a pleasure worth of 12 if Dorothy receives the iPhone (no matter which fairy performs the delivery). However, each fairy will suffer some cost from flying at night. This cost is random between 0 and 36 with continuous uniform distribution. Each fairy knows his own cost but not the other fairy's cost.

5. Suppose you are one of the fairies and you know that the other fairy's strategy is to travel if his cost is below 12. What will you do?

6. Find the Nash equilibrium in the simultaneous move game (with asymmetric information) between the fairies.

Consider a simultaneous-move game between two players (competitors), who, each chooses an effort level $x_i \geq 1$. If the effort profile is (x_1, x_2) then with probability $x_1/(x_1+x_2)$ player 1 wins and with probability $x_2/(x_1+x_2)$ player 2 wins. If player 1 wins she gets utility 60 and if player 2 wins he gets utility 40. Each player gets utility 0 from losing. In addition, each player suffers a cost equals to their effort. Therefore, the payoff to player i under profile (x_1, x_2) is

$$\frac{x_i}{x_1 + x_2} v_i - x_i$$

with $v_1 = 60, v_2 = 40$.

7. Explain, in one sentence, why the strategy $x_1 = 100$ for player 1 is dominated.

8. Find a Nash equilibrium in pure strategies in the game

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9. Consider a two-player simultaneous move zero-sum game. Prove that if $(\sigma_1, \sigma_2), (\sigma_1', \sigma_2')$ are two (pure or mixed) equilibria in the game then (σ_1, σ_2') is also an equilibrium.

10. Give an example for a two-player simultaneous-move game with two (pure or mixed) equilibria $(\sigma_1, \sigma_2), (\sigma_1', \sigma_2')$ such that (σ_1, σ_2') is not an equilibrium.