Eco 500 Fall 2020 midterm exam

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- Each solution should start with the immediate answer (a utility function in question 1, a demand function in question 2, etc) and then a **very short, no more than a few sentences** explanation, just to show what calculations you did.
- All monetary values are in dollars. When I say that the set of monetary outcomes is $(3, 7, 19)$ then the lottery $(1/2, 1/3, 1/6)$ is the lottery that gives \$3 with probability $1/2$, gives \$7 with probabilities $1/3$, and gives \$19 with probability $1/6$.
- When I write a Warlasian (Marshallian) demand function $x(p_1, p_2, w)$ the meaning is that $x_i(p_1, p_2, w)$ is the demand for good i when prices are p_1, p_2, and the wealth is w.

Part I: Consumer's Theory

1. An agent has the following preference relation over the set of outcomes Apple, Banana, Coconut, Dates$\}$

$$C \prec A \sim D \prec B.$$

   Give an example of a utility function that rationalizes this preference. (No need to add an explanation)

2. In a market for two goods, a consumer's utility over the bundle space $\mathbf{R} \times \mathbf{R}_+$ is given by $u(x_1, x_2) = x_1 + \log x_2$. Find the consumer's demand function $x(p_1, p_2, w)$.

3. In a market for three goods, a consumer's utility over the bundle space $\mathbf{R}^3_+$ is given by $u(x_1, x_2, x_3) = x_1 + x_2 + 2x_3$. Find the expenditure function.

4. Consider the following information about an agent's Warlasian demand function $x(p_1, p_2, w)$ in a market for two goods

$$x(1, 2, 6) = (2, 2), \text{ and } x(3, 5, 15) = (5, 0)$$

Is this information consistent with WARP? What can you deduce about the consumer's preference between the bundles $(2, 2), (5, 0)$?

5. In a market with two goods, consider a consumer with the Walrasian demand function $x(p_1, p_2, w)$ that satisfies Walras' Law and is homogeneous of degree 0. Assume that the demand for the first good when the price of the second good is 1 is given by

$$x_1(p_1, 1, w) = w/(2p_1 + 2).$$

Find the complete demand function $x_i(p_1, p_2, w)$ for $i = 1, 2$.

Part II: Choice under uncertaity

6. An agent's von-Neumann Morgenstern utility over the set candy colors {Blue, Red, Pink} is given by $u(B) = 4, u(R) = 7, u(P) = 16$. What is the agent's preference between the following lotteries?

$$L = (1/3, 1/3, 1/3), \ M = (1/2, 0, 1/2), \ R = (0, 1, 0).$$

7. An agent's utility over money is given by $u(x) = \sqrt{x}$. Find the certainty equivalent of a lotery that gives $10, 8,$ or 6 with probability $1/3$ to each outcome. (You don't need to come up with a number, you can write a

formula like log(4 + sin 5/18). Also if your formula is self-explanatory, no need to add an explanation)

8. An agent has von-Neumann Morgenstern utility from money given by $u(x) = \sqrt{x}$. For each of the following functions, does it induce the same preference like u over lotteries (there may be more than one correct answer, mark them all. No need to add an explanation)

   (a) $\sqrt{x + 7}$

   (b) $\sqrt{x} + 7$

   (c) $\sqrt{x * 7}$

   (d) $\sqrt{x} * 7$

   (e) $x + 7$.

9. An agent has the following strict preferences over lotteries with set of outcomes Asparagus, Broccoli, Celery, Domino's pizza$\}$.

$$(0.2, 0.2, 0.3, 0.3) \prec (0.2, 0.2, 0.4, 0.2), \text{ and}$$
$$(0.1, 0.3, 0.5, 0.1) \prec (0.1, 0.3, 0.4, 0.2).$$

   Prove that the agent's preference is inconsistent with expected utility maximization.

10. Consider the following two lotteries over the set of monetary prizes $X = (1, 4, 7, 12)$

$$L = (1/4,\ 1/4,\ 1/4,\ 1/4) \text{ and } R = (1/10,\ 5/10,\ 3/10,\ 1/10).$$

    Which of the following is correct? Prove your answer

    (a) A risk neutral agent will prefer L, but a risk averse agent might prefer R

(b) A risk neutral agent, and any risk averse agent, will prefer L

(c) A risk neutral agent will prefer R, but a risk averse agent might prefer L

(d) A risk neutral agent, and any risk averse agent, will prefer R

(e) None of the above