ECO 500 Final

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1. If the row player plays T, then playing L is better than C for the column player.

2. (B, C)

3. Gandalf votes against the bill, Radagast and Saruman vote for the bill.

   Outcome: (20, 10, 10)

4. $p = \frac{1}{2}$ The equation is $-10 + 20 \cdot (p^2 + 2p(1-p)) = 20 \cdot p^2$

5. To travel if the cost is below 8.

6. BR for i: $c_i = 12 - \frac{c_j}{3} \Rightarrow c_1 = c_2 = 9$

   NE: Both fairies' strategies are to travel if the cost is below 9.

7. If player 1 plays $x_1 \geq 100$, negative payoff is guaranteed regardless of what the opponent does.

8. $$\max_{x_i} \frac{x_i}{x_1 + x_2} v_i - x_i$$

   FOC: $$\begin{cases} \frac{60 x_2}{(x_1+x_2)^2} - 1 = 0 \\ \frac{40 x_1}{(x_1+x_2)^2} - 1 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{72}{5} \\ x_2 = \frac{48}{5} \end{cases}$$

   NE: $(x_1, x_2) = \left(\frac{72}{5}, \frac{48}{5}\right)$

9. Proof.

Both (σ_1, σ_2), (σ_1', σ_2') are NEs.

$$\begin{cases} \pi(\sigma_1', \sigma_2') \geq \pi(\sigma_1, \sigma_2') \geq \pi(\sigma_1, \sigma_2) \\ \pi(\sigma_1', \sigma_2') \leq \pi(\sigma_1, \sigma_2) \end{cases}$$

$\Rightarrow \quad \pi(\sigma_1', \sigma_2') = \pi(\sigma_1, \sigma_2') = \pi(\sigma_1, \sigma_2)$

A correct proof has two parts:

1) σ_1, σ_2' gives same expected payoff as (σ_1, σ_2) and (σ_1', σ_2')

$\forall\ s_1 \in S_1,\ s_2 \in S_2$

$$\begin{cases} \pi(s_1, \sigma_2') \leq \pi(\sigma_1', \sigma_2') = \pi(\sigma_1, \sigma_2') \\ \pi(\sigma_1, s_2) \geq \pi(\sigma_1, \sigma_2) = \pi(\sigma_1, \sigma_2') \end{cases}$$

$\Rightarrow \quad (\sigma_1, \sigma_2')$ is a NE

2) No deviation is profitable

$\square$

10.

		2	
		L	R
1	U	(2, 1)	(0, 0)
	D	(0, 0)	(1, 2)

where $(\sigma_1, \sigma_2) = (U, L)$, $(\sigma_1', \sigma_2') = (D, R)$ are two NEs

and $(\sigma_1, \sigma_2') = (U, R)$ is not a NE.