November 6

Cournot's competition

n firms

each firm decides on production quantity q_i

The inverse demand is given $P(\underbrace{q_1 + \dots + q_n}_{\text{total quantity produced}}) =$

profit for firm i $\quad q_i * P(q_1, \dots, q_n) - c_i(q_i)$

<u>In our game</u> $\quad n = 3$

$$\begin{cases} P(Q) = 30 - Q \\ c_i(q_i) = 6\, q_i \end{cases}$$

what would a monopoly do?

$$\max_{q} \; q \cdot (\underbrace{30 - q}_{\text{price}} - \underbrace{6}_{\text{cost of production}})$$

$q^* = 12$ $\quad$ profit of monopoly $= 144$

3 firms; If each produce 4 the total profit is 144.

what should I do if opponents produce X each?

$q \cdot (30 - 2 * X - q - 6) = q \cdot (24 - 2X - q) \quad q^* = 12 -$

price cost

Best response function; how should I play if I knew opponent's strategy

$$\boxed{BR(x) = 12 - x} \qquad \underbrace{BR(x, x)}_{\text{if all opponents play } x} = 12 - x$$

If I know that my opponents produce x, the best thing for me to do is to produce 12-

In an equilibrium $\underbrace{x = BR(x)}_{\text{fixed point}}$

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an n-player game is given by

$S_1, \dots, S_n$; set of pure strategies

and payoff functions $u_i : \underbrace{S_1 \times \dots \times S_n}_{\text{strategy-profile}} \to \mathbb{R}$

best-response correspondence BR_i is given by

$$BR_i(s_{-i}) = \operatorname{argmax} \{ u_i(s_i, s_{-i}) ; s_i$$

s_{-i}: An element of $\prod_{j \neq i} S_j$

A nash-equilibrium is given by a strategy profile $s^* = (s_1^*, \dots, s_n^*)$ such that

$$s_i^* \in BR_i(s_{-i}^*) \quad \text{for every } i$$

A strategy s_i for player i is <u>strictly dominant</u> if

$$s_i = BA(s_{-i}) \quad \text{for any } s_{-i} \in S_{-i}$$

Battle of the sexes

	Concert	play
Concert	3, 2	0, 0
play	0, 0	2, 3

BA_{row} (concert) = concert

BA_{row} (play) = play

		Foreign	
Domestic	old	new	mixed
old	60, 40	57, 43	55, 45
new	75, 25	47, 53	53, 47
mixed	20, 80	70, 30	50, 50

$BA_{domestic}$ (old) = new

$BA_{domestic}$ (new) = mixed

$BA_{domestic}$ (mixed) = old

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worst-case analysis in constant-sum games

... the worst thing that can happen is

If I play old the worst " " " " is
" " " new " " " " " " is
" " " mixed " " " " " " is

<u>Theorem</u> In a two-player constant-sum game, under a nash equilibrium profile, each player maximizes his worst-case payoff

$$s_i^* \in \arg\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$$

worst thing that can happen to me if I pick s_i

example in which worst case analysis an equilibri[um] gives different outcomes

	L	R
T	0, −2	0, −3
B	−1, −1	100, 100

(B, R) is a Nash equilibrium

but worst-case analysis say play (T, ...

<u>Penalty kick</u>

keeper

kicker	L	R
L	64, 36	94, 6
R	89, 11	44, 56

no pure strategy NE

worst case analysis for kicker

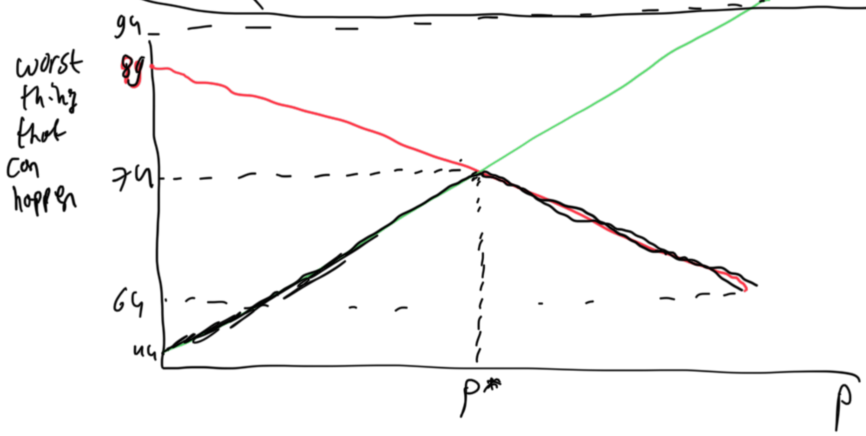
If kicker goes L. Worst thing is 64 = min(64, ...

'' '' '' R, '' '' '' 44 = min(89,

'' '' '' $(\frac{1}{2}, \frac{1}{2})$. Worst is $\min\left(\frac{64+89}{2}, \underbrace{\frac{94+44}{2}}\right) =$

'' '' '' $\begin{pmatrix} L & R \\ p, & 1-p \end{pmatrix}$. worst thing that can happen

$\min\left(p*64 + (1-p)*89,\ p*94 + (1-p)*44 \right)$



$$p \cdot 64 + (1-p) \cdot 89 = p \cdot 94 + (1-p) \cdot 44$$

$$p = 3/5$$

<u>at home</u>: Find the strategy that is optimal for kicker using worst-case analysis; check that these two strategies are an equilibrium.

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n players

$S_1, S_2, \ldots, S_n$ pure-strategy sets

$u_i : S_1 \times \ldots \times S_n \to \mathbb{R}$

We let $\Sigma_i = \Delta(S_i)$ be the set of <u>mixed stra</u>

We let

$$u_i(\underbrace{\sigma_1, \sigma_2, \ldots, \sigma_n}) = \sum u_i(s_1, \ldots, s_n) \cdot \underline{\sigma_1(s_1) \cdot \sigma_2(s_2) \ldots}$$

mixed-strategy profile $S=(s_1, \dots, s_n)$

$\sigma_i \in \Sigma_i$

players choose strategies simulta[neously] and independently

A mixed-strategy NE is a profile $(\sigma_1, \dots, \sigma_n)$ of mixed strategies, such that if my opponents follow this profile, then it is in my best interest to follow the profile

$$u_i(s_i, \sigma_{-i}) = \sum_{s_{-i}} u_i(s_i, s_{-i}) \prod_{j \neq i} \sigma_j(s_j)$$

my payoff if I use pure-strategy s_i and opponents use mixed strategy profile σ_{-i}