Signaling games

buyer

T types (of sender), $p \in \Delta(T)$ type = valu

Sender sends a message $m \in M$ $M = T$

Receiver observes m and take action $\underline{a \in A}$

price

prob. of gettin the product

$u(t, m, a)$ sender's payoff

$v(t, m, a)$ Receiver's payoff

<u>strategies</u> Sender: $\sigma : T \to \Delta(M)$ truth tel

Receiver: $\tau : M \to \Delta(A)$ mechani

Nash Equilibrium: σ^*, τ^* such that

$v(\sigma^*, \tau) \leq v(\sigma^*, \tau^*)$ for every τ

$\boxed{u(\sigma, \tau^*) \leq u(\sigma^*, \tau^*) \quad \text{for every } \sigma}$ I.C

$T = \{\underset{0.3}{\text{guilty}}, \underset{0.7}{\text{innocent}}\}$ $A = \{\text{Jail}, \text{free}\}$

payoff to Judge (Receiver) $\begin{cases} +1 & \text{correct decision} \\ -1 & \text{incorrect "} \end{cases}$

" " prosecutor (sender) $\begin{cases} 1 & \text{Jail} \\ 0 & \text{free} \end{cases}$

σ:

	Y	N
guilty	1	0
innocent	$\frac{3}{7}$	$\frac{4}{7}$

$\tau =$ if Y sent to jail

if N free

If message (outcome of the investigation) is N then the judge knows that innocent

If message is Y then the posterior probability

If message is y then the ... that guilty is $\dfrac{0.3*1}{0.3*1+0.7*\frac{3}{7}} = \frac{1}{2}$

Cheap talks games

$u(t,m,a)$ $\quad v(t,m,a)$ do not depend on m

If $u=v$ there is an equilibrium in which Sender reveals t, and Receiver's believes sender and pick $a \in \arg\max v(t,m,a)$

pooling equilibrium: under (σ^*,τ^*) Receiver has no information about t (different types have same dist. over m)

separating equilibrium: under (σ^*,τ^*) Receiver knows t (different types send different messages)

Eq condition: τ^* is best-response to σ^*

Refinement: - after observing m, R forms a posterior β_m belief about t.

- If m is on-path (if m could happen under σ^*) then this belief comes from Bayesian updating
- Receiver best-responds to his posterior belief

chooses $a \in \arg\max \sum_t \beta_m(t)\, v(t,m,a)$

$T=[0,1]$ uniform prior

$A=\mathbb{R}$

Receiver's cost function is $(a-t)^2$ (If R knew t he would chose $a=t$.

Sender's cost function is $\underline{(a-\theta-t)^2}$

θ is a parameter (bias)

If Sender could choose the action $a = t+\theta$

If $\theta=0$ there is a fully revealing (fully separating) equilibrium; Sender announces t, and Receiver believes him and picks $a=t$.

$M=T$ $\quad$ $\sigma^*(t)$ gives prob. 1 to t

$\tau^*(m)$ " " 1 to $a=m$

The belief of R after observing m is that $t=m$.

Let $\theta>0$ small.

If Receiver plays τ^* (after message m, Receiver picks a of m). Sender's best response when his type is t is $t+\theta$.

Babbling equilibrium: Sender sends $m\in M$ from a uniform distribution (regardless of her type)

Receiver's belief after every message is p

In our game, when Receiver's belief is uniform his best action is 1/2

Babbling equilibrium in our example

Sender babbles: send a random message, independent of her type

Receiver ignores the message and picks $a=1/2$

Assume θ is small.

What are Receiver and Sender's expected costs under babbling equ.

" " " " " " " " truth reveal

Under Truth revealing

Receiver plays $a=t$, Receiver's expected cost is 0

Sender's expected cost is θ^2

Under babbling $a = 1/2$

Receiver's expected cost: $\int_0^1 (t-1/2)^2 \, dt = 1/12$

Sender's expected cost: $\int_0^1 ((t+\theta)-1/2)^2 \, dt = 1/12 + \theta^2$

Suppose that Receiver has belief β over $[0,1]$

$\beta \in \Delta([0,1])$

Receiver should pick a that minimizes

$$\int (t-a)^2 \, d\beta(t)$$

for example if β is uniform over $[0,1]$ the optimal a is $1/2$

If X is a random variable (X is the type)

what is a that minimizes $E(X-a)^2$

the optimal a is EX

the expected cost if $a=EX$ is $E(X-EX)^2 = Var(X)$

If we choose another a then our cost is

$$E(X-a)^2 = E(X-EX + EX-a)^2$$

$$= Var\, X + (EX-a)^2$$

In our case, when Receiver's belief is $\beta \in \Delta([0,1])$

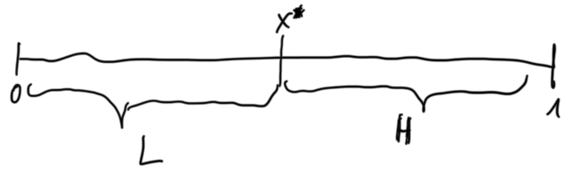
he will choose a to be the expectation of β.

he will choose a

Receiver's expected payoff is $Var(\beta)$

Sender's expected payoff " $Var(\beta) + \theta^2$

Example of a non-babbling equilibrium



σ: Sender's strategy; when $t \leq x^*$ say L

when $t > x^*$ say H

What is Receiver's best-response to σ?

τ: If the message is L, posterior belief is uniform $[0, x^*$

Receiver plays $x^*/2$.

If the message is H, posterior belief is $U[x^*, 1]$, Receiver plays $\frac{x^*+}{2}$

If I am Sender, my type is t, and I know that Receiver plays τ.

If I say L my cost $(t + \theta - x^*/2)^2$

If I say H " " $(t + \theta - (x^*+1)/2)^2$

Sender's best response: If t is closer to $\frac{x^*}{2} - \theta$ he says

" t is closer to $\frac{x^*+1}{2} - \theta$ he sa

$$\begin{cases} \text{If } t < \frac{x^*}{2}+\frac{1}{4}-\theta & \text{plays } L \\ \text{If } t > \frac{x^*}{2}+\frac{1}{4}-\theta & \text{'' } H \end{cases}$$

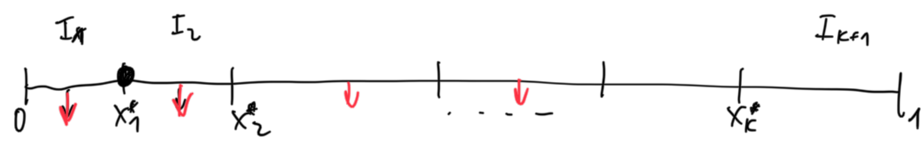
If $x^* = \frac{x^*}{2}+\frac{1}{4}-\theta$ we have an equilibrium

This happens if $\theta<\frac{1}{4}$ $x^* = 2\left(\frac{1}{4}-\theta\right)$

Receiver's expected cost under this equilibrium:

$$\underbrace{x^*}_{\text{prob that message is } L} \cdot \frac{x^{*2}}{12} + (1-x^*)\cdot\frac{(1-x^*)^2}{12} < \frac{1}{12}$$

Sender's expected cost is $\qquad + \theta^2$



If θ is sufficiently small we can construct an equilibrium which divides $[0,1]$ to $k+1$ intervals

Receiver's best response:

If Sender says that the interval is I_1 R plays $\frac{x_1}{2}$

'' '' '' '' '' '' '' I_2 R plays $\frac{x_1^*+}{2}$