Homework 1

ECO 501 Spring 2020

Due: Thursday February 6th

Question 1

Consider the following profile of preferences:

$A:$	a	b	c	d	$a:$	B	C	A	D	E
$B:$	d	b	c	a	$b:$	C	A	B	D	E
$C:$	d	c	a	b	$c:$	E	D	A	B	C
$D:$	a	d	c	b	$d:$	A	D	E	B	C
$E:$	a	b	d							

(a) Show that the men-proposing procedure (MPP) leads to

$$\mu_M = \langle Aa, Bb, Cc, Dd, E\rangle,$$

as the man-optimal stable matching, and that the woman-proposing procedure leads to

$$\mu_W = \langle Ad, Ba, Cb, Dc, E\rangle$$

as the woman-optimal stable matching.

(b) Let a misrepresent her preferences by stating

$$a: \quad B \quad C \quad D \quad E \quad A$$

while all others state according to the profile given above. Compute the new matching according to the MPP and show that a obtains a husband she prefers.

Question 2

Do exercise 1(c) and 4(a) from "Marriage Game by Shapley" posted in blackboard.

Question 3

Fix a profile of preferences and let μ and $\tilde{\mu}$ be two stable matchings for the profile. Let each man point to the better of his two wives in μ and $\tilde{\mu}$. Show

(a) no two men point to the same woman (so we get a matching).

(b) the matching obtained in (a) is stable. Notation: This matching is denoted $\mu \bigvee_{\mathrm{M}} \tilde{\mu}$; read "$\bigvee_{\mathrm{M}}$" as "better for men".

(c) Each woman obtains the *worse* of her two husbands in $\mu, \tilde{\mu}$. Notation: so the matching is also denoted $\mu \bigwedge_{\mathrm{W}} \tilde{\mu}$; read "$\bigwedge_{\mathrm{W}}$" as "worse for women".[1,2]

Question 4

Prove that if the result of the men-proposing procedure yields the same result as the women-proposing procedure, then this resulting matching is the unique stable matching.

Question 5

Do exercises 6 and 8 from "Housing-Shapley" posted in blackboard.

Question 6

Consider the housing market with the profile:

$$\begin{aligned} A: &\quad b \sim c \succ a \\ B: &\quad a \succ c \succ b \\ C: &\quad a \succ b \succ c \end{aligned}$$

Here A ranks b and c equally in the first place, and a in the second place.

Define strict-core[1] as in class and show that it is empty. (This example shows the need for strict preferences in our analysis.