Homework 2

Due Date: February 25th, 2020

1. Edgeworth Box. Let $e^1 = (1, 2)$ and $e^2 = (2, 3)$. Find (compute and picture) the Pareto set, core and competitive equilibrium for each of the following cases:
(a) $u^1(x, y) = x + y$, $u^2(x, y) = 2x + 3y$
(b) $u^1(x, y) = x^{1/2}y^{1/2}$, $u^2(x, y) = x^{2/3}y^{1/3}$
(c) $u^1(x, y) = \min\{x, y\}$, $u^2(x, y) = \min\{2x, 3y\}$
(d) $u^1(x, y) = x + y$, $u^2(x, y) = \min\{2x, 3y\}$
(e) $u^1(x, y) = x^{1/2}y^{1/2}$, $u^2(x, y) = 2x + 3y$

2. Let $u^h(x_1, ..., x_L) = x_1^{\alpha_1} x_2^{\alpha_2} ... x_L^{\alpha_L}$. Assume $\alpha_i \geq 0$ for all i and $\sum_{i=1}^{L} \alpha_i = 1$. Show that if $u^h(z)$ maximizes utility on $B^h(p) = \{y \in R_+^L | p \cdot y \leq p \cdot e^h\}$, then $p_l z_l = \alpha_l p \cdot e^h$ for all $l = 1, ..., L$.

3. Consider an exchange economy in which there are four agents and three goods. Agents' utility functions are $u^1(x, y, z) = x^{1/2}y^{1/4}z^{1/4}$, $u^2(x, y, z) = x^{1/3}y^{1/3}z^{1/3}$, $u^3(x, y, z) = x^{2/3}y^{1/4}z^{1/12}$ and $u^4(x, y, z) = x^{1/4}y^{1/4}z^{1/2}$ respectively. Their endowments are $e^1 = (1, 2, 0)$, $e^2 = (0, 2, 3)$, $e^3 = (1, 1, 1)$ and $e^4 = (1, 0, 0)$ respectively. Compute the competitve equilibrium for this economy.

4. Consider an exchange economy in which there is a commodity l such that
(a) Household 1 owns (i.e. is endowed with) only commodity l and likes only commodity l.
(b) Households $2, .., H$ each own commodity l but none of them like commodity l.
Show that a CE doesn't exist.

5. Consider an exchange economy in which there are two goods and two agents. Agent 1 has utility function $u^1(x, y) = -\sqrt{(x - 1/4)^2 + (y - 1/4)^2}$ and agent 2 has utility function $u^2(x, y) = \log(x) + \log(y)$. Endowments are $e^i = (1/2, 1/2)$ for $i = 1, 2$. Show that this economy has a competitive equilibrium which is not Pareto Optimal. Does this example contradict the First Welfare Theorem?

6. Consider traders $h \in H = \{1, 2, .., H\}$ with endowments $e^h \in R_{++}^K$ and monotonic, strictly concave utility function u^h. For any price vector $p \in R_{++}^K$, there will be a unique consumption bundle $y^h(p)$ in the budget set $B^h(p) = \{x \in R_+^K : p \cdot x \leq p \cdot e^h\}$ which maximizes u^h. (Note: the uniqueness follows from strict concavity). Define the aggregate excess demand function $z : R^K_{++} \longrightarrow R^K$ by $z(p) = \sum_{h \in H}[y^h(p) - e^h]$. The function $z(.)$ has the gross substitute (GS) property if whenever p' and p are such that, for some l, $p'_l > p_l$ and $p'_k = p_k$ for $k \neq l$, we have $z_k(p') > z_k(p)$ for $k \neq l$.

(a) Using the fact that the aggregate excess demand functions are homogenneous of degree zero, prove that $z_l(p') < z_l(p)$.

(b) Prove that if the aggregate excess demand function $z(.)$ satisfies the gross subsitute property, then there is at most one normalized price vector p such that $z(p) = 0$. (Note: normalized means $\sum_{l=1}^{K} p_l = 1$.)