Homework 4

Due on Monday 6th before midnight - Recitation Tuesday 7th at 9:00am

1. Consider a situation in which there are 3 agents. Assume that the set of alternatives, X, is the interval $[0, 1]$, and that each individual's preference is *single-peaked*, that is, for each i there is an alternative a_i^* such that if $a_i^* \geq b > c$ or $c > b \geq a_i^*$, then $b \succ_i c$. Consider the following voting procedure to chose a social alternative. Each agent writes in a sealed bid his vote for one alternative. The votes are counted and the median is calculated, which is then chosen as the social alternative. Show that the voting procedure described above is strategyproof, i.e., for each agent it is a dominant strategy to vote for her preferred alternative.

2. Consider an agency relationship in which the principal contracts the agent, whose effort determines the result. Assume that the uncertainty present is represented by three states of nature. The agent can choose between two effort levels. The results are shown in the following table.

	states of Nature		
	ϵ_1	ϵ_2	ϵ_3
$e = 6$	60,000	60,000	30,000
$e = 4$	30,000	60,000	30,000

The principal and the agent both believe that the probability of each state is one third. The objective functions of the principal and the agent are, respectively:

$$B(x, w) = x - w$$
$$U(w, e) = \sqrt{w} - e^2,$$

where $x = x(e, \epsilon)$ is the monetary result of the relationship and $w = w(x)$ is the monetary pay-off that the agent receives. Assume that the agent will only accept the contract if he obtains an expected utility of at least 114.

(a) What can be deduced from the participants' objective functions?

(b) What would be the effort and the wage in a situation of symmetric information?

(c) What happens in a situation of asymmetric information? What pay-off scheme allows an effort of $e = 4$ to be obtained? What pay-off scheme allows the effort level of $e = 6$ to be obtained? Which effort level that the principal prefer?

3. There's a fixed, finite supply of cars, and infinitely many buyers in the market. The quality distribution of cars as well as the valuation of different quality cars by buyers and sellers are given in the following table:

	q_1	q_2	q_3
Buyer	1100	1800	2500
Seller	1000	1500	2000
Fraction	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

(a) Suppose sellers observe the quality of the car, but buyers do not. Compute the market equilibrium prices.

(b) Suppose sellers have an option to credibly disclose the quality of the car. A seller chooses: He either sends a signal fully and truthfully disclosing the quality of the car, or does not disclose it at all. The cost of the signal to the sellers is 400. What will the equilibrium be in this case?