

	C	D
C	(1 year, 1 year)	(20 years, 0)
D	(0, 20)	(10, 10)

D is a dominant strategy

(10, 10) in prison

playing rationally yields an outcome which is bad for everyone

price war model

2 firms (players)

each chooses price

demand is  $(5-p)^+$  consumers choose store with lower price

price can be  $\{0, 1, 2, 3, 4, \dots\}$

production cost is  $c=1$  per item

$S_1 = S_2 = \{0, 1, 2, 3, \dots\}$

$$u_i(p_i, p_{-i}) = \begin{cases} (5-p_i)^+ * (p_i-1) & p_i < p_{-i} \\ \frac{1}{2} (5-p_i)^+ * (p_i-1) & p_i = p_{-i} \\ 0 & p_i > p_{-i} \end{cases}$$

$\downarrow$  my payoff       $\downarrow$  my price       $\downarrow$  opponent price

	1	2	3	U	5
1	0,0	1,0	0,0	0,0	0,0
2	0,0	1.5, 1.5	3,0	3,0	3,0
3	0,0	0,3	2,2	4,0	4,0
4	0,0	0,3	0,4	4.5, 4.5	3,0
5	0,0	0,3	0,4	0,3	0,0

3 is not dominant: If opponent plays 2 the best thing for me is to play 2, not 3

2 is not dominant: If opponent plays 5 I want to play 3, not 2

### Berthard Competition

"People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices."

Adam Smith (wealth of the nations)

	H	L
H	10,10	0,12

L (Defect) is dominant  
a nice outcome 6,6

L	12, 0	6, 6
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0.11

MC: Match the competition;  
my price is H, but if competition prices L  
then my price reduces to L

	H	L	MC
H	10, 10	0, 12	10, 10
L	12, 0	6, 6	6, 6
MC	10, 10	6, 6	10, 10

IS L dominant? No If opponent plays MC  
I prefer H over L

MC is not dominant: If opponent plays H  
I want to play L, not H

H is dominated by MC: MC sometimes give  
same payoff, sometimes higher payoff than H.

After eliminating H, MC is weakly dominant

Marketing war:

or both not advertise

If both firm advertise, each get  
revenue 50

If only one firm advertise, that firm gets  
80 and the other firm 20 revenue.

advertising cost 20

	not advertise	advertise
not advertise	50, 50	20, 60
Advertise	60, 20	30, 30

prisoner's dilemma

	how much time save on own task	payoff
Alice	2	0
Bob	5	-3
craig	2	0
David	1	1
Evan	3	
Faruk	4	

total minutes save for payoff is 1

each player's payoff:  $(-1) * \text{own effort} + 2 * 1$

What is best for society?  
if everyone chooses 7 everyone gets 7

$$S_1 = S_2 = \dots = S_6 = \{1, 2, \dots, 7\}$$

$$u_i(S_i, S_{-i}) = (-1) * S_i + 2 * \min(S_1, \dots, S_6)$$

is 7 dominated? no. If everybody else chooses 7, then 7 is the best option

## Nash equilibrium

A game in normal form is given by

$S_1, S_2, \dots, S_n$  strategy sets

payoff functions  $U_i: S_1 \times \dots \times S_n \rightarrow \mathbb{R}$

A profile  $(s_1^*, s_2^*, \dots, s_n^*)$  is a Nash equilibrium

if  $U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)$  for every  $s_i$

Nash eq; Self-enforcing agreement: If I agree on playing this profile, then no player has the incentive to unilaterally deviate

	S	H
S	(5, 5)	0, 3
H	3, 0	(3, 3)

	C	D
C	3, 3	0, 4
D	4, 0	(1, 1)

		Penalty	kicks	game
		L	keeper	R
Kicker	L	64, 36	94, 6	
	R	99, 11	44, 56	

A mixed strategy for player  $i$  is

a probability distribution over  $S_i$

If every player uses mixed strategies  
we calculate expected payoff.

		keeper	
		L	R
Kicker	P L	0, 1	1, 0
	1-P R	1, 0	0, 1

mixed strategy  
equilibrium;

every player plays  $(\frac{1}{2}, \frac{1}{2})$

If I the kicker and opponent plays  $(\frac{1}{2}, \frac{1}{2})$

If I play  $(p, 1-p)$  I get

$$\frac{1}{2} * (1-p) + \frac{1}{2} * p = \frac{1}{2}$$

I have no incentive to deviate.