Lecture Note: Revealed preference

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1 Revealed Preferences

1.0.1 Introduction

Up to now we have developed a theory of consumer behavior taking preferences as the primitives. Given rational preferences, we deduce a decision maker's optimal choices. However, in reality, we cannot directly observe decision makers' preferences. We have to discover decision makers' preferences from their behavior. This approach implicitly assumes that the preferences will remain unchanged while we observe the behavior. It may not be reasonable over very long spans.

1.0.2 Choice rule

In the second approach of modeling decision making, we take consumers' choices as the primitives of the theory. A choice strucutre $(\beta, C(.))$ consists of two ingredients

- β is a set of nonempty subsets of X. Each element of β is a set $B \subset X$. We call elements B budget sets. It needs not include all possible subsets of X. For example, $X = \{x, y, z\}$ and $\beta = \{\{x, y\}, \{x, y, z\}\}$. One way to think about B is that it is a choice experiment that is posted to the decision maker.

- $C(.)$ is a *choice rule*, which indicates the individual's choice for a given budget set B. That is, $C(B) \subset B$. The set $C(B)$ can contain more than one element. In this case, $C(B)$ indicates all the alternatives the individual *might* choose. One way to interpret this is to let the individual make choices repeatedly given the same budget set and he ends up choosing different alternatives.

Let's revisit the example with $X = \{x, y, z\}$ and $\beta = \{\{x, y\}, \{x, y, z\}\}$. Consider two choice rules

Choice rule 1 $C_1(\{x, y\}) = \{x\}$ and $C_1(\{x, y, z\}) = \{x\}$. In this case we see x is chosen no matter what budget the decision maker is given.

Choice rule 2 $C_2(\{x, y\}) = \{x\}$ and $C_2(\{x, y, z\}) = \{x, y\}$. In this case, x is chosen when the individual faces budget $\{x, y\}$. And, either x or y is chosen when the individual faces budget $\{x, y, z\}$.

Now, we'd like to impose some reasonable restrictions regarding an individual's choice behavior. For example, if we see an individual chooses x over y when facing the budget $\{x, y\}$, it would be surprising to see the individual chooses y over x when facing budget $\{x, y, z\}$.

Definition The choice structure $(\beta, C(.))$ satisfies the *weak axiom of revealed preference* if the following property holds: If for some $B \in \beta$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \beta$ with $x, y \in B^{'}$ and $y \in C(B')$, we must also have $x \in C(B')$.

In words, *weak axiom of revealed preference* says if a decision maker chooses x when y is available, then there does not exist a budget set B given which the decision maker chooses y but does not choose x. Let's check whether the two examples presented earlier satisfies *weak axiom of revealed preference.*

Choice rule 1 Clearly, $C_1(B)$ satisfies *weak axiom of revealed preference.*

Choice rule 2 Because $y \in C_2(\{x, y, z\})$, *weak axiom of revealed preference* implies given $\{x, y\}$, if the decision maker chooses x, he must also choose y. So, $C_2(\{x, y\}) = \{x\}$ violates *weak axiom of revealed preference.*

1.0.3 The Relationship between Preference Relations and Choice Rules

There are two fundamental questions regarding relathionship between the two different approaches of modeling decision maker's choices.

i) Does a rational preference relation $\succsim$ implies that the individual's choice satisfies the weak axiom?

ii) If an individual's choice structure $(\beta, C(.))$ satisfies the weak axiom, is there necessarily a rational preference relation $\succsim$ that is consistent with these choices?

The answer to i) is "yes" and the answer to ii) is "maybe".

Let

$$C^*(B, \succsim) = \{x \in B : x \succsim y \text{ for every } y \in B\}.$$

So elements of $C^*(B. \succsim)$ are rational decision maker's choices given budget set B. Assume $C^*(B, \succsim)$ is nonempty for all B. We say that the rational preference relation $\succsim$ generates the choice structure $C^*(B. \succsim)$.

Proposition Suppose that $\succsim$ is a ratioanl preference relation. Then the choice structure generated by $\succsim$, $(\beta, C^*(. \succsim))$, satisfies the weak axiom.

Proof: Consider two bundles $x, y \in B$. Suppose $x \in C^*(B, \succsim)$. This means $x \succsim y$. Consider another budget B' that contains both x and y. If $y \in C^*(B', \succsim)$, we have $x \in C^*(B', \succsim)$. This is because

$y \succsim z$ for every $z \in B'$ and $x \succsim y$, by transitivity $x \succsim z$ for every $z \in B'$. This is exactly the requirement of weak axiom.

Now, we give an example in which the decision maker's choice satisfies weak axiom but we cannot find a rational preference that is consistent with his choices.

Example Suppose $X = \{x, y, z\}$, $\beta = \{\{x, y\}, \{y, z\}, \{x, z\}\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, and $C(\{x, z\}) = \{z\}$. This choice rule satisfies the weak axiom but we cannot find a rational preference that is consistent with $C(.)$. Given budget $\{x, y\}$, to rationalize choice $\{x\}$, we must have $x \succ y$. Similarly, given budget $\{y, z\}$, to rationalize choice $\{y\}$, we must have $y \succ z$. Finally, given budget $\{x, z\}$, to rationalize choice $\{z\}$, we must have $z \succ x$. However, $\succsim$ violates transitivity.

Remark : When there are more budget sets in β, the weak axiom puts more restrictions on choice behavior and it is mroe likely for us to find a rational preference relation $\succsim$ consistent with the choice. For example, if $\beta = \{\{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}\}$, the choices in the example violates the weak axiom. This is because the weak axiom requires $C(\{x,y,z\}) = \{x,y,z\}$. But given $C(\{x,y,z\}) = \{x,y,z\}$, we must have $C(\{x,y\}) = \{x,y\}$. A contradiction.

1.0.4 Revealed Peference and Demand

In the following discussion we will **assume that the consumer always has only one optimal choice**; this is not essential, but it will simplify the analysis.

Definition 1 *A demand function $x(p,w)$ satisfies the* **weak axiom of revealed preferences** *if, for each pair of price-wealth (p,w) and (p',w') the following is true:*

$$\textit{If } px(p',w') \leq w \textit{ and } x(p',w') \neq x(p,w) \textit{ then } p'x(p,w) > w'.$$

The idea behind *weak axiom of revealed preferences* is very simple. It says that if the consumer chooses $x(p,w)$ when she could have chosen $x(p',w')$, the consumer must prefer $x(p,w)$ to $x(p',w')$. So, when $x(p',w')$ is chosen over $x(p,w)$, it must because $x(p,w)$ is not affordable. **Figure illustration**. The earlier discussion has shown that when a decision maker's choice satisfies *weak axiom of revealed preferences,* there does not necessarily exist a rational preference relation $\succsim$ that is consistent with the decision maker's choice.

However, we can impose even stronger restrictions on observed behavior for it to be compatible with rational preferences. In Definition 1 we only consider direct comparisons between two choices, x and x'. Transitivity requires something more. If we find out that x is preferred to x', and that x' is preferred to x'' then it must be the case that x is preferred to x''. We then have the following definition.

Definition A demand function $x(p,w)$ satisfies the **strong axiom of revealed preferences** if, for each sequence

$$\left(p^{(1)},w^{(1)}\right),\left(p^{(2)},w^{(2)}\right),\ldots,\left(p^{(k)},w^{(k)}\right)$$

with $x\left(p^{(i)},w^{(i)}\right) \neq x\left(p^{(i+1)},w^{(i+1)}\right)$ for each $i<k$ then the following is true:

If $p^{(i)}x\left(p^{(i+1)},w^{(i+1)}\right) \leq w^{(i)}$ for each $i<k$ then $p^{(k)}x\left(p^{(1)},w^{(1)}\right) > w^{(k)}$.

- To understand the definition, observe that if $p^{(i)}x\left(p^{(i+1)},w^{(i+1)}\right)\leq w^{(i)}$ then the bundle $x\left(p^{(i+1)},w^{(i+1)}\right)$ can be bought when prices are $p^{(i)}$ and wealth is $w^{(i)}$. Since $x\left(p^{(i)},w^{(i)}\right)$ is chosen instead, this must imply $x\left(p^{(i)},w^{(i)}\right)\succ x\left(p^{(i+1)},w^{(i+1)}\right)$. When this is true for each $i<k$, this implies that we can build the chain:

$$x\left(p^{(1)},w^{(1)}\right)\succ x\left(p^{(2)},w^{(2)}\right)\succ\ldots\succ x\left(p^{(k)},w^{(k)}\right).$$

  So, $x\left(p^{(1)},w^{(1)}\right)$ is indirectly revealed preferred to $x\left(p^{(k)},w^{(k)}\right)$. The definition says that if $x\left(p^{(1)},w^{(1)}\right)$ is indirectly revealed preferred to $x\left(p^{(k)},w^{(k)}\right)$, $x\left(p^{(k)},w^{(k)}\right)$ cannot be directly revealed preferred to $x\left(p^{(1)},w^{(1)}\right)$. In other words, .$x\left(p^{(1)},w^{(1)}\right)$ cannot be affordable given $(p^{(k)},w^{(k)})$.

- It can be proved, although we are not going to do it here, that **if a demand function satisfies the strong axiom of revealed preferences then it is always possible to find a rational preference relation generating that demand function**. This is an important result is economic theory, and in fact it provides an alternative way to characterize rational preferences (namely, a consumer has rational preferences only if her demand function satisfies the strong axiom of revealed preferences).