Lecture Note 8: Imperfect Competition

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1 Introduction

In this lecture we discuss market forms in which there are few firms, and the firms do not behave competitively. By this we mean that the firm does not assume that it can sell any quantity desired at the current market price. Rather, in general the firm understands that there is an inverse relation between the quantity sold and the price at which the quantity can be sold.

We will start analyzing first the case of monopoly, which is the simplest one. Next we will consider oligopolistic industries. There is an enormous literature on oligopoly, and we will only provide an introduction to the subject.

2 The One-price Monopolist

- Consider an industry in which a single producer sells an homogeneous good.
- The market is characterized by a downward sloping demand curve, $p(q)$, and the monopolist is only allowed to announce a single price at which it has to serve all demand. Since the demand is downward sloping, the unit price at which it can sell its output depends on the amount that is being sold.
- The monopolist can produce a quantity q at a cost $c(q)$, with $c'(q) > 0$ and $c''(q) > 0$.
- The monopolist's problem is to maximize its profit with respect to a price and a quantity level, subject to the constraint that the price-quantity pair lies on the demand curve, that is:

$$\max_{q \geq 0} \; p(q)\, q - c(q) \tag{1}$$

where q denotes the quantity produced, $p(\cdot)$ is the inverse demand curve and $c(\cdot)$ denotes the cost function.

- The profit function is concave in q if $p''(q) \leq 0$. To see this

$$\begin{aligned} \pi'(q) &= p(q) + p'(q)q - c'(q) \\ \pi''(q) &= 2p'(q) + p''(q)q - c''(q) \leq 0 \end{aligned}$$

  if $p''(q) \leq 0$. When does this condition hold? Consider that there is a continnum of consumers and each one wants to consume 1 unit of the good. A consumer's valuation from the good is a random variable $\theta \in [\underline{\theta}, \overline{\theta}]$ with distribution functions $F(\theta)$ and $f(\theta)$. For any quantity q, the marginal consumer's willingness to pay is $p = F^{-1}(1-q)$. This requires cdf function to be strictly increasing which is satisifed when $f(\theta)$ is positive in the full support. $p''(q) \leq 0$ implies that

$$\begin{aligned} p'(q) &= \frac{-1}{f(F^{-1}(1-q))} \\ p''(q) &= -\frac{f'(p)}{f^3(p)} \leq 0. \end{aligned}$$

  So, $p''(q) \leq 0$ if and only if $f'(p) \geq 0$, which seems to be a strong condition. Let's relax the condition a little bit. The second order condition is satisifed iff

$$2p'(q) + p''(q)q \leq c''(q). \tag{2}$$

  A sufficient condition for (2) to hold is

$$\frac{p''(q)q}{p'(q)} \geq -2. \tag{3}$$

  Use the random valuation example, (3) implies

$$\frac{f'(p)}{f(p)} \frac{[1-F(p)]}{f(p)} \geq -2.$$

  What is the meaning of this expression in statistics? What types of distribution function satisfiy this condition?

- The first-order condition for this program is:

$$\underbrace{p(q) + q\,p'(q)}_{MR(q)} \quad \leq \quad \underbrace{c'(q)}_{MC(q)} \tag{4}$$

with equality when $q > 0$.

- The optimal point involves $q > 0$ whenever $p\,(0) > c'\,(0)$ and $c\,(0) = 0$, that is whenever the price at which the very first unit can be sold is greater than the cost of producing that unit (notice that $c\,(0) = 0$ is not an entirely innocuous assumption, as it implies absence of fixed costs).

- Observe that for each q, the marginal revenue function $MR(q)$ lies below the demand function because $p(q)$. This is because when the monoplist sells one more unit, he gains the price from selling to the marginal consumer but bears a cost of lowering the price charged to the existing units. This can be interpreted as the opportunity cost, which is measued by $q\,p'(q)$.

- When the first order condition holds with equality, we can factor out $p(q)$ from $MR(q)$ and obtain:

$$MR(q) = p(q)\left[1 + \frac{q\,p'(q)}{p(q)}\right] \equiv p(q)\left[1 + \frac{1}{\varepsilon(q)}\right], \tag{5}$$

where

$$\varepsilon(q) \equiv p(q)/\left(q\,p'(q)\right)$$

is the elasticity of the demand. Hence $MR(q) \geq 0$ if and only if $\mid \varepsilon(q) \mid \geq 1$. Thus the monopolist *never produces at a point where the elasticity is smaller than one (in absolute value)*. To see the intuition. Suppose that the monoplist produces at a point where the elasticity is smaller than one. Then, by undercutting the output and raising the price, the monopolist will increase total revenue. This action will also reduce total cost. As a result, profit increases

- Also equations 4 and 5 imply

$$c'(q^m) = p(q^m)\left[1 + \frac{1}{\varepsilon(q^m)}\right]$$

or, solving for $\frac{1}{|\varepsilon(q)|}$,

$$\frac{p(q^m) - c'(q^m)}{p(q^m)} = \frac{-1}{\varepsilon(q^m)} \tag{6}$$

- The ratio $\frac{p(q^m)-c'(q^m)}{p(q^m)}$ is usually called the Lerner index, and it gives a measure of the market power existing in an industry. If the industry is competitive then the market price equals the marginal cost of each active firm, so that $p(q) = c'(q)$ and the value of the Lerner index is zero. Equation (6) tells us that market power is linked to the elasticity of demand. If the demand is perfectly elastic, i.e. $\mid \varepsilon(q) \mid = \infty$, even a monopolist cannot charge more than the competitive price. On the other hand, if the demand is rigid, i.e. $\mid \varepsilon(q) \mid$ is small, than the monopolist can charge a price much higher than the marginal cost.

- Monopoly generates the well-understood deadweight loss in social welfare, since the monopolist has an incentive to restrain production in order to maintain a higher price. Let q^c denote the competitive output. So, $p(q^c) = c'(q^c)$. When $\mid \varepsilon(q) \mid < \infty$, $p(q^m) > c'(q^m)$, which implies $q^m < q^c$. The deadweight loss is

  $$\int_{q^m}^{q^c} (p(q) - c'(q))dq > 0.$$

  When Lerner index is greater, the deadweight loss is larger.

  The negative impact on social welfare can be reduced if the monopolist is allowed to use price discrimination.

3 Is this the best selling mechanism?

- Mechanism design approach, dynamic setting...

4 The Discriminating Monopolist

- **Perfect price discrimination.** If a monopolist had **perfect information** about the consumers and resale were not allowed, then efficiency could be restored by allowing the monopolist to charge different prices to different consumers. In this case the monopolist would end up producing exactly the competitive quantity, and it would be able to appropriate the entire social surplus generated by its production activity. This kind of perfect discrimination is however difficult in many circumstances (and it may not be allowed by a public authority because of its undesirable distributional consequences). Let's revisit the unit demand example and assume that the marginal cost is constant. The monopolist will charge each consumer her valuation for the good. Hence, the monopolist's profit is

$$\max_{I(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} I(\theta)(\theta - c)f(\theta)d\theta,$$

where $I(\theta)$ is the indicator function. The solution is

$$I(\theta) = \left\{ \begin{array}{cc} 1 & \text{for } \theta \geq c \\ [0,1] & \text{for } \theta = c \\ 0 & \text{for } \theta < c \end{array} \right\}$$

- Under prefect price discrimination, the monopolist does not face the tradeoff between gain from the marginal consumer and the loss from inframarginal consumers. Hence, the marginal revenue curve is identical to the firm's demand curve. As a result, marginal revenue equals marginal cost at the competitive output level..

- **Two-part tariff** is a useful mechanism for sellers to extract consumer surplus when consumers need more than one unit of the good. For example, health club, Sam's club, football tickets, air conditioner plan (Is this a proper example?). In a two-part tariff, the monopolist charges a fixed fee T together with a price per unit p. Assume that consumers are identical. The optimal two-part tariff is

$$\begin{aligned} p &= c'(q^c) \\ T &= \int_0^{q^c} (p(q) - c'(q^c))dq. \end{aligned}$$

So, p implements the efficient consumption while T extracts the entire consumer surplus. The monopoly profit is

$$
\begin{aligned}
& T+\int_0^{q^c}(c'(q^c)-c(q))dq \\
= \; & \int_0^{q^c}(p(q)-c'(q^c))dq+\int_0^{q^c}(c'(q^c)-c(q))dq \\
= \; & \int_0^{q^c}(p(q)-c(q))dq \\
= \; & \text{Social Surplus from efficient consumption}
\end{aligned}
$$

- Sometimes however partial discrimination is possible. We will see this in a special example.

4.1 Durable goods monopoly

- Durable goods monopolist faces a changing demand overtime. Consumers only need one unit of the good and exist marktet after purchases. For example, iphone, computer, house, car...
- Suppose that the monopolist charges the static monopoly price in the first period and sells to consumers whose valuations are above the price.
- In period two, the monopolist faces a residual demand. The remaining consumers' maximum willingness to pay are lower than those who have purchased in the first period.
- Suppose that the monopolist charges the optimal static price. This price must be lower than the first period. This create a problem when consumers are strategic.
- The marginal consumer in the first period will prefer to wait and enjoy the lower price instead of making the purchase in the first period. In other words, the durable goods monopolist is competing against itself overtime.
- When the discount factor is one, the only optimal price is to charge marginal cost.

- The monopolist would be better off by committing not to reduce the price below the myopic monopoly price. In practice, durable goods monopolist use various means to make their commitment credible. For example, most favored customer clause, planned obsolescence.

5 Oligopoly: Static Theories

We have already discussed the Bertrand and the Cournot models.

- In Cournot model, firms choose quantity instead of price.

$$\pi_i\left(q_i, \sum_{j\neq i} q_j\right) = \left(1 - q_i - \sum_{j\neq i} q_j\right) q_i - cq_i$$

The first order conditions (which are also sufficient) are:

$$1 - Q - q_i - c = 0 \qquad \Longrightarrow \qquad q_i = \frac{1 - \sum_{j\neq i} q_j - c}{2}$$

Summing up the FOC, we have

$$\begin{aligned} Q &= \frac{(1-c)n}{2} - \frac{(n-1)Q}{2} \\ Q &= \frac{(1-c)n}{n+1} \end{aligned}$$

Substitute Q,

$$\begin{aligned} 2q_i &= 1 - Q + q_i - c \\ q_i &= 1 - c - \frac{(1-c)n}{n+1} \\ &= \frac{(1-c)}{n+1} \end{aligned}$$

$$
\begin{aligned}
q_1^* &= ..q_n^* = \frac{1-c}{n+1} \\
p^* &= 1 - \frac{(1-c)}{1+1/n}. \\
\pi^* &= [1 - \frac{(1-c)}{1+1/n} - c]\frac{1-c}{n+1} \\
&= \frac{(1-c)^2}{(n+1)^2}
\end{aligned}
$$

As $n \to \infty$, $p^* \to c$.

- The cournot makes more realistic predictions, but it relies on the unappealing assumption that firms select quantities, rather than prices, with the price set by some external auctioneer.

Various directions have been explored to obtain more realistic predictions out of the Bertrand model; in this note we explore two important modifications of the standard model, namely the existence of non-constant marginal costs (the existence of capacity constraints is an example of such situation) and the homogeneity of the goods sold.

- In summary, in Bertrand model, the prediction is $p_1 = p_2 = c$. This makes the unrealistic prediction that oligopolistic firms have no monopoly power.

5.1 Non-Constant Marginal Cost

- The prediction that price competition yields the competitive outcome even with two firms is not robust to changes in the form of the cost function. Consider the following example. An industry has n firms, each firm having a cost function $c_i(q_i) = \frac{c}{2}q_i^2$. The market demand function is $Q^d(p) = a - bp$.

- We first derive the competitive equilibrium in which each firm is a price taker. The supply function of each firm is

$$p = c_i'(q_i) = cq_i \to q_i = \frac{1}{c}p$$

, and the market supply would be

$$Q^s(p) = \frac{n}{c}p.$$

This gives the competitive equilibrium price

$$p^{ce} = \frac{ac}{(n+bc)}$$

; each firm produces

$$q^{ce} = \frac{1}{c}p^{ce} = \frac{a}{(n+bc)}$$

$$\begin{aligned}
\text{Profit margin} &= p^{ce} - AC \\
&= \frac{ac}{(n+bc)} - \frac{c}{2}\frac{a}{(n+bc)} \\
&= \frac{1}{2}\frac{ac}{(n+bc)}.
\end{aligned}$$

. Notice that, since there is a finite number of firms in the industry, at this price each firm makes a strictly positive profit.

- Is this a Bertrand equilibrium? More precisely, is it a Bertrand equilibrium for each firm to announce $p_i = p^{ce}$ and sell q^{ce}? By slightly undercutting the price a firm can obtain the whole market demand, that is nq^{ce}. However, it is not obvious that this would be profitable, since the extra demand has to be served at an increasing cost. Announcing the competitive price is an equilibrium if

$$p^{ce}q^{ce} - \frac{c}{2}(q^{ce})^2 \geq p^{ce}(nq^{ce}) - \frac{c}{2}(nq^{ce})^2.$$

After simplifications and rearrangements the condition is shown to be equivalent to

$$\frac{1}{2}(n-1)^2 \geq 0.$$

which is always satisfied. We conclude that in this case the competitive price is an equilibrium.

- Differently from the constant marginal cost case, however, this is not the unique equilibrium. Let $\underline{p}_n$ be the lowest price at which profits are zero when demand is served by n firms, that is $\underline{p}_n$ solves

$$\underline{p}_n\left(\frac{a-b\underline{p}_n}{n}\right)=\frac{c}{2}\left(\frac{a-b\underline{p}_n}{n}\right)^2 \qquad \Longrightarrow \qquad \underline{p}_n=\frac{ac}{2n+bc}.$$

  Furthermore, let $\overline{p}_n$ be the price equating the profit when demand is served by n firms to the profit when demand is served by one firm, that is

$$\overline{p}_n\left(\frac{a-b\overline{p}_n}{n}\right)-\frac{c}{2}\left(\frac{a-b\overline{p}_n}{n}\right)^2=\overline{p}_n\left(a-b\overline{p}_n\right)-\frac{c}{2}\left(a-b\overline{p}_n\right)^2.$$

  or

$$\overline{p}_n=\frac{ac}{2\frac{n}{n+1}+bc}$$

  We claim that for each price $p\in\left[\underline{p}_n,\overline{p}_n\right]$ it is a Bertrand equilibrium that all firms announce that price. Increasing the price cannot be profitable, since the profit in that case is zero. Undercutting is not profitable because, when $p<\overline{p}_n$ the profit obtained serving the whole market is less than what is obtained in the equilibrium. It is important to observe that the set $\left[\underline{p}_n,\overline{p}_n\right]$ still has a non-empty interior (so that there is still a multiplicity of equilibria) when n goes to infinity. In fact, as $n\to\infty$ we have $\left[\underline{p}_n,\overline{p}_n\right]\to\left[0,\frac{ac}{2+bc}\right]$.

- The competitive outcome is not an equilibrium any more when the cost function has constant marginal cost but a (sufficiently tight) capacity constraint (this is sometimes called the Bertrand-Edgeworth model). This can be seen as follows. Suppose that there are two firms, and that c is the constant marginal cost for both firms. Market demand is given by a strictly decreasing function $Q(p)$. The competitive equilibrium has $p^{ce}=c$ and the total quantity produced is $Q(c)$. Thus each firm produces $Q(c)/2$. Absent capacity constraints, we know that the unique Bertrand equilibrium is for both firms to announce c.

- Suppose that firm 1 has total capacity of production k_1 such that $Q(c)/2<k_1<Q(c)$. Is it still an equilibrium for both firms to

announce c? The answer is no. In that case the profit of firm 2 is zero. Now suppose that firm 2 announces $c+\varepsilon$. The consumers will want to buy from firm 1, but since firm 1 has a capacity constraint of k_1, not all demand can be served. Thus, a residual demand remains, and at least some of the consumers will be willing to pay at least $c+\varepsilon$. But this implies that firm 2 can obtain a strictly positive profit, therefore announcing c is not an equilibrium.

- Notice the to eliminate the equilibrium in which both firms announce c it is enough to assume that only one firm is capacity constrained. This gives incentives to the other firms to increase the price. All we need is a capacity $k_1 < Q(c)$, so that firm 1 is unable to serve all the demand at the competitive price.

- It is not obvious that an equilibrium exists, but if it exists it must involve both firms announcing a price higher than c. We have already seen that it cannot be an equilibrium for both firms to announce c. But if one firm is announcing $p_i > c$ then the best response of the other cannot be c, since by raising the price it could make positive profits.In general the equilibrium depends on the rationing rule in place. When there are capacity constraints, some consumer may be unable to buy from the preferred firm. It is therefore necessary to specify how the goods are allocated in this case.

- An important observation here is that a firm is better off when there are capacity constraints. Therefore, it is in the interest of the firms to limit the capacity. This begs the question: What level of capacity would be chosen if the firms were to play an initial stage of capacity choice and a second stage in which they set the price and serve the demand?

- A very interesting results due to Kreps and Scheinkman is that, under efficient rationing (consumers with the highest valuation are served first), the final outcome of such a game is the Cournot outcome. This in turn provides a justification for using the Cournot model. To see how this can be obtained, consider the simple model in which two firms choose first a capacity k_i at a constant marginal cost c, and then they can produce any quantity up to k_i at a zero marginal cost. Let

$P(Q)$ be the inverse demand. What is the subgame perfect equilibrium of this game?

Observe first that, since building capacity is costly, in equilibrium all capacity will be used. This implies that, if (k_1^*, k_2^*) is chosen at the first stage, the equilibrium price at the second stage will be $P(k_1 + k_2)$. Now, if firm 1 conjectures that firm 2 will choose k_2^* then the problem is

$$\max_{k_1 \geq 0} \qquad P(k_1 + k_2^*) k_1 - ck_1.$$

This is exactly the same problem which is to be solved in the Cournot model. Therefore, the equilibrium will be equivalent to the equilibrium of a Cournot model in which the firms choose the quantity produced and have a constant marginal cost of production.

5.2 Product Differentiation

In this case an equilibrium has to satisfy the property that each firm is maximizing its profit given the residual demand curve. Differently from the Bertrand case with homogeneous products, the residual demand curve is in general well-behaved.

- We now have n different products, one for each firm. Let $\mathbf{p} = (p_1, \ldots, p_n)$ be the vector of prices. We have a system of n demands $D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})$, where the demand of each good may depend on the price of the other goods. Let $c_i(q_i)$ be the cost function of firm i. A Nash equilibrium is a vector $\mathbf{p}^* = (p_1^*, \ldots, p_n^*)$ such that for each firm i the price p_i^* solves

$$\max_{p_i} p_i D_i\left(p_i, \mathbf{p}_{-i}^*\right) - c_i\left(D_i\left(p_i, \mathbf{p}_{-i}^*\right)\right).$$

  The solution obtained in general allows for positive profits for the firms, and the profits are higher when the products are less substitutable.

We first consider two spatial models, hotelling and cirular city, in which firms have local competition. i.e. a firm only complete with those close to its location.

5.2.1 Hotelling model

- A popular model of product differentiation is the 'Hotelling model'. The idea is that firms are selling the same product but at different locations. If consumers have transportation costs, they will see the products of different firms as different, as they imply different transportation costs. Furthermore, consumers will not rank the products of different firms in the same way, since they are at different locations.

- Model

  - A mass 1 of consumers distributed uniformly on a segment of length 1. Each consumer wants exactly one unit of the product, and obtains a benefit v from it.
  - There are two firms, located at points 0 and 1 respectively, with constant marginal cost c.
  - A consumer t distance away from firm i receives utility $v - td - p_i$ from purchasing the good from firm i, where t is the unit cost and d is the distance traveled to get to the firm. This means that when the two firms announce the same price, consumers with an index less than $\frac{1}{2}$ prefer to buy from firm 0 while consumers with index greater than $\frac{1}{2}$ prefer to buy from firm 1. Notice however that now when a firm increases the price it will not automatically lose all the demand, since consumers close enough will prefer to pay the higher price to avoid the transportation cost.
  - Assume for the moment that all consumers want to buy.

- Given a pair of prices (p_0, p_1), a consumer located at z prefers to buy at firm 0 if

$$v - p_0 - tz > v - p_1 - t(1 - z)$$

where z and $(1 - z)$ are the distances to be traveled in order to buy from firm 0 and firm 1 respectively. Define $\widehat{z}(p_0, p_1)$ as the value such that

$$v - p_0 - t\widehat{z} = v - p_1 - t(1 - \widehat{z}) \qquad \Longrightarrow \qquad \widehat{z}(p_0, p_1) = \frac{t + p_1 - p_0}{2t}.$$

Then the total demand for firm 0 and firm 1 at prices (p_0, p_1) is given by

$$D_0(p_0, p_1) = \widehat{z}(p_0, p_1) \qquad \Longrightarrow \qquad D_0(p_0, p_1) = \frac{t + p_1 - p_0}{2t}.$$

$$D_1(p_0, p_1) = (1 - \widehat{z}(p_0, p_1)) \qquad \Longrightarrow \qquad D_1(p_0, p_1) = \left(\frac{t - p_1 + p_0}{2t}\right).$$

The optimal price for firm 0 is obtained solving

$$\max_{p_0} \qquad \frac{t + p_1 - p_0}{2t}(p_0 - c)$$

which yields the first order condition

$$p_0 = \frac{c + t + p_1}{2}.$$

For firm 1 we have a similar condition

$$p_1 = \frac{c + t + p_0}{2}.$$

Solving the two equation we have that at the Nash equilibrium in which both firms produce and the whole market is served the prices are

$$p_0 = p_1 = c + t.$$

Notice that the prices depend positively on the transportation cost t. High values of t imply a high degree of differentiation, since consumers tend to value less a product when sold by the firm which is more far away.

- In order to make sure that the whole market is served, we have to check that all consumers prefer to buy rather than not to buy and get a utility of zero. The consumer who is worse off at this equilibrium is the one located at $\frac{1}{2}$. When buying, her utility is $v - (c + t) - t\frac{1}{2} = v - \frac{3}{2}t - c$. Therefore, a condition needed to make sure that the whole market will be served in equilibrium is that benefits are high enough with respect to transportation and production costs, that is $v > \frac{3}{2}t + c$.

5.2.2 Circular city model

- The hotelling model allows us to study price competition between two firms with differentiated product. However, it is not suitable to analyze firms' entry decision and how the price competition change in the number of firms in the market. We next introduce Salop's (1979) circular city model
- A continuum of consumers with mass one are located uniformly on a circle with a parimeter equal to 1.
- Firms are located around the same circle. Each firm is allowed to locate in only one location
- A firm needs to pay a fixed cost f to enter the market. After entry, firms pay a marginal cost c. So, a firm's profit is $(p_i - c)D_i - f_i$ from entry and 0 from staying out of the market.
- Consumers wish to buy one unit of the good with transportation cost t. Assume that consumers' valuation for the good is sufficient high and are willing to buy in equilibrium.

Timing

Stage1 potential entrants simultaneously choose whether or not to enter. Let n denote the number of entering firms. Entrants are automatically located equidistance from one another on the circle.

Stage2 Firms compete in prices given their locations.

We solve the game by backward induction. In stage 2, given the number of firms n, the distance between any two firms is $\frac{1}{n}$. We focus on symmetric equilibrium in which all firms charge identical price. A firm competes with its two neighbours. Consider firm $i's$ demand when it charges p_i and all other firms charge p. A consumer located at the distance $x \in (0, \frac{1}{n})$ from i is indifferent between purchasing from i and its cloest neigbour if

$$\begin{aligned} p_i + tx &= p + t(\frac{1}{n} - x) \\ x &= \frac{p - p_i + \frac{t}{n}}{2t}. \end{aligned}$$

So, firm $i's$ demand is

$$D_i(p_i, p) = 2x = \frac{p - p_i + \frac{t}{n}}{t}.$$

Firm $i's$ best response is

$$p_i = \arg\max(p_i - c)\left(\frac{p - p_i + \frac{t}{n}}{t}\right)$$

which yields

$$p_i = \frac{p + \frac{t}{n} + c}{2}.$$

Let $p_i = p$ in equilibrium, it follows that

$$\begin{aligned} p &= \frac{t}{n} + c. \\ \pi(p) &= \frac{t}{n^2} - f \end{aligned}$$

Consider firms' entry decision in stage one. Since entry is free, firms will enter iff it makes a positive profit. The zero profit condition pinns down the number of firms in equilibrium.

$$\begin{aligned} \pi(p) &= 0 \\ n &= \sqrt{\frac{t}{f}} \\ p &= c + \sqrt{tf} \end{aligned}$$

- For a fixed n, $p \to c$ as $t \to 0$. This is because products become more and more similar and competition drives the price down to marginal cost.

- as $f \to 0$, $p \to c$ and the equilibrium converges to the competitive equilibrium. This is because the number of entering firms tends to infinity.

5.2.3 Random Utility model (perloff and Salop (1985))

6 Oligopoly: Dynamic Theories

When the firms meet repeatedly in the market then the set of equilibria expands considerably.

- Let $(S_1, \ldots, S_n, \pi_1, \ldots, \pi_n)$ be the stage game, and assume that the game is infinitely repeated and that future payoffs are discounted according to the factor δ. This means that if firm i thinks that future payoffs are given by the sequence $\{\pi_t^i\}_{t=0}^{\infty}$ then the corresponding payoff is $\sum_{t=0}^{\infty} \delta^t \pi_t^i$.

- An history h_t is a sequence $(s(1),, \ldots, s(t-1))$ where $s(k) \in \times_{i=1}^{n} S_i$ is the strategy profile played at period k. In a repeated game a strategy has to specify the action to be taken at each period and after each possible history, that is for each period t and possible history h_t we have to specify $\sigma_t^i(h_t) \in \Sigma_i$. A subgame perfect equilibrium is a collection of strategies $\{(\sigma_t^{*1}(\cdot), \ldots, \sigma_t^{*n}(\cdot))\}_{t=1}^{\infty}$ such that $\sigma_t^{*i}(h_t)$ maximizes the expected payoff of firm i from period t onward, given the strategies of the other players.

6.1 Bertrand and Collusion

- Suppose now that the firms are playing the Bertrand game twice. Can we find an equilibrium in which collusion is sustained, that is, firms charge the monopoly price p^m and split the profit equally? Consider that one manager proposes to the other manager that they both charge the monopoly price in period 1. If no one breaks the agreement, they continue to charge the monopoly price in period one. Otherwise, they revert the price equals to marginal cost. This proposal is not credible because in the second period, both firms will optimally charge the marginal cost regardless of the history.

- Now, consider that n firms compete in price and the game is repeated infinitely. Consider the following trigger strategies.

Period 1 All firms choose p^m.

Period $t > 1$ Choose p^m if everybody in all previous periods choose p^m. Otherwise, choose $p_i = c$.

- This strategy can be described as follows. The firms start colluding, announcing the monopoly price and splitting the market. They keep colluding as long as no firm breaks the agreement in any period. When that happens, the firms start playing the static Bertrand equilibrium in each period. This is sometimes called a 'trigger strategy', since any deviation from p^m triggers reversion to the static Bertrand equilibrium.

- For such a strategy to be a subgame perfect equilibrium, it must be the case that for **each period and each history** the strategy prescribed maximizes the profit of the firm. Here the set of histories can be conveniently divided into two subsets:

  i) null history

  ii) histories in which some firm did not charge p^m in some past period,

  iii) histories in which both firms always charge p^m .

- It is clear that in ii) the proposed strategy is optimal, essentially for the same reason why it is optimal in the static game: if the other firms are going to announce c in every period than the profit can be at most zero, and therefore announcing c is optimal.

- Now, we show that trigger strategy is optimal after history iii). Suppose now that p^m has been unanimously announced in the past. Let π^m be the monopoly profit. If the strategy is followed, than the firm can get a share $\frac{1}{n}$ of the monopoly profit in each period. The total payoff from following the equilibrium strategy is therefore

$$\frac{\pi^m}{n} + \delta\frac{\pi^m}{n} + \delta^2\frac{\pi^m}{n} + \ldots = \sum_{t=0}^{\infty}\delta^t\frac{\pi^m}{n} = \frac{\pi^m}{n\,(1-\delta)}.$$

  Any deviation from this strategy causes the other firms to play $p = c$ for each following period, which means that in each period following a deviation the profit will be zero. The best deviation therefore maximizes the current profit. In this case, the firm may slightly undercut the monopoly profit, announcing $p^m - \varepsilon$, and get all the market. Since ε can be arbitrarily close to zero, the profit from the deviation can be arbitrarily close to π^m. Summing up, the best possible deviation yields a payoff

$$\pi^m + \delta \times 0 + \delta^2 \times 0 + \ldots = \pi^m.$$

For the strategy described above to be an equilibrium, it must be the case that the payoff obtained following the equilibrium strategy is higher than the payoff obtained from the deviation. The condition is therefore

$$\frac{\pi^m}{n(1-\delta)} \geq \pi^m \qquad \Longrightarrow \qquad \delta \geq 1 - \frac{1}{n}.$$

Thus, if $\delta \geq 1 - \frac{1}{n}$ then it is possible to sustain collusion in an industry with n firms.

- Lastly, the no deviation condition after history i) is exactly the same as that after history iii).

- *Sufficiency of one-shot-deviation.* In the above analysis, we only consider a one-shot-deviation. This is without loss of generality. In general, a deviator can deviate more than once. For example, it first slightly undercuts p^m and then charge a price different from c. But this deviation is less profitable than one-shot-deviation because charging c is the deviator's best response given that the rival will charge c perpetually.

Remark

i) Notice that it is harder to sustain collusion when there are more firms. To see this, write the no deviation condition as follows:

$$\underbrace{\frac{\delta}{1-\delta}\frac{\pi^m}{n}}_{\text{discounted future loss}} \geq \underbrace{\frac{(n-1)\pi^m}{n}}_{\text{current gain}}$$

When n goes up, the firm's current gain becomes larger and its future loss becomes smaller. As a result, it is more difficult to sustain collusion.

ii) The level of monopoly profit does not affect the cutoff discount factor necessary to support collusion. This is because when π^m increases, the current gain from deviation and the future loss from deviation increases by the same proportion.

- The theory of repeated games can therefore explain how self-interested firms may be able to collude. The main point is that, in a repeated game, the immediate gain obtained from undercutting has to be weighed against the loss of future profit as a consequence of breaking the collusive agreement. One interesting point is that collusion is easier to sustain the harsher is the punishment for deviating. The punishment in turn has to be *credible*, that is the strategies played in the punishment phase must form a subgame perfect equilibrium. One simple way to obtain a credible punishment is to repeat in every period the static equilibrium (this is an equilibrium in any subgame). In the Bertrand case this correspond to the worst possible punishment, since in the equilibrium of the static game each firm obtains zero.

- The main problem in the theory of repeated games is that there are many outcomes which can be justified as subgame perfect equilibrium. Consider any price $p^* \in (c, p^m)$, and call $\pi^* = \pi(p^*) > 0$ the aggregate profit of the firms when p is announced. Can we find a subgame perfect equilibrium in which the price p^* is announced in every period? The answer is yes, provided δ is sufficiently high. All we have to do is to modify the trigger strategy used to support the monopoly price putting p^* instead of p^m. The 'no deviation' condition becomes

$$\frac{\pi^*}{n(1-\delta)} \geq \pi^* \qquad \Longrightarrow \qquad \delta \geq 1 - \frac{1}{n}.$$

  which is exactly the same condition we found before.

- This implies that when $\delta \geq 1 - \frac{1}{n}$ any price in the interval $[c, p^m]$ can be supported as the outcome of a subgame perfect equilibrium. When $\delta < 1 - \frac{1}{n}$, the unique SPNE is $p_{1t} = p_{2t} = ... = p_{nt} = c$, $\forall t$.

- One puzzling aspect of the conditions for collusion found above is that they only depend on δ, not on the level of profit π^m. Intuitively, we would expect that when monopoly profits are higher it should be easier for the firms to collude. In fact, we can get this result if we abandon the extreme stationarity assumption (the demand is identical in every period) that we have imposed.

- Consider the following extension of the model.

  - In each period demand can be high (demand function $D_h(p)$) or low (demand function $D_l(p)$), each state having a probability of $\frac{1}{2}$.
  - The current state is observed by each firm, so that at any given period t the firm decides the price as a function of the history up to that point and the current state of the world.
  - For simplicity assume that there are just two firms, each with constant marginal cost c.

- Consider trigger strategy. After a deviation in state s, both firms revert to static N.E. perpetually.

- We want to find two prices (p_h, p_l) such that on the equilibrium path each firm announces the price p_s when the state is $s \in \{l, h\}$. Let π_s^m be the monopoly profit corresponding to state s. Suppose first that the firms try to coordinate on an equilibrium in which π_s^m is jointly achieved in state s. The expected discounted payoff in state s is

$$W_s = \frac{\pi_s^m}{2} + \delta\left(\frac{1}{2}\frac{\pi_h^m}{2} + \frac{1}{2}\frac{\pi_l^m}{2}\right) + \delta^2\left(\frac{1}{2}\frac{\pi_h^m}{2} + \frac{1}{2}\frac{\pi_l^m}{2}\right) + \ldots =$$

$$= \frac{\pi_s^m}{2} + \frac{\delta}{1-\delta}\left(\frac{\pi_h^m}{4} + \frac{\pi_l^m}{4}\right).$$

- The best deviation in state s is to announce a price below p_s^m and get arbitrarily close to the monopoly profit in state s, π_s^m; after the deviation, the price will be c in every period and in every state of the world, thus yielding a profit of zero. Thus, in order to support the equilibrium with maximal collusion, it must be the case that for each state s we have

$$\frac{\pi_s^m}{2} + \frac{\delta}{1-\delta}\left(\frac{\pi_h^m}{4} + \frac{\pi_l^m}{4}\right) \geq \pi_s^m \qquad \Longrightarrow \qquad \frac{\delta}{1-\delta}\left(\frac{\pi_h^m}{4} + \frac{\pi_l^m}{4}\right) \geq \frac{\pi_s^m}{2}.$$

Since $\pi_h^m > \pi_l^m$ it is sufficient to check the condition for $s = h$. Then the condition becomes

$$\delta \geq \delta^* = \frac{2\pi_h^m}{3\pi_h^m + \pi_l^m}$$

- Notice that the threshold value δ^* which makes maximal collusion possible for each $\delta \geq \delta^*$ does in fact depend on the level of profits, namely it is **increasing** in π_h^m and **decreasing** in π_l^m. The intution is best seen by comparing a deviating firm's current gain and future loss. We rewrite the no deviation condition as

$$\underbrace{\frac{\delta}{1-\delta}\left(\frac{\pi_h^m}{4} + \frac{\pi_l^m}{4}\right)}_{\text{discounted future loss}} \geq \underbrace{\frac{\pi_h^m}{2}}_{\text{current gain}}$$

  The intuition here is the following. We consider state h because this is the state in which a firm is more tempted to deviate. When π_h^m increases, the firm's current gain and discounted future loss both increase. However, the firm's future loss increases less than proportionally than its current gain because there is a 50% chance that the market demand is low in the future. So, firms are more tempted to deviate when π_h^m increases.

- So, we consider a change in π_l^m. When π_l^m increases,the firm's current gain from deviation remains unchanged but its future loss becomes larger. Hence, the higher is π_l^m, the easier it is to resist the temptation of deviating in state h.

- It is easy to see that the value δ^* is strictly greater than $\frac{1}{2}$ which is the threshold value obtained before for the stationary model. Now suppose that $\delta \in \left(\frac{1}{2}, \delta^*\right)$, thus implying that full collusion (i.e., monopoly prices in each state of the world) cannot be a subgame perfect equilibrium. The question then becomes: What is the highest expected profit that the firms can reach as the outcome of a subgame perfect equilibrium?

- Notice that if in state h the firms announce a price $p_h < p_h^m$ such that $\pi_h(p_h) = \pi_l^*$, then the problem becomes equivalent to supporting an equilibrium outcome in a stationary environment in which the firms obtain a total profit of π_l^* in every period. We already know that when $\delta \geq \frac{1}{2}$ such an equilibrium can be supported. More in general, the problem is to find the highest p_h such that

$$\frac{\pi_h(p_h)}{2} + \frac{\delta}{1-\delta}\left(\frac{\pi_h(p_h)}{4} + \frac{\pi_l^m}{4}\right) \geq \pi_h(p_h)$$

for a given $\delta \in \left(\frac{1}{2}, \delta^*\right)$.

- In this equilibrium the outcome is that the firms collude at the monopoly price in periods of low demand, while they collude less (practice a price closer to the competitive level) in periods with high demand.

6.2 Cournot and Collusion

- Consider Cournot model with two firms.
- Each firm's marginal cost is c.
- market price is $p = a - (q_1 + q_2)$
- Static Cournot equilibrium is

$$\begin{aligned}
q_1^S &= q_2^S = \frac{a-c}{3} \\
Q^S &= \frac{2(a-c)}{3}, \quad p^s = \frac{a+2c}{3} \\
\pi_1^S &= \pi_2^S = \frac{(a-c)^2}{9}
\end{aligned}$$

- Monopoly output is

$$Q^m = \arg\max(a - Q - c)Q,$$

which yields

$$\begin{aligned}
Q^m &= \frac{a-c}{2} < Q^S \\
\pi^m &= \frac{(a-c)^2}{4} > \pi_1^S + \pi_2^S = \frac{2(a-c)^2}{9}
\end{aligned}$$

- Consider the trigger strategy in which both firms choose $Q^m/2$ in the first period and continue to choose $Q^m/2$ as long as no one has deviated from this quantity in the past. If anyone deviates in the past, the game is reverted to static N.E. First, consider a history in which no one has deviated from $Q^m/2$ in the past. A firm's discounted profit from producing $Q^m/2$ is

$$\begin{aligned}\pi(q_1^c, q_2^c) &= \frac{\pi^m}{2} + \delta\frac{\pi^m}{2} + \delta^2\frac{\pi^m}{2} + ... \\ &= \frac{\pi^m}{2(1-\delta)} \\ &= \frac{(a-c)^2}{8(1-\delta)}.\end{aligned}$$

If a firm wishes to deviate, what is its most profitable deviation? Given that the other firm produces $Q^m/2$, a firm's most profitable deviation in quantity is

$$\begin{aligned}q^d &= \arg\max(a - q - Q^m/2 - c)q \\ &= \frac{a - c - Q^m/2}{2} \\ &= \frac{3(a-c)}{8}.\end{aligned} \tag{7}$$

The deviating firm's profit in the current period is

$$\begin{aligned}\pi^d &= (a - Q^m/2 - q^d - c)q^d \\ &= \frac{9(a-c)^2}{64},\end{aligned} \tag{8}$$

and its discounted profit from deviation is

$$\begin{aligned}\pi(q_1^d, q_2^c) &= \pi^d + \delta\pi_1^S + \delta^2\pi_1^S + ... \\ &= \pi^d + \frac{\delta\pi_1^S}{1-\delta}.\end{aligned}$$

The no-deviation condition is thus

$$\begin{aligned}\pi(q_1^c, q_2^c) &\geq \pi(q_1^d, q_2^c) \\ \delta &\geq \frac{9}{17}.\end{aligned}$$

- In summary, when $\delta \in [\frac{9}{17}, 1)$, $Q^m/2$ is sustainable. Can a discount factor $\delta < \frac{9}{17}$ sustain $q_i^* \in (Q^m/2, q_i^s)$ for $i = 1, 2$?

- Consider $\delta \in (0, \frac{9}{17})$. We again consider trigger strategy and let the collusive output be q^* for each firm. Let π^* denote a firm's equilibrium stage profit. So,

$$\pi^* = (a - 2q^* - c)q^*.$$

  The firm's discounted equilibrium profit is

$$\begin{aligned}\pi(q_1^*, q_2^*) &= \pi^* + \delta\pi^* + \delta^2\pi^* + ... \\ &= \frac{\pi^*}{1-\delta}.\end{aligned}$$

  Replacing $Q^m/2$ in (7) and (8) with q_i^*, it can be verified that the firm's most profitable deviation in quantity is $q^d = \dfrac{a-c-q^*}{2}$ and its most profitable deviation payoff is $\pi(q_1^d, q_2^*) = \dfrac{(a-c-q^*)^2}{4}$.

- Trigger strategy is a SPNE if

$$\begin{aligned}\pi(q_1^*, q_2^*) &\geq \pi(q_1^d, q_2^*) \\ (a - 2q^* - c)q^* &\geq \frac{(1-\delta)(a-c-q^*)^2}{4} + \frac{\delta(a-c)^2}{9}. \qquad (9)\end{aligned}$$

- (9) is a quadratic function in q^* and the smallest q^* which satisfies (9) is

$$q^*(\delta) = \frac{(9-5\delta)(a-c)}{3(9-\delta)}.$$

  So, q^* decreases in δ. In particular, $q^* \to Q^m/2$ when $\delta \to \frac{9}{17}$ and $q^* \to q^s$ when $\delta \to 0$.

- So, for $\delta \in (0, \frac{9}{17})$, firm i's collusive stage game profit is

$$\frac{(a - 2q^*(\delta) - c)q^*(\delta)}{4}$$

  which is increasing in δ and lies in the interval $(\pi_i^S, \frac{\pi^m}{2})$.

Carrot and stick

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