

Sealed bid second price private value auction
Sealed bid - each bidder places a bid
in an envelope

second price:

After bids are submitted
highest bidder gets the item for sale
pays the second highest bid

<u>example</u>	<u>value</u>	<u>bid</u>	
Alice	80	60	Alice gets the item
Bob	65	40	she pays 55
Craig	10	10	
David	50	55	her profit: $80 - 55$

Sealed Bid Auction: Second Price

Sorry!

You did not win the auction.

This is a 2nd Price auction.

You bid **\$21**.

Your Value

\$25

Winning Bid

\$93

Price Paid

\$75

Your Payoff

\$0

Sealed Bid Auction: Second Price

Sorry!

You did not win the auction.

This is a 2nd Price auction.

You bid **\$75**.

Your Value

\$89

Winning Bid

\$83

Price Paid

\$75

Your Payoff

\$0

My value 89

If I had bid 84 I would make
profit $89 - 83 = 6$

Suppose my value is 80

Suppose highest bid of opponents is	how much I should bid
40	anything above 40 (profit 40)
70	70 (profit 10)
78	78 (profit 2)
80	whatever I do (profit 0)
82	below 82 (profit 0)
90	90 (0)

↙
bidding own value will always give me
the maximal profit that I could possibly
make.

Dominant strategy

First price auction

the player who submit the highest bid
wins, pays own bid

Player #	Answer
1	Bid slightly below value, only 1 or 2 below, as betting at my value will always return zero profits
2	I bid at where below my valuation.
3	$0.9 \times \text{my value}$
4	Bid right below your value. Not knowing your opponent's value will make it difficult to choose an optimal amount to bid, but you should never bid above your own value.
5	I will bid slightly below my value of the item.
6	I won't bid more than my value because that would result in a negative profit. I should also bid below my value to determine since bidding my value will result in 0 profit.
7	My winning strategy would be to bid as close to my value on the price as possible. I still get a profit in that case as opposed to a payoff of 0
8	To bid my own value.

my value is 80

for me having \$100 and no painting
gives me some utility like \$20 + painting

how should we bid if knew opponents' highest bid

highest opponent bid	I should bid
40	41
70	71
78	79
83	we can only make profit 0 and we must bid below 83

There is no dominant strategy

Bidding above 80 is completely stupid

let's assume that the maximal opponent's bid has distribution $U(0, 100)$

my value is 80. how much should I bid?

Assume I bid b

expected profit = $\left(\frac{b}{100} \right) * (80 - b)$

prob. that I win

profit if I win

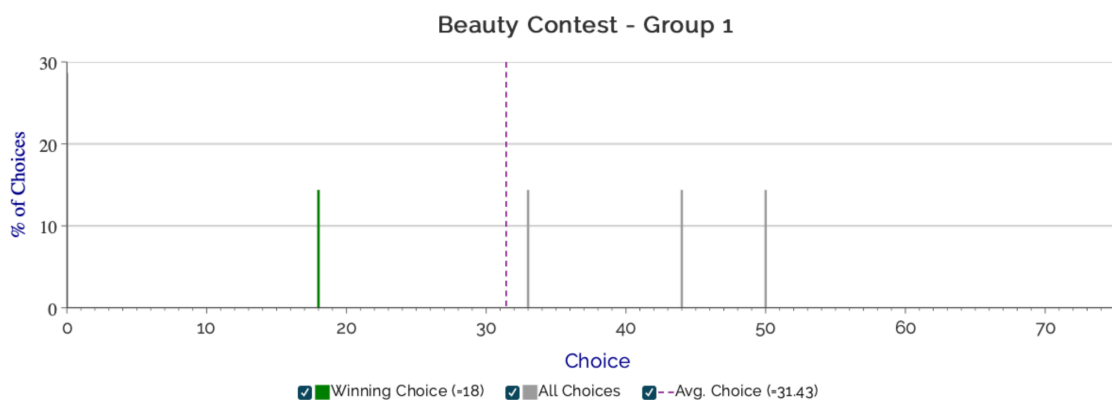
Jane's strategy; $\frac{78}{100} * (80 - 78)$

bid 60 ; $\frac{60}{100} * (80 - 60)$

optimal bid; 40

In a first price auction you should shade your value

Group #	# of Players	Avg. Choice	P	P * Avg. Choice	Winning Choice	# of Winners
1	8	31.43	0.67	20.95	18	1



Keynes' Beauty Contest

Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole ...

—
Suppose numbers picked by players are
10 30 45 90 83
average = 51.6
 $\frac{2}{3} \times \text{average} = 33$

if the average is 100 \rightarrow I should pick 67

if opponents are monkeys, pick random numbers between 0 and 100 \rightarrow we chose around 33

if my opponents think that their opponents are monkeys, then my opponents will pick 33.
Therefore I should pick 22

An n -player (normal-form) game is given by sets S_1, \dots, S_n of (pure) strategies and for every player i a payoff function

$$u_i : S_1 \times \dots \times S_n \rightarrow \mathbb{R}$$

$S_i =$ set of strategies of player i

$u_i =$ payoff function to player i

In Kake's game:

$$S_1 = S_2 = \dots$$

$$= S_n = \{0, 1, \dots, 100\}$$

$$u_i(s_1, \dots, s_n) = \begin{cases} 100 & i \in \arg\min \left| s_i - \frac{2}{3} * s_1 \right| \\ 0 & \text{otherwise} \end{cases}$$

	L	M	R
T	(1, 3)	(2, 4)	(3, 6)
B	(2, 2)	(1, 5)	(7, 0)

$$S_2 = \{L, M, R\}$$

$$S_1 = \{T, B\}$$

I player 1 chooses T \rightarrow player 1 gets 3
 player 2 chooses R \rightarrow player 2 gets 6

A we say that strategy s_i' of player i dominates strategy s_i of player i

if for every $s_{-i} \in \prod_{j \neq i} S_j$

(whatever s_{-i} the opponent choose)

$$\underbrace{u_i(s_i', s_{-i})}_{\text{my payoff if I play } s_i'} \geq \underbrace{u_i(s_i, s_{-i})}_{\text{my payoff if I play } s_i}$$

In second price auction, bidding own value dominates every other strategy. Such a strategy is called dominant strategy.

First rule of rationality:

- If you have a dominant strategy \rightarrow play
- don't play dominated strategies

In Keynes fractional beauty contest

$$S_1 = S_2 = \dots = S_n = \{0, 1, \dots, 100\}$$

you win if you are closest to $\frac{2}{3} \times \text{average}$

choosing 100 is dominated

Everyone choose a number between 0 and 100

↓
average is between 0 and 100

↓
 $\frac{2}{3} \times \text{average}$ is between 0 and 67

67 is definitely better than 100



100 is dominated by 67 because
67 is closer to $\frac{2}{3}$ of the average regardless of
... ..

what opponents are using

rational players should not pick above 67

If everyone is rational everyone picks number between 0 and 67

↓

average is between 0 and 67

$\frac{2}{3} \times$ average " " 0 and $\frac{2}{3} \times 67 = 45$



if all other players are rational we should play below 45

Sequential elimination of dominated strategies:

- eliminate dominated strategies to create a smaller game (eliminate every number above 67)
- repeat

If only one strategy remain for each player, the game is dominant solvable

This is based on the assumption that

- all players are rational
- all players know that the opponents are rational
- " " " " all players know that

- will " " " "
er engine is rational

Common knowledge of rationality