ECO 511: Midterm 2

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1. (See Chapter 10 Definition 118 in Dirk Krueger's note) The household problem is

   Definition. *A recursive competitive equilibrium is a value function*

2.

$$
\begin{aligned}
E_t(c_0 + b_1) &= E_t[b_0(1+r) + z_0] \\
E_t(c_1 + b_2) &= E_t[b_1(1+r) + z_1] \\
E_t(c_2 + b_3) &= E_t[b_2(1+r) + z_2] \\
&\vdots \\
E_t(c_t + b_{t+1}) &= E_t[b_t(1+r) + z_t]
\end{aligned}
$$

   Iterating forward on the budget constraint, combining the markovian consumption and the nPg condition

$$
\begin{aligned}
\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t c_{t+j} = (1+r)b_0 + \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t z_{t+j} \\
c_t = rb_0 + \frac{r}{1+r}\left[\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t z_{t+j}\right]
\end{aligned}
$$

   If $t \to \infty, z_{t+j} \to z$, then consumption will be finite.

3. $\underline{b} = \infty, r = \frac{1}{\beta} - 1$

   r and $\frac{1}{\beta} - 1$ represent return on investment and discount factor respectively.

   Euler equation gives the consumption is constant. Thus, this can be the equilibrium r.

4. This cannot be the equilibrium r (See Proposition 1 in LS 17.3.1 and Proposition 2 in LS 17.6)

   In summary, under certainty, the optimal consumption sequence converges to a finite limit as long as the discounted value of future income is bounded across all starting dates t. Surprisingly enough, that result is overturned when there is uncertainty.