Problem Set 12

1. Consider the RBC model. Utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t\right)$$

and constraints are

$$\begin{aligned} C_t + I_t &= A_t F\left(K_t, X_t N_t\right) \\ I_t &= K_{t+1} - (1-\delta) K_t \\ C_t &\geq 0, K_{t+1} \geq 0, 0 \leq N_t \leq 1 \end{aligned}$$

Assume the following functional forms

$$\begin{aligned} F\left(K_t, X_t N_t\right) &= K_t^{\alpha} \left(X_t N_t\right)^{1-\alpha} \\ u\left(C_t, 1 - N_t\right) &= \mu \log C_t + (1-\mu) \log\left(1 - N_t\right) \end{aligned}$$

Also assume $X_t = \gamma^t$ and that A_t follows a Markov process with two states $(A_L, A_H) = (0.9, 1.1)$ and transition matrix

$$\Pi = \begin{bmatrix} \Pr ob\left(A_H | A_H\right) & \Pr ob\left(A_L | A_H\right) \\ \Pr ob\left(A_H | A_L\right) & \Pr ob\left(A_L | A_L\right) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

   (a) Use BEA data for 1947-2017 (from www.bea.gov, you will need both NIPA data and Fixed Asset Tables) to determine the long run average values of the labor income share $\frac{wN}{Y}$, the investment to GDP ratio $\frac{I}{Y}$ and the capital to output ratio $\frac{K}{Y}$. You should submit an Excel file with the following

      i. The exact data/tables used from the BEA. (For each table you download and use, save it in a separate worksheet within the same Excel file).

      ii. A worksheet where you use the data to compute the three ratios for each year (make sure we can see the formula used in each cell, not just numbers...) and which also shows:

         A. Three graphs of these ratios over the sample period

         B. The mean values of these ratios (used for calibration below).

   (b) Also compute the average annual growth rate of real GDP per capita over the same period (you can find this variable easily at fred.stlouisfed.org).

   (c) Use all of the above information together with the observation that a third of time is devoted to working (i.e. $N^* = \frac{1}{3}$) to calibrate model parameters γ, α, δ, β and μ. This means find the values of parameters that imply the non-stochastic steady state of the transformed, stationary version of this economy delivers exactly these data observations. Note: you will need to transform the economy to stationary first to get the right steady state conditions.

(d) Write the Bellman equation for the transformed planner's problem

Optional Bonus Questions:

1. Using the calibrated parameter values, solve the Bellman equation numerically.

2. Provide plots (in separate graphs) for the value function and the three policy functions for capital, labor and consumption. Each of those plots should have two lines (one for each shock level z).