FINAL EXAM

1. A consumer maximizes utility over an infinite horizon

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $0 < \beta < 1$ and $\sigma > 0$. The consumer's income at t comprises exogenous labor income y_t as well as income from assets. The net interest rate paid by assets A_t held in period t is denoted by r_t. Each period t, the consumer chooses how much to consume c_t and how much to save A_{t+1}. By choosing $A_{t+1} < 0$ the consumer can also borrow at the same interest rate. Assume the consumer starts with no initial assets at $t = 0$.

   (a) Consider first a partial equilibrium version of this model, where the interest rate is exogenous and constant, $r_t = r$.

      i. Write the consumer's sequential budget constraint.
      ii. Show that the sequential budget constraint implies the following. State explicitly any assumption used.

      $$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t$$

      iii. Formulate the consumer's problem recursively. That is, display the Bellman equation and identify the state and control variables. State explicitly any assumption used.

   (b) From now on, let us consider a general equilibrium version of this setup. Suppose that there is a large number of these consumers in the economy, all of them identical. They trade the assets between them and the interest rate, r_t, adjusts every period to clear the asset market (so the interest rate is neither exogenous nor necessarily constant any more). Assume assets are in zero net supply for all t.

      i. Carefully define a competitive equilibrium with sequential trade in this economy.
      ii. Carefully define the corresponding competitive equilibrium with date-0 trade.
      iii. Find the equilibrium interest rate and determine under what conditions it will be constant.
      iv. Suppose consumers are given the opportunity to trade an additional asset at time t which promises to pay x_s units of the consumption good for all $s > t$. What will be the price of this asset?

2. Consider the following RBC model with exogenous labor augmenting technological growth. The production function is

$$Y_t = A_t K_t^\alpha (X_t N_t)^{1-\alpha}$$

1

where Y_t is output, K_t is capital and N_t is labor. X_t represents the deterministic component of productivity and grows at an exogenous rate $\gamma - 1 > 0$ (i.e. $X_{t+1} = \gamma X_t$, $X_0 = 1$). A_t represents a stochastic productivity shock that follows an AR(2) process

$$\ln A_t = \rho_1 \ln A_{t-1} + \rho_2 \ln A_{t-2} + \varepsilon_t, \ \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$$

The total time endowment is normalized to 1 so

$$L_t + N_t = 1$$

where L_t is leisure. The output of the economy can be used for consumption C_t or investment I_t and the capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Finally, preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \theta \ln L_t\right]$$

where $\beta \in (0, 1)$ is the discount factor and $\theta > 0$.

(a) Obtain the growth rates of all variables at the balanced growth path and transform the model to make it stationary using an appropriate rescaling of variables. State clearly the transformed constrained maximization problem of the social planner.

(b) Provide a dynamic programming representation of the planner's problem stating clearly the Bellman equation, state variables and choice variables.

(c) Consider now an equilibrium version of this economy where consumers rent capital and labor to the firm every period at rates r_t and w_t respectively. Define a competitive equilibrium with sequential trade for this economy.

(d) Use equilibrium conditions of this economy to calibrate the parameters in order to match the following long run properties of the actual economy (quarterly calibration). Note: Don't worry too much about getting specific numbers, it is enough to show you know how to solve for them

    i. The share of labor in total income is $\frac{2}{3}$.

    ii. 20% of available time is devoted to working

    iii. The average growth rate of per capita output is 0.4%

    iv. 79% of total output is used for consumption purposes

    v. The capital to output ratio is 7.35

(e) Briefly explain how you could use data to obtain a measure for the stochastic productivity shock A_t and estimates for the parameters ρ_1, ρ_2