Problem Set 7

1. (Population growth).Consider an economy where *aggregate* consumption, investment, output and capital are denoted by C_t, I_t, Y_t and K_t respectively. Let $n_t \in [0,1]$ be the fraction of time devoted to work by the average person and E_t denote the total number of the employed so that the aggregate labor input is $E_t n_t$. The economy's aggregate resource constraint, production function and capital accumulation are given below

$$\begin{aligned} C_t + I_t &= Y_t \\ Y_t &= F(K_t, E_t n_t) \\ I_t &= K_{t+1} - (1-\delta)K_t \\ & K_0 \text{ given} \end{aligned}$$

The production function satisfies the standard conditions, in particular it exhibits constant returns to scale..Assume the population M_t grows at a constant rate g and the employment to population ratio $\frac{E_t}{M_t}$ is constant. Finally, the average person consumes $\frac{C_t}{M_t}$ and their utility is given by

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{M_t} v(1-n_t)\right)^{1-\sigma} - 1}{1-\sigma}$$

where $v(.)$ is an increasing function of leisure. A planner maximizes the average individual's utility choosing all the aggregates.

   (a) Find the growth rates of all aggregate variables at the balanced growth path.

   (b) Are those growth rates consistent with the planner's optimality conditions?

   (c) Use an appropriate change of variables to transform the model to a stationary one. Provide an interpretation for the transformation you use (i.e. what do the transformed variables mean?)

   (d) Is this economy consistent with Kaldor's facts of growth? (Address each fact and show/explain why or why not)

2. (Bellman Guess-and-Verify) Take the Brock-Mirman version of the standard Cass-Koopmans model (i.e. $u(c) = \ln c$, $f(k) = k^{\alpha}$ and $\delta = 1$) and guess the value function is linear in k, i.e.

$$V^{guess}(k) = A + Bk$$

for some unknown coefficients A, B.

(a) Use the guess in the right-hand side of the Bellman equation to solve the maximization problem and find the optimal policy functions. (You will need to be careful to impose non-negativity constraints on consumption and capital choice and determine whether they bind or not. Hint: You will need to consider different cases depending on the value of B.)

(b) Use the optimal policy functions to evaluate the implied value function $V^{implied}$ (on the left hand side). Can you find A, B such that $V^{implied}(k) = V^{guess}(k)$ for all k? Explain.