Assignment 1

Haixiang Zhu

August 28, 2020

1. (a) The steay state condition:

$$\begin{aligned}&\Delta k = sy - \delta k = 0\\ \Rightarrow &0.3k^{\frac{1}{2}} = 0.1k\\ \Rightarrow &\begin{cases} k^* = 9\\ y^* = (k^*)^{\frac{1}{2}} = 3\\ i^* = \delta k^* = 0.9\\ c^* = y^* - i^* = 2.1\end{cases}\end{aligned}$$

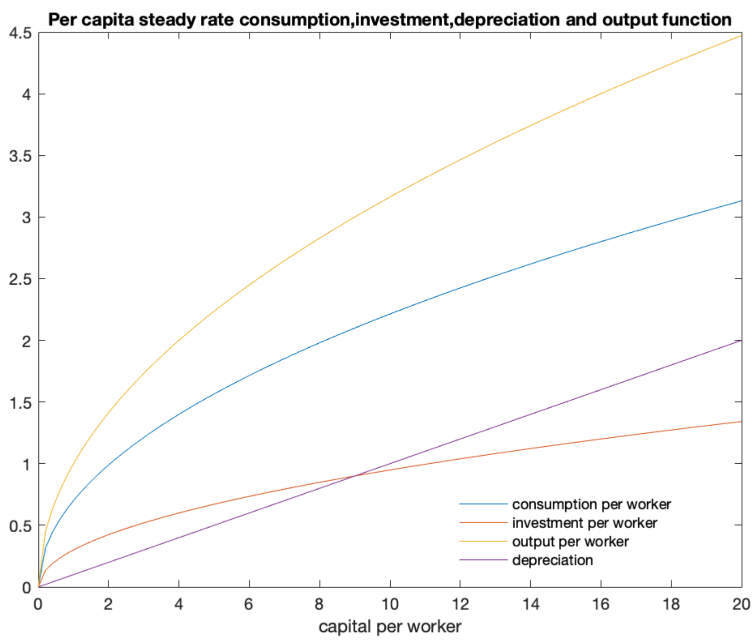

Figure 1: The steady state of Solow Model

(b) The MATLAB code is in attached file "solow_1_bc.m".

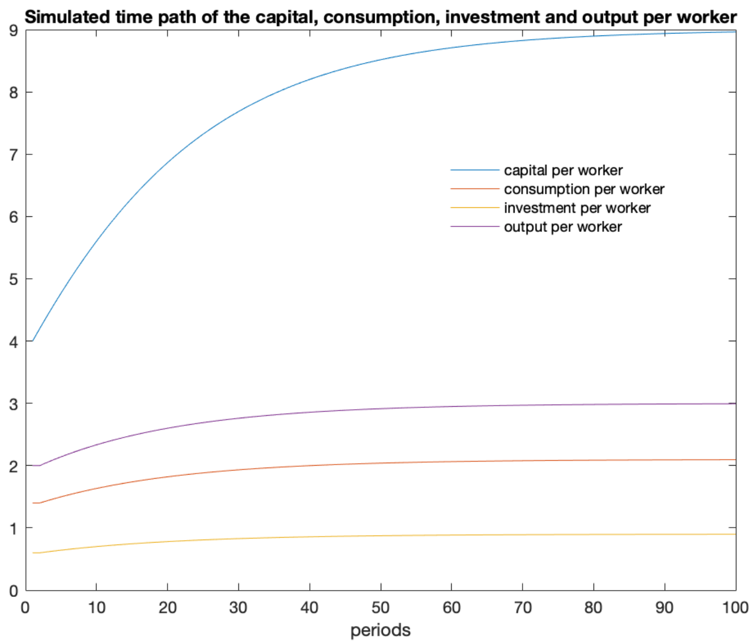

Figure 2: Simulation of Solow Model with k_0=4

(c) The MATLAB code is in attached file "solow_1_bc.m".

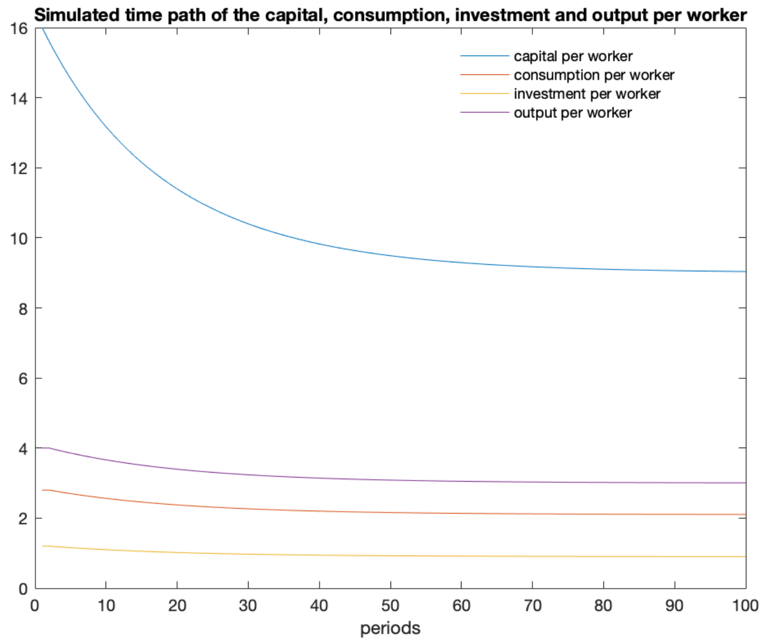

Figure 3: Simulation of Solow Model with k_0=16

(d) At the steady state of this economy:

$$\begin{cases} \dfrac{i}{y} = \dfrac{\delta k}{y} = \dfrac{sy}{y} = 0.08 \\ \dfrac{k}{y} = 2.5 \end{cases}$$

$$\Rightarrow \begin{cases} s^* = 0.08 \\ \delta^* = 0.032 \end{cases}$$

2. (a) The MATLAB code is in attached file "solow_2_abc.m".
With the number of samples N =1000,
an approximation of the golden rule consumption per worker = 2.499997
an approximation of the golden rule savings rate = 0.499499
an approximation of the golden rule capital per worker = 24.949975

(b) The MATLAB code is in attached file “solow_2_abc.m”.

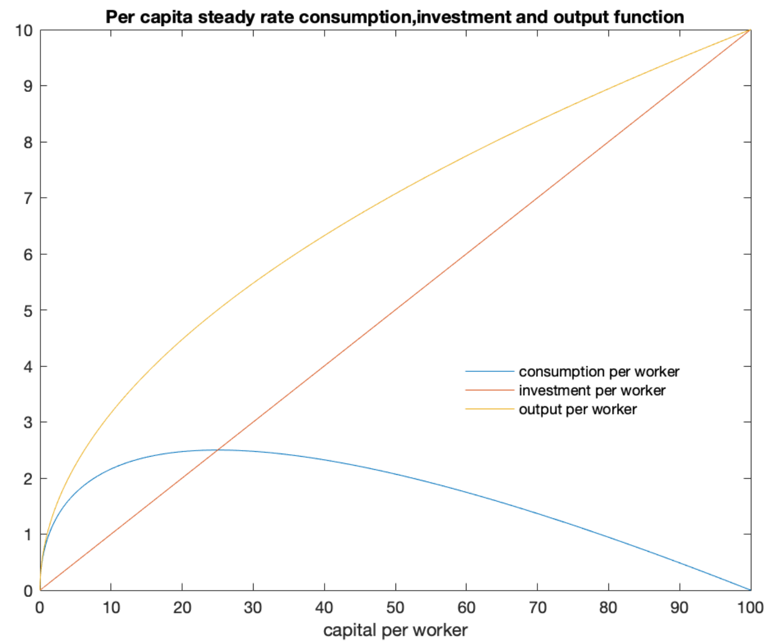

Figure 4: The steady state of Solow Model

(c) The MATLAB code is in attached file "solow_2_abc.m".

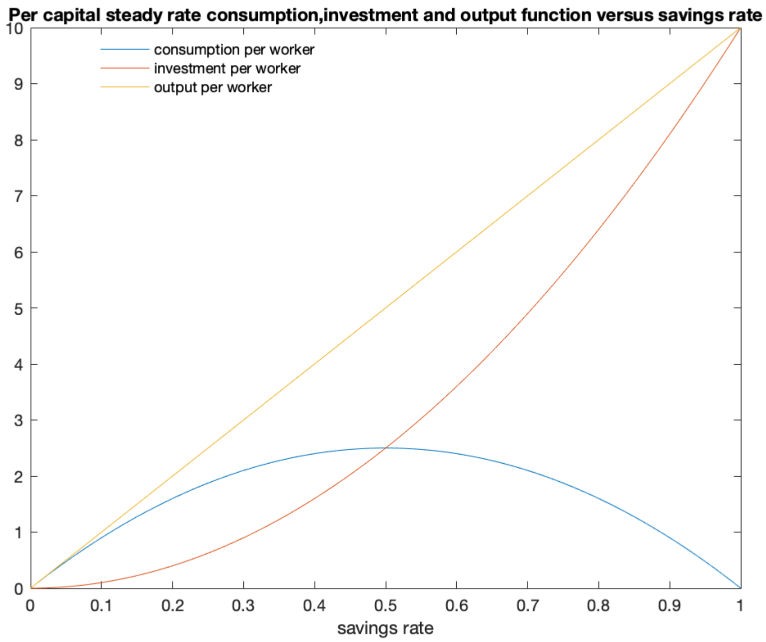

Figure 5: The steady state of Solow Model

(d) According to Golden Rule:

$$
\begin{aligned}
& y' = \delta \\
\Rightarrow & \frac{1}{2} k^{-\frac{1}{2}} = 0.1 \\
\Rightarrow & \begin{cases} k_{gold} = 25 \\ y_{gold} = 5 \\ i_{gold} = 2.5 \\ c_{gold} = 2.5 \end{cases} \\
\Rightarrow & s_{gold} = \frac{i_{gold}}{y_{gold}} = 0.5
\end{aligned}
$$

which is aligned with the numerical results in part(a).

3. According to Golden Rule:

$$
\begin{aligned}
& y' = \delta \\
\Rightarrow & \alpha k^{\alpha-1} = \delta \\
\Rightarrow & k^{1-\alpha} = \frac{\alpha}{\delta} \\
\Rightarrow & s = \frac{\delta k}{y} = \frac{\delta k}{k^{\alpha}} \\
\Rightarrow & s = \delta k^{1-\alpha} = \delta \cdot \frac{\alpha}{\delta} = \alpha
\end{aligned}
$$