CE with Uncertainty: Infinite Horizon and Asset Pricing

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We consider here the extension to infinite horizon, which is crucial in order to consider assets that are long lived such as stocks. Our ultimate objective is to describe the Lucas (1978) tree model, which forms the backbone of all modern asset pricing models, and the application of this model to the equity premium puzzle by Mehra and Prescott (1985).

As long as enough assets are available in the sequential trade equilibrium to complete the markets, the allocations will coincide with the date-0 trade equilibrium. Given this equivalence, we can use two alternative approaches to pricing assets.

1. Define a sequential trade equilibrium with all the contingent claims needed to complete markets. Also allow agents to trade any other asset that we want to price. Since additional assets are redundant, their introduction does not affect consumption allocations but it allows one to explicitly derive the new asset's equilibrium price using the corresponding first order condition.

2. Define a date-0 trade equilibrium (no assets) and obtain the equilibrium relative prices of consumption, sometimes referred to as the *pricing kernel*. Then use these relative prices to obtain the price of any asset by adding up the value of the consumption payments that the asset promises.

Keep in mind that these two approaches are often used interchangeably in what follows: we often switch back and forth between concepts that relate to the date-0 trade equilibrium, such as relative prices of consumption, and concepts that refer to the sequential trade equilibrium such as assets and asset prices.

We simplify things by focusing on a representative agent economy. This provides a significant simplification because it implies that consumption allocations are trivially given by market clearing (no trade) and we can focus on the endogenously determined asset prices. The representative agent simplification also implies that we can define a sequential trade equilibrium with no assets available and markets will still be *effectively* complete, in the sense that allocations will coincide with those in the date-0 trade equilibrium (since there is no trade anyway). As a result, the first approach described above can be followed without explicitly introducing to the model all the required contingent claims. Instead, we only need to introduce the asset that we are interested in pricing, for example a stock or a bond. This is the way the model is presented in the papers by Lucas (1978) and by Mehra and Prescott (1985): a representative agent, that lives in a sequential trade world and can only buy a stock (or a bond) as an asset.

We introduce the date-0 trade equilibrium, discuss how to price assets at time 0 and at any other time t and then move on to the Lucas (1978) tree model and the equity premium puzzle.

1 Equilibrium with date-0 trade

First, let's introduce some notation. In any period t, the endowment can take a value from a set $S_t = S$, so this set remains the same for all t. We denote a generic element of that set by $s_t \in S$. The state of nature in period t is described by the whole history of realizations $s^t = (s_0, s_1, ..., s_t) = (s^{t-1}, s_t) \in S^t$, where S^t is the t-times Cartesian product of S. Each event s^t has an associated probability denoted by $\pi(s^t)$. All variables in the model are stochastic processes, i.e. they will be indexed by a pair of date and event (t, s^t). Thus, the exogenous endowment in date-event (t, s^t) is denoted $w_t(s^t)$. Similarly, the endogenous consumption allocations and prices are denoted $c_t(s^t)$ and $p_t(s^t)$. When considering the number of goods, keep in mind that goods are differentiated according to date and event. A definition of a date-0 trade equilibrium in this setup follows.

Definition (representative agent): Given a stochastic process for endowments (i.e. values $w_t(s^t)$ and associated probabilities $\pi(s^t)$ for all (t, s^t)), a competitive equilibrium with date-0 trade consists of stochastic processes for consumption allocations $c_t^*(s^t)$ and prices $p_t^*(s^t)$ such that

1. Given prices $p_t^*(s^t)$, allocations are optimal for the household

$$\left\{\left\{c_t^*(s^t)\right\}_{s^t\in S^t}\right\}_{t=0}^{\infty} = \arg \max_{\left\{\left\{c_t(s^t)\right\}_{s^t\in S^t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t\in S^t} \beta^t \pi(s^t) u(c_t(s^t)) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\sum_{t=0}^{\infty}\sum_{s^t\in S^t} p_t^*(s^t) c_t(s^t) = \sum_{t=0}^{\infty}\sum_{s^t\in S^t} p_t^*(s^t) w_t(s^t)$$

2. All markets clear, i.e. for every date-event

$$c_t^*(s^t) = w_t(s^t) \text{ for all } \left(t, s^t\right)$$

The notation E_0 means expectation conditional on information available at time 0 (i.e. conditional on w_0 being known). Solving for the equilibrium is easy. The representative agent assumption allows us to obtain allocations trivially from market clearing and consumers' first order conditions give us prices as[1]

$$p_t^*(s^t) = \beta^t \pi(s^t) \frac{u_c(w_t(s^t))}{u_c(w_0)} \text{ for all } \left(t, s^t\right)$$

2 Asset Pricing

$p_t^*(s^t)$ is the price of the consumption good at (t, s^t) in terms of the good at $(0, s_0)$. From a sequential trade perspective, this is also the price $q_0\left(s^t\right)$ of a contingent claim bought by giving up $q_0\left(s^t\right)$ units of consumption at $t = 0$ and promising to pay one unit of the consumption good at the date-event (t, s^t) and zero in any other date-event. This 'pricing kernel' can be used as a basis to construct asset prices for assets with a more complicated payoff structure.

Example 1 *Asset bought at $t = 0$ which pays at $t = 1$ some amount d if state s_1 occurs and some amount d' if state s_1' occurs (and 0 otherwise). The corresponding histories are $s^1 = \{s_0, s_1\}$ and $s^{1'} = \{s_0, s_1'\}$*

$$Price = \beta\pi(s^1)\frac{u_c(w_1(s^1))}{u_c(w_0)}d + \beta\pi(s^{1\prime})\frac{u_c(w_1(s^{1\prime}))}{u_c(w_0)}d'$$

Example 2 *More generally, suppose an asset pays some amount $d_1(s^1)$ in every possible event in period $t = 1$ (possibly different for each event)*

$$\begin{aligned} Price &= \sum_{s^1} \beta\pi(s^1)\frac{u_c(w_1(s^1))}{u_c(w_0)}d_1(s^1) \\ &= E_0\left[\beta\frac{u_c(w_1)}{u_c(w_0)}d_1\right] \end{aligned}$$

Example 3 *An asset that pays random amounts in period 1 $(d_1(s^1))$ and in period 2 $(d_2(s^2))$. The value of the period 1 payment promised by this asset is as above. The value of the payment the asset will yield in period 2 is*

$$\sum_{s^2} \beta^2\pi(s^2)\frac{u_c(w_2(s^2))}{u_c(w_0)}d_2(s^2) = E_0\left[\beta^2\frac{u_c(w_2)}{u_c(w_0)}d_2\right]$$

[1] The normalization $p_0 = 1$ is used.

and the price of the asset will be given by adding up these two payments

$$Price = E_0 \left[\sum_{t=1}^{2} \beta^t \frac{u_c(w_t)}{u_c(w_0)} d_t \right]$$

Example 4 *A stock is an asset that promises to pay a dividend $d_t(s^t)$ in every future period, potentially different depending on the state in that period. Using similar reasoning to the previous example, the stock price is*

$$Price = E_0 \left[\sum_{t=1}^{\infty} \beta^t \frac{u_c(w_t)}{u_c(w_0)} d_t \right]$$

This last example provides a method of pricing the value of a company's stock.

2.1 Time-t pricing

All asset prices above were quoted in units of the consumption good in period 0. We can easily obtain prices in units of the consumption good at any date-event (t, s^t).[2] All that needs to be done is to re-normalize prices to be written relative to the price of the (t, s^t) consumption good. The relative price of consumption goods at any date/event (j, s^j) where $j > t$ relative to the consumption good at (t, s^t) is given by

$$\begin{aligned} \frac{p_j(s^j)}{p_t(s^t)} &= \frac{\beta^j \pi(s^j) \frac{u_c(w_j(s^j))}{u_c(w_0)}}{\beta^t \pi(s^t) \frac{u_c(w_t(s^t))}{u_c(w_0)}} \\ &= \beta^{j-t} \frac{\pi(s^j)}{\pi(s^t)} \frac{u_c(w_j(s^j))}{u_c(w_t(s^t))} \end{aligned}$$

Thus, the stock price at (t, s^t) is

$$q_t^s(s^t) = \sum_{j=t+1}^{\infty} \sum_{s^j | s^t} \beta^{j-t} \frac{\pi(s^j)}{\pi(s^t)} \frac{u_c(w_j(s^j))}{u_c(w_t(s^t))} d(s^j)$$

where note that we are adding up payments for all future events s^j that follow s^t, i.e. conditional on s^t which has already occurred and is known. Observe that now the probability term is a conditional probability, $\pi(s^j | s^t) = \frac{\pi(s^j)}{\pi(s^t)}$. We write conditional expectations as E_t , t denoting the information available when forming expectations and write

$$q_t^s(s^t) = E_t \sum_{j=t+1}^{\infty} \beta^{j-t} \frac{u_c(w_j)}{u_c(w_t(s^t))} d_j$$

[2] From a sequential trade perspective, the corresponding object of interest is the price of an asset available to be traded at any date-event.

Note that the term $u_c(w_t(s^t))$ can be taken out of the summation since it does not depend on j and also out of the expectation since it is not uncertain given information up to time t.

A risk free asset can also be priced in the same way. In particular, a one-period discount bond can be priced at (t, s^t) by looking at the value of the payments it yields. It yields a payment only in period $t+1$ and the payment is the same (equal to 1) regardless of the state of nature $s^{t+1}|s^t$

$$q_t^{rf}(s^t) = \beta E_t \left[\frac{u_c(w_{t+1})}{u_c(w_t(s^t))} \right]$$

3 The Lucas (1978) Tree Model

Consider the following setup. There is one productive unit in the economy (a tree) which produces $w_t(s^t)$ goods in every date-event. This production is exogenous. Households own shares of the tree which means they are entitled to a share of the tree's dividend $d_t(s^t)$. The goods produced are perishable and the firm (aka the tree) simply pays its output as a dividend to its shareholders, i.e. $d_t(s^t) = w_t(s^t)$. The households can trade their shares with other households at a market price $q_t^s(s^t)$.

Assuming identical agents and denoting the amount of shares purchased at date-event (t, s^t) by $z_{t+1}(s^t)$, we can write the representative agent's sequential budget constraint for any (t, s^t) as

$$c_t\left(s^t\right) + q_t^s\left(s^t\right) z_{t+1}\left(s^t\right) = d_t\left(s^t\right) z_t\left(s^{t-1}\right) + q_t^s\left(s^t\right) z_t\left(s^{t-1}\right)$$

Thus, a household buys shares z_{t+1} at a price q_t^s and in the following period these shares deliver a dividend d_{t+1} per share and can be sold at the prevailing market price q_{t+1}^s. The total supply of shares is 1. The total demand for shares is the aggregate amount agents want to buy. Assuming a continuum of identical households of measure one, share market clearing requires that for all date-events[3]

$$z_{t+1}\left(s^t\right) = 1$$

Given market clearing, equilibrium allocations are determined as $c_t(s^t) = w_t(s^t)$ for all date-events and there is no trade in stocks in any date-event. Markets are effectively complete despite the absence of all contingent claims because, even in a date-0 trade equilibrium with all the

[3] Reminder about the "continuum of measure one" trick: If we had a N agents indexed by i, aggregate demand would be $\sum_{i=1}^{N} z_{i,t+1}$. With identical agents this equals Nz_{t+1} and each agent holds an equal share $\frac{1}{N}$. With a continuum of identical agents, aggregate demand is $\int z_{i,t+1} di = z_{t+1} \int di$. All we are doing is normalizing the index of the continuum of agents to lie in the interval $[0, 1]$, so $\int_0^1 di = 1$.

possible trades allowed, identical agents would not trade. The only real explanatory power of the model is in determining equilibrium, specifically by finding what the prices need to be so that aggregate share demand is exactly equal to one. In practical terms, this means looking at the household's first order condition (the demand) with respect to shares $z_{t+1}(s^t)$ to determine prices

$$q_t^s(s^t)\, u_c(c_t(s^t)) = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)\, u_c(c_{t+1}(s^{t+1})) \left[d_{t+1}(s^{t+1}) + q_{t+1}^s(s^{t+1})\right]$$

The sum on the right hand side shows up because $z_{t+1}(s^t)$ is chosen at date event (t, s^t) and it appears in the budget constraint for every date-event $(t+1, s^{t+1})$ that follows from s^t (i.e. for every $s^{t+1} = (s^t, s_{t+1})$ given s^t). Replacing c_t with w_t and using the expectation notation

$$q_t^s(s^t) = \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t(s^t))} \left[d_{t+1} + q_{t+1}^s\right]$$

Recall that we could alternatively obtain the stock price by using the date-0 trade pricing kernel $\beta^{j-t} \frac{u_c(w_j)}{u_c(w_t(s^t))}$ and adding up the value of the dividends

$$q_t^s(s^t) = E_t \sum_{j=1}^{\infty} \beta^j \frac{u_c(w_{t+j})}{u_c(w_t(s^t))} d_{t+j}$$

The two approaches yield the same stock price. This can be shown by rolling forward the relation between the current price and the future price[4]

$$\begin{aligned}
q_t^s &= \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t)} d_{t+1} + \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t)} q_{t+1}^s \\
&= \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t)} d_{t+1} + \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t)} \left[\beta E_{t+1} \frac{u_c(w_{t+2})}{u_c(w_{t+1})} \left[d_{t+2} + q_{t+2}^s\right]\right] \\
&= \beta E_t \frac{u_c(w_{t+1})}{u_c(w_t)} d_{t+1} + \beta^2 E_t \frac{u_c(w_{t+2})}{u_c(w_t)} d_{t+2} + \beta^2 E_t \frac{u_c(w_{t+2})}{u_c(w_t)} q_{t+2}^s
\end{aligned}$$

where we have used the law of iterated expectation to replace $E_t E_{t+1} X_{t+1} = E_t X_{t+1}$. If we keep substituting forward q_{t+j}^s we can obtain an infinite forward looking sum expression as

$$q_t^s = E_t \sum_{j=1}^{\infty} \beta^j \frac{u_c(w_{t+j})}{u_c(w_t)} d_{t+j} + \lim_{T\to\infty} \beta^T E_t \frac{u_c(w_{t+T})}{u_c(w_t)} q_{t+T}^s$$

[4] I drop the argument (s^t) to reduce notation in what follows.

As long as there are no price bubbles, i.e. as long as $\lim_{T\to\infty} \beta^T E_t \frac{u_c(w_{t+T})}{u_c(w_t)} q^s_{t+T} = 0$, this gives the required result. The infinite sum expression gives the stock price in terms of exogenously given endowments (and dividends), i.e. we have solved for the equilibrium stock price. Notice that stock prices depend on expectations about the future.

We can also express the (ex post) return on the stock as

$$\frac{d_{t+1}\left(s^{t+1}\right) + q^s_{t+1}\left(s^{t+1}\right)}{q^s_t\left(s^t\right)}$$

The corresponding return on the risk free one period bond is

$$\frac{1}{q^{rf}_t\left(s^t\right)}$$

Whereas the return on the bond is known when the bond is bought (it depends on s^t only, not on s_{t+1}), the return on the stock is uncertain. The expected (ex ante) return on the stock at time t is

$$E_t \frac{d_{t+1}\left(s^{t+1}\right) + q^s_{t+1}\left(s^{t+1}\right)}{q^s_t\left(s^t\right)}$$

The difference between the expected return of the stock and the return on the risk free bond is known as the "equity premium". It is the additional return earned on average by holding equity (stocks) over and above the return earned by holding risk free assets like the bond.

4 An Application: The Equity Premium Puzzle

The equity premium is the difference in the returns of equity (stocks, a risky asset) and safe assets. Because agents are risk averse, they prefer certain payments to uncertain payments with the same expected value. Therefore, we expect to see a positive equity premium in equilibrium. Investors are compensated for the risk of holding stocks with returns that are, on average (in expectation), higher than the safe return.

The Lucas tree model provides a theory that can be used to make predictions about the size of the equity premium. This requires making choices regarding parameters, such as β and the utility function, and the exogenous variables. Mehra and Prescott (1985) show how to make reasonable choices and how to compute the equity premium implied by the theoretical model. They also look at the data to determine the size of the equity premium in practice. The theoretical prediction turns out to be much lower than the actual size of the equity premium. This is known as the "equity premium puzzle". Some more detail on this is provided below, but you should read the original paper for a full description.

4.1 The Equity Premium in the Data

To compute the equity premium in the data one needs to obtain the (expected) return on equity and the return on riskless assets, making sure these correspond to the theoretical returns in the model. The model is expressed in real terms, i.e. there is no money and no nominal variables. Therefore, we look for *real* returns in the data.

What is the actual world counterpart for the riskless assets used in the model? The bond in the model has a certain return and it pays its face value in the following period. US Treasury Bills are as close to riskless as one can get in practice. This is because the probability of the US government defaulting on its debt is very close to zero. Note, however, that even these bonds have some risk in their *real* return. Such bonds pay a certain amount in nominal terms and, since inflation is uncertain, the real return on the bond is also uncertain. However, inflation risk can be thought of as small for very short term bonds. Mehra and Prescott (1985) focus on 90 day Treasury Bills for the period 1931-1978. Since their investigation extends back to 1889, for the period before 1931 they use similar assets: Treasury Certificates for 1920-1930 and 60 to 90 day Prime commercial paper for the period before 1920. The nominal yield of these assets is contained in the data series RF. To convert those to real they need to adjust for inflation. Series PC contains data on the consumption deflator (a measure of consumption prices). Using this series, one can construct a measure of inflation as

$$\frac{PC_{t+1} - PC_t}{PC_t}$$

The real return (yield) in any year t is then (approximately) the nominal yield minus inflation.

What is the actual world counterpart for the stocks used in the model? In practice there are many different firms whose stocks are traded in the stock market whereas the model abstracts from this heterogeneity and lumps all production into one productive unit. In this one firm environment, holding equity of that firm can be thought of as holding the *market portfolio.* The return on that asset is then the return on the market portfolio. In taking the model to the data, the price of the stock will then correspond to an aggregate index of stocks. There are many different indices, but the leading indicator of US equities is the Standard and Poor's 500 index which consists of 500 stocks based on market size. This will correspond to "the stock price". Finally, dividends should also be computed for the same set of companies and both stock prices and dividends should be measured in real terms. So, Mehra and Prescott take the S&P index and divide it by the consumption deflator. This gives series P which are real stock prices and we also have series D that includes real dividends (both annual). The (net) return is then obtained by

$$\frac{P_{t+1} - P_t + D_{t+1}}{P_t}$$

where the first part captures capital gains and the second dividends. The above formula gives the realized (ex post) return for every year in the sample. Averaging over all years gives an expected return of 6.98%. The corresponding average of (real) returns on the riskless security is 0.8%. Therefore the equity premium in the data is 6.18%.[5]

4.2 Calibration - The Equity Premium in the Model

The idea is to choose exogenous parameters to make the model look "similar" to the US. Loosely speaking, this process of choosing parameters is called calibration. For the Mehra & Prescott paper, the calibration exercise is quite brief.[6] Assuming a CES utility

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \sigma > 0$$

the only exogenous parameters are β and σ and the main issue is how to choose the exogenous variable, i.e. the stochastic process for the endowments. In fact, given the small set of parameters needed, Mehra & Prescott prefer to be (almost) agnostic about the 'correct' values of β and σ and consider instead a wide range of values to see if the model can produce a realistic equity premium for any reasonable value. The discount factor β is simply restricted to be between zero and one, $0 < \beta < 1$. For the second utility parameter, we know it should be positive $\sigma > 0$. Remember that σ captures two different concepts: the degree of relative risk aversion $\left(-\frac{u_{cc}c}{u_c} = \sigma\right)$ and the intertemporal elasticity of substitution $(-\frac{d\log\frac{c_{t+1}}{c_t}}{d\log\frac{u_{c,t+1}}{u_{ct}}} = \frac{1}{\sigma})$. In determining a reasonable upper bound for σ, Mehra and Prescott use the findings from a wide variety of studies empirical, theoretical, micro and macro. Some of these findings are listed below.

1. From a finance perspective, studies by Arrow and also Friend and Blume on the portfolio decisions of individuals find that the degree of risk aversion is somewhere between 1 and 2.

[5] Note that this corresponds to the *unconditional* expectation of the equity premium.

[6] See the Real Business Cycles discussion later in the semester for a more comprehensive calibration exercise.

2. From an international perspective, Kehoe studying the shocks in the terms of trade, and how the trade balance responds to them, finds estimates close to 1.

3. From a macro perspective, Kydland and Prescott find that, in order for their model to mimic the business cycle they need something between 1 and 2.

4. From a life-cycle perspective, Tobin and Dolde find that to mimic observed patterns of life cycle savings they need 1.5.

5. Formal econometric estimates tend to vary a lot, Mehra and Prescott cite Altug's estimate that is close to 0.

Given that they want to consider as wide a range as possible, they take a very conservative guess for the upper bound (ten) and consider the prediction of the model for any value $0 < \sigma < 10$. Even though this was based on evidence in the 1980s, it's still hard to find a modern study that argues for higher values of σ.

The last component that is missing to compute equity premia in the model is the exogenous process for endowments. These are needed in order to obtain the *stochastic discount factor* $\beta^j \frac{u_{c,t+j}}{u_{ct}}$ which is another name for the pricing kernel used to value dividends at different date-events. This discount factor depends on the endogenous consumption process. Only because of the representative agent assumption is it the case that consumption equals endowments. In practice, consumption does not equal output so it makes more sense to look at the actual process of consumption to use for asset pricing. The series used for this is series C: per capita real consumption on non-durables and services obtained from the national income accounts. Given this time series, a stochastic process that matches important moments of the distribution of this series is constructed. Here is a very brief description of how this is achieved.

We need to fit a parsimonious process to the consumption data, one that allows for an analytical solution of the model.[7] Note first that consumption grows in the data (it is a non-stationary process) but the growth rate of consumption is a stationary process. Let y_t denote consumption in year t and let x_t denote the growth factor

$$y_{t+1} = x_{t+1} y_t$$

[7] In practice, not much has been gained by complicating this process along many dimensions.

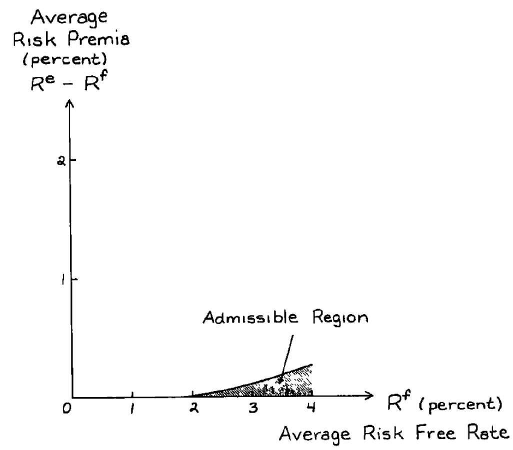

Fig. 4. Set of admissible average equity risk premia and real returns.

Figure 1: From Mehra and Prescott (1985).

The growth x_t is stationary, so we will fit a simple stationary stochastic process for the growth of consumption. Assume there are only two possible values of consumption growth, high and low. That is, consumption growth is modelled as a discrete random variable. The associated probability distribution allows for serial correlation and is specified as a Markov chain. This allows for a recursive representation of the process and, as a result, of the equilibrium. Since the exogenous process will be a state variable in the Bellman equation, this simply specifies the law of motion of the exogenous state variable to have a recursive form.

There are three dimensions along which the observed process is matched: the mean, standard deviation and persistence of consumption growth. The theoretical process replicates these data properties by appropriate choices of the parameters in the transition matrix of the Markov chain and the two values of x, x^H and x^L.

To summarize, Mehra and Prescott assume this calibrated process for the endowments and compute the equity premium in the model for different combinations of β and σ in the range specified. Each combination yields endogenously some risk free rate and some equity premium. They then plot these combinations of risk free rates and equity premia that can be justified by the model for different parameter choices (see Figure). The equity premium is found to be at most 0.3. In fact, it is 0.3 when β and σ are chosen in a way that also implies a risk free rate of 4%. If we restrict further the possible choices of β and σ to those that imply a realistic risk free rate of 0.8% (as in the data), the equity premium is essentially 0. That is the equity premium puzzle: the levels of equity premia observed cannot be explained by the Lucas tree model.

4.3 Discussion

The importance of the Mehra and Prescott (1985) paper lies in pointing out a direction for research that economists should look into. Remember that we learn by making simple assumptions and comparing to the data. If in some respect our predictions do not match the data, then we can start relaxing the assumptions one by one. If we can find one assumption that, if relaxed, would give us the observed equity premium, then we could say that we have understood (at least one of) the causes of equity premia.

The puzzle is clearly related to risk. There are a number of ways in which the Mehra and Prescott model could be wrong, in the sense of not capturing what happens in actual economies: Do we capture the amount of risk correctly? Do we capture the opportunities to share risk correctly? Do we capture the tolerance for risk in individuals' preferences correctly?

A vast literature has arisen after Mehra and Prescott's original paper, which tries to "explain" the puzzle. The approach is to change some aspect of the original model along some dimension and determine whether this allows the theory to produce the levels of risk premia that we see in the data. Broadly speaking, attempts tend to fall under one of the categories delineated by the three questions above. Several examples are mentioned in Krusell's notes. The bottom line is that it is very difficult to generate large equity premia by moving to heterogeneous agents and incomplete markets, changing the endowment process or changing preferences alone. Recent contributions that have managed to generate large premia do so by including many of those aspects all together. Even studies that can generate large premia tend to obtain counterfactual predictions along other dimensions, i.e. they generate new 'puzzles' that need to be investigated and explained.

5 References

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