PROBLEM SET 2

1.Consider a model with a representative consumer (no population growth) with preferences as:

$$\sum_{t=0}^{\infty} \beta^t \left[lnc_t + \gamma lng_t\right]$$

Here c_t denotes consumption per worker at time t and g_t is the level of public expenditure per worker at time t. Let $\beta = 0.96$ and $\gamma = 0.3$. The production function is given by $y_t = k_t^{0.3} n_t^{0.7}$. Capital depreciates at rate $\delta = 0.05$ (5%). There exists a government that imposes taxes $\left(\tau^K, \tau^L\right)$, on labor and capital income respectively. With these resources it finances a level of public expenditure, g_t, that is given.

(a) Define a competitive equilibrium.

(b) Characterize the competitive equilibrium as much as you can. Compute the steady state as a function of fiscal policies.

(c) Set up the social planner's problem, including the choice of government expenditure.

(d) Characterize the planner's solution as a dynamic system with appropriate initial and final conditions. Compute its steady state.

(e) Could you propose a specific fiscal policy such that the solution in part (b) coincides with the solution of part (d)?

2. Consider an overlapping generations exchange economy where every period $t = 1, 2, ...$ a new generation is born and lives for two periods. All generations have the same mass of people (say measure 1).

Every new generation has preferences

$$u\left(c_{1,t}, c_{2,t+1}\right) = logc_t^t + logc_{t+1}^t$$

and endowments (ω_1, ω_2). In addition there is a generation who lives only for the first period and its preferences are given by $logc_1^0$ and they have endowment ω_2. There is no fiat money.

(a) Define a competitive equilibrium for this economy.

(b) Calculate the unique competitive equilibrium. (You do not have to prove that it is unique.)

(c) Define a Pareto efficient allocation for this economy.

(d) Suppose $(\omega_1, \omega_2) = (3, 1)$. Is the competitive equilibrium Pareto efficient? Prove it.

(e) Suppose $(\omega_1, \omega_2) = (1, 3)$. Is the competitive equilibrium Pareto efficient? Prove it.

3. Consider an overlapping generation economy in which each generation has a representative consumer who lives two periods and has preferences: $log\left(c_{1,t}\right) + log\left(c_{2,t+1}\right)$

This consumer has one unit of labor when young and none when old. Output is produced using capital and labor with a Cobb-Douglas technology. Population is constant and normalized to one for each new generation born. There is an initial old consumer who has an endowment of $\bar{k}_1$ units of capital.

(a) Define a sequential markets equilibrium.

(b) Derive the demand functions for $\left(c_{1,t}, c_{2,t+1}\right)$ as a function of the wage and the interest rate.

(c) Show there exists a steady state in which the net return on capital is zero (the marginal product of capital is equal to δ, $\forall t$).

(d) Argue that no equilibrium capital path can converge to the steady state of part (c), unless the initial old consumer has an endowment of fiat money as well as of capital.

(e) Solve the problem of maximizing steady state utility subject to the feasibility condition in steady state. Compare the result to your answers to parts c,d.