Problem Set 8

Consider an infinite horizon exchange economy with two consumers, with utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

and endowments $\{w_{it}\}_{t=0}^{\infty}$ for $i = 1, 2$.

1. Carefully define a competitive equilibrium with date-0 trading for this economy.

2. Obtain all equilibrium conditions.

3. Solve for all endogenous variables. This means, find c_{it} and p_t in terms of exogenous variables and parameters.

4. Provide **intuitive explanations** for the following

   (a) How the relative price $\frac{p_s}{p_t}$ depends on the ratio of aggregate endowments $\frac{w_{1s}+w_{2s}}{w_{1t}+w_{2t}}$, on the discount factor β and on the utility parameter σ.

   (b) Individual consumption in any period t is a constant (independent of t) fraction of the aggregate endowment in that period. What does this fraction depend on, i.e. who gets a larger fraction of the aggregate endowment?

5. Compute allocations and prices for the following specific cases ($y > 0$ is a given parameter)

   (a) $w_{1t} = 2y$ for all t and $w_{2t} = y$ for all t.

   (b) Suppose now that the endowments fluctuate deterministically: consumer 1's endowment stream is $\{2y, y, 2y, y, 2y, y, ...\}$ and consumer 2's endowment stream is $\{y, 2y, y, 2y, y, 2y, ...\}$.

   (c) $w_{1t} = 2y$ for all t but consumer 2's endowment stream is $\{y, 3y, y, 3y, y, 3y, ...\}$

6. Find social welfare weights ξ_1, ξ_2 with $\xi_1+\xi_2 = 1$ that ensure the planner's problem

$$\begin{aligned} & \sum_{i=1}^{2} \xi_i \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma} \\ & s.t \\ c_{1t} + c_{2t} \quad = \quad & w_{1t} + w_{2t} \text{ for all } t \end{aligned}$$

   implements the competitive equilibrium allocations for general $\{w_{it}\}_{t=0}^{\infty}$. Explain intuitively what these weights depend on. (To build intuition you may want to use the specific cases from part 5).