Problem Set 10

1. Consider a competitive equilibrium model for a production economy, with two households indexed by $i = 1, 2$. Households maximize the following utility

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{it}, 1 - n_{it}\right)$$

In addition to the usual goods and services traded (consumption/investment good, labor and capital services), suppose agents can also trade a one-period discount bond like the one in Problem Set 8. Assume $b_{i,-1} = 0$.

   (a) First, assume all households are identical ($k_{i0} = k_0$) so we can focus on a representative household and drop the i subscripts everywhere.

      i. Carefully define a competitive equilibrium with sequential trading for this economy.

      ii. Provide an argument for why the non-negativity of capital $k_{t+1} \geq 0$ will not bind in equilibrium.

      iii. In equilibrium, how is the bond price q_t^b related to the return on capital? How is it related to the prices p_t of consumption goods in the date-0 trade equilibrium discussed in class?

      iv. What is the effect of introducing the asset on the equilibrium allocations and prices?

   (b) From now on assume two households, $i = 1, 2$, with identical utility, but potentially different initial capital stock k_{i0}.

      i. Carefully define a competitive equilibrium with sequential trading for this economy.

      ii. Show that, in equilibrium, the capital non-negativity constraint will not bind for any of the two households. This means, it cannot bind for both at the same time but also it cannot bind for one of them only. (Hint: Use no-arbitrage to argue the second). Explain intuitively why this is the case - would this be true if there were no financial asset available?

      iii. Carefully define a competitive equilibrium with date-0 trading for this economy.

      iv. Show that the date-0 trade budget implies the following equilibrium restriction on allocations

$$\sum_{t=0}^{\infty} \beta^t \frac{u_{cit}}{u_{ci0}} c_{it} = \left(1 - \delta + r_0\right) k_{i0} + \sum_{t=0}^{\infty} \beta^t \frac{u_{cit}}{u_{ci0}} w_t n_{it} \qquad (1)$$

      v. Prove that any date-0 trade equilibrium allocation $\left\{c_t^*, n_t^*, k_{t+1}^*\right\}_{t=0}^{\infty}$ can be implemented as a sequential trade equilibrium allocation too. Where would your proof fail if there were no financial asset available?