PROBLEM SET 1

1. Download US nominal GDP, Real GDP, Gross Capital Formation (Investment), Consumption of Fixed Capital (Depreciation), Working Age Population (16-64) and Total Hours Worked for as long a series as you can find. Express output and investment in constant dollars of a base year. Do feel free to perform this exercise for some different economy (provided there is similar data available).

(a) Use the series of REAL investment to construct a series of the capital stock, use:

$$I_t = K_{t+1} - (1-\delta) K_t$$

$$K_0 = \bar{K}_0$$

Choose your depreciation rate δ and the initial capital $\bar{K}_0$ such that $\frac{K_0}{Y_0} = \frac{1}{10}\sum_{t=1}^{10} \frac{K_t}{Y_t}$ and $\delta K_0/Y_0$ equals the average ratio of depreciation to GDP in the data for the first ten periods.

(b) Repeat (a) where the first condition is instead $K_1/K_0 = (K_{10}/K_0)^{1/10}$. Plot and compare both series of capital.

(c) Compute the capital income share. Explain your reasoning for how you compute that number.

(d) Use the data that you constructed to perform a growth accounting exercise and decompose output per working age person following the expression:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_t}{N_t}\right)$$

Plot and discuss your findings.

2. Consider the standard neoclassical growth model. Preferences are given by $U(c) = \sum_{t=0}^{\infty} \beta^t u(c_t)$, where the utility function takes the form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.

Technology is given by $Y_t = K_t^{\alpha} (E_t L_t)^{1-\alpha}$, $E_{t+1} = (1+x)E_t$, $L_{t+1} = (1+n)L_t$.

(a) Write down the social planner's problem.

(b) Express all the allocations in terms of efficiency units of labor and rewrite the social planner's problem.

(c) Characterize the solution as a sequence of equations (with appropriate initial and final conditions).

(d) Solve in the computer (feel free to choose your computing language) for this sequence of equations and unknowns, assuming $\beta = 0.97, \sigma = 3, \alpha = 0.3, x = 0.019, n = 0.012, \delta = 0.045$, and an initial capital stock 50% below its balanced growth path value. Plot the evolution of consumption, investment and output.

(e) Write down the same problem in recursive form.

(f) Establish a grid (100 points or so should be enough) going from 30% of the steady state capital stock to 150% of the steady state capital stock and write down a code to compute the value function and policy function associated to such a problem. [The easiest languages are Python, Matlab or similar, but feel free to choose any other]

(g) Plot the evolution of the main aggregates (consumption, investment, and output) starting with an initial capital stock 50% below its balanced growth path value, and check that your answer coincides with your solution to part (d).

3. Consider an economy with a representative infinitely lived consumer who has the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[\gamma log c_t + (1-\gamma) log\left(n_t \bar{h} - \ell_t\right)\right]$$

The set of feasible consumption and production plans satisfy:

$$c_t + k_{t+1} - (1-\delta) k_t \leq \lambda^t A k_t^{\alpha} \ell_t^{1-\alpha}$$

with k_0 given.

Here n_t is an exogenously given sequence of population sizes, and $\bar{h}$ is the endowment of hours available for work or leisure in one period.

(a) Define a sequential markets equilibrium.

(b) Assume that in the equilibrium in part (a), population and hours worked are constant. Suppose that both consumption and the capital stock grow at, possibly different, constant rates in equilibrium. Prove that they have to grow at the same rate. Derive the relationship between this rate of growth and λ.

(c) Use your answer to part (b) to define a balanced growth path for this economy.

(d) Suppose now that there is an economy with roughly constant population. In the year 2010 its national income and product accounts were:

Product		Income	
Consumption	80	Labor Income	70
Investment	20	Capital Income	20
		Depreciation	10
GDP	100	GDP	100

Between 2010 and 2020 all of these numbers grew at roughly ttwo percent per year in real terms. Hours worked per working age person were roughly constant at 25 hours per week. Either calibrate the model economy to match this set of balanced growth observations or carefully specify a procedure to do so.