Competitive Equilibrium with Uncertainty: Two Period example

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Until this point, we have abstracted from uncertainty. This section introduces uncertainty in a competitive equilibrium framework.

The equilibria we considered were defined in a deterministic setup where endowments were known in advance. The equilibrium with sequential trade involved an asset, a risk free bond, and we have seen how the price of the asset is determined in equilibrium. The asset allowed for the exchange of goods *across time* and one asset was enough to complete the market, i.e. to allow for the exchange of all consumption goods. Introducing additional assets was redundant in the sense that equilibrium allocations remain the same whether the additional asset is added or not.

With uncertainty, the second fundamental role of financial assets can be analyzed, namely insurance. Insurance refers to the ability to exchange goods *across states of nature*. For example, car insurance allows one to exchange goods in case of no accident with goods in the case of an accident: you pay a premium when there is no accident in exchange for receiving a payment in case of an accident. Most, if not all, financial assets have some degree of uncertainty in their returns. When an investor buys stocks in a company, she doesn't know the return that she will get out of that asset since both the dividends and (especially) the capital gains are uncertain. We model this uncertainty by introducing uncertain endowments with a well-defined probability distribution.

Incorporating uncertainty in the equilibrium models we have seen can be thought of as a direct extension of those environments once we realize that goods will now be differentiated depending on the date *and the state of nature* in which they are available. Once this concept is understood, the rest follows as a natural extension. So, the main idea is as follows: just as we considered goods available at different points in time as differentiated goods, we can consider goods available at different states of nature as differentiated goods. If aggregate income turns out to be low in the following year, a unit of the consumption good will have different value to the consumer (and hence market price) compared to the same unit if aggregate income turns out to be high. Overall, goods will now be differentiated along three dimensions: type (e.g. consumption good vs labor good), time (e.g. consumption good in period 1 vs consumption good in period 10) and states of nature (e.g. consumption good in period 10 if the income is low vs consumption good in period 10 if income is high).

The extension to uncertainty and the results on how to price different financial assets can be illustrated perfectly in a simple two period example. This is done in section 1. Section 2 moves to the infinite horizon case. The case of infinite horizon does not add many new insights, but it does require the use of heavier notation and it will be the natural setup in which to consider long-lived financial assets such as stocks

1 A two-period example

Suppose there are two periods, $t = 0$ and $t = 1$, and two (types of) consumers $i = 1, 2$. Each consumer receives a deterministic endowment w_{i0} in period $t = 0$. In contrast, endowments for period $t = 1$ are uncertain. To keep things simple, we assume the two agents' second period endowments follow a discrete probability distribution with two states of nature

$$\begin{aligned} w_{i1} &= w_i^H \text{ with probability } \pi^H \\ &= w_i^L \text{ with probability } \pi^L \end{aligned}$$

We allow the possible values of the endowment for each agent to be different, but maintain the same probabilities. Each agent's endowment w_{i1} in period 1 is a random variable and we make the extra simplifying assumption that w_{11} and w_{21} are independent random variables. We also assume that, even though each consumer does not know their endowment for period 1 ex ante, they know the probability distribution endowments.

There are four possible states of nature in period 1 given by all possible pairs of endowments for the two agents. We introduce the following notation: Let S_1 be the set containing all possible period 1 states of nature, so $S_1 = \left\{ \left\{ w_1^H, w_2^H \right\}, \left\{ w_1^L, w_2^H \right\}, \left\{ w_1^H, w_2^L \right\}, \left\{ w_1^L, w_2^L \right\} \right\}$. A generic element of that set will be denoted by s_1. Given independence of the endowment distributions, the associated probability $\pi(s_1)$ is simply the product of the two probabilities. So, $\pi\left(\left\{ w_1^H, w_2^H \right\}\right) = \left(\pi^H\right)^2$, $\pi\left(\left\{ w_1^H, w_2^L \right\}\right) = \pi\left(\left\{ w_1^L, w_2^H \right\}\right) = \pi^H \pi^L$ and $\pi\left(\left\{ w_1^L, w_2^L \right\}\right) = \left(\pi^L\right)^2$. All variables in period 1 are indexed by s_1.[1] In particular, $w_i(s_1)$ is the endowment of agent i in case state $s_1 \in S_1$ occurs and $c_i(s_1)$ is the corresponding consumption choice in that date/event. Note that consumption, and all other endogenous variables, is now a random variable itself. There are four consumption goods in period 1 differentiated by the event in which they are available. In addition, there is one consumption good in period 0. Correspondingly, there will be five market clearing conditions since markets have to clear in any state of nature

$$
\begin{aligned}
c_{10} + c_{20} &= w_{10} + w_{20} \\
c_{11}(s_1) + c_{21}(s_1) &= w_{11}(s_1) + w_{21}(s_1) \text{ for all } s_1 \in S_1
\end{aligned}
$$

We assume that agents maximize their expected utility

$$E_0(u(c_{i0}) + \beta u(c_{i1}(s_1)))$$

where expectations are taken over the conditional probability distribution of consumption, conditional on information available at $t = 0$.[2] In this example, the expected utility of agent i can be written as

$$u(c_{i0}) + \beta \sum_{s_1 \in S_1} \pi(s_1) u(c_{i1}(s_1))$$

As always, we will consider different equilibrium concepts depending on what can be traded and when. We start with the date-0 trade concept and consider the sequential trade equilibrium concept afterwards.

1.1 Date-0 trade equilibrium

In a date-0 trade equilibrium budget constraints are given by

$$p_0 c_{i0} + \sum_{s_1 \in S_1} p_1(s_1) c_{i1}(s_1) = p_0 w_{i0} + \sum_{s_1 \in S_1} p_1(s_1) w_{i1}(s_1)$$

where p_0 is the price of the consumption good at time 0 and $p_1(s_1)$ denotes the price of the consumption good associated with the date/event $(1, s_1)$. Recall that in the date-0 markets *all* goods can be traded at some price. There is nothing probabilistic about whether trade (and payment for the trade) takes place. Trade happens and the payments

[1] We use s_1 as an argument of each variable even though here it takes values out of a discrete set. This helps in moving to the case of distributions with a continuous support.

[2] Hence the notation E_0. More generally, we denote by E_t an expectation conditional on information at t.

occur before the resolution of uncertainty and this is reflected in the budget constraint. As always, one could think of this as trade in contracts that promise delivery of consumption goods in certain date/events. The only thing that is uncertain is whether these contracts will actually be delivering goods or not. The contracts associated with the event that actually occurs will deliver goods and the corresponding consumption of those goods will happen. The rest of the contracts will not deliver anything and become void. Thus, trade is certain but consumption is uncertain.

Notice that, just like we saw in the case of no uncertainty, agents can exchange goods across time periods. With uncertainty, we also allow agents to exchange goods across different states of nature, which opens the possibility for insuring against bad times. We are now in a position to define an equilibrium with date 0 trade.

DEFINITION: Given endowment processes$\{w_{i0}\}_{i=1}^{2}$, $\left\{\{w_{i1}(s_1)\}_{s_1\in S_1}\right\}_{i=1}^{2}$ and their corresponding probability distributions, an equilibrium with date-0 trade is a set of prices for each date/event p_0^*, $\{p_1^*(s_1)\}_{s_1\in S_1}$ and quantities for each date/event $\{c_{i0}^*\}_{i=1}^{2}$, $\left\{\{c_{i1}^*(s_1)\}_{s_1\in S_1}\right\}_{i=1}^{2}$ such that

1. Given prices, quantities are optimal for agent i. This means that for $i = 1, 2$

$$\left\{c_{i0}^*, \{c_{i1}^*(s_1)\}_{s_1\in S_1}\right\} = \arg \max_{c_{i0},\{c_i(s_1)\}_{s_1\in S_1}} u(c_{i0})+\beta \sum_{s_1\in S_1} \pi(s_1)u(c_{i1}(s_1))$$

st.

$$p_0^* c_{i0} + \sum_{s_1\in S_1} p_1^*(s_1)c_{i1}(s_1) = p_0^* w_{i0} + \sum_{s_1\in S_1} p_1^*(s_1)w_{i1}(s_1)$$

2. All markets clear

$$\begin{aligned} c_{10}^* + c_{20}^* &= w_{10} + w_{20} \\ c_{11}^*(s_1) + c_{21}^*(s_1) &= w_{11}(s_1) + w_{21}(s_1) \text{ for all } s_1 \in S_1 \end{aligned}$$

The mechanics of finding equilibria under uncertainty are exactly the same as before. The only difference is that now we have goods indexed by date and event and there are some (given) probabilities that will appear in the first order conditions. In fact, a characterization of the equilibrium shows that the main insights obtained in the deterministic setup carry over to this setup. The household's first order conditions give

$$\begin{aligned} u'(c_{i0}) &= \lambda_i p_0 \\ \beta\pi(s_1)u'\left(c_{i1}(s_1)\right) &= \lambda_i p_1(s_1) \text{ for all } s_1 \in S_1 \end{aligned}$$

Normalizing the price of the consumption good at $t = 0$ to be equal to one ($p_0 = 1$), we can write the price of the consumption good in date/event $(1, s_1)$ as

$$p_1(s_1) = \frac{\beta\pi(s_1)u'\left(c_{i1}(s_1)\right)}{u'(c_{i0})}$$

The price equals the marginal rate of substitution between the $(1, s_1)$-good and the numeraire good.

Following steps parallel to the deterministic case, it is straightforward to show that, with CES utility, the price will depend on the relative scarcity of the good in that date/event. Additionally, the price depends on the discount factor in the usual fashion. The new aspect here is the probability $\pi(s_1)$ with which event s_1 occurs. If it is a very probable event, the price of a consumption good in that date/event (from the point of view of $t = 0$) is high. This is intuitive: if it is very unlikely this will occur, the price is low since there is not much value to a good that will probably not be available. For example, think of the price of medical insurance. If you have high probability of becoming ill (old age, family history of illness, unhealthy lifestyle etc.) then the price of an asset that pays if you get sick (i.e. the medical insurance premium) will be high and vice versa.[3]

1.2 Sequential Trade

The date-0 equilibrium provides a way to "price" different consumption goods (differentiated by date/event), but it is not the natural setup to think about assets and asset pricing. In fact, there are no assets in the date-0 equilibrium, assets only appear explicitly in a sequential trade equilibrium. However, the consumption good prices from the date-0 trade equilibrium can be used to price assets in a hypothetical sequential trade economy, as long as the sequential trade economy yields the same equilibrium allocations. This idea is made precise in this example by defining carefully the sequential trade equilibrium.

Recall that in a date-0 equilibrium markets are complete, in the sense that everything can be traded at some price. Individuals can trade today's consumption good for tomorrow's. They can also trade tomorrow's consumption good in state s_1 with tomorrow's consumption good in state $s_1' \neq s_1$. The first is an example of trade across time and the second is an example of trade across states of nature. In a sequential trade economy, consumption goods cannot be directly traded because

[3]Provided, of course, that these individual characteristics are observable and the insurance provider is allowed to price discriminate on the basis of these characteristics.

markets open sequentially and only one consumption good is available to bring to the markets. The idea will be to use financial assets to achieve the same exchange indirectly.

In the absence of any assets, the sequential trade economy's budget constraint would be simply

$$\begin{aligned} c_{i0} &= w_{i0} \\ c_{i1}(s_1) &= w_{i1}(s_1) \text{ for all } s_1 \in S_1 \end{aligned}$$

i.e. there will (can) be no trade.[4] We say that markets are *incomplete*, in the sense that many exchanges of goods are not possible whether explicitly or implicitly. Contrast this with the case of the date-0 trade equilibrium where *all* exchanges are allowed, i.e. markets are *complete*. Because the possible exchanges are different in the two setups, in general the two equilibria will feature different allocations. An exception to this would occur if the date-0 trade equilibrium happened to involve no trade (as would be the case with identical households). In that case the sequential trade equilibrium would actually deliver the same allocations as the date-0 trade one. In these special cases we will say that markets are *effectively complete* in the sequential trade equilibrium.

To allow some exchange of goods, at least indirectly, we can use financial assets. Suppose we introduce a risk free bond, like we did in the case of no uncertainty. A consumer can buy b_{i1} units of the bond in period 0 at a price q^b and the bond promises to pay one unit of the consumption good in period 1. The budget constraints are then

$$\begin{aligned} c_{i0} + q^b b_{i1} &= w_{i0} \\ c_{i1}(s_1) &= w_{i1}(s_1) + b_{i1} \quad \text{for all } s_1 \in S_1 \end{aligned}$$

Just like in the case with no uncertainty, this asset allows households to effectively trade consumption goods across time by using the bond. In the absence of uncertainty, this was enough to allow the same possible trades of goods as in the date-0 trade equilibrium. An agent could use the date-0 endowment to buy an asset that would deliver date-1 goods.

[4] This is because we have assumed only one *type* of good per period. One could define an equilibrium where there are both apples and oranges in every date/event and, in that case you could still trade apples with oranges within any given date/event, but not across date/events. That is, the budgets would be

$$\begin{aligned} c_{i0}^A + p_0^O c_{i0}^O &= w_{i0}^A + p_0^O w_{i0}^O \\ c_{i1}^A(s_1) + p_1^O(s_1) c_{i1}^O(s_1) &= w_{i1}^A(s_1) + p_1^O(s_1) w_{i1}^O(s_1) \text{ for all } s_1 \in S_1 \end{aligned}$$

where p_0^O, $p_1^O(s_1)$ denote the relative price of oranges in terms of apples in each date/event.

In fact, the agent could use those goods at $t = 1$ to then buy an asset that delivers date-2 goods and so on. By a clever trading strategy, an agent could exchange some of his $t = 0$ goods for goods in any other period t. Put differently, this asset allowed for the exchange of goods across time just like in the date-0 trade equilibrium.

With uncertainty, this asset will not be enough to complete the markets because the date-0 trade equilibrium allows for trade not only across time but also across states of nature. The risk free bond we discussed does not allow for trade across states of nature because it pays the same (one unit of consumption) in all future states. This is indicated by the fact that the same b_{i1} appears in the second period budget for all possible s_1. So the bond can facilitate exchange of a good at time $t = 0$ with a good at $t = 1$ but it cannot help to trade a good at date/event $(1, s_1)$ with a good at date/event $(1, s_1')$, i.e. across states of nature. Once again, except in very special cases, the allocation will not be the same as the date-0 trade because markets are not complete. To have complete markets, one needs to add enough assets so that all trades are possible. The simplest way to do this is by using assets known as contingent claims.

A contingent claim is an asset that is bought in the current period at a market price (quoted in units of the current consumption good) and pays one unit of the consumption good in the next period *only if* one particular state of nature happens. If any other state of nature occurs, it pays nothing. We will need to add as many of those contingent claims as the possible next period states of nature to enable all possible trades. We use $a_1(s_1)$ to denote the number of contingent claims bought in period 0 and paying out in period 1 only if s_1 occurs. One unit is bought at price $q(s_1)$ and pays 1 unit of the consumption good in period 1 if state $s_1 \in S_1$ occurs and 0 otherwise (i.e. if state $s_1' \in S_1$ with $s_1' \neq s_1$ occurs). Clearly, there are as many such assets as possible states of nature. In the current example, we would need to add four different contingent claims. The difference with a date-0 trade equilibrium is again the households' budget constraint

$$c_{i0} + \sum_{s_1 \in S_1} q(s_1) a_{i1}(s_1) = w_{i0}$$

$$c_{i1}(s_1) = w_{i1}(s_1) + a_{i1}(s_1) \quad \text{for all } s_1 \in S_1$$

and the fact that asset markets must also clear (in any date/event)

$$a_{11}(s_1) + a_{21}(s_1) = 0 \text{ for all } s_1 \in S_1$$

The sequential trade equilibrium with all those contingent claims can be defined as follows.

DEFINITION: Given endowment processes$\{w_{i0}\}_{i=1}^{2}$, $\left\{\{w_{i1}(s_1)\}_{s_1\in S_1}\right\}_{i=1}^{2}$ and their corresponding probability distributions, an equilibrium with sequential trade is a set of contingent claim prices $\{q^*(s_1)\}_{s_1\in S_1}$ and consumption and asset allocations $\{c_{i0}^*\}_{i=1}^{2}$, $\left\{\{c_{i1}^*(s_1)\}_{s_1\in S_1}, \{a_{i1}^*(s_1)\}_{s_1\in S_1}\right\}_{i=1}^{2}$ such that

1. Given prices, allocations are optimal for agent i. This means that for $i = 1, 2$

$$\begin{aligned}&\left\{c_{i0}^*, \{c_{i1}^*(s_1)\}_{s_1\in S_1}, \{a_{i1}^*(s_1)\}_{s_1\in S_1}\right\}\\ &= \arg \max_{c_{i0}, \{c_i(s_1)\}_{s_1\in S_1}, \{a_{i1}(s_1)\}_{s_1\in S_1}} u(c_{i0}) + \beta \sum_{s_1\in S_1} \pi(s_1) u(c_{i1}(s_1))\end{aligned}$$

   st.

$$c_{i0} + \sum_{s_1\in S_1} q^*(s_1) a_{i1}(s_1) = w_{i0}$$
$$c_{i1}(s_1) = w_{i1}(s_1) + a_{i1}(s_1) \quad \text{for all } s_1 \in S_1$$

2. All markets clear, specifically goods markets

$$\begin{aligned} c_{10}^* + c_{20}^* &= w_{10} + w_{20} \\ c_{11}^*(s_1) + c_{21}^*(s_1) &= w_{11}(s_1) + w_{21}(s_1) \text{ for all } s_1 \in S_1\end{aligned}$$

   and asset markets

$$a_{11}^*(s_1) + a_{21}^*(s_1) = 0 \text{ for all } s_1 \in S_1$$

Note that the agent is choosing a whole portfolio of assets .They will buy 4 different assets, potentially different quantities of each asset denoted by $a_{i1}(s_1)$ and at different prices per unit $q(s_1)$. The agent pays $\sum_{s_1\in S_1} q^*(s_1)a_{i1}(s_1)$ to buy these assets and this can be positive or negative meaning that the agent can be an overall saver or a borrower in period 0. In the next period, only one of these assets will pay, a different one in each possible state. This allows the agent to trade across states of nature. For example, to give a good in date/event $(1, s_1)$ in exchange for goods in $(1, s_1')$ simply sell (buy negative amounts) of $a_{i1}(s_1)$ and buy positive amounts of $a_{i1}(s_1')$ in such a way as to ensure no time 0 goods are needed, i.e. $q(s_1) a_{i1}(s_1) + q(s_1') a_{i1}(s_1') = 0$. As usual the relative price $\frac{q(s_1)}{q(s_1')}$ between these two goods will be determined in equilibrium.

We can use the Lagrangian of the household problem to obtain the prices as functions of allocations

$$L = u(c_{i0}) + \beta \sum_{s_1 \in S_1} \pi(s_1) u(c_{i1}(s_1)) + \lambda_{i0} \left[w_{i0} - c_{i0} - \sum_{s_1 \in S_1} q(s_1) a_{i1}(s_1) \right]$$
$$... + \sum_{s_1 \in S_1} \lambda_{i1}(s_1) \left[w_{i1}(s_1) + a_{i1}(s_1) - c_{i1}(s_1) \right]$$

Notice that, for each agent i, there is one budget constraint in period 0, with associate multiplier λ_{i0}, and *four* budget constraints in period 1 with associated four multipliers $\lambda_{i1}(s_1)$. The foc give

$$u'(c_{i0}) = \lambda_{i0}$$
$$\beta\pi(s_1) u'(c_{i1}(s_1)) = \lambda_{i1}(s_1) \text{ for all } s_1$$
$$\lambda_{i0} q(s_1) = \lambda_{i1}(s_1) \text{ for all } s_1$$

Again notice there are five first order conditions for consumption and four first order conditions for the four different assets. The price of contingent claims is given by

$$q(s_1) = \frac{\beta\pi(s_1) u'\left(c_{i1}(s_1)\right)}{u'(c_{i0})} \text{ for all } s_1 \in S_1$$

Showing that the equilibrium allocations with sequential trade coincide with those in the date-0 trade equilibrium can be done following the same steps as in the case of no uncertainty. The crucial part is showing the equivalence of the budget sets, i.e. that any allocation that is feasible for the household in the date-0 trade can be obtained by the household in the sequential trade for some asset choices that satisfy market clearing. One would construct the asset trades needed as

$$a_{i1}(s_1) = c_{i1}(s_1) - w_{i1}(s_1) \text{ for all } s_1$$

notice that each asset market clears (because goods markets clear) and then show that this also satisfies for the first period budget, i.e.

$$c_{i0} + \sum_{s_1 \in S_1} q^*(s_1) \left[c_{i1}(s_1) - w_{i1}(s_1) \right] = w_{i0} \Rightarrow$$
$$c_{i0} + \sum_{s_1 \in S_1} q^*(s_1) c_{i1}(s_1) = w_{i0} + \sum_{s_1 \in S_1} q^*(s_1) w_{i1}(s_1)$$

This is indeed satisfied by the date-0 trade budget as long as the price of each contingent claim is set to equal the corresponding date-0 good price, i.e. $q^*(s_1) = p^*(s_1)$. This is also a clear illustration for why a risk free bond cannot implement every possible date-0 trade allocation. In that case we would need to construct the asset trades as

$$b_{i1} = c_{i1}(s_1) - w_{i1}(s_1) \text{ for all } s_1$$

But note this is impossible in general because the RHS is potentially different for each different s_1 but the LHS does not vary with s_1.

The fact that allocations will coincide also implies a direct relation between the asset prices here and the relative prices of consumption goods in the date-0 trade equilibrium. The price of the contingent claim $q(s_1)$ is equal to the relative price between the consumption good in date/event $(1, s_1)$ and the consumption good at time 0, $p_1(s_1)$. This is intuitive since the contingent claim is an instrument that achieves exactly this exchange of goods. So the prices of the date-0 trade equilibrium can be interpreted as prices of contingent claims and we will be using this result repeatedly below. It is important to keep this in mind because we often talk about asset pricing without explicitly defining a sequential trade equilibrium, but rather by looking directly at consumption good prices in a date-0 trade economy.

The fact that allocations in this sequential trade economy coincide with the date-0 economy is an illustration of the fact that the assets that we introduced are enough to complete the market (allow for all trades). In turn, this implies that adding more assets will make no difference in the allocations. We say the additional assets are redundant. However, we can find the prices of other assets, in fact *any* asset can now be priced. Consider for example introducing the risk free bond to this economy that already has all the contingent claims available. We can add this to the household budget, define the equilibrium with the additional asset and then characterize the equilibrium.[5] An additional first order condition will arise

$$\lambda_{i0} q^b = \sum_{s_1 \in S_1} \lambda_{i1}(s_1) \Rightarrow$$
$$q^b = \frac{\beta \sum_{s_1 \in S_1} \pi(s_1) u'(c_{i1}(s_1))}{u'(c_{i0})} = \beta \frac{E_0 u'(c_{i1})}{u'(c_{i0})}$$

The equilibrium price of the risk free bond is given by summing up the value of the goods it promises to pay. It promises one unit in every event next period, and one unit in event s_1 is valued at $\frac{\beta\pi(s_1)u'(c_{i1}(s_1))}{u'(c_{i0})}$. So it simply adds up the different payoffs, all written in the same units (the time zero good). Another way to see this, is that the returns of the risk free bond can be replicated by buying a portfolio of contingent claims.

[5] You should do this as an exercise.

Specifically, buying one unit of each of the contingent claims guarantees that the household will receive one unit of consumption in every event in period $t = 1$. The price paid for this portfolio is simply the sum of the prices of contingent claims $\sum_{s_1 \in S_1} q(s_1)$.

The point is that we could have used that idea to write the price of the bond without explicitly adding it and taking first order conditions. The next subsection exploits this idea to price any asset by the simple use of the prices of a date-0 equilibrium.

1.3 Asset Pricing

Given the prices and allocations in any date/event from the date-0 trade equilibrium, we can price any given asset without explicitly defining a sequential trade equilibrium, as long as markets are complete. The principle in this approach for asset pricing is simple: *The equilibrium price of an asset must equal the value of the goods it promises to pay.*

The simplest illustration of this principle was given above when pricing contingent claims. Here are some more examples:[6]

Example 1 *Suppose you can buy an asset in period $t = 0$ that will pay a dividend d (units of the consumption good) in period $t = 1$ state s_1. The price Q of such an asset must be the price of one unit of consumption in that date/event times the number d of units of consumption it promises to deliver*

$$Q = \frac{\beta \pi(s_1) u_c\left(c_{i1}(s_1)\right)}{u_c(c_{i0})} d$$

Example 2 *Suppose you can buy an asset in period $t = 0$ that entitles you to d units of consumption in period 1 event $s_1 \in S_1$ and d' units of consumption in period 1 event $s_1' \in S_1$, and 0 otherwise. Its price Q must be*

$$Q = \frac{\beta \pi(s_1) u_c\left(c_{i1}(s_1)\right)}{u_c(c_{i0})} d + \frac{\beta \pi(s_1') u_c\left(c_{i1}(s_1')\right)}{u_c(c_{i0})} d'$$

Note that both of the above assets are risky, in the following sense: the asset is bought at $t = 0$ at some price Q, but it could be that next period event $s_1'' \in S_1$occurs and the payout is zero. Put differently, the return of these assets is uncertain. Let us define the (gross) return as the answer to the following question: If I give 1 unit now, how many

[6] The examples that follow assume that consumption allocations in a date-0 trade equilibrium have already been computed as functions of exogenous endowments. The easiest case would be a representative agent economy where consumptions simply equal endowments.

units do I get tomorrow? So the return of a contingent claim is

$$\begin{aligned} R &= \frac{1}{q(s_1)} \text{ if } s_1 \text{ occurs} \\ &= 0 \text{ otherwise} \end{aligned}$$

The return differs depending on the state of nature tomorrow. We can also compute the expected (gross) return of each contingent claim

$$\begin{aligned} E(R) &= \pi(s_1)\frac{1}{q(s_1)} = \\ &= \frac{u'(c_{i0})}{\beta u'\left(c_{i1}(s_1)\right)} \end{aligned}$$

We can obtain the expected return for the previous two examples in the same way. The expected return for the first example is the same as that for a contingent claim (check this). For the second example we have

$$E(R) = \frac{\pi(s_1)d + \pi(s_1')d'}{\frac{\beta\pi(s_1)u_c(c_{i1}(s_1))}{u_c(c_{i0})}d + \frac{\beta\pi(s_1')u_c\left(c_{i1}(s_1')\right)}{u_c(c_{i0})}d'}$$

Example 3 *(one period discount bond) Suppose you buy an asset that ensures you will get one unit of the consumption good whatever happens next period, i.e. a risk-free asset. This is like buying one unit of each and every one of the contingent claims. The price* q^b *is*

$$q^b = \sum_{s_1 \in S_1} \frac{\beta\pi(s_1)u_c\left(c_{i1}(s_1)\right)}{u_c(c_{i0})} = E\left[\frac{\beta u_c\left(c_{i1}\right)}{u_c(c_{i0})}\right]$$

How does this relate to the (risk free) interest rate obtained on bank deposits? One dollar in the bank today guarantees R dollars next year, where R is the (gross) interest rate. Then

$$R = \frac{1}{\sum_{s_1 \in S_1} \frac{\beta\pi(s_1)u_c(c_{i1}(s_1))}{u_c(c_{i0})}}$$