Problem Set 6

Questions 1 and 2 take you through the steps of solving a Bellman equation numerically using Matlab. Question 3 asks you to use your solution to produce a simulation. You'll need to write and submit *separate* codes for each question.

1. (*Numerical Solution of the consumption/savings problem given a continuation value*). Consider the following maximization problem defining the function $V_2(.)$

$$\begin{aligned} V_2(k) &\equiv \max_{\{c,i,k'\}} \{\ln c + \beta V_1(k')\} \\ & \quad s.t. \\ c+i &= k^\alpha \\ k' &= (1-\delta)k+i \\ c,k' &\geq 0 \\ & \quad k \textit{ given} \end{aligned}$$

where $\beta = 0.96$, $\delta = 0.1$ and $\alpha = \frac{1}{3}$. Assume $V_1 : R_+ \rightarrow R$ is known and given by

$$V_1(k) = \alpha \ln k$$

   (a) Discretize the state space: Choose minimum and maximum values for the grid of k and split the interval into $N = 300$ (or more) equally spaced subintervals. Use those to construct the grid $kgrid = [k_1, k_2, k_3, ..., k_N]$.

   (b) Let $\tilde{V}_1(k)$ denote a function that is defined on $kgrid$ (instead of the whole R_+) such that $\tilde{V}_1(k) = V_1(k)$ for all $k \in kgrid$. Compute that function (a vector).

   (c) *For each* value of $k \in kgrid$, solve the maximization problem that defines $V_2(k)$. Note that this is maximization over a discrete state space so it can be achieved by evaluating the maximand at all values of $k' \in kgrid$ and choosing the value that yields the highest maximand. Be careful to ensure the choice is feasible, i.e. all constraints are satisfied. If some k' violates feasibility $(c, k' \geq 0)$, assign a very low negative number to the objective for this k' to ensure it is never chosen as the optimal choice.

   (d) Define the policy function (vector) for capital $g_2(k)$ and plot it.

   (e) Plot the functions $V_1(k)$ and $V_2(k)$ on the same graph. Are they different? In what scenario would the two functions coincide?

   (f) Repeat the process but now starting with the $V_2(k)$ computed before and solving for $V_3(k)$. That is solve for

$$\begin{aligned} V_3(k) &\equiv \max_{\{c,i,k'\}} \{\ln c + \beta V_2(k')\} \\ & \quad s.t. \\ c+i &= k^\alpha \\ k' &= (1-\delta)k + i \\ c, k' &\geq 0 \\ & \quad k \textit{ given} \end{aligned}$$

(g) Plot V_1, V_2 and V_3 on the same graph.

2. (*Iterating on the continuation value until convergence*). Consider the following planner's problem

$$\max_{\{c_t, k_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t &= k_t^\alpha \\ c_t &\geq 0 \\ k_{t+1} &\geq 0 \\ & k_0 \text{ given} \end{aligned}$$

where $\beta = 0.96$, $\delta = 0.1$ and $\alpha = \frac{1}{3}$.

(a) Formulate the Bellman equation for this problem and solve it using MATLAB. In particular, find the value function $V(k)$ and the policy functions for consumption $g^c(k)$ and capital $g^k(k)$. There are some degrees of freedom in the choice of N (the number of gridpoints) and ε (the tolerance level). Do the best you can in terms of N and ε, keeping in mind that the higher N and the lower ε, the better your approximation will be (but also the longer it will take to compute it). Note that for low values of N, the approximation can be so bad that your code does not converge. Provide the code and three plots of V, g^c and g^k with k on the horizontal axis.

3. Use the solution from question 2 to produce a simulated path for the economy for 100 periods starting from the lowest level of capital in the grid.

(a) Plot consumption, investment and output.

(b) What is the difference between a solution of this model and a simulation of this model?