Problem Set 11

Consider an exchange economy with two consumers indexed by $i = A, B$ and two periods indexed by $t = 0, 1$. Consumers know their period 0 endowment w_{i0} but they are uncertain about their endowment in period 1. There are two possible states of nature in period 1, i.e. the set of states of nature $S_1 = \{s_1, s_1'\}$ has two elements and we use s to denote a generic element of that set. Consumer i's endowments in period 1 are denoted by $w_{i1}(s)$. The two consumers have identical preferences: they each value consumption streams according to

$$u(c_{i0}) + \beta \sum_{s \in S_1} \pi(s) u(c_{i1}(s))$$

where $\pi(s)$ is the probability that state s occurs and $c_{i1}(s)$ is the consumption of consumer i in period 1, state s. Consumption goods are perishable (i.e. they cannot be stored and used for consumption in future periods). Suppose $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \sigma > 0$.

1. Consider a date-0 trade environment

   (a) Carefully define a competitive equilibrium with date-0 trading for this economy.

   (b) Find the equilibrium prices and consumption allocations in terms of exogenous endowments and parameters.

   (c) Suppose that the two states of nature occur with equal probability, that is $\pi(s) = \frac{1}{2}$ for all $s \in S_1$. Suppose also that the endowments are as follows

   $$\begin{aligned} w_{A0} &= w_{B0} = 5 \\ w_{A1}(s_1) &= w_{B1}(s_1') = 6 \\ w_{A1}(s_1') &= w_{B1}(s_1) = 4 \end{aligned}$$

   Compute prices and allocations for each date/event. *In equilibrium* (after all trade has taken place and prices and allocations have been decided in the date-0 markets), does consumer A prefer that state s_1 occurs or state s_1' occurs? Explain intuitively.

   (d) For any utility function $u(c)$ the coefficient of relative risk aversion is given by

   $$-\frac{cu''(c)}{u'(c)}$$

   (see Sargent and Ljungvist Chapter 13.11 for intuition on risk aversion). Compute the coefficient of relative risk aversion for the utility function given above. How does it relate to the elasticity of intertemporal substitution? In the above problem, what is the effect of the elasticity of intertemporal substitution (the consumption smoothing motive)? What is the effect of risk aversion (the risk pooling motive)? Explain.

(e) Now suppose that $\pi(s_1) = \frac{1}{3}$ and $\pi(s_1') = \frac{2}{3}$, but endowments are the same as in part c. Before any trade occurs, who do you think is better off A or B? Explain. Compute prices and allocations for each date/event. In equilibrium (after all trade has taken place and prices and allocations have been decided in the date-0 markets), who gets a higher consumption fraction?

2. Consider now a sequential trade environment

   (a) Introduce enough contingent claims to make markets complete and define a competitive equilibrium with sequential trading. You may assume that inherited assets are zero in the beginning.

   (b) Solve for the equilibrium price and quantity of the contingent claims.

   (c) For both specific cases specified in 1c and 1e

      i. What is the cost of the portfolio bought by each consumer at time 0? That is, are they net borrowers or net lenders and by how much?

      ii. What would be the equilibrium price of a discount (risk free) bond at $t = 0$ if it were also traded?

   (d) Suppose instead that only a risk free bond is available and no other assets (i.e. no contingent claims). Define an equilibrium with sequential trade and obtain the household first order conditions. Will the bond price be the same as before (you may explain why or why not without explicitly solving for the price in terms of exogenous variables).