

# MACRO COMP: PART I

1. Consider an infinite horizon economy with a tree which produces a quantity  $d_t$  of non-storable fruit every period. This fruit is the only endowment in the economy. There are  $N$  agents indexed by  $i$  who own and trade shares of the tree. They all have the same utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{it})$$

where  $c_{it}$  denotes consumption. The process for  $d_t$  is given by

$$\begin{aligned} d_t &= (1 - \rho) \bar{d} + \rho d_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2) \end{aligned}$$

where  $0 < \rho < 1$ ,  $\bar{d} > 0$  is large and  $\sigma > 0$  is small so that the possibility that  $d_t$  becomes non-positive can be ignored.

- (a) Carefully define an equilibrium with sequential trade in this economy. (*Think carefully: what is the source of heterogeneity?*)
- (b) Obtain all equilibrium conditions.
- (c) Use the equilibrium conditions to show that no share trade is an equilibrium, to determine consumption allocations and to solve for the stock price at  $t$  in terms of the time  $t$  dividend  $d_t$ .
- (d) Consider a planner that maximizes a weighted sum of individual utilities subject to feasibility. Is it possible to determine planner weights that would deliver the equilibrium allocations?
- (e) Suppose that agents have access to an insurance contract with an international company that guarantees them a constant consumption  $\bar{c}$ . Find the stock price at  $t$  in terms of the time  $t$  dividend  $d_t$  for this case.
- (f) Compare the stock price in cases c and e above. Which one is higher and under what conditions? Explain your result intuitively.