Econometrics Qualifying Exam: Part II
Do all 4 questions.

1. Consider the model,

$$\begin{aligned} y &= x\beta + u, \\ u &\sim (0, \Omega). \end{aligned}$$

Define

$$\widehat{u} = y - x\widehat{\beta}$$

where $\widehat{\beta}$ is the OLS estimator of β.

a. Prove that $x'\widehat{u} = 0$.

b. Consider adjusting the model to add one more regressor:

$$y = x\beta + \alpha\widehat{u} + e$$

where $\widehat{u}$ is defined above. What is the OLS estimate of α? Why is the R^2 for the adjusted model equal to 1?

2. Consider the model,

$$\begin{aligned} y_{1i} &= \beta_1 y_{2i} + \alpha_0 + \alpha_1 x_{1i} + \alpha_2 x_{2i} + u_{1i}, \\ y_{2i} &= \beta_2 y_{1i} + \gamma_0 + \gamma_1 x_{1i} + \gamma_3 x_{3i} + u_{2i} \end{aligned}$$

where (y_{1i}, y_{2i}) are endogenous variables, (u_{1i}, u_{2i}) are errors, and (x_{1i}, x_{2i}, x_{3i}) are exogenous variables with $E(x_{ji}u_{ki}) = 0$ for $j = 1, 2, 3$ and $k = 1, 2$.

a. Show that the structural parameters in the first equation are identified.

b. Show in detail how to estimate the structural parameters in the first equation using the identifying exogenous variable in the second equation as an instrument.

3. Consider the model,

$$\begin{aligned} y_{it}^* &= x_{it}\beta + u_i + \varepsilon_{it}, \\ u_i &\sim iidN\left(0, \sigma_u^2\right), \\ \varepsilon_{it} &\sim iidN(0, 1), \\ y_{it} &= 1\left(y_{it}^* > 0\right). \end{aligned}$$

Provide detail on how to estimate $\left(\beta, \sigma_u^2\right)$ using maximum likelihood estimation. Provide intuition on what identifies σ_u^2.

4. Consider the model,

$$\begin{aligned} y_{ij}^* &= x_{ij}\beta + \varepsilon_{ij}, \quad j = 1, 2, .., J, \\ \varepsilon_{ij} &\sim iidEV, \\ y_{ij} &= 1\left(y_{ij}^* > y_{ik}^* \forall k \neq j\right). \end{aligned}$$

a. What is

$$P_{ij} = \Pr\left(y_{ij} = 1\right)?$$

Hint: you don't have to derive it. Just write down the answer.

b. Let $y_i = (y_{i1}, y_{i2}, .., y_{iJ})$. Construct the covariance matrix of y_i.