Midterm Examination

Economics 522

7 November 2013
Exam begins at 1:00 and ends at 3:50

Please answer all 5 questions. If you cannot give a full answer to a question, please provide at least a partial answer so that you are eligible for partial credit.

1. A linear model is specified as $Y_i = \beta X_i + \epsilon_i$, with X_i being a *single* explanatory variable. The ϵ_i disturbance term is heteroskedastic, that is, $\text{Var}(\epsilon_i|X_i) = \sigma_{ii}$. Let $\hat{\beta}$ denote the ordinary least squares estimator and assume that this estimator is consistent.

   Let the scalar W be defined as
   $$W = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathrm{E}\left(\epsilon_i^2 X_i^2\right),$$
   and consider an estimator of it,
   $$\hat{W} = \frac{1}{n}\sum_{i=1}^{n} e_i^2 X_i^2.$$
   In the $\hat{W}$ expression, e_i^2 is the square of the ordinary least-squares residual for the i-th observation. You will recall that $\hat{W}$ appears in White's formula for correcting OLS standard errors given heteroskedasticity of unknown form.

   For the special case of a single X_i covariate, prove that $\hat{W} \xrightarrow{p} W$. Please be sure to state any additional assumptions that you need to complete the proof.

2. Consider a model $Y_i = \mathbf{X}_i'\beta + \epsilon_i$ in which $\mathrm{E}(\epsilon_i|\mathbf{X}_i) \neq 0$, that is, one or more of the $\mathbf{X}_i$ covariates is correlated with the disturbance term. The disturbances are uncorrelated over i and have the same variance σ^2.

   The β vector is of dimension k and you should assume that you have $m > k$ valid instruments in the $n \times m$ matrix $\mathbf{Z}$. In other words, this is a "over-identified" model.

   Consider the normalized quadratic form used in the Sargan test of the validity of the instruments,
   $$T = \frac{1}{\hat{\sigma}^2}\mathbf{e}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{e},$$
   in which $\mathbf{e}$ is the vector of instrumental-variables residuals and $\hat{\sigma}^2 = \mathbf{e}'\mathbf{e}/n$. Prove that the Sargan test statistic
   $$T \xrightarrow{d} \chi^2_{m-k}$$
   under the null hypothesis that all m instruments are valid. (Be sure to state any additional assumptions that you need in the proof.) After giving your proof, carefully discuss the limitations of this testing procedure.

3. Consider a linear data-generating process for a balanced panel with i=1, $\dots, N$ units and $t = 1, \dots, T$ time periods for each unit,

$$Y_{i,t} = \mathbf{X}_{i,t}' \beta_0 + \epsilon_{i,t}$$

In this model β_0 is the true value of the slope parameters and we assume that $\mathrm{E}(\epsilon_{i,t} \mid \mathbf{X}_{i,t}) = 0$. For each unit i, we allow the set of disturbances $\{\epsilon_{i,t}, \quad t = 1, \dots, T\}$ to be heteroskedastic and freely correlated. Across i, however, all of the random variables are assumed to be independent.

In your asymptotic analysis, let T be fixed and let $N \to \infty$. Discuss the conditions under which ordinary least squares provides a consistent estimator for β and derive the limiting distribution of $\sqrt{N}(\hat{\beta}_{OLS} - \beta_0)$.

4. A simple structural model of wages $Y_{i,t}$ for worker i at time t is

$$Y_{it} = \alpha + t \cdot \beta + D_{it}\delta + u_i + \epsilon_{it}.$$

This model contains a constant, a time trend, a dummy variable D_{it} which takes the value 1 if the worker has participated in a training program on or before time t, and an error-components disturbance term. If we think of the u_i component as representing the worker's motivation, among other things, we would suspect that more motivated workers (those with higher values of u_i) are also the kind of people who are more likely to take advantage of opportunities for job training. That is, it seems likely that u_i and D_{it} will be positively correlated.

   (a) Discuss how to estimate the program effect δ by the method of difference-in-differences. What are the important assumptions of this method?

   (b) Suppose the model is altered to $Y_{it} = \mathbf{X}_{i,t}' \gamma + D_{it}\delta + u_i + \epsilon_{it}$. Discuss how to estimate the program effect.

5. Consider a model $Y_i = \phi(\mathbf{X}_i, \theta) + \epsilon_i$ for which you know the functional form of $\phi()$ but not the true value of the θ parameter. The θ vector is of dimension k and let θ_0 denote its true value. Assume that $\mathrm{E}(\epsilon_i | \mathbf{X}_i) \neq 0$, that is, one or more of the $\mathbf{X}_i$ covariates is correlated with the disturbance term. You can assume that you have $m > k$ valid instruments $\mathbf{Z}_i$. The data series $\{(\mathbf{X}_i, \mathbf{Z}_i, \epsilon_i)\}$ is independent (over i) but not necessarily identically distributed (that is, the series is inid). In particular, the disturbance terms are heteroskedastic with variances σ_{ii}^2.

   Explain in detail how to estimate θ using the Generalized Method of Moments approach. What are the first-order conditions of $\hat{\theta}_{GMM}$, the GMM estimator? What is the limiting distribution of $\sqrt{n}(\hat{\theta}_{GMM} - \theta_0)$? How would you estimate the variance matrix of the limiting distribution?