Econ 521
Midterm
3/30/21

Do 5 out of 6 questions (20 points each):

1. Consider the model,

$$
\begin{aligned}
y_i &= X_i\beta + Z_i\gamma + u_i, \\
u_i &\sim iid\left(0, \sigma^2\right), \\
i &= 1, 2, .., n.
\end{aligned}
$$

Imagine estimating the model,

$$
y_i = X_i b + e_i,
$$

i.e., you do not include Z_i in your regression. Derive the asymptotic properties of the OLS estimator of b.

2. Consider three vectors, y, x, and z. Show geometrically the effects of projecting the residuals of a regression of y on z onto the space spanned by x.

3. Construct a model that allows you to estimate the differential effect of receiving a each of the possible COVID vaccines and how they vary by age group.

4. Consider the conjecture: after controlling for a reasonable set of explanatory variables, wages of men exhibit greater variance than wages of women as they age. In other words, the variance increases with age but at rates that vary by gender. Specify an equation to consider this conjecture and show how to estimate the parameters of interest.

5. Construct the Yule-Walker equations for and ARMA(2,2) process.

6. Consider the model,

$$
\begin{aligned}
y_{it} &= x_{it}\beta + u_i + e_{it}, \\
u_i &\sim iid(0, \sigma_u^2), \\
e_{it} &= \rho e_{it-1} + \varepsilon_{it}, \\
\varepsilon_{it} &\sim iid(0, \sigma_\varepsilon^2), \\
t &= 1, 2, .., T \\
i &= 1, 2, .., n.
\end{aligned}
$$

Provide details of how to construct the feasible GLS estimator of β.