Econ 772
Final Exam
Spring 2012

1 Section 1: Do 6 out 7 questions [60 points]

1. Consider the model,

$$\begin{aligned} y &= X\beta + u \\ u &\sim \left(0, \sigma^2 I\right) \end{aligned}$$

where all of the x-variables are exogenous. Consider a matrix Z such that $plim \frac{Z'X}{n}$ has full rank and $plim \frac{Z'u}{n} = 0$. Derive the small sample bias and asymptotic properties of the estimator of β that satisfies the orthogonality condition,

$$Z' \left(y - X\beta\right) = 0.$$

2. Consider the model,

$$y_i = \alpha_1 Male_i + \alpha_2 Female_i + \alpha_3 \left(Male_i * Educ_i\right) + \alpha_4 \left(Female_i * Educ_i\right) + u_i.$$

Assume that all observations are either male or female.
a) What is the interpretation of α_4?
b) What is the effect on y_i of changing gender?

3. Consider the model,

$$\begin{aligned} y_i &= X_i \beta_i + u_i, \\ \beta_i &\sim iidN\left(\beta^*, \Omega_\beta\right), \\ u &\sim N\left(0, \Omega_u\right) \end{aligned}$$

where $u = \left(u_1, u_2, .., u_n\right)'$. Provide intuition for what allows you to separately identify Ω_β and Ω_u.

4. Consider the model,

$$\begin{aligned} y_t &= \beta x_t + \gamma w_t + u_t, \\ x_t &= \alpha y_t + \delta z_t + e_t, \\ \begin{pmatrix} u_t \\ e_t \end{pmatrix} &\sim iid\left(0, \sigma^2 I\right) \end{aligned}$$

where all variables are all scalars and w_t and z_t are exogenous explanatory variables. Derive the $plim$ of the OLS estimator of (β, γ).

5. Consider the model,

$$\begin{aligned} y_{1i} + \beta_{12} y_{2i} &= \gamma_{10} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} + \gamma_{13} x_{3t} + u_{1t}, \\ \beta_{21} y_{1i} + y_{2i} &= \gamma_{20} + \gamma_{21} x_{1t} + \gamma_{22} x_{2t} + \gamma_{23} x_{3t} + u_{2t}. \end{aligned}$$

a) Determine whether each equation is identified.

b) Consider the restriction: $\gamma_{12} = \gamma_{13}$. Given this restriction, determine again whether each equation is identified.

6. Consider the model,

$$\begin{aligned} y_{it}^* &= X_{it}\beta + u_i + \varepsilon_{it}, \\ u_i &\sim iidN\left(0, \sigma_u^2\right), \\ \varepsilon_{it} &\sim iidLogistic, \\ y_{it} &= 1\left(y_{it}^* > 0\right). \end{aligned}$$

Construct the likelihood function.

7. Let

$$\underset{3\times 1}{u} \sim N\left(\mu, \Omega\right),$$

and consider two rectangular solids, A and B in R^3 that might or might not intersect. Provide details on how to construct a smooth simulator of $\Pr\left[u \in A \cup B\right]$.

2 Section 2: Do 2 out 3 questions [60 points]

1. Consider a model with parameter vector θ. Assume a prior distribution for θ of $\pi(\theta)$. Let x_1, x_2 be two iid random variables from $f(x \mid \theta)$. Provide the sketch of a proof that the posterior of θ is the same independent of the order in which one observes the data.

2. Assume that

$$\begin{aligned} y_i &= \alpha + \beta x_i + u_i \\ u_i &\sim iidN\left(0, \sigma^2\right). \end{aligned}$$

We want to test $H_0 : \beta = 0$ against $H_A : \beta \neq 0$ using a Lagrange Multiplier test. Work out all of the details of the corresponding test statistic, and derive its exact distribution.

3. Consider the model,

$$\begin{aligned} y_{i1}^* &= \alpha_1 y_{i2}^* + X_{it}\beta_1 + u_{i1}, \\ y_{i2}^* &= \alpha_2 y_{i1}^* + X_{it}\beta_2 + u_{i2} \\ u_i &\sim iidN\left(0, \Omega\right) \\ y_{ij} &= 1\left(y_{ij}^* > 0\right), j = 1, 2. \end{aligned}$$

a) Determine conditions for all of the parameters of the model to be identified.

b) Construct the likelihood function. Hint: part (b) will help a lot with answering part (a).

3 Section 3: Do 2 out 3 questions [60 points]

Your goal is to estimate the demand for different brands of cereal as a function of characteristics of the cereal, price of the cereal, prices of competitor brands, and characteristics of consumers.

1. Assume that you have data on cereal box purchases by individual consumers. In particular, you observe $\{y_i, x_i, p_i\}_{i=1}^n$ for n observations where y_i is a vector of purchase decisions for person i during a visit to a store, x_i is a set of observed characteristics for person i, and p_i is a vector of prices for each of the cereal choices (which vary over consumers). You also have access to information about the characteristics of each brand. Provide detail on a model of cereal consumption and how to estimate the parameters of the model.

2. Assume instead that you have data on aggregate city-specific cereal box purchases by consumers. In particular, you observe $\{y_i, x_i, p_i\}_{i=1}^n$ for n observations where y_i is a vector of sales units of each brand purchased at all stores in city i during a particular month, x_i is a set of observed characteristics of people living in city i, and p_i is a vector of prices for each of the cereal choices (which vary over cities). You also have access to information about the characteristics of each brand. Provide detail on a model of cereal consumption and how to estimate the parameters of the model.

3. Assume instead that you have data on cereal box purchases by consumers over time. In particular, you observe $\left\{[y_{it}, x_{it}, p_{it}]_{t=1}^T\right\}_{i=1}^n$ for n observations where y_{it} is a vector of purchase decisions for person i during month t, x_{it} is a set of observed characteristics for person i in month t, and p_{it} is a vector of prices for each of the cereal choices (which vary over consumers and time). Assume that a typical consumer purchases multiple boxes of cereal per month, that (at least) some consumer characteristics can change over time, and that prices change over time. You also have access to information about the characteristics of each brand which do not change over time. Provide detail on a model of cereal consumption and how to estimate the parameters of the model.