1 Econometrics: Answer 3 out of 4 questions. Each question is equally weighted.

1. Let

$$\begin{aligned} y_t &= X_t\beta + u_t, \\ u_t &= \sum_{i=1,2} \rho_i u_{t-i} + \varepsilon_t, \\ \varepsilon_t &\sim \left(0, \sigma^2\right). \end{aligned}$$

   Provide a consistent estimate of the covariance matrix of $u = (u_1, u_2, .., u_T)'$, show that it is consistent, and show that the Feasible GLS estimator of β is consistent.

2. Consider the model,

$$y_{1i} = \beta_0 + \beta_1 y_{2i} + \beta_2 x_{1i} + u_{1i},$$

$$y_{2i} = \alpha_0 + \alpha_2 x_{2i} + u_{2i},$$

$$\begin{pmatrix} u_{1i} \\ u_{2i} \end{pmatrix} \sim \left(0, \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right).$$

   Let $\widehat{\beta}$ be the OLS estimator of $\beta = (\beta_0, \beta_1, \beta_2)'$. Derive the asymptotic bias of $\widehat{\beta}$.

3. Let

$$\begin{aligned} x_i &\sim iidBernoulli\,(p) \\ p &\sim U\,(0,1)\,. \end{aligned}$$

   Use Bayes' Theorem to form a posterior for $p \mid x_1, x_2, .., x_n$.

4. Let

$$y_t = x_t\beta + u_t,$$

$$\begin{aligned} u_t &= \rho u_{t-1} + e_t, \\ e_t &= \exp\{\alpha v_t\}\,\varepsilon_t, \\ \varepsilon_t &\sim iidN\,(0,1)\,, \\ v_t &= \gamma v_{t-1} + \eta_t, \\ \eta_t &\sim iidN\left(0, \sigma_\eta^2\right). \end{aligned}$$

   Construct the likelihood function for $\{y_t, x_t\}_{t=1}^T$, and show, in detail, how to simulate it.