1 Econometrics: Answer 3 out of 4 questions. Each question is equally weighted. Total 40 points.

1. Let

$$\underset{T \times 1}{y} = X\beta + u,$$

$$u \sim (0, \sigma^2 I).$$

Define

$$\widehat{u} = y - X\widehat{\beta}$$

where $\widehat{\beta}$ is the GLS estimator of β.

a) Derive $plim \frac{1}{T} \widehat{u}' \widehat{u}$.

b) Define

$$\underset{T \times 1}{\iota} = (1, 1, .., 1)'.$$

Find the asymptotic distribution of $\sqrt{T} \iota' \widehat{u}$.

2. Consider the model,

$$y_{1i} = \beta_0 + \beta_1 y_{2i} + \beta_2 x_{1i} + u_{1i},$$

$$y_{2i} = \alpha_0 + \alpha_1 y_{1i} + \alpha_2 x_{2i} + u_{2i},$$

$$\begin{pmatrix} u_{1i} \\ u_{2i} \end{pmatrix} \sim \left(0, \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right).$$

Show how to estimate β_1 using indirect least squares (ILS). Provide intuition for why the ILS estimator of β_1 is unique.

3. Consider the model,

$$g(y_i, X_i \theta) = u_i,$$

$$u_i \sim iidF(\cdot),$$

$$E(u_i \mid X_i) = 0$$

$$i = 1, 2, .., n.$$

Describe how to estimate $g(\cdot)$ and θ semiparametrically.

4. Let

$$u_t = \rho u_{t-1} + e_t,$$

$$e_t = a_0 \varepsilon_t + a_1 \varepsilon_{t-1}$$

$$\varepsilon_t \sim iidN(0, \sigma^2 I).$$

Derive the distribution of u_t. Provide intuition for why you can't identify all of the parameters of the model.