

## Econometrics

Short Questions (Do two out of three) 15 points each

- 1) Let  $y = X\beta + u$  and  $Z$  be a set of instruments for  $X$ . When we estimate  $\beta$  with OLS we project  $y$  onto the space spanned by  $X$  along a path orthogonal to  $X$ . Write an analogous statement about the instrumental variables estimator of  $\beta$ , and then draw a picture to help explain your sentence.
- 2) Consider a model where an individual receives wage offers from distribution  $F(w; \theta)$  and accepts the first offer greater than  $\xi$ . You are given a data set of  $N$  iid observations,  $\{w_i\}_{i=1}^N$ , and the goal is to estimate  $\theta$  and  $\xi$  using MLE.
  - a) What is the MLE of  $\xi$ ? [Hint: focus on the fact that each person accepts the first offer greater than  $\xi$ ].
  - b) What goes wrong in the standard proof for consistency of the MLE of  $\xi$ ?
- 3) Consider the estimated model for log wages:

Variable	$\hat{\beta}$	$\hat{s}_{\hat{\beta}}$
Constant	1.234	0.35
Age	.0517	0.008
Age <sup>2</sup>	-0.003	0.001
Female	-0.211	0.073
Black	-0.153	0.043
Educ	0.084	0.008
Female*Educ	-0.009	0.005
Black*Educ	-0.008	0.003

For each of the questions below either provide the answer (it is not necessary to do any arithmetic) or report what information that is missing is needed and provide a formula:

- a) What is the estimated rate of return to education for women and its standard error?
- b) What is the estimate of the age where wage stops growing and its standard error?

Long Questions (Do one out of two) 30 points each

1) Let

$$\begin{aligned}q_t^d &= \alpha_0 + \alpha_1 p_t + \alpha_2 y_t + u_t^d \\q_t^s &= \beta_0 + \beta_1 p_t + \beta_2 w_t + u_t^s \\q_t^d &= q_t^s.\end{aligned}$$

a) Describe in detail how to simulate the asymptotic distribution of the 2SLS estimator of the structural parameters in the model.

b) Describe how you think your simulated parameter estimates would behave if  $\alpha_2 = 0$ .

2) Let

$$y_i = b(X_i, \delta) + u_i$$

for  $i = 1, 2, \dots, N$ . Assume that there are  $m$  elements in the parameter vector  $\delta$ . Let  $u' = (u_1, u_2, \dots, u_N)$  and assume that

$$u \sim N(0, \Omega).$$

You want to test  $H_0 : H(\delta) = 0$  against  $H_A : H(\delta) \neq 0$  where there are  $k < m$  nonlinear restrictions implied by  $H(\bullet)$ . Suggest a test statistic and derive its asymptotic distribution under  $H_0$ .