2. Econometrics Component (60 Points)

Instructions: Answer three out of the four following questions. We suggest you allocate one hour for the completion of this part of the examination.

1) Consider the model

$$y = X\beta + u$$

with

$$u \sim \left(0, \sigma^2 I\right).$$

Consider the test,

$$H_0 : A\beta = c \text{ vs. } H_A : A\beta \neq c.$$

Suggest a consistent estimate of β and use it to construct a test statistic. Derive the asymptotic distribution of the test statistic. Hint: it is not enough to specify a test statistic and assert its distribution; derive the distribution.

2) Consider the model

$$\begin{aligned} q_i^d &= \alpha_d + \beta_d p_i + \gamma_d z_i^d + u_i^d \\ q_i^s &= \alpha_s + \beta_s p_i + \gamma_s z_i^s + u_i^s \\ q_i^d &= q_i^s \end{aligned}$$

where q_i^d is demand for bananas, q_i^s is supply of bananas, p_i is price of bananas, and $\left(z_i^d, z_i^s\right)$ are two different exogenous variables. Under what conditions are all of the structural parameters identified? How might you test the identification assumption?

3) Show under reasonable conditions that the maximum likelihood estimator is consistent and derive its asymptotic distribution.

4) Consider the model

$$\begin{aligned} y_i &= m\left(x_i\right) + u_i, \\ u_i &\sim iid\left(0, \sigma^2\right), \\ i &= 1, 2, .., n \end{aligned}$$

where x_i is an exogenous scalar and $m\left(\cdot\right)$ is an unspecified function. Suggest how to estimate $m\left(\cdot\right)$ using a) kernel estimation, b) polynomial approximations, and c) spline functions in slopes. For each one, explain how your estimation procedure changes as $n \to \infty$ and why that provides a consistent estimate of $m\left(\cdot\right)$.