Econ 772
Final Exam
Spring 2011

1 Section 1: Do 6 out 7 questions [60 points]

1. Consider the model,

$$\begin{aligned} y &= X\beta + u \\ u &\sim (0, \Omega), \end{aligned}$$

and consider

$$H_0 : g(\beta) = 0 \quad \text{vs } H_A : g(\beta) \neq 0.$$

Show how to test this hypothesis using the OLS estimator of β.

2. Consider the model,

$$y_i = \alpha_1 Male_i + \alpha_2 Female_i + \alpha_3 Black_i + \alpha_4 White_i + \alpha_5 Asian_i + u_i. \quad (1)$$

a) Assume that all observations are either male or female, and all observations are either black, white, or Asian but only one of the three. What are the statistical properties of your OLS estimator of $(\alpha_1, \alpha_2, .., \alpha_5)$?
b) Now consider the possibility that observations can me mixed race. How should you change equation (1) so that it captures the effects of mixed race and in a way that has good statistical properties? Be precise about how you are assuming mixed race affects outcomes.

3. Consider the model,

$$\begin{aligned} y_i &= X_i \beta_i + u_i, \\ \beta_i &\sim iidN(\beta^*, \Omega_\beta), \\ u_i &\sim iidN\left(0, \sigma^2 I\right). \end{aligned}$$

Derive the GLS estimator of (β^*, Ω_β), and show that it is consistent.

4. Consider the model,

$$\begin{aligned} y_t &= \beta x_t + W_t \gamma + u_t, \\ x_t &= \alpha z_t + e_t, \\ \begin{pmatrix} u_t \\ e_t \end{pmatrix} &\sim iid(0, \Omega) \end{aligned}$$

where y_t, x_t, u_t, z_t, and e_t are all scalars and W_t is a vector of exogenous explanatory variables. Derive the *plim* of the OLS estimator of β.

5. Consider the model,

$$\begin{aligned} y_{1i} + \beta_{12} y_{2i} &= \gamma_{10} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} + u_{1t}, \\ \beta_{21} y_{1i} + y_{2i} &= \gamma_{20} + \gamma_{21} x_{1t} + \gamma_{23} x_{3t} + \gamma_{24} x_{4t} + u_{2t}. \end{aligned}$$

   How can one test the structure of the model?

6. Consider the density

$$f(u_1, u_2) = \frac{1\left(u_1^2 + u_2^2 \leq r^2\right)}{(2\pi r^2)}.$$

   Hint: this is a uniform density over a circle with radius r. Find the MLE of r.

7. Consider the model

$$\begin{aligned} y_i^* &= X_i \beta + u_i, \\ u_i &\sim iidN(0,1), \\ y_i &= 1(y_i^* > 0). \end{aligned}$$

   How can one use simulation to measure how quickly the statistical properties of the MLE estimator of β converge to the asymptotic properties? Be as precise as possible.

2 Section 2: Do 2 out 3 questions [60 points]

1. Consider the model,

$$y = X\beta + u$$

   where some of the variables in X may be endogenous. Let Z be a valid matrix of instruments for X. Using the geometry of projection matrices, show how the instrumental variables estimator of β works.

2. Consider the model,

$$\begin{aligned} y &= X\beta + u, \\ u &\sim N(0, \Omega). \end{aligned}$$

   Assume that you know what Ω is. Show that, if you have a prior for β that is normal, then the posterior for β, given (y, X), will also be normal.

3. Consider the model,

$$\begin{aligned} y_{it}^* &= X_{it} \beta + u_{it}, \\ u_i &\sim iidN(0, \Omega) \\ y_{it} &= 1(y_{it}^* > 0). \end{aligned}$$

   Construct a method of moments estimator for β.

3 Section 3: Do 2 out 3 questions [60 points]

1. Consider the utility function for individual i,

$$U_i(x_i) = \sum_{j=1}^{J} \beta_{ij} \log\left(x_{ij} - \gamma_j\right)$$

where

$$\beta_i \sim iidN(\beta^*, \Omega).$$

Each individual i maximizes utility subject to the budget constraint,

$$y_i \leq \sum_{j=1}^{J} p_{ij} x_{ij}.$$

Hint: the demand equation for each person for each good is

$$x_{ij} = \gamma_j + \frac{\beta_{ij} M_i}{p_{ij}}$$

where

$$M_i = y_i - \sum_{j=1}^{J} p_{ij}\gamma_j.$$

With data on $\{y_i, x_{i1}, x_{i2}, .., x_{iJ}\}_{i=1}^{n}$, discuss how to estimate $(\beta^*, \Omega, \gamma)$. Hint: do not add extra randomness into the model; enough randomness is already there.

2. Construct a model of an individual deciding whether to commit a crime. Consider the possibility that criminal behavior today depends on past criminal behavior. Note: the unit of observation in your model should be an individual. Provide as much detail as possible on how to estimate the parameters of your model.

3. Consider a model for the market for bananas,

$$\begin{aligned} q_t^d &= \alpha_0 + \alpha_1 p_t + \alpha_2 y_t + u_t^d, \\ q_t^s &= \beta_0 + \beta_1 p_t + \beta_2 w_t + u_t^s, \\ q_t^d &= q_t^s \end{aligned}$$

where y_t is a measure of income and w_t is a measure of weather relevant to the production of bananas. We know that the supply equation is identified because of the inclusion of y_t in the demand equation, and the demand equation is identified because of the inclusion of w_t in the supply equation. However, a skeptic suggests that w_t really should be included in the demand equation also because, when the weather changes, it affects the supply of substitutes for bananas such as pineapples. Discuss whether this is a reasonable arguement and, if so, what it implies about identification of the model parameters and how one can fix the problem.