1 Econometrics: Answer 3 out of 4 questions. Each question is equally weighted.

1. Let

$$y_t = \sum_{i=0}^{n} \beta_i x_{ti} + u_t$$

$$u_t \sim iid\left(0, \sigma^2\right).$$

Suggest how to estimate $\beta = (\beta_0, \beta_1, .., \beta_n)'$ subject to the restriction,

$$\beta_1 + 2\beta_3 = 4,$$

and show that it is consistent.

2. Consider the model,

$$y_{1i} = \beta_0 + \beta_1 y_{2i} + \beta_2 x_{1i} + u_{1i},$$

$$y_{2i} = \alpha_0 + \alpha_2 x_{2i} + u_{2i},$$

$$\left(\begin{array}{c} u_{1i} \\ u_{2i} \end{array} \right) \sim \left(0, \left(\begin{array}{cc} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{array} \right) \right).$$

Show that $\beta = (\beta_0, \beta_1, \beta_2)'$ is identified, and derive the asymptotic distribution of the 2SLS estimator of β.

3. Let

$$x_i \sim iidU\left(0, \theta\right), \ \ i = 1, 2, .., n.$$

Derive the MLE, the MOM estimator, and the Bayesian estimator of θ. For the Bayesian estimator, use an exponential prior.

4. Provide detailed instructions on how to do a Monte Carlo experiment to learn about the small sample properties of probit estimators.