Econ 521
Final Exam
Tuesday, May 11

Do 10 out of 12 problems.

1. Consider the process,

$$
\begin{aligned}
u_t &= \rho u_{t-1} + \varepsilon_t, \\
\varepsilon_t &\sim iid\left(0, \sigma_\varepsilon^2\right), \\
t &= 1, 2, .., T.
\end{aligned}
$$

   Define

$$\widehat{\rho} = \frac{\sum u_t u_{t-1}}{\sum u_t^2}.$$

   Derive $plim\widehat{\rho}$, and derive the asymptotic distribution of $\sqrt{T}\left(\widehat{\rho} - plim\widehat{\rho}\right)$.

2. Consider the model,

$$
\begin{aligned}
y_i &= X_i \beta + u_i, \\
u_i &\sim iid\left(0, \sigma_u^2\right), \\
i &= 1, 2, .., n.
\end{aligned}
$$

   We want to test

$$H_0 : \beta_2 = \beta_3 \text{ vs } H_A : \beta_2 \neq \beta_3.$$

   Construct a test statistic associated with this test, and derive its distribution under H_0.

3. Consider the model,

$$
\begin{aligned}
y_i &= \underset{1\times K}{X_i} \beta + u_i, \\
u_i &\sim iid\left(0, \sigma_u^2\right), \\
i &= 1, 2, .., n.
\end{aligned}
$$

   The data available to estimate β is $\{y_i, x_{i1}, w_i, x_{i3}, .., x_{iK}\}_{i=1}^{n}$ where

$$w_i = \alpha x_{i2} + e_i.$$

   What must be true about α and the distribution of e_i in order to get a consistent estimate of β?

4. We are interested in estimating the factors that affect demand for record albums by Neil Diamond. Let $y_{it} \sim Poisson\left(\lambda_{it}\right)$. Specify a model for λ_{it} that allows for variation in demand by gender, race, and age. Assume

that, in your available data, age is bracketed in ten year intervals; i.e., observed age of person i at time t, $OAge_{it}$, is

$$OAge_{it} = k \text{ iff } k-1 \leq Age_{it}/10 < k$$

where Age_{it} is unobserved actual age. Allow the effect of age to vary by race and gender. Hint: you can go to https://www.youtube.com/watch?v=1vhFnTjia_I if you think it would be helpful to listen to a Neil Diamond song.

5. Consider the model,

$$\begin{aligned} y_i &= X_i\beta + u_i, \\ u_i &\sim iid\left(0, \sigma_u^2\right), \\ i &= 1, 2, .., n. \end{aligned}$$

We want to test

$$H_0 : \beta_2 = \beta_3 = \beta_4 = \beta_5 \text{ vs } H_A : \beta_2 \neq \beta_3 \neq \beta_4 \neq \beta_5.$$

Construct a test statistic associated with this test, and derive its distribution under H_0. Also, derive its distribution under H_0 if there were some omitted variables in your estimation procedure.

6. Consider the model,

$$\begin{aligned} y_{it}^* &= x_{it}\beta + u_i + e_{it}, \\ u_i &\sim iidN\left(0, \sigma_u^2\right), \\ e_{it} &= \rho e_{it-2} + \varepsilon_{it}, \\ \varepsilon_{it} &\sim iidN\left(0, 1\right), \\ y_{it} &= 1\left(y_{it}^* > 0\right), \\ t &= 1, 2, .., T, \\ i &= 1, 2, .., n. \end{aligned}$$

Define

$$v_{it} = u_i + e_{it},$$

$v_i = \left(v_{i1}, v_{i2}, .., v_{iT}\right)'$, and $v = \left(v_1, v_2, .., v_n\right)'$. Derive the covariance matrix of v, and describe what covariation in the data would allow you to identify its terms.

7. Let

$$\begin{aligned} y &= X\beta + u, \\ u &\sim \left(0, \Omega\right). \end{aligned}$$

Let $\widehat{\Omega}$ be a consistent estimator of Ω. Prove that

$$E\left(X'\widehat{\Omega}^{-1}X\right)^{-1} X'\widehat{\Omega}^{-1}y = \beta.$$

8. Consider the model,

$$\begin{aligned} y_{1i} &= \beta_{12} y_{2i} + \alpha_{10} + \alpha_{11} x_{11i} + \alpha_{12} x_{12i} + u_{1i}, \\ y_{2i} &= \beta_{21} y_{1i} + \alpha_{20} + \alpha_{21} x_{11i} + u_{2i}. \end{aligned}$$

How can we estimate β_{12}?

9. Consider the model,

$$y = X\beta + Q\gamma + u$$

where y is a vector of dependent variables, X is a matrix of endogenous explanatory variables, Q is a matrix of exogenous explanatory variables, and u is a vector of errors. Construct the orthogonality used to estimate the structural parameters, (β, γ).

10. Consider the model,

$$\begin{aligned} y_i^* &= x_i \beta + u_i, \\ u_i &\sim iidN(0,1), \\ y_i &= 1(y_i^* > 0), \\ i &= 1, 2, .., n. \end{aligned}$$

Show how to use the log likelihood function for this model to construct an orthogonality condition for estimation.

11. Consider the model,

$$\begin{aligned} y_{ijt}^* &= x_{ijt}\beta + z_{it}\gamma_j + u_{ij} + \varepsilon_{ijt}, \\ u_i &= (u_{i1}, u_{i2}, .., u_{iJ})' \sim iidN(0, \Omega), \\ \varepsilon_{ijt} &\sim iidEV, \\ y_{ijt} &= 1\left(y_{ijt}^* > y_{ikt}^* \forall k \neq j\right), \\ j &= 1, 2, .., J, \\ t &= 1, 2, .., T, \\ i &= 1, 2, .., n. \end{aligned}$$

Construct the likelihood function for estimation of this model. Provide intuition for what covariation in the data identifies Ω.

12. Consider the model,

$$\begin{aligned} y_1^* &= \alpha y_2 + x_1\beta + u_1, \\ y_2^* &= \alpha y_1 + x_2\beta + u_2, \\ y_j &= k \text{ iff } \tau_k \leq y_j^* < \tau_{k+1},\ j = 1, 2;\ k = 1, 2, .., 4, \\ u &= \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \sim F. \end{aligned}$$

Assuming that $\alpha < 0$, show the regions of the support of u where there are multiple equilibria to the model.