1 Econometrics: Answer 3 out of 4 questions. Each question is equally weighted.

1. Let

$$\begin{aligned} y &= X\beta + u, \\ u &\sim (0, \Omega). \end{aligned}$$

   Show that the OLS estimator of β is consistent, and derive its asymptotic distribution.

2. Consider the model,

$$y_{1i} = \beta_0 + \beta_1 y_{2i} + \beta_2 x_{1i} + u_{1i},$$

$$y_{2i} = \alpha_0 + \alpha_1 y_{1i} + \alpha_2 x_{2i} + u_{2i},$$

$$\begin{pmatrix} u_{1i} \\ u_{2i} \end{pmatrix} \sim \left(0, \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right).$$

   Let $\widehat{\beta}$ be the OLS estimator of $\beta = (\beta_0, \beta_1, \beta_2)'$. Let t_{OLS} be a t-statistic with a 5% size to test $H_0 : \beta_1 = 3$ vs. $H_A : \beta_1 \neq 3$ using $\widehat{\beta}$ and ignoring the fact that y_{2i} is endogenous. Show how to compute $\Pr[\text{Reject } H_0 \mid H_0 \text{ is true}]$ using the flawed t-statistic.

3. Consider the model,

$$\begin{aligned} g(y_i, X_i, \theta) &= u_i, \\ u_i &\sim iidF(\cdot), \\ i &= 1, 2, .., n. \end{aligned}$$

   Sketch a proof that the MLE of θ is consistent, and derive its asymptotic distribution.

4. Let

$$u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix},$$

$$\begin{aligned} u_t &= Au_{t-1} + \varepsilon_t, \\ \varepsilon_t &\sim iidN\left(0, \sigma^2 I\right). \end{aligned}$$

   Derive the marginal distribution of u_t. Be specific about any assumptions you need to make about A and/or σ^2.