Homework 5

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1. First, we solve for the reduced form parameters

$$\begin{aligned} p_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} y_t - \frac{\alpha_2}{\alpha_1 - \beta_1} w_t + \frac{u_t^d - u_t^s}{\alpha_1 - \beta_1} \\ &= \pi_{0p} + \pi_{1p} y_t + \pi_{2p} w_t + v_{pt} \quad (1) \\ q_t &= \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 \beta_2}{\alpha_1 - \beta_1} y_t - \frac{\beta_1 \alpha_2}{\alpha_1 - \beta_1} w_t + \frac{\alpha_1 u_t^d - \beta_1 u_t^s}{\alpha_1 - \beta_1} \\ &= \pi_{0q} + \pi_{1q} y_t + \pi_{2q} w_t + v_{qt} \quad (2) \end{aligned}$$

Let $\pi_p = (\pi_{0p}, \pi_{1p}, \pi_{2p})'$, $\pi_q = (\pi_{0q}, \pi_{1q}, \pi_{2q})'$, $c = (0,0)'$ and $R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then, we set up Wald test on (1) and (2) with restriction $R\pi_p = c$ and $R\pi_q = c$ respectively.

2. (a) *Proof.*

$$\begin{aligned} \hat{\beta}_{OLS} &= (X'X)^{-1} x' y \\ &= \beta + (X'X)^{-1} X' u \end{aligned}$$

If any of the X variables are endogenous, i.e. $E(X'u) \neq 0$

$$E(\hat{\beta}_{OLS}) \neq \beta$$

Otherwise

$$E(\hat{\beta}_{OLS}) = \beta$$

□

(b) *Proof.*

$$\begin{aligned} \hat{\beta}_{IV} &= (Z'X)^{-1} Z' y \\ &= \beta + (Z'X)^{-1} Z' u \end{aligned}$$

Since $E(Z'u) = 0$

$$E(\hat{\beta}_{IV}) = \beta$$

whether or not the X variables are endogenous

□

(c) Let $d = \hat{\beta}_{IV} - \hat{\beta}_{OLS}$. Rewriting the hypothesis testing

$$H_0 : plim\, d = 0 \text{ vs. } H_A : plim\, d \neq 0$$

Denote D by the difference between the asymptotic covariance matrices of the two estimators above.

$$\begin{aligned}
D &= Asy.Var(\hat{\beta}_{IV}) - Asy.Var(\hat{\beta}_{OLS}) \\
&= \frac{\sigma^2}{n} plim \left(\frac{X'Z(Z'Z)^{-1}Z'X}{n} \right)^{-1} - \frac{\sigma^2}{n} plim \left(\frac{X'X}{n} \right)^{-1} \\
&= \frac{\sigma^2}{n} plim \left[\left(\frac{X'(I - M_Z)X}{n} \right)^{-1} - \left(\frac{X'X}{n} \right)^{-1} \right] \\
&= \frac{\sigma^2}{n} plim \left[\left(\frac{X'X - X'M_Z X}{n} \right)^{-1} - \left(\frac{X'X}{n} \right)^{-1} \right]
\end{aligned}$$

Therefore, test statistic is

$$H = d' D^{-1} d$$

3. (a) Taking the log of both sides

$$\log y_i = \log A_i + \alpha \log X_i + u_i$$

Therefore, OLS can be applied to estimate α.

(b) Denote c by the cost function. The firm's maximizing problem is

$$\begin{aligned}
\max \quad & r = y - c \\
s.t. \quad & y = AX^{\alpha} \\
& c = f(X)
\end{aligned}$$

X would be endogenous because the firm is adjusting X to maximizing its profit, i.e. X is no longer exogenous, $E(X'u) = \gamma \neq 0$

(c) A reasonable instrument for X_i could be drought, because it has direct effect on both output and cost and it's obviously exogenous.