Econ 772
Homework 7
Simulation

1) Consider the model

$$y_i = f(x_i, \theta) + u_i, \quad i = 1, 2, .., I,$$

and consider the test

$$H_0 : g(\theta) = 0 \text{ vs. } H_A : g(\theta) \neq 0.$$

Assume there is a test statistic $S(y, x)$ such that

$$plim S(y, x) = 0$$

under H_0. But it is not know what the distribution of $S(y, x)$ is under H_0. Suggest how we can use simulation to construct critical values for $S(y, x)$.

2) Use a computer package of your choice to simulate the following objects:
   a) Let

$$y = X\beta + u,$$
$$u \sim N(0, \Omega)$$

where Ω is the covariance matrix for an AR(1) process. Simulate the distribution of

$$\widehat{\beta} = (X'X)^{-1} X'y.$$

   b) Let

$$u \sim N(0, \sigma^2 I),$$
$$s^2 = \frac{1}{n} \sum_{i=1}^{n} u^2$$

with and without antithetic acceleration. Explain the difference in the variance of your simulators.
   c) Let

$$f(x) = \lambda \exp\{-\lambda(x - \theta)\}.$$

Let $\widehat{\theta}_n$ be the MLE of θ given a sample $\{x_i\}_{i=1}^{n}$. Simulate how the statistical properties of $\widehat{\theta}_n$ change with n.

3) Show that the GHK simulator is an importance sampler.