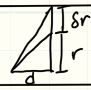
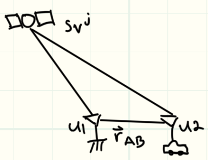
MECH 6970: GPS

2/22/23

Error Mitigation Through Differential GPS

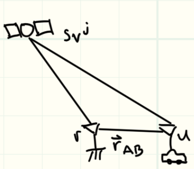
ERROR	Δ
SV Clock	≈ 0
SV Position	≈ 0  δr, r, d $\frac{d}{r}\delta r$
Iono	0.1–1m (dist < 100 km) ↳ depends on TEC b/w 2 paths
Tropo	0.1–0.2m ↳ depends on weather



ERROR SOURCE	GPS (m)	DGPS (m)
Iono	5	~0
Tropo	0.5	~0
SV Clock	1	~0
SV Ephemeris	1	~0
RCVR Noise	0.5	0.5
Multi-Path	0.5	0.5
SA	30	~0
TOTAL	~5-40	~1.0

- LAAS : Local Area augmentation system
  - ↳ local reference station
  - ↳ public or private
- WAAS : Wide area augmentation system
  - ↳ set of sparse reference stations
  - ↳ curve fit corrections based on locations
- SBAS : Satellite based augmentation system
  - ↳ WAAS with corrections uploaded to SV & broadcast back down
- FAA WAS : (Federal aviation association)
  - ↳ 25 reference stations
    - correct : SV Clock, Iono, SV Ephemeris
  - ↳ provides 1-2 m accuracy with high integrity

DGPS & Relative Positioning



$$\vec{X}_u^{corr} = \hat{x}_u^{GPS} + (\hat{x}_r^{GPS} - \vec{x}_r)$$

↑ position correction approach

Psuedorange DGPS

$$\rho_u^j = r_u^j + c(b_u - b^j) + I_u^j + T_u^j + \nu_{\rho_u}^j$$

$$\rho_r^j = r_r^j + c(b_r - b^j) + I_r^j + T_r^j + \nu_{\rho_r}^j$$

$$\vec{x}_u = \vec{x}_r + (\vec{r}_r - \vec{r}_u)$$

$$\vec{x}_u = \vec{x}_r + (\rho_r - c(b_r - b^j) - I_r - T_r - \nu_{\rho_r}) - (\rho_u - c(b_u - b^j) - I_u - T_u - \nu_{\rho_u})$$

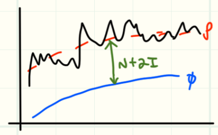
$$\vec{x}_u \approx \vec{x}_r + (\rho_r - \rho_u) - c(b_r - b_u) + \Delta\nu_\rho$$

* $(+\Delta I + \Delta T)$

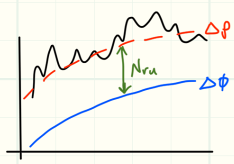
Carrier DGPS

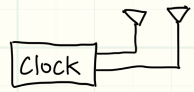
$$\phi^j = \frac{1}{\lambda}[r - I + T] + f_{L1}(b - b^j) + N + \nu_\phi$$

$$\phi_{ru} = \phi_r - \phi_u = \frac{\vec{r}_{ru}}{\lambda} + f_{L1}(b_r - b_u) + (N_r - N_u) + \nu_\phi$$



$$\rho_{ru} = \rho_r - \rho_u = \vec{r}_{ru} + c(b - b^j) + \Delta\nu_\rho$$





$\Rightarrow b_r = b_u$

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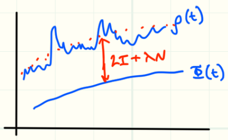
Code-Carrier Combination

$$\rho = r + c(b^j - b) + I + T + \nu_\rho$$
$$\Phi = \lambda\phi = r + c(b^j - b) - I + T + \lambda N + \nu_\Phi$$



- $\rho_{IF} = r + c(b^j - b) + T + \nu_\rho$
- $\Phi_{IF} = r + c(b^j - b) + T + \lambda N + \nu_\Phi$

} dual frequency meas. removes I

- 2 equations, 3 unknowns (ρ_{IF}, I, N)

- Over short periods:

$$\Delta\rho = \rho(t_i) - \rho(t_{i-1}) = \Delta r + c(\underbrace{\cancel{\Delta b^j}}_{\text{good clock/ephemeris}} - \Delta b) + \underbrace{\cancel{\Delta I}}_{\text{const}} + \underbrace{\cancel{\Delta T}}_{\text{const}} + \Delta\nu_\rho$$
$$\Delta\Phi = \Phi(t_i) - \Phi(t_{i-1}) = \Delta r + c(\underbrace{\cancel{\Delta b^j}}_{\text{good clock/ephemeris}} - \Delta b) - \underbrace{\cancel{\Delta I}}_{\text{const.}} - \underbrace{\cancel{\Delta T}}_{\text{const}} + \underbrace{\cancel{\lambda\Delta N}}_{\text{no cycle slip}} + \Delta\nu_\Phi$$

$$\boxed{\Delta\rho \simeq \Delta\Phi}$$

- $\bar\rho(t_i) = \frac{1}{M}\rho(t_i) + \frac{M-1}{M}\left[\rho(t_{i-1}) + \Phi(t_i) - \Phi(t_{i-1})\right]$ } recursive average

M: averaging window (in samples)
$\bar\rho$: carrier smoothed ρ, averaged over window M

- Limited in M (averaging window) by ΔI
  - small window $\approx$ 100 seconds
  - medium window $\approx$ 5–10 minutes
  - large widow > 15 minutes
    - ↳ start to experience code/carrier divergence

Dual Frequency

$$\rho_{L1} = \rho_{IF} + I + \nu_{\rho_{L1}} \qquad \Phi_{L1} = \rho_{IF} - I + \lambda_{L1}N_{L1} + \nu_{\Phi_{L1}}$$
$$\rho_{L2} = \rho_{IF} + \left(\frac{f_{L1}}{f_{L2}}\right)^2 I + \nu_{\rho_{L2}} \qquad \Phi_{L2} = \rho_{IF} - \left(\frac{f_{L1}}{f_{L2}}\right)^2 I + \lambda_{L2}N_{L2} + \nu_{\Phi_{L2}}$$

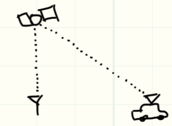
- 4 measurements, 4 unknowns $(\rho_{IF}, I, N_{L1}, N_{L2})$
- Because of Noise, can't difference all unknowns at one time step
  - $\sigma_{\rho_{L1}} \approx 0.5$ m (1–2 m)
  - $\sigma_{\Phi_{L1}} \approx 2$ mm
  - $\lambda \approx 19$ cm
- Over time, can solve for $\rho_{IF}, I, N_{L1}, N_{L2}$ (averaging)

Differential Code Carrier Smoothing

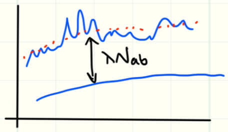
$$\Delta\rho_{ab} = r_{ab} + c b_{ab} + \Delta I_{ab} + \Delta T_{ab} + \nu_{\Delta\rho}$$
$$\Delta\Phi_{ab} = r_{ab} + c b_{ab} - \Delta I_{ab} + \Delta T_{ab} + \lambda N_{ab} + \nu_{\Delta\Phi}$$

- $\Delta I_{ab} \approx 0$
- $\Delta T_{ab} \approx 0$
- $N_{ab} = N_a - N_b = \text{const.}$

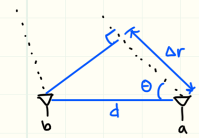
} common mode



$$\Delta\bar\rho_{ab}(t_i) = \frac{1}{M}\rho(t_i) + \frac{M-1}{M}\left[\Delta\bar\rho(t_{i-1}) + \Delta\Phi_{ab}(t_i) - \Delta\Phi_{ab}(t_{i-1})\right]$$



* Now no limit on M using differential measurements



$$\rho_a = \rho_b + \Delta r$$
$$\Phi_a = \Phi_b + \Delta r$$
$$\Delta r = d\cos(\theta) \doteq r_a^j - r_b^j$$

* $u_b^j \approx u_a^j$ because $L \gg d$ *

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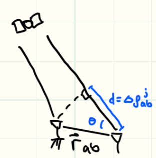
DGPS



$$\rho_a^j = r_a^j + c(b_a - b^j) + I_a + T_a + \nu_\rho$$
$$\rho_b^j = r_b^j + c(b_b - b^j) + I_b + T_b + \nu_\rho$$
$$\phi_a^j = r_a^j + c(b_a - b^j) + T_a - I_a + \lambda N_a + \nu_\phi$$
$$\phi_b^j = r_b^j + c(b_b - b^j) + T_b - I_b + \lambda N_b + \nu_\phi$$

- θ: Angle of Arrival
- $d = \Delta\rho_{ab}^j$: difference in psuedoranges

$$\boxed{d = |\vec r_{ab}|\cos\theta}$$
$$\boxed{|r_{ab}| = \frac{d}{\cos\theta}}$$
$$\boxed{\cos\theta = \frac{\Delta\rho^j}{|r_{ab}|}}$$

$$\Delta\rho_{ab}^j = \underbrace{\vec r_{ab}}_{\text{doesn't need clock correction from ephem}} + \underbrace{c b_{ab}}_{\text{if rcvrs have same clock, this goes away}} + \nu_{\Delta\rho} + \epsilon_{ab}$$
$$\Delta\phi_{ab}^j = \vec r_{ab} + c b_{ab} + \lambda N_{ab} + \nu_{\Delta\phi} + \epsilon'_{ab}$$

Single Difference

$$\rho_a^j - \rho_b^j = \Delta\rho_{ab}^j = \vec r_{ab} + c(b_{ab}) + \nu_{\Delta\rho} + \epsilon_{ab}$$
$$\Delta\phi_{ab}^j = \vec r_{ab} + c(b_{ab}) + \lambda N_{ab} + \nu_{\Delta\phi} + \epsilon'_{ab}$$

$$d = |\vec r_{ab}|\cos\theta \qquad \cos(\theta) = \frac{\Delta\rho^j}{|\vec r_{ab}|}$$
$$|r_{ab}| = \frac{d}{\cos\theta}$$

$$\Delta\rho^j = \begin{bmatrix} u_{ax}^j & u_{ay}^j & u_{az}^j & 1 \end{bmatrix}\begin{bmatrix} r_{ab,x} \\ r_{ab,y} \\ r_{ab,z} \\ c b_{ab} \end{bmatrix}$$

$$\Delta\phi^j = \begin{bmatrix} u_{ax}^j & u_{ay}^j & u_{az}^j & 1 \end{bmatrix}\begin{bmatrix} r_{ab,x} \\ r_{ab,y} \\ r_{ab,z} \\ c b_{ab} \end{bmatrix} + \lambda\left[N_{ab}^j\right]$$

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*** Must have 4 common satellites ***

↳ ρ does not have to be corrected

↳ $\sigma_{\rho_\Delta} = \sqrt{2}\,\sigma_\rho$

Double Difference

- Difference all $\Delta\rho_{ab}$ to a common $\Delta\rho_{ab}^{ref}$ S.V.

$$\nabla\Delta\rho_{ab}^{jr} = \Delta\rho_{ab}^{j} - \Delta\rho_{ab}^{r} \qquad \text{* r is constant (ref.)}$$
$$= r_{ab}^{j} + c(b_{ab}) - r_{ab}^{j} - c(b_{ab})$$

$$\nabla\Delta\rho_{ab}^{jr} = \begin{bmatrix}\Delta u_{ax}^{jr} & \Delta u_{ay}^{jr} & \Delta u_{az}^{jr}\end{bmatrix}\begin{bmatrix} r_{ab,x} \\ r_{ab,y} \\ r_{ab,z}\end{bmatrix}$$

- Gets rid of clock bias (3 unknowns)
- still need 4 SV (1 reference, 3 other)
- $\sigma_{\rho_{\Delta\nabla}} = 2\sigma_\rho$

$$\nabla\Delta\phi_{ab}^{jr} = \begin{bmatrix}\Delta u_{ax}^{jr} & \Delta u_{ay}^{jr} & \Delta u_{ay}^{jr}\end{bmatrix}\begin{bmatrix} r_{ab,x} \\ r_{ab,y} \\ r_{ab,z}\end{bmatrix} + \lambda\left[N_{ab}^{jr}\right]$$

$$\Delta\phi_{ab}^{j} - \Delta\rho_{ab}^{j} = \lambda N_{ab} + \nu_{\Delta\rho\Delta\phi}$$

$$N_{ab} = \left[\frac{\Delta\phi_{ab}^{j} - \Delta\rho_{ab}^{j}}{\lambda}\right]_{\text{rounded}}$$

* $\sigma(\phi-\rho) \approx 1$ m $\Rightarrow$ L1 $\approx$ 5 cycles

↳ moving average for better rounding

With dual Frequency

$\rho_{L1} = r + \nu$ $\qquad \phi_{L1} = r + \lambda_{L1}N_{L1}$ $\qquad$ * 3 unknowns

$\rho_{L2} = r + \nu$ $\qquad \phi_{L2} = r + \lambda_{L1}N_{L2}$ $\qquad$ * 4 equations

IN YE' OLD DAYS

- "Codeless" L2
  - ↳ $\rho_{L1} = r + \nu$ $\qquad$ * 3 unknowns
  - $\phi_{L1} = r + \lambda_{L1}N_{L1}$ $\qquad \phi_{L2} = r + \lambda_{L2}N_{L2}$ $\qquad$ * 3 equations
- "Wide Laning"
  - ↳ $\sin(\omega_1 t)\sin(\omega_2 t) = \underbrace{\sin((\omega_1-\omega_2)t)}_{\text{larger } \lambda} + \underbrace{\sin((\omega_1+\omega_2)t)}_{\text{smaller wavelength}}$
  - ↳ $\phi_{L1L2} = \phi_{L1} - \phi_{L2} = 86.2$ cm
    - bigger wavelength ⇒ smaller search space
- GPS WideLane

	Sum	Diff
L1 L2	10.7 cm	86.2 cm
L1 L5	10.89 cm	75 cm
L2 L5	12.47 cm	586 m

$$\left[\Delta\phi_{ab}^{j} - \Delta\rho_{ab}^{j}\right] = \lambda N_{ab}^{j}$$

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$$\Delta\rho = \vec{r} + cb_{ab} + \nu_{\Delta\rho} = \begin{bmatrix}u_{vx} & u_{vy} & u_{vz} & 1\end{bmatrix}\begin{bmatrix} r_{ax} \\ r_{ay} \\ r_{az} \\ cb\end{bmatrix}$$

$$\nabla\Delta\rho = \vec{r} + \nu_{\nabla\Delta\rho} = \begin{bmatrix}\Delta u_{vx} & \Delta u_{vy} & \Delta u_{vz}\end{bmatrix}\begin{bmatrix} r_{ax} \\ r_{ay} \\ r_{az}\end{bmatrix}$$

$$\Delta\phi = \vec{r} + \lambda N + cb_{ab} + \nu_{\Delta\phi}$$
$$\nabla\Delta\phi = \vec{r} + \lambda N + \nu_{\nabla\Delta\phi}$$

- Mathematically the second frequency improves the 'geometry' to solve for N without directly 'widelaning'

Integer Ambiguity

- Generally:

$$\begin{bmatrix}\Delta\rho_{L1} \\ \Delta\Phi_{L1} \\ \Delta\Phi_{L2}\end{bmatrix} = \begin{bmatrix} u_{vx} & u_{vy} & u_{vz} & 1 & 0 & 0 \\ & & & & 0 & 0 \\ \downarrow & \downarrow & \downarrow & & 0 & 0 \\ & & & \downarrow & \lambda_1 & 0 \\ & & & & 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} r_x \\ r_y \\ r_z \\ N_{L1}^{j} \\ N_{L2}^{j}\end{bmatrix}$$

OR

$$\begin{bmatrix}\Delta\rho_{L1} \\ \Delta\Phi_{L1} \\ \Delta\Phi_{L2}\end{bmatrix} = \begin{bmatrix} u_{vx} & u_{vy} & u_{vz} & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{bmatrix}\begin{bmatrix} r_x \\ r_y \\ r_z \\ cb\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_{L1} & 0 \\ 0 & \lambda_{L2}\end{bmatrix}\begin{bmatrix} N_1 \\ N_2\end{bmatrix}$$

$$y = G\vec{r} + \lambda\vec{N}$$

$L = \text{leftnull}(G)$ $\qquad$ * $L * G = 0$

$L(y = G\vec{r} + \lambda\vec{N}) \Rightarrow Ly = L\lambda\vec{N}$

$y' = H'\vec{N}$

$y' = Ly$

$H' = L\lambda$

$\hat{N} = (H'^T H')^{-1} H'^T y'$

OR USE DOUBLE DIFFERENCE

$$\begin{bmatrix}\nabla\Delta\rho_{L1} \\ \nabla\Delta\Phi_{L1} \\ \nabla\Delta\Phi_{L2}\end{bmatrix}_{ab}^{j} = \begin{bmatrix} u_{vx} & u_{vy} & u_{vz} \\ \downarrow & \downarrow & \downarrow \end{bmatrix}\begin{bmatrix} r_x \\ r_y \\ r_z\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}\underbrace{\begin{bmatrix} N_1 \\ N_2\end{bmatrix}_{ab}^{j}}_{\text{different from } \begin{bmatrix} N_1 \\ N_2\end{bmatrix}}$$

- For single diff., N may vary due to clock drift, with double diff., this isn't a problem since bias & drift are not present

- N const. for $\nabla\Delta\rho$ and $\nabla\Delta\Phi$ with fixed base line

Lambda Method

- $N = \begin{bmatrix} 2.6 \\ 1.4 \\ 10.8 \\ 3.5 \\ \vdots \end{bmatrix}$ * Rounding doesn't work

Round to 11

- $\sigma^2 (H'^T H')^{-1} = \begin{bmatrix} \sigma_{N1}^2 & & & \\ & \sigma_{N2}^2 & & \\ & & \sigma_{N3}^2 & \\ & & & \ddots \end{bmatrix}$ choose smallest variance

- Recalculate with 1 fewer N
  - ↳ Rinse and repeat
- $\Delta\Phi_{L1}' = \Delta\Phi_{L1} + \lambda\hat{N}$
- If $\sigma_{N3} = 0.1$
  - ↳ $N = 10.8 \pm 0.3$ (99%)
  - $10.5 \longleftrightarrow 11.1$
- Removing carrier with $\Delta\Phi_{L1}' = \Delta\Phi_{L1} + \lambda\hat{N}$ does not change the theoretical Noise
- Double Diff. : $N = N_{ab_{L1}}^{ij}$
  $= N_{ab_{L1}}^{i} - N_{ab_{L1}}^{j}$
  $= (N_a^i - N_b^i)_{L1} - (N_a^j - N_b^j)_{L1}$

* Lambda Method code provided by Delft University

- Because all $\hat{N}$'s are uncorrelated, $\sigma^2 (H^T H)^{-1}$ should be diagonal
- Find transformation to force $(H^T H)^{-1}$ to be diagonal in order to solve for $\hat{N}$
- Calculate best (most diagonal) with next best (second most diagonal). If ratio is good enough then it's a success!

$$\begin{bmatrix} \Delta\rho_{L1} \\ \Delta\Phi_{L1} \\ \Delta\Phi_{L2} \end{bmatrix} = \begin{bmatrix} u_{vx} & u_{vy} & u_{vz} & 1 & 0 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & \lambda_1 & 0 \\ & & & & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \\ cb \\ N_1 \\ N_2 \end{bmatrix}$$

↑ basically carrier smoothed DGPS (combined)

$$\hat{N} = (H'^T H')^{-1} H'^T y'$$

$$y = [\Delta\Phi - \lambda\hat{N}] = [u_{vx} \;\; u_{vy} \;\; u_{vz} \;\; 1] \begin{bmatrix} r_x \\ r_y \\ r_z \\ cb \end{bmatrix}$$

↳ only use carrier, cut $\Delta\rho$ rows

$$\hat{r} = (H_\Phi^T H_\Phi)^{-1} H_\Phi^T y_\Phi$$

- Combined $\Delta\rho$ and $\Delta\Phi$ ⟶ "Low Precision" (<1m, <0.5m)
- $\Delta\Phi - \lambda N$ ⟶ "float solution" (<1m, <0.5m)
- $\Phi - \lambda N_{fixed}$ ⟶ "high precision" (2-3 cm)
  - ↳ zero + fixed baseline

Triple Difference

- $\nabla\Delta\Phi_{ab}^{ij}(t_k) - \nabla\Delta\Phi_{ab}^{ij}(t_{k-1}) = \Delta\nabla\Delta\Phi_{ab}^{ij} = r(t_k) - r(t_{k-1})$
- $r(t_k) - r(t_{k-1}) = 0$ ⟶ great for static/fixed baseline
- $\Delta\nabla\Delta\Phi_{ab}^{ij} = [\Delta u_{vx} \;\; \Delta u_{vy} \;\; \Delta u_{vz} \;\; 1] \begin{bmatrix} \Delta r_x \\ \Delta r_y \\ \Delta r_z \\ N \end{bmatrix}$

differenced in time

- Static ⟶ $\Delta\nabla\Delta\Phi_{ab}^{ij} = N_{ab}^{ij}$

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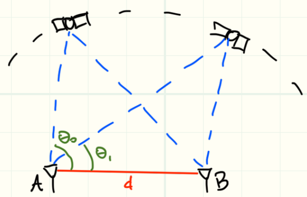
Carrier Model - Single Diff.



$d\cos(\theta_0) = \lambda(\Delta\Phi_0 + N_{ab})$
$d\cos(\theta_1) = \lambda(\Delta\Phi_1 + N_{ab})$

$\boxed{\begin{array}{l} d = \text{const.} \\ N_{ab} = \text{const.} \end{array}}$

$$\Delta\Phi = \left[\frac{\cos(\theta(t))}{\lambda} \quad -1\right]\begin{bmatrix} d \\ N \end{bmatrix}$$

$$\begin{bmatrix} \hat{d} \\ \hat{N} \end{bmatrix} = (H^T H)^{-1} H^T \Delta\Phi \qquad \frac{d\cos\theta_i}{\lambda} - N = \Delta\Phi^i$$

$$P = \begin{bmatrix} \sigma_{\hat{d}}^2 & \\ & \sigma_{\hat{N}}^2 \end{bmatrix} = \sigma_{\Delta\Phi}^2 (H^T H)^{-1}$$

$$\sigma_{\hat{N}}^2 = \sigma_{\Delta\Phi}^2 * \frac{\cos^2(\theta_0) + \cos^2(\theta_1)}{(\cos(\theta_0) - \cos(\theta_1))^2} = \sigma_{\Delta\Phi}^2 * \text{IDOP}$$

* For small IDOP, must wait for significant change in SV position * ~30-60 min

Precise Point Positioning (PPP)

↳ get DGPS-like performance without base station

- NEEDS
  1) More precise positioning
     - ↳ Real Time: Pos. Err. < 10 cm & Clk. Err < 5 ns
     - ↳ Post-Process: Pos. Err. < 5 cm & Clk. Err. < 0.1 ns
     - ↳ available on internet
     - ↳ ijscb.jplnasa.gov
  2) Ionosphere & Troposphere Models
     - ↳ Dual Frequency
  3) Psuedorange and Carrier Meas.
     - ↳ estimate N

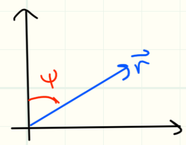
- Accuracy
  - Single Frequency
    - ↳ 0.5 - 1 m
    - ↳ settle time of 30-60 min
  - Dual Frequency
    - ↳ 2 - 20 cm
    - ↳ settle time < 10 min

$$\rho_{IF} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L1} + \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L2} = 2.546\rho_{L1} - 1.546\rho_{L2}$$

$$\rho_{IF} = r + cb + T_z m(el) + \varepsilon_{\rho_{IF}}$$
$$\Phi_{IF} = r + cb + T_z m(el) + \lambda_{IF} N + \varepsilon_{\Phi_{IF}}$$

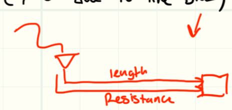
* N_{IF} is combination of N_{L1} and N_{L2} (not an integer)
* If static, T_z changes ~1 cm/hr
- Unknowns: x, y, z, b, T_z, N_{IF}

GPS for Attitude

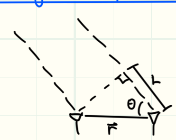


* convert $\vec{r}$ to ENU
* ϕ, θ, ψ = roll, pitch, yaw
* $L = \|\vec{r}\|$

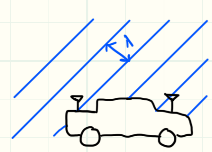
- With 3 antennas can get ϕ, θ, ψ
  - $\psi = \tan^{-1}\left(\frac{\Delta E}{\Delta N}\right)$ * course $\nu = \tan^{-1}\left(\frac{\dot{E}}{\dot{N}}\right)$
  - $\theta = \sin^{-1}\left(\frac{\Delta h}{L}\right)$
  - $\sigma \approx \frac{\sigma_{\Delta E}}{L}$
- Use carrier relative positioning
  - $\vec{r}$ is fixed (sometimes known)
    - ↳ estimating N is easier
    - ↳ easy to detect cycle slip (antenna should not move)
- Use the same clock
  - cb_{ab} = const. ($\neq 0$ due to line bias)



Angle of Arrival Attitude



$L = \vec{r}\cos(\theta)$
$\theta = \cos^{-1}\left(\frac{L}{|\vec{r}|}\right) \Leftarrow$ AoA



$\Delta\Phi = b^T A \hat{s} + \lambda N + B + \nu_{noise}$

- B : line bias
- b : baseline vector (body frame)
- $\hat{s}$: unit vector in ENU (known)
- A : Direction cosine matrix ENU→body
  - ↳ $f(\phi, \theta, \psi)$

- Solve for N, B, A
- Solving for θ, ϕ, ψ in A is a nonlinear iterative search known as Wahba's Problem