# System Response Review

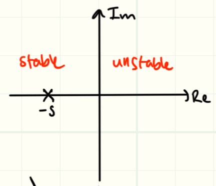
- $1^{st}$ order Response:

$$\dot{y} + sy = 0$$
$$y(t) = Ae^{st}$$

→ $\omega_{BW} = s$ → bandwidth

→ $\tau_c = \frac{1}{s}$ ($4\tau_c = 98\%$ steady state) → time constant

→ $\lambda = -s$ → eigenvalue



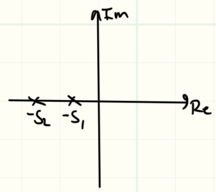

- $2^{nd}$ Order (overdamped) response:

$$\ddot{y} + (s_1+s_2)\dot{y} + s_1 s_2 y = u(t)$$
$$y_h = Ae^{s_1 t} + Be^{s_2 t}$$

→ $\omega_{BW} = s_1$ → slowest eigenvalue

→ $\tau_c = \frac{1}{s_1}$

→ $\lambda = -s_1, -s_2$




- $2^{nd}$ Order (underdamped) Response:

$$\ddot{y} + 2\zeta\omega_n \dot{y} + y = u(t)$$
$$y_h(t) = Ae^{-\zeta\omega_n t}\cos(\omega_D t + B)$$

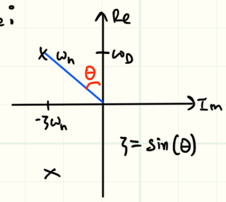
→ $t_{settle} = \frac{4.6}{\zeta\omega_n}$

→ $t_{peaks} = \frac{\pi}{\omega_D}$

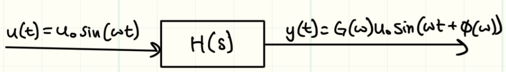
→ $t_{rise} = \frac{1.8}{\omega_n}$

→ $\%_{overshoot} = e^{\frac{-\zeta\omega_n}{\sqrt{1-\zeta^2}}}$

→ $\omega_{BW} \approx \omega_n$ ($\omega_{BW} = \omega_n$ if $\zeta = 0.707$)




## System Frequency Response




- where $G(\omega) = |H(j\omega)| = \frac{|num(j\omega)|}{|den(j\omega)|}$

$$\phi(\omega) = \angle H(j\omega) = \angle num(j\omega) - \angle den(j\omega)$$

# Control Loops

- Closed-Loop Transfer functions:

$$\frac{Y(s)}{R(s)} = \frac{KP}{1+KPS}$$ * ref → output

$$\frac{E(s)}{R(s)} = \frac{1}{1+KPS}$$ * ref → error

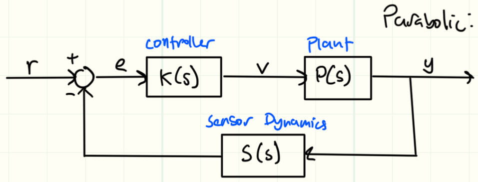
- Final value theorem:

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

- Laplace of Different Inputs: Step: $\frac{r}{s}$

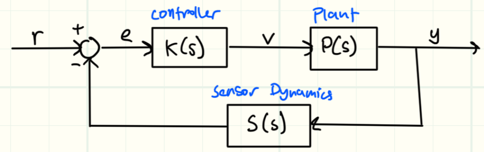
  Ramp: $\frac{r}{s^2}$

  Parabolic: $\frac{r}{s^3}$




## System Type

- Consider the following feedback loop




- System type is defined by the total number of pure integrators (eigenvalues at 0) of the open loop transmission $L(s) = K(s)P(s)S(s)$

  → sum of integrators in $K(s), P(s), S(s)$

  → dictates steady state error to various reference input types

- Type 0 (no integrators)

  → constant steady state error to a step (constant) input

  → INFINITE steady state error to ramp or parabolic input

- Type 1:

  → 0 SS error to step input

  → constant SS error to ramp input

  ↳ error depends on controller gain and ramp slope

  → infinite SS error to parabolic input

- Type 2:

  → 0 SS error to step or ramp input

  → Constant SS error to parabolic input

  ↳ error depends on controller gain and coefficient in front of parabolic

## Why Important In GPS?

→ Tracking of incoming signal is feedback loop

→ Reference input is a function of the line of sight signal

→ Two types of signal tracking

- tracking phase
- tracking frequency

- Tracking Phase

  → constant range results in a ramp reference

  ↳ phase is changing over time

  → constant range rate results in a parabolic reference

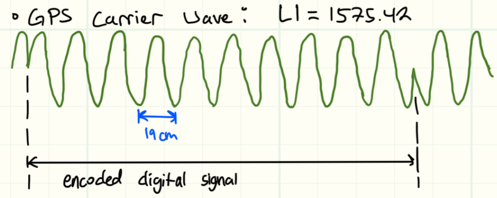
- Tracking Frequency

  → constant range rate results in constant reference

  → constant range acceleration results in ramp reference

# GPS Signal

- GPS Carrier Wave: L1 = 1575.42




digital code:

- SV #
- time
- position
- velocity

- -160 dBW ~ $10^{-16}$ W
  ~ equivant to viewing a 25W lightbulb from 10,000 miles away

# GPS Broadcast Signal Structure

- Each satellite transmits the precise time (UTC-USNO), the complete parameters of its orbit, and the major parameters of all other SV orbits
  ↳ ephemeris
- Navigation message (includes ephemeris) is 30s long and is transmitted in signal form at a rate of 50 $\frac{\text{bits}}{s}$
- This data transmission modulates the GPS carrier wave using binary phase-shift keying (BPSK)

## Gold Codes + Spread-Spectrum Transmission

- Gold codes are a family of unique binary sequences which have very low cross-correlation with other sequences in the family and low auto-correlation as well
- Modulating each GPS SV's signal by a unique Gold Code, known as the PRN number, spreads the signal over a wider bandwidth, which provides noise rejection and enables multiple access (CDMA: code division multiple access)
  ↳ allows SV's to transmit at same time and frequency without interfering with each other

## Carrier Wave

- L1 at 1575.42 MHz (154 * 10.23 MHz)
- L2 at 1227.60 MHz (120 * 10.23 MHz)
- L5 at 1176.45 MHz (115 * 1023 MHz)
- Modulated with code and navigation data using Binary Phase-Shift Keying (BPSK)
- C/A and P(y) are transmitted orthogonally on L1
  → and now on L2 (called L2C) with the newer SVs

## Code Signal

- Code division multiple access (CDMA)
- Course Acquisition - C/A
  → Gold Code (Period of 1ms)
- Precision Code – P(Y)
  → Anti-Spoofing mode
  → code reset each week
  → encrypted (authorized users only)

## Received GPS signal of $j^{th}$ Satellite

$$S_{L1}^{j}(t)=\sqrt{2P_{C1}^{j}}X^{j}(t)D^{j}(t)\cos(2\pi f_{L1}^{j}t+\theta_{L1})+\sqrt{2P_{Y1}^{j}}Y^{j}(t)D^{j}(t)\sin(2\pi f_{L1}^{j}t+\theta_{L1})$$

$$S_{L2}^{j}(t)=\sqrt{2P_{Y2}^{j}}Y^{j}(t)D^{j}(t)\sin(2\pi f_{L2}^{j}t+\theta_{L2})$$

- NOTE: received frequency includes doppler ($f_{L1}^{j}=f_{L1}+f_{doppler}^{j}$)
  → P: received signal power
  → X: C/A code, PRN, gold code (±1)
  → D: Data bit (± 1)

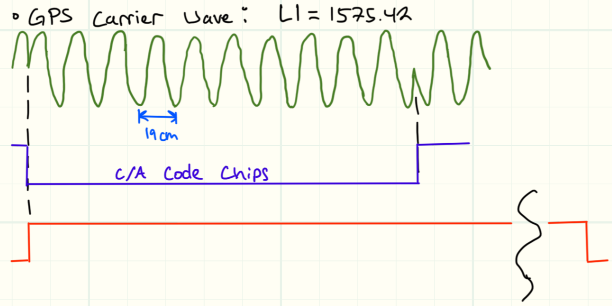
## Codes

- C/A Code Chip = $\frac{1}{1023}$ ms (≈ 1 μs or 300m)
  ↳ 1023 chips long = 1 ms
- Data bit = 20 ms (50 Hz)
  ↳ 20 C/A code repititions in a single data bit
  ↳ 12.5 minutes long (ephemeris + clock params repeat every 30s)
- P Code is $10^{14}$ chips (repeats every week)
  ↳ chip rate is 10.23 Mchip/sec (≈ 30m)
  ↳ requires C/A code to find place in P Code (or precise time)
  ↳ P code has been encrypted since 1994: P(Y) Code
    ↳ authorized users only

## GPS Signal

S = PRN * Data * carrier

- GPS Carrier Wave: L1 = 1575.42




- Data and Code are Modulo 2 Addition:
  → 0 + 0 = 0    * 0 is really -1
  → 1 + 1 = 0
  → 0 + 1 = 1
  → 1 + 0 = 0