Recursively evaluating some low values of $C(n_1, n_0)$.

We make use of equations 1, 2, 3, and 4.

```
c00 = (Cosh[γ])^(-1/2);

(* Evaluating Eq. 1 with n0 = 1 *)
c11 = (Cosh[γ])^-1 c00 ;

(* Evaluating Eq. 3 with n0 = 1 *)
c02 = Simplify[2^(-1/2) (- Sinh[γ]) c11 ];

(* Evaluating Eq. 1 with n0 = 3 *)
c13 = Simplify[(Cosh[γ])^-1 c02 3^(1/2)];

(* Evaluating Eq. 2 with n0 = 2 and n1 = 1 *)
c22a = Simplify[(Cosh[γ] 2^(1/2))^-1 (c11 2^(1/2) + c02 Sinh[γ])];

(* Evaluating Eq. 4 with n0 = 2 and n1 = 1 *)
c22b = Simplify[(- Sinh[γ] 2^(1/2))^-1 (c13 3^(1/2) - c02 Cosh[γ])];

c22 = c22b;

(* Evaluating Eq. 3 with n0 = 3 *)
c04 = Simplify[4^(-1/2) (- Sinh[γ]) c13];

(* Evaluating Eq. 1 with n0 = 5 *)
c15 = Simplify[Cosh[γ]^-1 c04 5^(1/2)];

(* Evaluating Eq. 2 with n0 = 4 and n1 = 1 *)
c24a = Simplify[(Cosh[γ] 2^(1/2))^-1 (c13 4^(1/2) + c04 Sinh[γ])];

(* Evaluating Eq. 4 with n0 = 4 and n1 = 1 *)
c24b = Simplify[(Sinh[γ] 2^(1/2))^-1 (c04 Cosh[γ] - c15 5^(1/2))];

c24 = c24a;

(* Evaluating Eq. 4 with n0 = 0 and n1 = 1 *)
c20 = Simplify[(Sinh[γ] 2^(1/2))^-1 (c00 Cosh[γ] - c11)];
```

Miscellaneous

Some calculations for solving the PDE of *F(x,y)*

```
p = (x + 1) (y - 1) / ((x - 1) (y + 1));
```

```
In[ ]:= Simplify[1/Cosh[γ] ((p + 1)^2 - Tanh[γ]^2 (p - 1)^2)/(4 p))^(-1/2), Assumptions → x > 0 && x < 1 && y > 0 && y < 1]
```

$$\text{Out[ ]=}\ \frac{\sqrt{(-1+x^2)(-1+y^2)}\ \text{Sech}[\gamma]}{\sqrt{(-1+x\,y)^2-(x-y)^2\ \text{Tanh}[\gamma]^2}}$$

Calculating the first 100 values of $D(n_1, n_0)$

F(x,y) is found in equation 12, and the form for $D(n_1, n_0)$ is given in equation 13. The 100 values determined below were used to generate Table 1.

```
In[1]:= F[x_, y_] := ((x y - 1)^2 Cosh[γ]^2 - (x - y)^2 Sinh[γ]^2)^(-1/2);
```

```
In[ ]:= d[n1_, n0_] :=
  Simplify[1/(n1! n0!) D[F[x, y], {x, n1}, {y, n0}] /. x → 0 /. y → 0, Assumptions → γ > 0]
```

```
In[ ]:= N[Table[d[n1, n0], {n1, 0, 9}, {n0, 0, 9}] /. γ → 1, 2]
```

```
Out[ ]= {{0.65, 0, 0.19, 0, 0.082, 0, 0.040, 0, 0.020, 0},
 {0, 0.27, 0, 0.24, 0, 0.17, 0, 0.12, 0, 0.076},
 {0.19, 0, 0.011, 0, 0.085, 0, 0.13, 0, 0.13, 0},
 {0, 0.24, 0, 0.055, 0, 0.00018, 0, 0.020, 0, 0.057},
 {0.082, 0, 0.085, 0, 0.12, 0, 0.061, 0, 0.012, 0},
 {0, 0.17, 0, 0.00018, 0, 0.046, 0, 0.082, 0, 0.067},
 {0.040, 0, 0.13, 0, 0.061, 0, 0.0011, 0, 0.019, 0},
 {0, 0.12, 0, 0.020, 0, 0.082, 0, 0.050, 0, 0.0067},
 {0.020, 0, 0.13, 0, 0.012, 0, 0.019, 0, 0.059, 0},
 {0, 0.076, 0, 0.057, 0, 0.067, 0, 0.0067, 0, 0.0097}}
```

Showing visually the first 21 *D(n,n)*

```
In[ ]:= nk = Table[d[n, n], {n, 0, 20}];
```

In[ ]:=
```
p = ListPlot[Table[{n - 1, nk[[n]]}, {n, 1, 21}] /. γ → 0.4,
    PlotRange → {{0, 21}, {0, 1}}, Frame → True, GridLines → Automatic,
    PlotStyle → Black, AxesStyle → {Black}, AspectRatio → 2/3, ImageSize → Medium];
Show[
 p]
```

Out[ ]=