VOLUME, SHAPE, AND ROUNDNESS OF ROCK PARTICLES

HAKON WADELL
University of Chicago

ABSTRACT

The present paper deals with the fundamental principles, conceptions, and ideas upon which some new sedimentological research methods are based.

Size classification of particles is commonly based on either the arithmetic or the geometric mean of the greatest, intermediate, and least diameters. The values obtained in both cases are dependent upon two factors: the shape and the volume of the particle. Two particles, one having diameters of extremely different values, the other being a sphere with the three diameters consequently of the same length, may both have the same "size" value as a result of the arithmetic or geometric mean of their diameters, yet they may differ distinctly in respect to both shape and volume. Shape and volume are factors independent of each other, and a distinction between the two is of importance for the interpretation of a vast number of phenomena.

No agreement has ever been reached in defining the three diameters of a particle of non-spherical shape. Some investigators require that the three diameters meet at right angles; others stipulate that the lines of measurement be at right angles to each other, but do not require a common point of crossing. In the last case the mean diameter is ordinarily computed as "the mean of length, breadth, and thickness" of the particle. Here again we enter upon ambiguous conceptions (breadth and thickness of an irregular solid?) which become still more confusing when dealing with particles with re-entrant angles.

Ordinarily the length is synonymized with the distance between the two most distant peripherical points situated opposite to each other. Maintaining this view and the rule that the diameters should run at right angles to each other, the mean diameter, for instance, of a cube of 2 mm. edge, equals the diagonal of the cube, i.e., 3.46 mm. The diameter of a sphere of the same volume as the cube amounts to 2.48 mm. It is evident that the high value of the mean diameter of the cube is due to the shape. The mean diameter has no significance whatsoever, except that it gives a general idea of the "size" of the particle. An attempt to use the mean diameter as a basis for calculating the volume of the particle gives values all out of proportion to the actual value of the volume. For instance, in the given example, 3.46^3 equals 41.42 cubic millimeters, or figuring on a sphere of a diameter of 3.46 mm. the volume amounts to about 22 cubic millimeters; whereas, the true volume of the cube equals 8 cubic millimeters. It is, of course, understood that the true volume of a particle of cubic shape may be readily obtained when the length of its edge is known, but, since rock particles, in general, do not attain such regular shape, the example tends to show the erroneous impression of the volume of a particle when the mean diameter, as commonly defined, is used as an expression for "size."

The "size" of a particle is best expressed by its simple volume value, because this value remains always unaffected by the shape. Sometimes the "size" may advantageously be expressed by the value of the *true nominal diameter*, which is the diameter of a sphere of the same volume as the particle (not to be confused with the *equivalent diameter* [equivalent radius, Odén, 1915] which is the diameter of a sphere having the same settling velocity as the particle).

SHAPE OF PARTICLES

A solid may be spherical, cylindrical, cubical, etc. To express its form it is necessary to adopt a standard shape for comparison.

Transportation of sedimentary particles either by wind or water involves rolling and suspension, and between these two an intermediate mode of dislodgment, saltation. The spherical form is a determinant factor in sorting the particles in traction by rolling, because spherical particles roll faster than non-spherical. When dealing with tractional particles the sphere may therefore appropriately be taken as a standard of comparison.

When dealing with particles in suspension the expression of shape must theoretically be seen from another angle, viz., the area of the grain surface. A sphere has the greatest relative volume with the smallest surface area and, therefore, has a greater settling velocity than any other shape of the same volume and density. Progressively greater departure from spherical shape means progressive increase of surface without change of volume and, therefore, also, a decrease of the settling velocity of the solid.

It is evident that the most exact shape expression, approximately reflecting the behavior of a particle in suspension, is the ratio of the surface area of a sphere, of the same volume as the particle, to the actual surface area of the particle; expressed by the formula

$$\frac{s}{S}=\text{Degree of true sphericity,}$$

where s is the surface area of a sphere of the same volume as the particle, and S is the actual surface area of the particle. The maximum value obtained by this formula is 1.000 which is the numerical value of the shape of a sphere. All other shapes have values less than 1.000.

The shape may also be expressed by the numerical value of the *standard surface area* which is the area of a solid having a volume of the numerical value 1 and the same shape as the particle. The numerical value of the standard surface area of a sphere is 4.8359, that of a cube 6.000, etc. Designating the true sphericity by the Greek letter ψ and the standard surface area with the Greek letter σ, the two conceptions are related as follows:

$$\frac{4.8359}{\sigma}=\psi$$

where *4.8359* is the numerical value of the standard surface area of a sphere.

The *sphericity* and the *standard surface area* as expressions for shape serve different purposes. The sphericity value has for several reasons been found to be best suited for expressing the shape of sedimentary particles.

The shape of particles has scientific, industrial, and commercial importance. Attempts have been made to analyze sediments on the basis of the sphericity of the individual particles composing the sediment. The results show that the shape of the particles is a very sensible factor in the accumulation of the sediment. Distinction between sediments mainly accumulated by traction and mainly settled direct from suspension is eventually made possible. The accumulation of the two types of sediments follows distinctly different laws, the operation of which influences the proportion of particles of different shapes by sorting. Correlation of oil sands on the basis of shape and roundness of the grains is suggested, but attempts have not as yet been made. However, others claim successful results by optical comparison with a more or less arbitrary set of standard "roundness" grades consisting of representative grains. By a clear mathematical distinction between shape and roundness, correlation on the basis of these properties is made possible without any influence of personal opinion. It is generally assumed that the shape of particles plays an important rôle in ore treatments and in respect to the properties of cement, concrete, pigment powder, and other industrial products. The commercial prices of granular products are frequently influenced by the compactibility of the material, which in turn depends largely on the form of the particles. Both the so-called *volume-weight* and the *sediment-volume* are to no small extent influenced by the shape of the particles. To what extent shape can be substituted for fineness without altering properties of material, and vice versa, is not known. I refer in the last case to the *specific surface*, the value of which evidently can be changed in two ways, either by changing the shape or else the fineness of the particles.

ROUNDNESS OF PARTICLES

A solid may possess a maximum degree of roundness and still not be a sphere, or have a high degree of sphericity and no roundness. The distinction is illustrated by the almost spherical, dodecahedral forms of garnet, which, if not worn, exhibits non-rounded, sharp corners with large obtuse angles.

A cylinder terminated at each end by a half sphere, Figure 1*b*, cannot become more rounded, and yet the cylinder is not a sphere. A cylinder of the described type may well be worn down to a sphere, but the radius of curvature of its ends must during the process of wear always remain equal to the radius of the maximum inscribed circle in the longitudinal section of the solid, assuming that no decrease of the degree of roundness takes place during the process of wear.

A corner may be defined as every such part of the outline of an area (projection area) which has a radius of curvature equal to or less than the radius of curvature of the maximum inscribed circle of the same area. Thus a corner has reached its maximum degree of roundness when the radius of its curvature equals the radius of the maximum inscribed circle; an increase of its radius of curvature over that of the inscribed circle results inevitably in one, two, or several new corners of smaller radii of curvature, i.e., less degree of roundness (Fig. 1*c*).

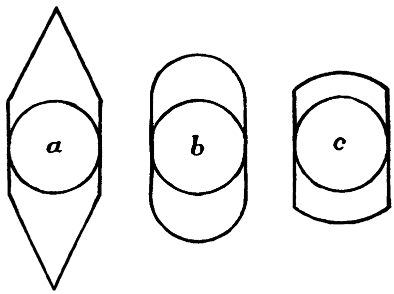

FIG. 1.—Illustration of the conception "roundness."

While sphericity is essentially a three-dimensional conception, roundness is obtained by measurements in one plane only. Strictly speaking, the total roundness of a solid is achieved by measurements in three planes at right angles to each other, but two planes are in most cases sufficient, while one plane is satisfactory when dealing with small sedimentary particles.

The curvature of the corners of a sand grain may easily be measured if submitted to sufficient magnification. The values obtained, however, would not be directly comparable with the same kind of values of a boulder. It is evident that two objects of such different size cannot be measured with equal delicacy. (If the point of a needle is submitted to sufficient magnification it appears blunt and its radius of curvature may readily be measured, while to the naked eye it is sharp.) In order to obtain comparable values a standard size must be adopted. Large objects such as boulders and cobbles must be reduced, and small ones like sand grains magnified to approximately the same size, i.e., the standard size, on which the measurements are performed. The average diameter of the standard size used for measurement of rock particles has been fixed to 70 mm., and microscopic particles have been enlarged to about that size by camera lucida or screen projection.

Since the radius of curvature of a corner may attain any value up to the maximum which equals the value of the maximum inscribed circle, the "roundness" of a corner may be expressed by the ratio $\frac{r}{R}$, where r is the radius of curvature of the corner and R is the radius of the maximum inscribed circle in the plane of measurement. The maximum value for $\frac{r}{R}$ is 1.000 for a corner of maximum roundness.

The total roundness of a solid in one plane may be obtained by taking the arithmetic mean of the roundness of the individual corners in that plane. Thus the formula for roundness reads:

$$\frac{\Sigma\left(\frac{r}{R}\right)}{N} = \text{Degree of roundness of a particle in one plane,}$$

where $\Sigma\left(\frac{r}{R}\right)$ is the sum of the roundness values of the corners, and N is the number of corners of the particle in the given plane. The maximum value for roundness achieved by this formula is 1.000 for a particle of maximum roundness in the given plane. Thus a sphere has a roundness value 1.000 and also a sphericity value 1.000.

A formula, sometimes giving a slightly different roundness value than that obtained by the one above, has also been used by me for determining the roundness of sedimentary rock particles. It reads:

$$\frac{N}{\Sigma\left(\frac{R}{r}\right)} = \text{Degree of roundness of a particle in one plane.}$$

The maximum value for roundness achieved by this formula is also 1.000 for a particle of maximum roundness in the given plane. The reason for the preference for the second formula is beyond the scope of the present paper.

Rounding of sedimentary particles is a special type of disintegration attributed to attrition and solution. Roundness is destroyed or diminished by fracturing and chipping, and high degree of roundness is, therefore, often an indication of gentle conditions of wear relative to the size, hardness, and toughness of the particle. Further discussion on the sedimentological significance of roundness of rock particles is also beyond the scope of the present paper.

THE IMAGE

The sphericity value expresses the shape, and the roundness value gives a summarized expression for certain detail characteristic of the solid; sphericity and roundness together express the image of the solid.

$$\frac{\text{Roundness}}{\text{Sphericity}} = \text{Image of solid,}$$

or substituting numerical values, e.g., $\frac{.48}{.83}$, we have *the numerical expression of an image, having 0.48 degree of roundness in a shape of 0.83 degree of sphericity.*

Although roundness and sphericity values meet in a maximum value 1.00 in the numerical expression of the image of a sphere, i.e., $\frac{1.00}{1.00}$, their values are not necessarily increased equally by wear toward that maximum. Sphericity and roundness have one thing in common, viz., their values increase by attrition, and sometimes by solution, of the solid, but they are not increased proportionally. Chipping of a particle may increase the sphericity, but it decreases the roundness. For this reason, and since equal numerical values may occur for roundness and sphericity, e.g., $\frac{.90}{.90}$, it is obvious that the image cannot be expressed as a ratio of the roundness value to the sphericity value. The double line separating the values is used to indicate that the image expression is not a ratio.

Figure 2 gives an illustration of sand grains of different image values. The particles were picked from different grade sizes of grains of St. Peter sandstone and enlarged to approximately the same size. The grains of the upper row vary slightly in respect to sphericity, but the roundness decreases from left to right. In the lower row the roundness varies considerably, but the sphericity decreases from left to right.

In judging the roundness, attention should be given to the general "sharpness" of the corners. The eye is more impressed by the expressiveness of the roundness values when grains of greatly different roundness values are compared. The first and second grains from the left in the upper row have the same sphericity, but different roundness values. The second, fourth, and sixth grains in the same row are of equal sphericity but show marked difference in "sharpness." Attention is also called to the first and second particles from the

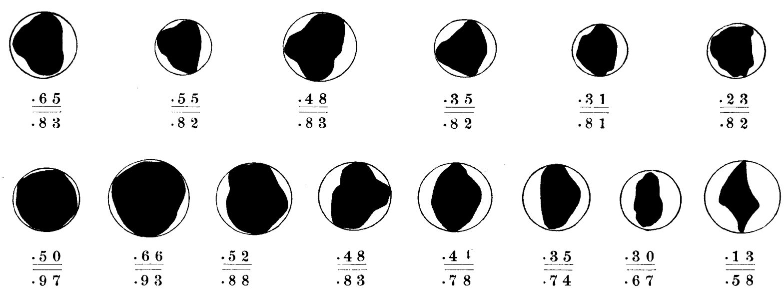

FIG. 2.—Images of sand grains illustrating the expressiveness of the image values

left in the lower row; the first has a lower roundness, but higher sphericity, than the other. The circles around the particles serve no other purpose than to facilitate the impression of the sphericity.

SUGGESTIONS FOR SEDIMENTOLOGICAL RESEARCH

The deposition of sediments is a complex process involving interactions which have thus far baffled mechanical analysis. The results achieved by current sedimentological analyses are influenced by a variety of factors such as the type of analysis employed—sedimentation, elutriation, and sieving—the statistical method used in compilation of the data, variation of the mineral composition, uniformity of the grain size, variation in shape of the particles, concentration of certain minerals in certain grade sizes, etc. When several such factors are summarized in a single graphic presentation, an insight into the true nature of the sediment is not achieved, and the actual changes which take place in a sediment from one locality to another is but faintly realized.

An attempt has been made by the present writer to obtain data on some of the important properties of the individual particles which compose a sediment, thereby making it possible to group a given number of investigated particles at will on the basis of any specified property. For instance, a classification of the particles may be achieved on the basis of their chemical composition and specific gravity; the obtained classes may be subdivided according to volume of the individual particles, and these subclasses then divided on the basis of the shape and roundness, etc., of the particles. By this method, or some intelligent variation of it, according to the purpose of the research and the nature of the sediment, data are secured which may furnish a clue to the origin of the deposit and the factors involved in the changes of a sediment from one locality to another.

An account of the results achieved and the methods used for obtaining numerical values of the various properties—volume, shape, roundness, etc.—of sedimentary particles has been prepared for publication in the near future.