

# Jacob's Bucket Problem (aka deriving circular caustics)

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## Abstract

*In this paper, I derive the equation for the first-order and second-order circular caustics—the envelope of once-reflected and twice-reflected light rays reflecting inside a circle or cylinder—for light rays emanating from a source inside the circle. When extended from a circle to a cylinder, this problem is equivalent to determining the equations for the shapes seen when shining a light into a reflective cylinder. In this paper, a computer simulation is also used for the more complex cylindrical problem.*

## Description of Problem

This is a problem I've had in the back of my mind for several years now.

Whenever you shine light into a cylindrical object with a glossy or reflective interior, these intricate cusped shapes appear on the base. The shapes appear to be similar to cardioids. The question is: what is the mathematical basis for these shapes?

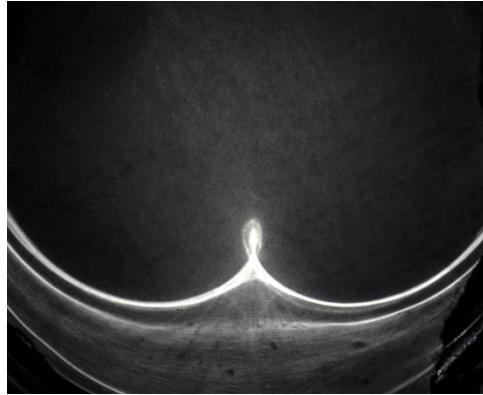


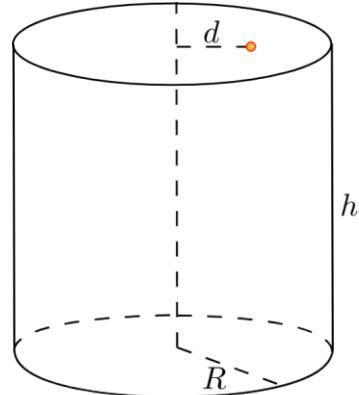
Image captured by laying down a piece of paper on the bottom of a metal rice cooker. The contrast is enhanced to improve the clarity of the image.



Image from Mathematics Stack Exchange

## Theoretical set-up

As with any problem, we need to make some simplifying assumptions! Suppose the light source is a height  $h$  above the base of the bucket (radius  $R$ ) and the bucket's walls extend arbitrarily high. Furthermore, the light source is assumed to be very small, scattering light evenly in all directions. Looking directly down into the bucket, the light source is offset from the center-axis by a distance  $d$ .



Orange dot: light source.

## Computer simulation

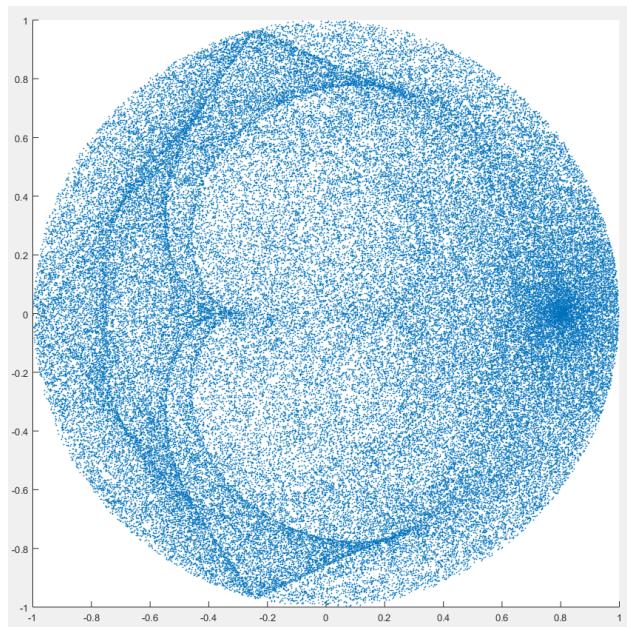
I used a MATLAB program to model the reflection of light inside the cylindrical bucket, plotting the places where the light hits the base of the bucket.

### Method

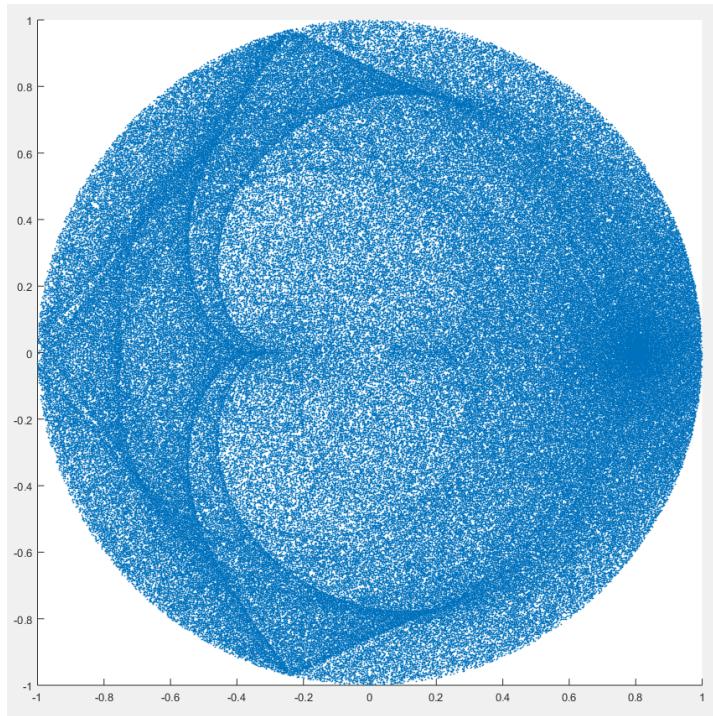
First, I randomized the direction of the light to go in all directions. Then, I move each particle of light in that direction until it hits a wall, after which the direction of the light particle is changed accordingly. Once the light particle hits the base of the bucket, I store the location of impact. Then, using all the locations of impact, I create a plot. I have the MATLAB code for this program in Appendix A.

### Results

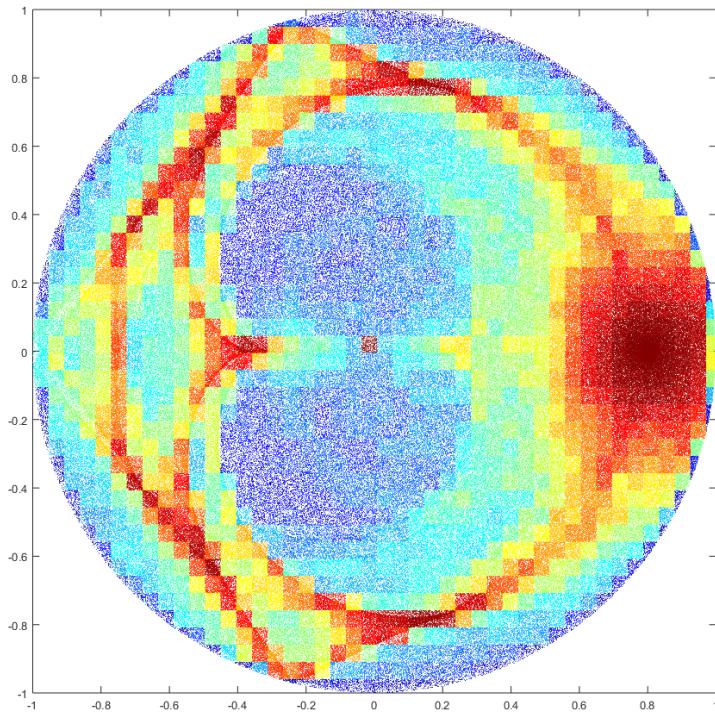
These are the results of the simulation for the parameters  $R = 1, d = 0.8, h = 2$ . Additional plots can be found in Appendix B.



100,000 points (MATLAB generated)



333,333 points (MATLAB generated)



1,000,000 points scatter-heat map (MATLAB generated)

Hopefully these three different visualizations of the data can give you some idea of the complex and interesting shapes, each of different intensity, formed in this extraordinarily simple configuration.

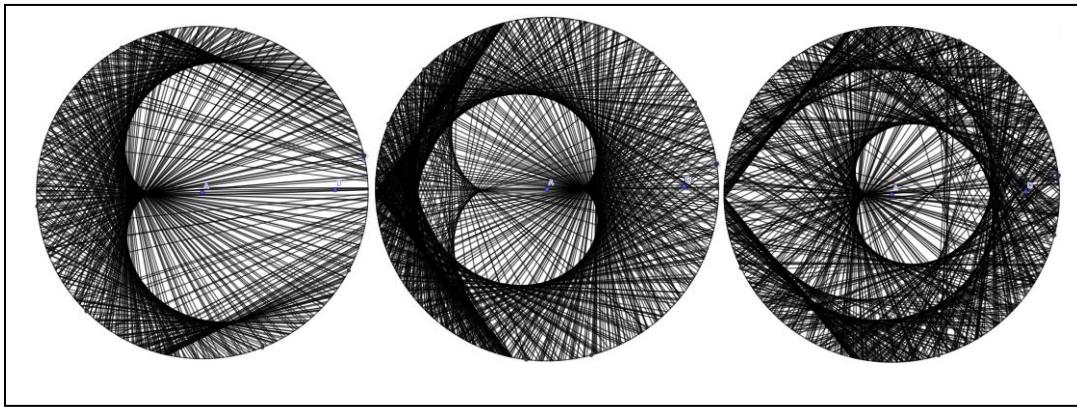
## Mathematical Derivation

### Ray Density

To understand the geometry of the shapes produced on the bottom of the bucket, it is first necessary to reduce the problem into a two-dimensional one. How do we determine where the light ray is stopped by the bottom of the bucket? Imagine it is simply travelling and reflecting in the plane of the circular cross section of the bucket. Then after a time  $t$  that is dependent on the height  $h$  and angle of depression  $\alpha$ , it will be stopped, and we record the point where it stops as a point of light on the bottom of the bucket.

So the shapes formed in the bucket are closely related to the possible configurations of reflected light rays within the circle! Given the set of all ray configurations (if you traced a ray and all its reflections within the circle), the "density" of lines in a certain area will increase the probability that the light ray is "stopped" within that area\*. If a point on the circle has a high likelihood of being the location where a light ray is "stopped," then it will form a shape on the bottom of the bucket. So the problem of the ray "density" is essentially\* equivalent to the problem of the shapes on the bucket.

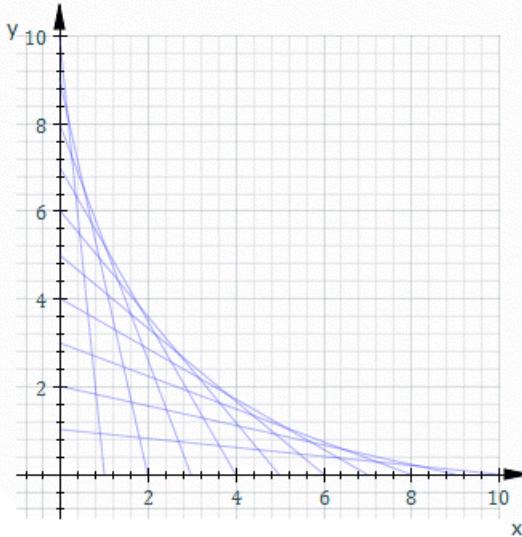
\*This is not exactly true, the probability that a light ray is "stopped" only after a very long time is exceedingly small because  $\alpha$  would have to be very near zero when in fact the light rays are distributed uniformly over  $\alpha$ . In other words, the "probabilities" we are speaking of are tied to the distribution of  $\alpha$ , which we will ignore here. This does not affect our analysis of the shapes formed on the bottom of the bucket, but it would affect the intensities of the shapes formed on the bottom of the bucket.



The plots above were generated in Geogebra with  $d = 0.8R$ . You can very clearly see that the shapes formed match those that form on the bottom of the bucket. The first image is the set of all possible rays following one reflection inside the circle, the second image is the set of all possible rays following exactly two reflections inside the circle, and the third image is the set of all possible rays following exactly three reflections inside the circle. You could continue on, but the successive images are not as visible on the bucket because it is not likely that a ray will have the space to reflect so many times.

### Envelopes and Caustics

The very loose idea of the ray "density" can be formalized with the idea of **envelopes**. For a set of curves (lines in this case), the envelope is a curve that is tangent to all the curves and formed by the points of tangency. When used to describe reflections of light rays, these envelopes are known as **caustics**.



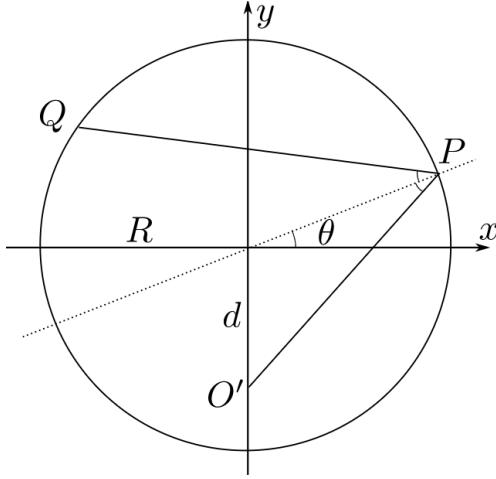
Envelope example (image from Wikipedia)

In the three images presented above, the envelopes are very clearly visible, and they very clearly correspond to the same shapes that appear in the simulation. Mathworld gives the mathematical definition as: for a one-parameter family of curves given by  $U(x, y, c) = 0$ , the envelope is the set of points  $(x, y)$  such that there exists a number  $c$  where  $\frac{\partial U}{\partial c} = 0$  and  $U(x, y, c) = 0$ .

### First-order Caustics

We are looking for a way to express the set of all possible ray reflections in a similar way as the range of a function of  $x, y$  and some other parameter:

Suppose the light source is located at the point  $(0, -d)$ , and the parameter  $\theta$  is the angle made between the  $x$ -axis and the ray from the origin to the point where the light hits the circle, as shown in the diagram.



Now we derive the equation for line  $PQ$  in terms of  $x$ ,  $y$ , and  $\theta$ .

Consider the vectors  $PQ$ ,  $PO$ , and  $PO'$ . Note that

$$PQ = PO' - PO = 2(\text{proj}_{PO} PO' - PO).$$

Since

$$\text{proj}_{PO} PO' = \frac{PO' \cdot PO}{|PO|^2} PO,$$

We find that

$$PQ = \frac{2PO' \cdot PO}{|PO|^2} PO - PO.$$

We can find

$$\begin{aligned} PO &= (-R \cos \theta, -R \sin \theta) \\ PO' &= (-R \cos \theta, -R \sin \theta - d) \end{aligned}$$

We can plug this in to find  $PQ$ :

$$\begin{aligned} PQ &= \frac{2(R^2 \cos^2 \theta + R^2 \sin^2 \theta + Rd \sin \theta)}{R^2} (-R \cos \theta, R \sin \theta) + (R \cos \theta, R \sin \theta + d) \\ PQ &= \left( 2 + \frac{2d}{R} \sin \theta \right) (-R \cos \theta, -R \sin \theta) + (R \cos \theta, R \sin \theta + d) \\ PQ &= \left( R \cos \theta \left( -1 - \frac{2d}{R} \sin \theta \right), R \sin \theta \left( -1 - \frac{2d}{R} \sin \theta \right) + d \right). \end{aligned}$$

Simplified, this is

$$PQ = (-R \cos \theta - 2d \sin \theta \cos \theta, -R \sin \theta + d(1 - 2 \sin^2 \theta))$$

$$PQ = (-R \cos \theta - d \sin 2\theta, -R \sin \theta + d \cos 2\theta).$$

Now, the line  $PQ$  is given by point slope form:

$$y - R \sin \theta = \frac{R \sin \theta - d \cos 2\theta}{R \cos \theta + d \sin 2\theta} (x - R \cos \theta).$$

Rearranging to slope-intercept form, the intercept is

$$\begin{aligned} & \frac{-R^2 \sin \cos \theta + Rd \cos \theta \cos 2\theta + R^2 \sin \theta \cos \theta + Rd \sin \theta \sin 2\theta}{R \cos \theta + d \sin 2\theta} \\ & \frac{Rd(\cos \theta \cos 2\theta + 2 \sin^2 \theta \cos \theta)}{R \cos \theta + d \sin 2\theta} \\ & \frac{Rd \cos \theta (\cos^2 \theta - \sin^2 \theta + 2 \sin^2 \theta)}{R \cos \theta + d \sin 2\theta}. \end{aligned}$$

That condenses nicely, so the slope-intercept form is simplified to

$$y = \frac{R \sin \theta - d \cos 2\theta}{R \cos \theta + d \sin 2\theta} x + \frac{dR \cos \theta}{R \cos \theta + d \sin 2\theta}$$

So, the set of all possible reflected rays can be given by

$$U(x, y, \theta) = (R \cos \theta + d \sin 2\theta)y + (-R \sin \theta + d \cos 2\theta)x - dR \cos \theta = 0.$$

Differentiating with respect to  $\theta$  and simultaneously solving the two resulting equations is unimaginably bad. So we employ a clever trick\*: reparametrizing  $U$  by making the substitution  $t = e^{i\theta}$ . With this substitution, we calculate:

$$\begin{aligned} \cos \theta &= \frac{1}{2}(t + t^{-1}) \\ \sin \theta &= \frac{i}{2}(t^{-1} - t) \\ \sin 2\theta &= \frac{i}{2}(t^{-2} - t^2) \\ \cos 2\theta &= \frac{1}{2}(t^2 + t^{-2}). \end{aligned}$$

With  $t$  as the new parameter, we substitute accordingly (with the help of a computer) and find

$$U(x, y, t) = \frac{1}{2} ((dx - idy)t^2 + (-dR + iRx + Ry)t + (-dR - iRx + Ry)t^{-1} + (dx + idy)t^{-2}) = 0.$$

By definition,  $t$  is never equal to zero, so there is no trouble if we reassign  $U$  to itself times  $2t^2$ :

$$U(x, y, t) = (dx - idy)t^4 + (Ry - dR + iRx)t^3 + (Ry - dR - iRx)t + (dx + idy)t^{-2} = 0.$$

From the definition of the envelope, we would like to solve the simultaneous system of equations

$$U = 0 \text{ and } \frac{\partial U}{\partial t} = 0.$$

Note that if there exists some  $t$  that satisfies this system,  $U(t)$  must have a double root. Furthermore, if  $U(t)$  has a double root, then its quartic discriminant must equal zero.

The expression for the quartic discriminant given a quartic

$$ax^4 + bx^3 + cx^2 + dx + e$$

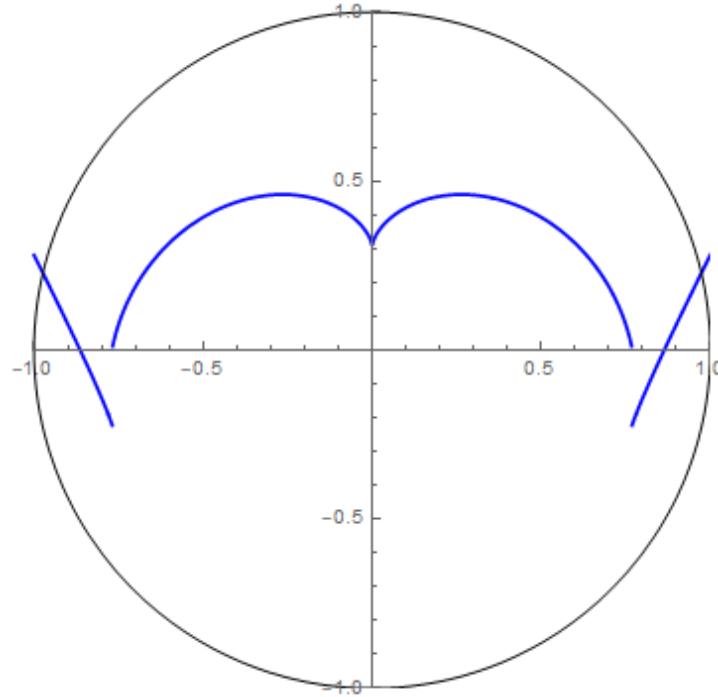
is

$$\begin{aligned} & 256a^3e^3 - 192a^2bde^2 - 128a^2c^2e^2 + 144a^2cd^2e - 27a^2d^4 + 144ab^2ce^2 - \\ & 6ab^2d^2e - 80abc^2de + 18abcd^3 + 16ac^4e - 4ac^3d^2 - 27b^4e^2 + \\ & 18b^3cde - 4b^3d^3 - 4b^2c^3e + b^2c^2d^2. \end{aligned}$$

Hence, we substitute accordingly (with the aid of a computer) and obtain

$$\begin{aligned} & -4d^6R^6 - 60d^6R^4x^2 - 12d^4R^6x^2 - 192d^6R^2x^4 + 312d^4R^4x^4 - \\ & 12d^2R^6x^4 + 256d^6x^6 - 192d^4R^2x^6 - 60d^2R^4x^6 - 4R^6x^6 + 24d^5R^6y - \\ & 192d^5R^4x^2y + 48d^3R^6x^2y + 384d^5R^2x^4y - 192d^3R^4x^4y + 24dR^6x^4y + \\ & 48d^6R^4y^2 - 60d^4R^6y^2 - 384d^6R^2x^2y^2 + 600d^4R^4x^2y^2 - 72d^2R^6x^2y^2 + \\ & 768d^6x^4y^2 - 576d^4R^2x^4y^2 - 72d^2R^4x^4y^2 - 12R^6x^4y^2 - 192d^5R^4y^3 + \\ & 80d^3R^6y^3 + 768d^5R^2x^2y^3 - 384d^3R^4x^2y^3 + 48dR^6x^2y^3 - 192d^6R^2y^4 + \\ & 288d^4R^4y^4 - 60d^2R^6y^4 + 768d^6x^2y^4 - 576d^4R^2x^2y^4 + 36d^2R^4x^2y^4 - \\ & 12R^6x^2y^4 + 384d^5R^2y^5 - 192d^3R^4y^5 + 24dR^6y^5 + 256d^6y^6 - \\ & 192d^4R^2y^6 + 48d^2R^4y^6 - 4R^6y^6 = 0. \end{aligned}$$

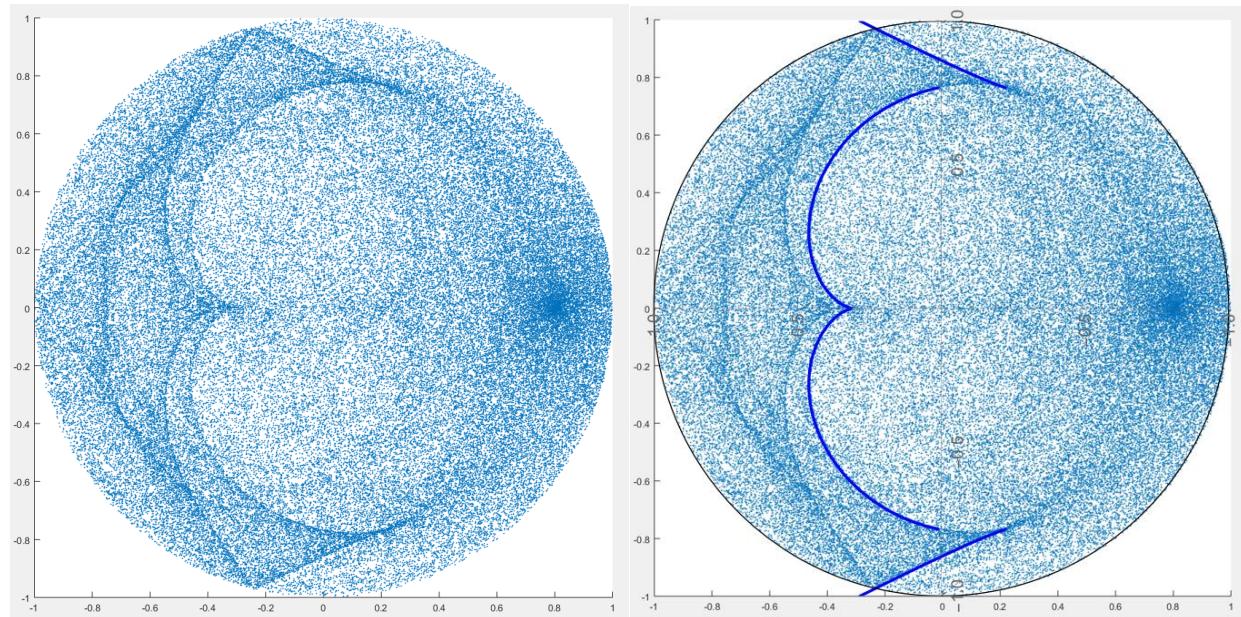
This is the equation for the envelope of  $U$ , and also the equation for the **first-order caustic** (the envelope of once-reflected rays). Below is the plot for this equation for  $d = 0.8$ , the parameter used for the previous methods.



$$R = 1 \text{ and } d = 0.8$$

(In Appendix C, I include the plots for  $d = \{.01, .02, \dots, 1\}$ .)

You can very clearly see how the plot for  $d = 0.8$  matches remarkably well with the plots from the simulation, which used the exact same parameters. This offers some “experimental” support this answer. If you don’t believe it, here is a side-by-side overlay comparison:



How about all the other shapes? We can chalk those down to higher-order caustics, the envelopes of light rays that have been reflected more than once. In reality (everyday buckets

and mugs), everything past the second order caustics aren't visible because the light source needs to be well within the bucket/mug.

### Second-order Caustics

Just as an exercise, we can use the same exact method from the previous part to derive the equation for the second-order caustic.

First, we need to find out what point  $Q$  is:

We solve

$$y = \frac{R \sin \theta - d \cos 2\theta}{R \cos \theta + d \sin 2\theta} x + \frac{dR \cos \theta}{R \cos \theta + d \sin 2\theta}$$

$$x^2 + y^2 = R^2$$

This has two solutions, so we take the one that isn't point  $P$ , and this is point  $Q$ :

$$\left( \frac{-R^3 \cos \theta + d^2 R \cos 3\theta - 2dR^2 \sin 2\theta}{d^2 + R^2 + 2dR \sin \theta}, \frac{2dR^2 \cos 2\theta - R^3 \sin \theta + d^2 R \sin 3\theta}{d^2 + R^2 + 2dR \sin \theta} \right).$$

Using the same method as previously, we find that the set of all possible second-reflections is given by

$$V(x, y, \theta) = 2R \cos \theta (d^4 + 6d^2 R^2 + R^4 - d^2(2d^2 + 9R^2) \cos 2\theta + d^4 \cos 4\theta + 8d^3 R \sin \theta + 7dR^3 \sin \theta - 5d^3 R \sin 3\theta) y - R(dR^3 - dR(3d^2 + 7R^2) \cos 2\theta + 5d^3 R \cos 4\theta + 3d^2 R^2 \sin \theta + 2R^4 \sin \theta - d^4 \sin 3\theta - 9d^2 R^2 \sin 3\theta + d^4 \sin 5\theta) x + 2dR^2 \cos \theta (R + d \sin \theta) (d^2 + R^2 + 2dR \sin \theta) = 0.$$

Just like we did above, we substitute  $t = e^{i\theta}$ , which gives us

$$\begin{aligned}
 V(x, y, t) = & \left( \frac{1}{2}id^4Rx + \frac{1}{2}d^4Ry \right) t^{10} + \left( -\frac{5}{2}d^3R^2x + \frac{5}{2}id^3R^2y \right) t^9 + \\
 & \left( -\frac{1}{2}d^3R^3 - \frac{1}{2}id^4Rx - \frac{9}{2}id^2R^3x - \frac{1}{2}d^4Ry - \frac{9}{2}d^2R^3y \right) t^8 + \\
 & \left( -\frac{1}{2}id^4R^2 - \frac{3}{2}id^2R^4 + \frac{3}{2}d^3R^2x + \frac{7}{2}dR^4x - \frac{3}{2}id^3R^2y - \frac{7}{2}idR^4y \right) t^7 + \\
 & \left( \frac{3}{2}d^3R^3 + dR^5 + \frac{3}{2}id^2R^3x + iR^5x + \frac{3}{2}d^2R^3y + R^5y \right) t^6 + (-dR^4x)t^5 + \\
 & \left( \frac{3}{2}d^3R^3 + dR^5 - \frac{3}{2}id^2R^3x - iR^5x + \frac{3}{2}d^2R^3y + R^5y \right) t^4 + \\
 & \left( \frac{1}{2}id^4R^2 + \frac{3}{2}id^2R^4 + \frac{3}{2}d^3R^2x + \frac{7}{2}dR^4x + \frac{3}{2}id^3R^2y + \frac{7}{2}idR^4y \right) t^3 + \\
 & \left( -\frac{1}{2}d^3R^3 + \frac{1}{2}id^4Rx + \frac{9}{2}id^2R^3x - \frac{1}{2}d^4Ry - \frac{9}{2}d^2R^3y \right) t^2 + \\
 & \left( -\frac{5}{2}d^3R^2x - \frac{5}{2}id^3R^2y \right) t + \left( -\frac{1}{2}id^4Rx + \frac{1}{2}d^4Ry \right) \\
 = & 0.
 \end{aligned}$$

As before, the equation for the envelope is given by the discriminant of this degree-10 equation. The resulting equation is too long to be placed inside this document, so I have placed it into Appendix D.

## Sources

All research is original except as noted here:

1. [Wolfram MathWorld: Envelope](#)
  - a. I used this page to find the mathematical definition of the envelope.
2. [Wolfram MathWorld: Circle Catacaustic](#)
  - a. This article contains parametrized formulas for the catacaustics with light sources on the edge of the circle, and outside the circle. It does not provide such formulas for cases where the light source exists inside the circle, which is the case in this paper. Nevertheless, this was a page I referenced in my research.
3. [Wikipedia: Discriminant](#)
  - a. I used this article to find the expression for the discriminant of a quartic and degree-10 polynomial.
4. [Wikipedia: Envelope](#)
  - a. \*This article introduced the “trick” substitution  $t = e^{i\theta}$  employed in the derivation.

## Software Used

1. MATLAB was used for the simulation and for all resulting plots.
2. Inkscape was used for all diagrams and the overlay picture.
3. Wolfram Mathematica was used for much of the heavy algebra, primarily for complex substitutions and expansions. Wolfram Mathematica was also used to plot the caustic equations.
4. GeoGebra, an online plotting application, was used where noted.
5. The scatter-heat map was created in MATLAB using a third-party script written by Jose Manuel Amigo. It can be downloaded [here](#).
6. AoPS TeXeR, a simple online LaTeX to \*.png tool, was for most of the math-setting.
7. Microsoft Word was used to create this document.

## Appendix A

MATLAB code for the computer simulation:

```
%Initial parameters
R = 1;
h = 2;
d=0.9;
%%%%%%%%%%%%

%Time steps
dt = 0.05; %crude time step
ddt = .005;%fine time step

final = [0 0];
for n = 1:100000
alpha = rand(1)*0.5*pi;
%angle between initial direction and the horizontal
plane

theta = rand(1)*2*pi;
%angle of initial direction in the horizontal plane

done = 0;
x = [d 0 h]; %initial photon position
v = [cos(theta)*cos(alpha),sin(theta)*cos(alpha),-
sin(alpha)]; %initial photo dir.
for k = 1:10000
x0 = x;
x = x0 + v*dt; %advance the photon's position
if x(1,1)^2 + x(1,2)^2 > R^2
%has the photon left the cylinder?

y = x0;
%since yes, go to previous position
%begin fine adjustment of position
for j = 1:10000
y0 = y;
y = y0 + v*ddt; %fine advancement
if y(1,1)^2 + y(1,2)^2 > R^2
%has the photon left?
v(:,[1 2]) = v(:,[1 2]) -
```

```

    2*dot(v(:, [1 2]), y(:, [1 2]))/
    norm(y(:, [1 2]))*y(:, [1 2]);
    %reflect

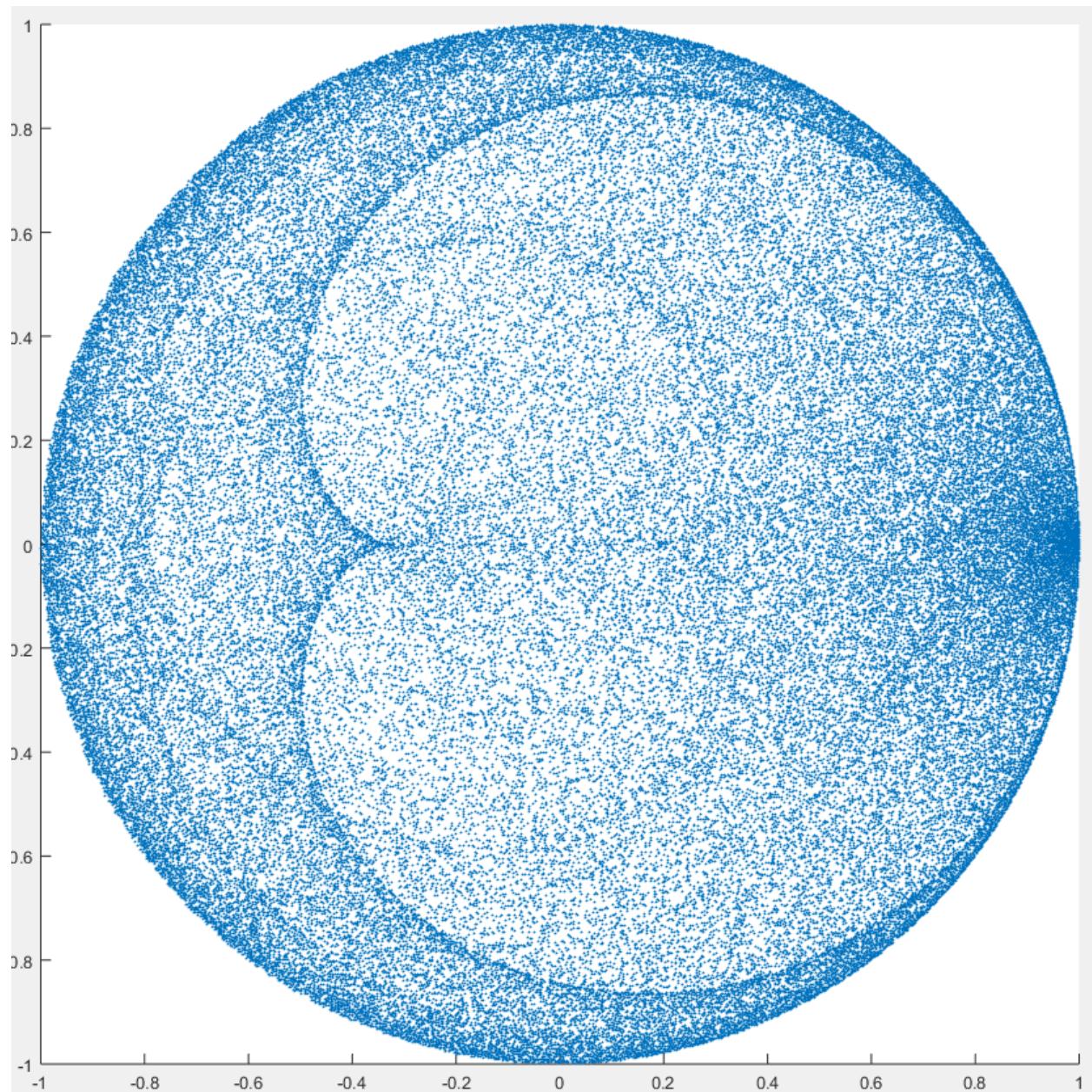
    x = y0;
    %new position set to the previous
    break
end
end
if x(1,3)<0
%has the photon left the base of the cylinder?
    y = x0;
    %since yes, return to previous position
    %fine adjust
    for j = 1:10000
        y0 = y;
        y = y0 + v*ddt;
        if y(1,3)<0
            final(n,[1 2]) = y0(:,[1 2]);
            %save where it hits the base
            done = 1; %finish up the loop
            break
        end
    end
end
if done == 1
    break
end
end

end
p = scatter(final(:,1)',final(:,2)',1); %scatterplot
hold on
axis([-1 1 -1 1]);
daspect([1 1 1]);
hold off
saveas(p, strcat('plot',num2str(d*100), '.png'), 'png');

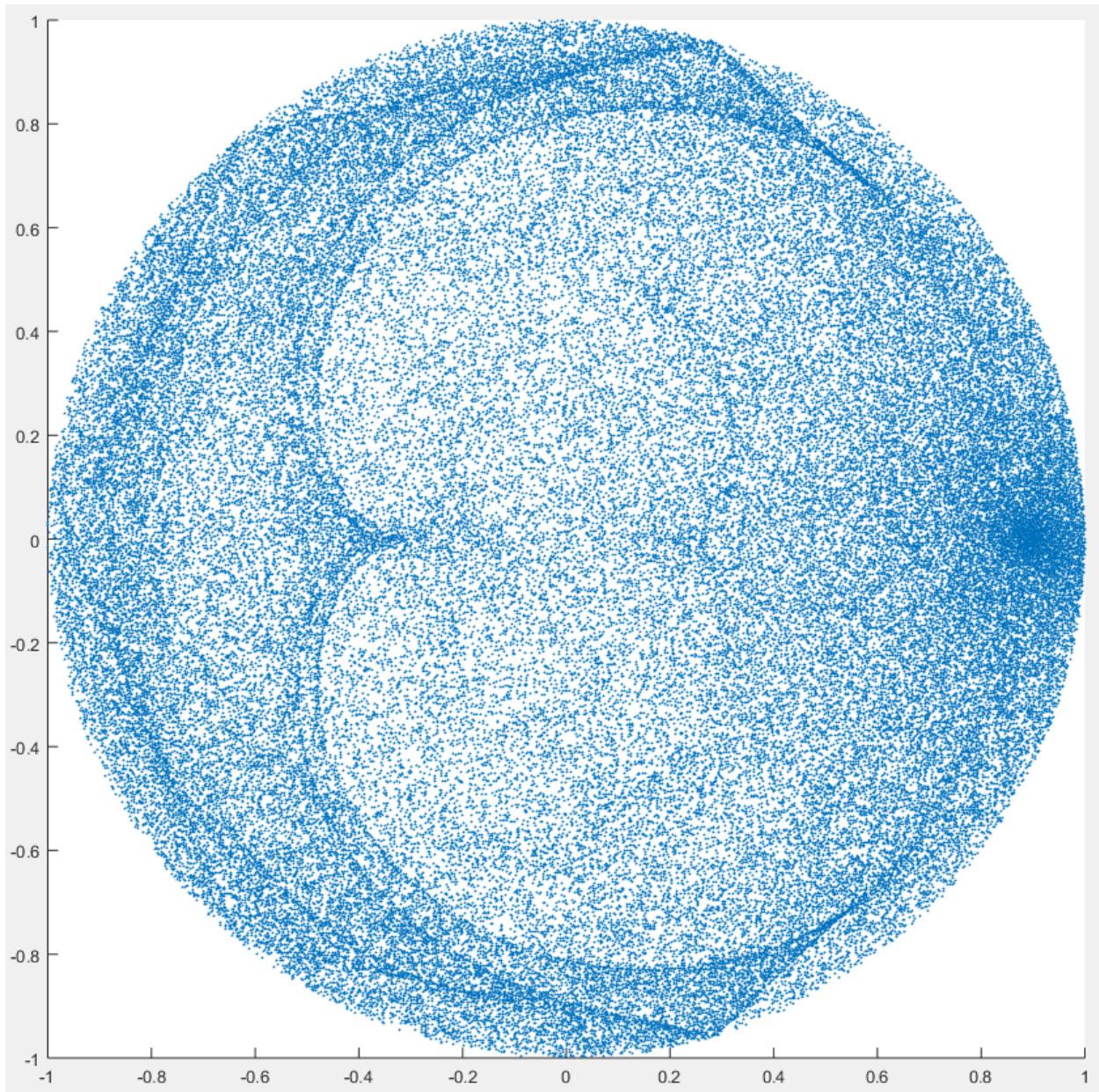
```

## Appendix B

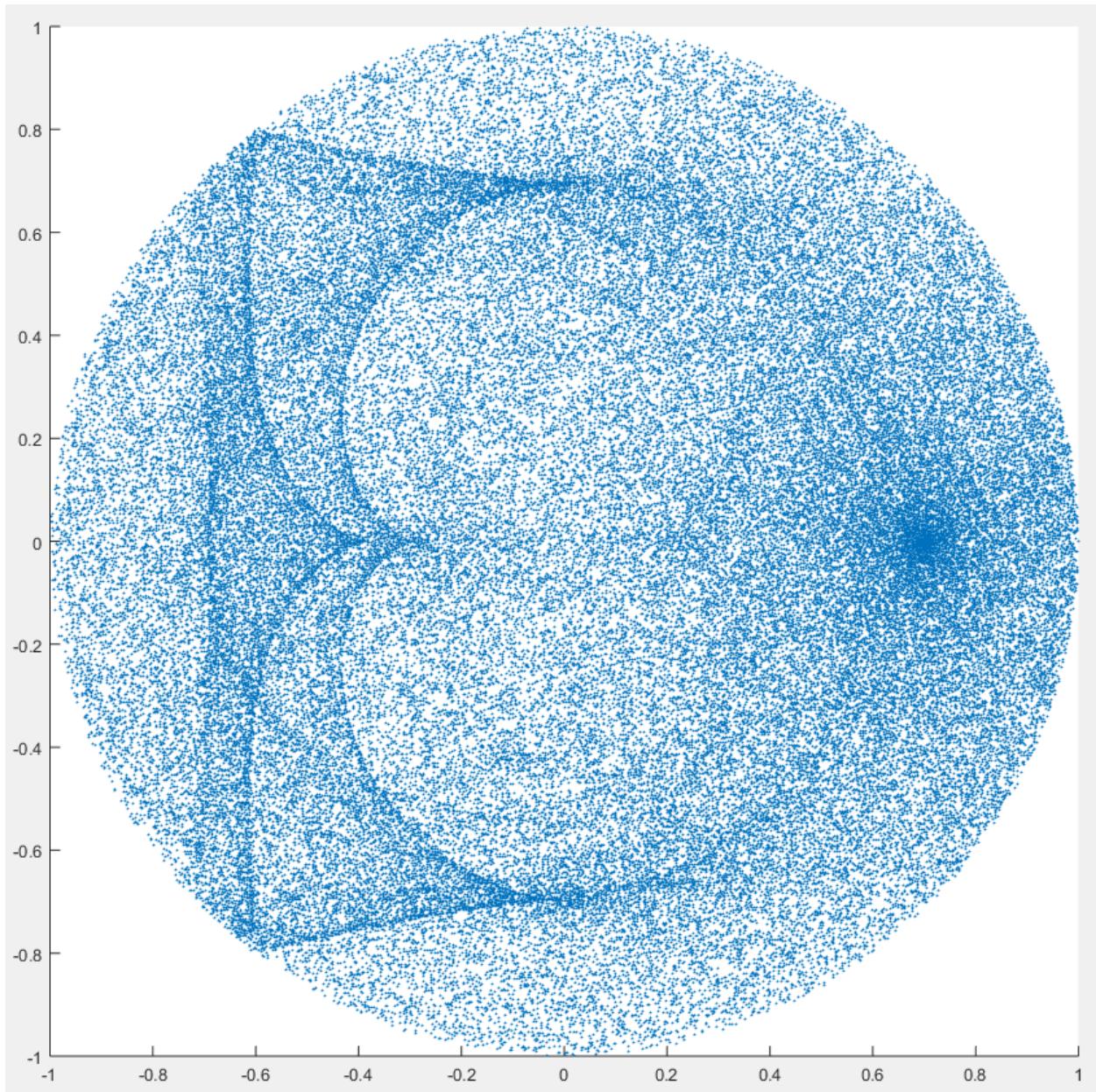
Plot for  $d = 1$ :



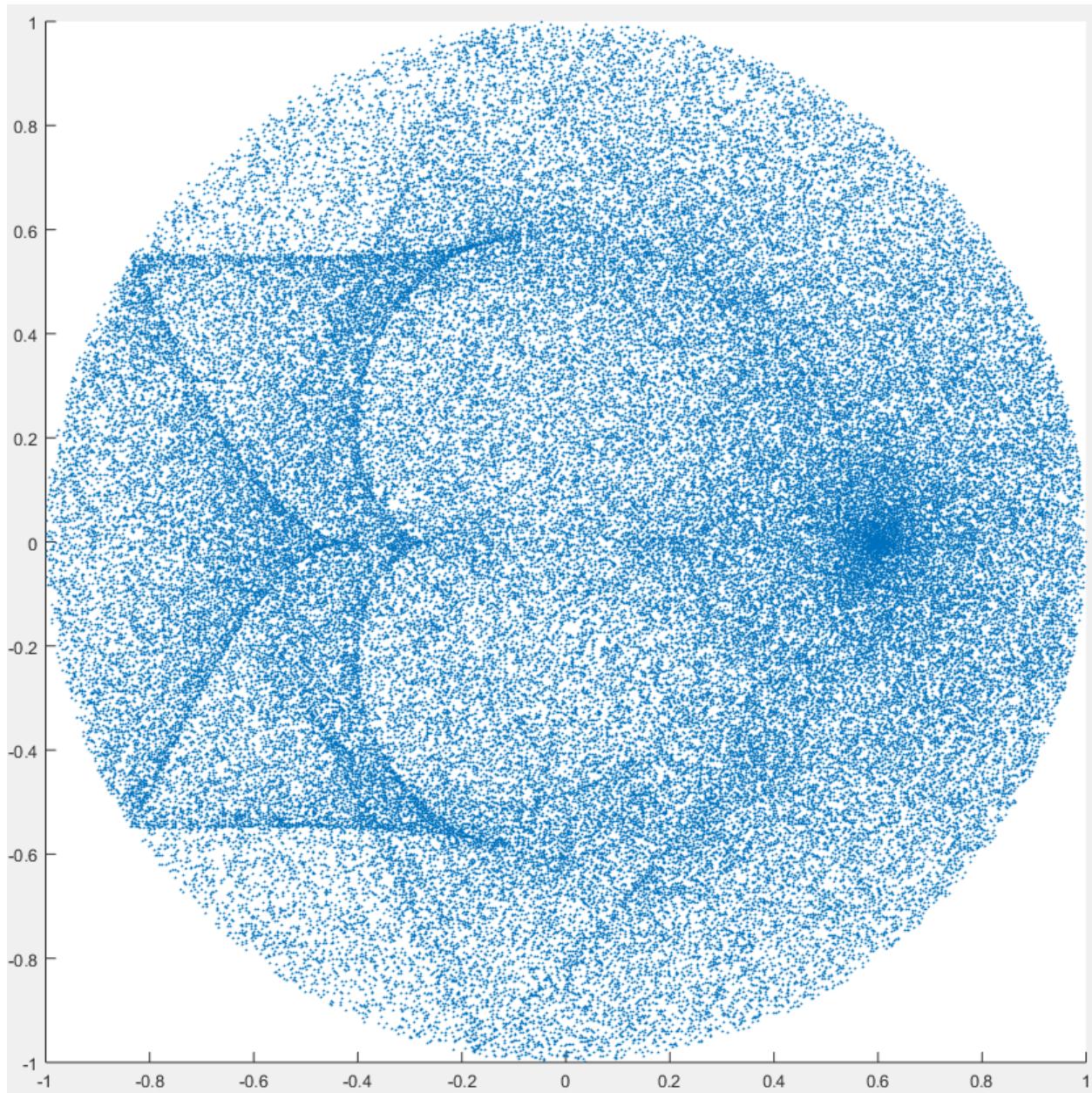
Plot for  $d = 0.9$ :



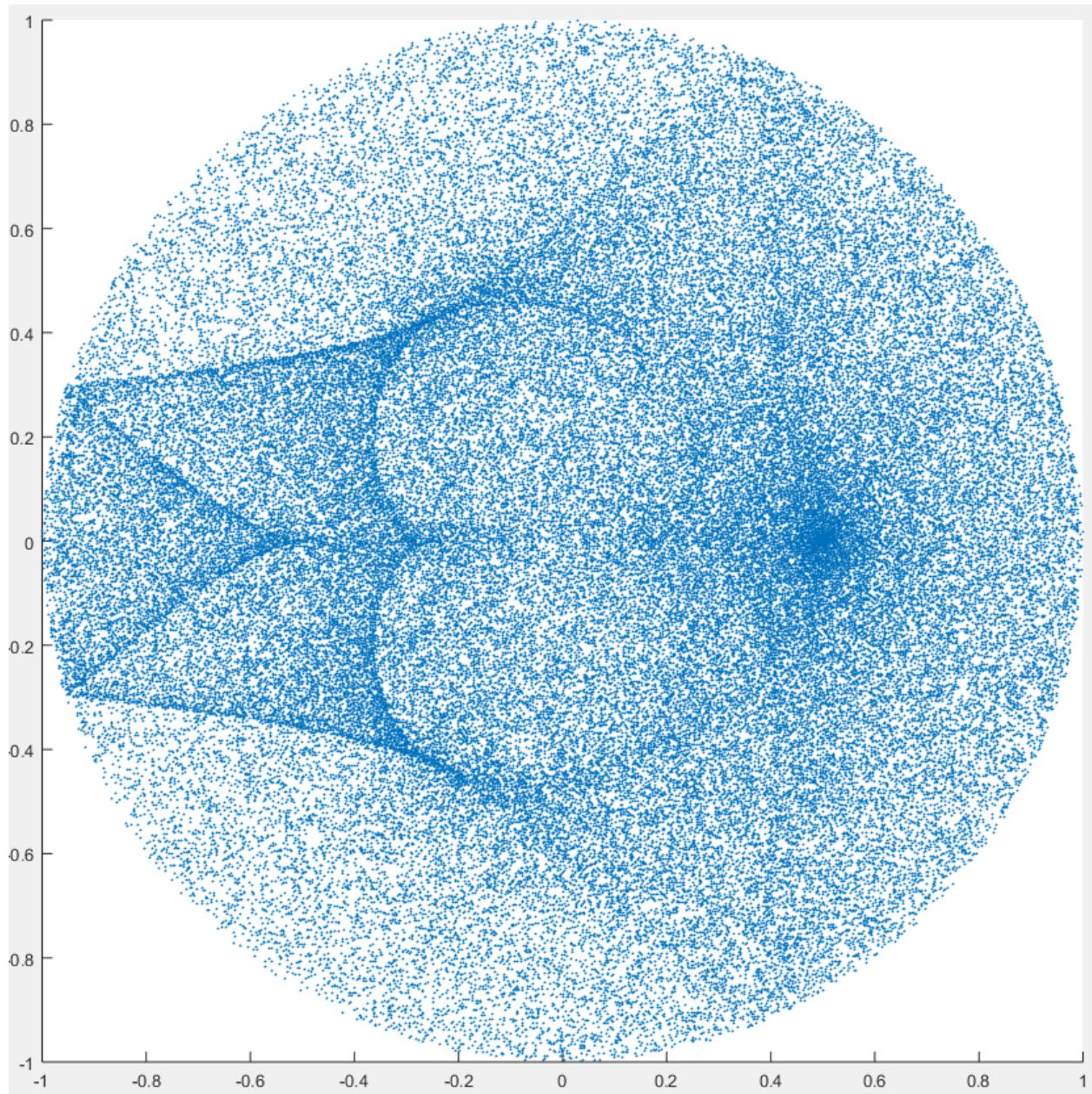
Plot for  $d = 0.7$ :



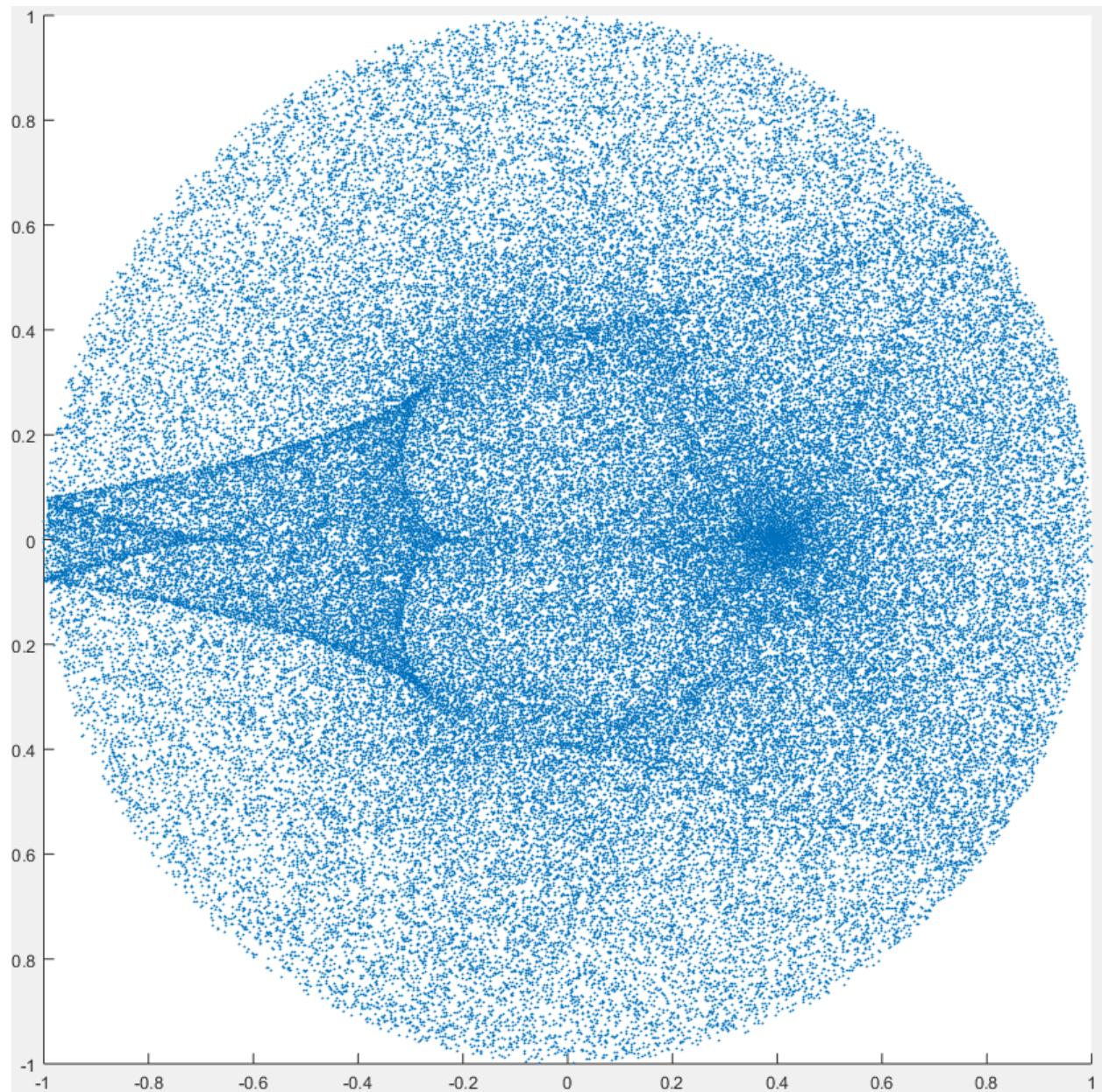
Plot for  $d = 0.6$ :



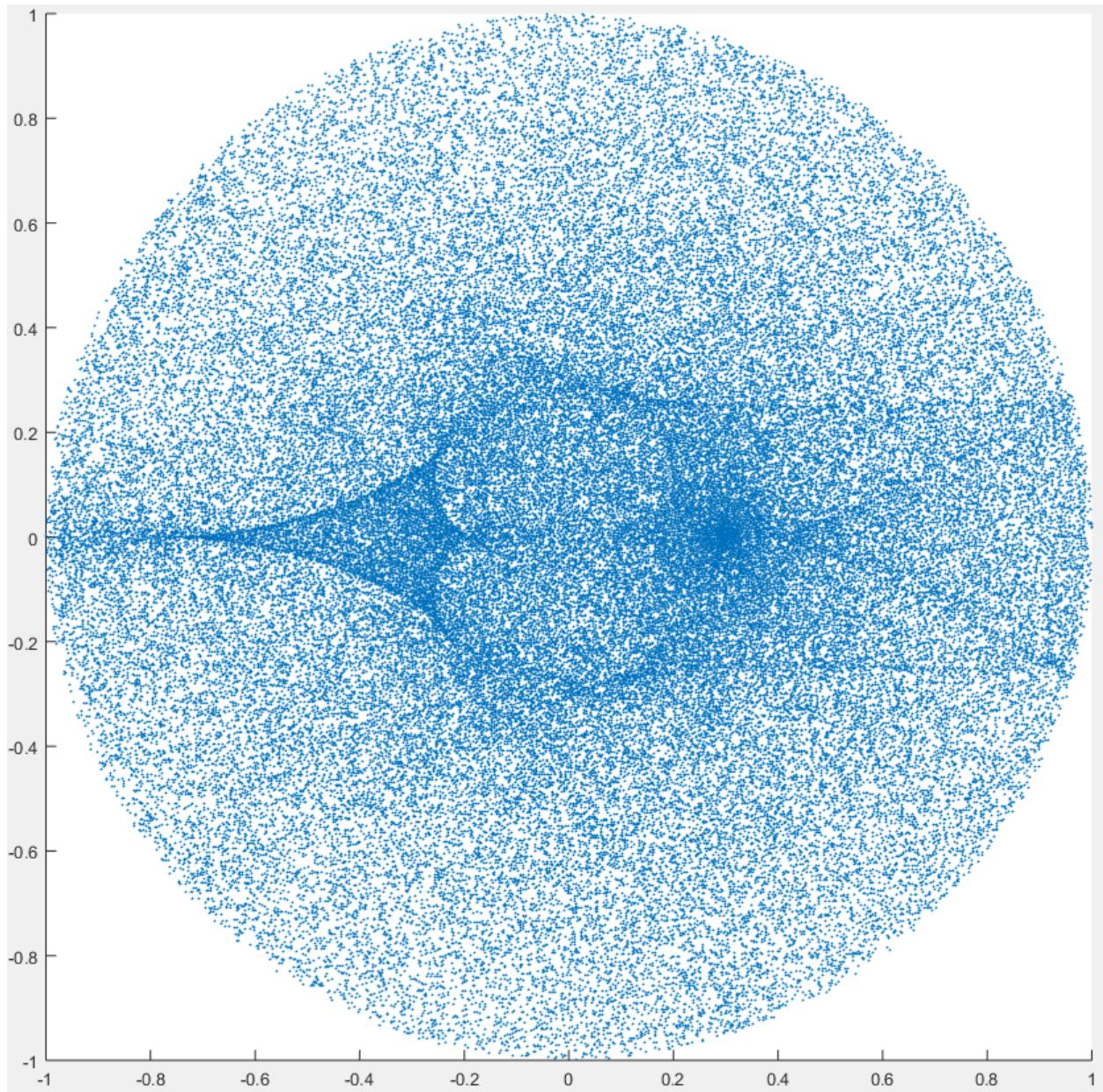
Plot for  $d = 0.5$ :



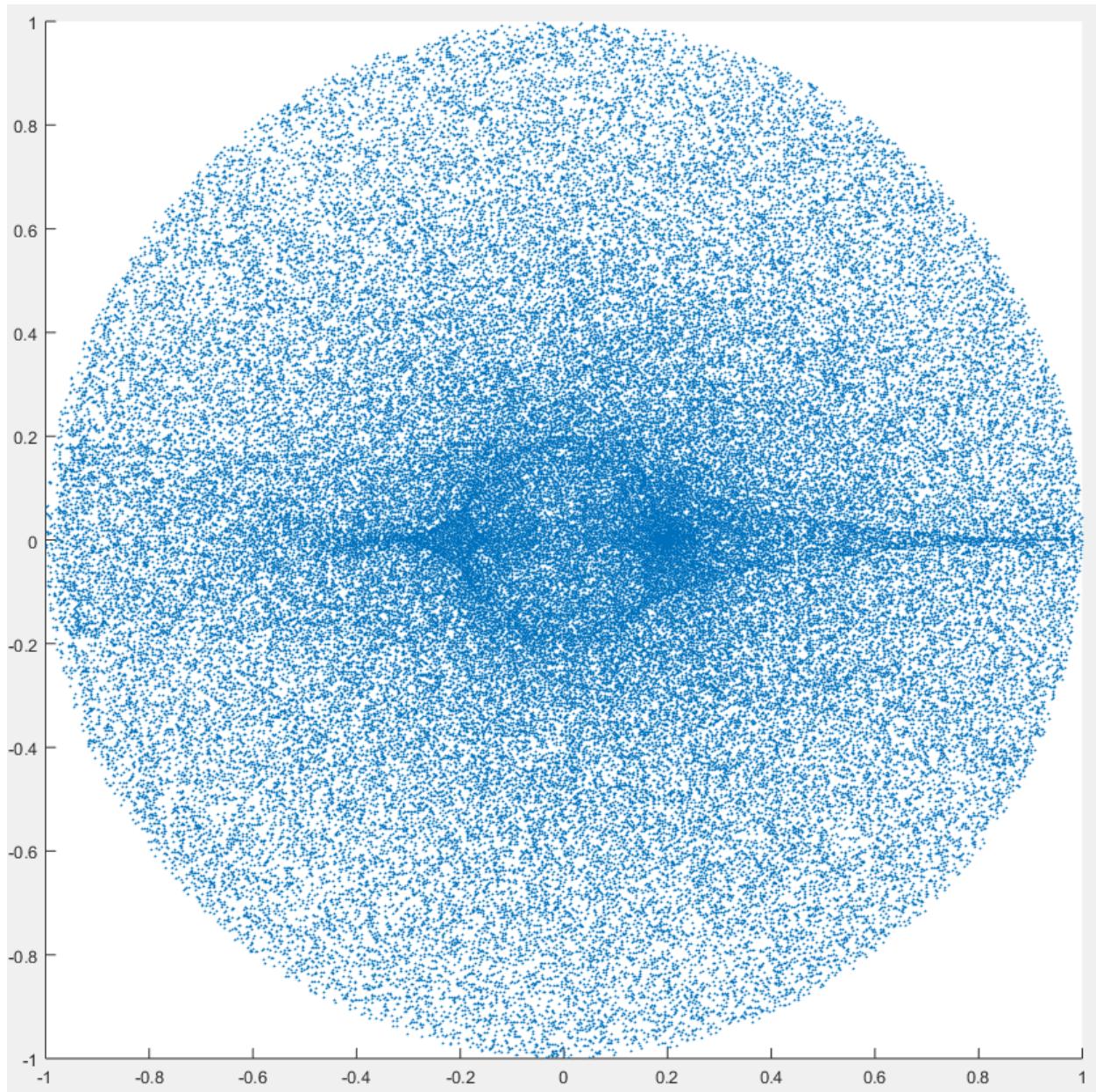
Plot for  $d = 0.4$ :



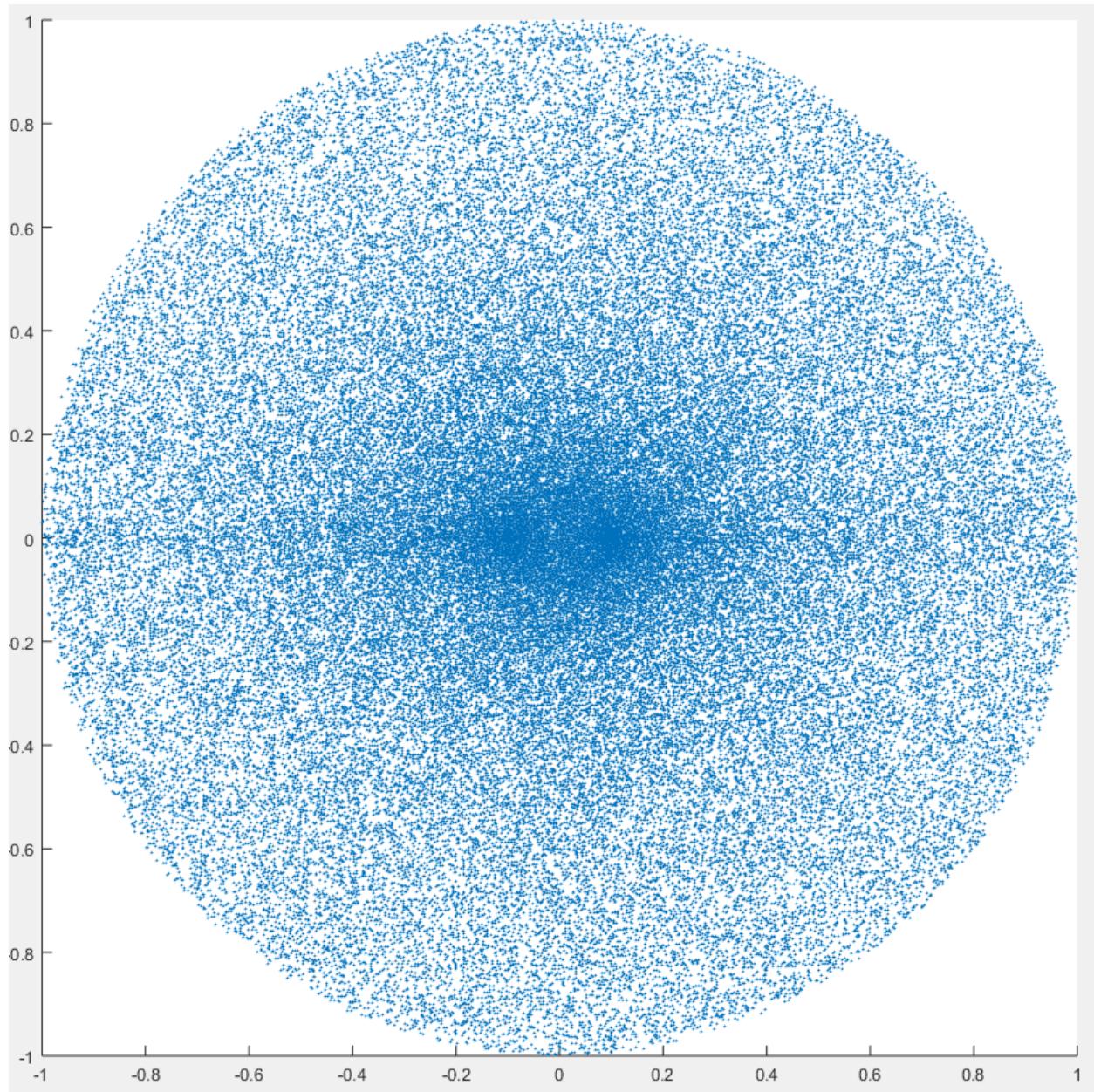
Plot for  $d = 0.3$ :



Plot for  $d = 0.2$ :

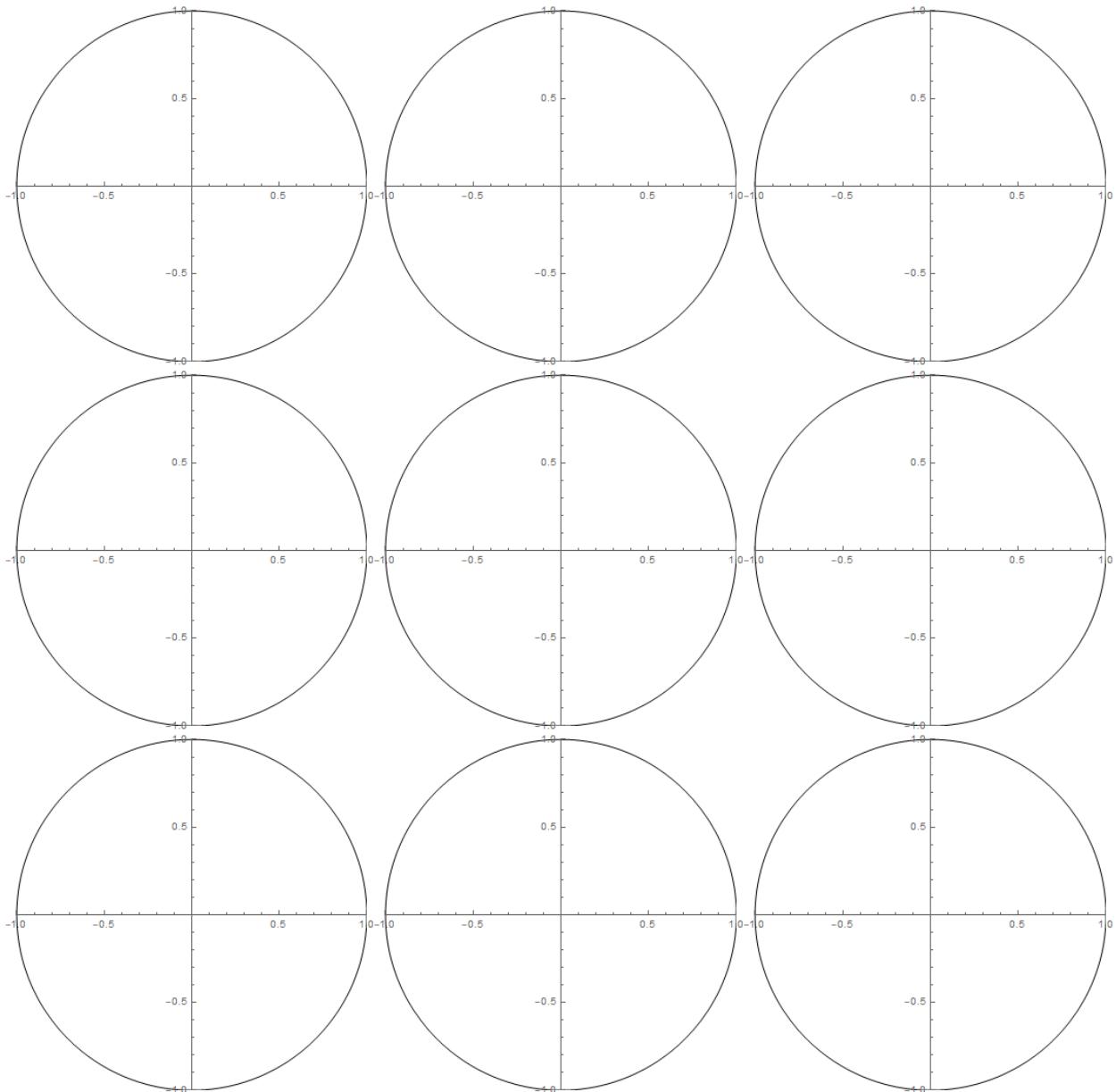


Plot for  $d = 0.1$ :

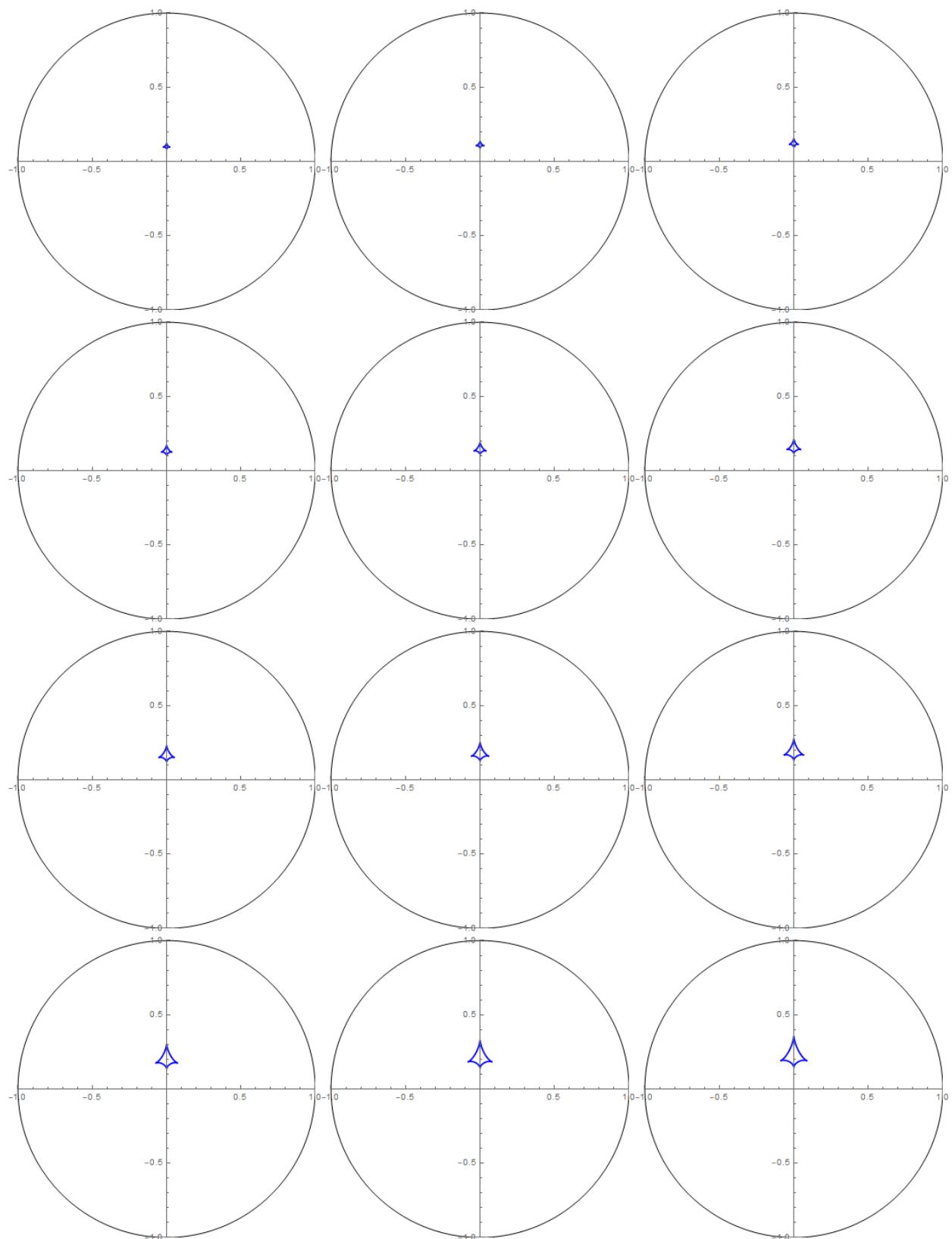


## Appendix C

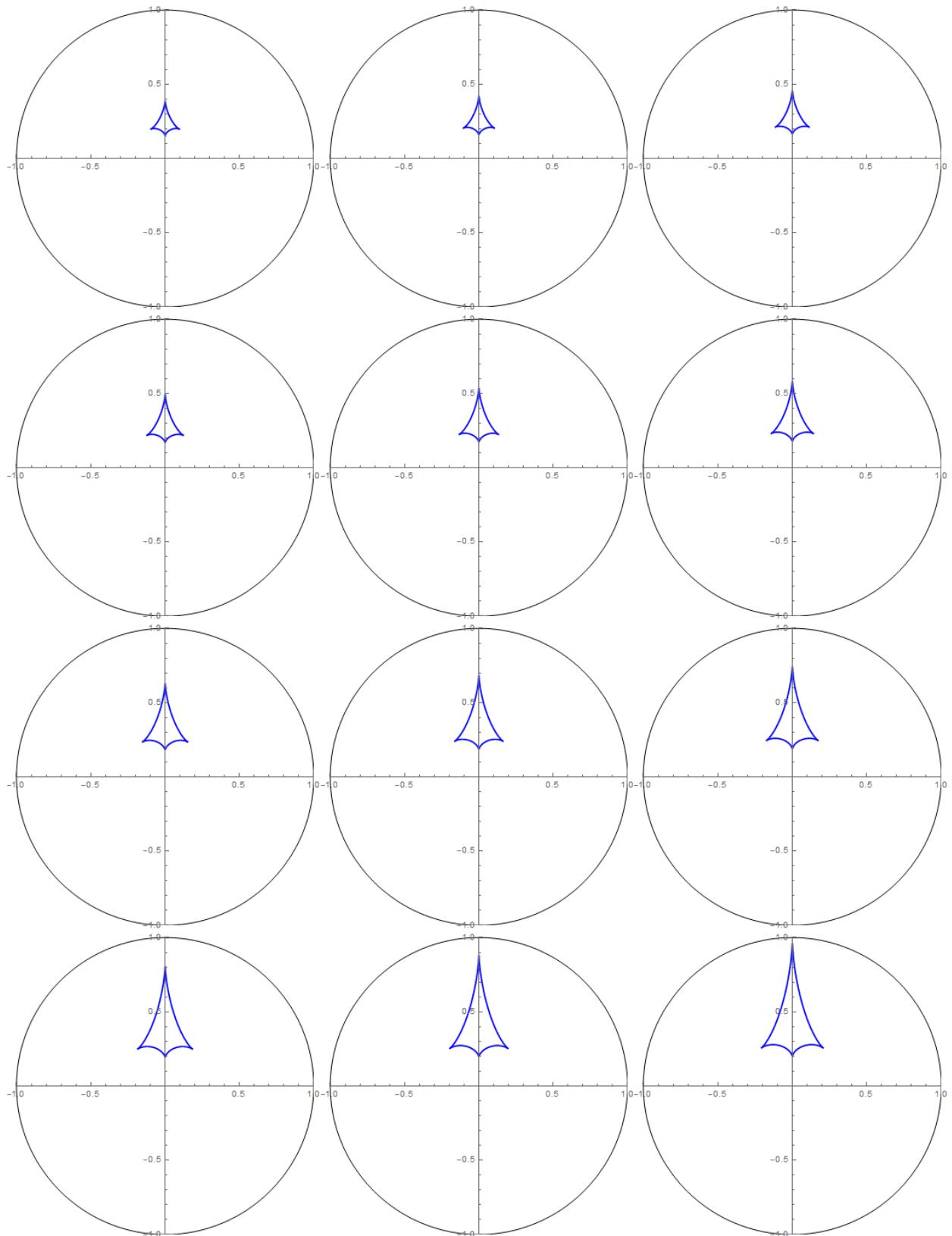
Here are all the plots for parameter  $d = \{.01, .02, \dots, .99, 1.00\}$ . The plots begin at  $d = .01$  and increase from left to right, top to bottom. For ease of notation, I have labelled them by  $i = 100d$ , from  $i = 1$  to  $i = 100$ . (I believe any vertical lines are artifacts.)



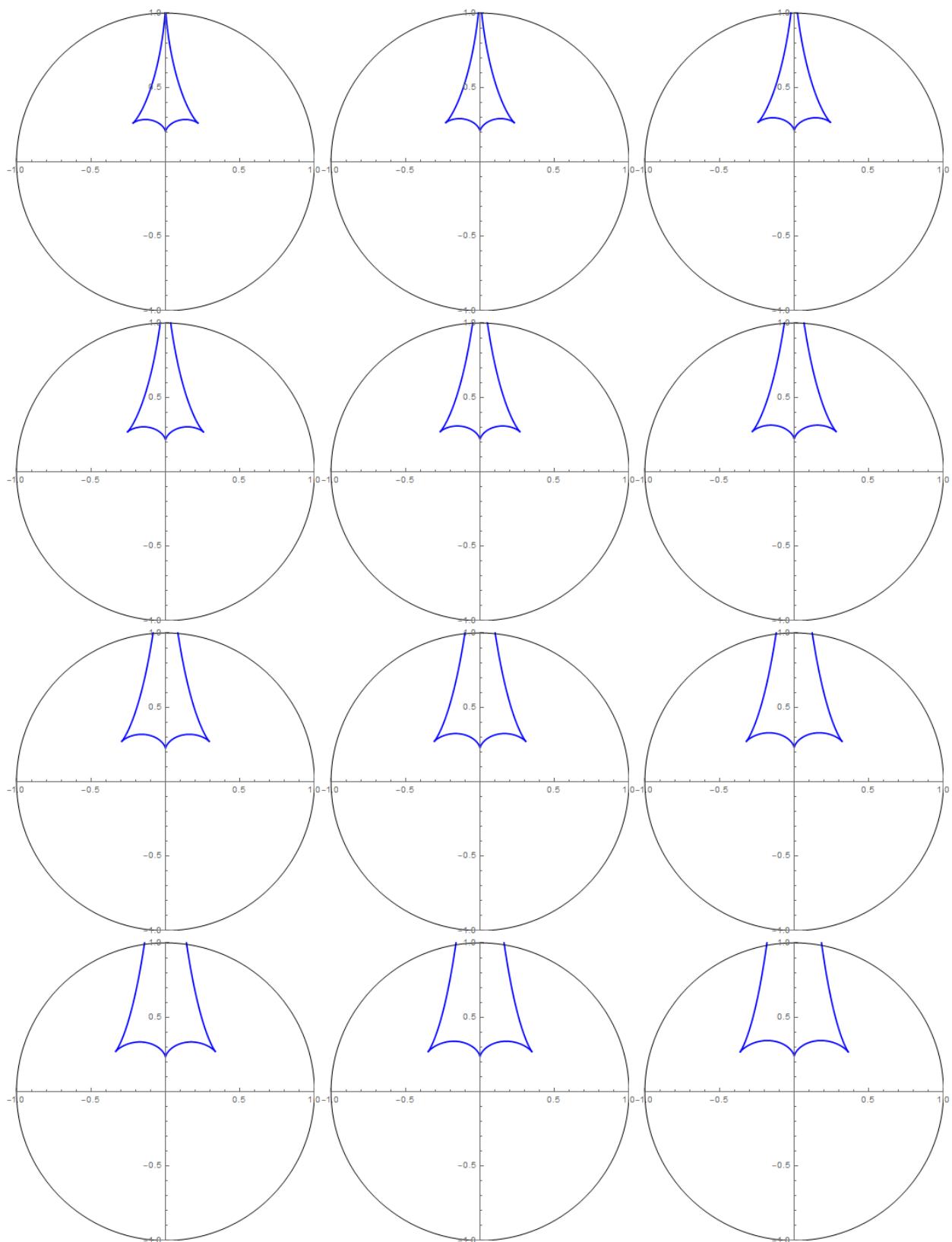
1 – 9 (nothing appears until 10)



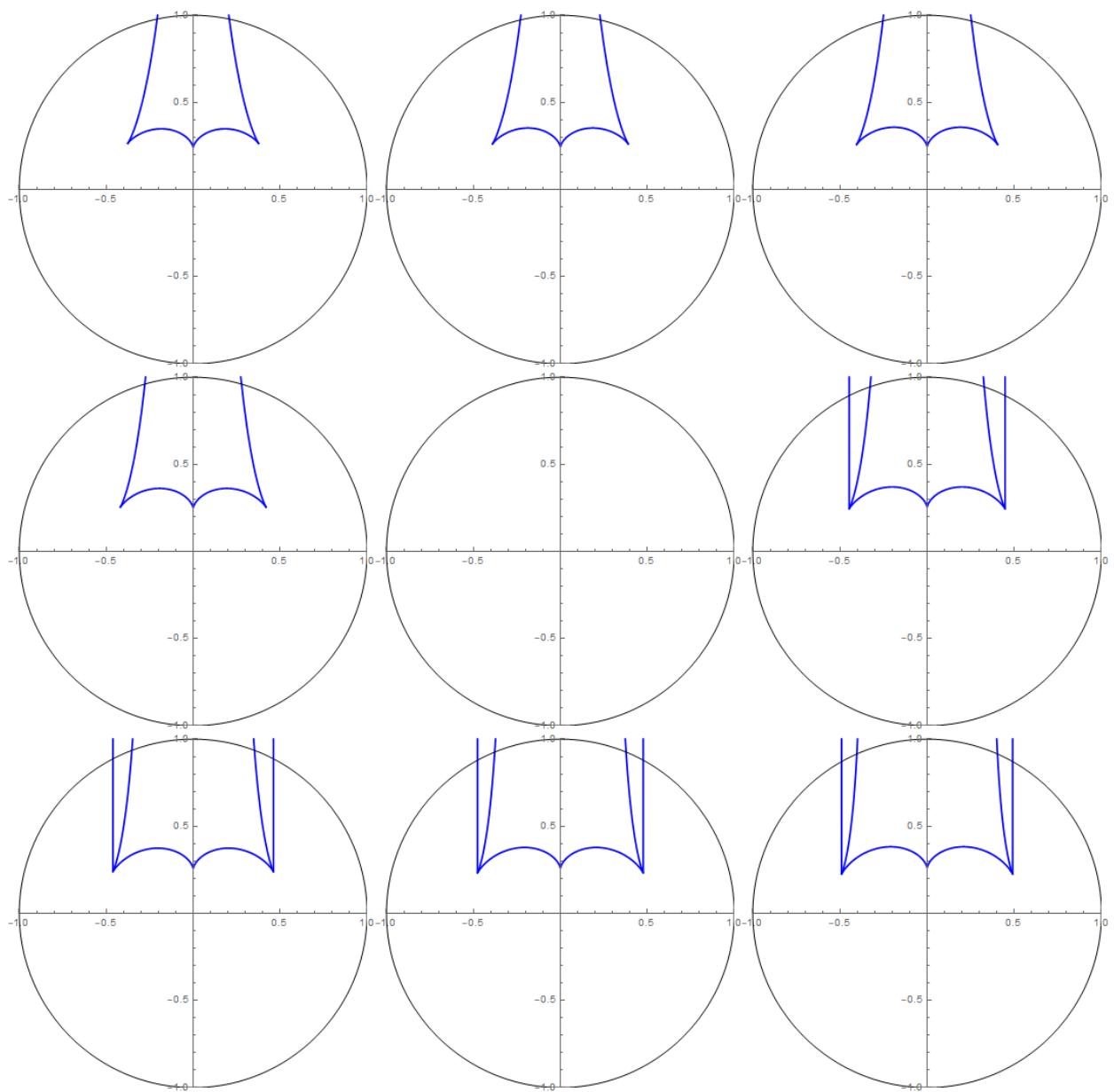
10 - 21



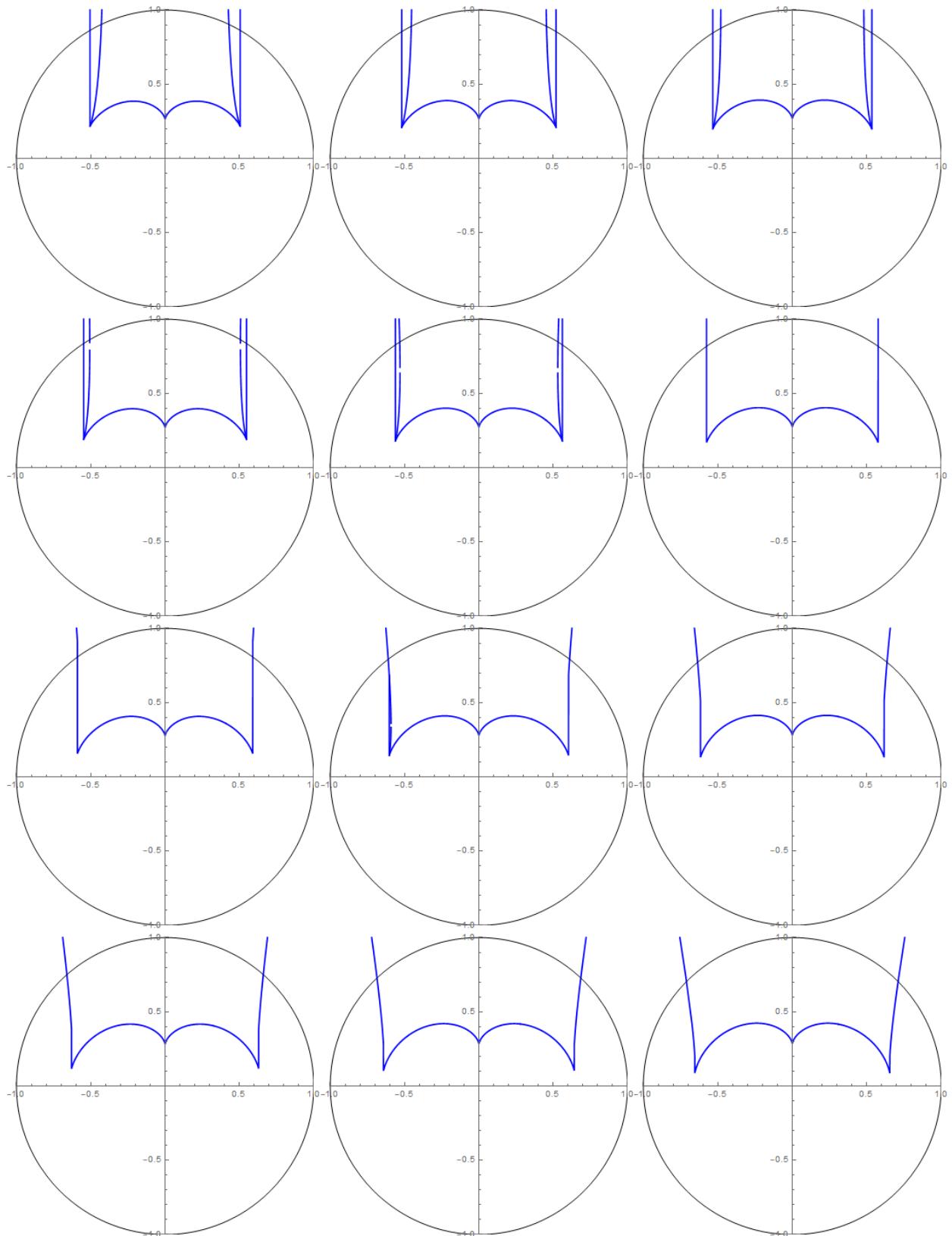
22-33



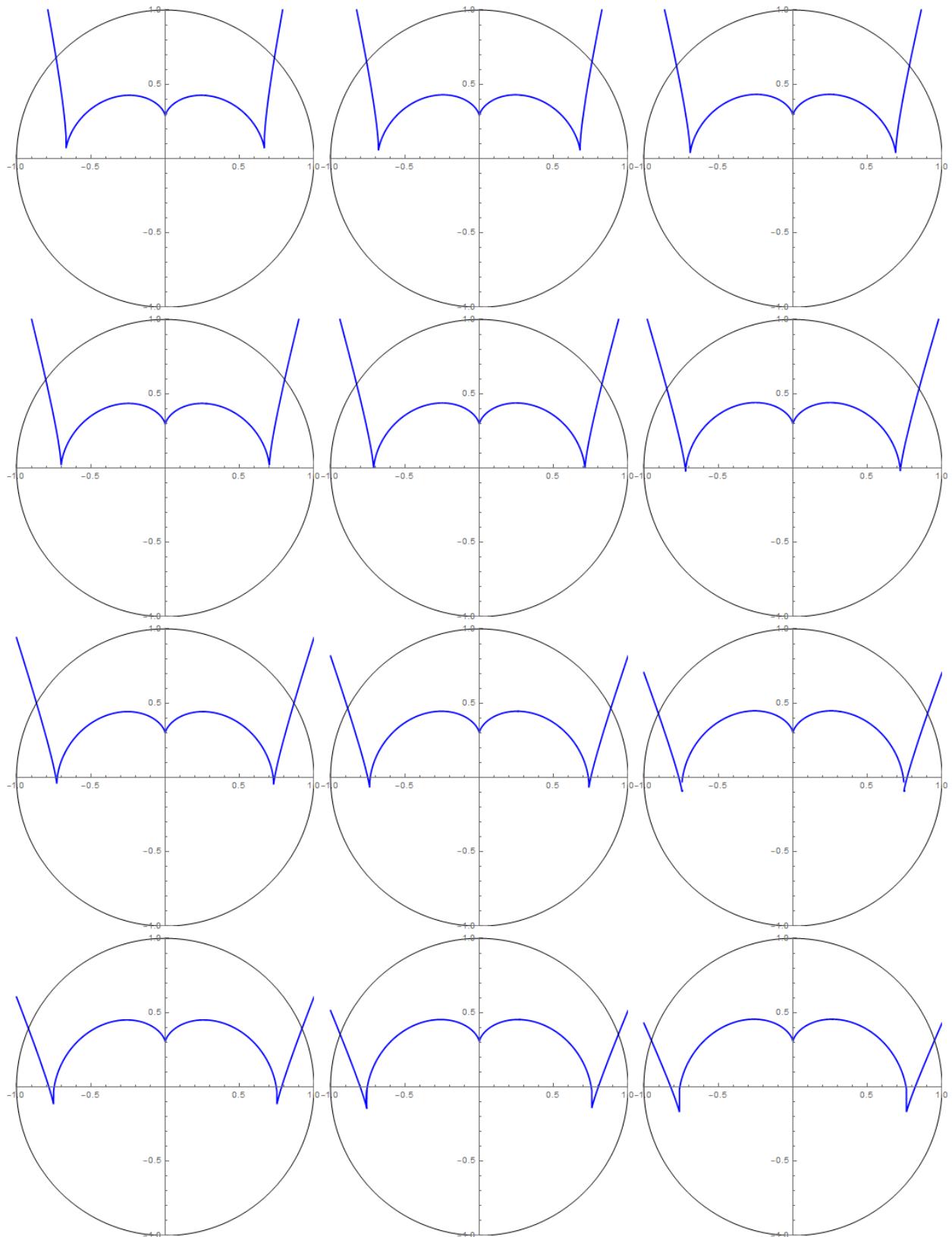
34-45



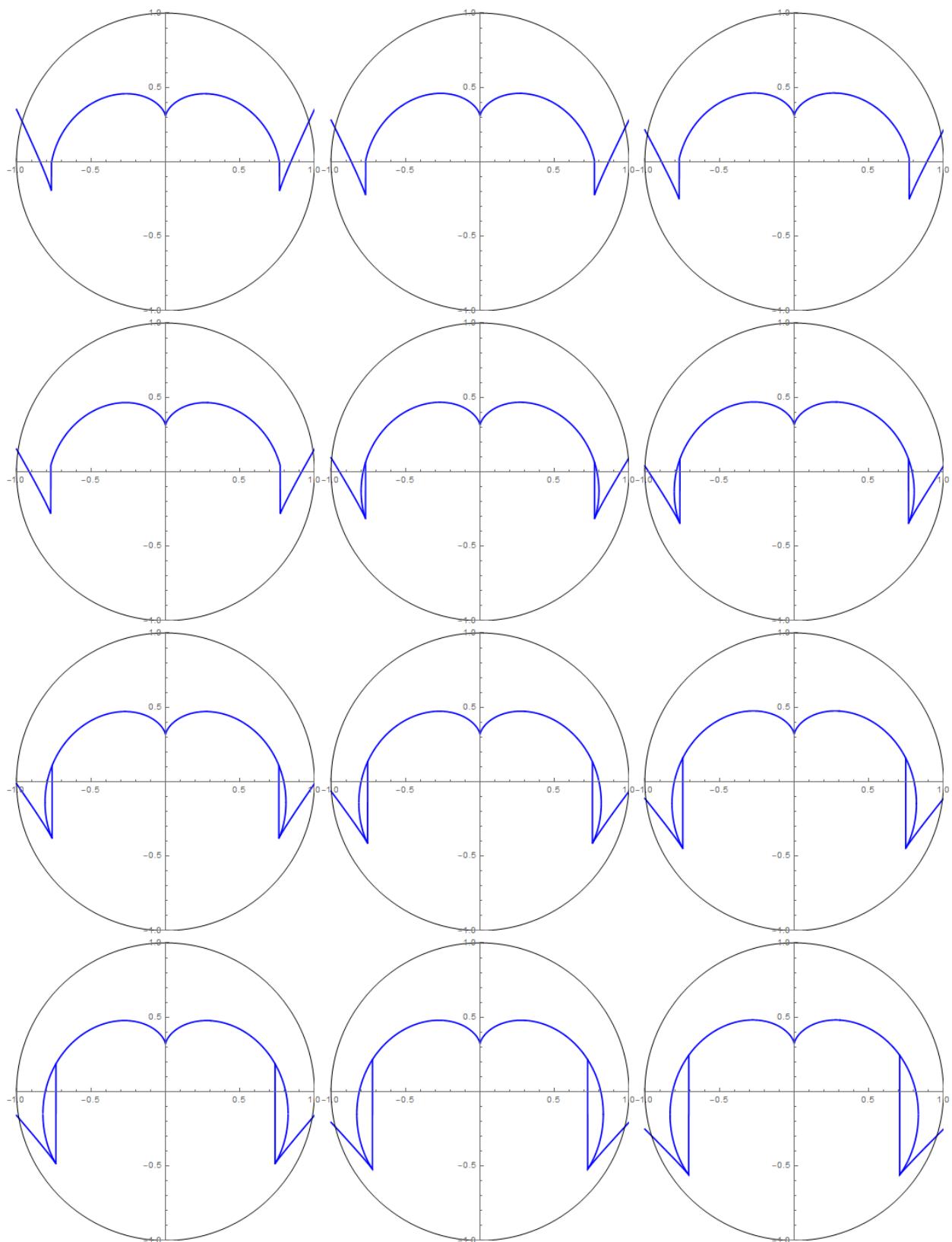
46 – 54 (50 is a special case)

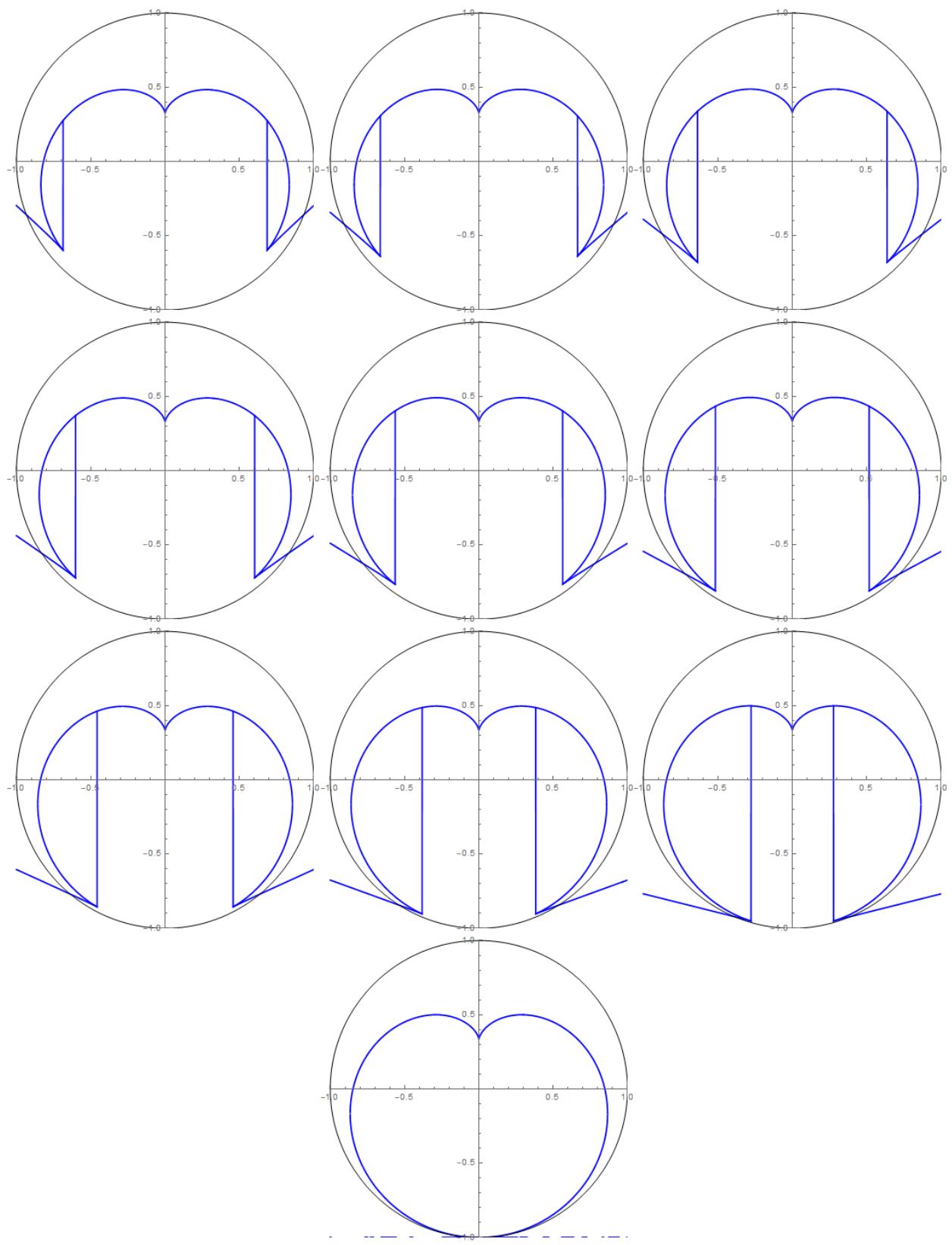


55 - 66



67 - 78





91 – 100

Jacob Nie

## Appendix D

Here is the equation for the envelope of all twice-reflected rays. It is given by the discriminant of the degree-10 polynomial derived above. It is listed here in plain text without mathsetting to facilitate copy and paste.

$$\begin{aligned}
 & (1/1024) (16 d^{66} R^{40} x^2 - 208 d^{64} R^{42} x^2 + 1248 d^{62} R^{44} x^2 - \\
 & 4576 d^{60} R^{46} x^2 + 11440 d^{58} R^{48} x^2 - 20592 d^{56} R^{50} x^2 + \\
 & 27456 d^{54} R^{52} x^2 - 27456 d^{52} R^{54} x^2 + 20592 d^{50} R^{56} x^2 - \\
 & 11440 d^{48} R^{58} x^2 + 4576 d^{46} R^{60} x^2 - 1248 d^{44} R^{62} x^2 + 208 \\
 & d^{42} R^{64} x^2 - 16 d^{40} R^{66} x^2 + 729 d^{38} R^{72} x^4 - 10773 d^{70} \\
 & R^{34} x^4 + 74334 d^{68} R^{36} x^4 - 316718 d^{66} R^{38} x^4 + 927955 d^{64} \\
 & R^{40} x^4 - 1968239 d^{62} R^{42} x^4 + 3092180 d^{60} R^{44} x^4 - 3603028 \\
 & d^{58} R^{46} x^4 + 3034031 d^{56} R^{48} x^4 - 1688115 d^{54} R^{50} x^4 + \\
 & 400686 d^{52} R^{52} x^4 + 245154 d^{50} R^{54} x^4 - 305851 d^{48} R^{56} x^4 + \\
 & 155687 d^{46} R^{58} x^4 - 43840 d^{44} R^{60} x^4 + 5824 d^{42} R^{62} x^4 + 64 \\
 & d^{40} R^{64} x^4 - 80 d^{38} R^{66} x^4 + 46197 d^{36} R^{72} x^6 - 678282 d^{70} \\
 & R^{32} x^6 + 4645563 d^{68} R^{34} x^6 - 19675076 d^{66} R^{36} x^6 + \\
 & 57566517 d^{64} R^{38} x^6 - 123057606 d^{62} R^{40} x^6 + 198038659 d^{60} \\
 & R^{42} x^6 - 243278568 d^{58} R^{44} x^6 + 228229287 d^{56} R^{46} x^6 - \\
 & 161003414 d^{54} R^{48} x^6 + 81694041 d^{52} R^{50} x^6 - 26140452 d^{50} \\
 & R^{52} x^6 + 2169167 d^{48} R^{54} x^6 + 2597574 d^{46} R^{56} x^6 - 1514103 \\
 & d^{44} R^{58} x^6 + 420368 d^{42} R^{60} x^6 - 64800 d^{40} R^{62} x^6 + 5088 \\
 & d^{38} R^{64} x^6 - 160 d^{36} R^{66} x^6 + 549259 d^{34} R^{72} x^8 - 8199625 \\
 & d^{70} R^{30} x^8 + 57249105 d^{68} R^{32} x^8 - 248241079 d^{66} R^{34} x^8 + \\
 & 748693819 d^{64} R^{36} x^8 - 1666929393 d^{62} R^{38} x^8 + 2838100009 d^{60} \\
 & R^{40} x^8 - 3777334951 d^{58} R^{42} x^8 + 3984336681 d^{56} R^{44} x^8 - \\
 & 3357705923 d^{54} R^{46} x^8 + 2269326059 d^{52} R^{48} x^8 - 1230348093 \\
 & d^{50} R^{50} x^8 + 533423137 d^{48} R^{52} x^8 - 183644275 d^{46} R^{54} x^8 + \\
 & 49586139 d^{44} R^{56} x^8 - 10281925 d^{42} R^{58} x^8 + 1577584 d^{40} R^{60} \\
 & x^8 - 166416 d^{38} R^{62} x^8 + 10048 d^{36} R^{64} x^8 - 160 d^{34} R^{66} x^8 \\
 & - 5602265 d^{32} R^{72} x^{10} + 84790660 d^{30} R^{28} x^{10} - 601195190 d^{68} \\
 & R^{30} x^{10} + 2651676476 d^{66} R^{32} x^{10} - 8147244538 d^{64} R^{34} x^{10} + \\
 & 18503296100 d^{62} R^{36} x^{10} - 32165306750 d^{60} R^{38} x^{10} + \\
 & 43723073500 d^{58} R^{40} x^{10} - 47072687948 d^{56} R^{42} x^{10} + \\
 & 40405569724 d^{54} R^{44} x^{10} - 27699253010 d^{52} R^{46} x^{10} + \\
 & 15124485220 d^{50} R^{48} x^{10} - 6531425030 d^{48} R^{50} x^{10} + 2204318668 \\
 & d^{46} R^{52} x^{10} - 571136234 d^{44} R^{54} x^{10} + 110739380 d^{42} R^{56} x^{10} \\
 & - 15503755 d^{40} R^{58} x^{10} + 1496720 d^{38} R^{60} x^{10} - 95696 d^{36} R^{62} \\
 & x^{10} + 4048 d^{34} R^{64} x^{10} - 80 d^{32} R^{66} x^{10} + 17232640 d^{30} R^{72} x^{24} \\
 & x^{12} - 266561820 d^{28} R^{26} x^{12} + 1935143010 d^{26} R^{28} x^{12} - \\
 & 8756190770 d^{24} R^{30} x^{12} + 27657747891 d^{22} R^{32} x^{12} - 64720751727 \\
 & d^{20} R^{34} x^{12} + 116199062936 d^{18} R^{36} x^{12} - 163537082760 d^{16} \\
 & R^{38} x^{12} + 182743195365 d^{14} R^{40} x^{12} - 163194394233 d^{12} R^{42} \\
 & x^{12} + 116627873466 d^{10} R^{44} x^{12} - 66475269978 d^{8} R^{46} x^{12} + \\
 & 29967593565 d^{6} R^{48} x^{12} - 10535289345 d^{4} R^{50} x^{12} + 2826881868
 \end{aligned}$$

$$\begin{aligned}
& d^{44} R^{52} x^{12} - 560814236 d^{42} R^{54} x^{12} + 78417003 d^{40} R^{56} x^{12} \\
& - 7146315 d^{38} R^{58} x^{12} + 356000 d^{36} R^{60} x^{12} - 1776 d^{34} R^{62} \\
& x^{12} - 768 d^{32} R^{64} x^{12} - 16 d^{30} R^{66} x^{12} - 24285184 d^{72} R^{22} \\
& x^{14} + 385785088 d^{70} R^{24} x^{14} - 2881002774 d^{68} R^{26} x^{14} + \\
& 13434361364 d^{66} R^{28} x^{14} - 43817348183 d^{64} R^{30} x^{14} + \\
& 106103399406 d^{62} R^{32} x^{14} - 197582816509 d^{60} R^{34} x^{14} + \\
& 289135653352 d^{58} R^{36} x^{14} - 336827237487 d^{56} R^{38} x^{14} + \\
& 314434794154 d^{54} R^{40} x^{14} - 235528588009 d^{52} R^{42} x^{14} + \\
& 141034898004 d^{50} R^{44} x^{14} - 66887317757 d^{48} R^{46} x^{14} + \\
& 24716858882 d^{46} R^{48} x^{14} - 6929499759 d^{44} R^{50} x^{14} + 1410347536 \\
& d^{42} R^{52} x^{14} - 192400333 d^{40} R^{54} x^{14} + 14694054 d^{38} R^{56} x^{14} \\
& - 271509 d^{36} R^{58} x^{14} - 23712 d^{34} R^{60} x^{14} - 624 d^{32} R^{62} x^{14} \\
& + 16252928 d^{72} R^{20} x^{16} - 265904128 d^{70} R^{22} x^{16} + 2047814912 \\
& d^{68} R^{24} x^{16} - 9861937948 d^{66} R^{26} x^{16} + 33272046827 d^{64} R^{28} \\
& x^{16} - 83484705205 d^{62} R^{30} x^{16} + 161398843859 d^{60} R^{32} x^{16} - \\
& 245718018061 d^{58} R^{34} x^{16} + 298486944119 d^{56} R^{36} x^{16} - \\
& 291288720281 d^{54} R^{38} x^{16} + 228716678047 d^{52} R^{40} x^{16} - \\
& 143983269209 d^{50} R^{42} x^{16} + 72007943761 d^{48} R^{44} x^{16} - \\
& 28142435519 d^{46} R^{46} x^{16} + 8364025081 d^{44} R^{48} x^{16} - 1805085191 \\
& d^{42} R^{50} x^{16} + 259353853 d^{40} R^{52} x^{16} - 20039987 d^{38} R^{54} x^{16} \\
& + 149973 d^{36} R^{56} x^{16} + 60873 d^{34} R^{58} x^{16} + 1296 d^{32} R^{60} \\
& x^{16} - 4194304 d^{72} R^{18} x^{18} + 70778880 d^{70} R^{20} x^{18} - 562728960 \\
& d^{68} R^{22} x^{18} + 2800239360 d^{66} R^{24} x^{18} - 9771135705 d^{64} R^{26} \\
& x^{18} + 25382000016 d^{62} R^{28} x^{18} - 50851456440 d^{60} R^{30} x^{18} + \\
& 80308309680 d^{58} R^{32} x^{18} - 101298990780 d^{56} R^{34} x^{18} + \\
& 102750711440 d^{54} R^{36} x^{18} - 83934217224 d^{52} R^{38} x^{18} + \\
& 55014888240 d^{50} R^{40} x^{18} - 28663462230 d^{48} R^{42} x^{18} + \\
& 11672754480 d^{46} R^{44} x^{18} - 3612869640 d^{44} R^{46} x^{18} + 810281616 \\
& d^{42} R^{48} x^{18} - 120213180 d^{40} R^{50} x^{18} + 9347760 d^{38} R^{52} x^{18} - \\
& 5560 d^{36} R^{54} x^{18} - 36720 d^{34} R^{56} x^{18} - 729 d^{32} R^{58} x^{18} + \\
& 216 d^{69} R^{36} x^2 y - 3000 d^{67} R^{38} x^2 y + 19504 d^{65} R^{40} x^2 y - \\
& 78832 d^{63} R^{42} x^2 y + 221832 d^{61} R^{44} x^2 y - 461032 d^{59} R^{46} \\
& x^2 y + 732160 d^{57} R^{48} x^2 y - 906048 d^{55} R^{50} x^2 y + 882024 \\
& d^{53} R^{52} x^2 y - 676104 d^{51} R^{54} x^2 y + 404976 d^{49} R^{56} x^2 y - \\
& 186160 d^{47} R^{58} x^2 y + 63544 d^{45} R^{60} x^2 y - 15192 d^{43} R^{62} x^2 \\
& y + 2272 d^{41} R^{64} x^2 y - 160 d^{39} R^{66} x^2 y + 6966 d^{71} R^{32} x^4 \\
& y - 89286 d^{69} R^{34} x^4 y + 521964 d^{67} R^{36} x^4 y - 1830100 d^{65} \\
& R^{38} x^4 y + 4240226 d^{63} R^{40} x^4 y - 6682546 d^{61} R^{42} x^4 y + \\
& 6918496 d^{59} R^{44} x^4 y - 3760328 d^{57} R^{46} x^4 y - 996710 d^{55} R^{48} \\
& x^4 y + 4042038 d^{53} R^{50} x^4 y - 3948516 d^{51} R^{52} x^4 y + \\
& 2106156 d^{49} R^{54} x^4 y - 525746 d^{47} R^{56} x^4 y - 88190 d^{45} R^{58} \\
& x^4 y + 123736 d^{43} R^{60} x^4 y - 45904 d^{41} R^{62} x^4 y + 8384 d^{39} \\
& R^{64} x^4 y - 640 d^{37} R^{66} x^4 y + 45204 d^{35} R^{68} x^4 y - 471336 \\
& d^{69} R^{32} x^6 y + 1823460 d^{67} R^{34} x^6 y - 1238304 d^{65} R^{36} x^6 y \\
& - 17677188 d^{63} R^{38} x^6 y + 90924936 d^{61} R^{40} x^6 y - 251208828
\end{aligned}$$

$$\begin{aligned}
& d^{59} R^{42} x^6 y + 476951280 d^{57} R^{44} x^6 y - 671482812 d^{55} R^{46} \\
& x^6 y + 724539816 d^{53} R^{48} x^6 y - 607822644 d^{51} R^{50} x^6 y + \\
& 397547904 d^{49} R^{52} x^6 y - 201353100 d^{47} R^{54} x^6 y + 77643576 \\
& d^{45} R^{56} x^6 y - 22112628 d^{43} R^{58} x^6 y + 4419504 d^{41} R^{60} x^6 y \\
& - 567240 d^{39} R^{62} x^6 y + 39360 d^{37} R^{64} x^6 y - 960 d^{35} R^{66} x^6 \\
& y + 3847990 d^{31} R^{28} x^8 y - 59421970 d^{29} R^{30} x^8 y + 429369186 \\
& d^{67} R^{32} x^8 y - 1927578430 d^{65} R^{34} x^8 y + 6019901830 d^{63} R^{36} \\
& x^8 y - 13875469890 d^{61} R^{38} x^8 y + 24435373810 d^{59} R^{40} x^8 y - \\
& 33574281598 d^{57} R^{42} x^8 y + 36432793410 d^{55} R^{44} x^8 y - \\
& 31401806150 d^{53} R^{46} x^8 y + 21505050710 d^{51} R^{48} x^8 y - \\
& 11646725610 d^{49} R^{50} x^8 y + 4938409138 d^{47} R^{52} x^8 y - \\
& 1613103430 d^{45} R^{54} x^8 y + 396394710 d^{43} R^{56} x^8 y - 70894810 \\
& d^{41} R^{58} x^8 y + 8829520 d^{39} R^{60} x^8 y - 722976 d^{37} R^{62} x^8 y + \\
& 35200 d^{35} R^{64} x^8 y - 640 d^{33} R^{66} x^8 y - 23768320 d^{71} R^{26} \\
& x^{10} y + 364399400 d^{69} R^{28} x^{10} y - 2618255944 d^{67} R^{30} x^{10} y + \\
& 11707050232 d^{65} R^{32} x^{10} y - 36476089544 d^{63} R^{34} x^{10} y + \\
& 84025646560 d^{61} R^{36} x^{10} y - 148159689040 d^{59} R^{38} x^{10} y + \\
& 204226869032 d^{57} R^{40} x^{10} y - 222788997496 d^{55} R^{42} x^{10} y + \\
& 193465636832 d^{53} R^{44} x^{10} y - 133792893520 d^{51} R^{46} x^{10} y + \\
& 73339920680 d^{49} R^{48} x^{10} y - 31540556632 d^{47} R^{50} x^{10} y + \\
& 10462138496 d^{45} R^{52} x^{10} y - 2607412432 d^{43} R^{54} x^{10} y + \\
& 469124440 d^{41} R^{56} x^{10} y - 57220040 d^{39} R^{58} x^{10} y + 4251544 \\
& d^{37} R^{60} x^{10} y - 156232 d^{35} R^{62} x^{10} y + 2144 d^{33} R^{64} x^{10} y - \\
& 160 d^{31} R^{66} x^{10} y + 52554240 d^{29} R^{24} x^{12} y - 811382016 d^{69} \\
& R^{26} x^{12} y + 5877772356 d^{67} R^{28} x^{12} y - 26531437236 d^{65} R^{30} \\
& x^{12} y + 83568550818 d^{63} R^{32} x^{12} y - 194907352482 d^{61} R^{34} x^{12} \\
& y + 348530711088 d^{59} R^{36} x^{12} y - 488067660168 d^{57} R^{38} x^{12} y + \\
& 541894772718 d^{55} R^{40} x^{12} y - 479827292334 d^{53} R^{42} x^{12} y + \\
& 338944098636 d^{51} R^{44} x^{12} y - 190033687428 d^{49} R^{46} x^{12} y + \\
& 83621294718 d^{47} R^{48} x^{12} y - 28330465278 d^{45} R^{50} x^{12} y + \\
& 7164506664 d^{43} R^{52} x^{12} y - 1285164336 d^{41} R^{54} x^{12} y + \\
& 149237106 d^{39} R^{56} x^{12} y - 9201906 d^{37} R^{58} x^{12} y + 142296 d^{35} \\
& R^{60} x^{12} y + 2352 d^{33} R^{62} x^{12} y + 192 d^{31} R^{64} x^{12} y - \\
& 49463296 d^{29} R^{22} x^{14} y + 774532096 d^{69} R^{24} x^{14} y - 5695934208 \\
& d^{67} R^{26} x^{14} y + 26125934888 d^{65} R^{28} x^{14} y - 83705443676 d^{63} \\
& R^{30} x^{14} y + 198791334864 d^{61} R^{32} x^{14} y - 362360076028 d^{59} R^{34} \\
& x^{14} y + 517828500184 d^{57} R^{36} x^{14} y - 587347762404 d^{55} R^{38} x^{14} \\
& y + 531834073888 d^{53} R^{40} x^{14} y - 384500958556 d^{51} R^{42} x^{14} y + \\
& 220755783768 d^{49} R^{44} x^{14} y - 99471421244 d^{47} R^{46} x^{14} y + \\
& 34472024144 d^{45} R^{48} x^{14} y - 8889641748 d^{43} R^{50} x^{14} y + \\
& 1614209704 d^{41} R^{52} x^{14} y - 186449836 d^{39} R^{54} x^{14} y + 10872768 \\
& d^{37} R^{56} x^{14} y - 116628 d^{35} R^{58} x^{14} y + 1536 d^{33} R^{60} x^{14} y - \\
& 216 d^{31} R^{62} x^{14} y + 16777216 d^{29} R^{20} x^{16} y - 267567104 d^{69} \\
& R^{22} x^{16} y + 2004199936 d^{67} R^{24} x^{16} y - 9363394304 d^{65} R^{26} \\
& x^{16} y + 30554278966 d^{63} R^{28} x^{16} y - 73892953898 d^{61} R^{30} x^{16} y
\end{aligned}$$

$$\begin{aligned}
& + 137122548262 d^{59} R^{32} x^{16} y - 199397464058 d^{57} R^{34} x^{16} y + \\
& 229983124462 d^{55} R^{36} x^{16} y - 211546518898 d^{53} R^{38} x^{16} y + \\
& 155138238398 d^{51} R^{40} x^{16} y - 90155780962 d^{49} R^{42} x^{16} y + \\
& 40987849058 d^{47} R^{44} x^{16} y - 14261859262 d^{45} R^{46} x^{16} y + \\
& 3663766898 d^{43} R^{48} x^{16} y - 653737966 d^{41} R^{50} x^{16} y + 72258554 \\
& d^{39} R^{52} x^{16} y - 3803686 d^{37} R^{54} x^{16} y + 45354 d^{35} R^{56} x^{16} y \\
& - 6966 d^{33} R^{58} x^{16} y + 16 d^{66} R^{40} y^2 - 208 d^{64} R^{42} y^2 + \\
& 1248 d^{62} R^{44} y^2 - 4576 d^{60} R^{46} y^2 + 11440 d^{58} R^{48} y^2 - \\
& 20592 d^{56} R^{50} y^2 + 27456 d^{54} R^{52} y^2 - 27456 d^{52} R^{54} y^2 + \\
& 20592 d^{50} R^{56} y^2 - 11440 d^{48} R^{58} y^2 + 4576 d^{46} R^{60} y^2 - \\
& 1248 d^{44} R^{62} y^2 + 208 d^{42} R^{64} y^2 - 16 d^{40} R^{66} y^2 + 1458 \\
& d^{72} R^{32} x^2 y^2 - 21546 d^{70} R^{34} x^2 y^2 + 150732 d^{68} R^{36} x^2 \\
& y^2 - 663932 d^{66} R^{38} x^2 y^2 + 2065174 d^{64} R^{40} x^2 y^2 - 4820894 \\
& d^{62} R^{42} x^2 y^2 + 8757944 d^{60} R^{44} x^2 y^2 - 12665224 d^{58} R^{46} \\
& x^2 y^2 + 14783054 d^{56} R^{48} x^2 y^2 - 14029158 d^{54} R^{50} x^2 y^2 + \\
& 10843404 d^{52} R^{52} x^2 y^2 - 6799260 d^{50} R^{54} x^2 y^2 + 3422042 \\
& d^{48} R^{56} x^2 y^2 - 1355122 d^{46} R^{58} x^2 y^2 + 407984 d^{44} R^{60} x^2 \\
& y^2 - 87968 d^{42} R^{62} x^2 y^2 + 12112 d^{40} R^{64} x^2 y^2 - 800 d^{38} \\
& R^{66} x^2 y^2 + 55674 d^{72} R^{30} x^4 y^2 - 784401 d^{70} R^{32} x^4 y^2 + \\
& 5123679 d^{68} R^{34} x^4 y^2 - 20488910 d^{66} R^{36} x^4 y^2 + 55648912 \\
& d^{64} R^{38} x^4 y^2 - 107117483 d^{62} R^{40} x^4 y^2 + 146175809 d^{60} \\
& R^{42} x^4 y^2 - 132040324 d^{58} R^{44} x^4 y^2 + 52502450 d^{56} R^{46} x^4 \\
& y^2 + 52419081 d^{54} R^{48} x^4 y^2 - 121243551 d^{52} R^{50} x^4 y^2 + \\
& 125677890 d^{50} R^{52} x^4 y^2 - 86755084 d^{48} R^{54} x^4 y^2 + 42805235 \\
& d^{46} R^{56} x^4 y^2 - 15196993 d^{44} R^{58} x^4 y^2 + 3781360 d^{42} R^{60} \\
& x^4 y^2 - 620048 d^{40} R^{62} x^4 y^2 + 59104 d^{38} R^{64} x^4 y^2 - \\
& 2400 d^{36} R^{66} x^4 y^2 + 2459638 d^{72} R^{28} x^6 y^2 - 37214248 d^{70} \\
& R^{30} x^6 y^2 + 264903510 d^{68} R^{32} x^6 y^2 - 1179281392 d^{66} R^{34} \\
& x^6 y^2 + 3681157198 d^{64} R^{36} x^6 y^2 - 8561932488 d^{62} R^{38} x^6 \\
& y^2 + 15389511838 d^{60} R^{40} x^6 y^2 - 21876048688 d^{58} R^{42} x^6 y^2 \\
& + 24951660786 d^{56} R^{44} x^6 y^2 - 23023536536 d^{54} R^{46} x^6 y^2 + \\
& 17235929282 d^{52} R^{48} x^6 y^2 - 10445300112 d^{50} R^{50} x^6 y^2 + \\
& 5084899066 d^{48} R^{52} x^6 y^2 - 1960452376 d^{46} R^{54} x^6 y^2 + \\
& 585281898 d^{44} R^{56} x^6 y^2 - 130756432 d^{42} R^{58} x^6 y^2 + 20744272 \\
& d^{40} R^{60} x^6 y^2 - 2145888 d^{38} R^{62} x^6 y^2 + 123232 d^{36} R^{64} x^6 \\
& y^2 - 2560 d^{34} R^{66} x^6 y^2 - 26903330 d^{72} R^{26} x^8 y^2 + \\
& 415373815 d^{70} R^{28} x^8 y^2 - 3016918217 d^{68} R^{30} x^8 y^2 + \\
& 13695403109 d^{66} R^{32} x^8 y^2 - 43540098295 d^{64} R^{34} x^8 y^2 + \\
& 102935673575 d^{62} R^{36} x^8 y^2 - 187531854305 d^{60} R^{38} x^8 y^2 + \\
& 269174294701 d^{58} R^{40} x^8 y^2 - 308547637307 d^{56} R^{42} x^8 y^2 + \\
& 284522633005 d^{54} R^{44} x^8 y^2 - 211530594275 d^{52} R^{46} x^8 y^2 + \\
& 126466349815 d^{50} R^{48} x^8 y^2 - 60341031101 d^{48} R^{50} x^8 y^2 + \\
& 22666156837 d^{46} R^{52} x^8 y^2 - 6561090275 d^{44} R^{54} x^8 y^2 + \\
& 1416966695 d^{42} R^{56} x^8 y^2 - 217508575 d^{40} R^{58} x^8 y^2 + \\
& 22086992 d^{38} R^{60} x^8 y^2 - 1349264 d^{36} R^{62} x^8 y^2 + 47440 d^{34}
\end{aligned}$$

$$\begin{aligned}
& R^{64} x^8 y^2 - 1040 d^{32} R^{66} x^8 y^2 + 93383680 d^{72} R^{24} x^{10} y^2 \\
& - 1475978900 d^{70} R^{26} x^{10} y^2 + 10975412668 d^{68} R^{28} x^{10} y^2 - \\
& 51004227700 d^{66} R^{30} x^{10} y^2 + 165941036006 d^{64} R^{32} x^{10} y^2 - \\
& 401241533686 d^{62} R^{34} x^{10} y^2 + 746961116536 d^{60} R^{36} x^{10} y^2 - \\
& 1094190073904 d^{58} R^{38} x^{10} y^2 + 1277904534730 d^{56} R^{40} x^{10} y^2 - \\
& 1198123268498 d^{54} R^{42} x^{10} y^2 + 903345893788 d^{52} R^{44} x^{10} \\
& y^2 - 546037684868 d^{50} R^{46} x^{10} y^2 + 262459494154 d^{48} R^{48} x^{10} \\
& y^2 - 98897723210 d^{46} R^{50} x^{10} y^2 + 28568963728 d^{44} R^{52} x^{10} \\
& y^2 - 6114132088 d^{42} R^{54} x^{10} y^2 + 919025078 d^{40} R^{56} x^{10} y^2 - \\
& 88839418 d^{38} R^{58} x^{10} y^2 + 4691680 d^{36} R^{60} x^{10} y^2 - 88736 \\
& d^{34} R^{62} x^{10} y^2 - 944 d^{32} R^{64} x^{10} y^2 - 96 d^{30} R^{66} x^{10} y^2 \\
& - 145543168 d^{72} R^{22} x^{12} y^2 + 2364272896 d^{70} R^{24} x^{12} y^2 - \\
& 18066939384 d^{68} R^{26} x^{12} y^2 + 86267131334 d^{66} R^{28} x^{12} y^2 - \\
& 288319532420 d^{64} R^{30} x^{12} y^2 + 715953697641 d^{62} R^{32} x^{12} y^2 - \\
& 1368292886803 d^{60} R^{34} x^{12} y^2 + 2056717288792 d^{58} R^{36} x^{12} y^2 \\
& - 2463311258382 d^{56} R^{38} x^{12} y^2 + 2366535740335 d^{54} R^{40} x^{12} \\
& y^2 - 1826318049985 d^{52} R^{42} x^{12} y^2 + 1128142741518 d^{50} R^{44} \\
& x^{12} y^2 - 552800677832 d^{48} R^{46} x^{12} y^2 + 211514032487 d^{46} R^{48} \\
& x^{12} y^2 - 61611587685 d^{44} R^{50} x^{12} y^2 + 13116859444 d^{42} R^{52} \\
& x^{12} y^2 - 1903744486 d^{40} R^{54} x^{12} y^2 + 164294049 d^{38} R^{56} x^{12} \\
& y^2 - 5795439 d^{36} R^{58} x^{12} y^2 - 36480 d^{34} R^{60} x^{12} y^2 - 6432 \\
& d^{32} R^{62} x^{12} y^2 + 105906176 d^{72} R^{20} x^{14} y^2 - 1769488384 d^{70} \\
& R^{22} x^{14} y^2 + 13908176384 d^{68} R^{24} x^{14} y^2 - 68317226188 d^{66} \\
& R^{26} x^{14} y^2 + 234949378070 d^{64} R^{28} x^{14} y^2 - 600582620716 d^{62} \\
& R^{30} x^{14} y^2 + 1182188981282 d^{60} R^{32} x^{14} y^2 - 1831467141472 \\
& d^{58} R^{34} x^{14} y^2 + 2262688841894 d^{56} R^{36} x^{14} y^2 - \\
& 2244568610180 d^{54} R^{38} x^{14} y^2 + 1790642947522 d^{52} R^{40} x^{14} y^2 \\
& - 1144874486912 d^{50} R^{42} x^{14} y^2 + 581406985042 d^{48} R^{44} x^{14} \\
& y^2 - 230795692724 d^{46} R^{46} x^{14} y^2 + 69761620150 d^{44} R^{48} x^{14} \\
& y^2 - 15374519600 d^{42} R^{50} x^{14} y^2 + 2284806514 d^{40} R^{52} x^{14} y^2 \\
& - 192626204 d^{38} R^{54} x^{14} y^2 + 4444518 d^{36} R^{56} x^{14} y^2 + 314460 \\
& d^{34} R^{58} x^{14} y^2 + 10368 d^{32} R^{60} x^{14} y^2 - 29360128 d^{72} R^{18} \\
& x^{16} y^2 + 503840768 d^{70} R^{20} x^{16} y^2 - 4072271872 d^{68} R^{22} x^{16} \\
& y^2 + 20593964288 d^{66} R^{24} x^{16} y^2 - 73006224262 d^{64} R^{26} x^{16} \\
& y^2 + 192609347195 d^{62} R^{28} x^{16} y^2 - 391798973029 d^{60} R^{30} x^{16} \\
& y^2 + 628069552691 d^{58} R^{32} x^{16} y^2 - 803951732389 d^{56} R^{34} x^{16} \\
& y^2 + 827362714711 d^{54} R^{36} x^{16} y^2 - 685608883817 d^{52} R^{38} x^{16} \\
& y^2 + 455868538879 d^{50} R^{40} x^{16} y^2 - 240995597141 d^{48} R^{42} x^{16} \\
& y^2 + 99650094289 d^{46} R^{44} x^{16} y^2 - 31370458511 d^{44} R^{46} x^{16} \\
& y^2 + 7185889561 d^{42} R^{48} x^{16} y^2 - 1101699743 d^{40} R^{50} x^{16} y^2 \\
& + 92903197 d^{38} R^{52} x^{16} y^2 - 1390563 d^{36} R^{54} x^{16} y^2 - \\
& 247563 d^{34} R^{56} x^{16} y^2 - 6561 d^{32} R^{58} x^{16} y^2 + 216 d^{69} R^{36} \\
& y^3 - 3000 d^{67} R^{38} y^3 + 19504 d^{65} R^{40} y^3 - 78832 d^{63} R^{42} y^3 \\
& + 221832 d^{61} R^{44} y^3 - 461032 d^{59} R^{46} y^3 + 732160 d^{57} R^{48} y^3 \\
& - 906048 d^{55} R^{50} y^3 + 882024 d^{53} R^{52} y^3 - 676104 d^{51} R^{54} y^3
\end{aligned}$$

$$\begin{aligned}
& + 404976 d^{49} R^{56} y^3 - 186160 d^{47} R^{58} y^3 + 63544 d^{45} R^{60} y^3 \\
& - 15192 d^{43} R^{62} y^3 + 2272 d^{41} R^{64} y^3 - 160 d^{39} R^{66} y^3 + \\
& 13932 d^{71} R^{32} x^2 y^3 - 203140 d^{69} R^{34} x^2 y^3 + 1392592 d^{67} \\
& R^{36} x^2 y^3 - 5971864 d^{65} R^{38} x^2 y^3 + 17983380 d^{63} R^{40} x^2 \\
& y^3 - 40470092 d^{61} R^{42} x^2 y^3 + 70700968 d^{59} R^{44} x^2 y^3 - \\
& 98253584 d^{57} R^{46} x^2 y^3 + 110269588 d^{55} R^{48} x^2 y^3 - 100686300 \\
& d^{53} R^{50} x^2 y^3 + 74808448 d^{51} R^{52} x^2 y^3 - 44890456 d^{49} R^{54} \\
& x^2 y^3 + 21411052 d^{47} R^{56} x^2 y^3 - 7904724 d^{45} R^{58} x^2 y^3 + \\
& 2166920 d^{43} R^{60} x^2 y^3 - 412288 d^{41} R^{62} x^2 y^3 + 48128 d^{39} \\
& R^{64} x^2 y^3 - 2560 d^{37} R^{66} x^2 y^3 - 190872 d^{71} R^{30} x^4 y^3 + \\
& 3528644 d^{69} R^{32} x^4 y^3 - 29654236 d^{67} R^{34} x^4 y^3 + 151901888 \\
& d^{65} R^{36} x^4 y^3 - 534456264 d^{63} R^{38} x^4 y^3 + 1376526300 d^{61} \\
& R^{40} x^4 y^3 - 2695872660 d^{59} R^{42} x^4 y^3 + 4110938728 d^{57} R^{44} \\
& x^4 y^3 - 4951671296 d^{55} R^{46} x^4 y^3 + 4746575548 d^{53} R^{48} x^4 \\
& y^3 - 3626872964 d^{51} R^{50} x^4 y^3 + 2200005808 d^{49} R^{52} x^4 y^3 - \\
& 1048273928 d^{47} R^{54} x^4 y^3 + 385077348 d^{45} R^{56} x^4 y^3 - \\
& 105790284 d^{43} R^{58} x^4 y^3 + 20698536 d^{41} R^{60} x^4 y^3 - 2656792 \\
& d^{39} R^{62} x^4 y^3 + 191936 d^{37} R^{64} x^4 y^3 - 5440 d^{35} R^{66} x^4 \\
& y^3 + 15643180 d^{71} R^{28} x^6 y^3 - 241923080 d^{69} R^{30} x^6 y^3 + \\
& 1753480524 d^{67} R^{32} x^6 y^3 - 7910548456 d^{65} R^{34} x^6 y^3 + \\
& 24876920300 d^{63} R^{36} x^6 y^3 - 57873833160 d^{61} R^{38} x^6 y^3 + \\
& 103145106940 d^{59} R^{40} x^6 y^3 - 143874098472 d^{57} R^{42} x^6 y^3 + \\
& 159067859028 d^{55} R^{44} x^6 y^3 - 140273945240 d^{53} R^{46} x^6 y^3 + \\
& 98762558180 d^{51} R^{48} x^6 y^3 - 55292872440 d^{49} R^{50} x^6 y^3 + \\
& 24383239972 d^{47} R^{52} x^6 y^3 - 8334784088 d^{45} R^{54} x^6 y^3 + \\
& 2154429780 d^{43} R^{56} x^6 y^3 - 405758200 d^{41} R^{58} x^6 y^3 + \\
& 52660080 d^{39} R^{60} x^6 y^3 - 4329024 d^{37} R^{62} x^6 y^3 + 198016 d^{35} \\
& R^{64} x^6 y^3 - 3840 d^{33} R^{66} x^6 y^3 - 113616640 d^{71} R^{26} x^8 y^3 + \\
& + 1745410680 d^{69} R^{28} x^8 y^3 - 12577960328 d^{67} R^{30} x^8 y^3 + \\
& 56465328184 d^{65} R^{32} x^8 y^3 - 176849449000 d^{63} R^{34} x^8 y^3 + \\
& 410083194720 d^{61} R^{36} x^8 y^3 - 729040197040 d^{59} R^{38} x^8 y^3 + \\
& 1015083745384 d^{57} R^{40} x^8 y^3 - 1120947653592 d^{55} R^{42} x^8 y^3 + \\
& 987832576640 d^{53} R^{44} x^8 y^3 - 695279647920 d^{51} R^{46} x^8 y^3 + \\
& 389198116840 d^{49} R^{48} x^8 y^3 - 171585499384 d^{47} R^{50} x^8 y^3 + \\
& 58604136832 d^{45} R^{52} x^8 y^3 - 15114244080 d^{43} R^{54} x^8 y^3 + \\
& 2830287640 d^{41} R^{56} x^8 y^3 - 362005480 d^{39} R^{58} x^8 y^3 + \\
& 28652328 d^{37} R^{60} x^8 y^3 - 1200424 d^{35} R^{62} x^8 y^3 + 25440 d^{33} \\
& R^{64} x^8 y^3 - 800 d^{31} R^{66} x^8 y^3 + 293038080 d^{71} R^{24} x^{10} y^3 \\
& - 4532007936 d^{69} R^{26} x^{10} y^3 + 32905553808 d^{67} R^{28} x^{10} y^3 - \\
& 148962520320 d^{65} R^{30} x^{10} y^3 + 470887784108 d^{63} R^{32} x^{10} y^3 - \\
& 1103045744692 d^{61} R^{34} x^{10} y^3 + 1982751333248 d^{59} R^{36} x^{10} y^3 \\
& - 2793733768184 d^{57} R^{38} x^{10} y^3 + 3124398209140 d^{55} R^{40} x^{10} \\
& y^3 - 2790074376284 d^{53} R^{42} x^{10} y^3 + 1990462884696 d^{51} R^{44} \\
& x^{10} y^3 - 1128971055888 d^{49} R^{46} x^{10} y^3 + 503622116244 d^{47} \\
& R^{48} x^{10} y^3 - 173460437420 d^{45} R^{50} x^{10} y^3 + 44786747344 d^{43}
\end{aligned}$$

$$\begin{aligned}
& R^{52} x^{10} y^3 - 8266087736 d^{41} R^{54} x^{10} y^3 + 1005376076 d^{39} R^{56} \\
& x^{10} y^3 - 68844868 d^{37} R^{58} x^{10} y^3 + 1814600 d^{35} R^{60} x^{10} y^3 \\
& - 15168 d^{33} R^{62} x^{10} y^3 + 1152 d^{31} R^{64} x^{10} y^3 - 312328192 \\
& d^{71} R^{22} x^{12} y^3 + 4907637760 d^{69} R^{24} x^{12} y^3 - 36228115200 \\
& d^{67} R^{26} x^{12} y^3 + 166859005640 d^{65} R^{28} x^{12} y^3 - 537013217520 \\
& d^{63} R^{30} x^{12} y^3 + 1281582371988 d^{61} R^{32} x^{12} y^3 - \\
& 2348447861980 d^{59} R^{34} x^{12} y^3 + 3375232482600 d^{57} R^{36} x^{12} y^3 \\
& - 3852055633920 d^{55} R^{38} x^{12} y^3 + 3511402364860 d^{53} R^{40} x^{12} \\
& y^3 - 2557281853412 d^{51} R^{42} x^{12} y^3 + 1480175233680 d^{49} R^{44} \\
& x^{12} y^3 - 673131968600 d^{47} R^{46} x^{12} y^3 + 235844697420 d^{45} R^{48} \\
& x^{12} y^3 - 61681722660 d^{43} R^{50} x^{12} y^3 + 11433221128 d^{41} R^{52} \\
& x^{12} y^3 - 1370586960 d^{39} R^{54} x^{12} y^3 + 88073700 d^{37} R^{56} x^{12} \\
& y^3 - 1834140 d^{35} R^{58} x^{12} y^3 + 35320 d^{33} R^{60} x^{12} y^3 - 1512 \\
& d^{31} R^{62} x^{12} y^3 + 117440512 d^{71} R^{20} x^{14} y^3 - 1888518144 d^{69} \\
& R^{22} x^{14} y^3 + 14261795840 d^{67} R^{24} x^{14} y^3 - 67168549376 d^{65} \\
& R^{26} x^{14} y^3 + 220932779268 d^{63} R^{28} x^{14} y^3 - 538524075496 d^{61} \\
& R^{30} x^{14} y^3 + 1007136271372 d^{59} R^{32} x^{14} y^3 - 1475869711680 \\
& d^{57} R^{34} x^{14} y^3 + 1715378148068 d^{55} R^{36} x^{14} y^3 - \\
& 1590073606264 d^{53} R^{38} x^{14} y^3 + 1175259898956 d^{51} R^{40} x^{14} y^3 \\
& - 688569323312 d^{49} R^{42} x^{14} y^3 + 315812863340 d^{47} R^{44} x^{14} y^3 \\
& - 111004258008 d^{45} R^{46} x^{14} y^3 + 28884222724 d^{43} R^{48} x^{14} y^3 - \\
& 5252661472 d^{41} R^{50} x^{14} y^3 + 601575084 d^{39} R^{52} x^{14} y^3 - \\
& 34925000 d^{37} R^{54} x^{14} y^3 + 689316 d^{35} R^{56} x^{14} y^3 - 55728 d^{33} \\
& R^{58} x^{14} y^3 + 729 d^{72} R^{32} y^4 - 10773 d^{70} R^{34} y^4 + 76398 d^{68} \\
& R^{36} y^4 - 347214 d^{66} R^{38} y^4 + 1137219 d^{64} R^{40} y^4 - 2852655 \\
& d^{62} R^{42} y^4 + 5665764 d^{60} R^{44} y^4 - 9062196 d^{58} R^{46} y^4 + \\
& 11749023 d^{56} R^{48} y^4 - 12341043 d^{54} R^{50} y^4 + 10442718 d^{52} R^{52} \\
& y^4 - 7044414 d^{50} R^{54} y^4 + 3727893 d^{48} R^{56} y^4 - 1510809 d^{46} \\
& R^{58} y^4 + 451824 d^{44} R^{60} y^4 - 93792 d^{42} R^{62} y^4 + 12048 d^{40} \\
& R^{64} y^4 - 720 d^{38} R^{66} y^4 - 27243 d^{72} R^{30} x^2 y^4 + 466044 \\
& d^{70} R^{32} x^2 y^4 - 3786067 d^{68} R^{34} x^2 y^4 + 19399168 d^{66} R^{36} \\
& x^2 y^4 - 70194287 d^{64} R^{38} x^2 y^4 + 190247132 d^{62} R^{40} x^2 y^4 - \\
& 399653319 d^{60} R^{42} x^2 y^4 + 664181024 d^{58} R^{44} x^2 y^4 - \\
& 883472161 d^{56} R^{46} x^2 y^4 + 945554324 d^{54} R^{48} x^2 y^4 - \\
& 814226985 d^{52} R^{50} x^2 y^4 + 561242656 d^{50} R^{52} x^2 y^4 - \\
& 306386197 d^{48} R^{54} x^2 y^4 + 130148468 d^{46} R^{56} x^2 y^4 - 41851037 \\
& d^{44} R^{58} x^2 y^4 + 9756096 d^{42} R^{60} x^2 y^4 - 1534256 d^{40} R^{62} \\
& x^2 y^4 + 142240 d^{38} R^{64} x^2 y^4 - 5600 d^{36} R^{66} x^2 y^4 + \\
& 3915339 d^{72} R^{28} x^4 y^4 - 60271317 d^{70} R^{30} x^4 y^4 + 438751738 \\
& d^{68} R^{32} x^4 y^4 - 2008332008 d^{66} R^{34} x^4 y^4 + 6481747705 d^{64} \\
& R^{36} x^4 y^4 - 15671120763 d^{62} R^{38} x^4 y^4 + 29422993644 d^{60} R^{40} \\
& x^4 y^4 - 43863006642 d^{58} R^{42} x^4 y^4 + 52612857245 d^{56} R^{44} x^4 \\
& y^4 - 51115625275 d^{54} R^{46} x^4 y^4 + 40270281194 d^{52} R^{48} x^4 y^4 \\
& - 25624872052 d^{50} R^{50} x^4 y^4 + 13047801455 d^{48} R^{52} x^4 y^4 - \\
& 5234158981 d^{46} R^{54} x^4 y^4 + 1615438320 d^{44} R^{56} x^4 y^4 -
\end{aligned}$$

$$\begin{aligned}
& 370234770 d^{42} R^{58} x^4 y^4 + 59691600 d^{40} R^{60} x^4 y^4 - 6204176 \\
& d^{38} R^{62} x^4 y^4 + 355744 d^{36} R^{64} x^4 y^4 - 8000 d^{34} R^{66} x^4 \\
& y^4 - 51374265 d^{72} R^{26} x^6 y^4 + 809678274 d^{70} R^{28} x^6 y^4 - \\
& 6021520437 d^{68} R^{30} x^6 y^4 + 28076591304 d^{66} R^{32} x^6 y^4 - \\
& 91968255171 d^{64} R^{34} x^6 y^4 + 224701884870 d^{62} R^{36} x^6 y^4 - \\
& 424272563037 d^{60} R^{38} x^6 y^4 + 632779179636 d^{58} R^{40} x^6 y^4 - \\
& 755353608477 d^{56} R^{42} x^6 y^4 + 726636627678 d^{54} R^{44} x^6 y^4 - \\
& 564259665255 d^{52} R^{46} x^6 y^4 + 352590137952 d^{50} R^{48} x^6 y^4 - \\
& 175846616361 d^{48} R^{50} x^6 y^4 + 69015540042 d^{46} R^{52} x^6 y^4 - \\
& 20857985943 d^{44} R^{54} x^6 y^4 + 4699360260 d^{42} R^{56} x^6 y^4 - \\
& 752102814 d^{40} R^{58} x^6 y^4 + 79536912 d^{38} R^{60} x^6 y^4 - 5013744 \\
& d^{36} R^{62} x^6 y^4 + 171936 d^{34} R^{64} x^6 y^4 - 3360 d^{32} R^{66} x^6 \\
& y^4 + 209072640 d^{72} R^{24} x^8 y^4 - 3386393876 d^{70} R^{26} x^8 y^4 + \\
& 25840813736 d^{68} R^{28} x^8 y^4 - 123394314276 d^{66} R^{30} x^8 y^4 + \\
& 413044770677 d^{64} R^{32} x^8 y^4 - 1028788131405 d^{62} R^{34} x^8 y^4 + \\
& 1975084833118 d^{60} R^{36} x^8 y^4 - 2986793220698 d^{58} R^{38} x^8 y^4 + \\
& 3604593920583 d^{56} R^{40} x^8 y^4 - 3495335570311 d^{54} R^{42} x^8 y^4 + \\
& 2727836092540 d^{52} R^{44} x^8 y^4 - 1707959984848 d^{50} R^{46} x^8 y^4 + \\
& 850936340163 d^{48} R^{48} x^8 y^4 - 332569491563 d^{46} R^{50} x^8 y^4 + \\
& 99716442366 d^{44} R^{52} x^8 y^4 - 22173370450 d^{42} R^{54} x^8 y^4 + \\
& 3470032721 d^{40} R^{56} x^8 y^4 - 351292701 d^{38} R^{58} x^8 y^4 + \\
& 19956256 d^{36} R^{60} x^8 y^4 - 511232 d^{34} R^{62} x^8 y^4 + 6800 d^{32} \\
& R^{64} x^8 y^4 - 240 d^{30} R^{66} x^8 y^4 - 367632384 d^{72} R^{22} x^{10} y^4 \\
& + 6144164352 d^{70} R^{24} x^{10} y^4 - 48269277444 d^{68} R^{26} x^{10} y^4 + \\
& 236783290896 d^{66} R^{28} x^{10} y^4 - 812492382543 d^{64} R^{30} x^{10} y^4 + \\
& 2070201076240 d^{62} R^{32} x^{10} y^4 - 4057481067371 d^{60} R^{34} x^{10} y^4 \\
& + 6251682947752 d^{58} R^{36} x^{10} y^4 - 7672094201083 d^{56} R^{38} x^{10} \\
& y^4 + 7550009304344 d^{54} R^{40} x^{10} y^4 - 5967216960991 d^{52} R^{42} \\
& x^{10} y^4 + 3775029407136 d^{50} R^{44} x^{10} y^4 - 1895015745597 d^{48} \\
& R^{46} x^{10} y^4 + 743408084928 d^{46} R^{48} x^{10} y^4 - 222438772609 d^{44} \\
& R^{50} x^{10} y^4 + 48854718248 d^{42} R^{52} x^{10} y^4 - 7394696617 d^{40} \\
& R^{54} x^{10} y^4 + 688224664 d^{38} R^{56} x^{10} y^4 - 30832241 d^{36} R^{58} \\
& x^{10} y^4 + 375808 d^{34} R^{60} x^{10} y^4 - 25488 d^{32} R^{62} x^{10} y^4 + \\
& 294125568 d^{72} R^{20} x^{12} y^4 - 5058109440 d^{70} R^{22} x^{12} y^4 + \\
& 40827151104 d^{68} R^{24} x^{12} y^4 - 205527107928 d^{66} R^{26} x^{12} y^4 + \\
& 723087502089 d^{64} R^{28} x^{12} y^4 - 1887850975287 d^{62} R^{30} x^{12} y^4 + \\
& 3789952288440 d^{60} R^{32} x^{12} y^4 - 5980484564202 d^{58} R^{34} x^{12} y^4 \\
& + 7517112465759 d^{56} R^{36} x^{12} y^4 - 7578886155477 d^{54} R^{38} x^{12} \\
& y^4 + 6139719043458 d^{52} R^{40} x^{12} y^4 - 3983494489260 d^{50} R^{42} \\
& x^{12} y^4 + 2051970193767 d^{48} R^{44} x^{12} y^4 - 826283725809 d^{46} \\
& R^{46} x^{12} y^4 + 253621596612 d^{44} R^{48} x^{12} y^4 - 56955249402 d^{42} \\
& R^{50} x^{12} y^4 + 8714858385 d^{40} R^{52} x^{12} y^4 - 786304419 d^{38} R^{54} \\
& x^{12} y^4 + 26922978 d^{36} R^{56} x^{12} y^4 + 496776 d^{34} R^{58} x^{12} y^4 + \\
& 36288 d^{32} R^{60} x^{12} y^4 - 88080384 d^{72} R^{18} x^{14} y^4 + 1545076736 \\
& d^{70} R^{20} x^{14} y^4 - 12753686528 d^{68} R^{22} x^{14} y^4 + 65813438208
\end{aligned}$$

$$\begin{aligned}
& d^{66} R^{24} x^{14} y^4 - 237885439831 d^{64} R^{26} x^{14} y^4 + 639438673906 \\
& d^{62} R^{28} x^{14} y^4 - 1324342233873 d^{60} R^{30} x^{14} y^4 + \\
& 2160149422844 d^{58} R^{32} x^{14} y^4 - 2811854875739 d^{56} R^{34} x^{14} y^4 + \\
& + 2941188198078 d^{54} R^{36} x^{14} y^4 - 2476193248529 d^{52} R^{38} x^{14} y^4 \\
& y^4 + 1672276634288 d^{50} R^{40} x^{14} y^4 - 897889188729 d^{48} R^{42} \\
& x^{14} y^4 + 377241443854 d^{46} R^{44} x^{14} y^4 - 120833127163 d^{44} R^{46} \\
& x^{14} y^4 + 28264776876 d^{42} R^{48} x^{14} y^4 - 4470648401 d^{40} R^{50} \\
& x^{14} y^4 + 404305538 d^{38} R^{52} x^{14} y^4 - 10756323 d^{36} R^{54} x^{14} \\
& y^4 - 658584 d^{34} R^{56} x^{14} y^4 - 26244 d^{32} R^{58} x^{14} y^4 + 6966 \\
& d^{71} R^{32} y^5 - 113854 d^{69} R^{34} y^5 + 870628 d^{67} R^{36} y^5 - \\
& 4141764 d^{65} R^{38} y^5 + 13743154 d^{63} R^{40} y^5 - 33787546 d^{61} R^{42} \\
& y^5 + 63782472 d^{59} R^{44} y^5 - 94493256 d^{57} R^{46} y^5 + 111266298 \\
& d^{55} R^{48} y^5 - 104728338 d^{53} R^{50} y^5 + 78756964 d^{51} R^{52} y^5 - \\
& 46996612 d^{49} R^{54} y^5 + 21936798 d^{47} R^{56} y^5 - 7816534 d^{45} R^{58} \\
& y^5 + 2043184 d^{43} R^{60} y^5 - 366384 d^{41} R^{62} y^5 + 39744 d^{39} R^{64} \\
& y^5 - 1920 d^{37} R^{66} y^5 - 517356 d^{31} R^{30} x^2 y^5 + 8421512 d^{69} \\
& R^{32} x^2 y^5 - 64179052 d^{67} R^{34} x^2 y^5 + 304331024 d^{65} R^{36} x^2 \\
& y^5 - 1006387284 d^{63} R^{38} x^2 y^5 + 2464635768 d^{61} R^{40} x^2 y^5 - \\
& 4631882892 d^{59} R^{42} x^2 y^5 + 6827932384 d^{57} R^{44} x^2 y^5 - \\
& 7998461900 d^{55} R^{46} x^2 y^5 + 7493814328 d^{53} R^{48} x^2 y^5 - \\
& 5619807908 d^{51} R^{50} x^2 y^5 + 3357307408 d^{49} R^{52} x^2 y^5 - \\
& 1580230652 d^{47} R^{54} x^2 y^5 + 574986888 d^{45} R^{56} x^2 y^5 - \\
& 156907044 d^{43} R^{58} x^2 y^5 + 30602496 d^{41} R^{60} x^2 y^5 - 3937528 \\
& d^{39} R^{62} x^2 y^5 + 288320 d^{37} R^{64} x^2 y^5 - 8512 d^{35} R^{66} x^2 \\
& y^5 + 23508150 d^{71} R^{28} x^4 y^5 - 364221682 d^{69} R^{30} x^4 y^5 + \\
& 2648767892 d^{67} R^{32} x^4 y^5 - 12009681544 d^{65} R^{34} x^4 y^5 + \\
& 38028373314 d^{63} R^{36} x^4 y^5 - 89262493278 d^{61} R^{38} x^4 y^5 + \\
& 160876459416 d^{59} R^{40} x^4 y^5 - 227489605148 d^{57} R^{42} x^4 y^5 + \\
& 255664774378 d^{55} R^{44} x^4 y^5 - 229846941038 d^{53} R^{46} x^4 y^5 + \\
& 165486282676 d^{51} R^{48} x^4 y^5 - 95042267216 d^{49} R^{50} x^4 y^5 + \\
& 43127960590 d^{47} R^{52} x^4 y^5 - 15211903554 d^{45} R^{54} x^4 y^5 + \\
& 4065335904 d^{43} R^{56} x^4 y^5 - 791737548 d^{41} R^{58} x^4 y^5 + \\
& 105841232 d^{39} R^{60} x^4 y^5 - 8843584 d^{37} R^{62} x^4 y^5 + 398720 \\
& d^{35} R^{64} x^4 y^5 - 7680 d^{33} R^{66} x^4 y^5 - 216917760 d^{71} R^{26} x^6 \\
& y^5 + 3339842680 d^{69} R^{28} x^6 y^5 - 24144536848 d^{67} R^{30} x^6 y^5 + \\
& 108849381096 d^{65} R^{32} x^6 y^5 - 342763516064 d^{63} R^{34} x^6 y^5 + \\
& 800172136424 d^{61} R^{36} x^6 y^5 - 1434254413728 d^{59} R^{38} x^6 y^5 + \\
& 2016775774664 d^{57} R^{40} x^6 y^5 - 2253307166240 d^{55} R^{42} x^6 y^5 + \\
& 2013171853368 d^{53} R^{44} x^6 y^5 - 1439759454848 d^{51} R^{46} x^6 y^5 + \\
& 820904537336 d^{49} R^{48} x^6 y^5 - 369607858464 d^{47} R^{50} x^6 y^5 + \\
& 129291521912 d^{45} R^{52} x^6 y^5 - 34259558624 d^{43} R^{54} x^6 y^5 + \\
& 6616325784 d^{41} R^{56} x^6 y^5 - 877592288 d^{39} R^{58} x^6 y^5 + \\
& 72881648 d^{37} R^{60} x^6 y^5 - 3318768 d^{35} R^{62} x^6 y^5 + 80320 d^{33} \\
& R^{64} x^6 y^5 - 1600 d^{31} R^{66} x^6 y^5 + 678067200 d^{71} R^{24} x^8 y^5 \\
& - 10508287488 d^{69} R^{26} x^8 y^5 + 76500598680 d^{67} R^{28} x^8 y^5 -
\end{aligned}$$

347467031328 d^65 R^30 x^8 y^5 + 1102829628494 d^63 R^32 x^8 y^5 -  
 2595860127886 d^61 R^34 x^8 y^5 + 4692738674828 d^59 R^36 x^8 y^5 -  
 6656120864036 d^57 R^38 x^8 y^5 + 7501126875658 d^55 R^40 x^8 y^5 -  
 6757570118858 d^53 R^42 x^8 y^5 + 4869576377952 d^51 R^44 x^8 y^5 -  
 2793900592392 d^49 R^46 x^8 y^5 + 1262944497282 d^47 R^48 x^8 y^5 -  
 441809177282 d^45 R^50 x^8 y^5 + 116265051484 d^43 R^52 x^8 y^5 -  
 22007447396 d^41 R^54 x^8 y^5 + 2783211302 d^39 R^56 x^8 y^5 -  
 206213926 d^37 R^58 x^8 y^5 + 6985952 d^35 R^60 x^8 y^5 - 111120  
 d^33 R^62 x^8 y^5 + 2880 d^31 R^64 x^8 y^5 - 836468736 d^71 R^22  
 x^10 y^5 + 13201225728 d^69 R^24 x^10 y^5 - 97913206272 d^67 R^26  
 x^10 y^5 + 453265526928 d^65 R^28 x^10 y^5 - 1466737181284 d^63  
 R^30 x^10 y^5 + 3520773556424 d^61 R^32 x^10 y^5 - 6491731384324  
 d^59 R^34 x^10 y^5 + 9391625507536 d^57 R^36 x^10 y^5 -  
 10793588566124 d^55 R^38 x^10 y^5 + 9912561493432 d^53 R^40 x^10 y^5  
 - 7276852887876 d^51 R^42 x^10 y^5 + 4248416724864 d^49 R^44 x^10  
 y^5 - 1950626160756 d^47 R^46 x^10 y^5 + 691066779064 d^45 R^48  
 x^10 y^5 - 183262512812 d^43 R^50 x^10 y^5 + 34642886224 d^41 R^52  
 x^10 y^5 - 4295787268 d^39 R^54 x^10 y^5 + 298785992 d^37 R^56 x^10  
 y^5 - 8505868 d^35 R^58 x^10 y^5 + 179664 d^33 R^60 x^10 y^5 - 4536  
 d^31 R^62 x^10 y^5 + 352321536 d^71 R^20 x^12 y^5 - 5728976896 d^69  
 R^22 x^12 y^5 + 43732137472 d^67 R^24 x^12 y^5 - 208116261120 d^65  
 R^26 x^12 y^5 + 691455345578 d^63 R^28 x^12 y^5 - 1701891014078  
 d^61 R^30 x^12 y^5 + 3212967579168 d^59 R^32 x^12 y^5 -  
 4751600383036 d^57 R^34 x^12 y^5 + 5572205454790 d^55 R^36 x^12 y^5  
 - 5210638386954 d^53 R^38 x^12 y^5 + 3885017684548 d^51 R^40 x^12  
 y^5 - 2296426741648 d^49 R^42 x^12 y^5 + 1063133516406 d^47 R^44  
 x^12 y^5 - 377606593970 d^45 R^46 x^12 y^5 + 99535906424 d^43 R^48  
 x^12 y^5 - 18440895756 d^41 R^50 x^12 y^5 + 2183585626 d^39 R^52  
 x^12 y^5 - 137638342 d^37 R^54 x^12 y^5 + 3555300 d^35 R^56 x^12 y^5  
 - 195048 d^33 R^58 x^12 y^5 - 36720 d^72 R^30 y^6 + 572163 d^70  
 R^32 y^6 - 4264183 d^68 R^34 y^6 + 20213002 d^66 R^36 y^6 - 68276682  
 d^64 R^38 y^6 + 174307009 d^62 R^40 y^6 - 347790469 d^60 R^42 y^6 +  
 552942780 d^58 R^44 y^6 - 707745324 d^56 R^46 y^6 + 732131829 d^54  
 R^48 y^6 - 611289393 d^52 R^50 y^6 + 409424314 d^50 R^52 y^6 -  
 217461946 d^48 R^54 y^6 + 89940807 d^46 R^56 y^6 - 28168147 d^44  
 R^58 y^6 + 6395104 d^42 R^60 y^6 - 979008 d^40 R^62 y^6 + 88224  
 d^38 R^64 y^6 - 3360 d^36 R^66 y^6 + 2648800 d^72 R^28 x^2 y^6 -  
 41698390 d^70 R^30 x^2 y^6 + 311690566 d^68 R^32 x^2 y^6 -  
 1470406684 d^66 R^34 x^2 y^6 + 4905869508 d^64 R^36 x^2 y^6 -  
 12288479394 d^62 R^38 x^2 y^6 + 23931641074 d^60 R^40 x^2 y^6 -  
 37010215216 d^58 R^42 x^2 y^6 + 46011112472 d^56 R^44 x^2 y^6 -  
 46249738106 d^54 R^46 x^2 y^6 + 37608714858 d^52 R^48 x^2 y^6 -  
 24633059308 d^50 R^50 x^2 y^6 + 12873727204 d^48 R^52 x^2 y^6 -  
 5285977422 d^46 R^54 x^2 y^6 + 1665618558 d^44 R^56 x^2 y^6 -  
 388816264 d^42 R^58 x^2 y^6 + 63694624 d^40 R^60 x^2 y^6 - 6706304

$$\begin{aligned}
& d^{38} R^{62} x^2 y^6 + 388384 d^{36} R^{64} x^2 y^6 - 8960 d^{34} R^{66} x^2 \\
& y^6 - 48809840 d^{72} R^{26} x^4 y^6 + 785951407 d^{70} R^{28} x^4 y^6 - \\
& 5983736183 d^{68} R^{30} x^4 y^6 + 28612067972 d^{66} R^{32} x^4 y^6 - \\
& 96251803120 d^{64} R^{34} x^4 y^6 + 241783727729 d^{62} R^{36} x^4 y^6 - \\
& 469727098841 d^{60} R^{38} x^4 y^6 + 721127575570 d^{58} R^{40} x^4 y^6 - \\
& 886148986814 d^{56} R^{42} x^4 y^6 + 877377152353 d^{54} R^{44} x^4 y^6 - \\
& 700949505113 d^{52} R^{46} x^4 y^6 + 450396995776 d^{50} R^{48} x^4 y^6 - \\
& 230868820676 d^{48} R^{50} x^4 y^6 + 93101581327 d^{46} R^{52} x^4 y^6 - \\
& 28914886631 d^{44} R^{54} x^4 y^6 + 6700643114 d^{42} R^{56} x^4 y^6 - \\
& 1105309342 d^{40} R^{58} x^4 y^6 + 120883664 d^{38} R^{60} x^4 y^6 - 7889360 \\
& d^{36} R^{62} x^4 y^6 + 271648 d^{34} R^{64} x^4 y^6 - 4640 d^{32} R^{66} x^4 \\
& y^6 + 247106560 d^{72} R^{24} x^6 y^6 - 4117021404 d^{70} R^{26} x^6 y^6 + \\
& 32312316772 d^{68} R^{28} x^6 y^6 - 158663536104 d^{66} R^{30} x^6 y^6 + \\
& 545954799132 d^{64} R^{32} x^6 y^6 - 1397277388400 d^{62} R^{34} x^6 y^6 + \\
& 2755054944148 d^{60} R^{36} x^6 y^6 - 4276706028984 d^{58} R^{38} x^6 y^6 + \\
& 5295229759100 d^{56} R^{40} x^6 y^6 - 5265166150712 d^{54} R^{42} x^6 y^6 + \\
& 4211431171740 d^{52} R^{44} x^6 y^6 - 2701574344664 d^{50} R^{46} x^6 y^6 + \\
& 1378745811092 d^{48} R^{48} x^6 y^6 - 552045285648 d^{46} R^{50} x^6 y^6 + \\
& 169692084668 d^{44} R^{52} x^6 y^6 - 38750396680 d^{42} R^{54} x^6 y^6 + \\
& 6251938740 d^{40} R^{56} x^6 y^6 - 658542380 d^{38} R^{58} x^6 y^6 + \\
& 39935152 d^{36} R^{60} x^6 y^6 - 1193088 d^{34} R^{62} x^6 y^6 + 21280 d^{32} \\
& R^{64} x^6 y^6 - 320 d^{30} R^{66} x^6 y^6 - 504995840 d^{72} R^{22} x^8 y^6 + \\
& 8759206400 d^{70} R^{24} x^8 y^6 - 71177756184 d^{68} R^{26} x^8 y^6 + \\
& 360117194260 d^{66} R^{28} x^8 y^6 - 1271309042800 d^{64} R^{30} x^8 y^6 + \\
& 3325451238655 d^{62} R^{32} x^8 y^6 - 6678777680495 d^{60} R^{34} x^8 y^6 + \\
& 10528146145342 d^{58} R^{36} x^8 y^6 - 13200885055930 d^{56} R^{38} x^8 y^6 \\
& + 13258535020445 d^{54} R^{40} x^8 y^6 - 10685962137565 d^{52} R^{42} x^8 \\
& y^6 + 6889975823640 d^{50} R^{44} x^8 y^6 - 3524465869132 d^{48} R^{46} x^8 \\
& y^6 + 1409508563305 d^{46} R^{48} x^8 y^6 - 430548925465 d^{44} R^{50} x^8 \\
& y^6 + 96869086550 d^{42} R^{52} x^8 y^6 - 15145568170 d^{40} R^{54} x^8 y^6 \\
& + 1489843099 d^{38} R^{56} x^8 y^6 - 76633635 d^{36} R^{58} x^8 y^6 + \\
& 1596320 d^{34} R^{60} x^8 y^6 - 52800 d^{32} R^{62} x^8 y^6 + 450887680 d^{72} \\
& R^{20} x^{10} y^6 - 8080031744 d^{70} R^{22} x^{10} y^6 + 67568837632 d^{68} \\
& R^{24} x^{10} y^6 - 350769421208 d^{66} R^{26} x^{10} y^6 + 1267822831876 d^{64} \\
& R^{28} x^{10} y^6 - 3390016219166 d^{62} R^{30} x^{10} y^6 + 6952092118822 \\
& d^{60} R^{32} x^{10} y^6 - 11182349969216 d^{58} R^{34} x^{10} y^6 + \\
& 14301587594944 d^{56} R^{36} x^{10} y^6 - 14649742652818 d^{54} R^{38} x^{10} \\
& y^6 + 12043211455418 d^{52} R^{40} x^{10} y^6 - 7921931551612 d^{50} R^{42} \\
& x^{10} y^6 + 4134919026836 d^{48} R^{44} x^{10} y^6 - 1687080307714 d^{46} \\
& R^{46} x^{10} y^6 + 525165460058 d^{44} R^{48} x^{10} y^6 - 119963504008 d^{42} \\
& R^{50} x^{10} y^6 + 18834055784 d^{40} R^{52} x^{10} y^6 - 1795071502 d^{38} \\
& R^{54} x^{10} y^6 + 76601478 d^{36} R^{56} x^{10} y^6 - 214116 d^{34} R^{58} x^{10} \\
& y^6 + 72576 d^{32} R^{60} x^{10} y^6 - 146800640 d^{72} R^{18} x^{12} y^6 + \\
& 2653421568 d^{70} R^{20} x^{12} y^6 - 22521425920 d^{68} R^{22} x^{12} y^6 + \\
& 119283173632 d^{66} R^{24} x^{12} y^6 - 441799283076 d^{64} R^{26} x^{12} y^6 +
\end{aligned}$$

$$\begin{aligned}
& 1215098094101 d^{62} R^{28} x^{12} y^6 - 2571553314509 d^{60} R^{30} x^{12} y^6 \\
& + 4281020628930 d^{58} R^{32} x^{12} y^6 - 5681511176926 d^{56} R^{34} x^{12} y^6 \\
& + 6053329726223 d^{54} R^{36} x^{12} y^6 - 5186923027143 d^{52} R^{38} x^{12} y^6 \\
& + 3563012751064 d^{50} R^{40} x^{12} y^6 - 1945187019100 d^{48} R^{42} x^{12} y^6 \\
& + 831059171331 d^{46} R^{44} x^{12} y^6 - 270964232843 d^{44} R^{46} x^{12} y^6 \\
& + 64718767418 d^{42} R^{48} x^{12} y^6 - 10545949998 d^{40} R^{50} x^{12} y^6 \\
& + 1014145465 d^{38} R^{52} x^{12} y^6 - 36825537 d^{36} R^{54} x^{12} y^6 \\
& - 762804 d^{34} R^{56} x^{12} y^6 - 61236 d^{32} R^{58} x^{12} y^6 \\
& - 281280 d^{30} R^{60} y^7 + 4421532 d^{69} R^{32} y^7 - 32701356 d^{67} R^{34} y^7 \\
& + 151190832 d^{65} R^{36} y^7 - 489608208 d^{63} R^{38} y^7 + 1179034404 d^{61} R^{40} y^7 \\
& - 2187219060 d^{59} R^{42} y^7 + 3193944936 d^{57} R^{44} y^7 \\
& - 3718273416 d^{55} R^{46} y^7 + 3471778596 d^{53} R^{48} y^7 - 2600757588 d^{51} R^{50} y^7 \\
& + 1554849504 d^{49} R^{52} y^7 - 733309824 d^{47} R^{54} y^7 + 267553116 d^{45} R^{56} y^7 \\
& - 73229388 d^{43} R^{58} y^7 + 14323464 d^{41} R^{60} y^7 - 1847976 d^{39} R^{62} y^7 + 135744 d^{37} R^{64} y^7 - 4032 d^{35} R^{66} y^7 \\
& + 15478720 d^{71} R^{28} x^2 y^7 - 240387004 d^{69} R^{30} x^2 y^7 \\
& + 1754919956 d^{67} R^{32} x^2 y^7 - 8000295936 d^{65} R^{34} x^2 y^7 + 25514640712 d^{63} R^{36} x^2 y^7 - 60430212132 d^{61} R^{38} x^2 y^7 \\
& + 110108612956 d^{59} R^{40} x^2 y^7 - 157727673688 d^{57} R^{42} x^2 y^7 + 179939591104 d^{55} R^{44} x^2 y^7 - 164548182084 d^{53} R^{46} x^2 y^7 \\
& + 120746498444 d^{51} R^{48} x^2 y^7 - 70808700560 d^{49} R^{50} x^2 y^7 + 32861705448 d^{47} R^{52} x^2 y^7 - 11869928860 d^{45} R^{54} x^2 y^7 \\
& + 3251552772 d^{43} R^{56} x^2 y^7 - 649307032 d^{41} R^{58} x^2 y^7 + 88934608 d^{39} R^{60} x^2 y^7 - 7585216 d^{37} R^{62} x^2 y^7 + 344192 d^{35} R^{64} x^2 y^7 \\
& - 6400 d^{33} R^{66} x^2 y^7 - 206736640 d^{71} R^{26} x^4 y^7 + 3191057960 d^{69} R^{28} x^4 y^7 - 23148407848 d^{67} R^{30} x^4 y^7 \\
& + 104826673936 d^{65} R^{32} x^4 y^7 - 331954734032 d^{63} R^{34} x^4 y^7 + 780273661000 d^{61} R^{36} x^4 y^7 - 1410118914088 d^{59} R^{38} x^4 y^7 \\
& + 2002093765088 d^{57} R^{40} x^4 y^7 - 2262126554272 d^{55} R^{42} x^4 y^7 + 2047178733704 d^{53} R^{44} x^4 y^7 - 1485532820200 d^{51} R^{46} x^4 y^7 \\
& + 860927786576 d^{49} R^{48} x^4 y^7 - 394719035152 d^{47} R^{50} x^4 y^7 + 140872791416 d^{45} R^{52} x^4 y^7 - 38168017432 d^{43} R^{54} x^4 y^7 \\
& + 7558685440 d^{41} R^{56} x^4 y^7 - 1032976448 d^{39} R^{58} x^4 y^7 + 89248912 d^{37} R^{60} x^4 y^7 - 4316080 d^{35} R^{62} x^4 y^7 + 109760 d^{33} R^{64} x^4 y^7 \\
& - 1600 d^{31} R^{66} x^4 y^7 + 832972800 d^{71} R^{24} x^6 y^7 - 12940710912 d^{69} R^{26} x^6 y^7 + 94502303760 d^{67} R^{28} x^6 y^7 \\
& - 43087166664 d^{65} R^{30} x^6 y^7 + 1373811711072 d^{63} R^{32} x^6 y^7 - 3251130413160 d^{61} R^{34} x^6 y^7 + 5914057412544 d^{59} R^{36} x^6 y^7 \\
& - 8448505721544 d^{57} R^{38} x^6 y^7 + 9598437809376 d^{55} R^{40} x^6 y^7 - 8726113158984 d^{53} R^{42} x^6 y^7 + 6352599533280 d^{51} R^{44} x^6 y^7 \\
& - 3686606527032 d^{49} R^{46} x^6 y^7 + 1688013488928 d^{47} R^{48} x^6 y^7 - 599258752440 d^{45} R^{50} x^6 y^7 + 160489716672 d^{43} R^{52} x^6 y^7 \\
& - 31074068760 d^{41} R^{54} x^6 y^7 + 4064138400 d^{39} R^{56} x^6 y^7 - 320469144 d^{37} R^{58} x^6 y^7 + 12643728 d^{35} R^{60} x^6 y^7 - 245760 d^{33} R^{62} x^6 y^7 \\
& + 3840 d^{31} R^{64} x^6 y^7 - 1228636160 d^{71} R^{22}
\end{aligned}$$

$$\begin{aligned}
& x^8 y^7 + 19500185600 d^{69} R^{24} x^8 y^7 - 145502000640 d^{67} R^{26} x^8 \\
& y^7 + 677850271120 d^{65} R^{28} x^8 y^7 - 2208178197280 d^{63} R^{30} x^8 \\
& y^7 + 5337829247540 d^{61} R^{32} x^8 y^7 - 9914543602900 d^{59} R^{34} x^8 \\
& y^7 + 14453644955120 d^{57} R^{36} x^8 y^7 - 16744297880960 d^{55} R^{38} \\
& x^8 y^7 + 15505996755340 d^{53} R^{40} x^8 y^7 - 11482613087660 d^{51} \\
& R^{42} x^8 y^7 + 6765950451000 d^{49} R^{44} x^8 y^7 - 3137683193320 d^{47} \\
& R^{46} x^8 y^7 + 1124202154060 d^{45} R^{48} x^8 y^7 - 302231335340 d^{43} \\
& R^{50} x^8 y^7 + 58215403840 d^{41} R^{52} x^8 y^7 - 7446201200 d^{39} R^{54} \\
& x^8 y^7 + 553396340 d^{37} R^{56} x^8 y^7 - 19099220 d^{35} R^{58} x^8 y^7 + \\
& 422280 d^{33} R^{60} x^8 y^7 - 7560 d^{31} R^{62} x^8 y^7 + 587202560 d^{71} \\
& R^{20} x^{10} y^7 - 9698738176 d^{69} R^{22} x^{10} y^7 + 75130308608 d^{67} \\
& R^{24} x^{10} y^7 - 362508110848 d^{65} R^{26} x^{10} y^7 + 1220183330936 d^{63} \\
& R^{28} x^{10} y^7 - 3040345917340 d^{61} R^{30} x^{10} y^7 + 5806806478868 \\
& d^{59} R^{32} x^{10} y^7 - 8682596340184 d^{57} R^{34} x^{10} y^7 + \\
& 10289346589184 d^{55} R^{36} x^{10} y^7 - 9718889648468 d^{53} R^{38} x^{10} y^7 \\
& + 7317389579500 d^{51} R^{40} x^{10} y^7 - 4367328394208 d^{49} R^{42} x^{10} \\
& y^7 + 2042050515544 d^{47} R^{44} x^{10} y^7 - 733214264324 d^{45} R^{46} \\
& x^{10} y^7 + 195817707628 d^{43} R^{48} x^{10} y^7 - 36948809720 d^{41} R^{50} \\
& x^{10} y^7 + 4514916016 d^{39} R^{52} x^{10} y^7 - 305411468 d^{37} R^{54} x^{10} \\
& y^7 + 9395988 d^{35} R^{56} x^{10} y^7 - 390096 d^{33} R^{58} x^{10} y^7 + \\
& 643840 d^{72} R^{28} y^8 - 10441696 d^{70} R^{30} y^8 + 80593233 d^{68} R^{32} \\
& y^8 - 393114989 d^{66} R^{34} y^8 + 1356585182 d^{64} R^{36} y^8 - \\
& 3512361726 d^{62} R^{38} y^8 + 7060059259 d^{60} R^{40} y^8 - 11245922311 \\
& d^{58} R^{42} y^8 + 14365579332 d^{56} R^{44} y^8 - 14799943444 d^{54} R^{46} \\
& y^8 + 12305036887 d^{52} R^{48} y^8 - 8223139275 d^{50} R^{50} y^8 + \\
& 4377401678 d^{48} R^{52} y^8 - 1828626542 d^{46} R^{54} y^8 + 585875997 d^{44} \\
& R^{56} y^8 - 139056001 d^{42} R^{58} y^8 + 23169712 d^{40} R^{60} y^8 - \\
& 2481600 d^{38} R^{62} y^8 + 145824 d^{36} R^{64} y^8 - 3360 d^{34} R^{66} y^8 - \\
& 23098880 d^{72} R^{26} x^2 y^8 + 380409952 d^{70} R^{28} x^2 y^8 - \\
& 2964607973 d^{68} R^{30} x^2 y^8 + 14517592088 d^{66} R^{32} x^2 y^8 - \\
& 50023012945 d^{64} R^{34} x^2 y^8 + 128688037112 d^{62} R^{36} x^2 y^8 - \\
& 255937526825 d^{60} R^{38} x^2 y^8 + 401999917744 d^{58} R^{40} x^2 y^8 - \\
& 505065493205 d^{56} R^{42} x^2 y^8 + 510909329008 d^{54} R^{44} x^2 y^8 - \\
& 416749091759 d^{52} R^{46} x^2 y^8 + 273269516104 d^{50} R^{48} x^2 y^8 - \\
& 142907015459 d^{48} R^{50} x^2 y^8 + 58800812056 d^{46} R^{52} x^2 y^8 - \\
& 18644989571 d^{44} R^{54} x^2 y^8 + 4417510112 d^{42} R^{56} x^2 y^8 - \\
& 746848231 d^{40} R^{58} x^2 y^8 + 84022688 d^{38} R^{60} x^2 y^8 - 5656064 \\
& d^{36} R^{62} x^2 y^8 + 197008 d^{34} R^{64} x^2 y^8 - 2960 d^{32} R^{66} x^2 \\
& y^8 + 162231040 d^{72} R^{24} x^4 y^8 - 2794062656 d^{70} R^{26} x^4 y^8 + \\
& 22619049430 d^{68} R^{28} x^4 y^8 - 114324138106 d^{66} R^{30} x^4 y^8 + \\
& 404139915029 d^{64} R^{32} x^4 y^8 - 1060707437725 d^{62} R^{34} x^4 y^8 + \\
& 2141281055056 d^{60} R^{36} x^4 y^8 - 3398203260404 d^{58} R^{38} x^4 y^8 + \\
& 4296023170627 d^{56} R^{40} x^4 y^8 - 4356801801755 d^{54} R^{42} x^4 y^8 + \\
& 3551317795870 d^{52} R^{44} x^4 y^8 - 2320210923914 d^{50} R^{46} x^4 y^8 + \\
& 1205700430363 d^{48} R^{48} x^4 y^8 - 491666799491 d^{46} R^{50} x^4 y^8 +
\end{aligned}$$

$$\begin{aligned}
& 154064009068 d^{44} R^{52} x^4 y^8 - 35940074680 d^{42} R^{54} x^4 y^8 + \\
& 5949615629 d^{40} R^{56} x^4 y^8 - 649070389 d^{38} R^{58} x^4 y^8 + \\
& 41644720 d^{36} R^{60} x^4 y^8 - 1372592 d^{34} R^{62} x^4 y^8 + 25120 d^{32} \\
& R^{64} x^4 y^8 - 240 d^{30} R^{66} x^4 y^8 - 404541440 d^{72} R^{22} x^6 y^8 + \\
& 7378161920 d^{70} R^{24} x^6 y^8 - 62521940886 d^{68} R^{26} x^6 y^8 + \\
& 327775847020 d^{66} R^{28} x^6 y^8 - 1193095807765 d^{64} R^{30} x^6 y^8 + \\
& 3205249165830 d^{62} R^{32} x^6 y^8 - 6590759206515 d^{60} R^{34} x^6 y^8 + \\
& 10610353012368 d^{58} R^{36} x^6 y^8 - 13559870068485 d^{56} R^{38} x^6 y^8 \\
& + 13859544049650 d^{54} R^{40} x^6 y^8 - 11354539359735 d^{52} R^{42} x^6 \\
& y^8 + 7436242623420 d^{50} R^{44} x^6 y^8 - 3862663455703 d^{48} R^{46} x^6 \\
& y^8 + 1569143992810 d^{46} R^{48} x^6 y^8 - 487507237905 d^{44} R^{50} x^6 \\
& y^8 + 111904188840 d^{42} R^{52} x^6 y^8 - 17976184575 d^{40} R^{54} x^6 y^8 \\
& + 1848714846 d^{38} R^{56} x^6 y^8 - 104618095 d^{36} R^{58} x^6 y^8 + \\
& 2727520 d^{34} R^{60} x^6 y^8 - 63120 d^{32} R^{62} x^6 y^8 + 411566080 d^{72} \\
& R^{20} x^8 y^8 - 7847321600 d^{70} R^{22} x^8 y^8 + 68876051200 d^{68} R^{24} \\
& x^8 y^8 - 371707747340 d^{66} R^{26} x^8 y^8 + 1386986582605 d^{64} R^{28} \\
& x^8 y^8 - 3808778827235 d^{62} R^{30} x^8 y^8 + 7989871542250 d^{60} R^{32} \\
& x^8 y^8 - 13105570070300 d^{58} R^{34} x^8 y^8 + 17051274366295 d^{56} \\
& R^{36} x^8 y^8 - 17735111868565 d^{54} R^{38} x^8 y^8 + 14782607325200 \\
& d^{52} R^{40} x^8 y^8 - 9849018616450 d^{50} R^{42} x^8 y^8 + 5203730640875 \\
& d^{48} R^{44} x^8 y^8 - 2149024410445 d^{46} R^{46} x^8 y^8 + 677678300690 \\
& d^{44} R^{48} x^8 y^8 - 157241001640 d^{42} R^{50} x^8 y^8 + 25260861425 \\
& d^{40} R^{52} x^8 y^8 - 2519909275 d^{38} R^{54} x^8 y^8 + 124222740 d^{36} \\
& R^{56} x^8 y^8 - 1777230 d^{34} R^{58} x^8 y^8 + 90720 d^{32} R^{60} x^8 y^8 - \\
& 146800640 d^{72} R^{18} x^{10} y^8 + 2770862080 d^{70} R^{20} x^{10} y^8 - \\
& 24444514304 d^{68} R^{22} x^{10} y^8 + 134048680192 d^{66} R^{24} x^{10} y^8 - \\
& 512386921847 d^{64} R^{26} x^{10} y^8 + 1450367853484 d^{62} R^{28} x^{10} y^8 - \\
& 3151591953905 d^{60} R^{30} x^{10} y^8 + 5376033824872 d^{58} R^{32} x^{10} y^8 \\
& - 7297738846541 d^{56} R^{34} x^{10} y^8 + 7940748580076 d^{54} R^{36} x^{10} \\
& y^8 - 6939866067277 d^{52} R^{38} x^{10} y^8 + 4857029445320 d^{50} R^{40} \\
& x^{10} y^8 - 2699606350207 d^{48} R^{42} x^{10} y^8 + 1173922922756 d^{46} \\
& R^{44} x^{10} y^8 - 389813828611 d^{44} R^{46} x^{10} y^8 + 95066621912 d^{42} \\
& R^{48} x^{10} y^8 - 15940408495 d^{40} R^{50} x^{10} y^8 + 1619217764 d^{38} \\
& R^{52} x^{10} y^8 - 72241407 d^{36} R^{54} x^{10} y^8 + 16632 d^{34} R^{56} x^{10} \\
& y^8 - 91854 d^{32} R^{58} x^{10} y^8 + 3765760 d^{71} R^{28} y^9 - 58666432 \\
& d^{69} R^{30} y^9 + 430263402 d^{67} R^{32} y^9 - 1973584418 d^{65} R^{34} y^9 + \\
& 6343285868 d^{63} R^{36} y^9 - 15166082124 d^{61} R^{38} y^9 + 27941886670 \\
& d^{59} R^{40} y^9 - 40537885414 d^{57} R^{42} y^9 + 46909882344 d^{55} R^{44} \\
& y^9 - 43573380136 d^{53} R^{46} y^9 + 32517723238 d^{51} R^{48} y^9 - \\
& 19412580174 d^{49} R^{50} y^9 + 9178575692 d^{47} R^{52} y^9 - 3379705964 \\
& d^{45} R^{54} y^9 + 944251938 d^{43} R^{56} y^9 - 192432874 d^{41} R^{58} y^9 + \\
& 26923936 d^{39} R^{60} y^9 - 2347680 d^{37} R^{62} y^9 + 108288 d^{35} R^{64} \\
& y^9 - 1920 d^{33} R^{66} y^9 - 98344960 d^{71} R^{26} x^2 y^9 + 1522233600 \\
& d^{69} R^{28} x^2 y^9 - 11083766744 d^{67} R^{30} x^2 y^9 + 50431266160 d^{65} \\
& R^{32} x^2 y^9 - 160636268152 d^{63} R^{34} x^2 y^9 + 380235458808 d^{61}
\end{aligned}$$

$$\begin{aligned}
R^{36} x^2 y^9 - & 692836929952 d^{59} R^{38} x^2 y^9 + 993060842560 d^{57} \\
R^{40} x^2 y^9 - & 1134177236712 d^{55} R^{42} x^2 y^9 + 1038844029704 d^{53} \\
R^{44} x^2 y^9 - & 763941399936 d^{51} R^{46} x^2 y^9 + 449234825104 d^{49} \\
R^{48} x^2 y^9 - & 209253709288 d^{47} R^{50} x^2 y^9 + 75976871944 d^{45} \\
R^{52} x^2 y^9 - & 20977273152 d^{43} R^{54} x^2 y^9 + 4243921888 d^{41} R^{56} \\
x^2 y^9 - & 595097752 d^{39} R^{58} x^2 y^9 + 53204472 d^{37} R^{60} x^2 y^9 - \\
2696392 d^{35} R^{62} x^2 y^9 + & 69600 d^{33} R^{64} x^2 y^9 - 800 d^{31} R^{66} \\
x^2 y^9 + & 572582400 d^{71} R^{24} x^4 y^9 - 8921756928 d^{69} R^{26} x^4 y^9 \\
+ 65392271940 d^{67} R^{28} x^4 y^9 - & 299466456684 d^{65} R^{30} x^4 y^9 + \\
959806941062 d^{63} R^{32} x^4 y^9 - & 2285077631854 d^{61} R^{34} x^4 y^9 + \\
4185292854824 d^{59} R^{36} x^4 y^9 - & 6025114759736 d^{57} R^{38} x^4 y^9 + \\
6904112674810 d^{55} R^{40} x^4 y^9 - & 6336253141634 d^{53} R^{42} x^4 y^9 + \\
4660807699548 d^{51} R^{44} x^4 y^9 - & 2735621274204 d^{49} R^{46} x^4 y^9 + \\
1268275224762 d^{47} R^{48} x^4 y^9 - & 456568588658 d^{45} R^{50} x^4 y^9 + \\
124278714592 d^{43} R^{52} x^4 y^9 - & 24560934224 d^{41} R^{54} x^4 y^9 + \\
3308351558 d^{39} R^{56} x^4 y^9 - & 274543966 d^{37} R^{58} x^4 y^9 + \\
12027032 d^{35} R^{60} x^4 y^9 - & 257520 d^{33} R^{62} x^4 y^9 + 2880 d^{31} \\
R^{64} x^4 y^9 - & 1065205760 d^{71} R^{22} x^6 y^9 + 17032616960 d^{69} R^{24} \\
x^6 y^9 - & 128080546560 d^{67} R^{26} x^6 y^9 + 601518206920 d^{65} R^{28} \\
x^6 y^9 - & 1975914798020 d^{63} R^{30} x^6 y^9 + 4817532145440 d^{61} R^{32} \\
x^6 y^9 - & 9027215146980 d^{59} R^{34} x^6 y^9 + 13278976890680 d^{57} \\
R^{36} x^6 y^9 - & 15525246856860 d^{55} R^{38} x^6 y^9 + 14512226798160 \\
d^{53} R^{40} x^6 y^9 - & 10850026990660 d^{51} R^{42} x^6 y^9 + \\
6456686797080 d^{49} R^{44} x^6 y^9 - & 3025610794180 d^{47} R^{46} x^6 y^9 + \\
1096513928960 d^{45} R^{48} x^6 y^9 - & 298797205260 d^{43} R^{50} x^6 y^9 + \\
58599836040 d^{41} R^{52} x^6 y^9 - & 7712927060 d^{39} R^{54} x^6 y^9 + \\
606652560 d^{37} R^{56} x^6 y^9 - & 23939020 d^{35} R^{58} x^6 y^9 + 545120 \\
d^{33} R^{60} x^6 y^9 - & 7560 d^{31} R^{62} x^6 y^9 + 587202560 d^{71} R^{20} x^8 \\
y^9 - & 9927475200 d^{69} R^{22} x^8 y^9 + 78538339840 d^{67} R^{24} x^8 y^9 - \\
386258264320 d^{65} R^{26} x^8 y^9 + & 1322934217770 d^{63} R^{28} x^8 y^9 - \\
3349200245030 d^{61} R^{30} x^8 y^9 + & 6490709826500 d^{59} R^{32} x^8 y^9 - \\
9836593380120 d^{57} R^{34} x^8 y^9 + & 11803067594110 d^{55} R^{36} x^8 y^9 - \\
11279192334410 d^{53} R^{38} x^8 y^9 + & 8586102404880 d^{51} R^{40} x^8 y^9 - \\
5179262494900 d^{49} R^{42} x^8 y^9 + & 2447550614470 d^{47} R^{44} x^8 y^9 - \\
888791294010 d^{45} R^{46} x^8 y^9 + & 240534054260 d^{43} R^{48} x^8 y^9 - \\
46212106160 d^{41} R^{50} x^8 y^9 + & 5817824850 d^{39} R^{52} x^8 y^9 - \\
418599190 d^{37} R^{54} x^8 y^9 + & 14601720 d^{35} R^{56} x^8 y^9 - 487620 \\
d^{33} R^{58} x^8 y^9 - & 4362240 d^{72} R^{26} y^{10} + 73553664 d^{70} R^{28} y^{10} \\
- 586669200 d^{68} R^{30} y^{10} + & 2938388787 d^{66} R^{32} y^{10} - \\
10346611239 d^{64} R^{34} y^{10} + & 27173816778 d^{62} R^{36} y^{10} - \\
55116443466 d^{60} R^{38} y^{10} + & 88200300609 d^{58} R^{40} y^{10} - \\
112795165509 d^{56} R^{42} y^{10} + & 116051741052 d^{54} R^{44} y^{10} - \\
96227910636 d^{52} R^{46} y^{10} + & 64120793685 d^{50} R^{48} y^{10} - \\
34075205073 d^{48} R^{50} y^{10} + & 14252932602 d^{46} R^{52} y^{10} - 4598134842 \\
d^{44} R^{54} y^{10} + & 1109999943 d^{42} R^{56} y^{10} - 191636883 d^{40} R^{58}
\end{aligned}$$

$$\begin{aligned}
& y^{10} + 22085664 d^{38} R^{60} y^{10} - 1526880 d^{36} R^{62} y^{10} + 53904 d^{34} \\
& R^{64} y^{10} - 720 d^{32} R^{66} y^{10} + 55910400 d^{72} R^{24} x^2 y^{10} - \\
& 1002113024 d^{70} R^{26} x^2 y^{10} + 8398198208 d^{68} R^{28} x^2 y^{10} - \\
& 43750931124 d^{66} R^{30} x^2 y^{10} + 158835822686 d^{64} R^{32} x^2 y^{10} - \\
& 426852077802 d^{62} R^{34} x^2 y^{10} + 880125405076 d^{60} R^{36} x^2 y^{10} - \\
& 1423733443688 d^{58} R^{38} x^2 y^{10} + 1831669244826 d^{56} R^{40} x^2 y^{10} \\
& - 1887997455526 d^{54} R^{42} x^2 y^{10} + 1562724218536 d^{52} R^{44} x^2 \\
& y^{10} - 1036201948756 d^{50} R^{46} x^2 y^{10} + 546416878722 d^{48} R^{48} \\
& x^2 y^{10} - 226198828310 d^{46} R^{50} x^2 y^{10} + 72034006164 d^{44} R^{52} \\
& x^2 y^{10} - 17116666096 d^{42} R^{54} x^2 y^{10} + 2898922550 d^{40} R^{56} x^2 \\
& y^{10} - 326365242 d^{38} R^{58} x^2 y^{10} + 21986608 d^{36} R^{60} x^2 y^{10} - \\
& 777824 d^{34} R^{62} x^2 y^{10} + 13712 d^{32} R^{64} x^2 y^{10} - 96 d^{30} R^{66} \\
& x^2 y^{10} - 186814464 d^{72} R^{22} x^4 y^{10} + 3658284288 d^{70} R^{24} x^4 \\
& y^{10} - 32689027488 d^{68} R^{26} x^4 y^{10} + 178571903094 d^{66} R^{28} x^4 \\
& y^{10} - 671728317468 d^{64} R^{30} x^4 y^{10} + 1853916750871 d^{62} R^{32} \\
& x^4 y^{10} - 3899268054929 d^{60} R^{34} x^4 y^{10} + 6400101031084 d^{58} \\
& R^{36} x^4 y^{10} - 8318944992202 d^{56} R^{38} x^4 y^{10} + 8632609641929 \\
& d^{54} R^{40} x^4 y^{10} - 7171344462139 d^{52} R^{42} x^4 y^{10} + \\
& 4758758735094 d^{50} R^{44} x^4 y^{10} - 2503976113704 d^{48} R^{46} x^4 y^{10} \\
& + 1030804997097 d^{46} R^{48} x^4 y^{10} - 324961566679 d^{44} R^{50} x^4 \\
& y^{10} + 75909636296 d^{42} R^{52} x^4 y^{10} - 12486350578 d^{40} R^{54} x^4 \\
& y^{10} + 1333496983 d^{38} R^{56} x^4 y^{10} - 81144989 d^{36} R^{58} x^4 y^{10} + \\
& 2411968 d^{34} R^{60} x^4 y^{10} - 44064 d^{32} R^{62} x^4 y^{10} + 223346688 \\
& d^{72} R^{20} x^6 y^{10} - 4709695488 d^{70} R^{22} x^6 y^{10} + 44250312192 \\
& d^{68} R^{24} x^6 y^{10} - 250844544348 d^{66} R^{26} x^6 y^{10} + 971474443302 \\
& d^{64} R^{28} x^6 y^{10} - 2746724389440 d^{62} R^{30} x^6 y^{10} + \\
& 5899331660814 d^{60} R^{32} x^6 y^{10} - 9867330932976 d^{58} R^{34} x^6 y^{10} \\
& + 13052630235294 d^{56} R^{36} x^6 y^{10} - 13773011035056 d^{54} R^{38} x^6 \\
& y^{10} + 11628326423142 d^{52} R^{40} x^6 y^{10} - 7839104768664 d^{50} R^{42} \\
& x^6 y^{10} + 4188336544626 d^{48} R^{44} x^6 y^{10} - 1749096224544 d^{46} \\
& R^{46} x^6 y^{10} + 558219233706 d^{44} R^{48} x^6 y^{10} - 131410539216 d^{42} \\
& R^{50} x^6 y^{10} + 21555643338 d^{40} R^{52} x^6 y^{10} - 2235332592 d^{38} \\
& R^{54} x^6 y^{10} + 122176098 d^{36} R^{56} x^6 y^{10} - 2629452 d^{34} R^{58} x^6 \\
& y^{10} + 72576 d^{32} R^{60} x^6 y^{10} - 88080384 d^{72} R^{18} x^8 y^{10} + \\
& 1779957760 d^{70} R^{20} x^8 y^{10} - 16625029120 d^{68} R^{22} x^8 y^{10} + \\
& 95718792960 d^{66} R^{24} x^8 y^{10} - 381663157670 d^{64} R^{26} x^8 y^{10} + \\
& 1121199140981 d^{62} R^{28} x^8 y^{10} - 2517993636855 d^{60} R^{30} x^8 y^{10} \\
& + 4424067821260 d^{58} R^{32} x^8 y^{10} - 6167998800730 d^{56} R^{34} x^8 \\
& y^{10} + 6876583551435 d^{54} R^{36} x^8 y^{10} - 6145338130789 d^{52} R^{38} \\
& x^8 y^{10} + 4390716357430 d^{50} R^{40} x^8 y^{10} - 2488252511160 d^{48} \\
& R^{42} x^8 y^{10} + 1102438042955 d^{46} R^{44} x^8 y^{10} - 373054579985 \\
& d^{44} R^{46} x^8 y^{10} + 92902003296 d^{42} R^{48} x^8 y^{10} - 16013365210 \\
& d^{40} R^{50} x^8 y^{10} + 1709194645 d^{38} R^{52} x^8 y^{10} - 88656435 d^{36} \\
& R^{54} x^8 y^{10} + 1177470 d^{34} R^{56} x^8 y^{10} - 91854 d^{32} R^{58} x^8 \\
& y^{10} - 18677760 d^{71} R^{26} y^{11} + 290007040 d^{69} R^{28} y^{11} -
\end{aligned}$$

$$\begin{aligned}
& 2120191360 d^{67} R^{30} y^{11} + 9695695368 d^{65} R^{32} y^{11} - 31071690728 \\
& d^{63} R^{34} y^{11} + 74076386072 d^{61} R^{36} y^{11} - 136091921592 d^{59} R^{38} \\
& y^{11} + 196885975784 d^{57} R^{40} y^{11} - 227199192584 d^{55} R^{42} y^{11} + \\
& 210470209560 d^{53} R^{44} y^{11} - 156681280184 d^{51} R^{46} y^{11} + \\
& 93353379704 d^{49} R^{48} y^{11} - 44097589848 d^{47} R^{50} y^{11} + \\
& 16253604104 d^{45} R^{52} y^{11} - 4561982696 d^{43} R^{54} y^{11} + 940399032 \\
& d^{41} R^{56} y^{11} - 134928152 d^{39} R^{58} y^{11} + 12436424 d^{37} R^{60} y^{11} \\
& - 654888 d^{35} R^{62} y^{11} + 16864 d^{33} R^{64} y^{11} - 160 d^{31} R^{66} y^{11} \\
& + 208650240 d^{71} R^{24} x^2 y^{11} - 3262783488 d^{69} R^{26} x^2 y^{11} + \\
& 24018892416 d^{67} R^{28} x^2 y^{11} - 110562060504 d^{65} R^{30} x^2 y^{11} + \\
& 356469647252 d^{63} R^{32} x^2 y^{11} - 854419927204 d^{61} R^{34} x^2 y^{11} + \\
& 1576795771616 d^{59} R^{36} x^2 y^{11} - 2288932931072 d^{57} R^{38} x^2 y^{11} \\
& + 2646797014732 d^{55} R^{40} x^2 y^{11} - 2453039154332 d^{53} R^{42} x^2 \\
& y^{11} + 1823472479400 d^{51} R^{44} x^2 y^{11} - 1082372182008 d^{49} R^{46} \\
& x^2 y^{11} + 507905158956 d^{47} R^{48} x^2 y^{11} - 185278233692 d^{45} R^{50} \\
& x^2 y^{11} + 51201788368 d^{43} R^{52} x^2 y^{11} - 10309999088 d^{41} R^{54} \\
& x^2 y^{11} + 1425675380 d^{39} R^{56} x^2 y^{11} - 123523300 d^{37} R^{58} x^2 \\
& y^{11} + 5847464 d^{35} R^{60} x^2 y^{11} - 132288 d^{33} R^{62} x^2 y^{11} + \\
& 1152 d^{31} R^{64} x^2 y^{11} - 542294016 d^{71} R^{22} x^4 y^{11} + 8759602176 \\
& d^{69} R^{24} x^4 y^{11} - 66554588928 d^{67} R^{26} x^4 y^{11} + 315867811368 \\
& d^{65} R^{28} x^4 y^{11} - 1048663077616 d^{63} R^{30} x^4 y^{11} + \\
& 2584238967644 d^{61} R^{32} x^4 y^{11} - 4894541333668 d^{59} R^{34} x^4 y^{11} \\
& + 7277227281544 d^{57} R^{36} x^4 y^{11} - 8599307447744 d^{55} R^{38} x^4 \\
& y^{11} + 8123794875508 d^{53} R^{40} x^4 y^{11} - 6138223653276 d^{51} R^{42} \\
& x^4 y^{11} + 3691767887808 d^{49} R^{44} x^4 y^{11} - 1748915147304 d^{47} \\
& R^{46} x^4 y^{11} + 641238698884 d^{45} R^{48} x^4 y^{11} - 177085368668 d^{43} \\
& R^{50} x^4 y^{11} + 35336034184 d^{41} R^{52} x^4 y^{11} - 4776088816 d^{39} \\
& R^{54} x^4 y^{11} + 394662188 d^{37} R^{56} x^4 y^{11} - 17217508 d^{35} R^{58} \\
& x^4 y^{11} + 400776 d^{33} R^{60} x^4 y^{11} - 4536 d^{31} R^{62} x^4 y^{11} + \\
& 352321536 d^{71} R^{20} x^6 y^{11} - 6187679744 d^{69} R^{22} x^6 y^{11} + \\
& 50565364736 d^{67} R^{24} x^6 y^{11} - 255728024064 d^{65} R^{26} x^6 y^{11} + \\
& 897402820996 d^{63} R^{28} x^6 y^{11} - 2320824864208 d^{61} R^{30} x^6 y^{11} + \\
& 4583219305332 d^{59} R^{32} x^6 y^{11} - 7063235857808 d^{57} R^{34} x^6 y^{11} \\
& + 8603733367892 d^{55} R^{36} x^6 y^{11} - 8334669431088 d^{53} R^{38} x^6 \\
& y^{11} + 6424512402308 d^{51} R^{40} x^6 y^{11} - 3921098288432 d^{49} R^{42} \\
& x^6 y^{11} + 1874236960908 d^{47} R^{44} x^6 y^{11} - 688669500592 d^{45} \\
& R^{46} x^6 y^{11} + 188900420188 d^{43} R^{48} x^6 y^{11} - 36944417136 d^{41} \\
& R^{50} x^6 y^{11} + 4785202844 d^{39} R^{52} x^6 y^{11} - 363680336 d^{37} R^{54} \\
& x^6 y^{11} + 13966764 d^{35} R^{56} x^6 y^{11} - 390096 d^{33} R^{58} x^6 y^{11} + \\
& 7864320 d^{72} R^{24} y^{12} - 148094976 d^{70} R^{26} y^{12} + 1290921472 d^{68} \\
& R^{28} y^{12} - 6944051776 d^{66} R^{30} y^{12} + 25889224227 d^{64} R^{32} y^{12} - \\
& 71154679031 d^{62} R^{34} y^{12} + 149576514650 d^{60} R^{36} y^{12} - \\
& 246095982714 d^{58} R^{38} y^{12} + 321443252081 d^{56} R^{40} y^{12} - \\
& 335955108437 d^{54} R^{42} y^{12} + 281719522188 d^{52} R^{44} y^{12} - \\
& 189167799548 d^{50} R^{46} y^{12} + 101017819877 d^{48} R^{48} y^{12} -
\end{aligned}$$

$$\begin{aligned}
& 42370256769 d^{46} R^{50} y^{12} + 13687721258 d^{44} R^{52} y^{12} - 3306935498 \\
& d^{42} R^{54} y^{12} + 571821015 d^{40} R^{56} y^{12} - 66237635 d^{38} R^{58} y^{12} \\
& + 4656464 d^{36} R^{60} y^{12} - 174048 d^{34} R^{62} y^{12} + 2896 d^{32} R^{64} \\
& y^{12} - 16 d^{30} R^{66} y^{12} - 45088768 d^{72} R^{22} x^2 y^{12} + 983228416 \\
& d^{70} R^{24} x^2 y^{12} - 9411486720 d^{68} R^{26} x^2 y^{12} + 53930846144 \\
& d^{66} R^{28} x^2 y^{12} - 210139227365 d^{64} R^{30} x^2 y^{12} + 595883967676 \\
& d^{62} R^{32} x^2 y^{12} - 1280641480693 d^{60} R^{34} x^2 y^{12} + \\
& 2139667026960 d^{58} R^{36} x^2 y^{12} - 2823425885137 d^{56} R^{38} x^2 y^{12} \\
& + 2968851577900 d^{54} R^{40} x^2 y^{12} - 2496042774225 d^{52} R^{42} x^2 \\
& y^{12} + 1675158021888 d^{50} R^{44} x^2 y^{12} - 891339451535 d^{48} R^{46} \\
& x^2 y^{12} + 371238027092 d^{46} R^{48} x^2 y^{12} - 118566286255 d^{44} R^{50} \\
& x^2 y^{12} + 28138184944 d^{42} R^{52} x^2 y^{12} - 4728601611 d^{40} R^{54} x^2 \\
& y^{12} + 522004420 d^{38} R^{56} x^2 y^{12} - 33681099 d^{36} R^{58} x^2 y^{12} + \\
& 1094720 d^{34} R^{60} x^2 y^{12} - 16752 d^{32} R^{62} x^2 y^{12} + 66584576 \\
& d^{72} R^{20} x^4 y^{12} - 1687773184 d^{70} R^{22} x^4 y^{12} + 17508625664 \\
& d^{68} R^{24} x^4 y^{12} - 105602229568 d^{66} R^{26} x^4 y^{12} + 426739250015 \\
& d^{64} R^{28} x^4 y^{12} - 1244561056561 d^{62} R^{30} x^4 y^{12} + \\
& 2737203569432 d^{60} R^{32} x^4 y^{12} - 4665509071462 d^{58} R^{34} x^4 y^{12} \\
& + 6268264715609 d^{56} R^{36} x^4 y^{12} - 6702352735715 d^{54} R^{38} x^4 \\
& y^{12} + 5725099133182 d^{52} R^{40} x^4 y^{12} - 3900933471812 d^{50} R^{42} \\
& x^4 y^{12} + 2105587411057 d^{48} R^{44} x^4 y^{12} - 888410753639 d^{46} \\
& R^{46} x^4 y^{12} + 286719869260 d^{44} R^{48} x^4 y^{12} - 68414433110 d^{42} \\
& R^{50} x^4 y^{12} + 11438472439 d^{40} R^{52} x^4 y^{12} - 1226721509 d^{38} \\
& R^{54} x^4 y^{12} + 72497598 d^{36} R^{56} x^4 y^{12} - 1918560 d^{34} R^{58} x^4 \\
& y^{12} + 36288 d^{32} R^{60} x^4 y^{12} - 29360128 d^{72} R^{18} x^6 y^{12} + \\
& 671612928 d^{70} R^{20} x^6 y^{12} - 6882455552 d^{68} R^{22} x^6 y^{12} + \\
& 42623399168 d^{66} R^{24} x^6 y^{12} - 180351839097 d^{64} R^{26} x^6 y^{12} + \\
& 556761765970 d^{62} R^{28} x^6 y^{12} - 1304363261659 d^{60} R^{30} x^6 y^{12} + \\
& 2377112752956 d^{58} R^{32} x^6 y^{12} - 3422091413429 d^{56} R^{34} x^6 y^{12} \\
& + 3925108259566 d^{54} R^{36} x^6 y^{12} - 3598011941667 d^{52} R^{38} x^6 \\
& y^{12} + 2630531362784 d^{50} R^{40} x^6 y^{12} - 1522587931631 d^{48} R^{42} \\
& x^6 y^{12} + 688149739854 d^{46} R^{44} x^6 y^{12} - 237469866841 d^{44} R^{46} \\
& x^6 y^{12} + 60396111436 d^{42} R^{48} x^6 y^{12} - 10692960303 d^{40} R^{50} \\
& x^6 y^{12} + 1194183602 d^{38} R^{52} x^6 y^{12} - 69655593 d^{36} R^{54} x^6 \\
& y^{12} + 1558872 d^{34} R^{56} x^6 y^{12} - 61236 d^{32} R^{58} x^6 y^{12} + \\
& 31457280 d^{71} R^{24} y^{13} - 494075904 d^{69} R^{26} y^{13} + 3656107008 d^{67} \\
& R^{28} y^{13} - 16931322240 d^{65} R^{30} y^{13} + 54964022058 d^{63} R^{32} y^{13} \\
& - 132750972834 d^{61} R^{34} y^{13} + 247042276668 d^{59} R^{36} y^{13} - \\
& 361869136860 d^{57} R^{38} y^{13} + 422498710062 d^{55} R^{40} y^{13} - \\
& 395576136774 d^{53} R^{42} y^{13} + 297206721240 d^{51} R^{44} y^{13} - \\
& 178394210904 d^{49} R^{46} y^{13} + 84699747366 d^{47} R^{48} y^{13} - \\
& 31289192334 d^{45} R^{50} y^{13} + 8769979644 d^{43} R^{52} y^{13} - 1796609628 \\
& d^{41} R^{54} y^{13} + 254389890 d^{39} R^{56} y^{13} - 22877514 d^{37} R^{58} y^{13} \\
& + 1150512 d^{35} R^{60} y^{13} - 26928 d^{33} R^{62} y^{13} + 192 d^{31} R^{64} \\
& y^{13} - 148897792 d^{71} R^{22} x^2 y^{13} + 2440069120 d^{69} R^{24} x^2 y^{13}
\end{aligned}$$

$$\begin{aligned}
& - 18806661120 d^{67} R^{26} x^2 y^{13} + 90526941440 d^{65} R^{28} x^2 y^{13} - \\
& 304749843260 d^{63} R^{30} x^2 y^{13} + 761285594888 d^{61} R^{32} x^2 y^{13} - \\
& 1461121356060 d^{59} R^{34} x^2 y^{13} + 2200571568160 d^{57} R^{36} x^2 y^{13} \\
& - 2633022484820 d^{55} R^{38} x^2 y^{13} + 2517664582680 d^{53} R^{40} x^2 \\
& y^{13} - 1924738656412 d^{51} R^{42} x^2 y^{13} + 1170954479760 d^{49} R^{44} \\
& x^2 y^{13} - 561091744460 d^{47} R^{46} x^2 y^{13} + 208174347320 d^{45} R^{48} \\
& x^2 y^{13} - 58254742580 d^{43} R^{50} x^2 y^{13} + 11819603328 d^{41} R^{52} \\
& x^2 y^{13} - 1637637820 d^{39} R^{54} x^2 y^{13} + 141354920 d^{37} R^{56} x^2 \\
& y^{13} - 6673940 d^{35} R^{58} x^2 y^{13} + 158160 d^{33} R^{60} x^2 y^{13} - \\
& 1512 d^{31} R^{62} x^2 y^{13} + 117440512 d^{29} R^{64} x^2 y^{13} - 2217918464 \\
& d^{69} R^{22} x^2 y^{13} + 19167193600 d^{67} R^{24} x^2 y^{13} - 101336174336 \\
& d^{65} R^{26} x^2 y^{13} + 368674835638 d^{63} R^{28} x^2 y^{13} - 982369960946 \\
& d^{61} R^{30} x^2 y^{13} + 1989380405632 d^{59} R^{32} x^2 y^{13} - \\
& 3132239900660 d^{57} R^{34} x^2 y^{13} + 3886592233498 d^{55} R^{36} x^2 y^{13} \\
& - 3826418169574 d^{53} R^{38} x^2 y^{13} + 2992140507356 d^{51} R^{40} x^2 \\
& y^{13} - 1850196635872 d^{49} R^{42} x^2 y^{13} + 895319961770 d^{47} R^{44} \\
& x^2 y^{13} - 333061830238 d^{45} R^{46} x^2 y^{13} + 92618618984 d^{43} R^{48} \\
& x^2 y^{13} - 18436503172 d^{41} R^{50} x^2 y^{13} + 2453872454 d^{39} R^{52} x^2 \\
& y^{13} - 195907210 d^{37} R^{54} x^2 y^{13} + 8126076 d^{35} R^{56} x^2 y^{13} - \\
& 195048 d^{33} R^{58} x^2 y^{13} - 4194304 d^{29} R^{62} x^2 y^{14} + 109576192 d^{27} \\
& R^{24} x^2 y^{14} - 1149984768 d^{26} R^{26} x^2 y^{14} + 6968116736 d^{24} R^{28} x^2 y^{14} - \\
& 28187873168 d^{22} R^{30} x^2 y^{14} + 82115921985 d^{20} R^{32} x^2 y^{14} - \\
& 180125948861 d^{18} R^{34} x^2 y^{14} + 305874175214 d^{16} R^{36} x^2 y^{14} - \\
& 409076085678 d^{14} R^{38} x^2 y^{14} + 435159323339 d^{12} R^{40} x^2 y^{14} - \\
& 369703033271 d^{10} R^{42} x^2 y^{14} + 250587650196 d^{8} R^{44} x^2 y^{14} - \\
& 134663309924 d^{4} R^{46} x^2 y^{14} + 56679370823 d^{2} R^{48} x^2 y^{14} - \\
& 18319716699 d^{44} R^{50} x^2 y^{14} + 4411857278 d^{42} R^{52} x^2 y^{14} - 756219902 \\
& d^{40} R^{54} x^2 y^{14} + 86003853 d^{38} R^{56} x^2 y^{14} - 5828881 d^{36} R^{58} x^2 y^{14} + \\
& 202528 d^{34} R^{60} x^2 y^{14} - 2688 d^{32} R^{62} x^2 y^{14} + 8388608 d^{29} R^{64} x^2 y^{14} \\
& y^{14} - 325058560 d^{27} R^{66} x^2 y^{14} + 3914522624 d^{25} R^{68} x^2 y^{14} \\
& - 25492277248 d^{23} R^{70} x^2 y^{14} + 107943081824 d^{21} R^{72} x^2 y^{14} - \\
& 324888306262 d^{19} R^{74} x^2 y^{14} + 731019343850 d^{17} R^{76} x^2 y^{14} - \\
& 1267937318512 d^{15} R^{78} x^2 y^{14} + 1727546670044 d^{13} R^{80} x^2 y^{14} \\
& - 1869053000282 d^{11} R^{82} x^2 y^{14} + 1613138073118 d^{9} R^{84} x^2 y^{14} \\
& y^{14} - 1109681026220 d^{5} R^{86} x^2 y^{14} + 604528434952 d^{4} R^{88} x^2 y^{14} \\
& x^2 y^{14} - 257500926794 d^{46} R^{46} x^2 y^{14} + 83978379142 d^{44} R^{48} \\
& x^2 y^{14} - 20294275832 d^{42} R^{50} x^2 y^{14} + 3453517180 d^{40} R^{52} x^2 \\
& y^{14} - 381487814 d^{38} R^{54} x^2 y^{14} + 23976498 d^{36} R^{56} x^2 y^{14} - \\
& 720684 d^{34} R^{58} x^2 y^{14} + 10368 d^{32} R^{60} x^2 y^{14} - 4194304 d^{29} \\
& R^{18} x^2 y^{14} + 129499136 d^{27} R^{20} x^2 y^{14} - 1582993408 d^{25} R^{22} \\
& x^2 y^{14} + 11056456448 d^{23} R^{24} x^2 y^{14} - 51137386096 d^{21} R^{26} \\
& x^2 y^{14} + 169200006491 d^{19} R^{28} x^2 y^{14} - 419215883623 d^{17} R^{30} \\
& x^2 y^{14} + 800249698934 d^{15} R^{32} x^2 y^{14} - 1198069799134 d^{13} \\
& R^{34} x^2 y^{14} + 1421185137073 d^{11} R^{36} x^2 y^{14} - 1341463598189 \\
& d^{52} R^{38} x^2 y^{14} + 1006425339088 d^{50} R^{40} x^2 y^{14} -
\end{aligned}$$

$$\begin{aligned}
& 596187827144 d^{48} R^{42} x^4 y^{14} + 275249461549 d^{46} R^{44} x^4 y^{14} - \\
& 96943053233 d^{44} R^{46} x^4 y^{14} + 25186567246 d^{42} R^{48} x^4 y^{14} - \\
& 4577222726 d^{40} R^{50} x^4 y^{14} + 533024743 d^{38} R^{52} x^4 y^{14} - \\
& 34206363 d^{36} R^{54} x^4 y^{14} + 999756 d^{34} R^{56} x^4 y^{14} - 26244 d^{32} \\
& R^{58} x^4 y^{14} - 16777216 d^{71} R^{22} y^{15} + 281018368 d^{69} R^{24} y^{15} - \\
& 2211643392 d^{67} R^{26} y^{15} + 10859522048 d^{65} R^{28} y^{15} - 37252773824 \\
& d^{63} R^{30} y^{15} + 94732044444 d^{61} R^{32} y^{15} - 184895164844 d^{59} R^{34} \\
& y^{15} + 282897747296 d^{57} R^{36} y^{15} - 343544707584 d^{55} R^{38} y^{15} + \\
& 333092952452 d^{53} R^{40} y^{15} - 258000899156 d^{51} R^{42} y^{15} + \\
& 158920212984 d^{49} R^{44} y^{15} - 77069811416 d^{47} R^{46} y^{15} + \\
& 28941529124 d^{45} R^{48} y^{15} - 8205675732 d^{43} R^{50} y^{15} + 1691876144 \\
& d^{41} R^{52} y^{15} - 239925008 d^{39} R^{54} y^{15} + 21534012 d^{37} R^{56} y^{15} \\
& - 1084588 d^{35} R^{58} y^{15} + 26104 d^{33} R^{60} y^{15} - 216 d^{31} R^{62} y^{15} \\
& + 16777216 d^{71} R^{20} x^2 y^{15} - 383778816 d^{69} R^{22} x^2 y^{15} + \\
& 3733962752 d^{67} R^{24} x^2 y^{15} - 21405655040 d^{65} R^{26} x^2 y^{15} + \\
& 82598275008 d^{63} R^{28} x^2 y^{15} - 230157909868 d^{61} R^{30} x^2 y^{15} + \\
& 482741768428 d^{59} R^{32} x^2 y^{15} - 781858076376 d^{57} R^{34} x^2 y^{15} + \\
& 992972481800 d^{55} R^{36} x^2 y^{15} - 996836370244 d^{53} R^{38} x^2 y^{15} + \\
& 792598251588 d^{51} R^{40} x^2 y^{15} - 497327849408 d^{49} R^{42} x^2 y^{15} + \\
& 243892768496 d^{47} R^{44} x^2 y^{15} - 91913644980 d^{45} R^{46} x^2 y^{15} + \\
& 25919670964 d^{43} R^{48} x^2 y^{15} - 5250778936 d^{41} R^{50} x^2 y^{15} + \\
& 717412296 d^{39} R^{52} x^2 y^{15} - 59897372 d^{37} R^{54} x^2 y^{15} + 2648220 \\
& d^{35} R^{56} x^2 y^{15} - 55728 d^{33} R^{58} x^2 y^{15} - 25165824 d^{70} R^{22} \\
& y^{16} + 382205952 d^{68} R^{24} y^{16} - 2724446208 d^{66} R^{26} y^{16} + \\
& 12104353536 d^{64} R^{28} y^{16} - 37535971296 d^{62} R^{30} y^{16} + \\
& 86205860787 d^{60} R^{32} y^{16} - 151803638151 d^{58} R^{34} y^{16} + \\
& 209314850394 d^{56} R^{36} y^{16} - 228735818298 d^{54} R^{38} y^{16} + \\
& 199176719313 d^{52} R^{40} y^{16} - 138161986677 d^{50} R^{42} y^{16} + \\
& 75894801996 d^{48} R^{44} y^{16} - 32611826364 d^{46} R^{46} y^{16} + \\
& 10740901413 d^{44} R^{48} y^{16} - 2627069313 d^{42} R^{50} y^{16} + 454476714 \\
& d^{40} R^{52} y^{16} - 51542922 d^{38} R^{54} y^{16} + 3405303 d^{36} R^{56} y^{16} - \\
& 111651 d^{34} R^{58} y^{16} + 1296 d^{32} R^{60} y^{16} + 8388608 d^{70} R^{20} x^2 \\
& y^{16} - 162529280 d^{68} R^{22} x^2 y^{16} + 1429020672 d^{66} R^{24} x^2 y^{16} \\
& - 7644468736 d^{64} R^{26} x^2 y^{16} + 28026636256 d^{62} R^{28} x^2 y^{16} - \\
& 75009686379 d^{60} R^{30} x^2 y^{16} + 152122747760 d^{58} R^{32} x^2 y^{16} - \\
& 239172201671 d^{56} R^{34} x^2 y^{16} + 295443086328 d^{54} R^{36} x^2 y^{16} - \\
& 288562096823 d^{52} R^{38} x^2 y^{16} + 222929176184 d^{50} R^{40} x^2 y^{16} - \\
& 135481563555 d^{48} R^{42} x^2 y^{16} + 63998146576 d^{46} R^{44} x^2 y^{16} - \\
& 23027031073 d^{44} R^{46} x^2 y^{16} + 6113452128 d^{42} R^{48} x^2 y^{16} - \\
& 1139823413 d^{40} R^{50} x^2 y^{16} + 138018200 d^{38} R^{52} x^2 y^{16} - \\
& 9598077 d^{36} R^{54} x^2 y^{16} + 332856 d^{34} R^{56} x^2 y^{16} - 6561 d^{32} \\
& R^{58} x^2 y^{16} - 16777216 d^{69} R^{22} y^{17} + 249561088 d^{67} R^{24} y^{17} - \\
& 1736245248 d^{65} R^{26} y^{17} + 7498528256 d^{63} R^{28} y^{17} - 22498592960 \\
& d^{61} R^{30} y^{17} + 49723464438 d^{59} R^{32} y^{17} - 83728858174 d^{57} R^{34} \\
& y^{17} + 109582180084 d^{55} R^{36} y^{17} - 112673646228 d^{53} R^{38} y^{17} +
\end{aligned}$$

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$$\begin{aligned} & 91361297170 d^{51} R^{40} y^{17} - 58282201978 d^{49} R^{42} y^{17} + \\ & 29001166584 d^{47} R^{44} y^{17} - 11080090424 d^{45} R^{46} y^{17} + 3169674938 \\ & d^{43} R^{48} y^{17} - 653424210 d^{41} R^{50} y^{17} + 91564756 d^{39} R^{52} y^{17} \\ & - 7965748 d^{37} R^{54} y^{17} + 371838 d^{35} R^{56} y^{17} - 6966 d^{33} R^{58} \\ & y^{17} - 4194304 d^{68} R^{22} y^{18} + 62390272 d^{66} R^{24} y^{18} - 433754112 \\ & d^{64} R^{26} y^{18} + 1870340864 d^{62} R^{28} y^{18} - 5596784240 d^{60} R^{30} \\ & y^{18} + 12319356297 d^{58} R^{32} y^{18} - 20624818981 d^{56} R^{34} y^{18} + \\ & 26777706046 d^{54} R^{36} y^{18} - 27233931582 d^{52} R^{38} y^{18} + \\ & 21758520355 d^{50} R^{40} y^{18} - 13605541807 d^{48} R^{42} y^{18} + 6588237396 \\ & d^{46} R^{44} y^{18} - 2424398756 d^{44} R^{46} y^{18} + 658016447 d^{42} R^{48} \\ & y^{18} - 125816115 d^{40} R^{50} y^{18} + 15804814 d^{38} R^{52} y^{18} - 1178062 \\ & d^{36} R^{54} y^{18} + 46197 d^{34} R^{56} y^{18} - 729 d^{32} R^{58} y^{18}) = 0. \end{aligned}$$