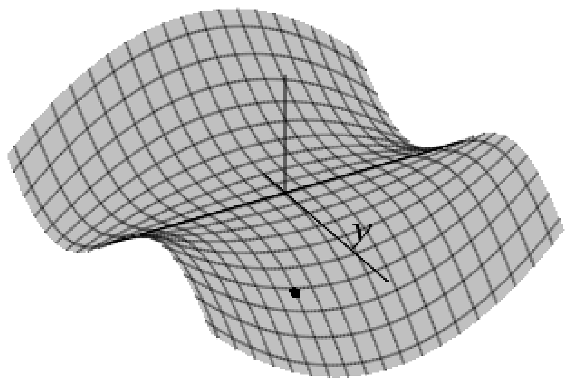
MATH 2210 HOMEWORK WORKSHEET 6

Name: ________________________

Partial Derivatives

1. Determine the signs of the partial derviatves for the function f whose graph is shown. The point $\big(1, 2, f(1,2)\big)$ is marked.



(a) $f_x(1,2)$

(b) $f_y(1,2)$

(c) $f_{xx}(1,2)$

(d) $f_{yy}(1,2)$

(e) $f_{xy}(1,2)$

2. Find the first partial derivatives of the following functions.

(a) $f(x, y) = x^2y - 3y^4$

(b) $u(r, \theta) = \sin(r \cos \theta)$

3. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ if

$$yz + x \ln y = z^2.$$

4. Find all the second partial derivatives of the function

$$w(u,v) = \sqrt{1 + uv^2}.$$

Tangent Planes and Linear Approximations

5. Find an equation of the tangent plane to the surface given by

$$z = \frac{x}{y^2}$$

at the point $(-4, 2, -1)$.

6. Verify the linear approximation

$$\frac{y-1}{x+1} \approx x + y - 1$$

at $(0, 0)$.

7. Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$, and $f_y(2,5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

8. Find the differential of the function $u = \sqrt{x^2 + 3y^2}$.

9. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

The Chain Rule

10. Use the chain rule to find $\frac{dz}{dt}$ and $\frac{dw}{dt}$.

(a) $z = xy^3 - x^2y, \quad \text{where} \quad x = t^2 + 1, \quad \text{and} \quad y = t^2 - 1.$

(b) $w = \ln\sqrt{x^2 + y^2 + z^2}, \quad \text{where} \quad x = \sin t, \quad y = \cos t, \quad \text{and} \quad z = \tan t.$

11. Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ if

$$z = \sqrt{x}\mathrm{e}^{xy}, \quad \text{where} \quad x = 1 + st, \quad \text{and} \quad y = s^2 - t^2.$$

12. Use the chain rule to find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ if

$$w = xy + yz + zx, \quad \text{where} \quad x = r\cos\theta, \quad y = r\sin\theta, \quad \text{and} \quad z = r\theta,$$

when $r = 2$ and $\theta = \dfrac{\pi}{2}$.

13. Use the equations

$$\frac{\partial z}{\partial x} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial z}}, \qquad \frac{\partial z}{\partial y} = -\frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial z}}$$

to compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ where

$$yz + x\ln y = z^2.$$

14. The radius of a right circular cone is increasing at a rate of 1.8 in/s while its height is decreasing at a rate of 2.5 in/s. At what rate is the volume of the cone changing when the radius is 120 in and the height is 140 in?

15. Show that any function of the form

$$z = f(x + at) + g(x - at)$$

is a solution of the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

Hint: Let $u = x + at$ *and* $v = x - at$.