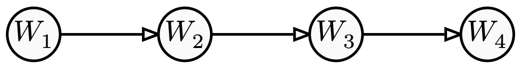
The Back Door Criterion

by Carlos Rubio

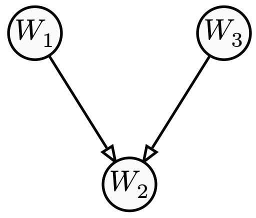
In order to understand the backdoor criterion (or admisible sets), let's review the concept of d-separation.

Consider a directed acyclic graph that contains a path p like the following:



Define S as any of set of nodes that *blocks* the path p: meaning, **one of the arrow emiting nodes of p is in S**, or S ***excludes* at least one of the collision nodes** of p **and their descendant**. For p, any $S \in 2^{\{W_1, W_2, W_3\}} = \{\{W_1\}, \{W_2\}, ...\{W_1, W_3\}, ...\}$ blocks p.

Another example: let p be the following path.



For p, $S = \emptyset$ blocks p, as p contains the collision node W_2.

Two things to note about this definition:

1. Note that an "arrow emiting node in a path" includes collider nodes that emit arrows out of the path as well as nodes that have descendants in the path. Therefore, **if a node is emiting an arrow there are only to possibilities: it has descendants in the path**, or **it's a collider of two other nodes.**
2. The reasoning behind the second part of the definition is that, as long as the path includes a collider, the propagation of the effect will be restricted. I like to think about this preposition as, given a collider in a path, such collider *implicitly blocks* the path, and for purposes explained later, we will not include it.

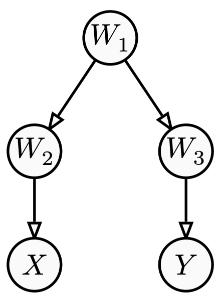
Now, consider a causal model G, were $X, Y \in G$ are considered the exposure variable and the outcome variables, respectively. A set S **adjusts the** G, or **to be admissible for adjustment**, if

1. **None of its nodes are descendants of** X, and
2. **It blocks all the backdoor paths from** Y **"to"** X. By backdoor, we mean all **paths that include nodes** X **and** Y, and X **has an arrow point to it**.

In practice, the nodes/variables in S must be observable variables. If not, it's impossible to condition on them in practise.

Note that the set S must block **all** paths in order to be admissible. **If the set doesn't block all paths** (which includes opening a path), **it is not an admissible set.**

Consider the path p:



Note that $p = X \leftarrow W_2 \leftarrow W_1 \rightarrow W_3 \rightarrow Y$ is the only backdoor path.

The sets S that can be consider admissible for adjustment are

1. $\{W_1\}$,
2. $\{W_2\}$

3. $\{W_3\}$
4. $\{W_1, W_2\}$
5. $\{W_1, W_3\}$
6. $\{W_2, W_3\}$
7. $\{W_1, W_2, W_3\}$

While in theory all these sets are admisible, in practice the **best option is to condition for either of the minimal sets.** Therefore, either of the first three sets is admissible.

Now, let's consider Pearl's classic butterfly causal model. I reproduced it using the causal graph simulator hosted in *dagitty.net*. I also include some of the metadata in the screenshot in order to compare my arguments with the results from the algorithm.

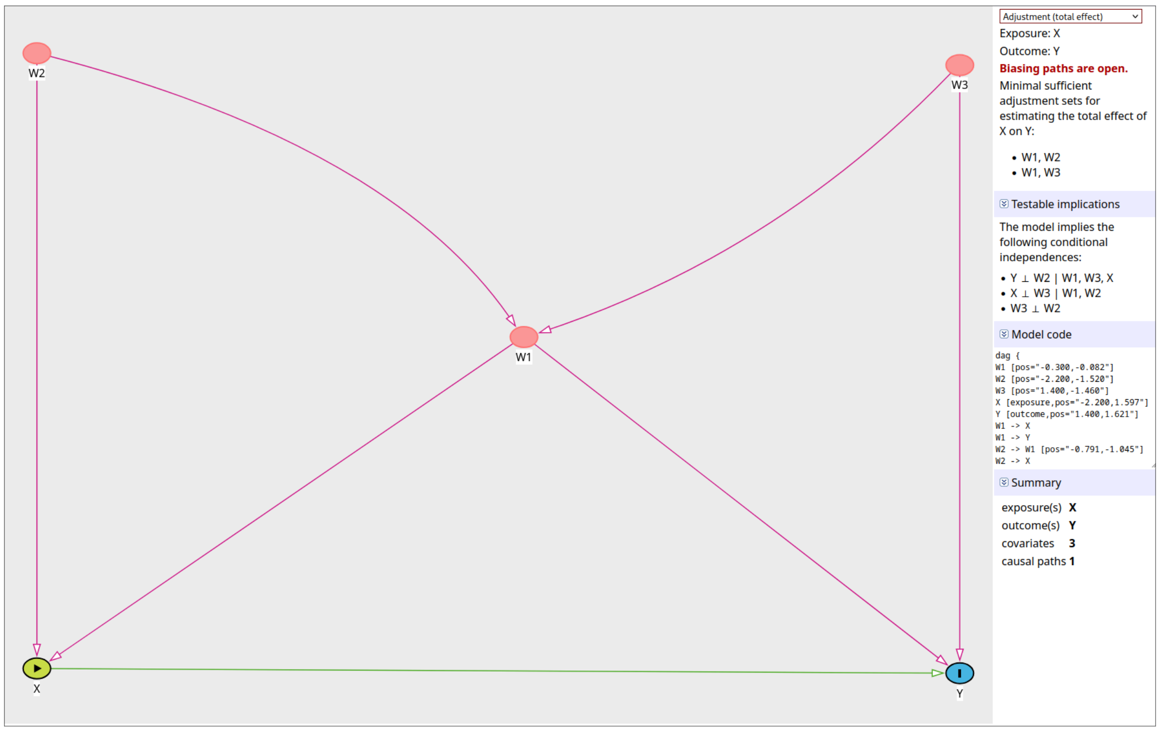


Figure 1: No adjustments made.

In this model, the backdoor baths are

1. $X \leftarrow W_1 \rightarrow Y$
2. $X \leftarrow W_2 \rightarrow W_1 \rightarrow Y$
3. $X \leftarrow W_1 \leftarrow W_3 \rightarrow Y$

Let's consider we might consider $S = \{W_2\}$ or $S = \{W_3\}$ for adjustment. This sets only block 2 out of 3 backdoor paths. Then, we might consider the set $S = \{W_1\}$ for adjustment, as it blocks all the foremention paths. But in doing so it opens the path $p = X \leftarrow W_2 \rightarrow W_1 \leftarrow W_3 \rightarrow Y$ that was being *implictly blocked by* the collider W_1,

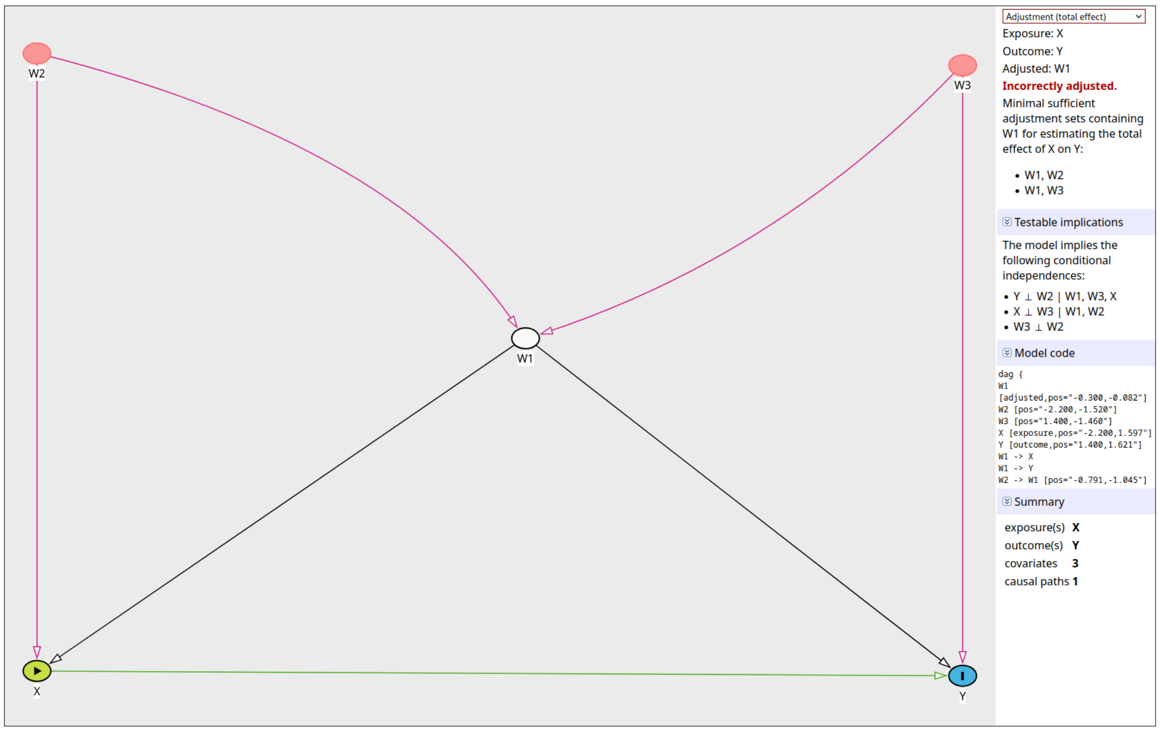


Figure 2: W_1 is insufficient.

The solution is blocking p without having to exclude W_1 from S. We can do this by including either W_2 or W_3.

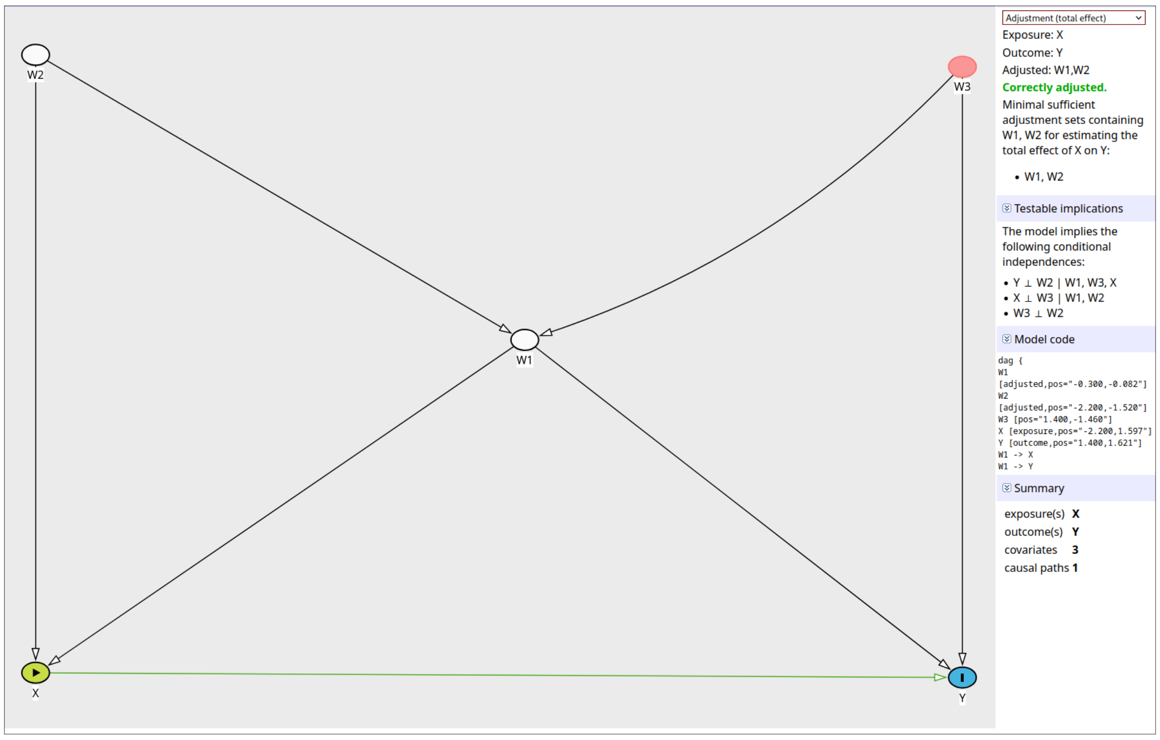


Figure 3: $S = \{W_1, W_2\}$ is admissible.