MATH 2210 HOMEWORK WORKSHEET 7

Name: _____

Directional Derivatives and the Gradient Vector

1. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

(a) $f(x,y) = \dfrac{x}{x^2+y^2}, \quad (1,2), \quad \mathbf{v} = \langle 3,5 \rangle.$

(b) $f(x,y,z) = xy^2 \tan^{-1} z, \quad (2,1,1), \quad \mathbf{v} = \langle 1,1,1 \rangle.$

2. Find the maximum rate of change of

$$f(x, y, z) = x \ln(yz)$$

at the point $\left(1, 2, \frac{1}{2}\right)$ and the direction in which it occurs.

3. Find equations of (a) the tangent plane and (b) the normal line to the surface

$$x = y^2 + z^2 + 1$$

at the point $(3, 1, -1)$. *Hint: Recall that the normal line is the line through the point that is perpendicular (i.e. orthogonal) to the surface.*

Maximum and Minimum Values

4. Suppose $(0, 2)$ is a critical point of a function g with continuous second derivatives. In each case below, what can you say about g? Explain your reasoning.

(a) $g_{xx}(0, 2) = -1, \quad g_{xy}(0, 2) = 6, \quad g_{yy}(0, 2) = 1.$

(b) $g_{xx}(0,2) = -1, \quad g_{xy}(0,2) = 2, \quad g_{yy}(0,2) = -8.$

(c) $g_{xx}(0,2) = 4, \quad g_{xy}(0,2) = 6, \quad g_{yy}(0,2) = 9.$

5. Find the local maximum and minimum values and saddle point(s) of the function.

(a) $f(x, y) = y(\mathrm{e}^x - 1)$

(b) $f(x, y) = xy\mathrm{e}^{-(x^2+y^2)/2}$