Text: Motion of a Race Car on a Curved Track

A race car is moving along a curved track at a speed of $\boldsymbol{v}(t) = 50\frac{m}{s}$. The track has a radius of curvature $\rho = 200m$ at a particular point P. At this instant, the car's speed is increasing at a rate of $\frac{dv}{dt} = 10\frac{m}{s^2}$.

Tangential Acceleration ($a_{\boldsymbol{T}}\boldsymbol{T}$):

The tangential component of acceleration is responsible for the change in the car's speed. Since the car is speeding up at a rate of $\frac{dv}{dt} = 10\frac{m}{s^2}$, the tangential acceleration is:

$$a_{\boldsymbol{T}} = \frac{dv}{dt} = 10\frac{m}{s^2}$$

Normal Acceleration ($a_{\boldsymbol{N}}\boldsymbol{N}$):

The normal (or centripetal) component of acceleration is responsible for the change in the car's direction. It is given by:

Substituting the given values: an=(50 m/s)2200 m=2500200=12.5 m/s2. an=200m(50m/s)2=2002500 =12.5m/s2.

Total Acceleration (a):

The total acceleration is the vector sum of the tangential and normal components. Its magnitude is: a=at2+an2=(10)2+(12.5)2=100+156.25=256.25≈16 m/s2. a=at2+an2

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The direction of the total acceleration can be found using the angle θθ it makes with the tangential direction: θ=tan−1(anat)=tan−1(12.510)≈51.3∘. θ=tan−1(atan)=tan−1(1012.5)≈51.3∘.

Interpretation:

The car is speeding up at $10\frac{m}{s^2}$ due to the tangential acceleration.

The car is also changing direction, which requires a centripetal acceleration of 12.5 m/s212.5m/s2.

The total acceleration is 16 m/s216m/s2, directed at an angle of 51.3∘51.3∘ relative to the tangential direction.

Questions for Understanding:

What would happen to the normal acceleration if the car's speed doubled?

If the radius of curvature were smaller, how would that affect the normal acceleration?

How would the total acceleration change if the car were moving at a constant speed?