Section 2.1

Evaluate the determinant of the given matrix. If th matrix is invertible, find its inverse using $A^{-1} = \frac{1}{\det(A)} adj(A)$.

Exercise 5

$$A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= 3(5) - (-2)(4) \\ &= 15 + 8 \\ &= 23 \end{aligned}$$

$$\begin{aligned} A^{-1} &= \frac{1}{\det(A)} \begin{bmatrix} 4 & -5 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{23} & -\frac{5}{23} \\ -\frac{3}{23} & \frac{2}{23} \end{bmatrix} \end{aligned}$$

Exercise 7

$$A = \begin{bmatrix} -5 & 7 \\ -7 & 2 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= -5(2) - (-7)(7) \\ &= -10 + 49 \\ &= 39 \end{aligned}$$

$$\begin{aligned} A^{-1} &= \frac{1}{\det(A)} \begin{bmatrix} 2 & -7 \\ 7 & -5 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2}{39} & -\frac{7}{39} \\ \frac{7}{39} & -\frac{5}{39} \end{bmatrix} \end{aligned}$$

Use the arrow technique to evaluate the determinant.

Exercise 11

$$A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= -2(5)(2) + 1(-7)(1) + 4(3)(6) - (-2)(-7)(6) - (1)(3)(2) - (4)(5)(1) \\ &= -20 - 7 + 72 - 84 - 6 - 20 \\ &= -20 - 12 - 7 - 6 - 20 \\ &= -32 - 13 - 20 \\ &= -45 - 20 \\ &= -65 \end{aligned}$$

Exercise 13

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= 3(-1)(-4) - 3(5)(9) \\ &= 12 - 135 \\ &= -123 \end{aligned}$$

Find all values of λ for which $\det(A) = 0$

Exercise 15

$$A = \begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= (\lambda - 2)(\lambda + 4) + 5 \\ &= \lambda^2 + 2\lambda - 3 \\ &= (\lambda + 3)(\lambda - 1) \end{aligned}$$

Therefore, $\lambda = -3, 1$

Exercise 17

$$A = \begin{bmatrix} \lambda - 1 & 0 \\ 2 & \lambda + 1 \end{bmatrix}$$

Answer

$$\det(A) = (\lambda - 1)(\lambda + 2)$$

Therefore, $\lambda = -2, 1$

Evaluate the determinant of the given matrix A by inspection.

Exercise 25

Evaluate $\det(A)$ by a cofactor expansion along a row or a column of you choice.

$$A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} \\ &= 3(4+20) - 3(4+4) + 5(20-4) - 3(2) + 3(8) - 5(2-8) \\ &= 72 - 24 + 80 - 6 + 24 + 30 \\ &= 72 + 80 + 24 \\ &= 80 + 96 \\ &= 176 \end{aligned}$$

Exercise 27

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer

$$\det(A) = -1$$

Exercise 29

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

Answer

$$\det(A) = 0$$

Section 2.2

Verify that $\det(A) = \det(A^T)$

Exercise 1

$$A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= -2(4) - 3 \\ &= -11 \end{aligned}$$

$$A^T = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{aligned} \det(A^T) &= -2(4) - 3 \\ &= -11 \end{aligned}$$

Exercise 3

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}$$

Answer

$$\begin{aligned} \det(A) &= -2(12 + 12) + (6 - 20) + 3(-3 - 10) \\ &= -48 - 14 - 39 \\ &= -50 - 11 - 40 \\ &= -101 \end{aligned}$$

$$A^T = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 2 & -3 \\ 3 & 4 & 6 \end{bmatrix}$$

$$\begin{aligned} \det(A^T) &= -2(12 + 12) + (6 - 20) + 3(-3 - 10) \\ &= -48 - 14 - 39 \\ &= -50 - 11 - 40 \\ &= -101 \end{aligned}$$

Find the determinant of the given elementary matrix by inspection,

Exercise 5

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer

$$\det(A) = -5$$

Exercise 7

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer

$$\det(A) = -1$$

Evaluate the determinant of the matrix by first reducing the matrix to row echelon form and then using some combination of row operations and cofactor expansion.

Exercise 11

$$\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

Answer

$$\begin{vmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & -2 & 3 \\ 0 & 11 & -8 \\ 0 & 1 & 5 \end{vmatrix} = 33\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{8}{11} \\ 0 & 1 & 5 \end{vmatrix}$$

$$= 33\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{8}{11} \\ 0 & 0 & \frac{63}{11} \end{vmatrix} = \left(\frac{363}{63}\right)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{8}{11} \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{363}{63}$$

Exercise 13

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Answer

$$\begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & 1 & -2 & -6 & -8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = -2$$

Evaluate the determinant, given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 6$$

Exercise 15

$$\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$$

Answer

$$\det(A) = 1$$

Exercise 17

$$\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$$

Answer

$$\det(A) = -12$$

Exercise 21

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4d \end{vmatrix}$$

Answer

$$\det(A) = -3$$

Exercise 31

It can be proven that if a square matrix M is partitioned into **block triangular form** as

$$M = \begin{bmatrix} A & 0 \\ C & B \end{bmatrix} \text{ or } M = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$$

in which A and B are square, then $\det(M) = \det(A)\det(B)$. Use this result to compute the determinant of

$$M = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 8 & 6 & 7 \\ 2 & 5 & 0 & 4 & 7 & 5 \\ -1 & 3 & 2 & 6 & 9 & -2 \\ \hline 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 8 & -4 \end{array}\right]$$

Answer

$$\begin{aligned} \det(M) &= 2(5-4)(3)(-4) \\ &= -24 \end{aligned}$$