**Name:** _____

# Functions of Several Variables

**1.** A company makes three sizes of cardboard boxes: small, medium, and large. It costs $2.50 to make a small box, $4.00 for a medium box, and $4.50 for a large box. Fixed costs are $8,000$.

**(a)** Express the cost of making $x$ small boxes, $y$ medium boxes, and $z$ large boxes as a function of three variables: $C = f(x, y, z)$.

**(b)** Find $f(3000, 5000, 4000)$ and interpret it.

**(c)** What is the domain of $f$?

**2.** Find and sketch the domain of the function.

$$f(x, y) = \sqrt{x^2 + y^2 - 4}$$

**3.** Match the function with its graph (labeled I-IV and listed on the following page). Give reasons for your choices.
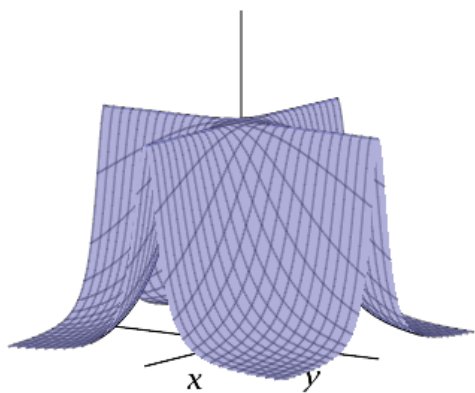
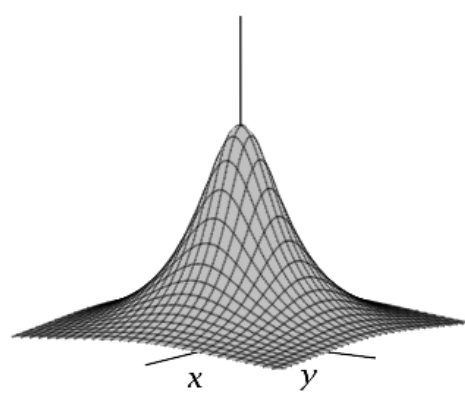**(a)** $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$

**(b)** $f(x, y) = \dfrac{1}{1 + x^2 y^2}$

**(c)** $f(x, y) = \ln(x^2 + y^2)$

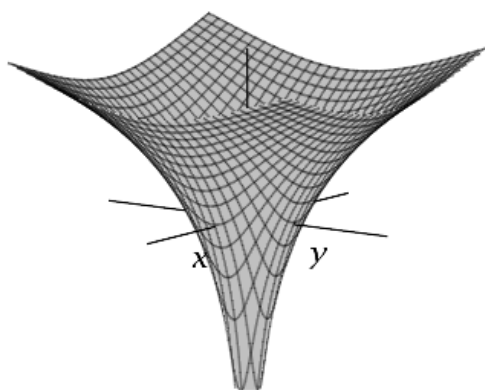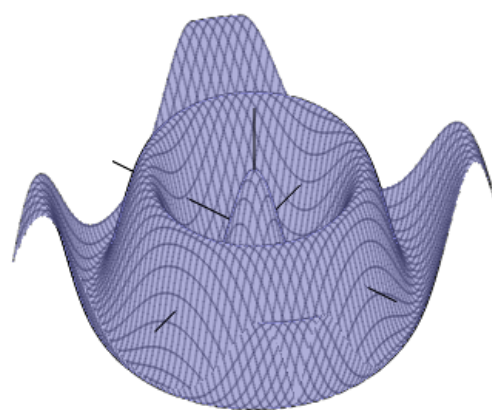**(d)** $f(x, y) = \cos\left(\sqrt{x^2 + y^2}\right)$

**I.**



**II.**



**III.**



**IV.**

# Limits and Continuity

**4.** Find the limit if it exists, or show that the limit does not exist.

**(a)** $\lim_{(x,y)\to(3,2)} (x^2y^3 - 4y^2)$

**(b)** $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^4 + y^4}$

**(c)** $\lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2 + y^6}$

**5.** Determine the set of points at which the function is continuous.

$$F(x, y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2}.$$

**6.** Use polar coordinates to find the limit. [If $(r, \theta)$ are polar coordinatess of the point $(x, y)$ with $r \geq 0$, note that $r \to 0^+$ as $(x, y) \to (0, 0)$.]

$$\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln(x^2 + y^2).$$