4. Exercise for the lecture ”numerical fluid dynamics” SoSe 18

Task 1

Calculate the first derivative of the function x^2 and x^3 at $x = 1$ with forward differences of 1. order and central differences of 2. order on a mesh with mesh size h. what? How is the error behaving as a function of h?

Task 2

The interval $[0 \ldots 4\pi]$ is discretized in 100 grid points, so that the mesh size is $h = {}^{4\pi}/_{100}$. Write a program that calculates and saves for every grid point the analytical derivative, central finite difference of 2. order and the forward differences for the function $\sin(5x)$. Which types of error do occur for both approximation methods?

Task 3

Calculate analytically the errors for the discretizations of functions of the form e^{ikx}. Use central finite differences of 2. order and forward differences. Which is the largest wavenumber k, which it is reasonable to consider. Compare these results to the previously calculated ones.

Task 4

Given is the diffusion problem for the temperature field $T(t, x)$ with $x \in [0, 1]$ for $t > 0$

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial^2 T}{\partial x^2} + Q \\ \text{with boundary conditions } T(t, 0) &= T(t, 1) = 0, \\ \text{initial conditions } T(0, x) &= 0, \\ \text{and the source term } Q(x) &= \sin(\pi x). \end{aligned}$$

- Find the analytical solution of the problem.
- Write a program in order to solve the diffusion problem with an FTCS scheme. The space is discretized in x_j with $j = 0 \ldots N$ and $x_0 = 0$, $x_N = 1$ at the boundaries. At the end of the time integration, compare the numerical and analytical solution.
- Use this program with $N = 10$ and a time step of $\Delta t = 0.01$. Integrate over 100 and 101 time steps. How does the numerical solution behave?
- How large is the largest time step theoretically? Demonstrate this criterion with the simulation.

- Use the time step $\Delta t = 2 \times 10^{-3}$. How many time steps does it take to reach a stationary state? How large is the error of the numerical simulation after 10^3 time steps?
- By what factor does the number of operations grow if the required accuracy is increased by two orders of magnitude?