7. Exercise for the lecture "numerical fluid dynamics" SoSe 18

Task 1

A velocity field $\vec{v}$ is produced by a fan in a rectangle with side length L. Two opposite sides of the rectangle are thermally insulated and the other two are kept at temperatures T_0 and $T_0 + \Delta T$. In this task you are going to calculate the heat transport between the two plates with constant temperature. The equation for the temperature field $T(t, x, y)$ is

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T$$

$$T(t, x, 0) = T_0,\ T(t, x, L) = T_0 + \Delta T,\ \frac{\partial T}{\partial x}(t, 0, y) = \frac{\partial T}{\partial x}(t, L, y) = 0$$

$$\vec{v} = v(\pi \sin(2\pi x/L) \cos(\pi y/L), -2\pi \cos(2\pi x/L) \sin(\pi y/L)) = v\vec{v}_0.$$

- Does $\vec{v}_0$ have zero divergence? Plot $\vec{v}_0$.
- The zero temperature is shifted to T_0 and use ΔT, L and L^2/κ as scale for temperature, length and time. The equation for the dimensionless variables are

$$\frac{\partial T}{\partial t} + Pe\ \vec{v}_0 \cdot \nabla T = \nabla^2 T$$

$$T(t, x, 0) = 0,\ T(t, x, 1) = 1,\ \frac{\partial T}{\partial x}(t, 0, y) = \frac{\partial T}{\partial x}(t, 1, y) = 0$$

Compute the expression for Pe and v_0.

- Write a program that solves the dimensionless equations with an FTCS-scheme. Useful parameters are $T(0, x, y) = y$, $Pe = 2$, $\Delta t = 0.001$, $N_x = N_y = 10$.
- The question arises whether the program is working free of error. Because we do not have an analytical solution for the initial problem, we choose a $T^*(x, y)$ that fulfills the boundary conditions and insert it into the equation. T^* is only a solution if on adds a source term $Q(x, y)$

$$Pe\ \vec{v}_0 \nabla T^* = \nabla^2 T^* + Q$$

What is the expression for Q if $T^* = \cos(\pi x) \sin(\pi y) + y$ is chosen? Implement Q in your program and test if the time iteration converges to the stationary solution T^*.

- Visualize the temperature field for $Pe = 2$, $N_x = N_y = 30$, $\Delta t = 2 \times 10^{-4}$ at times $t = 0.005, 0.05$ and 0.5. Repeat the task for $Pe = 10$. Then choose the initial conditions $T(0, x, y) = 0$ and afterwards $T(0, x, y) = 1$ and visualize it at times $t = 0.0005, 0.005, 0.02, 0.05$ and 0.5.