2. Exercise for the lecture "numerical fluid dynamics" SoSe 18

Part for programming newcomers or to refresh knowledge:

Task 0

Chose a programming language that suits the purpose. We recommend c/c++ as a language for beginners, because one has to be more careful and do more by hand. Also it is easier to move from c to python than the other way around.
A PRO tip from the Prof. is Fortran or Julia for those who like an adventure.

Task 1

Write a program that outputs two columns into a file. The columns are (x, y)-coordinates of a list of points. The left column is x_i with $0 \leq x_i \leq \pi$, $x_1 = 0$ and $x_{i+1} - x_i = 0.1$. The right column contains $y_i = \sin x_i$. Plot this list of points with for example "gnuplot" or any other plotting tool if your language does not have plotting capability on its own. Compare that result to the built in version of sin in for example "gnuplot".

Task 2

Create a script or batch file for for example "gnuplot" or any other plotting tool that is capable of plotting 3D contour plots with 20 isolines. Maybe read Task 3 first.

Task 3

Write a program that outputs a file with a format that is readable by the script from the last task. Write out a triple (x, y, z) with $z = x^2 + y^2$ in the area $[-10, 10] \times [-10, 10]$. For for example "gnuplot" this means that every line contains one triplet (x, y, z). The data has to be arranged in blocks which are separated by a blank line. In every block x is held constant.

Part for people who know or pretend to know what they are doing:

Task 1

Integrate the following system of equations

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= 998x + 1998y \\
\frac{\mathrm{d}y}{\mathrm{d}t} &= -999x - 1999y \\
x(0) &= 1, y(0) = 0
\end{aligned}
$$

until $t = 1$. All variables are dimensionless. Use the explicit and implicit Euler method with timestep sizes of $\mathrm{d}t = {}^{1}/_{10}, {}^{1}/_{500}$ and ${}^{1}/_{550}$. Inspect the time evolution of the solution. What is the maximum timestep for stable results?

Task 2

The simple harmonic oscillator is governed by the following second order ordinary differential equation (ODE)

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -k \cdot x$$

where M represents the mass of the oscillating body, x its displacement from the equilibrium position and k the spring constant. Use the explicit Euler, RK2 (Runge-Kutta 2. Order) and RK4[1] integration schemes to integrate the system in time to compute the position and velocity up to $t = 50\mathrm{T}$, where T is the temporal period of the system. The mass M starts from rest at a distance of $x(0) = 1\mathrm{m}$ from its equilibrium position. Plot the phase diagram (position vs momentum) for the system and also plot the variation (in time) of the total energy

$$E_{\mathrm{tot}} = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2$$

for each of the time integration methods. Which of these methods would you choose for time integration and why? Check the solutions behavior for different timestep sizes. For simplicity use

$$\begin{aligned} k &= 1\frac{\mathrm{N}}{\mathrm{m}} \text{ and} \\ M &= 1\mathrm{kg}. \end{aligned}$$

Task 3

Integrate the Lorenz system

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \sigma \cdot (y - x) \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -xz + rx - y \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= xy - bz \end{aligned}$$

for $\sigma = 10$, $b = {}^{8}/_{3}$, $r = 28$ with initial conditions $x(0) = y(0) = z(0) = 1$ until $t = 100$ and plot the attractor. All variables are dimensionless.

[1] Look up the method or use a built in version.