6. Exercise for the lecture "numerical fluid dynamics" SoSe 18

This exercise is for two weeks! So take your time and work through it carefully.

Task 1

Given is the diffusion-advection-problem for the temperature field $T(t,x)$ in the interval $0 \le x \le 1$ for $t \ge 0$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = D\frac{\partial^2 T}{\partial x^2}$$
$$T(t,0) = 0,\ T(t,1) = 1,\ T(0,x) = x,$$

v and D are constant. The field at $t = 0$ corresponds to the diffusion solution without advection ($v = 0$).

• Compute the analytical solution for the stationary problem ($\partial T/\partial t = 0$) for arbitrary v and D.

• Write a program for solving the problem with an FTCS-scheme. Here the x-axis is discretized in N intervals and $v = 1$ for the following calculations. Integrate until $t = 20$ and compare the results to the stationary solution. Integrate

- for $D = 1$, $N = 20$ with the time steps $\Delta t = 10^{-3}$ and $2 \cdot 10^{-3}$,
- for $D = 0.1$, $N = 20$ with $\Delta t = 10^{-2}$ and $2 \cdot 10^{-2}$ and
- for $D = 0.01$, $N = 20$ with $\Delta t = 10^{-2}$.

Think about these questions and revisit them after the lecture:
How do these results match the stability criteria? What do you think about the spatial discretization of the last part? What stability boundaries are to be expected for $D = 0.01$ and $N = 100$. Is this the case?

• Now use a new scheme with explicit advection and implicit diffusion. Therefore a tridiagonal system has to be solved. Write a subroutine

```
int tridiag(int n, float a[n+1], float b[n+1],
float c[n+1], float d[n+1], float T[n+1]);
```

where n is the resolution, a, b, c are the diagonals, d the right hand side and T the result. Also write a test routine

```
int test_tridiag(int n, float v, float D, float dt,
float d[n+1],float T[n+1]);
```

that tests whether T (the returned value of tridiag()) is the actual solution. Use this scheme after the test for the case $D = 0.01$, $N = 100$. What is the largest possible time step?

• Do the von-Neumann-stability analysis for the problem, which means calculate the growth factor ξ (in the usual notation). Take a look at the special case $v = 0$ and $D = 0$. How does the mode with the largest possible wavenumber for the grid behave?

• Use "upwinding" for the advection. Observations?