5. Exercise for the lecture "numerical fluid dynamics" SoSe 18

Task 1

The temperature profile in the ground varies as a function of time because of the time dependent solar irradiation. The temperature profile is modeled by the following equation

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial^2 T}{\partial x^2} \text{ with} \\ T(t,0) &= 0,\ T(t,1) = \sin(10\pi t) \text{ and} \\ T(0,x) &= 0. \end{aligned}$$

• Modify the program from Task 4 of the last sheet and integrate with $N = 100$ and $\Delta t = 2 \times 10^{-5}$ until $t = 2$. Afterwards continue integrating for $1/4$, $1/2$ and $3/4$ period of the external temperature variation. Plot the spatial temperature variation for those times.

• Repeat the task for $T(t,1) = \sin(50\pi t)$.

Task 2

A hot metal sphere with radius r_0 is quenched in cold water of temperature T_W. The metal has the thermal diffusivity κ and an initial temperature of T_0. The cooling process is described by the radial symmetric temperature profile $T(t,r)$, with the following equations:

$$\begin{aligned} \frac{\partial T}{\partial t} &= \kappa \nabla^2 T \\ T(t,r_0) &= T_W,\ T(0,r) = T_0 \end{aligned}$$

The Laplacian in spherical coordinates for radially symmetric T is:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial T}{\partial r}),\ \vec{e}_r \cdot \nabla T = \frac{\partial T}{\partial r}$$

• Write the problem in dimensionless variables, so that the water temperature becomes $T_W = 0$, the initial temperature $T_0 = 1$ and the radius $r_0 = 1$. In addition the diffusion equation should be free of parameters.

• Formulate the problem *entirely* in spherical coordinates. How must the point $r = 0$ be handled?

• Use, like in the last exercise, the FTCS-scheme with $N = 100$ and $\Delta t = 2 \times 10^{-5}$. Plot the solution after 10^3, 5×10^3 and 10^4 steps. Do this for first and second order schemes at the boundaries.

• Use the program to test how large the largest time step can be.

• For Earth one can assume that $r_0 = 6.4 \times 10^6$m and $\kappa = 10^{-6}\mathrm{m}^2\,\mathrm{s}^{-1}$. Assume as well that Earth was for the most part covered by water and that it had after creation a temperature of 6000K above the water temperature. Nowadays a temperature gradient near the surface of $\mathrm{d}T/\mathrm{d}r = -0.033\mathrm{K}\,\mathrm{m}^{-1}$ can be observed. How old would the earth be if it had reached this surface gradient only by diffusive cooling?