A Review and Demonstration of Image Recovery by a Nonlocal Operator

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***Abstract*—Image recovery is a crucial problem for many fields including autonomous car, medical imaging, astronomical imaging, to forensic sciences specially when image is degraded and some noise is added. Since the application of image reconstruction is very sensitive, it needs more attention and research. Moreover, in order to apply optimization based technique, mathematical validation is required to formulate the loss function. In this project, image recovery by a nonlocal operator has been explored where loss function is constructed based on structural similarity and preserving originality of the image. In my project, I have included H^1 semi-norm like regularization functional as the nonlocal operator in the loss function and found good results.**

***Index Terms*—image recovery, regularization, nonlocal operator**

I. Introduction

Image recovery can be thought of as a problem of image denoising and restoring from a corrupted image while maintaining all the attributes of the image. Generally, an image can be corrupted by the camera if not taken properly. One such case is an image taken from the autonomous car. An autonomous car takes an image for the purpose of self-navigation but is affected by motion blur since it takes a picture while moving. Also, it adds noise due to exposure mismatch and correct ISO settings. Therefore, the image needs to be denoised automatically while keeping the image looks like the original. The method which restores the image from the noisy corrupted image is popular as image restoration. Image recovery has a number of applications other than in the autonomous car. It is used in medical imaging, astronomical imaging, to forensic science. The presence of noise in medical images is common and can lead to misdiagnosis. An X-ray image can be degraded due to secondary radiation from the X-ray machine and other objects. In tomography, like CT scan, the main source of noise is the quantum noise from X-ray photons and electric noise from the X-ray machine. Since diagnosis is a serious issue and needs to be handled with great care, image restoration technique plays a great role in diagnosis. Astronomical images are taken using the high precision camera but still, it cannot get rid of the noise. Random fluctuation in the arrival of photons when that high precision sensor captures them introduces noise. Sometimes, the sensor which is capturing those signals is responsible for introducing noise. Lastly, in forensics, surveillance camera videos, fingerprints, etc. are used which are of very bad qualities. In the above-described field, noise and image degradation is inevitable and very common. Moreover, the application of those fields is very sensitive and requires a clean image. Therefore, restoration image to their clean version is very important in those cases. There are a variety of image restoration techniques available to solve those issues. Recently, optimization-based problem solving has become popular and in [1] author proposed local operator-based image denoising where every pixel is formed based on its similar structures. This method has considered two objectives in the loss function – reducing structural dissimilarities of the optimized image with the noising degraded image and reducing the gap between the pixel and their neighbors by reducing derelict energy. The proposed method proves to be very efficient in deblurring and denoising images even in the presence of high noise.

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II. Background

Let's first formulate the problem of image recovery. Suppose, I have received an image f degraded by some system K and n noise added to it. So, f can be expressed as equation (1):

$$f = Ku + n \qquad (1)$$

Finding u from f is called the image recovery here. Here, K is some operation done by the system and n is the amount of noise added. From the nature, it is an inverse problem and there a number of applications of it discussed in the introduction section. In [1], author mainly mentioned two main applications - image deconvolution and tomographic reconstruction. It is obviously a deconvolutional operation when K is a convolution opreration and f is a degraded image in equation 1. Moreover, when K is an attenuated Random Transform and f is the photon counts, then recovering u can be considered as a tomographic reconstruction problem. There are three different methods has been discussed in [1] for image recovery – linear models, nonlinear models and nonlocal methods.

A. Linear Model

Two special linear models have been discussed in [1]. One of them is Tikhonov regularization with an identity where we need to find minimizer u (the recovered image) which minimizes the following loss function (equation 2) [2]:

$$u = argmin \int (f - Ku)^2 \ + \mu \int u^2 \qquad (2)$$

The first term on the right side of the equation is the reconstruction loss and the second term of the L_2 norm. μ is a regularizing parameter. If we take the derivative of equation 2, equation (3) is found where K is the degradation of the image and K* is the adjoint of the K.

$$K * (Ku - f) + \mu u = 0 \qquad (3)$$

Another way to minimize u is to replace L_2 norm by H^1 semi norm (equation 4) [3].

$$u = argmin \int (f - Ku)^2 \ + \nu \int |\Delta u|^2 \qquad (4)$$

In this case, equation (4) (Euler-Lagrange equation) can minimize equation (3)

$$K * (Ku - f) - \Delta u = 0 \qquad (4)$$

where Δu is the Laplacian of u. K is the fast Fourier transform of degradation and can be found by a single step. Therefore, implementation of linear model is very fast. A conjugate gradient algorithm can minimize both model's loss function very easily. But both of those minimization smear out the edges when μ and v are large. Moreover, high frequency signal amplification in the presence of noise in Tikhonov regularization's is its another demerit.

B. Non-linear Model

The second method is the nonlinear method, In [1], TV regularization method has been discussed where the regularizing term is TV (Total Variation) proposed by Rudin [4] and Lagrange multiplier is a multiplier of the restoration loss.

$$u = argmin \int |\Delta u| \ + \frac{\lambda}{2} \int (f - Ku)^2 \qquad (5-a)$$

and the minimization of equation 5-a required solving a PDE of equation 5-b.

$$\begin{cases} \text{u}_t\text{=}\Delta \frac{\Delta u}{|\Delta u|} + \lambda k * (f - Ku), & \text{in } \Omega \qquad (1) \\ \dfrac{\partial u}{\partial n} = 0, & \text{on } \partial\Omega \qquad (2) \end{cases}$$

C. Non-local methods

1) Overview: The third method is the Nonlocal method which has been evolved from the Yaroslavsky filter originally proposed for texture analysis [5]. The idea of the method is to restore a pixel by the average of other similar structured pixels. The similarity of the two pixels is calculated from the patches around these two pixels. The similarity of two pixels is called weight function and the formula to calculate the weight function is given as equation 6.

$$w_f(x, y) = exp^{(-\frac{(G_a * |f(x+.) - f(y+.)|^2))(0)}{h^2})} \qquad (6)$$

Where Ga is the Gaussian kernel with standard deviation a and h is a filtering parameter. Here f is the noisy degraded reference image and $|f(x + .) - f(y + .)|$ is the substraction of the patches around x and y in f. Weight is significant when y looks like x and non-significant otherwise. Again, the formula to restore a pixel x from all the neighboring pixels, y is given below:

$$NL_f(x) = \frac{1}{C(x)} \int w_v(x, y) f(y) dy \qquad (7)$$

where $C(x) = \int_\Omega w_v(x, y) dy$
This similarity measure can be introduced in the loss function to reduce noise in the image. Moreover, when the noisy image is considered to construct the weight function, it helps to restore edges where reducing noises.

2) Nonlocal Operators: Based on the overview, loss function needs to be formulated for the image restoration. In [1], author proposed, two types of regularization functions : Tikhonov and standard H^1 semi norm. Consider all pixels $x, y \epsilon R^2$ where $\Omega \subset R^2$ and $w(x, y)$ is the weight function, then nonlocal gradient is:

$$\Delta_w u(x, y) = (u(y) - u(x))\sqrt{w(x, y)} \qquad (8)$$

From there, for H^1 semi norm like regularization can be expressed as equation 9:

$$J_{NL/H^1}(u) = \frac{1}{4} \int |\Delta_w u|^2$$

$$= \int \int_{\Omega \times \Omega} (u(x) - u(y))^2 w(x, y) dx dy \qquad (9)$$

and the Euler-Lagrange of equation (9) is:

$$L_{NL/H^1}(u) = - \int_\Omega (u(y) - u(x)) w(x, y) dy$$

Similarly for TV like regularization,

$$J_{NL/TV}(u) = \int |\Delta_w u|$$

$$= \int \sqrt{\int_{\Omega \times \Omega} (u(x) - u(y))^2 w(x, y) dy} dx \qquad (8)$$

and the corresponding Euler-Lagrange of equation is:

$$L_{NL/H^1}(u) = - \int_\Omega (u(y) - u(x)) w(x, y) [\frac{1}{|\Delta_w u(x)|} + \frac{1}{|\Delta_w u(y)|}] dy$$

3) Computing Weights: For computing weights, image pre-processed by H^1 semi norm regularization has been used.

Reconstruction of a single pixel from all pixels in the image is energy and time inefficient and therefore, only local neighborhood is considered for a certain pixel of window size (21,21).

III. Algorithm

The step-by-step procedure of image recovery is shown in figure 1. In every optimization algorithm, an initial solution is required. Here, as suggested by [1], denoised image by a regularized filter is taken as an initial solution. When a pixel is corrupted, the neighbouring pixels still carry a significant information of its clean value. Therefore, a way to restore this noisy pixel is to reconstruct it from its neighbors. To do so, a weight matrix is calculated for all neighbors of that pixel which represents the similarities of all the neighbors with the considered pixels as explained in the previous section. Weight matrix of a pixel contain the similarity of every pixels of some neighbourhood with that pixel where the considered pixel is in the center. This reconstruction algorithm needs to minimize a loss function consists of a reconstruction loss term and a nonlocal operator loss term. The nonlocal operator los in this project is Euler Lagrange functional for H^1 norm. These two terms also reduce some context dissimilarities for the noisy image. In every iteration, the image is updated based on the loss. This process is repeated for several times defined by the user to find an optimum solution.

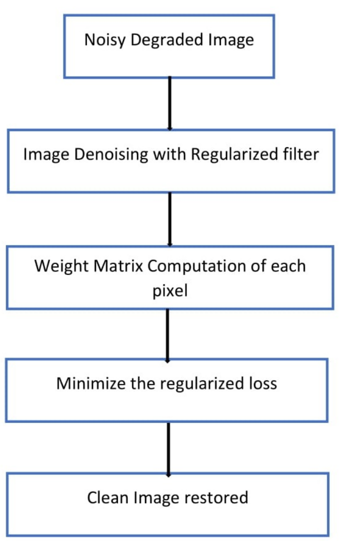

Fig. 1: Block Diagram of the Image Recovery steps.

Weight Matrix computation of every pixel of an image is a little bit tricky and therefore, it has been shown as a flowchart in figure 2. The input of this method is the regularized image, RI, and the patch size, PS. The first task is to take a patch at a pixel position k and all the neighboring patches to calculate the weight matrix. It creates a problem at the boundary

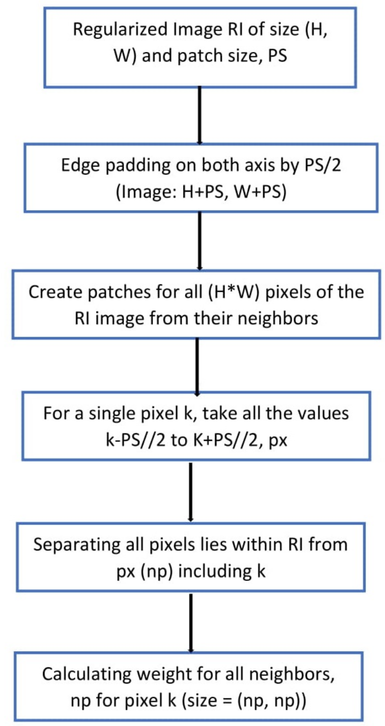

Fig. 2: Weight matrix computation of a single pixel

since at the boundary it is not possible to create patches for the pixel itself and its neighbor. This is shown in figure 3. For pixel not at the boundary (orange) has 8 neighbors total (marked as bluish). It is possible to find patches for all 8 neighbors and the pixel itself. But for the pixels at the boundary (black), it is not possible to find patches for all 5 neighbors and the pixel itself. The problem can be fixed by edge padding i.e., copping pixels at the edge (Figure 4). In figure 4, the edge padding has been done in the up, bottom, right, left by PS/2 where PS is the patch height and width. Now, it is evident from figures 3 and 4 that not all pixels have the same number of neighbors. Therefore, firstly, possible indexes of neighbors of k are taken around k. Suppose pixel k is at row r and column c. So, neighbors for pixel k will be from row r - PS//2 to r + PS//2, and column will be from c -PS//2 to c - PS//2. From those indexes, it is checked that which pixels are within RI and discard the other index of the pixels. Lastly, weight between pixel k and all of their neighbors are calculated using equation 6. After obtaining the initial image, a patch of all pixels, and weight matrix multiplied by the gaussian kernel, the task is to minimize the loss function. The total loss function can be expressed as equation (9):

$$Loss(u) = J(u) + \frac{\lambda}{2}\int(f - Ku)^2 \qquad (9)$$

where $J(u)$ is from equation 9.

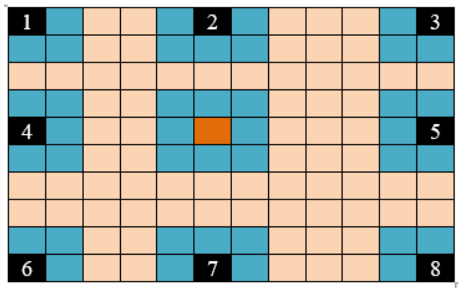

Fig. 3: Neighbours of some pixels in an unpadded image

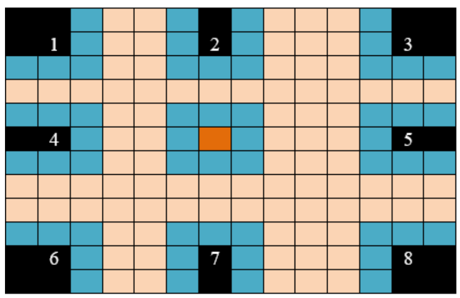

Fig. 4: Neighbours of some pixels in a edge padded image

When Loss(u) is differentiated with u, equation (10) i found which is used here update u.

$$u^{k+1} = u^k - dt(Lu^k + \lambda K^*(Ku^k - f)) \qquad (10)$$

where dt is the learning rate, λ is the Lagrange mutiplier.

To implement the weight update by equation 10, the respective patch needs to find out having all of its neighbors. Then, all pixels in the patch need to be subtracted from the considered pixel. Next, the resultant patch needs to be multiplied by the weight matrix to sum the value. This will restore just a single pixel. This process will be implemented for all the pixels in the image which will return regularization/nonlocal operator loss term term (Lu^k). The reconstruction loss of equation 10 is formulated using the degradation model K in frequency domain and image (noisy and the initial) in frequency domain. After calculating the loss term in frequency domain, inverse fourier operation is performed to convert the whole frequency domain into spatial domain. Then, the initial image is updated based on equation 10. This process for continued for some steps defined by the users to find an optimum solution.

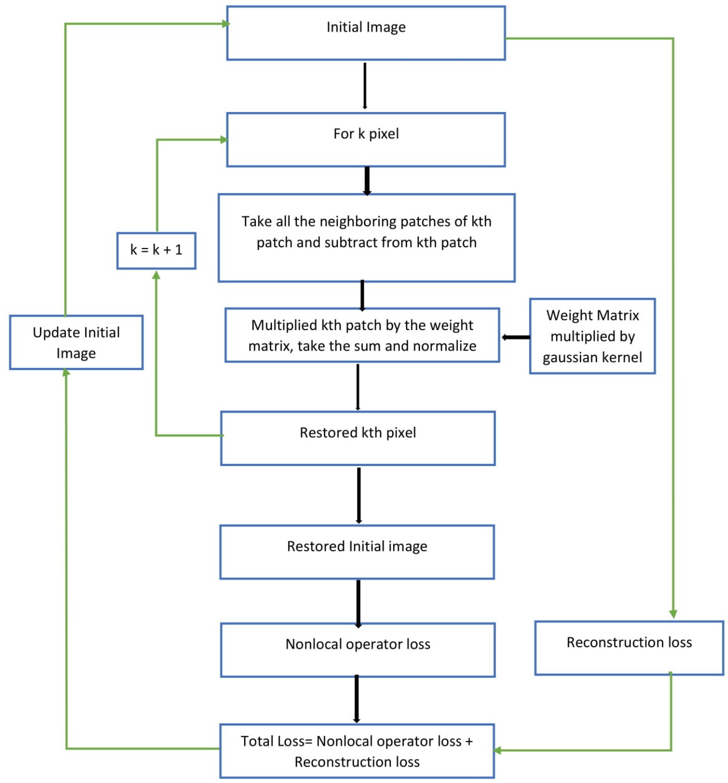

Fig. 5: Optimization procedure of reconstruction

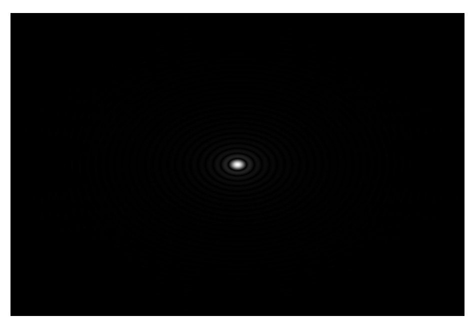

Fig. 6: Blurring Filter in Frequency domain

IV. RESULTS

First of all, noisy image has been created from clean image by a blurring filter and adding noise. The blurring filter has been a black image with a circular disk of radius 30 pixels having value 1. This is shown in figure-6. Figure-6 is a centered 2D frequency response. Conventionally, X axis represents the real axis of the frequency and Y axis represents the imaginary axis. Therefore, frequencies are conjugate of each other around X axis. In this figure, the center portion is white (True) upto a certain radius and outside this radius range, the value is zero. Therefore, low frequencies are permitted (white close to the center) and high frequencies are cancelled (black far away from the center). In image, high frequencies are edges. Therefore, this filter removes all the edges in the image and smooth the image. Lets demonstrate that with an example. Suppose, figure 7 is our original image that has gone through blurring operation defined by figure 6. Figure 8 is showing the filtered image. It is evident that all the edges has been removed and therefore, the image is blurred. Then, some Gaussian noise is added to the blurred image. In figure 9, gaussian noise of mean 0 and standard deviation 0.0001 is added to figure 8 (blurry image). The adding of noise making direct deconvolution difficult since when the noisy image is divided by the system response in frequency domain, noise can be multiplied a large factor if system response has very low value at some frequency. Therefore, optimization algorithm has been used here. Here, I have a used least square method with H^1 norm regularization to create the initial image which is shown in figure 10. MATLAB is used to create the initial image.

Fig. 7: Original Image

Fig. 8: Blurred image

Fig. 9: Noise added blurred image

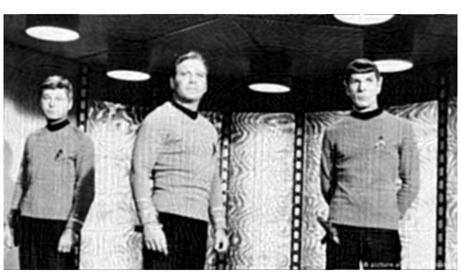

Fig. 10: Initial image reconstructed by H^1 semi norm

Then, the regularized filtered image is read with opencv-python and resized to (100,200) to reduce computation. Also, the degradation of the model, K read from MATLAB and resized to the same size of the image. As explained in the methodology, image is edge padded with size (Patch Height//2,Patch Width//2) on the both axis. The patch size was 21 there. After finding the padded image, a patch is created for every pixel of size (Patch Height,Patch Width). So, for a (H,W) image size, patch is a four dimensional array of size (H,W,Patch Height,Patch Width) where Patch[i,j,k,l] is the $(k,l)^th$ value of a patch of size (Patch Height,Patch Width) where patch is created for the pixel position (i,j). Then, weight matrix is calculated for all the pixels in the image and multiplied by some Gaussian kernel of standard deviation of 1 pixel and mean 0. Lastly, image is constructed based on the loss function of 9 and the image is updated based on equation 10. The training procedure (loss vs epoch curve) is shown in figure 11 for a Star Trek image [6]. Figure (12) shows the reconstructed image. This shows that most of the work is done in the initialization stage and not much needs to be done in optimization. Then noise is noise of standrad deviation 0.001 and 0.01 is added and the resultant image is shown in figure 12, 13, 14. It is observed that most of the denoising is done by initialization.

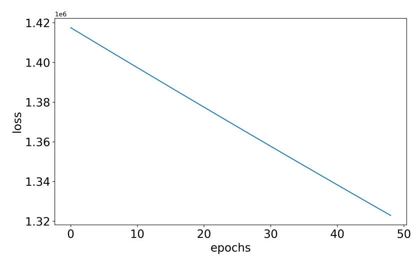

Fig. 11: Loss reduction with the number of epochs

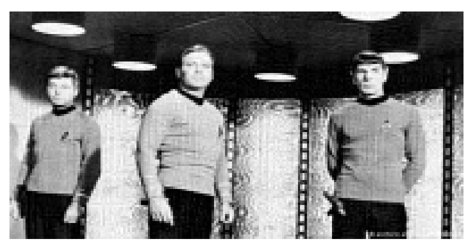

Fig. 12: Recovered image

V. Conclusions

Image recovery problem is formulated here by a nonlocal operator loss and reconstruction loss. For initialization, H1 semi norm regularization is used and the image is reconstructed by minimizing the loss function by a number of epochs. It appears that adding more noise makes image deconvolution difficult.

Acknowledgment

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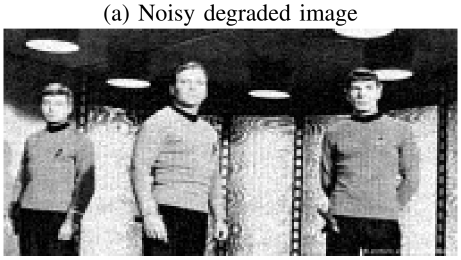
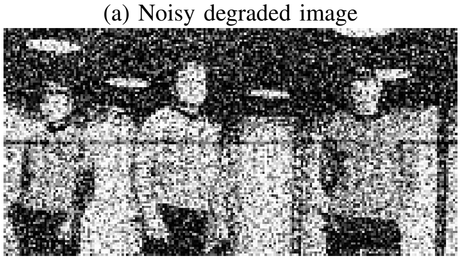
(a) Noisy degraded image



(b) Reconstructed Image

Fig. 13: Image restoration from a degraded and gaussian noise added with 0.001 std

(a) Noisy degraded image



(b) Reconstructed Image

Fig. 14: Image restoration from a degraded and gaussian noise added with 0.01 std

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