

Merge Sort

Merging Operation:

Given 2 sorted arrays $P[]$ and $Q[]$ we have to create a third sorted array $R[]$ such that $R = P \cup Q$.

4	8	13	18	20
---	---	----	----	----

i

$P[]$

6	7	14	24	25
---	---	----	----	----

j

$Q[]$

4	6	7	8	13	14	18	20	24
---	---	---	---	----	----	----	----	----

k

$R[]$

Here we can see both $P[]$ and $Q[]$ are sorted array being indexed by i and j . We have to create a third sorted array $R[]$ such that $R = P \cup Q$. R is indexed by k . We compare $P[i]$ with $Q[j]$ and put the minimum of these two element in $R[k]$ and increment the index k . If the element is copied from $P[]$ to $R[]$ we increment i otherwise the element is copied from $Q[]$ to $R[]$ and we increment j . Thus we keep on incrementing index pointers i and j in alternate fashion till we exhaust one of the 2 arrays namely $P[]$ or $Q[]$. Lastly we copy the remaining elements of the unfinished array of $P[]$ or $Q[]$ in the array $R[]$.

If the size of the arrays $P[]$ and $Q[]$ are m and n respectively then the time to perform merge operation is $O(m+n)$ since the index pointer k fills up $m+n$ elements in the array $R[]$ without backtracking.

```

#include<stdio.h>

#define MAX 1000

void merge(int P[], int Q[], int R[], int m, int n)
{
    int i=0; int j=0; int k=0;

    while((i<m)&&(j<n))
    {
        if(P[i] < Q[j]) {R[k]=P[i]; i++; k++;}
        else {R[k] = Q[j]; j++; k++;}
    }

    while(i<m)
    { R[k] = P[i];
      i++; k++;
    }

    while(j<n)
    { R[k] = Q[j];
      j++; k++;
    }
}

void mergesort(int A[], int low, int high)
{
    int mid;
    int i, j, k, l, m, n;
    int P[MAX], Q[MAX], R[MAX];
    if (low < high)

```

```

    {
        mid = (low+high)/2;
        mergesort(A, low, mid);
        mergesort(A, mid+1, high);
        i=low; m=0;
        while(i<=mid) {
            P[m] = A[i]; i++; m++;
        }
        j=mid+1; n=0;
        while(j<=high){
            Q[n]= A[j]; j++; n++;
        }
        merge(P, Q, R, m, n);
        l=low; k=0;
        while(l<=high){
            A[l]=R[k]; l++; k++;
        }
    }
}

int main()

{
    int A[MAX]; int i,  size;

    char c;

    printf("Enter the number of elements of the array\n");

    scanf("%d", &size);

```

```

printf("Enter the elements of the array\n");

for (i=0; i<size; i++)
{
    scanf("%d", &A[i]);
}

mergesort(A,0,size-1);

for (i=0; i<size; i++)
{
    printf("%d\n", A[i]);
}

printf("Enter character\n");

c=getchar();

return(0);

}

```

Merge Sort

Merge sort splits the array $A[low..high]$ to be sorted in two halves $A[low..mid]$ and $A[mid+1..high]$ where $mid = (low+high)/2$ and recursively invokes mergesort function on these two halves. Then it invokes the merging routine to merge already sorted two arrays $A[low..mid]$ and $A[mid+1..high]$ to the original array $A[low..high]$.

This merge-sort function is invoked from the driver or main function with $low=0$ and $high=size$ where $size$ is the number of elements in the array $A[]$.

Time complexity Analysis:

If $T(n)$ is the time complexity of the program where $size = n$ then we can write

$T(n) = 2T(n/2) + c.n$ since we have to spend $c.n$ time for the merging operation on 2 sub-arrays of size $n/2$ and $2T(n/2)$ term comes for 2 recursive invocations.

Thus $T(n) \in O(n \log n)$.