Connection to Linear Algebra

$Xa = y \longrightarrow$ Linear Transformation

$Xa = \lambda a \longrightarrow$ Eigen-values

$Xa = 0$

* **Geometric Vectors**

Vectors are arrows pointing in space, where the length of the arrow represents the mag & dirn.

* **Scalar vector**

$\vec{v} = v_1, v_2, v_3, \ldots\ldots v_N$

$\vec{\omega} = \omega_1, \omega_2, \omega_3 \ldots \omega_N$

$\vec{v}\cdot\vec{\omega} = v_1\omega_1 + v_2\omega_2 + \ldots\ldots\ldots v_N\omega_N = v^T\omega \qquad \vec{v}, \vec{\omega} \in R^N$

If dot product is positive – More aligned
negative – opposite directions
zero – orthogonal

$\vec{v}\cdot\vec{\omega} = |\vec{v}||\vec{\omega}|\cos\theta$

* **Types of matrices**
  - Square
  - Symmetric
  - Triangular
  - Diagonal

* **Determinant**
  - If $|x| = 0$ matrix is not invertible
  - 2 rows or 2 columns are zeroes $\Rightarrow$ det = 0
  - two rows or two columns are identical / linearly dependent

Orthogonal matrix : $XX^T = I = X^TX$

$\Rightarrow \boxed{X^{-1} = X^T}$

classmate

Date
Page

* Orthogonal matrix

Transformations by orthogonal matrices are special because the length of a vector v is not changed when transforming it with orth matrix x.

$$\|xv\|^2 = (xv)^T(xv)$$

$$= v^T \underbrace{x^T x}\, v \qquad (x^T x = I)$$

$$= v^T v$$

$$= \|v\|^2$$

$$\cos\theta = \frac{U^T V}{\|U\| \, \|V\|}$$

rotation about origin : $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Permutation : $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Matrix with a vector

$x a = y$

Particular and General Solution:

$$a = \begin{bmatrix} 42 \\ 8 \\ 0 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 8 \\ 2 \\ -1 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} -4 \\ 12 \\ 0 \\ -1 \end{bmatrix}$$

* Matrix Method - I

$X a = Y$ $\quad a \to$ unknown

$a = x^{-1} y = \frac{(\text{adj } x)}{|x|} \times y$ $\quad$ only possible if $x \to$ square + invertible

If $|x| \neq 0 \Rightarrow$ Sys of eqns consistent and has unique solution

$x a = y$

$$\Leftrightarrow x^T x a = x^T y \Leftrightarrow a = (x^T x)^{-1} x^T y$$

so $\quad x a = y$

$x^T x a = x^T y$ $\longrightarrow$ Moore Penrose Pseudo inverse.

$a = (x^T x)^{-1} x^T y$

* Matrix Method - II (QR decomposition)

matrix X (m×n) can be decomposed to Q (orthogonal matrix)

$$X = \underset{\substack{\text{orthogonal} \\ \text{matrix} \\ (m\times n)}}{Q} \cdot \underset{\substack{\text{upper triangle} \\ \text{matrix} \\ (n\times n)}}{R}$$

$Xa = Y$

$a = X^{-1}Y = (QR)^{-1}Y$

$\therefore\ a = R^{-1}Q^{-1}Y$

$\boxed{a = R^{-1}Q^{T}Y}$

R is invertible because a triangular matrix is invertible if its diagonal entries are strictly positive.

Q. what should be the nature of a and y for which the solution to the above equation $Xa = Y$ or $Xa = 0$ exists?

Vector Space.

* Vector Space

consists of
- vectors
- scalars
- defined operations.

- vector addition of two vectors is a vector.

  $\vec{v}, \vec{w} \in V \Rightarrow \vec{v} + \vec{w} \in V$

- Scalar multiplication ~~c∈F~~ produces new vector.

  $c \in F,\ \vec{v} \in V \quad c\vec{v} \in V$

* Vector Space

- Associativity of vectors: $\vec{u}, \vec{v}, \vec{w} \in V$

  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
- Existence of a zero vector

  $0 + \vec{v} = \vec{v}$ for all $\vec{v} \in V$
- Existence of negatives:

  $\vec{v} \in V$ there is $-\vec{v} \in V$ st. $\vec{v} + (-\vec{v}) = 0$
- Associativity: $a, b \in F$, $\vec{v} \in V$

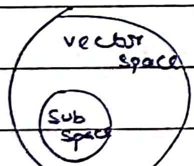
  $(ab)\vec{v} = a(b\vec{v})$
- Distributivity: $a, b \in F$ $\vec{v}, \vec{w} \in V$

  $(a+b)\vec{v} = a\vec{v} + b\vec{v}$

  $a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$
- Unitarity $\vec{v} \in V$

  $1\vec{v} = \vec{v}$

$R^3 \rightarrow$ ordered Triples of real numbers (x, y, z)



* Subspace

- A non-empty subset of a vector space
- Note 0 vector by default is a subspace.
- $\vec{0}$ is a subspace.

Independent vectors

- Do not have redundant relationships among them.
- No vector in the set can be expressed as a linear combination.

$\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots \vec{v_n} \in R^N$

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} + \cdots \cdots + c_n\vec{v_n} = 0$$

Basis

- Set of vectors, linearly independent and span the entire vector space.
- Any vector can be represented as a unique linear combination of the basis vectors.

IMP → No. of vectors in basis = Dimension of vector space.

- Every basis will have same number of vectors.

* Conclusion:

	concepts used
• Data representation :	matrices & vectors
• Feature engineering :	normalization, dim reductn
• Model representation :	linear eqns, matrix vector mul
• PCA :	eigen vectors, eigen values
• SVD :	data compression
• Optimization. :	Gradient descent.