1.) Given $a \in Z_p$

$$(a+p)^n \bmod p = a^n \bmod p$$

$$= \left({}^nC_0 a^0 p^n + {}^nC_1 a^1 p^{n-1} + {}^nC_2 a^2 p^{n-2} + \dots + {}^nC_n a^n p^0\right) \bmod p$$

$$= (0+0+0+\dots+a^n) \bmod p$$

$$= a^n (\bmod p)$$

2.) Z_5

$a = \{1, 2, 3, 4\}$

$a^{-1} = \{1, 3, 2, 4\}$

Z_{11} = $a = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$a^{-1} = \{1, 6, 4, 3, 9, 2, 8, 7, 5, 10\}$

3.) Euclidean Algorithm to find GCD

gcd (56245, 43159) = ?

$56245 = 1 \times 43159 + 13086$

$43159 = 3 \times 13086 + 3901$

$13086 = 3 \times 3901 + 1383$

$3901 = 2 \times 1383 + 1135$

$1383 = 1 \times 1135 + 248$

$1135 = 4 \times 248 + 143$

$248 = 1 \times 143 + 105$

$143 = 1 \times 105 + 38$

$105 = 2 \times 38 + 29$

$38 = 1 \times 29 + 9$

$29 = 3 \times 9 + 2$

$9 = 4 \times 2 + 1$

$2 = 2 \times 1 + 0$

$\gcd(56245, 43159) = 1$

4.) $\phi(3^4)$

$\because$ 3 is a prime w.r.t $\phi(p^\alpha) = p^\alpha - p^{\alpha-1}$

$\Rightarrow \phi(3^4) = 3^4 - 3^{4-1}$

$= 3^4 - 3^3 = 3^3(3-1)$

$= 27 \times 2 = 54$

$\phi(2^{10}) = 2^{10} - 2^9 = 1024 - 512 = 512$

5) $3^{100} \bmod (31319)$

$100 = 1100100$

$= 2^6 + 2^5 + 2^2$

$3^{100} = (3)^{2^6 + 2^5 + 2^2}$

$= \left(3^{2^6} \times 3^{2^5} \times 3^{2^2}\right) (\bmod\ 31319)$

$(3)^{2^0} (\bmod\ 31319) = 3$

$3^2 \bmod 31319 = 9$

$3^4 (\bmod\ 31319) = 81$

$3^{2^3} (\bmod\ 31319) = 6561$

$3^{2^4} (\bmod\ 31319) = 14415$

$3^{2^5} = (14415)^2 (\bmod\ 31319) = 21979$

$3^{2^6} = (21979)^2 (\bmod\ 31319) = 12185$

$3^{100} (\bmod\ 31319) = 25879 (\bmod\ 31319)$