## Review problems for Test 2

(1) Let $f(x) = x^2 + 1$

A. Find the average rate of change of $y$ with respect to $x$ over the interval $[1, 2]$

B. Find the instantaneous rate of change of $y$ with respect to $x$ at the point $x = 2$.

(2) Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 3x^2 - x$.

(3) Use the definition of the derivative to calculate $f'(3)$ if $f(x) = \sqrt{x^2 - 5}$.

(4) The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ where $r$ is the radius of the sphere. Find the instantaneous rate of change of $V$ with respect to $r$ when $r = 4$.

(5) Given that $f(2) = -1$ and $f'(2) = 5$, find an equation for the tangent line to the graph of $y = f(x)$ at the point where $x = 2$.

(6) Show that $f(x) = \begin{cases} x^2 - 5, & x \le 1 \\ x - 5, & x > 1 \end{cases}$ is continuous but not differentiable at $x = 1$.

(7) Find $\frac{dy}{dx}$:

A. $y = \frac{-8}{x^2} + \frac{1}{5}x^5$

B. $y = \frac{x^2 + 3x}{7 - 2x}$

C. $y = (x^2 - 2)(x^3 + 5x)$

D. $y = x \tan x$

E. $y = \frac{\sin x}{x^2}$

F. $y = \frac{\cot x}{1 + \csc x}$

G. $y = \frac{4}{x^2 - 3x}$

H. $y = \frac{3}{(x^2 - 2x + 2)^3}$

I. $y = \cos^3 2x$

J. $y = (x^2 - 1)^3(3x + 2)^4$

K. $y = \frac{(2x-3)^4}{(x^2+1)^3}$

L. $y = \left(\frac{3x-5}{x^2+4}\right)^7$

M. $y = \sqrt{2x}$

N. $3x - 2x^2y + y^2 = 7$

O. $\cot(xy) = x^2 - y$

(8) Find $\frac{d^2y}{dx^2}$: A. $x^2 - y^2 = 9$ ( in terms of y only)

B. $2x^3 + y^2 = 1$

(9) Find equations of the lines tangent to the graph of $y = x^3 - 3x^2 - 9x + 2$ which are horizontal.

(10) Given the curve $x^2 - xy + y^2 = 3$.

A. Write a general expression for the slope of the curve.

B. Find the coordinates of the points on the curve where the tangents are vertical.

C. Find the equations of all lines normal to the curve at $x = 1$.

(11) If $f(x) = |9 - x^2|$, find A. $f'(1)$ B. $f'(3)$ C. $f'(4)$

(12) If $y = (x^2 + 2x - 3)^4$, find $\frac{dy}{dx}$.

(13) Evaluate:

A. $\lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h}$

B. $\lim_{x \to 8} \frac{\sqrt[3]{x} - 2}{x - 8}$

(14) Find the value(s) of x for which the lines tangent to the graph of $f(x) = 8x^{\frac{1}{3}} - x^{\frac{4}{3}}$ are horizontal.

CALCULUS TEST #2 REVIEW SHEET 2.

1. *Use the definition of the derivative to find $f'(x)$ if $f(x)=2x^2+4$.*
2. Find $\frac{dy}{dx}$ Show all work.

   a. $y=\dfrac{x}{\left(x^2-6x\right)^2}$ c. $y=(x^2+1)(x^3-2x+4)$ e. $y=(x-4)^3(2x-5)^4$

   b. $y=x^7\sin x$ d. $y=\tan^3(2x)$ f. $y=\dfrac{\sqrt{x^2-4}}{\sqrt[3]{3x+1}}$

3. Evaluate:

   a. $\lim\limits_{h\to 0}\dfrac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}$ c. $\lim\limits_{x\to 2}\dfrac{x^5-32}{x-2}$

   b. $\lim\limits_{h\to 0}\dfrac{\sin^2(x+h)-\sin^2(x)}{h}$ d. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan x-\sqrt{3}}{x-\frac{\pi}{3}}$

4. If $f(x)=|4x-12|$, find a. $f'(1)$ b. $f'(4)$ c. $f'(3)$ (Show all work & use good notation)
5. Find the value(s) of x for which the lines tangent to the graph of $f(x)=6x^{\frac{1}{3}}-x^{\frac{4}{3}}$ are horizontal.
6. Find $\frac{d^2y}{dx^2}$ if $y=\cos x$
7. Find $\frac{dy}{dx}$ a. $\dfrac{x-y^2}{x+y}=x$ b. $5y^2+\sin y=x^2$
8. Find the slopes of the curve $y^2-x+1=0$ at the points (2,-1) and (2,1).
9. Use implicit differentiation to find $\frac{d^2y}{dx^2}$, if $4x^2-2y^2=9$
10. Given: $x^3+y^3=3xy$

    a. Use implicit differentiation to find $\frac{dy}{dx}$

    b. Find an equation for the tangent line to the curve at the point $\left(\frac{3}{2},\frac{3}{2}\right)$

    c. At what point(s) in the first quadrant (Quad. I) is the tangent line horizontal?

GO OVER YOUR NOTES AND HW FOR PROBLEMS ON CONTINUITY AND DIFFERENTIABILITY