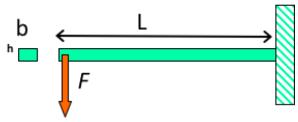
CENTRALE NANTES

Optimization for design in mechanical engineering

Application to the cantilever beam cross section optimization

Foundation master program (2019-2020)

The objective of the lab work is to make a global analysis and synthesis in order to find the best section for the problem the can support a concentrated force of F = 20kN applied to the free end of the cantilever beam (see the figure).



Beam parameter :

Young modulus	E= 2. 1.e11	Pa
Density (mass/volume)	ρ = Ro = 7800	Kg / m3
Admissible stress yield (max)	σ_a =sigma= 2 1.e8	Pa
Length of the beam	L = 2	m
Concentrated Force	F = 20 000	N
Admissible deflection (max)	da = Δ_a = 0.01	m
Admissible mass (max)	ma = μ_a = 300	kg

General equations of the flexion of the cantilever beam

stress :	σ_{max} <= σ_a	σ_{max} = (FL/ $\mathbf{I}$) * v (v=h/2)
Deflection	Δ < = Δ_a	$\Delta = FL^3 / (3 E \mathbf{I})$
Masse	m < = ma	m = ρ L S (S = section)

Where $\mathbf{I}$ represent the Second moment of the cross section and S is the section.
For the bounds it is accepted that the h>L/5

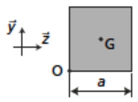
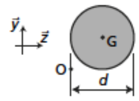
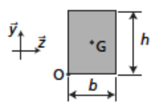
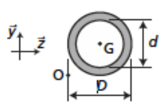
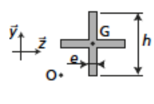
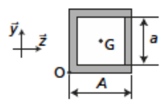
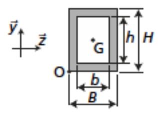
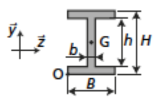
Example 1: for the beam of square section : $\mathbf{I} = a^4 / 12$ and $S = a^2$

so that : $\sigma_{max} = 6FL / a^3$ $\Delta = 4 FL^3 / (E a^4)$ and the Mass = ρ L a^2

Example 2: for the beam of square section : $\mathbf{I} = \pi D^4 / 64$ and $S = \pi D^2 / 4$

so that : $\sigma_{max} = 32 \ F L / \pi D^3$, $\Delta = 64 FL^3 / (3 E \pi D^4)$ and the Mass = ρ L $\pi D^2 / 4$

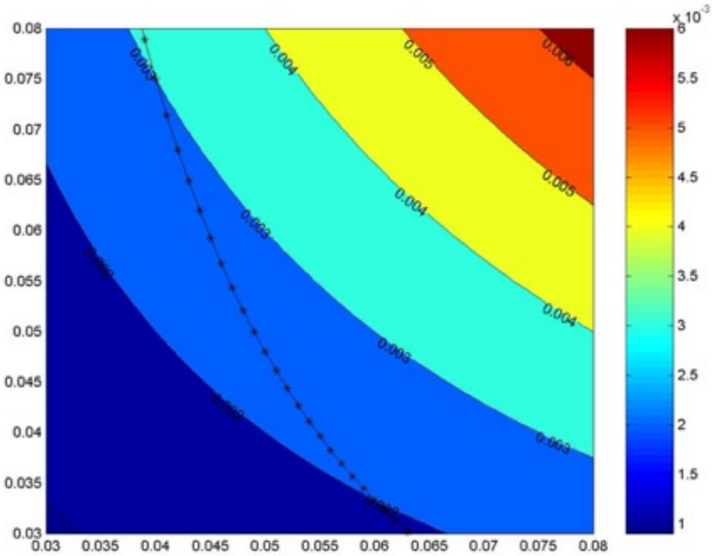
a/ Find by hand the maximum value of a (the square side length) and D (the diameter if the disc section).
b/ Find the “best” value of “a” for the beam with square section and the value “D” for the beam with disc section.
It can be helpful to plot the deflection and the mass with respect to a and D.
The objective is to have solid arguments a-in order to select the best section Square or disc?
The final objective of the lab work is to make a synthesis and select the best section among the following.

One variable				
Two variables				
four variables				

For each section write the equations of the mass, deflection stress yield
And for the problem with two variables plot the contour of the functions.
And for each profile find the optimum of the 2 problems
--- P1- Find the minim of the mass subject to the constraints (σ_{max} <= σ_a , Δ < = Δ_a)
--- P2- Find the minim of the deflection subject to the constraints (σ_{max} <= σ_a , mass < ma)

What is the best section ?

This figure is an example for the plot of the contour of mass including the stress yield constraint line for the problem of the beam



Introductive exercise

In order to resolve cantilever beam cross section optimization the it is necessary to understand the use of the contour function of Matlab and the use of Matlab optimization function. We start with the following mathematical function.

1/ Plot single variable function :

$f(x) = x^2 – 1$ [-2, 2]

$f(x) = 10 \sin(x) + 2 \sin(x/10)$ [-2pi, 2pi]

2/ Plot two variables function :

$f(x, y) = (x – 1)^2 + (y – 2)^2$ [-4, 4]

$f(x, y) = –x \sin (\mathrm{sqrt}(| x |)) – y \sin(\mathrm{sqrt}(|y|))$ [-512, 512] , step :0.5

3/ Using « fminconf » optimisation function of Matlab and find the optimum of :

$f(x, y) = (3x + 2y – 1)^2 + (x – y + 1)^2$ [-10, 10]

4/ Using « fminconf » optimisation function of Matlab and find the optimum of :

$f(x, y) = (3x + 2y – 1)^2 + (x – y + 1)^2$ [-10, 10]

subject to : $(x – 1)^2 + y^2 > 2$ and $x^2 + (y – 1)^2 > 2$

5/ Using « fminconf » optimisation function of Matlab and find the optimum of :

$f(x, y) = (3x + 2y – 1)^2 + (x – y + 1)^2$ [-10, 10]

subject to : $(x – 1)^2 + y^2 < 2$ and $x^2 + (y – 1)^2 < 2$