Assignment 1

$$1.\ \overset{f'_{approx}(x)}{\frac{f(x+h)-f(x)}{h}} = f'(x) + \frac{h}{2}f''(x) \cdots$$

$$\epsilon(h) = \left| \frac{f'_{exact}(x) - f'(x) \quad \frac{h}{2}f''(x)}{f'_{ex}(x)} \right|$$

$$= \left| \frac{\frac{h}{2}f''(x)}{f'(x)} \right| = \left| \frac{-\frac{h}{2} \cdot -\alpha^2 \cos(\alpha x)}{-\alpha \sin(\alpha x)} \right|$$ = error as a function of h

(Graphs included for $\alpha = 1, 10, 100$)

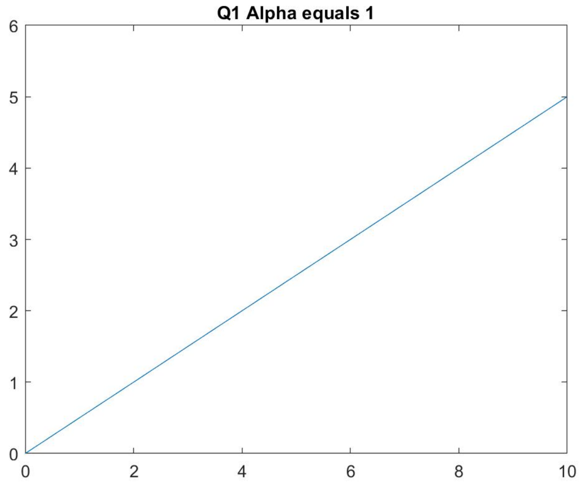
$$2.\ \frac{f(x+h)-f(x-h)}{2h} = \frac{f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) - \left(f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x)\right)}{2h}$$

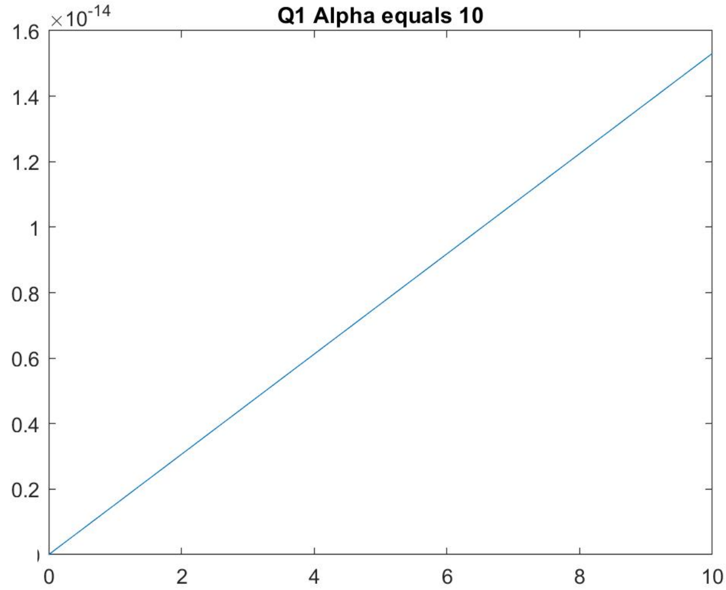
$$= f'(x) + \frac{h^2}{3!}f'''(x)$$

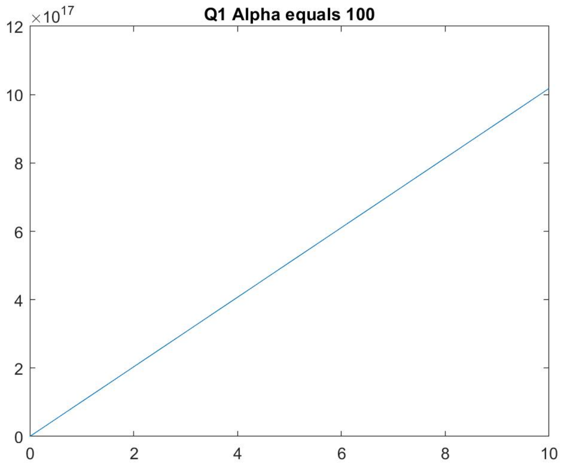
$$\epsilon(h) = \left| \frac{f'(x) - \left(f'(x) + \frac{h^2}{3!}f'''(x)\right)}{f'(x)} \right|$$

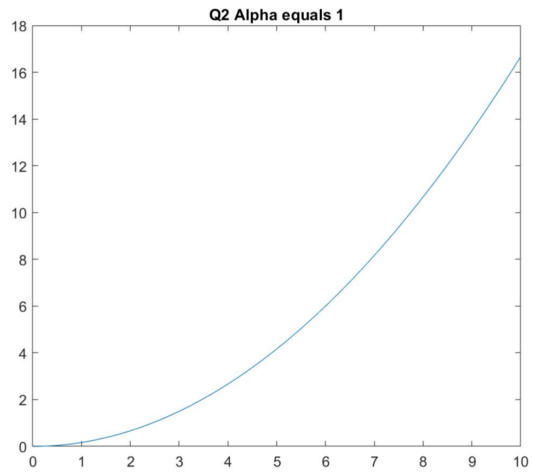
$$= \left| \frac{-\frac{h^2}{3!} \cdot \alpha^3 \sin(\alpha x)}{-\alpha \sin(\alpha x)} \right|$$: error as a function of h

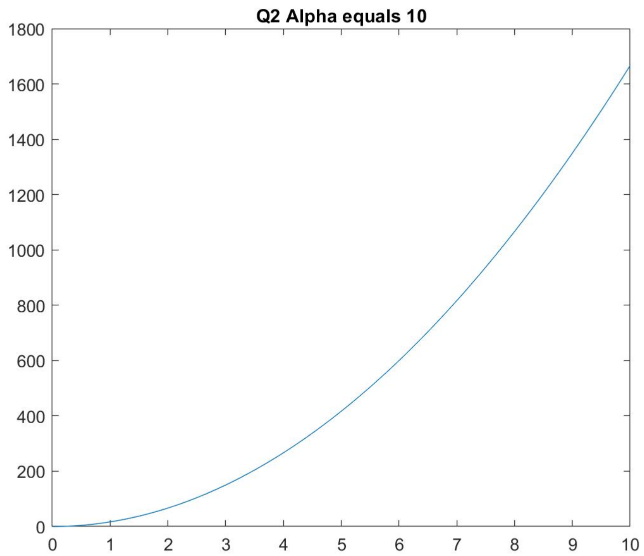
(Graphs included for $\alpha = 1, 10, 100$)

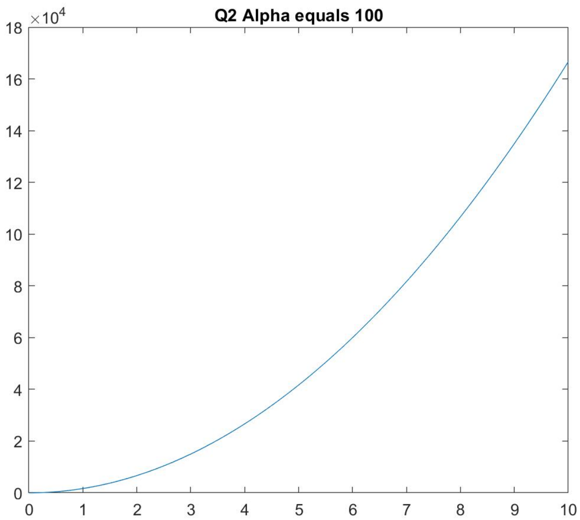

Q1 Alpha equals 1


Q1 Alpha equals 10
×10^-14


Q1 Alpha equals 100
×10^17


Q2 Alpha equals 1


Q2 Alpha equals 10


Q2 Alpha equals 100
×10^4

3. $\alpha f(x + h_1) + \beta f(x) + \gamma f(x + h_2)$

$= \alpha \left[f(x) + h f'(x) + \frac{h^2}{2} f''(x) \right] + \beta f(x) +$

$\gamma \left[f(x) + h f'(x) + \frac{h^2}{2} f''(x) \right]$

to make it so this approximates $f'(x)$ as close as possible we need to set up equations like so,

$\alpha + \beta + \gamma = 0$

$\alpha + \gamma = 1$

So, α & γ can be any value that adds up to 1, ex $\frac{1}{2} + \frac{1}{2}$ and β is -1.

$\alpha = \frac{1}{2} \quad \gamma = \frac{1}{2} \quad \beta = -1$