Assignment 2 CMPUT 617
Steven Weikai Lu

Question 4

Part 1:
Consider the maximum of a functional, which satisfy the following condition:

$$J[f] > J[f + \epsilon g] \tag{1}$$

for any arbitrary small ϵ and function f in the function space. Then we have:

$$\int f \cdot p_{xz} \cdot dxdz - log \int e^f \cdot p_x p_z dxdz > \int (f + \epsilon \cdot g) \cdot p_{xz} \cdot dxdz - log \int e^{f+\epsilon \cdot g} \cdot p_x p_z dxdz \tag{2}$$

Rearranging the terms we have:

$$\int f \cdot p_{xz} \cdot dxdz - \int (f + \epsilon \cdot g) \cdot p_{xz} \cdot dxdz > log \int e^f \cdot p_x p_z dxdz - log \int e^{f+\epsilon \cdot g} \cdot p_x p_z dxdz \tag{3}$$

$$-\int \epsilon \cdot g \cdot p_{xz} \cdot dxdz > log \frac{\int e^f \cdot p_x p_z dxdz}{\int e^{f+\epsilon \cdot g} \cdot p_x p_z dxdz} \tag{4}$$

Hence in a general sense, we can interpret this as:

$$-E_{xz}[\epsilon \cdot g] > log \frac{E_{xz}[e^f]}{E_{xz}[e^{f+\epsilon \cdot g}]} \tag{5}$$

More specifically, this condition states that $P_{XZ}(x, z)$ gives the proper weight of e^f over the expectation $E_{XZ}[e^f]$ as ϵ approaches 0. Hence mathematically,

$$P_{XZ}(x, z) = \frac{e^{f(x,z)} \cdot P_X(x) P_Z(z)}{\int e^{f(x,z)} \cdot P_X(x) P_Z(z) dxdz} \tag{6}$$

Part 2:
If we subsitute optimal P_{XZ} into $J[f]$, we have

$$J[f] = \int f \cdot \frac{e^f P_X P_Z}{\int e^f P_X P_Z dxdz} - \log \int e^f P_X P_Z dxdz \tag{7}$$

If there is a small perturbation lead to $J[f + \epsilon g]$, then we have

$$J[f + \epsilon g] = \int (f + \epsilon g) \cdot \frac{e^{f+\epsilon g} P_X P_Z}{E_{XZ}[e^{f+\epsilon g}]} - \log E_{XZ}[e^{f+\epsilon g}] \tag{8}$$