Multi-view Intact Space Learning

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This paper talks about multi-view learning. Given the i-th training data point, it has m different views, $\{z_i^v, 1 \leq v \leq m\}$, but we assume that different views are genereated from the same single one intact, x_i. Each view is obtained by applying a linear transformaion to x_i,

$z_i^v \leftarrow W_v x_i$, where W_v is the transformation matrix.

We want to find the hidden x_i from observed m views, $z_i^v, 1 \leq v \leq m$, and use x_i as the new presenation of the i-th data point.

Thus we want to learn both W_v and x_i to resoncstruct the observed z_i^v, and the reconstruction error is measured by the loss function of

Cauchy estimator, $\rho(x) = \log(1 + (x/c)^2)$

$$\min_{x,W} \log\left(1 + \frac{\|z_i^v - W_v x_i\|^2}{c^2}\right)$$

Considering n data points of m views,

$$\min_{x,W} \frac{1}{mn} \sum_{v=1}^{m} \sum_{i=1}^{n} \log\left(1 + \frac{\|z_i^v - W_v x_i\|^2}{c^2}\right)$$

We also went to regularized both W_v and x_i

$$\min_{x,W} \frac{1}{mn} \sum_{v=1}^{m} \sum_{i=1}^{n} \log\left(1 + \frac{\|z_i^v - W_v x_i\|^2}{c^2}\right) + C_1 \sum_{v=1}^{m} \|W_v\|_F^2 + C_2 \sum_{i=1}^{n} \|x_i\|_2^2$$

$$\min_{x,W} \frac{1}{mn}\sum_{v=1}^{m}\sum_{i=1}^{n}\log\left(1+\frac{\|z_i^v - W_v x_i\|^2}{c^2}\right) + C_1\sum_{v=1}^{m}\|W_v\|_F^2 + C_2\sum_{i=1}^{n}\|x_i\|_2^2$$

We also use alternate optimization strategy to opimize this problem:

- Fix W, solve x for the i-th data point:

$$\min_{x} \mathcal{J} = \frac{1}{m}\sum_{v=1}^{m}\log\left(1+\frac{\|z^v - W_v x\|^2}{c^2}\right) + C_2\|x\|_2^2.$$

$\Downarrow$ Setting the gradient of $\mathcal{J}$ with respect to x to 0,

$$\sum_{v=1}^{m} -\frac{2W_v^T(z^v - W_v x)}{c^2 + \|z^v - W_v x\|_2^2} + 2mC_2 x = 0$$

$\Downarrow$

$$\left(\sum_{v=1}^{m}\frac{W_v^T W_v}{c^2 + \|z^v - W_v x\|_2^2} + mC_2\right) x = \sum_{v=1}^{m}\frac{W_v^T z^v}{c^2 + \|z^v - W_v x\|_2^2}$$

$\Downarrow$

$$x = \left(\sum_{v=1}^{m}\frac{W_v^T W_v}{c^2 + \|z^v - W_v x\|_2^2} + mC_2\right)^{-1}\left(\sum_{v=1}^{m}\frac{W_v^T z^v}{c^2 + \|z^v - W_v x\|_2^2}\right)$$

$$r^v = z^v - W_v x$$

$$Q = \left[\frac{1}{c^2 + \|r^1\|^2}, \cdots, \frac{1}{c^2 + \|r^m\|^2}\right]$$

$\Downarrow$

$$x = \left(\sum_{v=1}^{m} W_v^T Q_v W_v + mC_2\right)^{-1}\sum_{v=1}^{m} W_v^T Q_v z^v$$

$\Downarrow$

for $k = 1, \cdots$ **do**

Weight function Q is chosen through Eq. (13)

$$Q = \left[\frac{1}{c^2 + \|r^1\|^2}, \cdots, \frac{1}{c^2 + \|r^m\|^2}\right], \qquad (13)$$

Using Eq. (14) to obtain the estimate x^k

$$x = \left(\sum_{v=1}^{m} W_v^T Q_v W_v + mC_2\right)^{-1}\sum_{v=1}^{m} W_v^T Q_v z^v. \qquad (14)$$

Update the residuals $\{r_v\}_{v=1}^{m}$

$$r^v = z^v - W_v x$$

end for

- Fix x, solve W

$$\min_{W} \mathcal{J} = \frac{1}{n}\sum_{i=1}^{n} \log\left(1 + \frac{\|z^i - Wx_i\|^2}{c^2}\right) + C_1\|W\|^2$$

Still, we set the gradient of J with regard to W to 0

$$\frac{1}{n}\sum_{i=1}^{n} \frac{2(z^i - Wx_i)x_i^T}{1 + \frac{\|z^i - Wx_i\|_2^2}{c^2}} + C_1 W = 0$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} \frac{2c^2(z^i - Wx_i)x_i^T}{c^2 + \|z^i - Wx_i\|^2} + C_1 W = 0$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} \frac{2c^2 z^i x_i^T}{c^2 + \|z^i - Wx_i\|_2^2} - \frac{1}{n}\sum_{i=1}^{n} \frac{2c^2 W x_i x_i^T}{c^2 + \|z^i - Wx_i\|_2^2} + C_1 W = 0$$

$$r^v = z^v - W_v x$$

$$Q = \left[\frac{1}{c^2 + \|r^1\|^2}, \cdots, \frac{1}{c^2 + \|r^m\|^2}\right]$$

$$W = \sum_{i=1}^{n} z_i Q_i x_i^T \left(\sum_{i=1}^{n} x_i Q_i x_i^T + nC_1\right)^{-1}$$

Hallas!

Inspirations:

1. Intact space learning for multiview learning provides a new multiview repsresentation method. It can be easily extened to supervised learning problems, but adding a hinge loss, or a multiview loss to the objective.

2. When we have a close form solution of x, but in the right hand, there is an inverse matrix x, we can play tricks and fix the x in inverse matrix.