Student Questions, Fall 2017
MATH 110, Reviews

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1. Bases. Dimensions

(1) Is every field that is a vector space dim 1 over itself?
(2) Implementing and proving the Replacement Theorem.
(3) How to check if a set spans?
(4) What are the fastest ways to extend/contract bases?
(5) Example of a change of bases.
(6) How does the notion of span extend to infinite bases? (Fourier basis, etc.)
(7) Fourier basis.
(8) What results do not hold in infinite-dimensional vector spaces? (E.g., orthogonality.)
(9) Prove that the kernel expands and the image shrinks.*

2. Matrices and Linear Transformations

(10) Matrix representation of a Linear transformation.
(11) Can we go over rotations?

3. Diagonalizability

(12) Diagonalizability criteria.
(13) I believe you said not to worry about whether $\text{char}_T(\lambda)$ splits$_{/\mathbb{R}}$ for some T because we can always make it split$_{/\mathbb{C}}$ and then go back to $\mathbb{R}$ at the end. Can we get some clarification?
(14) If the characteristic polynomial of some linear operator splits, then which of these are not possible sequences for $\dim \text{Ker}(T^i)$, $i = 1, 2, 3, 4$?
(a) 2, 3, 4, 5; (b) 3, 4, 4, 4; (c) 1, 3, 4, 4; (d) 1, 4, 4, 4. *

4. Jordan Canonical Form (JCF)

(15) More complex calculation of JCF:
- from last HW on JCF;
- including a dot diagram;
- not skipping finding eigenvalues, [generalized] eigenvectors and eigenspaces?
- Can you walk through the bullet proof method for finding JCF?
- Go over a specific example of JCF where bullet-proof algorithm is needed. *
- What do you mean by "going backwards" and how do we actually find these components?

(16) Theory of T-cyclic spaces? Summarize.
(17) Concept: JCF?**
(18) When can we put a matrix in JCF?
(19) PST: How to find a Jordan block?*

(20) Process to find Jordan Basis and JCF. (No Jordan "canonical" basis!)
(21) What does JCF apply to? Where is it useful?
(22) How to extrapolate T-cyclic subspaces for given transformations: similar to the one problem on Midterm 2? *
(23) Problem about T-invariant and T-cyclic.****
(24) A matrix example of JCF (bulletproof form) and applying the theory to computation.*
(25) When do we use bullet-proof JCF method (bottom up) vs. top down? How do we know? When to know we can use the bulletproof form and when to use shortcuts?*

5. Cayley Hamilton Theorem

(26) Why does "it" [matrix, transformation] satisfy its own equation [char. poly]?
(27) Relations between JCF III, invariant spaces, and CHT?
(28) Cayley-Hamilton Theorem.*

6. Orthogonality and Inner products

(29) How is the CBS inequality used to imply the Triangle inequality?
(30) Geometrically explain the (orthogonal?) complements in an inner product space.
(31) Why is $V = W \oplus W^{\perp}$, where W is a subspace of V?
(32) Projections and reflections as they relate to inner products.
(33) How do we prove that a space is an inner product space?
(34) The Gram-Schmidt process for complex vectors.
(35) Anything about inner products.
(36) Explain more the concepts of orthogonal bases and orthonormal complements.
(37) Show how to find an orthonormal basis.
(38) Why does having the same angle between basis vectors make a positive definite matrix? How does that connect to inner product spaces?*

7. Normal and Self-Adjoint Operators

(39) Does $[T^*]_\beta = [T]_\beta^*$ for β **not** orthonormal?
(40) Let $T : V \to V_{\langle\, , \,\rangle}/_{\mathbb{C}}$ be a unitary operator. $[T]_\beta$, where β is **not** orthogonal/orthonormal basis, is not necessarily an unitary matrix. Does $[T]_\beta$ still preserve length?
(41) What are the sufficient and necessary conditions for a normal or self-adjoint matrix ... (?) to be diagonalizable in an orthonormal basis?
(42) Why are symmetric/hermitian matrices diagonalizable in an orthonormal basis?
(43) Is every square matrix$_{/\mathbb{C}}$ normal?
(44) Why does a normal matrix imply diagonalizability over $\mathbb{C}$ but not over $\mathbb{R}$?
(45) How to visualize T^*?
(46) Geometric meaning of a normal transformation.
(47) How to use special (i.e., Hermitian) matrices?
(48) Do symmetric matrices$_{/\mathbb{C}}$ (not hermitian) have special properties? E.g., $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$.

(49) Does every linear transformation have a unique adjoint? (in finite-dim vs. in ∞-dim)
(50) Is the metric of a self-adjoint operator symmetrical (Hermitian) in any basis? Only orthogonal basis? Some orthogonal basis?
(51) What is the relationship between self-adjoint and complex numbers?
(52) If T is a self-adjoint operator and we apply Gram-Schmidt to a basis consisting of eigenvectors [for T], do we still have a basis of eigenvectors [for T]?
(53) How do we prove something is self-adjoint?
(54) What are the algorithms to show two matrices are unitarily equivalent?
(55) Diagonalizability of self-adjoint operator.*
(56) How to prove that normal operators are orthogonally diagonalizable?

8. Quadratic Forms

(57) How to prove that $A \in M_{n\times n}(\mathbb{R})$ is positive-definite iff each principal submatrix $A^{(\mathrm{k})}$ has a positive determinant?
(58) Quadratic forms.