Real Analysis: Exercise 1

Due date: 2025-11-04

Question 1. Let $A = \{1, 2, apple\}$, $B = \{2, apple, 1, apple\}$, $C = \{2, 1\}$, $D = \{1, 2, pear\}$.

1. Is $1 \in A$?
2. Is $10 \in A$?
3. Is $A = B$?
4. Is $A = C$?
5. Is $C \subseteq A$?
6. Is $C \subset A$?
7. Is $A \subseteq C$?
8. Is $D \subseteq B$?

Question 2. In this question, the universal set is

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, fish, fowl\}.$$

Let $A = \{1, 2, 3\}$, $B = \{2, 3, fish\}$ and $C = \{2, fowl, 7, 8\}$. Work out each of the following sets using only the definition of union, intersection and complement.

1. $A \cap (B \cup C)$.
2. $(A \cap B) \cup (A \cap C)$.
3. $A \cup (B \cap C)$.
4. $(A \cup B) \cap (A \cup C)$.
5. C^c.
6. $(C^c)^c$.
7. $(A \cap C)^c$.
8. $A^c \cup C^c$.
9. $(A \cup B)^c$.
10. $A^c \cap B^c$.

Question 3. Express each of the following sets as a single interval.

1. $(1, 3) \cup (2, 15)$.
2. $[1, 8] \cap [4, 16]$.
3. $[66, 76] - [72, 100]$.
4. $[0, \infty) \cup (-10, 10)$.

5. $[27, 29] - (26, 28)$

Question 4. If X, Y are sets containing m, n elements respectively, how many elements are there in the Cartesian product $X \times Y$?

Question 5.
1. Find, if possible, sets A and B, each with infinitely many elements and such that $A \cap B = \{0\}$ and $A \cup B = \mathbb{Z}$.
2. Find, if possible, sets C and D such that $C \cup D = \{b, i, g\}$ and $C \cap D = \{s, m, a, l, l\}$.

Question 6. Find examples of two functions f, $g : \mathbb{R} \to \mathbb{R}$ such that $f \circ g$ and $g \circ f$ are not equal. Justify your answer.

Question 7. Let $f : X \to Y$ and $g : Y \to X$ be two functions such that

$$g \circ f(x) = x, \quad \forall\, x \in X.$$

Show that f is injective and g is surjective.

Question 8. Let $f : X \to Y$ and $g : Y \to Z$. Is it possible for $g \circ f$ to be a bijection if neither f or g are bijections? If not prove it, and if so give an example to justify it.