Introduction to interest rate models

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https://github.com/styluck/mat_fin

Outline

固定收益

- Fixed-Income Securities 证券

- Fixed-Income Markets

- Fixed-Income Valuation

fixed-income security

- Fixed-income security: promises to make a series of interest payments in fixed amounts and to repay the principal amount at maturity
- Bonds are rated based on their relative probability of default (failure to make promised payments).

违约

债券

fixed-income security

特征

- The features of a fixed-income security include specification of:
- The issuer of the bond. 发行方
- The maturity date of the bond. 到期日
- The par value (principal value to be repaid). 平价，证券的面值
- Coupon rate and frequency. 票息率
- Currency in which payments will be made

货币

Issuers of Bonds

公司 企业

- Corporations: corporate bonds 公司债

  国库
- Sovereign national governments: U.S. Treasury bonds 美国国债
- Non-sovereign governments: state bonds 地方债

  拟政府实体
- Quasi-government entities: Federal National Mortgage 中国：铁道部债券 Association (Fannie Mae) 房利美
- Supranational entities: World Bank, the European Investment Bank, and the International Monetary Fund (IMF)
- Special purpose entities: asset-backed securities

  资产抵押证券

Bond Maturity

到期日 本金

- The **maturity date** of a bond is **the date on which the principal is to be repaid**. Once a bond has been issued, the time remaining until maturity is referred to as the **term to maturity** or **tenor** of a bond.
- Bonds that have no maturity date are called **perpetual bonds**
- Bonds with original maturities of one year or less are referred to as **money market securities**. 货币市场：商业票据，银行间拆借
- Bonds with original maturities of more than one year are referred to as **capital market securities** 资本市场

Par Value 平价，面值

本金

- The **par value** of a bond is the **principal amount** that will be repaid at maturity.
- 溢价 **Premium**: a bond that is selling for more than its par value
- 折价 **Discount**: a bond that is selling at less than its par value
- **At par**: a bond that is selling for exactly its par value
- **zero-coupon bonds** or **pure discount bonds**: a bond pays no interest prior to maturity

-950 1000

PV FV

抵押物 信用增益

Collateral and credit enhancements

- **Unsecured bonds**: a claim to the overall assets and cash flows of the issuer. 信用债
- **Secured bonds**: backed by a claim to specific assets of a corporation 利率债

  collateral
- **Mortgage-backed security (MBS)**: The underlying assets are a pool of mortgages. The most common type of securitized bond

  按揭贷款

子弹

Bullet structure 结构

- A typical bond has a **bullet structure** 先息后本
- Consider a $1,000 face value 5-year bond with an annual coupon rate of 5%. With **a bullet structure**, the bond's promised payments at the end of each year would be as follows.

Year	1	2	3	4	5
PMT	$50	$50	$50	$50	$1,050
Principal remaining	$1,000	$1,000	$1,000	$1,000	$0

Fully amortizing structure 等额本息

- Fully amortizing structure: the principal is fully paid off when the last periodic payment is made.
- Consider a $1,000 face value 5-year bond with an annual coupon rate of 5%. With a Fully amortizing, the bond's promised payments at the end of each year would be as follows.

Year	1	2	3	4	5
PMT	$230.97	$230.97	$230.97	$230.97	$230.98
Principal remaining	$819.03	$629.01	$429.49	$219.99	$0

浮动利率债券

Floating-rate bonds and LIBOR

- Some bonds pay periodic interest that depends on a current market rate of interest. These bonds are called floating-rate notes (FRN) or floaters.
- The most widely used reference rate for floating-rate bonds was the **London Interbank Offered Rate (LIBOR)** HIBOR SHIBOR Hong Kong Shanghai
- LIBOR is not based on actual transactions, and has been subject to manipulation by bankers reporting their expected interbank lending rates, has led to an effort to replace LIBOR with market-determined rates.
- As an example, consider a floating-rate note that pays the London Interbank Offered Rate (LIBOR) plus a margin of 0.75% (75 basis points) annually. If 1-year LIBOR is 2.3% at the beginning of the year, the bond will pay 2.3% + 0.75% = 3.05% of its par value at the end of the year. The new 1-year rate at that time will determine the rate of interest paid at the end of the next year. Most floaters pay quarterly and are based on a quarterly (90-day) reference rate.

2.3% + 0.75% = 3.05%

Reset date

- The values of floating rate notes (FRNs) are more stable than those of fixed-rate debt of similar maturity
- because the coupon interest rates are reset periodically based on a reference rate
- Recall that the coupon rate on a floating-rate note is the reference rate plus or minus a margin
- The coupon rate for the next period is set using the market reference rate (MRR) for the reset period

EXAMPLE: Valuation of a floating-rate note

- A $100,000 floating rate note is based on a 180-day MRR with a quoted margin of 120 basis points. On a reset date with 5 years remaining to maturity, the 180-day MRR is quoted as 3.0% (annualized) and the required rate of return (based on the issuer's current credit rating) is 4.5% (annualized). What is the market value of the floating rate note?

semi-annual

$$PV = \text{Coupon} \times \frac{1-(1+r)^{-n}}{r} + \frac{\text{Face Value}}{(1+r)^n}$$

$$\text{Coupon} = \frac{1}{2}(3.0\% + 1.2\%) = 2.1\%$$

$$n = 2 \times 5 = 10$$

$$r = \frac{1}{2} \cdot 4.5\% = 2.25\%$$

$$FV = 100{,}000$$

EXAMPLE: Valuation of a floating-rate note

- The current annualized coupon rate on the note is 3.0% + 1.2% = 4.2%, so the next semiannual coupon payment will be 4.2% / 2 = 2.1% of face value. The required return in the market (discount margin) as an effective 180-day discount rate is 4.5% / 2 = 2.25%.
- Using a face value of 100%, 10 coupon payments of 2.1%, and a discount rate per period of 2.25%, we can calculate the present value of the floating rate note as:
- $PV_{coupons} = C \times \frac{1-(1+r)^{-n}}{r}$=18.6201
- $PV_{face} = 100 \times (1 + 0.0225)^{-10} = 80.05$
- $PV = PV_{coupons} + PV_{face} = 98.6701$ × 1,000

到期收益率

Yield-to-maturity (YTM)

- The market discount rate appropriate for discounting a bond's cash flows is called the bond's **yield-to-maturity (YTM)**
- If we know a bond's yield-to-maturity, we can calculate its value, and if we know its value (market price), we can calculate its yield-to-maturity.

Yield-to-maturity (YTM)

- Consider a newly issued 10-year, $1,000 par value, 10% coupon, annual-pay bond. The coupon payments will be $100 at the end of each year the $1,000 par value will be paid at the end of year 10. First, let's value this bond assuming the appropriate discount rate is 10%. The present value of the bond's cash flows discounted at 10% is:

$$\frac{100}{1.1}+\frac{100}{1.1^2}+\frac{100}{1.1^3}+\ldots\ldots+\frac{100}{1.1^9}+\frac{1{,}100}{1.1^{10}} = 1{,}000$$

if coupon rate = discount rate

then Face Value = Present Value

Yield-to-maturity (YTM)

- Now let's value that same bond with a discount rate of 8%:

$$\frac{100}{1.08}+\frac{100}{1.08^2}+\frac{100}{1.08^3}+\ldots\ldots+\frac{100}{1.08^9}+\frac{1{,}100}{1.08^{10}}=1{,}134.20$$

- If the market discount rate for this bond were 8%, it would sell at a premium of $134.20 above its par value.
- **When bond yields decrease, the present value of a bond's payments, its market value, increases.**

Yield-to-maturity (YTM)

- If we discount the bond's cash flows at 12%, the present value of the bond is:

$$\frac{100}{1.12} + \frac{100}{1.12^2} + \frac{100}{1.12^3} + \ldots\ldots + \frac{100}{1.12^9} + \frac{1{,}100}{1.12^{10}} = 887.00$$

- If the market discount rate for this bond were 12%, it would sell at a discount of $113 to its par value.
- **When bond yields increase, the present value of a bond's payments, its market value, decreases.**

Yield-to-maturity (YTM)

- Calculating the value of a bond with semiannual coupon payments.
- Rather than \$100 per year, the security will pay \$50 every six months.
- With an annual YTM of 8%, we need to discount the coupon payments at 4% per period

$$\frac{50}{1.04} + \frac{50}{1.04^2} + \frac{50}{1.04^3} + \ldots\ldots + \frac{50}{1.04^{19}} + \frac{1{,}050}{1.04^{20}} = 1{,}135.90$$

The relationships between price and yield

- 1. At a point in time, a decrease (increase) in a bond's YTM will increase (decrease) its price.
- 2. If a bond's coupon rate is greater than its YTM, its price will be at a premium to par value. If a bond's coupon rate is less than its YTM, its price will be at a discount to par value.
- 3. The percentage decrease in value when the YTM increases by a given amount is smaller than the increase in value when the YTM decreases by the same amount (the price-yield relationship is convex).

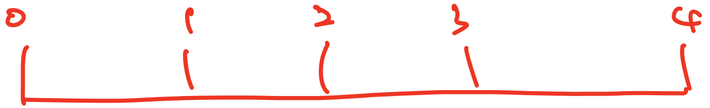
The relationships between price and yield

① 票息低

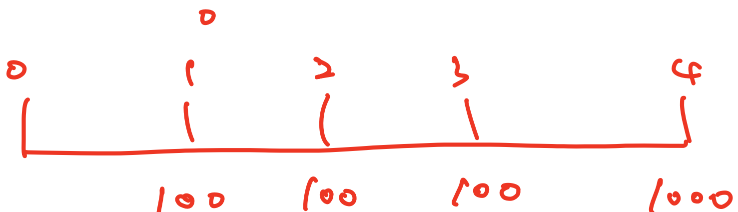
② 存续期长

对利率更敏感

- 4. Other things equal, the price of a bond with a lower coupon rate is more sensitive to a change in yield than is the price of a bond with a higher coupon rate.
- 5. Other things equal, the price of a bond with a longer maturity is more sensitive to a change in yield than is the price of a bond with a shorter maturity



$$PV = \frac{1000}{(1+r)^4}$$

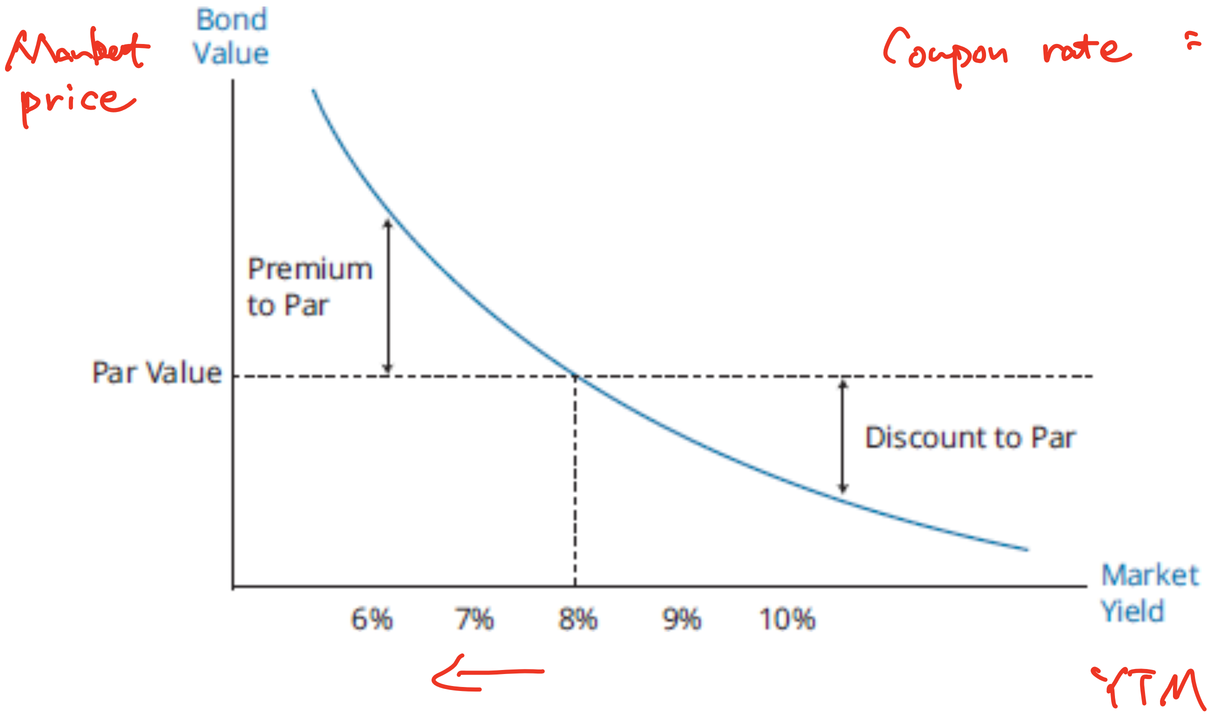


$$PV = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3} + \frac{1100}{(1+r)^4}$$

The relationships between price and yield

- Market Yield vs. Bond Value for an 8% Coupon Bond

Coupon rate = 8%



Relationship Between Price and Maturity

- Prior to maturity, a bond can be selling at a significant discount or premium to par value.
- Regardless of its required yield, the price will converge to par value as maturity approaches.

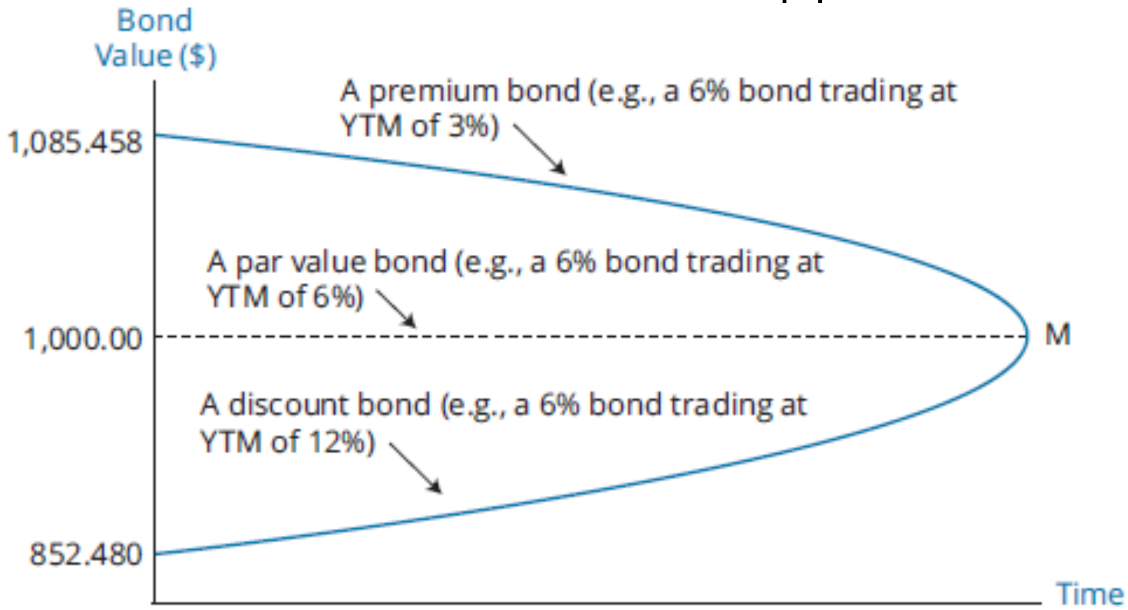
Relationship Between Price and Maturity

- Consider a bond with $1,000 par value and a 3-year life paying 6% semiannual coupons. The bond values corresponding to required yields of 3%, 6%, and 12% as the bond approaches maturity are

Time to Maturity (in years)	YTM = 3%	YTM = 6%	YTM = 12%
3.0	$1,085.46	$1,000.00	$852.48
2.5	1,071.74	1,000.00	873.63
2.0	1,057.82	1,000.00	896.05
1.5	1,043.68	1,000.00	919.81
1.0	1,029.34	1,000.00	945.00
0.5	1,014.78	1,000.00	971.69
0.0	1,000.00	1,000.00	1,000.00

Relationship Between Price and Maturity

- Consider a bond with $1,000 par value and a 3-year life paying 6% semiannual coupons. The bond values corresponding to required yields of 3%, 6%, and 12% as the bond approaches maturity are



The full price

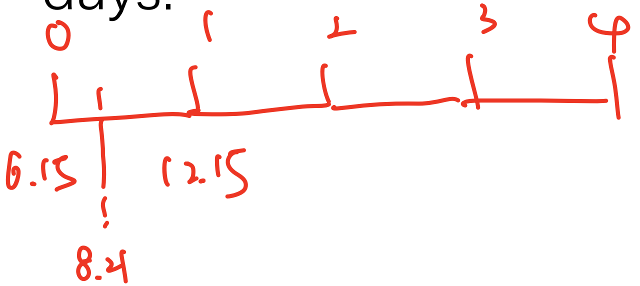
结算日

- For most bond trades, the settlement date, which is when cash is exchanged for the bond, will fall between coupon payment dates.
- The value of a bond between coupon dates can be calculated, using its current YTM, as the value of the bond on its last coupon date (PV) times (1 + YTM / # of coupon periods per year)t/T,
  - where
  - t is the number of days since the last coupon payment, and
  - T is the number of days in the coupon period.
- this value is referred to as the **full price** of the bond.

flat price $\leq$ full price

EXAMPLE: Calculating the full price of a bond

- A 5% bond makes coupon payments on June 15 and December 15 and is trading with a YTM of 4%. The bond is purchased and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price of the bond using actual days.



$$PV = \left[\frac{2.5\%}{1+2\%} + \frac{2.5\%}{(1+2\%)^2} + \frac{2.5\%}{(1+2\%)^3} + \frac{102.5\%}{(1+2\%)^4}\right] \times 1000$$

$$= 1019.04$$

$$1019.04 \times 1.02^{(67/183)} = 1026.46$$

EXAMPLE: Calculating the full price of a bond

- A 5% bond makes coupon payments on June 15 and December 15 and is trading with a YTM of 4%. The bond is purchased and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price of the bond using actual days.
- Step 1: Calculate the value of the bond on the last coupon date (coupons are semiannual, so we use 4 / 2 = 2% for the periodic discount rate):
- The formula for the present value of a bond is

$$P = C \times \frac{1-(1+r)^{-n}}{r} + \frac{F}{(1+r)^n} = 2.5 \times \frac{1-(1+0.02)^{-4}}{0.02} + \frac{100}{(1+0.02)^4}$$

EXAMPLE: Calculating the full price of a bond

- A 5% bond makes coupon payments on June 15 and December 15 and is trading with a YTM of 4%. The bond is purchased and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price of the bond using actual days.
- Step 2: Adjust for the number of days since the last coupon payment:
- Days between June 15 and December 15 = 183 days.
- Days between June 15 and settlement on August 21 = 67 days.
- Full price = $1{,}019.04 \times (1.02)^{67/183} = 1{,}026.46$.

Full price

The flat price

- The accrued interest since the last payment date can be calculated as the coupon payment times the portion of the coupon period that has passed between the last coupon payment date and the settlement date of the transaction.
- For the bond in the previous example, the accrued interest on the settlement date of August 21 is:

  应计利息

    - \$25 (67 / 183) = \$9.15
- The **full price** minus the accrued interest is referred to as the **flat price** of the bond
    - flat price = full price – accrued interest

The flat price

- So for the bond in our example,
  - the flat price = 1,026.46 – 9.15 = 1,017.31.

- So far, in calculating accrued interest, we used the actual number of days between coupon payments and the actual number of days between the last coupon date and the settlement date. This **actual/actual method** is **used most often with government bonds**.
- The **30/360 method** is **most often used for corporate bonds**. This method assumes that there are 30 days in each month and 360 days in a year.

EXAMPLE: Accrued interest

- An investor buys a $1,000 par value, 4% annual-pay bond that pays its coupons on May 15. The investor's buy order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.

$4\% \times \$1000 = \$40$

Actual/Actual Method

$16+30+31+10 = 87$

$\frac{87}{365} \times \$40 = \$9.53$

30/360

$15+30+30+10 = 85$

$\frac{85}{360} \times \$40 = \$9.44$

EXAMPLE: Accrued interest

- An investor buys a $1,000 par value, 4% annual-pay bond that pays its coupons on May 15. The investor's buy order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.
- The annual coupon payment is 4% × $1,000 = $40.
- Using the 30/360 method, interest is accrued for 30 – 15 = 15 days in May; 30 days each in June and July; and 10 days in August, or 15 + 30 + 30 + 10 = 85 days.

$$\text{accrued interest (30/360 method)} = \frac{85}{360} \times \$40 = \$9.44$$

EXAMPLE: Accrued interest

- An investor buys a $1,000 par value, 4% annual-pay bond that pays its coupons on May 15. The investor's buy order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.
- Using the actual/actual method, interest is accrued for 31 – 15 = 16 days in May; 30 days in June; 31 days in July; and 10 days in August, or 16 + 30 + 31 + 10 = 87 days.

$$\text{accrued interest (actual/actual method)} = \frac{87}{365} \times \$40 = \$9.53$$

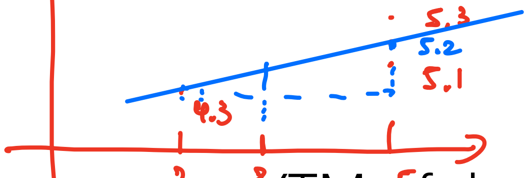
Matrix pricing

- Matrix pricing is a method of estimating the required yield-to-maturity (or price) of bonds that are currently not traded or infrequently traded.
- The procedure is to use the YTMs of traded bonds that have credit quality very close to that of a nontraded or infrequently traded bond and are similar in maturity and coupon, to estimate the required YTM.

EXAMPLE: Pricing an illiquid bond

- You are estimating the value of a nontraded 4% annual-pay, A+ rated bond that has three years remaining until maturity. You have obtained the following yields-to-maturity on similar corporate bonds:
  - A+ rated, 2-year annual-pay, YTM = 4.3%
  - A+ rated, 5-year annual-pay, YTM = 5.1%
  - A+ rated, 5-year annual-pay, YTM = 5.3%
- Estimate the value of the nontraded bond.

EXAMPLE: Pricing an illiquid bond



$$\frac{(5.2-4.3)}{5-2} = \frac{x-4.3}{3-2}$$

$\Rightarrow x = 4.6\%$

线性插值法

simple *linear interpolation*.

- Answer:
- Step 1: Take the average YTM of the 5-year bonds: (5.1 + 5.3) / 2 = 5.2%.
- Step 2: Interpolate the 3-year YTM based on the 2-year and average 5-year YTMs:

4.3% + (5.2% – 4.3%) × [(3 years – 2 years) / (5 years – 2 years)]
= 4.6%

- Step 3: Price the nontraded bond with a YTM of 4.6%:
- N = 3; PMT = 40; FV = 1,000; I/Y = 4.6; CPT → PV = –983.54
- The estimated value is $983.54 per $1,000 par value.

$$PV = \text{Coupon} \times \frac{1-(1+r)^{-n}}{r} + \frac{F}{(1+r)^n} = 983.54$$

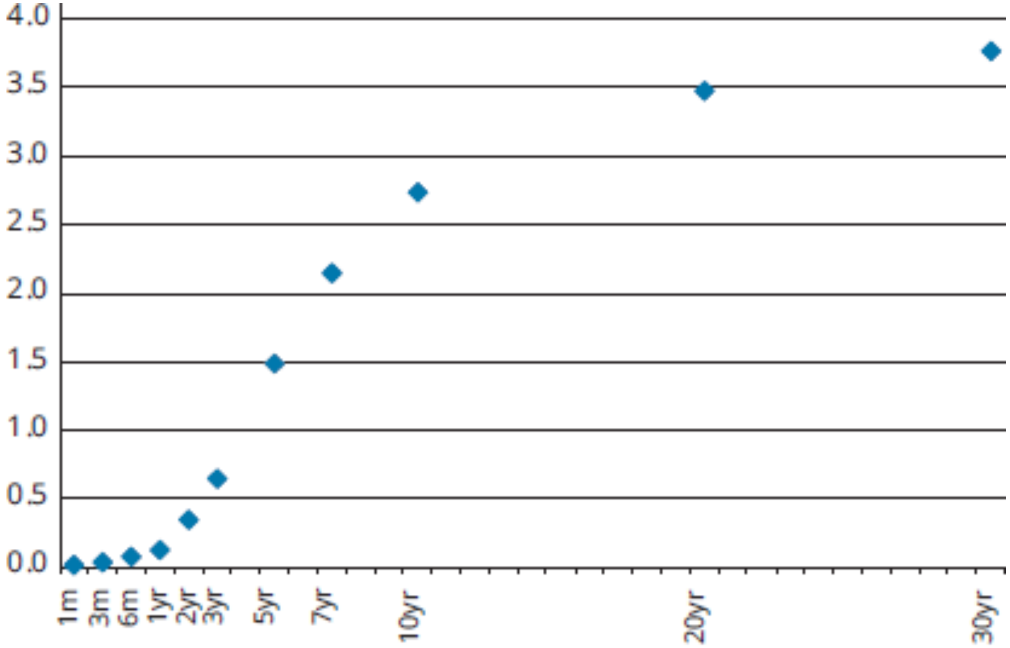
Yield curve

- A **yield curve** shows yields by maturity. Yield curves are constructed for yields of various types
- it's very important to understand exactly which yield is being shown.
- **The term structure** of interest rates refers to the yields at different maturities (terms) for securities or interest rates.

期限结构

Yield curve

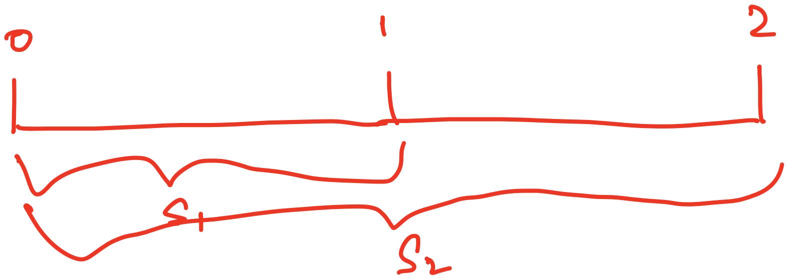
收益率曲线



1 month	0.02	5 year	1.49
3 month	0.04	7 year	2.15
6 month	0.08	10 year	2.74
1 year	0.13	20 year	3.48
2 year	0.35	30 year	3.77
3 year	0.65		

Source: www.treasury.gov/resource-center

Spot rates



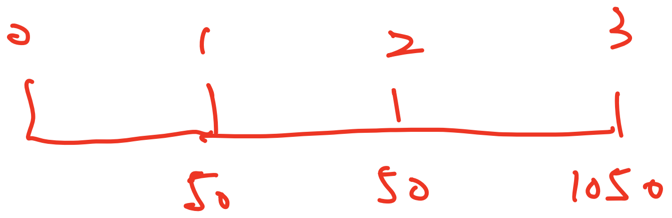
即期利率

- **Spot rates** are the market discount rates for **a single payment** to be received in the future.
- The discount rates for zero-coupon bonds are spot rates and we sometimes refer to spot rates as **zero-coupon rates** or simply **zero rates**.

$$\frac{CPN_1}{1+S_1}+\frac{CPN_2}{(1+S_2)^2}+\ldots\ldots+\frac{CPN_N+FV_N}{(1+S_N)^N}=PV$$

EXAMPLE: Valuing a bond using spot rates

- Given the following spot rates, calculate the value of a 3-year, 5% annual-coupon bond.
    - Spot rates
    - 1-year: 3%
    - 2-year: 4%
    - 3-year: 5%



$$\frac{50}{(1+3\%)} + \frac{50}{(1+4\%)^2} + \frac{1050}{(1+5\%)^3} = 1001.8$$

EXAMPLE: Valuing a bond using spot rates

- Given the following spot rates, calculate the value of a 3-year, 5% annual-coupon bond.
- Spot rates:
  - 1-year: 3%
  - 2-year: 4%
  - 3-year: 5%

$$\frac{50}{1.03} + \frac{50}{(1.04)^2} + \frac{1{,}050}{(1.05)^3} = 48.54 + 46.23 + 907.03 = \$1{,}001.80$$

EXAMPLE: Valuing a bond using spot rates

- This price, calculated using spot rates, is sometimes called the no-arbitrage price of a bond because if a bond is priced differently there will be a profit opportunity from arbitrage among bonds.
- Because the bond value is slightly greater than its par value, we know its YTM is slightly less than its coupon rate of 5%. Using the price of 1,001.80, we can calculate the YTM for this bond as:

3-month : 0.5%

$$-1001.80 = 50 \times \frac{1-(1+r)^{-3}}{r} + \frac{1000}{(1+r)^3}$$

5-year : 5%

$r = 4.93\%$

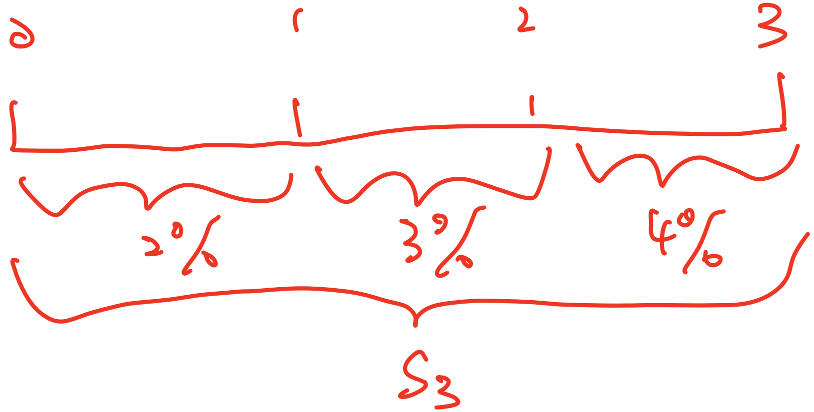
Forward rates

远期利率

- **Forward rates** are yields for future periods. The rate of interest on a 1-year loan that would be made two years from now is a forward rate.
- A forward yield curve shows the **future** rates for bonds or money market securities for the same maturities for annual periods in the future.
- Typically, the forward curve would show the yields of 1-year securities for each future year, quoted on a semiannual bond basis.

EXAMPLE: Computing spot rates from forward rates

- If the current 1-year spot rate is 2%, the 1-year forward rate one year from today (1y1y) is 3%, and the 1-year forward rate two years from today (2y1y) is 4%, what is the 3-year spot rate?



$$(1+S_3)^3 = (1+S_1)\cdot(1+1y1y)\cdot(1+2y1y)$$

$$S_3 = (1.02 \times 1.03 \times 1.04)^{\frac{1}{3}} - 1$$

$$= 2.997\%$$

EXAMPLE: Computing spot rates from forward rates

- If the current 1-year spot rate is 2%, the 1-year forward rate one year from today (1y1y) is 3%, and the 1-year forward rate two years from today (2y1y) is 4%, what is the 3-year spot rate?
- Answer:
- This relation is illustrated as $(1 + S_3)^3 = (1 + S1)(1 + 1y1y)(1 + 2y1y)$. Thus, $S_3 = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)]^{1/3} - 1$, which is the geometric mean return we covered in Quantitative Methods
  - $S_3 = [(1.02)(1.03)(1.04)]^{1/3} - 1 = 2.997\%$

EXAMPLE: Computing a forward rate from spot rates

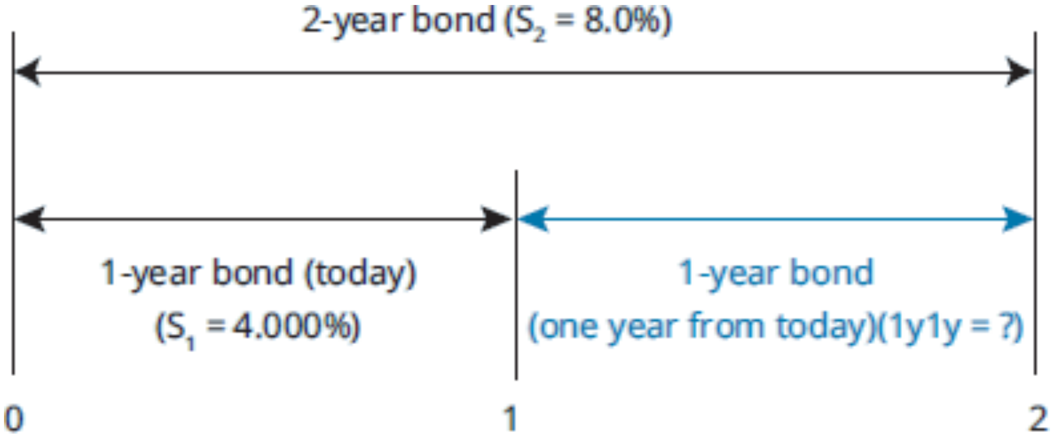
- The 2-period spot rate, S_2, is 8%, and the 1-period spot rate, S_1, is 4%. Calculate the forward rate for one period, one period from now, 1y1y.

$$(1+8\%)^2 = (1+4\%)(1+1y1y)$$

$$1y1y = 12.154\%$$

EXAMPLE: Computing a forward rate from spot rates

- The 2-period spot rate, S_2, is 8%, and the 1-period spot rate, S_1, is 4%. Calculate the forward rate for one period, one period from now, 1y1y.
- Answer:



- $(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, we can get

EXAMPLE: Computing a forward rate from spot rates

- The 2-period spot rate, S_2, is 8%, and the 1-period spot rate, S_1, is 4%. Calculate the forward rate for one period, one period from now, 1y1y.
- Answer: $$\frac{(1+S_2)^2}{(1+S_1)} = (1 + 1y1y)$$
- Or, because we know that both choices have the same payoff in two years: $$(1.08)^2 = (1.04)(1 + 1y1y)$$

$$(1 + 1y1y) = \frac{(1.08)^2}{(1.04)}$$

$$1y1y = \frac{(1.08)^2}{(1.04)} - 1 = \frac{1.1664}{1.04} - 1 = 12.154\%$$

Yield spread

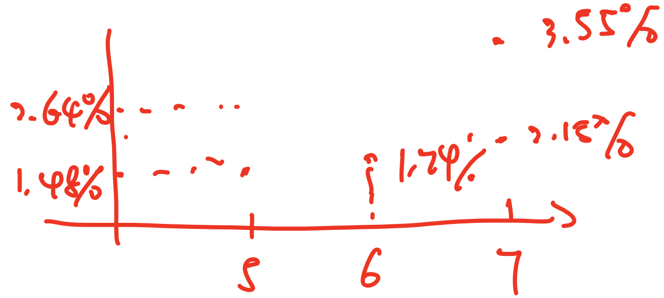
YTM

- A yield spread is the difference between the yields of two different bonds. Yield spreads are typically quoted in basis points.
- A yield spread relative to a benchmark bond is known as a benchmark spread. For example, if a 5-year corporate bond has a yield of 6.25% and its benchmark, the 5-year Treasury note, has a yield of 3.50%, the corporate bond has a benchmark spread of 625 – 350 = 275 basis points.

625 - 350 = 275

EXAMPLE: Estimating the spread for a new 6-year, A rated bond issue

Interpolation



- Consider the following market yields:
  - 5-year, U.S. Treasury bond, YTM 1.48%
  - 5-year, A rated corporate bond, YTM 2.64%
  - 7-year, U.S. Treasury bond, YTM 2.15%
  - 7-year, A rated corporate bond, YTM 3.55%
  - 6-year U.S. Treasury bond, YTM 1.74%
- Estimate the required yield on a newly issued 6-year, A rated corporate bond.

5-year spread: $2.64\% - 1.48\% = 1.16\%$

7-year spread: $3.55\% - 2.15\% = 1.40\%$

6-year spread: $\frac{1.40\% + 1.16\%}{2} = 1.28\%$

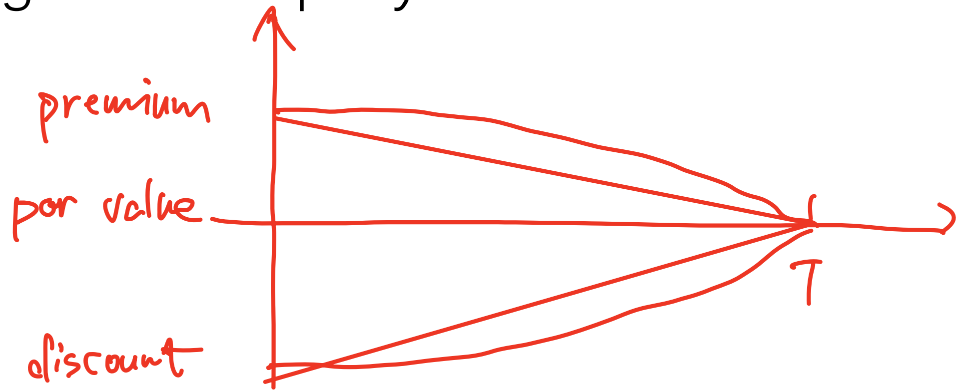
6-year A rated corporate bond : 1.74% + 1.28% = 3.02%

EXAMPLE: Estimating the spread for a new 6-year, A rated bond issue

- Answer:
- 1. Calculate the spreads to the benchmark (Treasury) yields.
  - Spread on the 5-year corporate bond is 2.64 – 1.48 = 1.16%.
  - Spread on the 7-year corporate bond is 3.55 – 2.15 = 1.40%.
- 2. Calculate the average spread because the 6-year bond is the midpoint of five and seven years:
  - average spread = (1.16 + 1.40) / 2 = 1.28%
- 3. Add the average spread to the YTM of the 6-year Treasury (benchmark) bond.
  - 1.74 + 1.28 = 3.02%, which is our estimate of the YTM on the newly issued 6-year,

Simple yield 简单收益率

- A bond's **simple yield** takes a discount or premium into account by assuming that any discount or premium declines evenly over the remaining years to maturity.
- The sum of the annual coupon payment plus (minus) the straight-line amortization of a discount (premium) is divided by the flat price to get the simple yield.



EXAMPLE: Computing simple yield

- A 3-year, 8% coupon, semiannual-pay bond is priced at 90.165. Calculate the simple yield

discount: $100 - 90.165 = 9.835$

straight-line amortization $\frac{9.835}{3} = 3.278$

$$\frac{8 + 3.278}{90.165} = 12.51\%$$

精确收益率: 12.00%

EXAMPLE: Computing simple yield

- A 3-year, 8% coupon, semiannual-pay bond is priced at 90.165. Calculate the simple yield
- **Answer:**
  - The discount from par value is 100 – 90.165 = 9.835. Annual straight-line amortization of the discount is 9.835 / 3 = 3.278.

$$\text{simple yield} = \frac{8 + 3.278}{90.165} = 12.51\%$$

Contingency provision

或有条款　　合同

- A **contingency provision** in a contract describes an action that may be taken if an event (the contingency) actually occurs.
- **Contingency provisions in bond** indentures are referred to as **embedded options**, embedded in the sense that they are an integral part of the bond contract and are not a separate security.

嵌入期权

Call option

买入期权 / 看涨期权

- A **call option** gives the issuer the right to redeem all or part of a bond issue at a specific price (call price) if they choose to.
- Example: consider a 6% 20-year bond issued at par on June 1, 2012, for which the indenture includes the following call schedule:
  - The bonds can be redeemed by the issuer at 102% of par after June 1, 2017.
  - The bonds can be redeemed by the issuer at 101% of par after June 1, 2020.
  - The bonds can be redeemed by the issuer at 100% of par after June 1, 2022

可回购债券

- **Callable bond**: a bond with embedded call options
- For a bond that is currently callable, the call price puts an **upper limit** on the value of the bond in the market.

Callable bond

- A call option has value to the issuer because it gives the issuer the **right to redeem** the bond and **issue a new bond** (borrow) if the market yield on the bond declines.
- This could occur either because **interest rates** in general have decreased or because the credit **quality of the bond** has increased (default risk has decreased).

Putable Bonds

callable bond → issuer has right to buy

Putable bond → bondholder has right to sell

卖出/看跌期权

- A put option gives the **bondholder** the **right to sell** the bond back to the issuing company at a prespecified price, typically par.
- Bondholders are likely to exercise such a put option when the fair value of the bond is less than the put price because **interest rates** have risen or the **credit quality of the issuer** has fallen.
- Unlike a call option, a put option has value to the **bondholder** because the **choice of whether to exercise** the option is the bondholder's

Yield-to-call and yield-to-worst

- For a callable bond, an investor's yield will depend on whether and when the bond is called. The **yield-to-call** can be calculated for each possible call date and price.
- The lowest of yield-to-maturity and the various yields-to-call is termed the **yield-to-worst**.

EXAMPLE: Yield-to-call and yield-to-worst

- Consider a 10-year, semiannual-pay 6% bond trading at 102 on January 1, 2014. The bond is callable according to the following schedule:
  - Callable at 102 on or after January 1, 2019.
  - Callable at 100 on or after January 1, 2022.
- Calculate the bond's YTM, yield-to-first call, yield-to-first par call, and yield-to-worst.

EXAMPLE: Yield-to-call and yield-to-worst

- Answer:
- The yield-to-maturity on the bond is calculated as:
- N = 20; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.867%
  2 × 2.867 = 5.734% = YTM
- To calculate the yield-to-first call, we calculate the yield-to-maturity using the number of semiannual periods until the first call date (10) for N and the call price (1,020) for FV:
- N = 10; PMT = 30; FV = 1,020; PV = –1,020; CPT → I/Y = 2.941%
  2 × 2.941 = 5.882% = yield-to-first call

EXAMPLE: Yield-to-call and yield-to-worst

- To calculate the yield-to-first par call (second call date), we calculate the yield-to-maturity using the number of semiannual periods until the first par call date (16) for N and the call price (1,000) for FV:
- N = 16; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.843%
  2 × 2.843 = 5.686% = yield-to-first par call
- The lowest yield, 5.686%, is realized if the bond is called at par on January 1, 2022, so the yield-to-worst is 5.686%.

YTM : 5.734% Yield to ... 5.686%

Yield-to-first call : 5.883%

REVISION

- a. describe basic features of a fixed-income security
- **Basic features of a fixed income security include the issuer, maturity date, par value, coupon rate, coupon frequency, and currency.**

REVISION

- b. describe how cash flows of fixed-income securities are structured
- **A bond with a bullet structure pays coupon interest periodically and repays the entire principal value at maturity.**
- **A bond with an amortizing structure repays part of its principal at each payment date. A fully amortizing structure makes equal payments throughout the bond's life.**

REVISION

- c. describe the use of interbank offered rates as reference rates in floating-rate debt.
- **Floating-rate notes have coupon rates that adjust based on a reference rate such as LIBOR.**

REVISION

- d. calculate a bond's price given a market discount rate
- **The price of a bond is the present value of its future cash flows, discounted at the bond's yield-to-maturity.**
- **For an annual-coupon bond with N years to maturity:**

$$\text{price} = \frac{\text{coupon}}{(1+\text{YTM})} + \frac{\text{coupon}}{(1+\text{YTM})^2} + \ldots + \frac{\text{coupon} + \text{principal}}{(1+\text{YTM})^N}$$

- **For a semiannual-coupon bond with N years to maturity:**

$$\text{price} = \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)^2} + \ldots + \frac{\text{coupon} + \text{principal}}{\left(1+\frac{\text{YTM}}{2}\right)^{N\times 2}}$$

REVISION

- e. identify the relationships among a bond’s price, coupon rate, maturity, and market discount rate (yield-to-maturity)
- **A bond’s price and YTM are inversely related. An increase in YTM decreases the price and a decrease in YTM increases the price.**
- **A bond will be priced at a discount to par value if its coupon rate is less than its YTM, and at a premium to par value if its coupon rate is greater than its YTM.**
- **Prices are more sensitive to changes in YTM for bonds with lower coupon rates and longer maturities, and less sensitive to changes in YTM for bonds with higher coupon rates and shorter maturities.**
- **A bond’s price moves toward par value as time passes and maturity approaches.**

REVISION

- f. define spot rates and calculate the price of a bond using spot rates.
- **Spot rates are market discount rates for single payments to be made in the future.**
- **The no-arbitrage price of a bond is calculated using (no-arbitrage) spot rates as follows:**

$$\text{no-arbitrage price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_2)^2} + \cdots + \frac{\text{coupon} + \text{principal}}{(1+S_N)^N}$$

REVISION

flat price = full price — accrued interest

- d. describe and calculate the flat price, accrued interest, and the full price of a bond.
- **The full price of a bond includes interest accrued between coupon dates. The flat price of a bond is the full price minus accrued interest.**
- **Accrued interest for a bond transaction is calculated as the coupon payment times the portion of the coupon period from the previous payment date to the settlement date.**
- **Methods for determining the period of accrued interest include actual days (typically used for government bonds) or 30-day months and 360-day years (typically used for corporate bonds).**

REVISION

- e. describe matrix pricing.
- **Matrix pricing is a method used to estimate the yield-to-maturity for bonds that are not traded or infrequently traded. The yield is estimated based on the yields of traded bonds with the same credit quality.**
- **If these traded bonds have different maturities than the bond being valued, linear interpolation is used to estimate the subject bond's yield.**

REVISION

- f. calculate annual yield on a bond for varying compounding periods in a year.
- **The effective yield of a bond depends on its periodicity, or annual frequency of coupon payments. For an annual-pay bond the effective yield is equal to the yield-to-maturity.**
- **For bonds with greater periodicity, the effective yield is greater than the yield-to-maturity.**
- **A YTM quoted on a semiannual bond basis is two times the semiannual discount rate.**

REVISION

- h. define the yield curve
- **A yield curve shows the term structure of interest rates by displaying yields across different maturities.**

REVISION

- i. define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.
- **Forward rates are current lending/borrowing rates for short-term loans to be made in future periods.**
- **A spot rate for a maturity of N periods is the geometric mean of forward rates over the N periods. The same relation can be used to solve for a forward rate given spot rates for two different periods.**
- **To value a bond using forward rates, discount the cash flows at times 1 through N by the product of one plus each forward rate for periods 1 to N, and sum them.**

REVISION

- i. define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.
- **For a 3-year annual-pay bond:**

$$\text{price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_1)(1+1y1y)} + \frac{\text{coupon} + \text{principal}}{(1+S_1)(1+1y1y)(1+2y1y)}$$

REVISION

- j. Describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender. Calculate the yield-to-call of a callable bond.
- **For a callable bond, a yield-to-call may be calculated using each of its call dates and prices. The lowest of these yields and YTM is a callable bond's yield-to-worst.**