Data and financial modelling

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https://github.com/styluck/mat_fin

Outline

- Organizing, Visualizing, and Describing Data
- **Probability Concepts**
- **Common Probability Distributions**
- Sampling and Estimation
- Hypothesis Testing
- Introduction to Linear Regression

PROBABILITY CONCEPTS

Unconditional and conditional probability

- **Unconditional probability** (a.k.a. marginal probability) refers to the probability of an event regardless of the past or future occurrence of other events.
- If we are concerned with the probability of an economic recession, regardless of the occurrence of changes in interest rates or inflation, we are concerned with the unconditional probability of a recession.

Unconditional and conditional probability

- A **conditional probability** is one where the occurrence of one event affects the probability of the occurrence of another event. In symbols we write "the probability of A occurring, given that B has occurred as Prob(A|B) or P(A|B).
- For example, we might be concerned with the probability of a recession given that the monetary authority has increased interest rates. This is a conditional probability. The key word to watch for here is "given." Using probability notation, "the probability of A given the occurrence of B" is expressed as P(A| B), where the vertical bar (I) indicates "given," or "conditional upon."

Addition rule of probability

- The **addition rule of probability** is used to determine the probability that at least one of two events will occur:

- P(A or B) = P(A) + P(B) - P(AB)

Joint probability

- The **joint probability** of two events is the probability that they will both occur.
- We can calculate this from the conditional probability that A will occur given B occurs (a conditional probability) and the probability that B will occur (the unconditional probability of B).
- This calculation is sometimes referred to as the **multiplication rule of probability**. Using the notation for conditional and unconditional probabilities, we can express this rule as:

$$P(AB) = P(A|\, B) \times P(B)$$

Conditional probability

- The conditional probability of A given B as follows

$$P(A|B) = \frac{P(AB)}{P(B)}$$

Bayes' formula

- **Bayes' formula** is used to update a given set of prior probabilities for a given event in response to the arrival of new information.

- updated probability $= \frac{\text{probability of new information for a given event}}{\text{unconditional probabilityof new information}}$ x prior probability of event

$$P(B|A)\ x\ P(A)\ =\ P(BA), and\ P(A|B)\ x\ P(B)\ =\ P(AB)$$

Example: Multiplication rule of probability

- Consider the following information:
- $P(I) = \mathbf{0.4}$, the probability of the monetary authority increasing interest rates (I) is 40%.
- $P(R|\, I) = \mathbf{0.7}$, the probability of a recession (R) given an increase in interest rates is 70%.
- What is $P(RI)$, the joint probability of a recession and an increase in interest rates?

Example: Multiplication rule of probability

- Applying the multiplication rule, we get the following result:
  - $P(RI) \; = \; P(R \mid I) \; x \; P(I)$
  - $P(RI) \; = \; 0.7 \; x \; 0.4$
  - $P(RI) \; = \; 0.28$

Independent events

- Independent events refer to events for which the occurrence of one has no influence on the occurrence of the others. The definition of independent events can be expressed in terms of conditional probabilities. Events A and B are independent if and only if:
    - $P(A \mid B) = P(A)$, or equivalently, $P(B \mid A) = P(B)$

The total probability rule

- The **total probability rule** is used to determine the unconditional probability of an event, given conditional probabilities:

$$P(A) = P(A|B_1)P(B_1) + P(A|\, B_2)P(B_2) + \ldots + P(A|\, BN)P(BN)$$

The expected value

- The expected value of a random variable is the weighted average of the possible outcomes for the variable. The mathematical representation for the expected value of random variable X, that can take on any of the values from x_j to x_n, is:

$$E(X) = \sum P(x_j)x_j = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$$

Portfolio

- A portfolio is a collection of **financial assets** such as stocks, bonds, cash, mutual funds, and other investments owned by an individual, institution, or entity.
- The purpose of a portfolio is typically to achieve specific financial goals, such as capital appreciation, income generation, or risk diversification.

Portfolio expected return

- The return for a portfolio is the weighted average of the returns of the individual assets in the portfolio.
- The expected return of a portfolio composed of n assets with weights,w_j, and expected returns, R_j (can be determined using the following formula:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) = w_1 E(R_1) \ + \ w_2 E(R_2) + \ldots + w_n E(R_n)$$

Covariance matrix

- A covariance matrix shows the covariances between returns on a group of asset

Asset	A	B	C
A	$Cov(R_A, R_A)$	$Cov(R_A, R_B)$	$Cov(R_A, R_C)$
B	$Cov(R_B, R_A)$	$Cov(R_B, R_B)$	$Cov(R_B, R_C)$
C	$Cov(R_C, R_A)$	$Cov(R_C, R_B)$	$Cov(R_C, R_C)$

- Note that the diagonal terms are the variances of each asset's returns, i.e., $Cov(R_A, R_A) = Var(R_A)$

Portfolio variance.

- To calculate the variance of portfolio returns, we use the asset weights, returns variances, and returns covariances.

$$\mathrm{Var}\left(R_p\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \mathrm{Cov}\left(R_i, R_j\right)$$

- The variance of a portfolio composed of two risky assets. A and B can be expressed as:

$$\mathrm{Var}\left(R_p\right) = w_A w_A \mathrm{Cov}\left(R_A, R_A\right) + w_A w_B \mathrm{Cov}\left(R_A, R_B\right) + w_B w_A \mathrm{Cov}\left(R_B, R_A\right) + w_B w_B \mathrm{Cov}\left(R_B, R_B\right)$$

COMMON PROBABILITY DISTRIBUTIONS

A probability function

- A **probability function**, denoted $p(x)$, specifies the probability that a random variable is equal to a specific value. More formally, $p(x)$ is the probability that random variable X takes on the value x, or $p(x) = P(X = x)$
- The two key properties of a probability function are:
    - $0 \leq p(x) \leq 1$.
    - $\sum p(X) = 1$

Discrete and continuous random variable

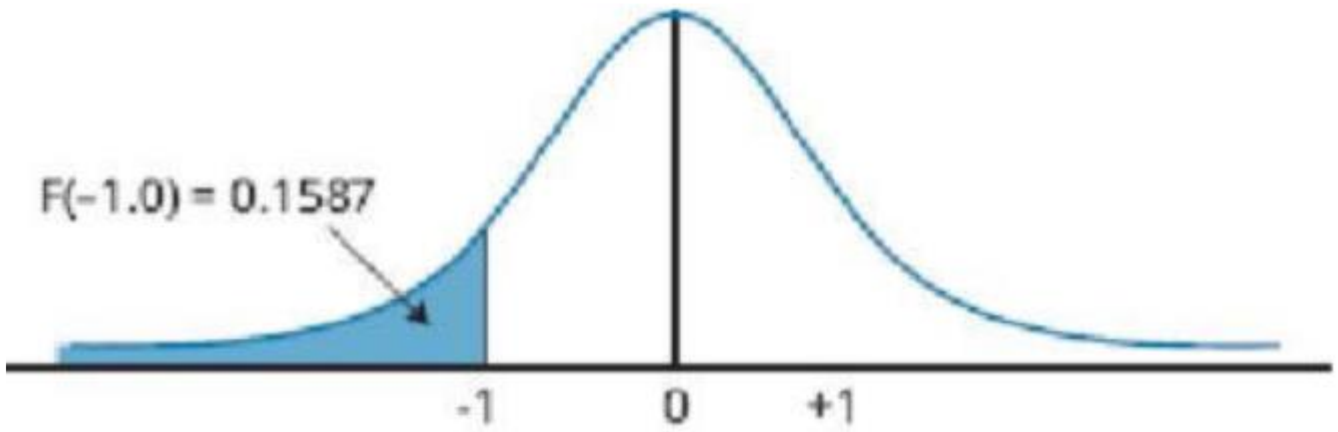
- A **discrete random variable** is one for which the number of possible outcomes can be counted, and for each possible outcome, there is a measurable and positive probability. An example of a discrete random variable is the number of days it will rain in a given month, because there is a countable number of possible outcomes, ranging from zero to the number of days in the month.

Discrete and continuous random variable

- A **continuous random variable** is one for which the number of possible outcomes is infinite, even if lower and upper bounds exist. The actual amount of daily rainfall between zero and 100 inches is an example of a continuous random variable because the actual amount of rainfall can take on an infinite number of values. Daily rainfall can be measured in inches, half inches, quarter inches, thousandths of inches, or even smaller increments. Thus, the number of possible daily rainfall amounts between zero and 100 inches is essentially infinite.

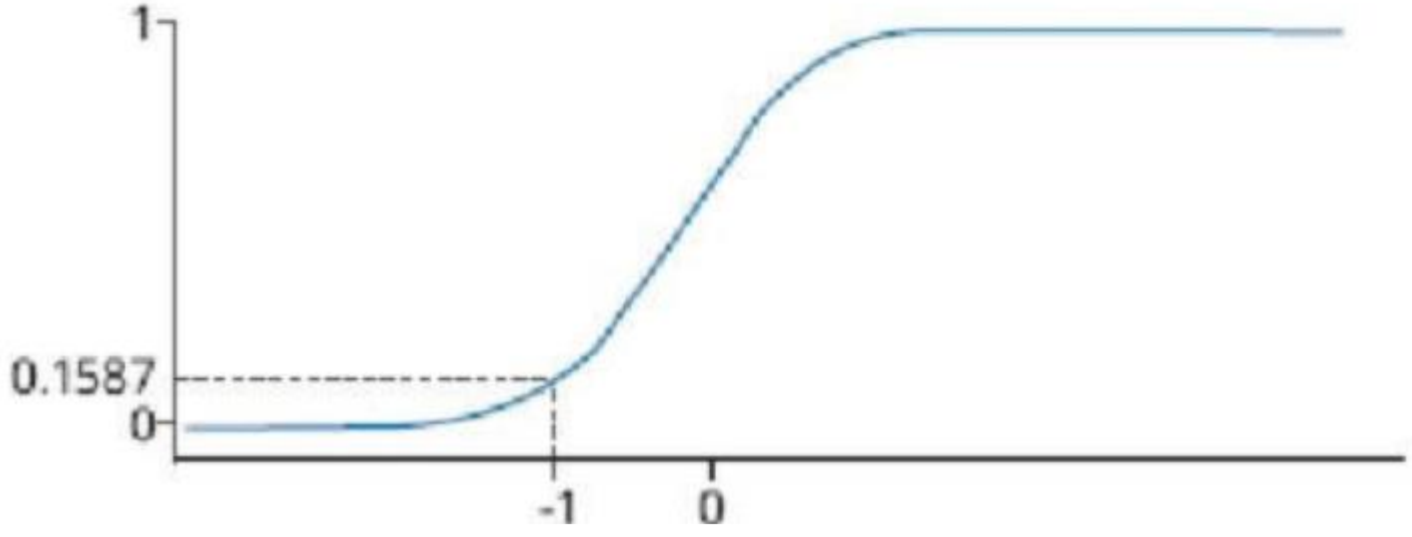
Probability density function

- A **probability density function** (PDF) is a mathematical function that describes the likelihood of a continuous random variable taking on a specific value within a given range. In simpler terms, it provides a way to model the probability distribution of a continuous random variable.



Cumulative distribution function

- A **cumulative distribution function** (CDF), or simply distribution function, defines the probability that a random variable, X, takes on a value equal to or less than a specific value, x.It represents the sum, or cumulative value, of the probabilities for the outcomes up to and including a specified outcome.
  - $F(x) = P(X \leq x)$.



continuous uniform distribution

- The continuous uniform distribution is defined over a range that spans between some lower limit, a, and some upper limit, b, which serve as the parameters of the distribution.
  - $P(X < a \text{ } or \text{ } X > b) = 0$
  - $P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$.
  - $a < x < b, P(X = x) = 0$.

Binomial random variable

- A **binomial random variable** may be defined as the number of "successes" in a given number of trials, whereby the outcome can be either "success" or "failure." The probability of success, p, is constant for each trial, and the trials are independent. A binomial random variable for which the number of trials is 1 is called a **Bernoulli random variable**.

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$$

Example: Binomial probability

- Assuming a binomial distribution, compute the probability of drawing three black beans from a bowl of black and white beans if the probability of selecting a black bean in any given attempt is 0.6. You will draw five beans from the bowl

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$$\boldsymbol{P(X=3) = p(3) = \frac{5!}{2!\,3!}(0.6)^3(0.4)^2 = (120/12)(0.216)(0.160) = 0.3456}$$

Expected value and Variance

- For a given series of n trials, the expected number of successes, or $E(X)$, is given by the following formula:

$$\text{expected value of } X = E(X) = np$$

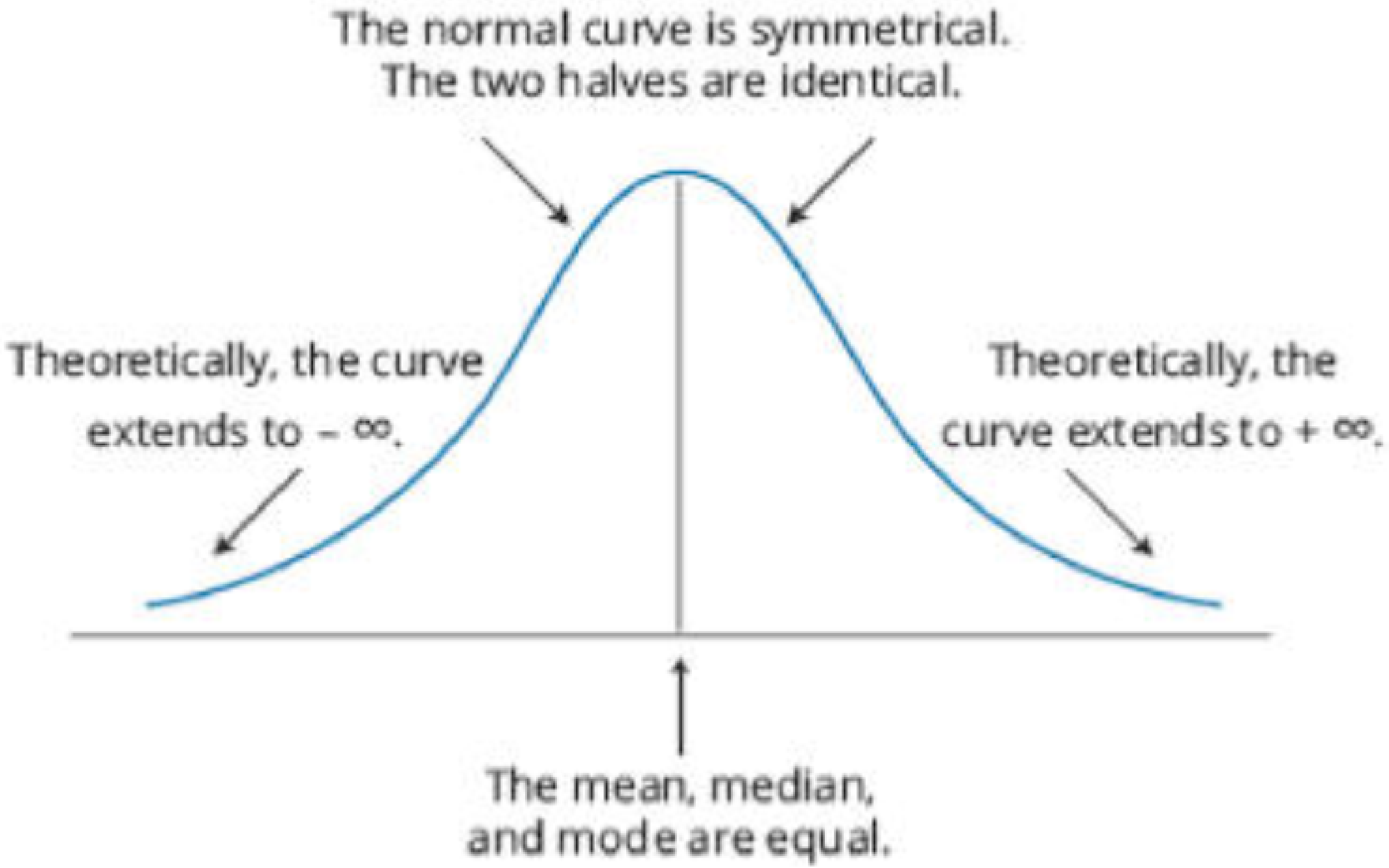
- The intuition is straightforward; if we perform n trials and the probability of success on each trial is p, we expect np successes.
- The variance of a binomial random variable is given by:

$$\text{variance of } X = np(1 - p)$$

Normal distribution

- The normal distribution has the following key properties:
- It is completely described by its mean, μ, and variance, σ^2, stated as $X \sim N(\mu, \sigma^2)$. In words, this says that "X is normally distributed with mean μ and variance σ^2."
- **Skewness = 0**, meaning that the normal distribution is symmetric about its mean, so that $P(X \leq \mu) = P(\mu \leq X) = 0.5$, and mean = median = mode.
- **Kurtosis = 3**; this is a measure of how flat the distribution is. Recall that excess kurtosis is measured relative to 3, the kurtosis of the normal distribution.

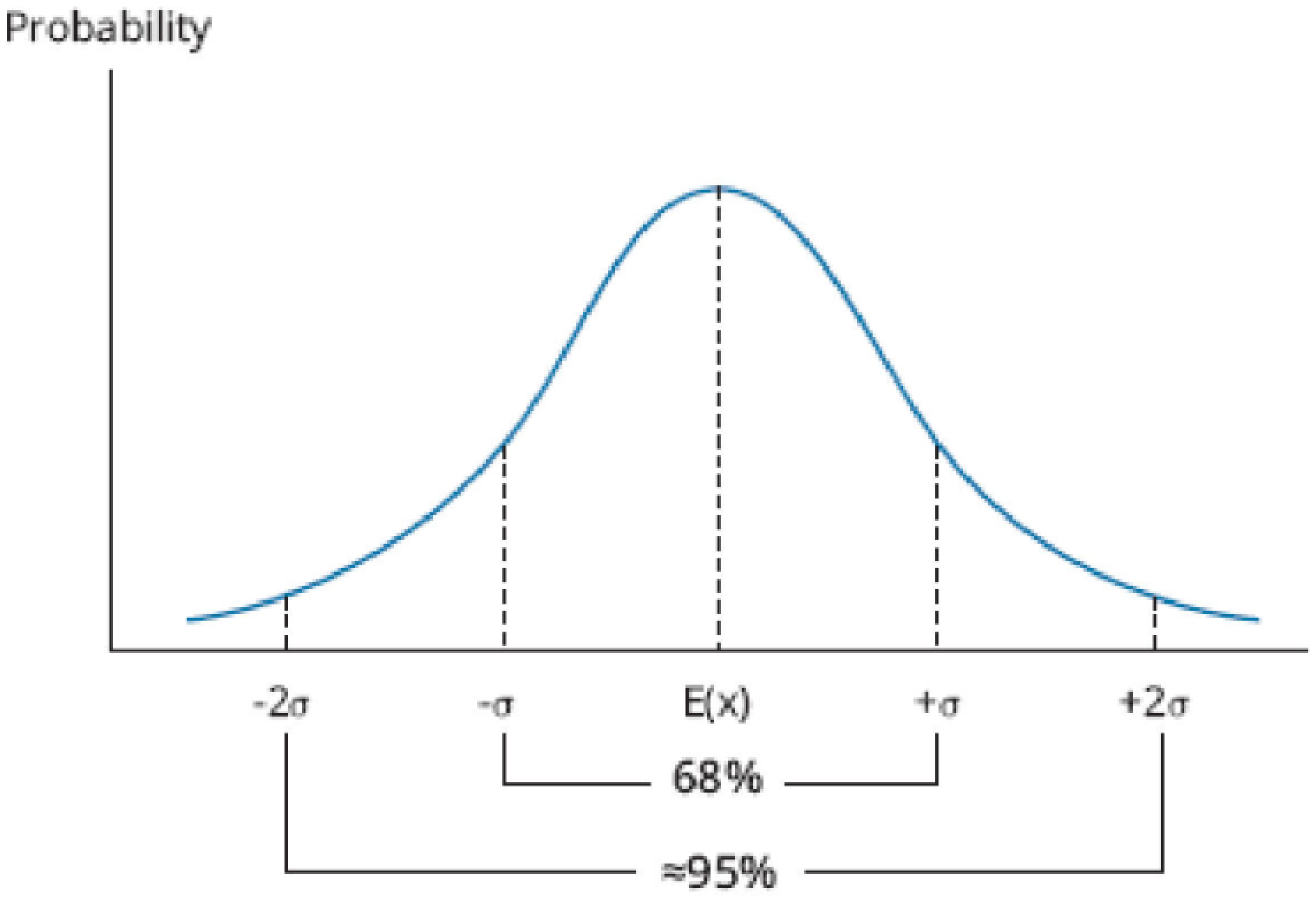
Normal distribution



Confidence interval

- A confidence interval is a range of values around the expected outcome within which we expect the actual outcome to be some specified percentage of the time.
- A 95% confidence interval is a range that we expect the random variable to be in 95% of the time.
- For a normal distribution, this interval is based on the expected value (sometimes called a point estimate) of the random variable and on its variability, which we measure with standard deviation.

Confidence interval



For any normally distributed random variable, 68% of the outcomes are within one standard deviation of the expected value (mean), and approximately 95% of the outcomes are within two standard deviations of the expected value.

Confidence interval

- In practice, we will not know the actual values for the mean and standard deviation of the distribution, but will have estimated them as X and s. The three confidence intervals of most interest are given by the following:
- The 90% confidence interval for X is $[-\ 1.65, +\ 1.65]$.
- The 95% confidence interval for X is $[-\ 1.96, +\ 1.96]$.
- The 99% confidence interval for X is $[-\ 2.58, +\ 2.58]$.

Example: Confidence intervals

- The average return of a mutual fund is 10.5% per year and the standard deviation of annual returns is 18%. If returns are approximately normal, what is the 95% confidence interval for the mutual fund return next year?

Example: Confidence intervals

- The average return of a mutual fund is 10.5% per year and the standard deviation of annual returns is 18%. If returns are approximately normal, what is the 95% confidence interval for the mutual fund return next year?
- Here μ and σ are 10.5% and 18%, respectively. Thus, the 95% confidence interval for the return, R, is:

$$10.5 \pm 1.96(18) = -24.78\% \; to \; 45.78\%$$

- Symbolically, this result can be expressed as:

$$P(-24.78 < R < 45.78) = 0.95 \; or \; 95\%$$

- The interpretation is that the annual return is expected to be within this interval 95% of the time, or 95 out of 100 years.

Standard normal distribution

- The standard normal distribution is a normal distribution that has been standardized so that it has a mean of zero and a standard deviation of 1 [i.e., $N \sim (0,1)$].
- To standardize an observation from a given normal distribution, the z-value of the observation must be calculated.
- The following formula is used to standardize a random variable:

$$z = \frac{\text{observation } - \text{pop}ulation\ mean}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Z-table

- We will refer to this table as the z-table, as it contains values generated using the cumulative distribution function for a standard normal distribution, denoted by $F(Z)$.
- The values in the z-table are the probabilities of observing a z-value that is less than a given value, z [i.e., $P(Z < z)$].
- For the negative z-values, we know from the symmetry of the standard normal distribution that $F(-Z) = 1 - F(Z)$.

Z-table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

Example: Using the z-table

- Considering again EPS distributed with $\mu = \$6$ and $\sigma = \$2$, what is the probability that EPS will be \$9.70 or more?

Example: Using the z-table

- Considering again EPS distributed with μ = \$6 and σ = \$2, what is the probability that EPS will be \$9.70 or more?
- Here we want to know P(EPS > \$9.70), which is the area under the curve to the right of the z-value corresponding to EPS = \$. The z-value for EPS = \$9.70 is: $z = \frac{x-\mu}{\sigma} = \frac{9.70-6}{2} = 1.85$
- That is, \$9.70 is 1.85 standard deviations above the mean EPS value of \$6. From the z-table, $F(1.85) = 0.9678$, but this is $P(EPS \leq 9.70)$. We want $P(EPS > 9.70)$, which is $1 - P(EPS \leq 9.70)$.

$P(EPS > 9.70) = 1 - 0.9678 = 0.0322$, or 3.2%

Example: Using the z-table

- Considering again EPS distributed with μ = \$6 and σ = \$2, what is the probability that EPS will be \$9.70 or more?

Roy's safety-first criterion

- **Roy's safety-first criterion** states that the optimal portfolio minimizes the probability that the return of the portfolio falls below some minimum acceptable level.
- This minimum acceptable level is called the **threshold level**. Symbolically, Roy's safety-first criterion can be stated as:

$$minimize\ P(R_p < R_L)$$

where:

R_p=portfolio return

R_L=threshold level return

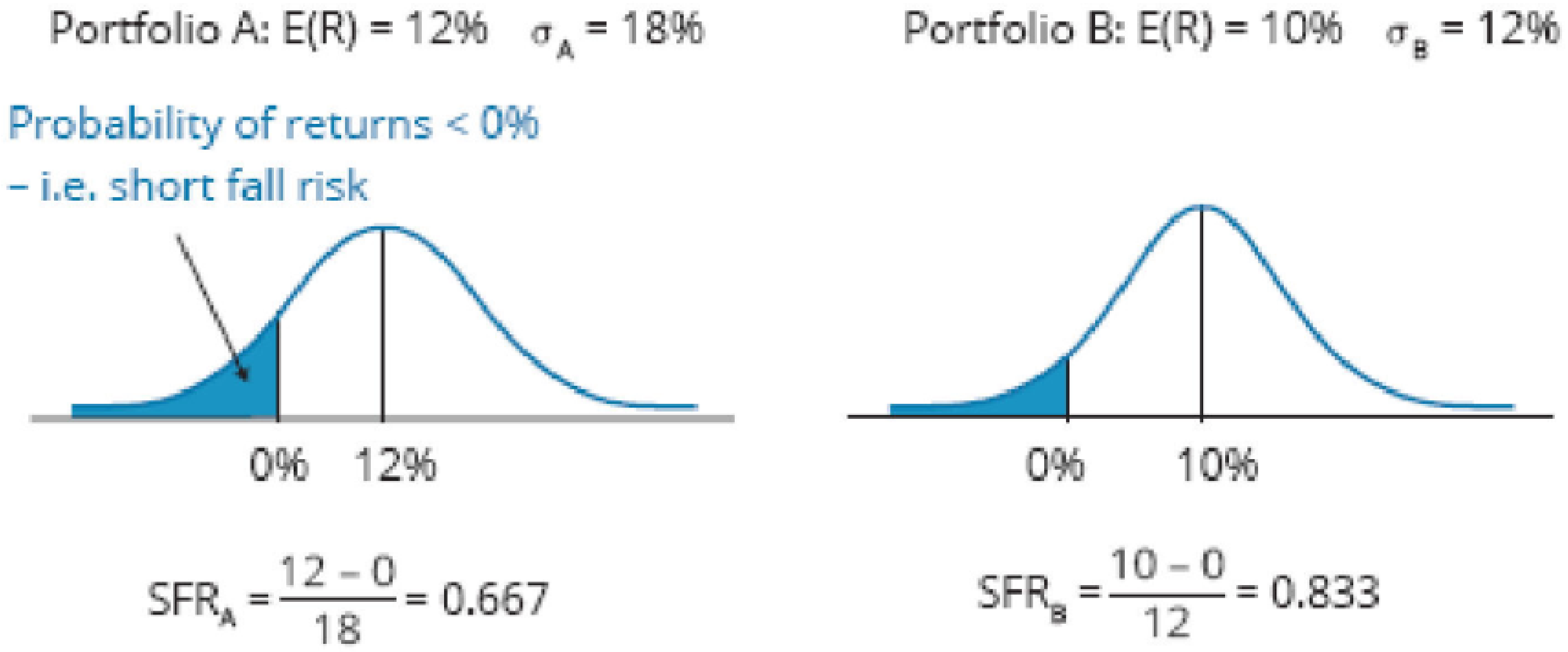
SFRatio

- The safety-first ratio:

$$SFRatio = \frac{E(R_P) - R_L}{\sigma_P}$$

- If portfolio returns are normally distributed, then Roy's safety-first criterion can be stated as maximize the SFRatio.
- Assuming that returns are normally distributed, the portfolio with the larger SFR using 0% as the threshold return (R_L) will be the one with the lower probability of negative returns.

SFRatio



Portfolio A: E(R) = 12% σA = 18%
Probability of returns < 0%
– i.e. short fall risk
0%
12%
SFRA = (12 – 0) / 18 = 0.667
Portfolio B: E(R) = 10% σB = 12%
0%
10%
SFRB = (10 – 0) / 12 = 0.833

Example: Roy's safety-first criterion

- For the next year, the managers of a $120 million college endowment plan have set a minimum acceptable end-of-year portfolio value of $123.6 million. Three portfolios are being considered which have the expected returns and standard deviation shown in the first two rows of the following table. Determine which of these portfolios is the most desirable using Roy's safety-first criterion and the probability that the portfolio value will fall short of the target amount.

Portfolio	Portfolio A	Portfolio B	Portfolio C
$E(R_p)$	9%	11%	6.6%
σ_p	12%	20%	8.2%

Example: Roy's safety-first criterion

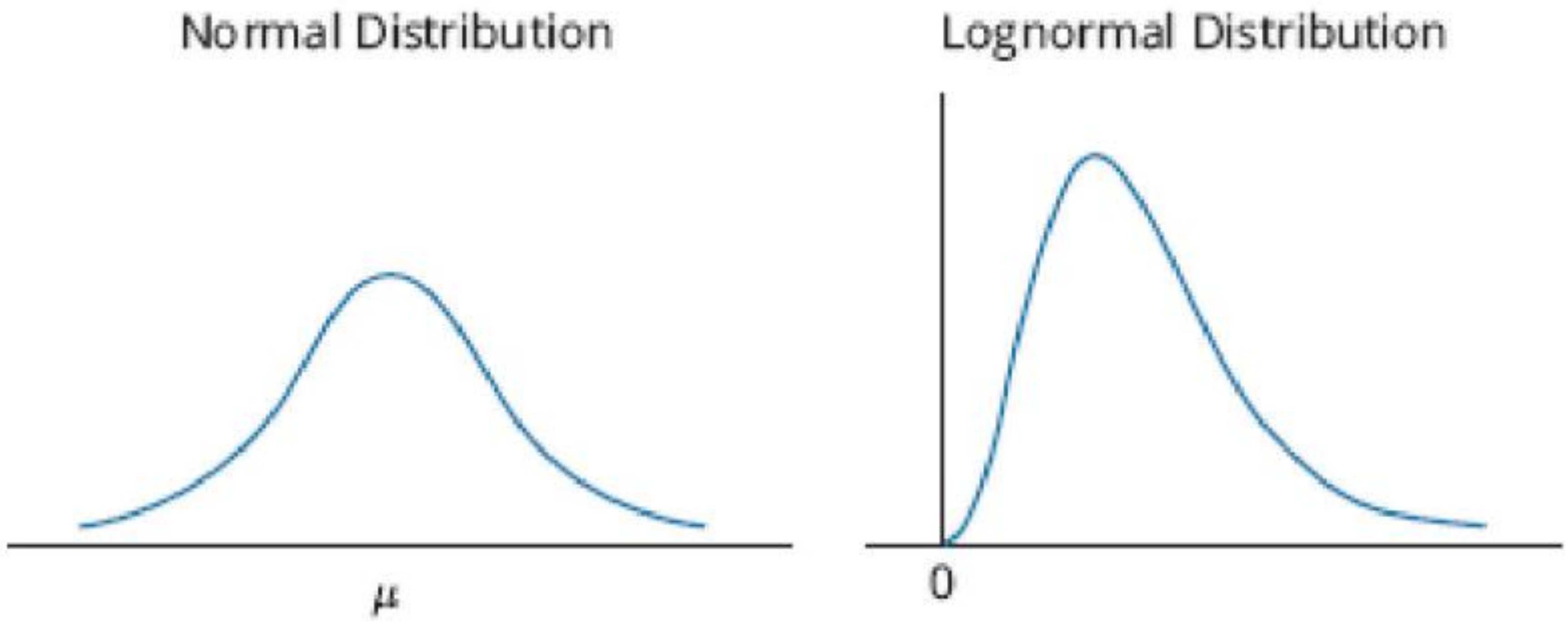
- The threshold return is R_L = (123.6 – 120) / 120 = 0.030 = 3%. The SFRs are shown in the table below. As indicated, the best choice is Portfolio A because it has the largest SFR.

Portfolio	Portfolio A	Portfolio B	Portfolio C
$E(R_p)$	9%	11%	6.6%
σ_p	12%	20%	8.2%
SFRatio	$0.5 = \frac{9-3}{12}$	$0.4 = \frac{11-3}{20}$	$0.44 = \frac{6.6-3}{8.2}$

Lognormal distribution

- The lognormal distribution is generated by the function e^x, where x is normally distributed.
- Since the natural logarithm, ln, of e^x is x, the logarithms of lognormally distributed random variables are normally distributed, thus the name.

Lognormal distribution



Lognormal distribution

- The lognormal distribution is skewed to the right.
- The lognormal distribution is bounded from below by zero so that it is useful for modelling asset prices, which never take negative values.
- If we used a normal distribution of returns to model asset prices over time, we would admit the possibility of returns less than –100%, which would admit the possibility of asset prices less than zero. Using a lognormal distribution to model price relatives avoids this problem.

Continuous compounding

- For a stated rate of 10%,
- semiannual compounding results in an effective yield of $\left(1+\frac{0.1}{2}\right)^2-1=10.25\%$
- Monthly compounding results in an effective yield of

$$\left(1+\frac{0.1}{12}\right)^{12}-1=10.47\%$$

- as the compounding periods get shorter and shorter, we get **continuous compounding**
  effective annual rate = $e^{0.1}-1$=10.5171%

Continuous compounding

- The continuously compounded rate of return is:

$$\ln\left(\frac{S_1}{S_0}\right) = \ln(1 + \text{HPR}) = R_{cc}$$

Example: Calculating continuously compounded returns

- A stock was purchased for $100 and sold one year later for $120. Calculate the investor’s annual rate of return on a continuously compounded basis.

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- A stock was purchased for $100 and sold one year later for $120. Calculate the investor's annual rate of return on a continuously compounded basis.

$$\ln\left(\frac{120}{100}\right) = 18.232\%$$

Holding period return

- The holding period return after T years, when the annual continuously compounded rate is R_{cc}, is given by:

$$\mathrm{HPR_T} = \mathrm{e}^{\mathrm{Rcc} \times \mathrm{T}} - 1$$

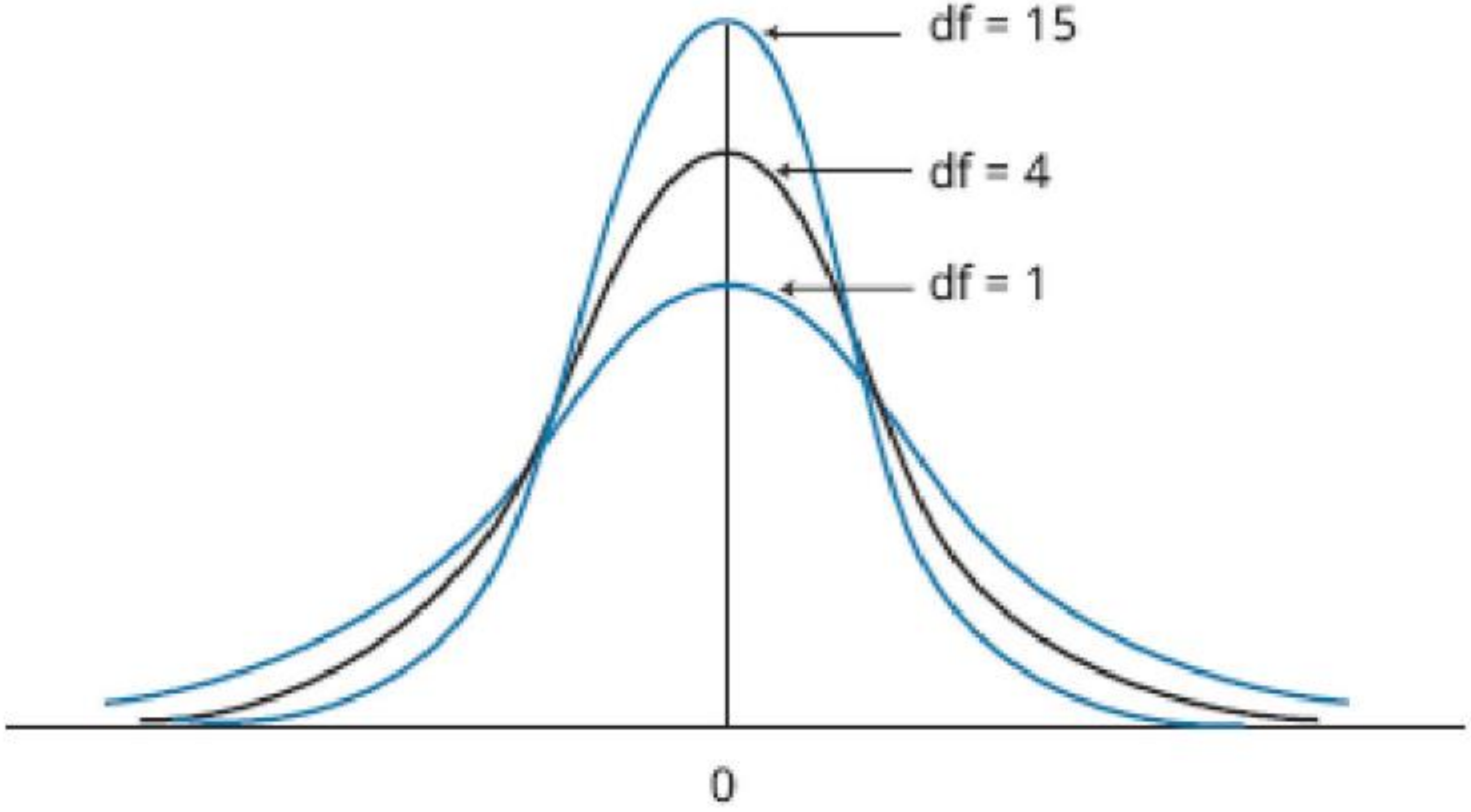
Student’s t-distribution

- Student’s t-distribution, or simply the t-distribution, is a bell-shaped probability distribution that is symmetrical about its mean. It is the appropriate distribution to use when constructing confidence intervals based on small samples ($n < 30$) from populations with unknown variance and a normal, or approximately normal, distribution.

Student’s t-distribution

- Student’s t-distribution has the following properties:
- It is symmetrical.
- It is defined by a single parameter, the **degrees of freedom (df),** where the degrees of freedom are equal to the number of sample observations minus 1, n – 1, for sample means.
- It has more probability in the tails (“fatter tails”) than the normal distribution.
- As the degrees of freedom (the sample size) gets larger, the shape of the t-distribution more closely approaches a standard normal distribution.

Student’s t-distribution



REVISION

- **a. describe the properties of a continuous uniform random variable, and calculate and interpret probabilities given the continuous uniform distribution function.**
- A discrete uniform distribution is one where there are n discrete, equally likely outcomes. For a discrete uniform distribution with n possible outcomes, the probability for each outcome equals 1/n.

REVISION

- **b. describe the properties of the continuous uniform distribution, and calculate and interpret probabilities given a continuous uniform distribution.**
- A continuous uniform distribution is one where the probability of X occurring in a possible range is the length of the range relative to the total of all possible values. Letting a and b be the lower and upper limit of the uniform distribution, respectively, then for:

$$a \le x_1 \le x_2 \le b,\ P(x_1 \le X \le x_2) = \frac{x_2 - x_1}{b - a}$$

REVISION

- **c. describe the properties of a Bernoulli random variable and a binomial random variable, and calculate and interpret probabilities given the binomial distribution function.**
- The binomial distribution is a probability distribution for a binomial (discrete) random variable that has two possible outcomes. For a binomial distribution, if the probability of success is p, the probability of x successes in n trials is:

$$p(x) = P(X = x) = \frac{n!}{(n - x)!x!} p^x (1 - p)^{n-x} = {}_nC_x \times p^x (1 - p)^{n-x}$$

REVISION

- **d. explain the key properties of the normal distribution**
- The normal probability distribution and normal curve have the following characteristics:
- The normal curve is symmetrical and bell-shaped with a single peak at the exact center of the distribution.
- Mean = median = mode, and all are in the exact center of the distribution.
- The normal distribution can be completely defined by its mean and standard deviation because the skew is always zero and kurtosis is always 3.

REVISION

- **e. calculate the probability that a normally distributed random variable lies inside a given interval**
- A confidence interval is a range within which we have a given level of confidence of finding a point estimate (e.g., the 90% confidence interval for X is [-1.65, 1.65]
- Confidence intervals for any normally distributed random variable are:
- 90%: $\mu \pm 1.65$ standard deviations.
- 95%: $\mu \pm 1.96$ standard deviations.
- 99%: $\mu \pm 2.58$ standard deviations.

REVISION

- **f. explain how to standardize a random variable.**
- The standard normal probability distribution has a mean of 0 and a standard deviation of 1.
- A normally distributed random variable X can be standardized as and Z will be normally distributed with mean = 0 and standard deviation 1

REVISION

- **g. calculate and interpret probabilities using the standard normal distribution**
- The z-table is used to find the probability that X will be less than or equal to a given value.
- $P(X < x) = F(x) =$, which is found in the standard normal probability table.
- $P(X > x) = 1 – P(X < x) = 1 – F(z)$.

REVISION

- **h. define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.**
- Shortfall risk is the probability that a portfolio's value (or return) will fall below a specific value over a given period of time.
- The safety-first ratio for portfolio P, based on a target return RT, is:

$$\text{SFRatio} = \frac{E(R_P) - R_T}{\sigma_P}$$

- Greater safety-first ratios are preferred and indicate a smaller shortfall probability. Roy's safety- first criterion states that the optimal portfolio minimizes shortfall risk.

REVISION

- **i. explain the relationship between normal and lognormal distributions and why the lognormal distribution is used to model asset prices.**
- If x is normally distributed, ex follows a lognormal distribution. A lognormal distribution is often used to model asset prices, since a lognormal random variable cannot be negative and can take on any positive value.

REVISION

- **j. calculate and interpret a continuously compounded rate of return, given a specific holding period return.**
- As we decrease the length of discrete compounding periods (e.g., from quarterly to monthly) the effective annual rate increases. As the length of the compounding period in discrete compounding gets shorter and shorter, the compounding becomes continuous, where the effective annual rate = $e^{i} - 1$.
- For a holding period return (HPR) over any period, the equivalent continuously compounded rate over the period is ln(1 + HPR).

REVISION

- **k. describe the properties of the Student's t-distribution, and calculate and interpret its degrees of freedom.**
- The t-distribution is similar, but not identical, to the normal distribution in shape—it is defined by the degrees of freedom and has fatter tails compared to the normal distribution.
- Degrees of freedom for the t-distribution are equal to n – 1. Student's t-distribution is closer to the normal distribution when degrees of freedom are greater, and confidence intervals are narrower when degrees of freedom are greater.