Introduction to Mathematical Finance

Mock Exam for Revision

1. The Federal Funds Rate refers to the interest rate at which banks and credit unions lend reserve balances to other depository institutions overnight on an uncollateralized basis. In the United States, this rate is set by the Federal Reserve, which is the central banking system of the country. Compute the PV of the following fixed-income securities to be matured one year from now with face value of $1,000 using a stated annual federal funds interest rate of 6% with a range of compounding periods.

$1000/(1+6%) = $943.40

a) Treasury bill (a risk-free bond with zero coupon), annually compounding
b) Corporate bond with a credit spread of +100 BP, making semiannual coupon payment of $35, semiannually compounding
c) The Federal Open Market Committee (FOMC) is the branch of the Federal Reserve System responsible for setting monetary policy in the United States. It meets regularly to review economic and financial conditions, including changes to the Federal Funds Rate. If the FOMC decided to hike the rate to 7% immediately after the purchasing of the security, calculate the profit & loss (P&L) of an investor who decide to buy-and-hold the Treasury bill.
d) If the FOMC decided to cut the rate to 5%, calculate the P&L of an investor who decide to sell the corporate bond now.

b) interest rate 6% + 100 BP = 7% 7%/2 = 3.5%

$$PV = \frac{\$35}{1+3.5\%} + \frac{\$1{,}000+\$35}{(1+3.5\%)^2} = \$1{,}000$$

c) $1000 − $943.40 = $56.60

P&L of a buy-and-hold investor is unchanged if there is no reinvestment risk of the security

d) $PV_{new} = \frac{\$35}{1+3\%} + \frac{\$1000+\$35}{(1+3\%)^2} = 1009.57$ interest rate = 5% + 100BP = 6%

P&L = $1009.57 − $1000 = $9.57

2. Consider a 10-year, annual-pay 6% bond trading at 102 on January 1, 2014. The bond is callable according to the following schedule:

- Callable at 102 on or after January 1, 2019 (first call).
- Callable at 100 on or after January 1, 2022 (first par call).

a) Calculate the bond's simple YTM,
b) yield-to-first call,
c) yield-to-first par call, and
d) yield-to-worst.

a) $YTM \approx \dfrac{(100-102)/10 + 6}{102} = 5.69\%$

b) $N = 2019 - 2014 = 5 \qquad FV = 102$

$$YTM = \frac{(FV-PV)/N + PMT}{PV} = \frac{6}{102} = 5.88\%$$

c) $$YTM = \frac{(100-102)/8 + 6}{102} = 5.63\%$$

d) $$\text{Yield-to-worst} = \min(5.69\%, 5.88\%, 5.63\%)$$
$$= 5.63\%$$

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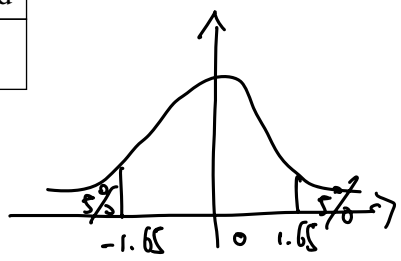
3. A pension fund manager with AUM (asset under management) of $100 million is considering a mutual fund's return with μ = 10.5% and σ = 18%,

a) what is the probability that the return will be -19.2% or less?
b) what is the 95% confidence interval for the mutual fund return next year?
c) If this manager has a minimum acceptable end-of-year AUM value of $ 102 million. He is considering two other assets: treasury bill with μ = 3% and σ = 1%, and corporate bond with μ = 6% and σ = 3.5%. According to the Roy's safety-first criterion, which asset is the most desirable one?
d) If the manager has decided to construct the following portfolio, calculate the expected return and the standard error of the portfolio, assuming the returns of the assets are independent.

Mutual fund	Treasury bill	Corporate bond
20%	30%	50%

Note: The 90% confidence interval is [-1.65, +1.65].
The 95% confidence interval is [-1.96, +1.96].
The 99% confidence interval is [-2.58, +2.58].



a) $t = \frac{-19.2\% - \mu}{\sigma} = \frac{-19.2\% - 10.5\%}{18\%} = -1.65$

Such probability is less than 5%.

b) $-1.96 = \frac{x_1 - 10.5\%}{18\%} \Rightarrow x_1 = -24.78\%$

$1.96 = \frac{x_2 - 10.5\%}{18\%} \Rightarrow x_2 = 45.78\%$

95% confidence interval : [-24.78%, 45.78%]

c) The minimum required rate of return is

$r = \frac{\$102 \text{ million}}{\$100 \text{ million}} - 1 = 2\%$

Mutual fund $\frac{10.5\% - 2\%}{18\%} = 0.47$

Treasury bill $\frac{3\% - 2\%}{1\%} = 1.00$

Corporate bond $\frac{6\% - 2\%}{3.5\%} = 1.14$ ← most desirable asset

d) Expected return :

$10.5\% \times 20\% + 3\% \times 30\% + 6\% \times 50\% = 6\%$

Variance $(18\%)^2 \times (20\%)^2 + (1\%)^2 \times (30\%)^2 + (3.5\%)^2 \times (50\%)^2 = 0.023$

std err $= \sqrt{0.023} = 4.01\%$

4. Mr. Michael Burry wants to use the EXCESS return on the S&P 500 to explain the variation in EXCESS returns on ABC common stock using linear regression.

a) what is the dependent variable and what is the independent variable?
b) Compute the slope coefficient and intercept term for the ABC regression example using the following information, given the risk-free interest rate of 1.5%:

Excess return deviations:	Absolute return means:
Cov(S&P 500, ABC) = 0.000541	Mean(S&P 500) = 4.5%
Var(S&P 500, ABC) = 0.000416	Mean(ABC) = 6.3%

c) If the Total sum of squares (SST) is 20,135 and the Sum of squared errors (SSE) is 12,423, what is the coefficient of determination (R^2) of this linear regression.
d) What is the Fstatistic of this linear regression, given the number of samples is 101.

a) dependent variable: ABC (Excess return) ; independent variable: S&P (Excess return)

b) $\beta_1 = \frac{cov(X,Y)}{var(X)} = \frac{0.000541}{0.000416} = 1.3005$

$\beta_1 = \bar{Y} - \beta_1 \bar{X} = (6.3\% - 1.5\%) - 1.3005 \cdot (4.5\% - 1.5\%)$

$= 0.8985$

c) $R^2 = \frac{SSR}{SST}$ $SST = SSE + SSR$ $\Rightarrow R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{12423}{20135} = 0.383$

d) $F = \frac{MSR}{MSE} = \frac{SSR/k}{SSE/(n-k-1)}$ $\frac{k=1}{n=101}$ $\frac{20135 - 12423}{12423 / 99} = 61.658$

5. Consider a four-year zero-coupon bond.

a) What is the Macaulay duration of this bond?
b) If the YTM of this bond is decided by risk premium (220BP) and inflation (4.3%), if the risk-free rate is 4%, what is the modified duration of this bond?
c) Calculate the money duration if an investor has $10 million (face value) position of this zero-coupon bond.
d) Calculate the price value of a basis point.

a) Mac Dur = $w_4 \cdot 4 = 100\% \cdot 4 = 4$ year

b) YTM = 4% + 4.3% + 2.20% = 10.5%

c) Mod Dur = Mac Dur / (1 + YTM %) = 3.619

$$PV = \frac{FV}{(1+YTM)^4} = \$670.7$$

Money duration = $3.619 \times \frac{\$670.7}{\$6000} \times \$10 \text{ million}$

$= 24.27$ \$ million

d) V_-

V_+

$$PVBP = \frac{V_- - V_+}{2}$$

V_-: YTM = 10.5% → 10.49% $PV = \frac{FV}{(1+10.49\%)^4} = 670.98$

V_+: YTM = 10.5% → 10.51% $PV = \frac{FV}{(1+10.51\%)^4} = 670.49$

$$PVBP = \frac{670.98 - 670.99}{2} = 0.2450$$