Modern Portfolio Theory

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https://github.com/styluck/matfin

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Portfolio Management overview

Portfolio perspective

- A portfolio perspective is to examine the risk and return of individual investments in isolation.
- An investor who holds all his wealth in a single stock because he believes it to be the best stock available is not taking the portfolio perspective—his portfolio is very risky compared to holding a diversified portfolio of stocks.

Diversification

- In the early 1950s, the research of Professor Harry Markowitz provided a framework for measuring the **risk-reduction benefits of diversification**.
- **Modern portfolio theory** concludes that the extra risk from holding only a single security is not rewarded with higher expected investment returns.
- Diversification allows an investor to **reduce portfolio risk** without necessarily reducing the **portfolio's expected return**.

Portfolio management process

- 1. **Planning step**: begins with an analysis of the investor's risk tolerance, return objectives, time horizon, tax exposure, liquidity needs, income needs, and any unique circumstances or investor preferences.
- This analysis results in an **investment policy statement** (IPS) that details the investor's investment objectives and constraints.

Portfolio management process

- 2. **Execution step**. analysis of the risk and return characteristics of various asset classes to determine how funds will be allocated to the various asset types.
- **Top-down analysis**, a portfolio manager will examine current economic conditions and forecasts of such macroeconomic variables as GDP growth, inflation, and interest rates, in order to identify the asset classes that are most attractive.
- **bottom-up security analysis**, a portfolio managers may attempt to identify the most attractive securities within the asset class.

Portfolio management process

- 3. **Feedback and rebalance step.** Over time, investor circumstances will change, risk and return characteristics of asset classes will change, and the actual weights of the assets in the portfolio will change with asset price.
- The manager must also measure portfolio performance and evaluate it relative to the return on the benchmark portfolio identified in the IPS.

Types of investors

- Individual investors
- Institutional investors:
  - **endowment funds**: providing financial support on an ongoing basis for a specific purpose. A typical foundation's investment objective is to fund the activity or research on a continuing basis without decreasing the real (inflation adjusted) value of the portfolio assets.
  - **bank**: to earn more on the bank's loans and investments than the bank pays for deposits of various types.
  - **Insurance companies**: invest customer premiums with the objective of funding customer claims as they occur.
  - **Investment companies**: mutual funds, private funds manage the pooled funds of many investors.
  - **Sovereign wealth funds**: pools of assets owned by a government.

Types of investors

Investor	Risk Tolerance	Investment Horizon	Liquidity Needs	Income Needs
Individuals	Depends on individual	Depends on individual	Depends on individual	Depends on individual
Banks	Low	Short	High	Pay interest
Endowments	High	Long	Low	Spending level
Insurance	Low	Long—life Short—P&C	High	Low
Mutual funds	Depends on fund	Depends on fund	High	Depends on fund
Defined benefit pensions	High	Long	Low	Depends on age

Pension plans

- A **defined contribution pension plan** is a retirement plan in which the firm contributes a sum each period to the employee’s retirement account. In any event, the firm makes no promise to the employee regarding the future value of the plan assets. The investment decisions are left to the employee, who assumes all of the investment risk.
- In a **defined benefit pension plan**, the firm promises to make periodic payments to employees after retirement. The benefit is usually based on the employee’s years of service and the employee’s compensation at, or near, retirement.

Asset management industry

- **Buy-side firms**: include both independent managers and divisions of larger financial services companies
- **Active management** attempts to outperform a chosen benchmark through manager skill, for example by using fundamental or technical analysis.
- **Passive management** attempts to replicate the performance of a chosen benchmark index. This may include traditional broad market index tracking or a **smart beta** approach that focuses on exposure to a particular market risk factor

Mutual Funds

- **Mutual funds**: Each investor owns shares representing ownership of a portion of the overall portfolio. The total net value of the assets in the fund (pool) divided by the number of such shares issued is referred to as the **net asset value (NAV)** of each share.
- With an **open-end fund**, investors can buy newly issued shares at the NAV. Newly invested cash is invested by the mutual fund managers in additional portfolio securities. Investors can redeem their shares (sell them back to the fund) at NAV as well.
- **Closed-end funds** are professionally managed pools of investor money that do not take new investments into the fund or redeem investor shares. The shares of a closed-end fund trade like equity shares (on exchanges or over-the-counter).

Types of Mutual Funds

- **Money market funds** invest in short-term debt securities and provide interest income with very low risk of changes in share value.
- **Bond mutual funds** invest in fixed-income securities. They are differentiated by bond maturities, credit ratings, issuers, and types.
- A great variety of stock mutual funds are available to investors. **Index funds** are passively managed; that is, the portfolio is constructed to match the performance of a particular index, such as the Standard & Poor's 500 Index. **Actively managed funds** refer to funds where the management selects individual security. **Exchange-traded funds (ETFs**) are similar to closed-end funds in that purchases and sales are made in the market rather than with the fund itself. es with the goal of producing returns greater than those of their benchmark indexes.

Types of Mutual Funds

- **Hedge funds** are pools of investor funds that are not regulated to the extent that mutual funds are. Hedge funds are limited in the number of investors who can invest in the fund and are often sold only to qualified investors who have a **minimum amount** of overall portfolio wealth.
- **Private equity** and **venture capital** funds invest in portfolios of companies, often with the intention to sell them later in public offerings. Managers of funds may take active roles in managing the companies in which they invest.

Portfolio Risk and Return

Holding period return

- **Holding period return** (HPR) is simply the percentage increase in the value of an investment over a given time period:

$$\text{holding period return} = \frac{\text{end-of-period value}}{\text{beginning-of-period value}} - 1 = \frac{P_t + Div_t}{P_0} - 1$$

$$= \frac{P_t - P_0 + Div_t}{P_0}$$

EXAMPLE: Return measures

- An investor purchased $1,000 of a mutual fund's shares. The fund had the following total returns over a 3-year period: +5%, –8%, +12%. Calculate the value at the end of the 3-year period, the holding period return, and the mean annual return.

EXAMPLE: Return measures

- An investor purchased \$1,000 of a mutual fund's shares. The fund had the following total returns over a 3-year period: +5%, –8%, +12%. Calculate the value at the end of the 3-year period, the holding period return, and the mean annual return.
- ending value = (1,000)(1.05)(0.92)(1.12) = \$1,081.92
- holding period return = (1.05)(0.92)(1.12) – 1 = 0.08192 = 8.192%, which can also be calculated as 1,081.92 / 1,000 – 1 = 8.192%

geometric mean return $= \sqrt[3]{(1.05)(0.92)(1.12)} - 1 = 0.02659 = 2.66\%$,
which can also be calculated as geometric mean return $= \sqrt[3]{1 + \text{HPR}} - 1 =$
$\sqrt[3]{1.08192} - 1 = 2.66\%$.

The money-weighted return and IRR

- The money-weighted rate of return is defined as the internal rate of return on a portfolio, taking into account all cash inflows and outflows.

EXAMPLE: Money-weighted rate of return

- Assume an investor buys a share of stock for \$100 at t = 0 and at the end of the year (t = 1), she buys an additional share for \$120. At the end of Year 2, the investor sells both shares for \$130 each. At the end of each year in the holding period, the stock paid a \$2.00 per share dividend. What is the money-weighted rate of return?

EXAMPLE: Money-weighted rate of return

- Determine the timing of each cash flow and whether the cash flow is an inflow (+), into the account, or an outflow (–), available from the account.

t = 0:	purchase of first share	=	+\$100.00	inflow to account
t = 1:	purchase of second share	=	+\$120.00	
	dividend from first share	=	–\$2.00	
	Subtotal, t = 1		+\$118.00	inflow to account
t = 2:	dividend from two shares	=	–\$4.00	
	proceeds from selling shares	=	–\$260.00	
	Subtotal, t = 2		–\$264.00	outflow from account

EXAMPLE: Money-weighted rate of return

- Net the cash flows for each time period and set the PV of cash inflows equal to the present value of cash outflows.

$$PV_{inflows} = PV_{outflows}$$

$$\$100 + \frac{\$118}{(1+r)} = \frac{\$264}{(1+r)^2}$$

- The money-weighted rate of return for this problem is 13.86%.

Time-weighted rate of return

- **Time-weighted rate of return** measures compound growth. It is the rate at which $1 compounds over a specified performance horizon. Time-weighting is the process of averaging a set of values over time.

EXAMPLE: Time-weighted rate of return

- An investor purchases a share of stock at t = 0 for $100. At the end of the year, t = 1, the investor buys another share of the same stock for $120. At the end of Year 2, the investor sells both shares for $130 each. At the end of both years 1 and 2, the stock paid a $2 per share dividend. What is the annual time-weighted rate of return for this investment?

EXAMPLE: Time-weighted rate of return

- Break the evaluation period into two subperiods based on timing of cash flows.

Holding period 1:	Beginning value	= \$100
	Dividends paid	= \$2
	Ending value	= \$120
Holding period 2:	Beginning value	= \$240 (2 shares)
	Dividends paid	= \$4 (\$2 per share)
	Ending value	= \$260 (2 shares)

EXAMPLE: Time-weighted rate of return

- Calculate the HPR for each holding period.

$$HPR_1 = [(\$120 + 2) / \$100] - 1 = 22\%$$

$$HPR_2 = [(\$260 + 4) / \$240] - 1 = 10\%$$

- Find the compound annual rate that would have produced a total return equal to the return on the account over the 2-year period.

$$(1 + \text{time-weighted rate of return})^2 = (1.22)(1.10)$$

$$\text{time-weighted rate of return} = [(1.22)(1.10)]^{0.5} - 1 = 15.84\%$$

Variance (Standard Deviation) of Returns

- Population variance, σ^2

$$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{T}$$

- Sample variance, s^2

$$s^2 = \frac{\sum_{t=1}^{T}\left(R_t - \overline{R}\right)^2}{T - 1}$$

Covariance and Correlation of Returns

- The sample covariance is

$$\text{Cov}_{1,2} = \frac{\sum_{t=1}^{n}\left\{\left[R_{t,1} - \overline{R}_1\right]\left[R_{t,2} - \overline{R}_2\right]\right\}}{n-1}$$

- Correlation is

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2}$$

The variance of returns for a portfolio

- The variance of returns for a portfolio of two risky assets is

$$\text{Var}_{\text{portfolio}} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}_{12}$$

- or

$$\text{Var}_{\text{portfolio}} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2$$

EXAMPLE: Portfolio risk as correlation varies

- Consider two risky assets that have returns variances of 0.0625 and 0.0324, respectively. The assets’ standard deviations of returns are then 25% and 18%, respectively. Calculate the variances and standard deviations of portfolio returns for an equal-weighted portfolio of the two assets when their correlation of returns is 1, 0.5, 0, and -0.5

EXAMPLE: Portfolio risk as correlation varies

$$\text{variance}_{\text{portfolio}} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\,w_1\,w_2\,\rho_1\,\sigma_1\,\sigma_2$$

$$\sigma_{\text{portfolio}} = \sqrt{\text{variance}_{\text{portfolio}}}$$

$$\sigma_{\text{portfolio}} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\,w_1\,w_2\,\rho_{12}\,\sigma_1\,\sigma_2}$$

EXAMPLE: Portfolio risk as correlation varies

- ρ = correlation = +1:

$$\sigma = \text{portfolio standard deviation} = 0.5(25\%) + 0.5(18\%) = \mathbf{21.5\%}$$

$$\sigma^2 = \text{portfolio variance} = 0.215^2 = 0.046225$$

- ρ = correlation = 0.5:

$$\sigma^2 = (0.5^2)0.0625 + (0.5^2)0.0324 + 2(0.5)(0.5)(\mathbf{0.5})(0.25)(0.18) = 0.034975$$

$$\sigma = \mathbf{18.70\%}$$

EXAMPLE: Portfolio risk as correlation varies

- ρ = correlation = 0:

$$\sigma^2 = (0.5^2)0.0625 + (0.5^2)0.0324 = 0.023725$$

$$\sigma = \mathbf{15.40\%}$$

- ρ = correlation = –0.5:

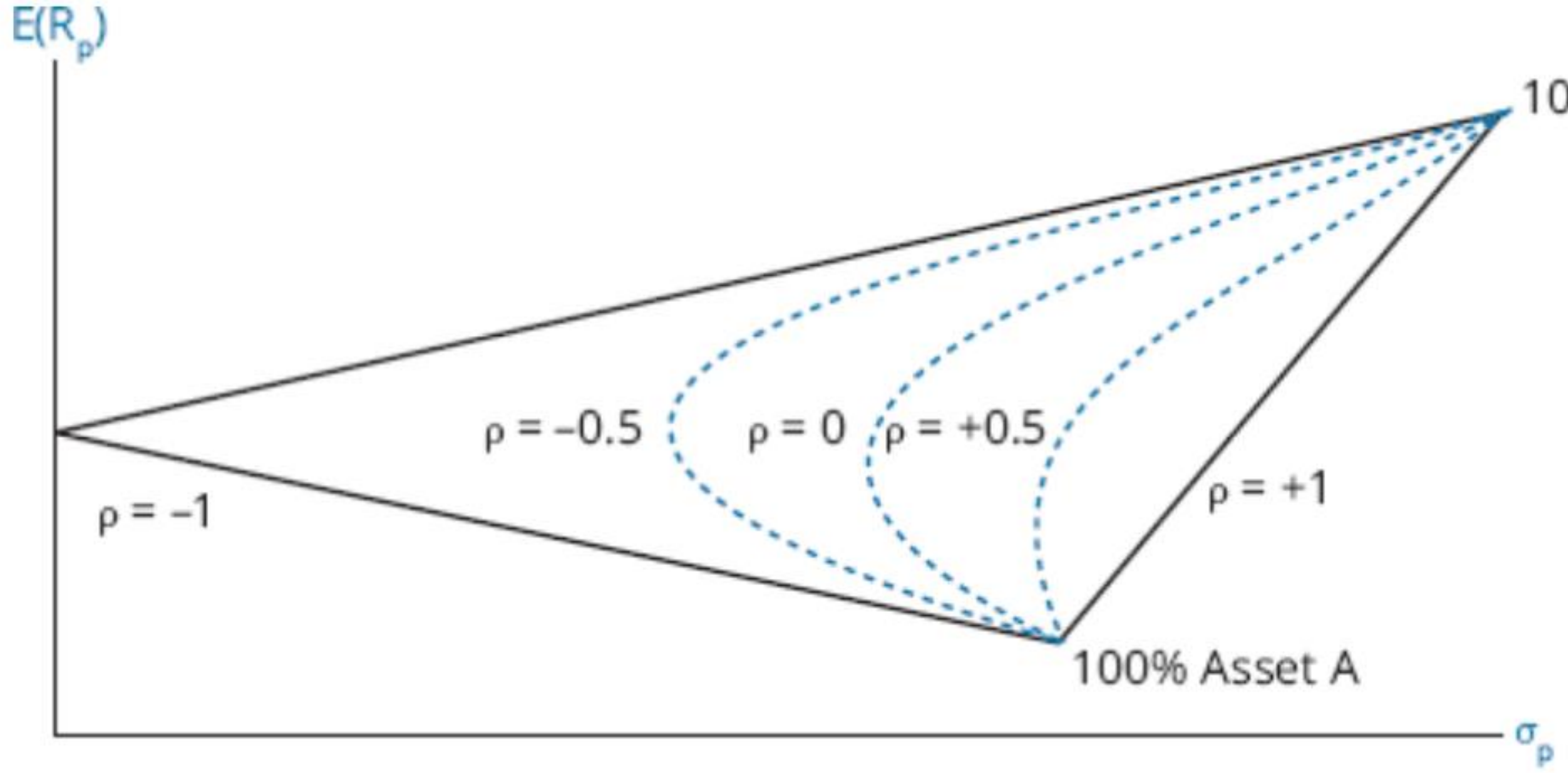
$$\sigma^2 = (0.5^2)0.0625 + (0.5^2)0.0324 + 2(0.5)(0.5)(\mathbf{-0.5})(0.25)(0.18) = 0.012475$$

$$\sigma = \mathbf{11.17\%}$$

Portfolio risk as correlation varies

- Note that portfolio risk decreases as the correlation between the assets' returns decreases. This is an important result of the analysis of portfolio risk: The lower the correlation of asset returns, the greater the risk reduction (diversification) benefit of combining assets in a portfolio. If asset returns were perfectly negatively correlated, portfolio risk could be eliminated altogether for a specific set of asset weights.
- From these analyses, the risk reduction benefits of investing in assets with low return correlations should be clear. The desire to reduce risk is what drives investors to invest in not just domestic stocks, but also bonds, foreign stocks, real estate, and other asset classes.

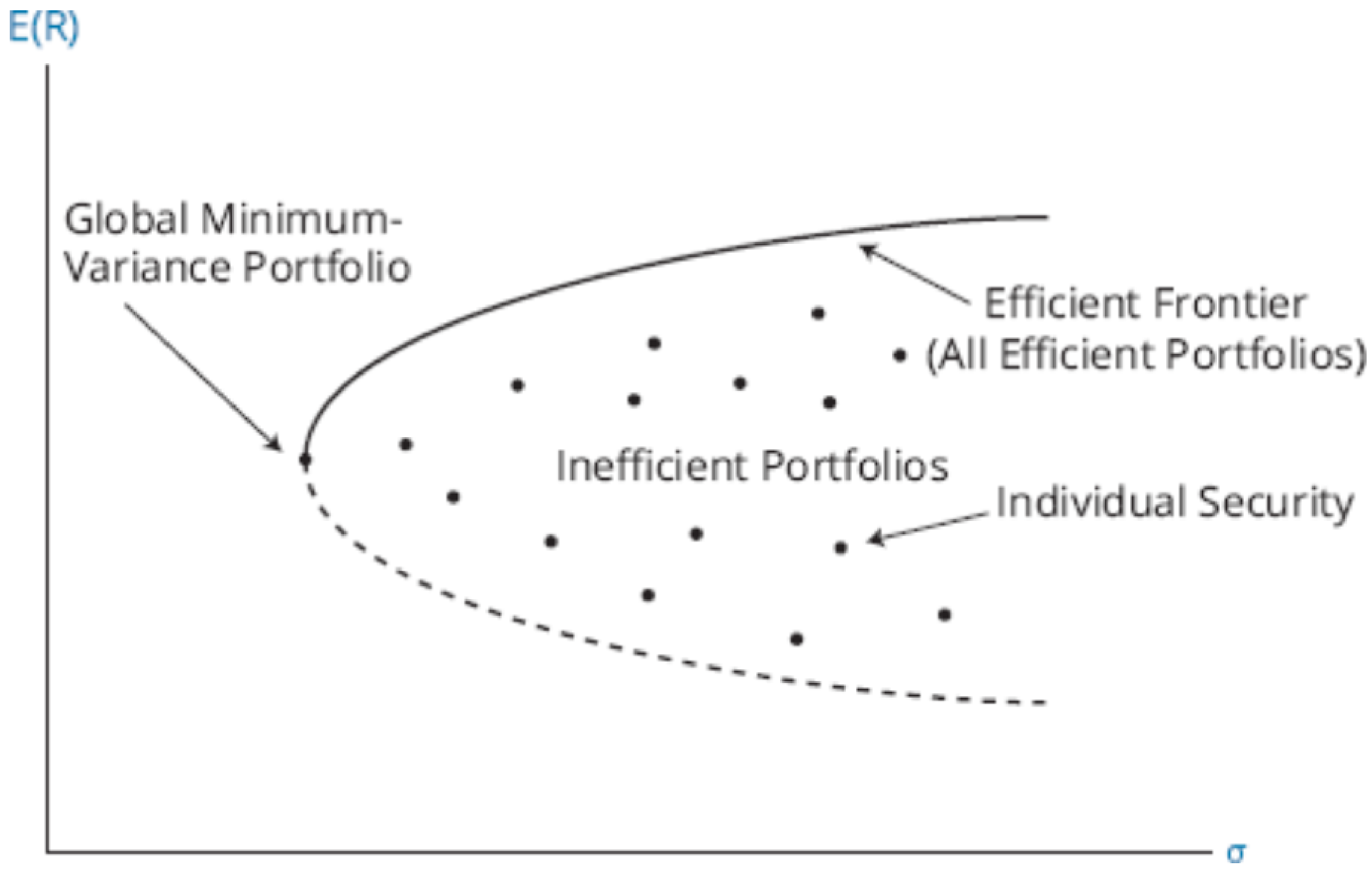
Portfolio risk as correlation varies



THE EFFICIENT FRONTIER

- For each level of expected portfolio return, we can vary the portfolio weights on the individual assets to determine the portfolio that has the least risk. These portfolios that have the lowest standard deviation of all portfolios with a given expected return are known as **minimum variance portfolios**. Together they make up the **minimum-variance frontier**.
- Assuming that investors are risk averse, investors prefer the portfolio that has the greatest expected return when choosing among portfolios that have the same standard deviation of returns. Those portfolios that have the greatest expected return for each level of risk (standard deviation) make up the **efficient frontier**. The portfolio on the efficient frontier that has the least risk is the **global minimum-variance portfolio**.

THE EFFICIENT FRONTIER



Risk aversion

- A **risk-averse investor** is simply one that dislikes risk (i.e., prefers less risk to available online. more risk). Given two investments that have equal expected returns, a risk-averse investor will choose the one with less risk (standard deviation, σ). Financial models assume all investors are risk averse.
- A **risk-seeking (risk-loving) investor** would actually prefer more risk to less and, given equal expected returns, would prefer the more risky investment.
- A **risk-neutral investor** would have no preference regarding risk and would therefore be indifferent between any two investments with equal expected returns

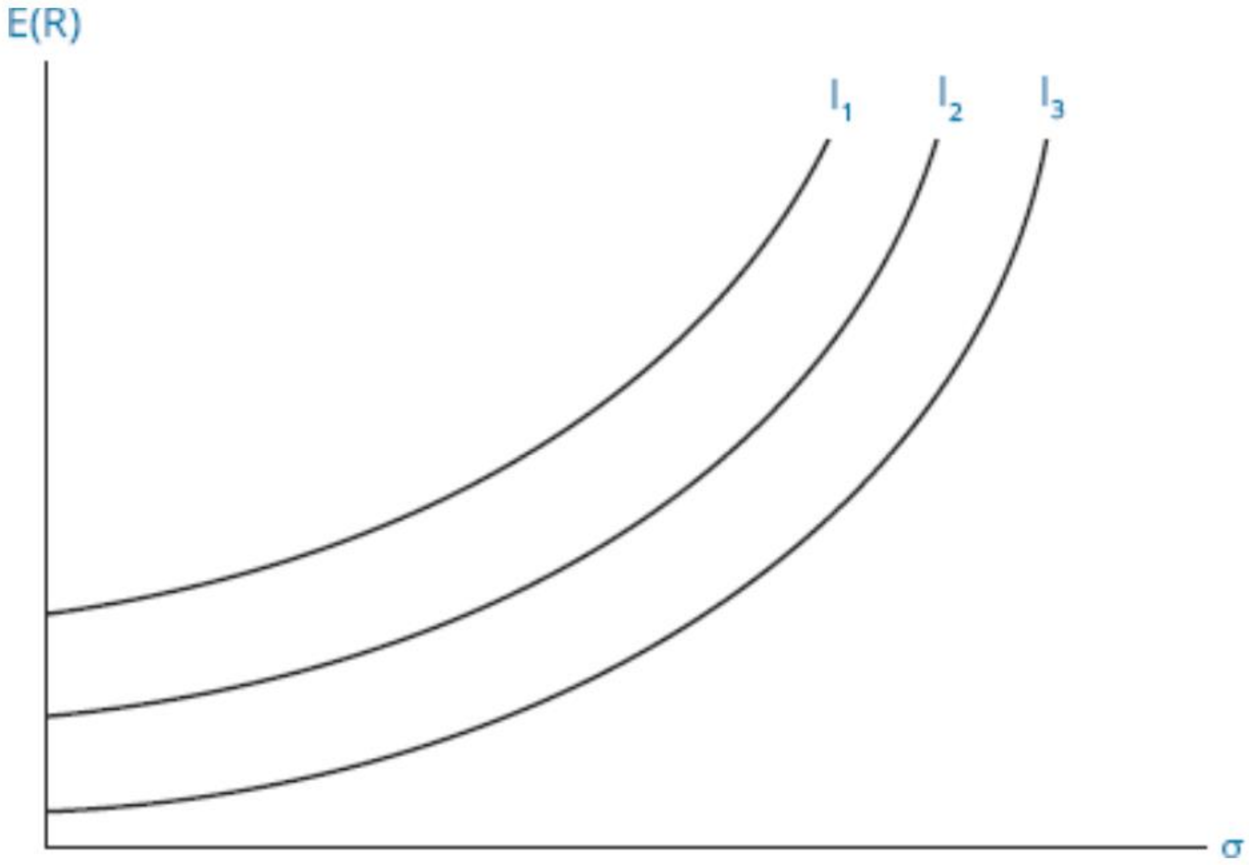
Risk aversion

- Consider this gamble: A coin will be flipped; if it comes up heads, you receive $100; if it comes up tails, you receive nothing. The expected payoff is 0.5($100) + 0.5($0) = $50.
- A risk-averse investor would choose a payment of $50 (a certain outcome) over the gamble.
- A risk-seeking investor would prefer the gamble to a certain payment of $50.
- A risk-neutral investor would be indifferent between the gamble and a certain payment of $50.

Investors’ utility functions and indifference curve

- **Investors’ utility functions** represent their preferences regarding the tradeoff between risk and return (i.e., their degrees of risk aversion).
- An **indifference curve** is a tool from economics that, in this application, plots combinations of risk (standard deviation) and expected returns among which an investor is indifferent.

Investors' utility functions and indifference curve



Two-fund separation theorem

- In our previous illustration of efficient portfolios available in the market, we included only risky assets.
- Now we will introduce a **risk-free asset** into our universe of available assets, and we will examine the risk and return characteristics of a portfolio that combines a portfolio of risky assets and a risk-free asset.

Two-fund separation theorem

- We can calculate the expected return and standard deviation of a portfolio with weight WA allocated to risky Asset A and weight WB allocated to risky Asset B using the following formulas:

$$E(R_{portfolio}) = W_A E(R_A) + W_B E(R_B)$$

$$\sigma_{portfolio} = \sqrt{W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \rho_{AB} \sigma_A \sigma_B}$$

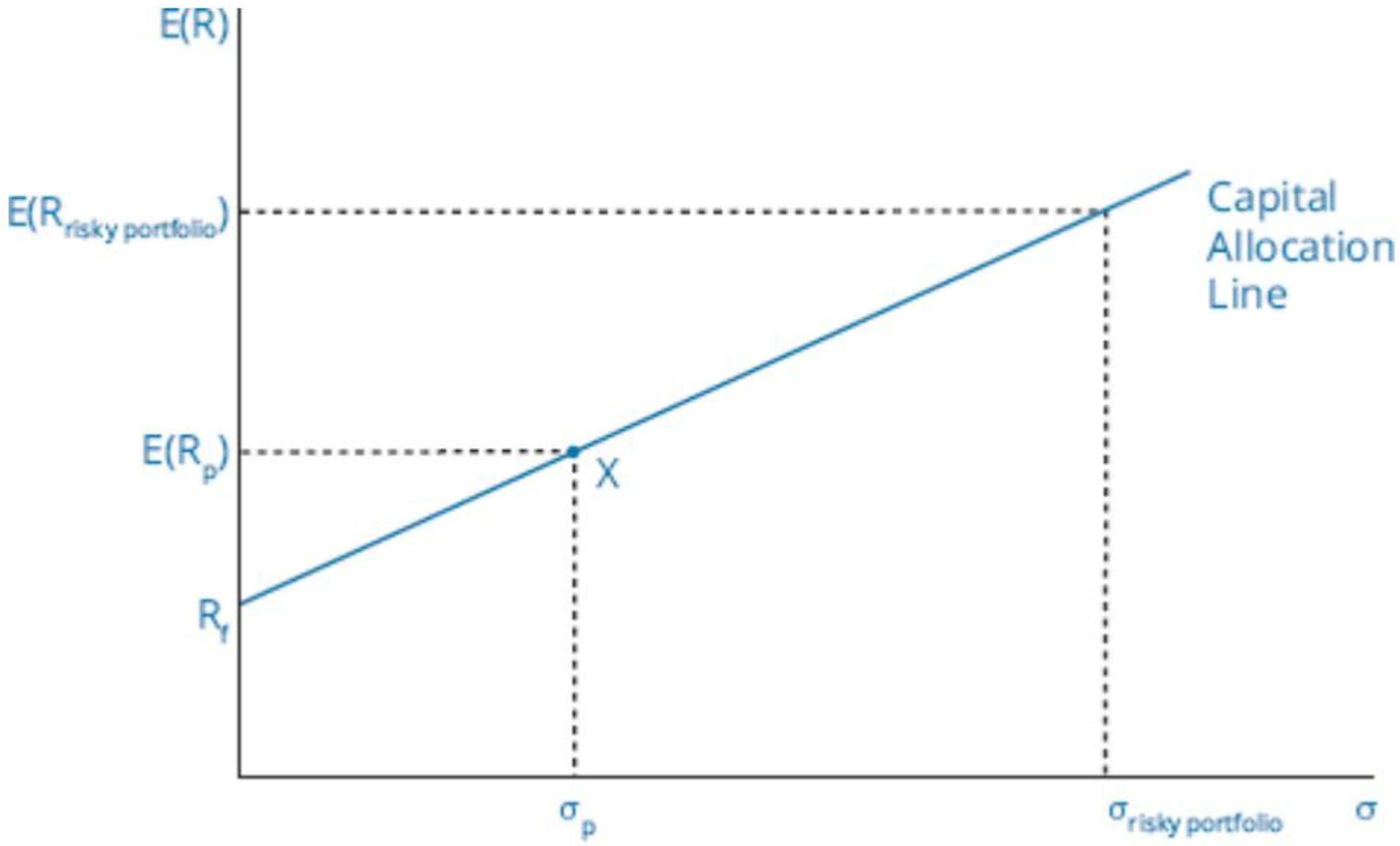
- Because a risk-free asset has **zero standard deviation** and **zero correlation** of returns with those of a risky portfolio, this results in the reduced equation:

$$\sigma_{portfolio} = \sqrt{W_A^2 \sigma_A^2} = W_A \sigma_A$$

Two-fund separation theorem

- Combining a risky portfolio with a risk-free asset is the process that supports the **two-fund separation theorem**, which states that all investors' optimal portfolios will be made up of some combination of the optimal portfolio of risky assets and the risk-free asset.
- The line representing these possible combinations of risk-free assets and the optimal risky asset portfolio is referred to as the **capital allocation line**.

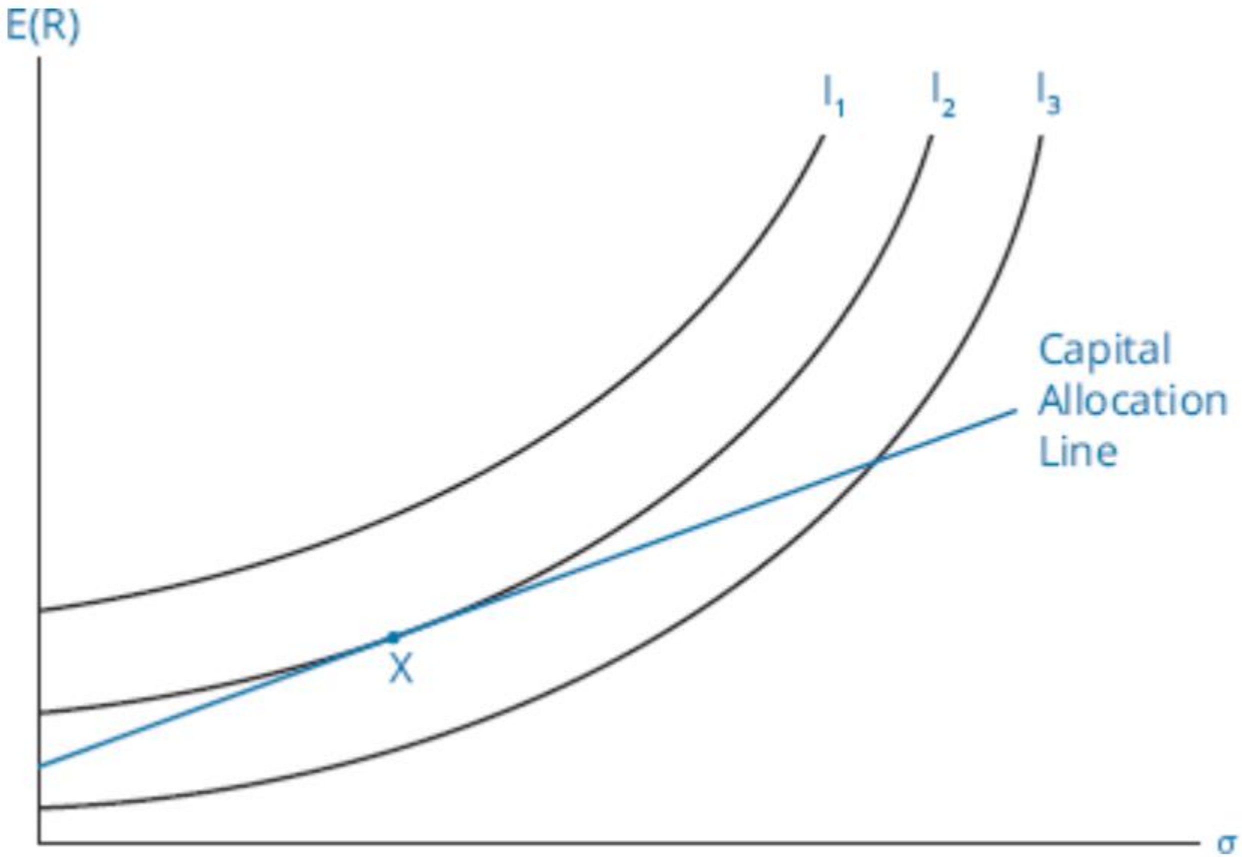
Two-fund separation theorem



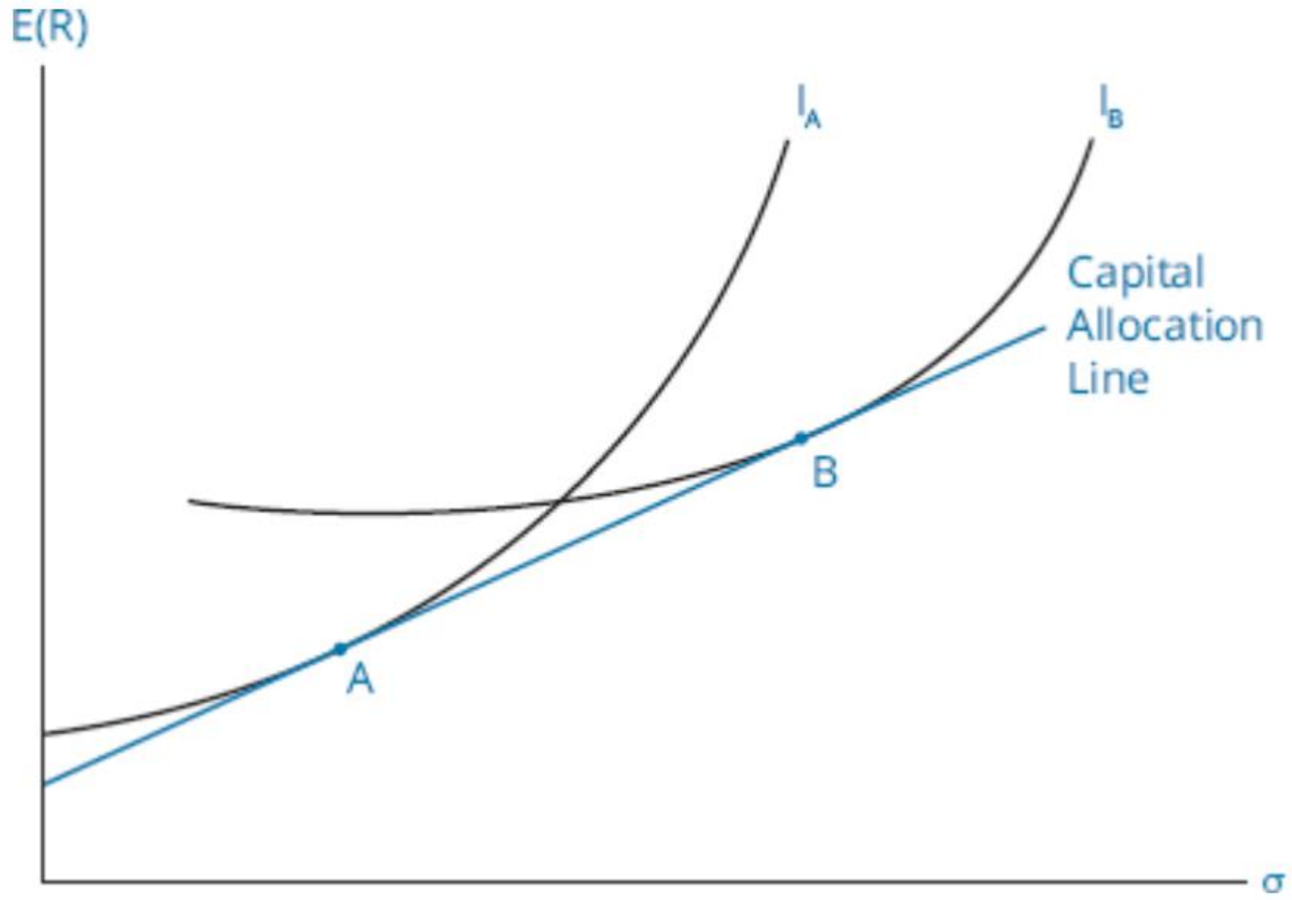
Capital allocation line

- Now that we have constructed a set of the possible efficient portfolios (the capital allocation line),
- we can combine this with indifference curves representing an individual’s preferences for risk and return to illustrate the logic of selecting an optimal portfolio (i.e., one that maximizes the investor’s expected utility).

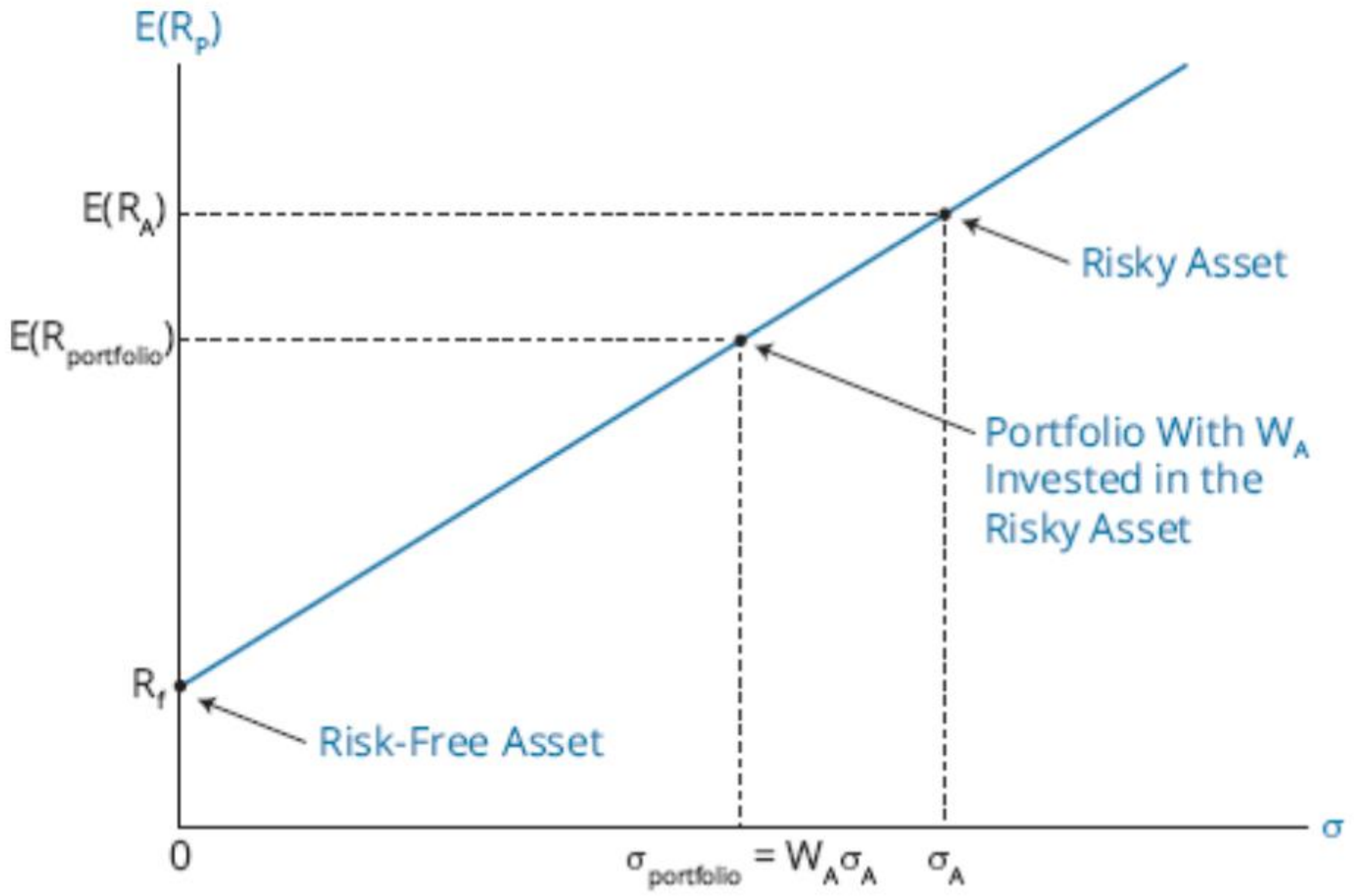
Capital allocation line



Capital allocation line



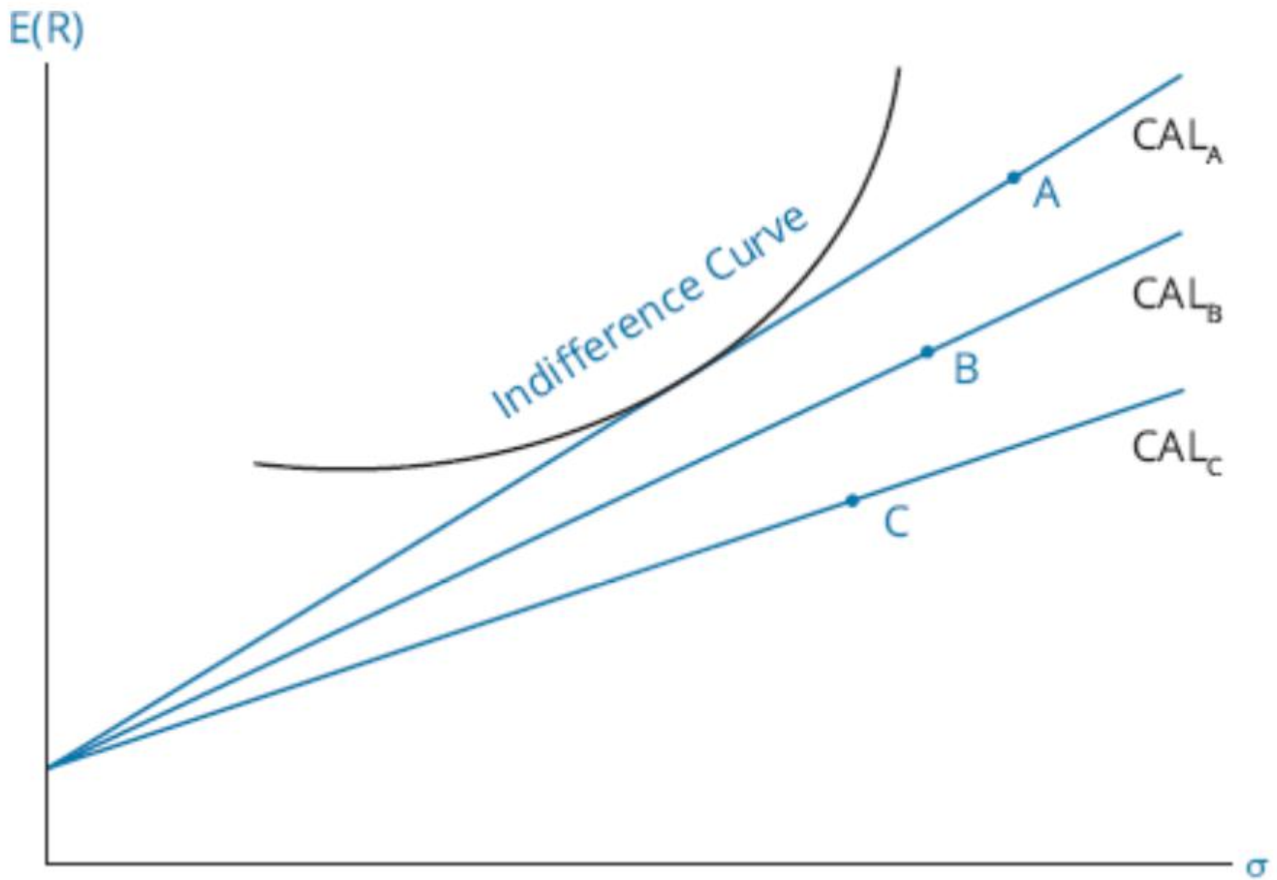
The capital allocation line



The capital allocation line

- The line of possible portfolio risk and return combinations given the risk-free rate and the risk and return of a portfolio of risky assets is referred to as the **capital allocation line (CAL).**
- For an individual investor, the best CAL is the one that offers the **most-preferred set of possible portfolios** in terms of their risk and return.

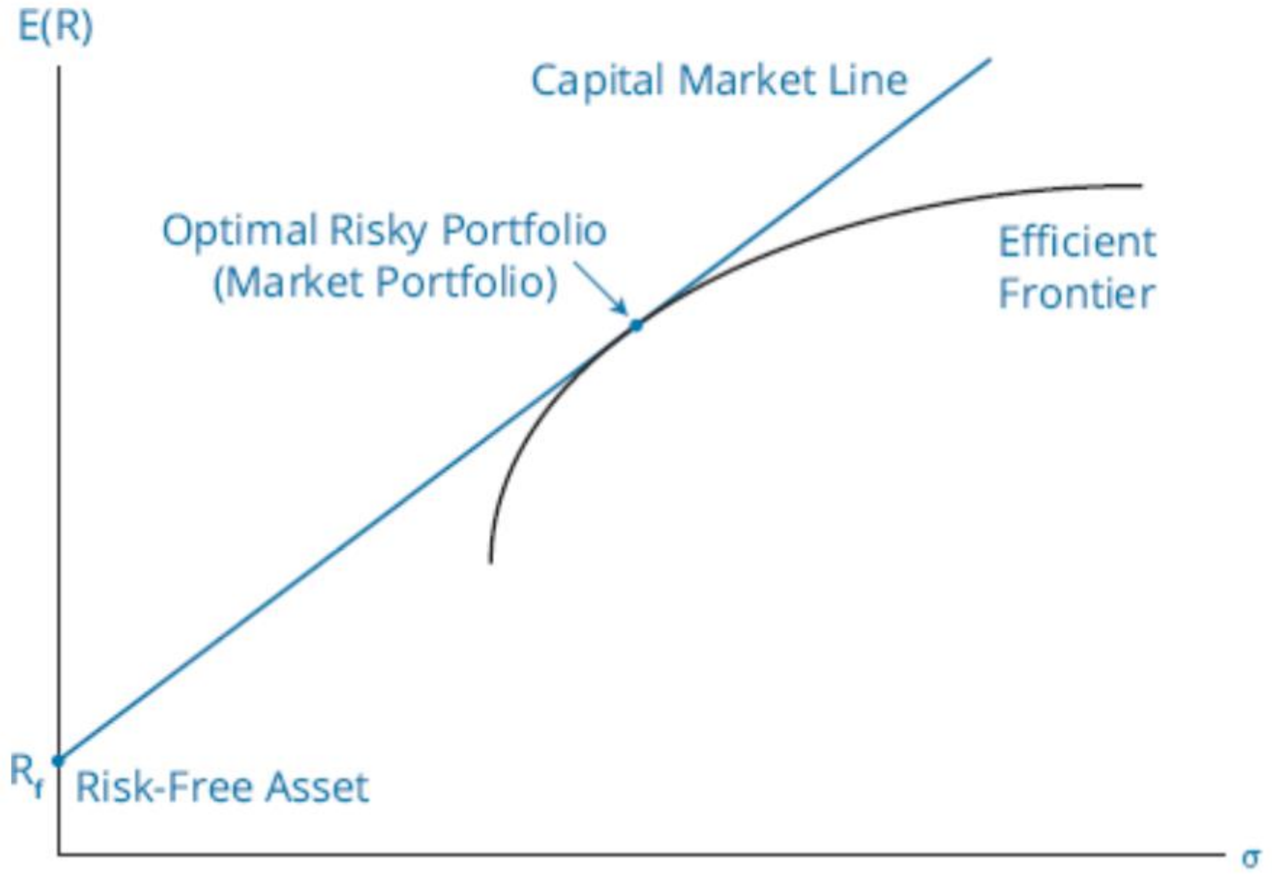
The capital allocation line



The capital allocation line and market portfolio

- If each investor has different expectations about the expected returns of, standard deviations of, or correlations between risky asset returns, each investor will have a different optimal risky asset portfolio and a different CAL.
- A simplifying assumption underlying modern portfolio theory (and the capital asset pricing model, which is introduced later in this reading) is that investors have **homogeneous expectations** (i.e., they all have the same estimates of risk, return, and correlations with other risky assets for all risky assets). Under this assumption, all investors face the same efficient frontier of risky portfolios and will all have the same optimal risky portfolio and CAL.

The capital allocation line and market portfolio



The capital market line

- This optimal CAL for all investors is termed the **capital market line** (CML).
- Along this line, expected portfolio return, $E(R_P)$, is a linear function of portfolio risk, σ_P . The equation of this line is as follows:

$$E(R_P) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M}\right)\sigma_P$$

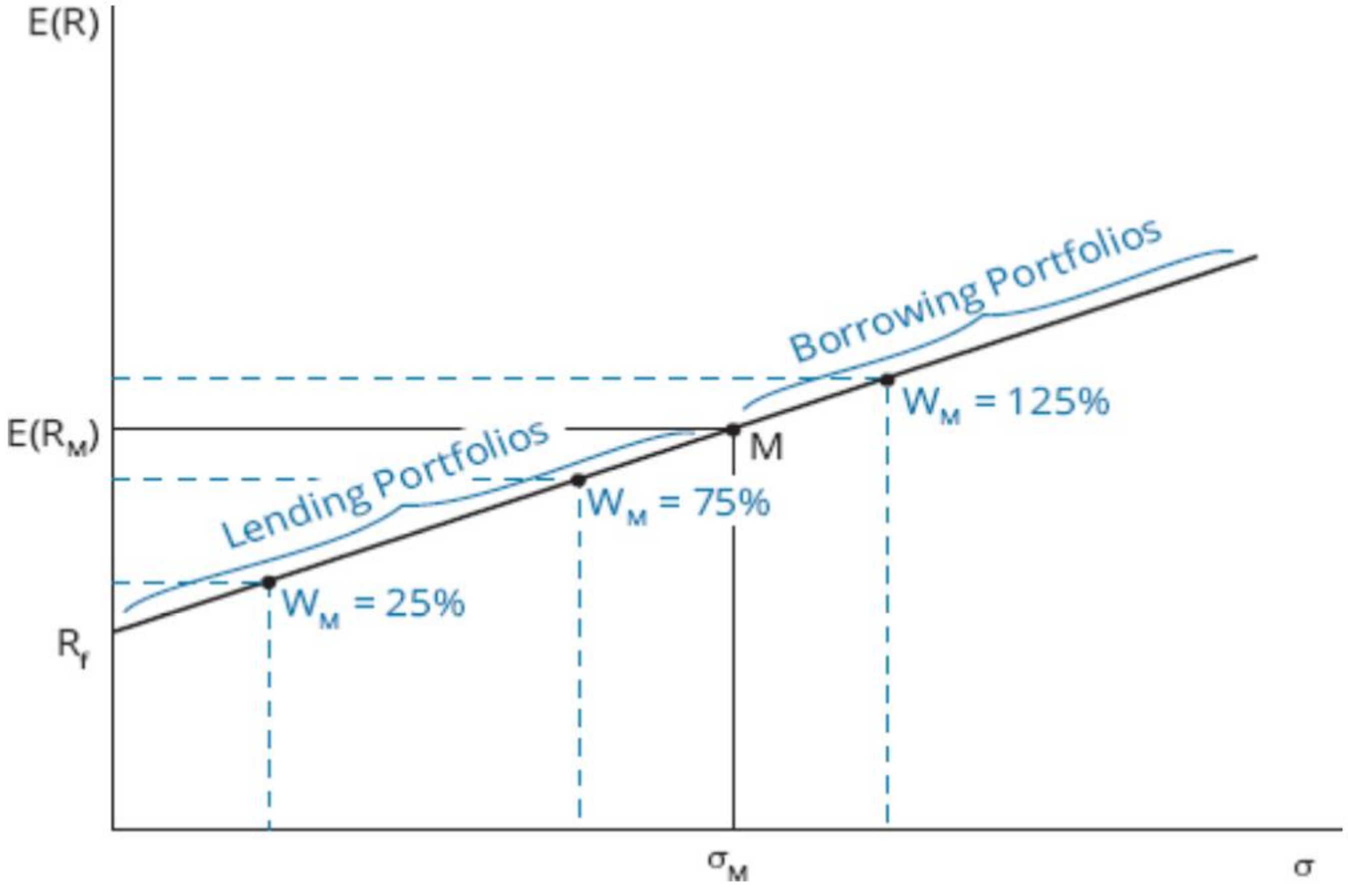
The capital market line

- If we rewrite the CML equation as

$$E(R_P) = R_f + \left(E(R_M) - R_f\right)\left(\frac{\sigma_P}{\sigma_M}\right)$$

- The intuition of this relation is straightforward. An investor who chooses to take on no risk ($\sigma_P = 0$) will earn the risk-free rate, R_f. The difference between the expected return on the market and the risk-free rate is termed the **market risk premium**.
- we can see that an investor can expect to get one unit of market risk premium in additional return (above the risk-free rate) for every unit of market risk, σ_M, that the investor is willing to accept.

The capital market line



Passive and active portfolio management

- Investors who believe market prices are **informationally efficient** often follow a **passive investment strategy** (i.e., invest in an index of risky assets that serves as a proxy for the market portfolio and allocate a portion of their investable assets to a risk-free asset, such as short-term government securities).
- many investors and portfolio managers believe their estimates of security values are correct and market prices are incorrect. Such investors will not use the weights of the market portfolio but will invest more than the market weights in securities that they believe are undervalued and less than the market weights in securities which they believe are overvalued. This is referred to as **active portfolio management** to differentiate it from a passive investment strategy that utilizes a market index for the optimal risky asset portfolio.

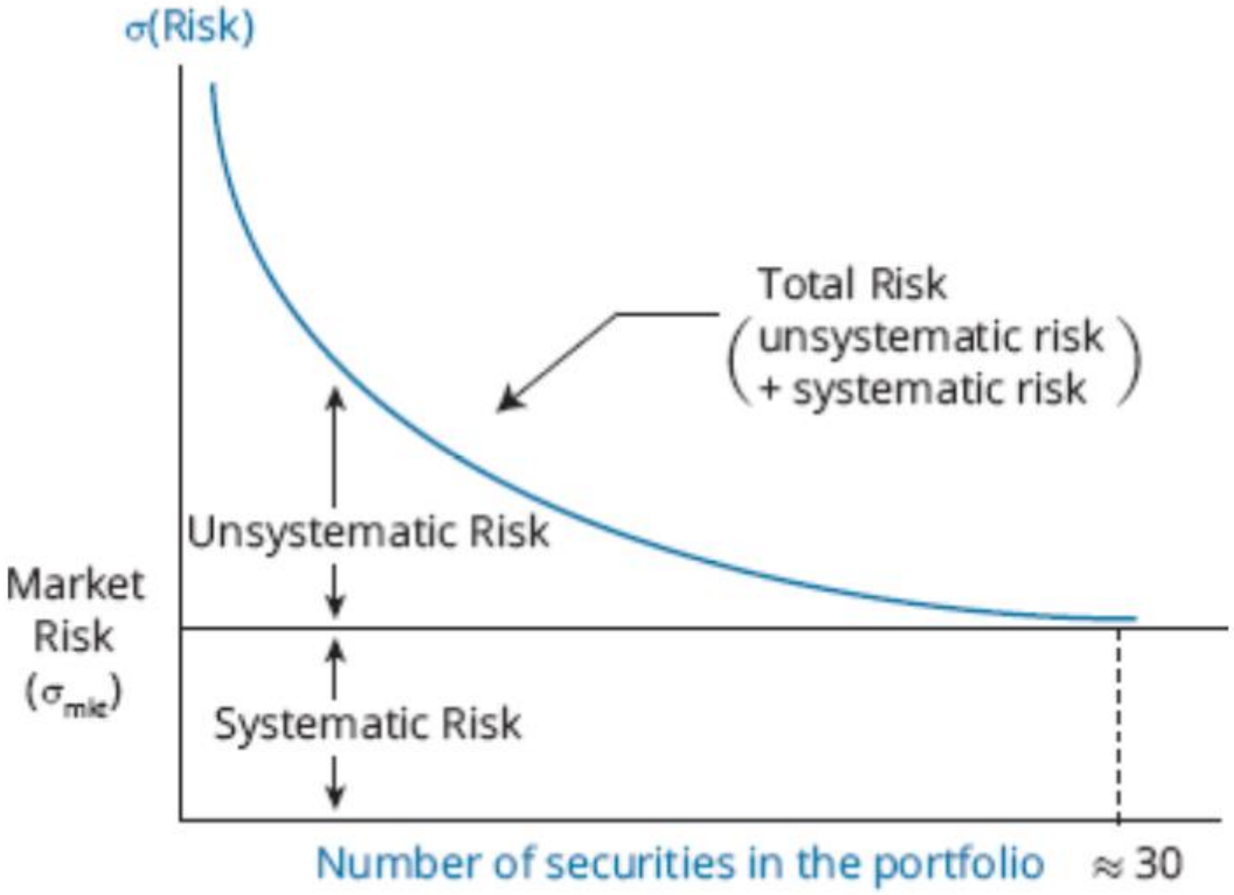
The systematic and nonsystematic risk

- When an investor diversifies across assets that are not perfectly correlated, the portfolio's risk is less than the weighted average of the risks of the individual securities in the portfolio. The risk that is eliminated by diversification is called **unsystematic risk** (also called unique, diversifiable, or firm-specific risk). Because the market portfolio contains all risky assets, it must be a well-diversified portfolio. All the risk that can be diversified away has been.
- The risk that remains cannot be diversified away and is called the **systematic risk** (also called nondiversifiable risk or market risk).

The systematic and nonsystematic risk

- The concept of systematic risk applies to individual securities as well as to portfolios. Some securities' returns are highly correlated with overall market returns. Examples of firms that are highly correlated with market returns are luxury goods manufacturers such as Ferrari automobiles and Harley Davidson motorcycles. These firms have high systematic risk (i.e., they are very responsive to market, or systematic, changes). Other firms, such as utility companies, respond very little to changes in the systematic risk factors. These firms have very little systematic risk. Hence, total risk (as measured by standard deviation) can be broken down into its component parts: unsystematic risk and systematic risk. Mathematically:
    - total risk = systematic risk + unsystematic risk

The systematic and nonsystematic risk



Return generating models

- **Return generating models** are used to estimate the expected returns on risky securities based on specific factors. For each security, we must estimate the sensitivity of its returns to each specific factor. Factors that explain security returns can be classified as macroeconomic, fundamental, and statistical factors.
- **Multifactor models** most commonly use macroeconomic factors such as GDP growth, inflation, or consumer confidence, along with fundamental factors such as earnings, earnings growth, firm size, and research expenditures.

Multifactor models

- The general form of a multifactor model with k factors is as follows:

$$E(R_i) - R_f = \beta_{i1} \times E(\text{Factor } 1) + \beta_{i2} \times E(\text{Factor } 2) + \ldots + \beta_{ik} \times E(\text{Factor } k)$$

- This model states that the expected excess return (above the risk-free rate) for Asset i is the sum of each factor sensitivity or factor loading (the βs) for Asset i multiplied by the expected value of that factor for the period. The first factor is often the expected excess return on the market, $E(R_m - R_f)$. Statistical factors often have **no basis in finance theory** and are suspect in that they may represent only relations for **a specific time period** which have been identified by **data mining** (repeated tests on a single dataset).

The beta

- A simplified form of a multifactor model is the single-index market model. The form of the market model is as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

where:

R_i = return on Asset *i*
R_m = market return
β_i = slope coefficient
α_i = intercept
e_i = abnormal return on Asset *i*

The beta

- The sensitivity of an asset's return to the return on the market index in the context of the market model is referred to as its beta. Beta is a standardized measure of the covariance of the asset's return with the market return. Beta can be calculated as follows:

$$\beta_i = \frac{\text{covariance of Asset } i\text{'s return with the market return}}{\text{variance of the market return}} = \frac{\text{Cov}_{im}}{\sigma_m^2}$$

The beta

- We can use the definition of the correlation between the returns on Asset i with the returns on the market index:

$$\rho_{im} = \frac{\mathrm{Cov}_{im}}{\sigma_i \sigma_m}$$

$$\text{to get } \mathrm{Cov}_{im} = \rho_{im} \sigma_i \sigma_m$$

- Substituting for Cov_{im} in the equation for β_i, we can also calculate beta as:

$$\beta_i = \frac{\rho_{im} \sigma_i \sigma_m}{\sigma_m^2} = \rho_{im}\left(\frac{\sigma_i}{\sigma_m}\right)$$

EXAMPLE: Calculating an asset's beta

- The standard deviation of the return on the market index is estimated as 20%.
- 1. If Asset A's standard deviation is 30% and its correlation of returns with the market index is 0.8, what is Asset A's beta?
- 2. If the covariance of Asset A's returns with the returns on the market index is 0.048, what is the beta of Asset A?

EXAMPLE: Calculating an asset's beta

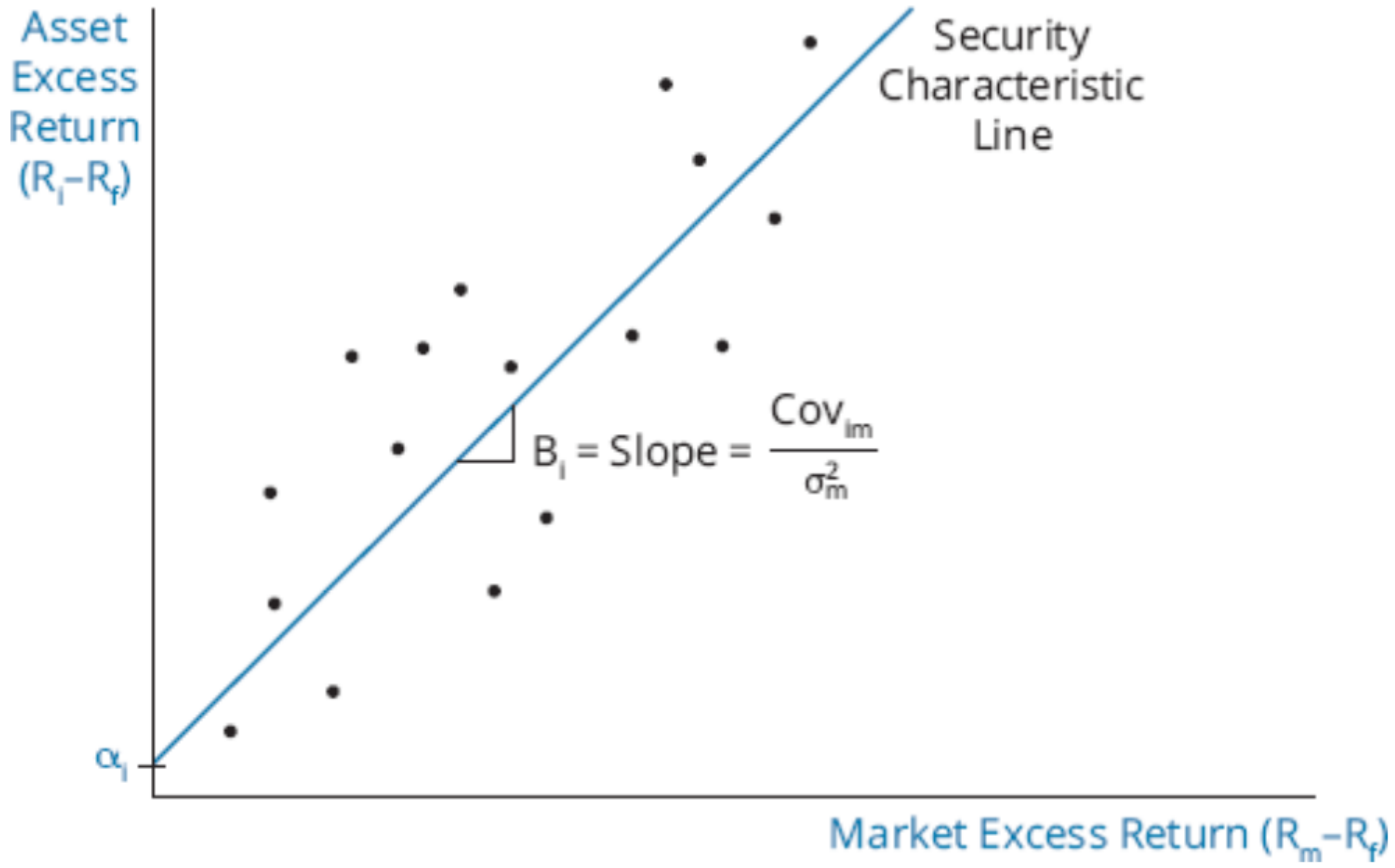
- 1.

$$\beta_i = \rho_{im}\left(\frac{\sigma_i}{\sigma_m}\right), \text{we have: } \beta_i = 0.80\left(\frac{0.30}{0.20}\right) = 1.2.$$

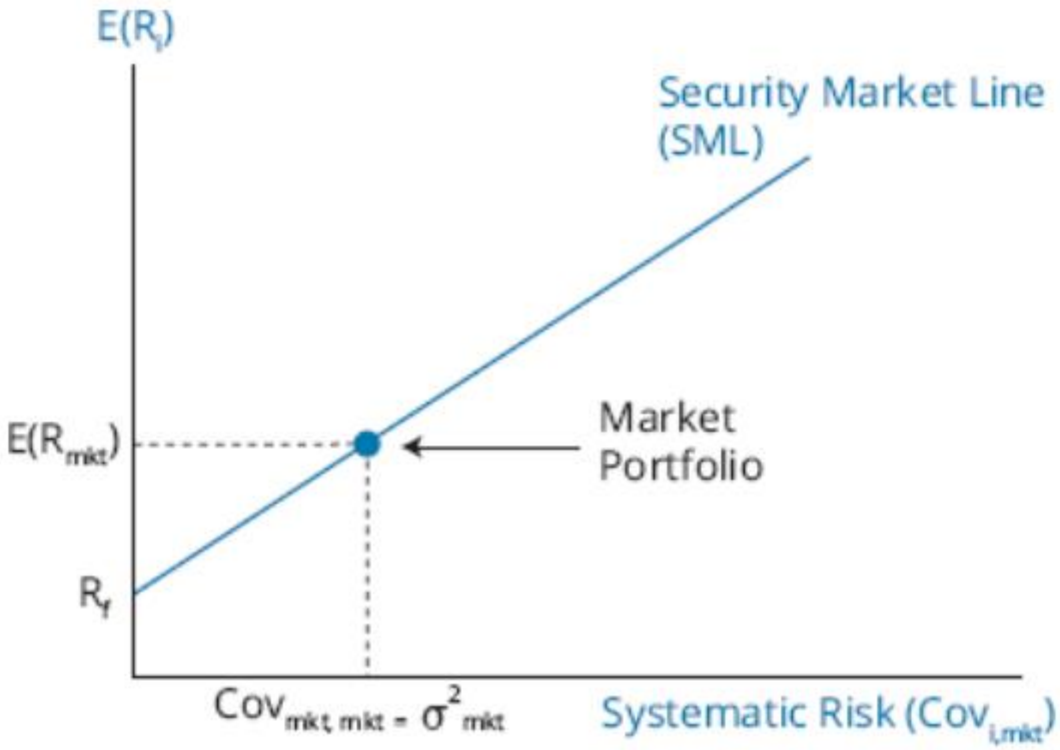
- 2.

$$\beta_i = \frac{Cov_{im}}{\sigma_m^2}, \text{ we have } \beta_i = \frac{0.048}{0.2^2} = 1.2.$$

The beta and the security characteristic line



THE CAPM AND THE SML



THE CAPM AND THE SML

- The equation of the SML is:

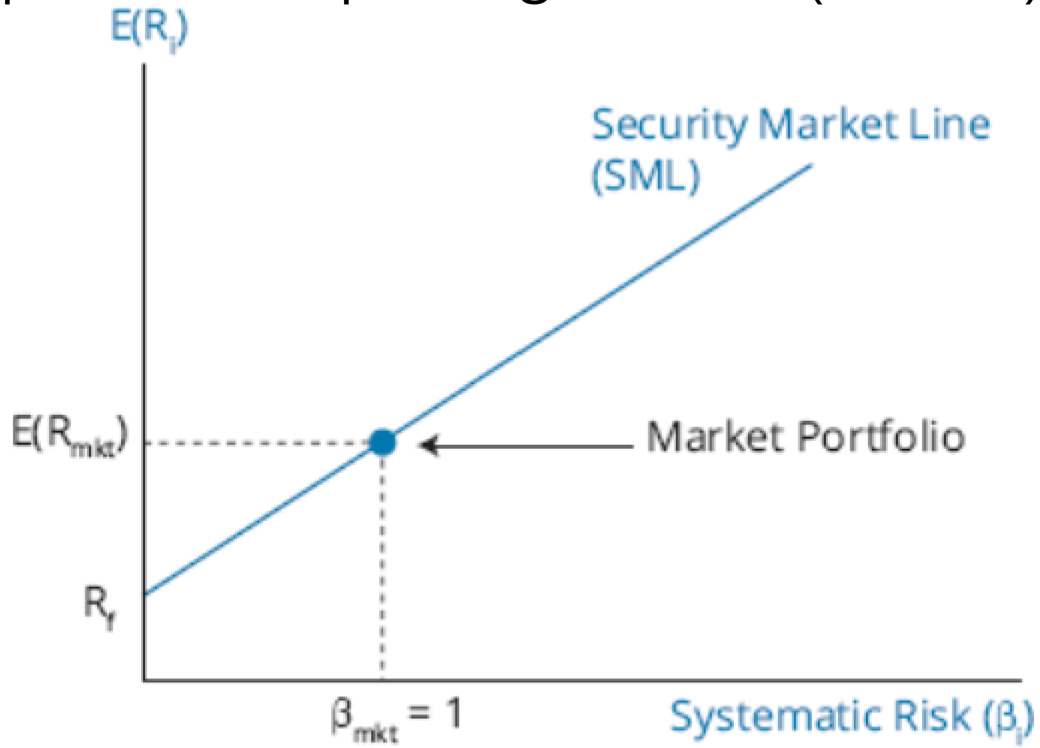
$$E(R_i) = R_f + \frac{E(R_{mkt}) - R_f}{\sigma^2_{mkt}}(Cov_{i,mkt})$$

- which can be rearranged and stated as:

$$E(R_i) = R_f + \frac{Cov_{i,mkt}}{\sigma^2_{mkt}}[E(R_{mkt}) - R_f]$$

THE CAPM

- This is the most common means of describing the SML, and this relation between beta (systematic risk) and expected return is known as the capital asset pricing model (CAPM).



THE CAPM

- Formally, the CAPM is stated as:

$$E(R_i) = R_f + \beta_i[E(R_{mkt}) - R_f]$$

- The CAPM holds that, in equilibrium, the expected return on risky asset $E(Ri)$ is the risk-free rate (R_f) plus a beta-adjusted market risk premium, $\beta_i[E(R_{mkt}) - Rf]$. Beta measures systematic (market or covariance) risk.

EXAMPLE: Capital asset pricing model

- The expected return on the market is 8%, the risk-free rate is 2%, and the beta for Stock A is 1.2. Compute the rate of return that would be expected (required) on this stock.

EXAMPLE: Capital asset pricing model

- The expected return on the market is 8%, the risk-free rate is 2%, and the beta for Stock A is 1.2. Compute the rate of return that would be expected (required) on this stock.

$$E(R_A) = 2\% + 1.2(8\% - 2\%) = 9.2\%$$

Note: $\beta_A > 1$, so $E(R_A) > E(R_{mkt})$

THE CAPM

- The assumptions of the CAPM are:
- **Risk aversion**. To accept a greater degree of risk, investors require a higher expected return.
- **Utility maximizing investors**. Investors choose the portfolio, based on their individual preferences, with the risk and return combination that maximizes their (expected) utility.
- **Frictionless markets**. There are no taxes, transaction costs, or other impediments to trading.
- **One-period horizon**. All investors have the same one-period time horizon.

THE CAPM

- The assumptions of the CAPM are:
- **Homogeneous expectations**. All investors have the same expectations for assets’ expected returns, standard deviation of returns, and returns correlations between assets.
- **Divisible assets**. All investments are infinitely divisible.
- **Competitive markets**. Investors take the market price as given and no investor can influence prices with their trades.

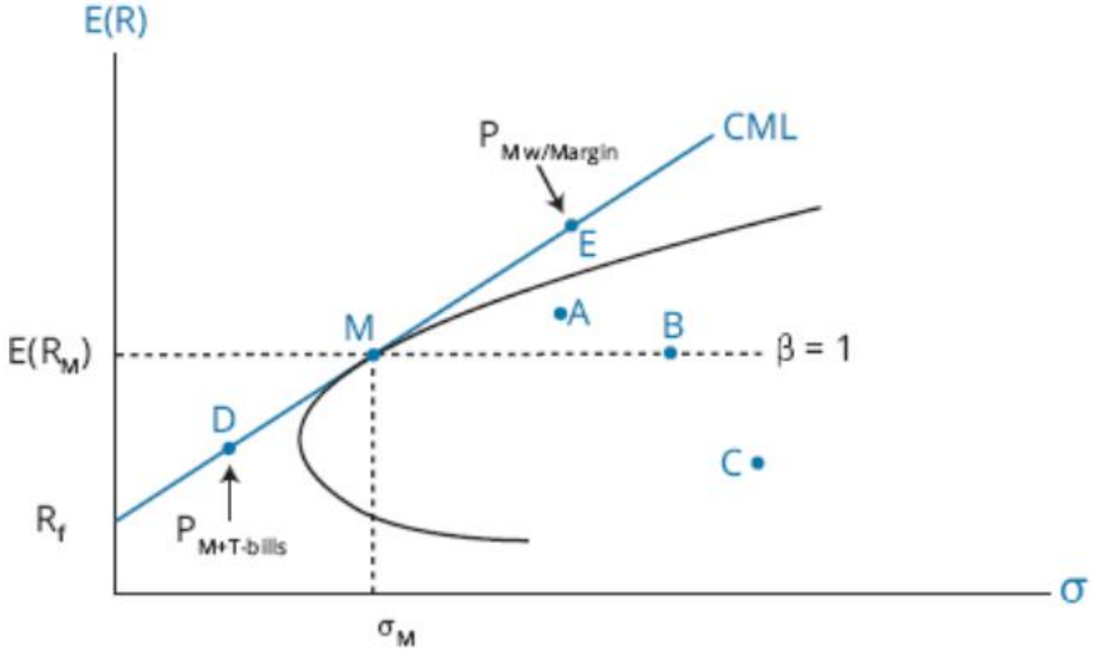
Comparing the CML and the SML

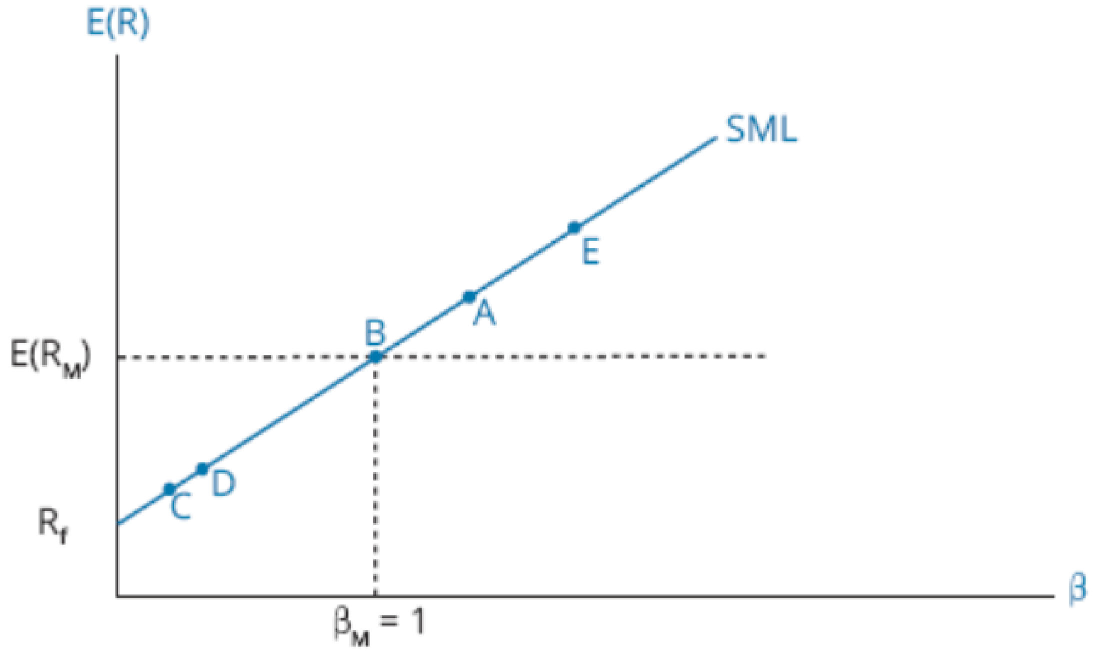
- It is important to recognize that the CML and SML are very different. Recall the equation of the CML:

$$E(R_P) = R_f + \sigma_P \left\{ \frac{[E(R_M) - R_f]}{\sigma_M} \right\}$$

- The CML uses total risk = σ_p on the x-axis. Hence, only efficient portfolios will plot on the CML.
- On the other hand, the SML uses beta (systematic risk) on the x-axis. So in a CAPM world, all properly priced securities and portfolios of securities will plot on the SML

Comparing the CML and the SML





EXAMPLE: Identifying mispriced securities

- The following figure contains information based on analyst's forecasts for three stocks. Assume a risk-free rate of 7% and a market return of 15%. Compute the expected and required return on each stock, determine whether each stock is undervalued, overvalued, or properly valued, and outline an appropriate trading strategy.

Forecast Data

Stock	Price Today	E(Price) in 1 Year	E(Dividend) in 1 Year	Beta
A	$25	$27	$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

EXAMPLE: Identifying mispriced securities

- Expected and required returns computations are shown in the following figure.

Forecasts vs. Required Returns

Stock	Forecast Return	Required Return
A	(\$27 – \$25 + \$1) / \$25 = 12.0%	0.07 + (1.0)(0.15 – 0.07) = 15.0%
B	(\$45 – \$40 + \$2) / \$40 = 17.5%	0.07 + (0.8)(0.15 – 0.07) = 13.4%
C	(\$17 – \$15 + \$0.5) / \$15 = 16.6%	0.07 + (1.2)(0.15 – 0.07) = 16.6%

EXAMPLE: Identifying mispriced securities

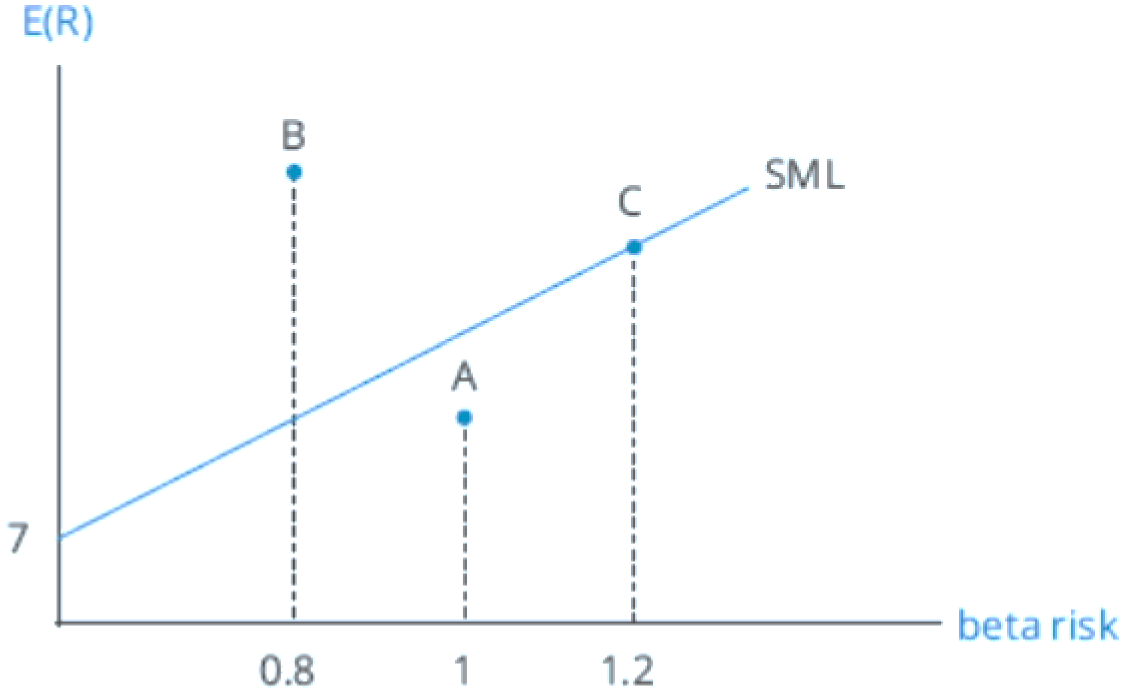
- Stock A is overvalued. It is expected to earn 12%, but based on its systematic risk, it should earn 15%. It plots below the SML.
- Stock B is undervalued. It is expected to earn 17.5%, but based on its systematic risk, it should earn 13.4%. It plots above the SML.
- Stock C is properly valued. It is expected to earn 16.6%, and based on its systematic risk, it should earn 16.6%. It plots on the SML.

EXAMPLE: Identifying mispriced securities

- The appropriate trading strategy is:
- Short sell Stock A.
- Buy Stock B.
- Buy, sell, or ignore Stock C.

EXAMPLE: Identifying mispriced securities

- We can do this same analysis graphically. The expected return/beta combinations of all three stocks are graphed in the following figure relative to the SML.



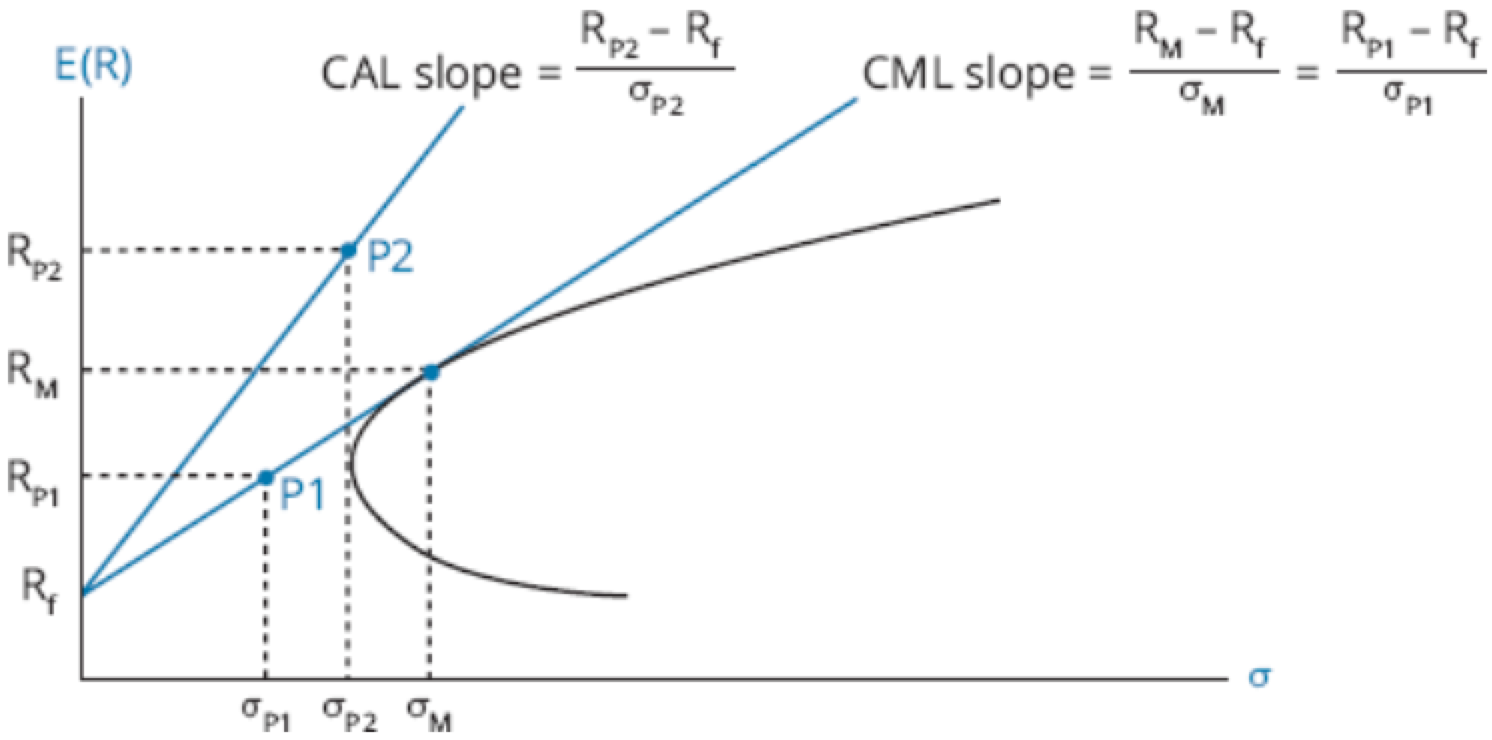
Portfolio Evaluation and Risk Management

Sharpe ratio

- To consider both risk and return in evaluating portfolio performance, the most commonly used is the **Sharpe ratio**.
- The Sharpe ratio of a portfolio is its excess returns per unit of total portfolio risk. Higher Sharpe ratios indicate better **risk-adjusted portfolio performance**.

$$\text{Sharpe ratio} = \frac{E[R_{\text{portfolio}}] - R_f}{\sigma_{\text{portfolio}}}$$

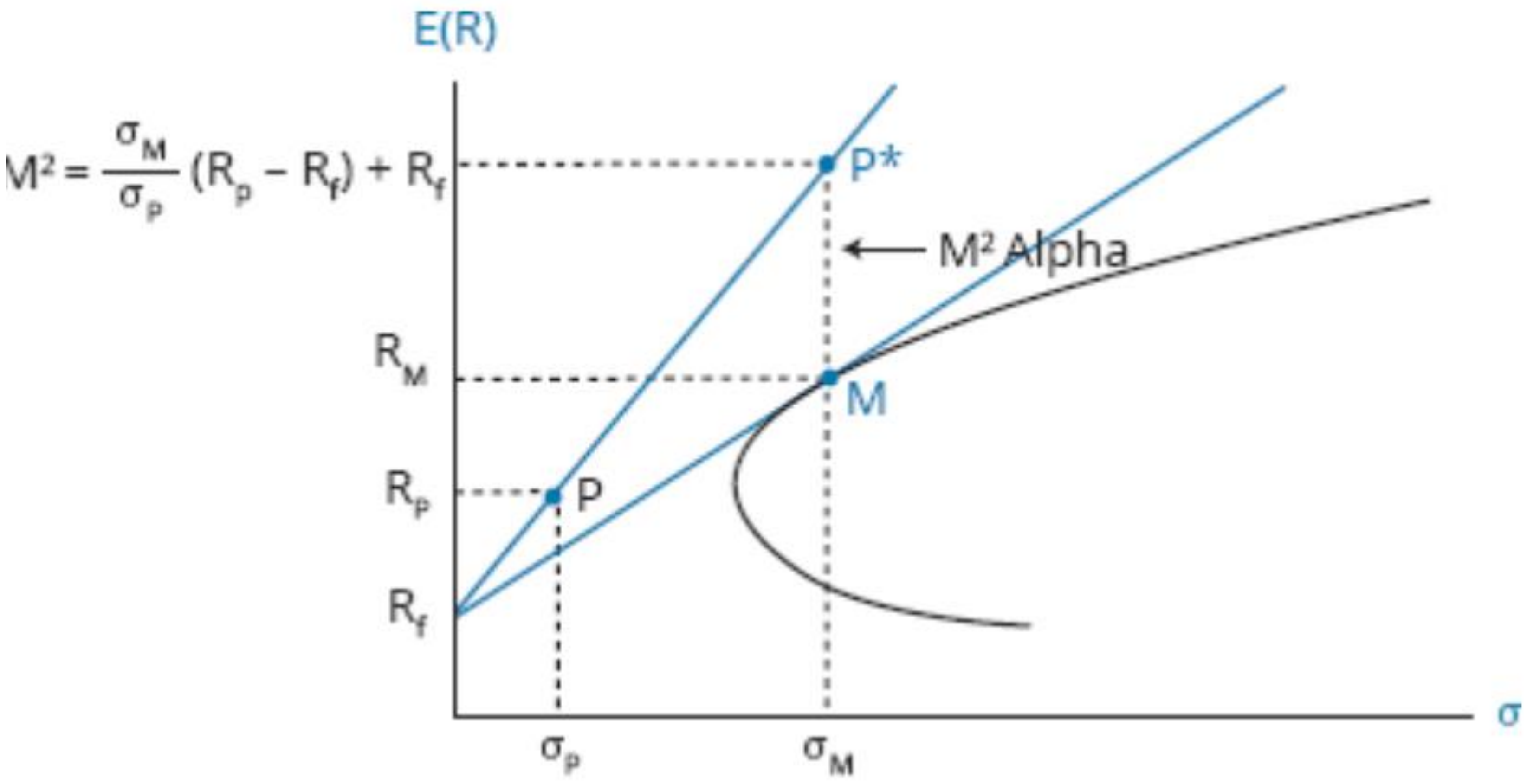
Sharpe ratio



M-squared

- For a portfolio of risky assets, **M-squared (M^2)** is an alternative to the Sharpe ratio as a risk-adjusted rate of return, expressed as a percentage rather than as a slope.

M-squared



E(R)
$M^2 = \frac{\sigma_M}{\sigma_p}(R_p - R_f) + R_f$
P*
M² Alpha
R_M
M
R_p
P
R_f
σ
σ_p
σ_M

M-squared

- Given a Portfolio P, we can calculate the return on a Portfolio P^* that is leveraged (when $\sigma_M > \sigma_P$), or deleveraged (when $\sigma_M < \sigma_P$), so that P* has the same risk (standard deviation of returns) as the market portfolio. The return on P^* is

$$R_f + \frac{\sigma_M}{\sigma_P}(R_P - R_f)$$

- and we refer to that as the M^2 measure for Portfolio P.
- The extra return on the Portfolio P^* above the return on the market portfolio, $(P^* – R_M)$, is referred to as **M^2 alpha**.

M-squared an Sharpe ratio

- The M^2 measure produces the same risk-adjusted portfolio rankings as the Sharpe ratio, but is stated in percentage terms. Note that M^2 can be derived from the Sharpe ratio (SR) for Portfolio P,
  - $SR = (R_P – R_f)/\sigma_P$,
- as SR (σ_M) + R_f, so that if the Sharpe ratio of Portfolio P is greater than the slope of the CML, $M^2 > R_m$ and M^2 alpha > 0.

Example: M2 measure

- Consider a Portfolio P with return of 10% and standard deviation of returns of 20%, when R_f = 5%, R_M = 11% and σ_M = 30%. Compute the M2 measure and M2 alpha.

Example: M2 measure

- Consider a Portfolio P with return of 10% and standard deviation of returns of 20%, when R_f = 5%, R_M = 11% and σ_M = 30%. Compute the M2 measure and M2 alpha.

- The Sharpe ratio of Portfolio P = (10 – 5)/20 = 0.25, and M^2 = 0.25(0.30) + 0.05 = 12.5%. Comparing that to R_M = 11%, we can see that M^2 alpha is 1.5%.

Treynor measure and Jensen's alpha

- The **Treynor measure** is calculated as

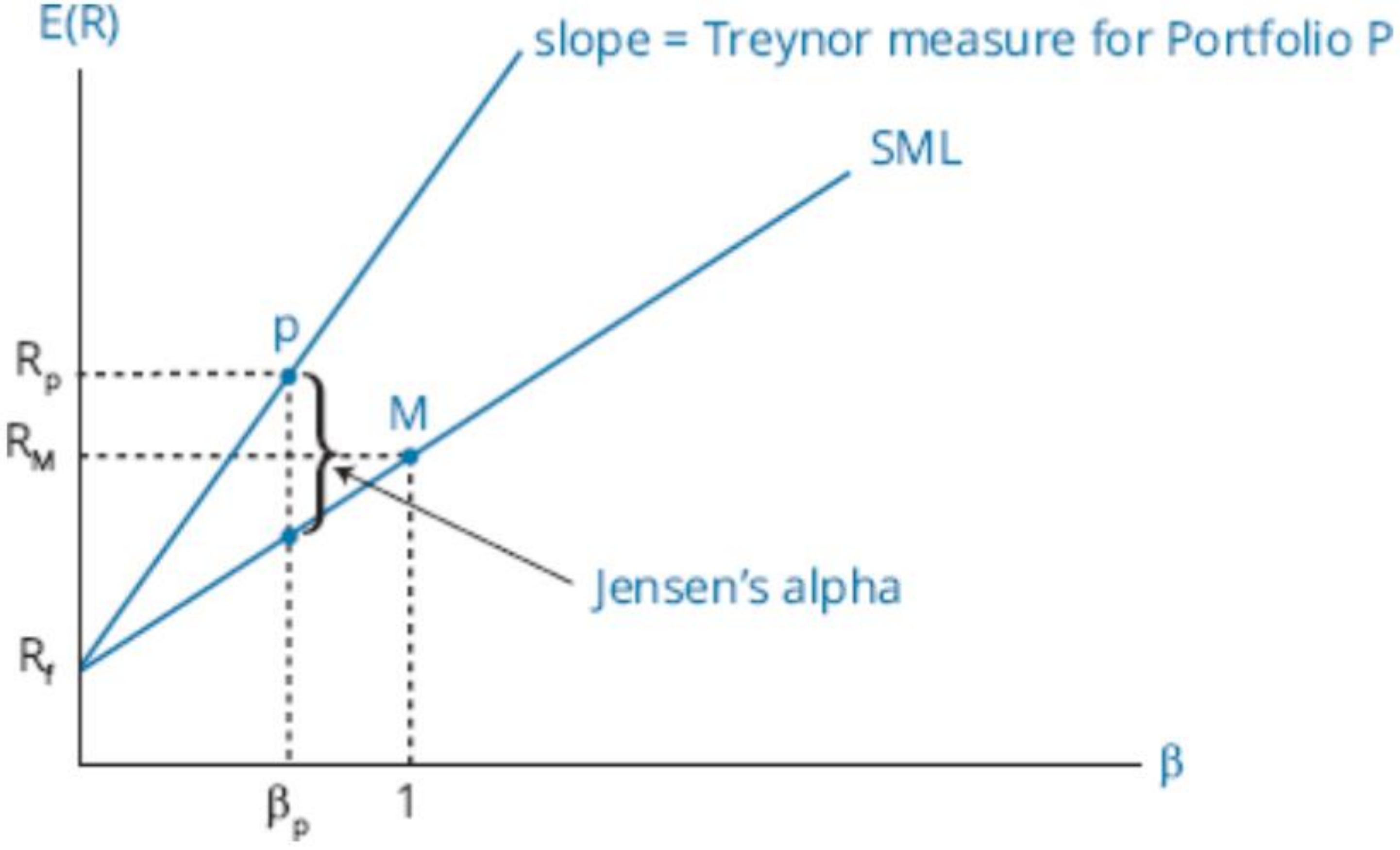
$$\frac{R_P - R_f}{\beta_P}$$

- interpreted as excess returns per unit of systematic risk, and represented by the slope of a line. **Jensen's alpha** for Portfolio P is calculated as

$$\alpha_P = R_p - [R_f + \beta_P(R_M - R_f)]$$

- and is the percentage portfolio return above that of a portfolio (or security) with the same beta as the portfolio that lies on the SML

Treynor measure and Jensen's alpha



Which to pick?

- If the total risk (including any nonsystematic risk) is the relevant measure and risk adjustment using total risk, as with the **Sharpe** and **M2** measures, is appropriate.
- If the fund portfolio is well diversified (has no unsystematic risk), then performance measures based on systematic (beta) risk, such as the **Treynor measure** and **Jensen's alpha**, are appropriate.

RISK MANAGEMENT

- The **risk management** process seeks to 1) identify the risk tolerance of the organization, 2) identify and measure the risks that the organization faces, and 3) modify and monitor these risks.
- The process **does not seek to minimize or eliminate** all of these risks. The organization may increase its exposure to risks it decides to take because it is better able to manage and respond to them.

RISK MANAGEMENT

- Risk (uncertainty) is not something to be avoided by an organization or in an investment portfolio. Returns above the risk-free rate are earned **by taking on risk**.
- The organization may decrease its exposure to risks that it is less well able to manage and respond to by making organizational changes, purchasing insurance, or entering into hedging transactions. Through these choices the firm aligns the risks it takes with its **risk tolerances** for these various types of risk.

RISK MANAGEMENT

- While returns for any period are not under the control of managers, the specific risks and overall level of risk the organization takes are under their control.
- We can think of risk management as determining organizational risks, determining the optimal bundle of risks for the organization, and implementing risk mitigation strategies to achieve that bundle of risks.

Measures of risk

- Measures of risk for specific asset types include standard deviation, beta, and duration.
- **Standard deviation** is a measure of the volatility of asset prices and interest rates. Standard deviation may not be the appropriate measure of risk for non-normal probability distributions, especially those with negative skew or positive excess kurtosis (fat tails).
- **Beta** measures the market risk of equity securities and portfolios of equity securities. This measure considers the risk reduction benefits of diversification and is appropriate for securities held in a well-diversified portfolio, whereas standard deviation is a measure of risk on a stand-alone basis.
- **Duration** is a measure of the price sensitivity of debt securities to changes in interest rates.

Tail risk

- **Tail risk** is the uncertainty about the probability of extreme (negative) outcomes. Commonly used measures of tail risk (sometimes referred to as downside risk) include **Value at Risk** and **Conditional VaR**.

Value at risk (VaR)

- **Value at risk (VaR)** is the minimum loss over a period that will occur with a specific probability. Consider a bank that has a one-month VaR of $1 million with a probability of 5%. That means that a one-month loss of at least $1 million is expected to occur 5% of the time.
- Note that this is not the maximum one-month loss the bank will experience; it is the minimum loss that will occur 5% of the time. VaR does not provide a maximum loss for a period. VaR has become accepted as a risk measure for banks and is used in establishing minimum capital requirements.

Conditional VaR (CVaR)

- **Conditional VaR (CVaR)** is the expected value of a loss, given that the loss exceeds a minimum amount. Relating this to the VaR measure presented previously, the CVaR would be the expected loss, given that the loss was at least $1 million.
- It is calculated as the **probability-weighted average loss** for all losses expected to be at least $1 million. CVaR is similar to the measure of loss given default that is used in estimating risk for debt securities.

REVISION

- **a. calculate and interpret Holding period return measures and describe their appropriate uses.**
- Holding period return is used to measure an investment's return over a specific period.
- Arithmetic mean return is the simple average of a series of periodic returns. Geometric mean return is a compound annual rate.

REVISION

- **b. compare the money-weighted and time-weighted rates of return and evaluate the performance of portfolios based on these measures.**
- The money-weighted rate of return is the IRR calculated using periodic cash flows into and out of an account and is the discount rate that makes the PV of cash inflows equal to the PV of cash outflows.
  The time-weighted rate of return measures compound growth. It is the rate at which $1 compounds over a specified performance horizon.

REVISION

- **c. explain risk aversion and its implications for portfolio selection.**
- A risk-averse investor is one that dislikes risk. Given two investments that have equal expected returns, a risk-averse investor will choose the one with less risk. However, a risk-averse investor will hold risky assets if he feels that the extra return he expects to earn is adequate compensation for the additional risk. Assets in the financial markets are priced according to the preferences of risk-averse investors.
- A risk-seeking (risk-loving) investor prefers more risk to less and, given investments with equal expected returns, will choose the more risky investment.
- A risk-neutral investor would be indifferent to risk and would be indifferent between two investments with the same expected return regardless of the investments' standard deviation of returns.

REVISION

- **d. explain the selection of an optimal portfolio, given an investor's utility (or risk aversion) and the capital allocation line.**
- An indifference curve plots combinations of risk and expected return that provide the same expected utility. Indifference curves for risk and return slope upward because risk-averse investors will only take on more risk if they are compensated with greater expected returns. A more risk-averse investor will have steeper indifference curves.
- Flatter indifference curves (less risk aversion) result in an optimal portfolio with higher risk and higher expected return. An investor who is less risk averse will optimally choose a portfolio with more invested in the risky asset portfolio and less invested in the risk-free asset, compared to a more risk-averse investor.

REVISION

- **e. calculate and interpret portfolio standard deviation.**
- The standard deviation of returns for a portfolio of two risky assets is calculated as follows:

$$\sigma_{portfolio} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{1,2}\sigma_1\sigma_2}$$

REVISION

- **f. describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum variance portfolio.**
- For each level of expected portfolio return, the portfolio that has the least risk is known as a minimum-variance portfolio. Taken together, these portfolios form a line called the minimum variance frontier.
- On a risk versus return graph, the one risky portfolio that is farthest to the left (has the least risk) is known as the global minimum-variance portfolio.
- Those portfolios that have the greatest expected return for each level of risk make up the efficient frontier. The efficient frontier coincides with the top portion of the minimum variance frontier. Risk-averse investors would only choose a portfolio that lies on the efficient frontier.

REVISION

- **g. describe the implications of combining a risk-free asset with a portfolio of risky assets.**
- The availability of a risk-free asset allows investors to build portfolios with superior risk-return properties. By combining a risk-free asset with a portfolio of risky assets, the overall risk and return can be adjusted to appeal to investors with various degrees of risk aversion.

REVISION

- **h. explain the capital allocation line (CAL) and the capital market line (CML).**
- On a graph of return versus risk, the various combinations of a risky asset and the risk-free asset form the capital allocation line (CAL). In the specific case where the risky asset is the market portfolio, the combinations of the risky asset and the risk-free asset form the capital market line (CML).

REVISION

- **i. explain systematic and nonsystematic risk, including why an investor should not expect to receive additional return for bearing nonsystematic risk.**
- Systematic (market) risk is due to factors, such as GDP growth and interest rate changes, that affect the values of all risky securities. Systematic risk cannot be reduced by diversification.
- Unsystematic (firm-specific) risk can be reduced by portfolio diversification. Because one of the assumptions underlying the CAPM is that portfolio diversification to eliminate unsystematic risk is costless, investors cannot increase expected equilibrium portfolio returns by taking on unsystematic risk.

REVISION

- **j. explain return generating models (including the market model) and their uses.**
- A return generating model is an equation that estimates the expected return of an investment, based on a security's exposure to one or more macroeconomic, fundamental, or statistical factors.
- The simplest return generating model is the market model, which assumes the return on an asset is related to the return on the market portfolio in the following manner:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

REVISION

- **k. calculate and interpret beta.**
- Beta can be calculated using the following equation:

$$\beta_i = \frac{[\mathrm{Cov}(R_i, R_m)]}{\sigma_m^2} = \rho_{im}\left(\frac{\sigma_i}{\sigma_m}\right)$$

- where [Cov (R_i, R_m)] and $\rho_{i,m}$ are the covariance and correlation between the asset and the market, and σ_i and σ_m are the standard deviations of asset returns and market returns.
- The theoretical average beta of stocks in the market is 1. A beta of zero indicates that a security’s return is uncorrelated with the returns of the market.

REVISION

- **l. explain the capital asset pricing model (CAPM), including its assumptions, and the security market line (SML).**
- The capital asset pricing model (CAPM) requires several assumptions:
  - Investors are risk averse, utility maximizing, and rational.
  - Markets are free of frictions like costs and taxes.
  - All investors plan using the same time period.
  - All investors have the same expectations of security returns.
  - Investments are infinitely divisible.
  - Prices are unaffected by an investor’s trades.
- The security market line (SML) is a graphical representation of the CAPM that plots expected return versus beta for any security.

REVISION

- **m. calculate and interpret the expected return of an asset using the CAPM.**
- The CAPM relates expected return to the market factor (beta) using the following formula:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f]$$

REVISION

- **n. describe and demonstrate applications of the CAPM and the SML.**
- The CAPM and the SML indicate what a security’s equilibrium required rate of return should be based on the security’s exposure to market risk. An analyst can compare his expected rate of return on a security to the required rate of return indicated by the SML to determine whether the security is overvalued, undervalued, or properly valued.

REVISION

- **o. calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen's alpha**
- The Sharpe ratio measures excess return per unit of total risk and is useful for comparing portfolios on a risk-adjusted basis.

$$\text{Sharpe ratio} = \left(\frac{R_P - R_f}{\sigma_P}\right)$$

REVISION

- **o. calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen's alpha**
- Given a Portfolio P, we can calculate the return on a Portfolio P* that is leveraged or deleveraged, so that P* has the same risk as the market portfolio. The return on P* is the M-squared measure for portfolio P.

$$M^2 = R_f + \frac{\sigma_M}{\sigma_P}(R_P - R_f)$$

- M-squared alpha is the extra return on Portfolio P* above the market portfolio.

REVISION

- **o. calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen's alpha**
- The Treynor measure measures a portfolio's excess return per unit of systematic risk. Jensen's alpha is the difference between a portfolio's return and the return of a portfolio on the SML that has the same beta:

$$\text{Treynor measure} = \frac{R_P - R_f}{\beta_P}$$

$$\text{Jensen's alpha} = \alpha_P = R_P - [R_f + \beta_P(R_M - R_f)]$$

REVISION

- **p. define risk management.**
- Risk management is the process of identifying and measuring the risks an organization (or portfolio manager or individual) faces, determining an acceptable level of overall risk (establishing risk tolerance), deciding which risks should be taken and which risks should be reduced or avoided, and putting the structure in place to maintain the bundle of risks that is expected to best achieve the goals of the organization.

REVISION

- **q. describe methods for measuring and modifying risk exposures**
- Risk of assets is measured by standard deviation, beta, or duration.
- Tail risk is measured with value at risk (VaR) or conditional VaR.