Revision

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https://github.com/styluck/mat_fin

Chp1 Theory of Interest rate

RIVISION

- **a. interpret interest rates as required rates of return, discount rates, or opportunity costs.**
- An interest rate can be interpreted as the rate of return required in equilibrium for a particular investment, the discount rate for calculating the present value of future cash flows, or as the opportunity cost of consuming now, rather than saving and investing.

RIVISION

- **b. explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk.**
- The real risk-free rate is a theoretical rate on a single-period loan when there is no expectation of inflation. Nominal risk-free rate = real risk-free rate + expected inflation rate.
- Securities may have several risks, and each increases the required rate of return. These include default risk, liquidity risk, and maturity risk.
- The required rate of return on a security = real risk-free rate + expected inflation + default risk premium + liquidity premium + maturity risk premium.

RIVISION

- **c. calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows.**
- Future value: $FV = PV(1 + I/Y)^N$
- Present value: $PV = FV/(1 + I/Y)^N$
- An annuity is a series of equal cash flows that occurs at evenly spaced intervals over time. Ordinary annuity cash flows occur at the end of each time period. Annuity due cash flows occur at the beginning of each time period.
- Perpetuities are annuities with infinite lives (perpetual annuities):
    - $PV_{perp} = \frac{PMT}{I/Y}$

RIVISION

- **d. demonstrate the use of a time line in modeling and solving time value of money problems.**
- Constructing a time line showing future cash flows will help in solving many types of TVM problems.
- Cash flows occur at the end of the period depicted on the time line. The end of one period is the same as the beginning of the next period.
- For example, a cash flow at the beginning of Year 3 appears at time $t = 2$ on the time line.

RIVISION

- **e. calculate the solution for time value of money problems with different frequencies of compounding.**
- For non-annual time value of money problems, divide the stated annual interest rate by the number of compounding periods per year, m, and multiply the number of years by the number of compounding periods per year.

RIVISION

- f. calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.
- The EAR when there are m compounding periods =

$$\left(1+\frac{stated\ annual\ rate}{m}\right)^{m}-1$$

QUIZ 1

- 1. An interest rate is best
  - A. a discount rate or a measure of risk.
  - B. a measure of risk or a required rate of return.
  - C. a required rate of return or the opportunity cost of consumption.
- 2. An interest rate from which the inflation premium has been subtracted is known as:
  - A. a real interest rate.
  - B. a risk-free interest rate.
  - C. a real risk-free interest rate.

QUIZ 1

- 1. An interest rate is best
  - C. a required rate of return or the opportunity cost of consumption.
  - **Interest rates can be interpreted as required rates of return, discount rates, or opportunity costs of current consumption. A risk premium can be, but is not always, a component of an interest rate.**
- 2. An interest rate from which the inflation premium has been subtracted is known as:
  - A. a real interest rate.
  - **Real interest rates are those that have been adjusted for inflation.**

QUIZ 2

- 1. The amount an investor will have in 15 years if $1,000 is invested today at an annual interest rate of 9% will be closest
  - A. $1,350.
  - B. $3,518.
  - C. $3,642.
- 2. How much must be invested today, at 8% interest, to accumulate enough to retire a $10,000 debt due seven years from today?
  - A. $5,835.
  - B. $6,123.
  - C. $8,794.
- 3. An investor has just won the lottery and will receive $50,000 per year at the end of each of the next 20 years. At a 10% interest rate, the present value of the winnings is closest
  - A. $425,678.
  - B. $637,241.
  - C. $2,863,750.

QUIZ 2

- 1. The amount an investor will have in 15 years if $1,000 is invested today at an annual interest rate of 9% will be closest
  - **C. $3,642.**
  - $FV = 1000 \times (1.09)^{15}$ = $3,642.48
- 2. How much must be invested today, at 8% interest, to accumulate enough to retire a $10,000 debt due seven years from today?
  - **A. $5,835.**
  - $PV = 10000/(1+0.08)^{7}$ = $5,834.90
- 3. An investor has just won the lottery and will receive $50,000 per year at the end of each of the next 20 years. At a 10% interest rate, the present value of the winnings is closest
  - **A. $425,678.**
  - $PV = 50000 \times (1-(1+0.10)^{-20})/0.10$ = $425,678.19

QUIZ 2

- 4. An investor is to receive a 15-year, $8,000 annuity, with the first payment to be received today. At an 11% discount rate, this annuity's worth today is closest
  - A. $55,855.
  - B. $57,527.
  - C. $63,855.
- 5. If $1,000 is invested today and $1,000 is invested at the beginning of each of the next three years at 12% interest (compounded annually), the amount an investor will have at the end of the fourth year will be closest
  - A. $4,779.
  - B. $5,353.
  - C. $6,792.
- 6. Terry Corporation preferred stocks are expected to pay a $9 annual dividend forever. If the required rate of return on equivalent investments is 11%, a share of Terry preferred should be worth:
  - A. $81.82.
  - B. $99.00.
  - C. $122.22.

QUIZ 2

- 4. An investor is to receive a 15-year, $8,000 annuity, with the first payment to be received today. At an 11% discount rate, this annuity's worth today is closest
  - **C. $63,855.**
  - $PV = \mathrm{PMT} \times \frac{1-(1+r)^{-n}}{r} \times (1+r)$=8000×(1−(1.11)$^{-15}$)/0.11×(1+0.11) = $63,854.96
- 5. If $1,000 is invested today and $1,000 is invested at the beginning of each of the next three years at 12% interest (compounded annually), the amount an investor will have at the end of the fourth year will be closest
  - **B. $5,353.**
  - The key to this problem is to recognize that it is a 4-year annuity due, FV = **1573.52+1404.93+1254.40+1120=$5352.85**
- 6. Terry Corporation preferred stocks are expected to pay a $9 annual dividend forever. If the required rate of return on equivalent investments is 11%, a share of Terry preferred should be worth:
  - **A. $81.82.**
  - 9/0.11 = $81.82

QUIZ 3

- 1. What is the effective annual rate for a credit card that charges 18% compounded monthly?
  - A. 15.38%.
  - B. 18.81%.
  - C. 19.56%.
- 2. Given daily compounding, the growth of $5,000 invested for one year at 12% interest will be closest to:
  - A. $5,600.
  - B. $5,628.
  - C. $5,637.
- 3. An investor is looking at a $150,000 home. If 20% must be put down and the balance is financed at a stated annual rate of 9% over the next 30 years, what is the monthly mortgage payment?
  - A. $799.33.
  - B. $895.21.
  - C. $965.55

QUIZ 3

- 1. What is the effective annual rate for a credit card that charges 18% compounded monthly?
  - **C. 19.56%.**
  - EAR = $[(1 + (0.18/12)]^{12}$ -1 = 19.56%
- 2. Given daily compounding, the growth of $5,000 invested for one year at 12% interest will be closest to:
  - **C. $5,637.**
  - $A = P\left(1 + \frac{r}{n}\right)^{nt}$=5000(1+0.12/365)$^{365\times1}$=$5637.38
- 3. An investor is looking at a $150,000 home. If 20% must be put down and the balance is financed at a stated annual rate of 9% over the next 30 years, what is the monthly mortgage payment?
  - **C. $965.55**
  - $150000 \times 0.2 = \$30{,}000$ $PV = \frac{PMT\times(1-(1+r)^{-n})}{r}$, PMT = $\frac{120000\times0.0075\times(1+0.0075)^{360}}{(1+0.0075)^{360}-1}$=**$965.55**

Introduction to interest rate models

REVISION

- a. describe the use of interbank offered rates as reference rates in floating-rate debt.
- **Floating-rate notes have coupon rates that adjust based on a reference rate such as LIBOR.**

REVISION

- b. calculate a bond’s price given a market discount rate
- **The price of a bond is the present value of its future cash flows, discounted at the bond’s yield-to-maturity.**
- **For an annual-coupon bond with N years to maturity:**

$$\text{price} = \frac{\text{coupon}}{(1+\text{YTM})} + \frac{\text{coupon}}{(1+\text{YTM})^2} + \ldots + \frac{\text{coupon} + \text{principal}}{(1+\text{YTM})^N}$$

- **For a semiannual-coupon bond with N years to maturity:**

$$\text{price} = \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)^2} + \ldots + \frac{\text{coupon} + \text{principal}}{\left(1+\frac{\text{YTM}}{2}\right)^{N\times 2}}$$

REVISION

- c. identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity)
- **A bond's price and YTM are inversely related. An increase in YTM decreases the price and a decrease in YTM increases the price.**
- **A bond will be priced at a discount to par value if its coupon rate is less than its YTM, and at a premium to par value if its coupon rate is greater than its YTM.**
- **Prices are more sensitive to changes in YTM for bonds with lower coupon rates and longer maturities, and less sensitive to changes in YTM for bonds with higher coupon rates and shorter maturities.**
- **A bond's price moves toward par value as time passes and maturity approaches.**

REVISION

- d. describe and calculate the flat price, accrued interest, and the full price of a bond.
- **The full price of a bond includes interest accrued between coupon dates. The flat price of a bond is the full price minus accrued interest.**
- **Accrued interest for a bond transaction is calculated as the coupon payment times the portion of the coupon period from the previous payment date to the settlement date.**
- **Methods for determining the period of accrued interest include actual days (typically used for government bonds) or 30-day months and 360-day years (typically used for corporate bonds).**

REVISION

- e. describe matrix pricing.
- **Matrix pricing is a method used to estimate the yield-to-maturity for bonds that are not traded or infrequently traded. The yield is estimated based on the yields of traded bonds with the same credit quality.**
- **If these traded bonds have different maturities than the bond being valued, linear interpolation is used to estimate the subject bond's yield.**

REVISION

- f. calculate annual yield on a bond for varying compounding periods in a year.
- **The effective yield of a bond depends on its periodicity, or annual frequency of coupon payments. For an annual-pay bond the effective yield is equal to the yield-to-maturity.**
- **For bonds with greater periodicity, the effective yield is greater than the yield-to-maturity.**
- **A YTM quoted on a semiannual bond basis is two times the semiannual discount rate.**

REVISION

- g. define the yield curve
- **A yield curve shows the term structure of interest rates by displaying yields across different maturities.**

REVISION

- h. define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.
- **Forward rates are current lending/borrowing rates for short-term loans to be made in future periods.**
- **A spot rate for a maturity of N periods is the geometric mean of forward rates over the N periods. The same relation can be used to solve for a forward rate given spot rates for two different periods.**
- **To value a bond using forward rates, discount the cash flows at times 1 through N by the product of one plus each forward rate for periods 1 to N, and sum them.**

REVISION

- i. define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.
- **For a 3-year annual-pay bond:**

$$\text{price} = \frac{\text{coupon}}{(1+S_1)} + \frac{\text{coupon}}{(1+S_1)(1+1y1y)} + \frac{\text{coupon} + \text{principal}}{(1+S_1)(1+1y1y)(1+2y1y)}$$

REVISION

- j. Describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender. Calculate the yield-to-call of a callable bond.
- **For a callable bond, a yield-to-call may be calculated using each of its call dates and prices. The lowest of these yields and YTM is a callable bond's yield-to-worst.**

Data and financial modelling

REVISION

- **a. calculate and interpret target downside deviation.**
- **Target downside deviation** or semideviation is a measure of downside risk. Calculating target downside deviation is similar to calculating standard deviation, but in this case, we choose a target against which to measure each outcome and only include outcomes below that target when calculating the numerator.
- The formula for target downside deviation is:

$$s_{target} = \sqrt{\frac{\sum_{\text{all } X_i < B}^{n} (X_i - B)^2}{n - 1}}$$

where B is the target value

RIVISION

- **b. interpret skewness.**
- Skewness describes the degree to which a distribution is not symmetric about its mean. A right-skewed distribution has positive skewness. A left-skewed distribution has negative skewness.
- For a positively skewed, unimodal distribution, the mean is greater than the median, which is greater than the mode.
- For a negatively skewed, unimodal distribution, the mean is less than the median, which is less than the mode.

REVISION

- **c. interpret kurtosis.**
- Kurtosis measures the peakedness of a distribution and the probability of extreme outcomes (thickness of tails):
- Excess kurtosis is measured relative to a normal distribution, which has a kurtosis of 3.
- Positive values of excess kurtosis indicate a distribution that is leptokurtic (fat tails, more peaked), so the probability of extreme outcomes is greater than for a normal distribution.
- Negative values of excess kurtosis indicate a platykurtic distribution (thin tails, less peaked)

REVISION

- **d. interpret correlation between two variables.**
- Correlation is a standardized measure of association between two random variables. It ranges in value from -1 to +1 and is equal to

$$\frac{cov_{A,B}}{\sigma_A \sigma_B}$$

- Scatterplots are useful for revealing nonlinear relationships that are not measured by correlation.
- Correlation does not imply that changes in one variable cause changes in the other. Spurious correlation may result by chance or from the relationships of two variables to a third variable.

REVISION

- **e. calculate the probability that a normally distributed random variable lies inside a given interval**
- A confidence interval is a range within which we have a given level of confidence of finding a point estimate (e.g., the 90% confidence interval for X is [-1.65, 1.65]
- Confidence intervals for any normally distributed random variable are:
- 90%: $\mu \pm 1.65$ standard deviations.
- 95%: $\mu \pm 1.96$ standard deviations.
- 99%: $\mu \pm 2.58$ standard deviations.

REVISION

- **f. explain how to standardize a random variable.**
- The standard normal probability distribution has a mean of 0 and a standard deviation of 1.
- A normally distributed random variable X can be standardized as and Z will be normally distributed with mean = 0 and standard deviation 1

REVISION

- **g. define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.**
- Shortfall risk is the probability that a portfolio's value (or return) will fall below a specific value over a given period of time.
- The safety-first ratio for portfolio P, based on a target return RT, is:

$$\text{SFRatio} = \frac{E(R_P) - R_T}{\sigma_P}$$

- Greater safety-first ratios are preferred and indicate a smaller shortfall probability. Roy's safety- first criterion states that the optimal portfolio minimizes shortfall risk.

REVISION

- **h. explain the relationship between normal and lognormal distributions and why the lognormal distribution is used to model asset prices.**
- If x is normally distributed, ex follows a lognormal distribution. A lognormal distribution is often used to model asset prices, since a lognormal random variable cannot be negative and can take on any positive value.

REVISION

- **i. calculate and interpret a continuously compounded rate of return, given a specific holding period return.**
- As we decrease the length of discrete compounding periods (e.g., from quarterly to monthly) the effective annual rate increases. As the length of the compounding period in discrete compounding gets shorter and shorter, the compounding becomes continuous, where the effective annual rate $= e^i - 1$.
- For a holding period return (HPR) over any period, the equivalent continuously compounded rate over the period is $\ln(1 + HPR)$.

REVISION: Simple linear regression

- The following linear regression model is used to describe the relationship between two variables, X and Y:

$$Y_i = b_0 + b_1 X_i + \epsilon_i, \quad i = 1, \ldots, n$$

where:
Y_i= ith observation of the dependent variable, Y
X_i = ith observation of the independent variable, X
b_0= regression intercept term
b_1= regression slope coefficient
ϵ_i = residual for the ith observation (also referred to as the disturbance term or error term)

REVISION: Simple linear regression

- Based on this regression model, the regression process estimates an equation for a line through a scatter plot of the data that "best" explains the observed values for Y in terms of the observed values for X. The linear equation, often called the line of best fit or regression line, takes the following form:

$$\widehat{Y_i} = \widehat{b_0} + \widehat{b_1}X_i, \quad i = 1,2,3, \dots, n$$

where:
$\widehat{Y_i}$ = estimated value of Y_i given X_i
$\widehat{b_0}$ = estimated intercept term
$\widehat{b_1}$ = estimated slope coefficient

REVISION: Simple linear regression

- The sum of the squared vertical distances between the estimated and actual Y-values is referred to as **the sum of squared errors (SSE).**
- Thus, the regression line is the line that minimizes the SSE. This explains why simple linear regression is frequently referred to as **ordinary least squares (OLS) regression**, and the values determined by the estimated regression equation, , are called least squares estimates.

REVISION: Simple linear regression

- The intercept term $\hat{b}_0$ is the line's intersection with the Y-axis at $X = 0$. It can be positive, negative, or zero. A property of the least squares method is that the intercept term may be expressed as:

$$\hat{b}_0 = \bar{Y} - \hat{b}_1\bar{X}$$

where:

- $\bar{X} = \text{mean of } X$
- $\bar{Y} = \text{mean of } Y$

REVISION: Simple linear regression

- The estimated slope coefficient for the regression line describes the change in Y for a one unit change in X. It can be positive, negative, or zero, depending on the relationship between the regression variables. The slope term is calculated as:

$$\hat{b}_1 = \frac{\mathrm{Cov}_{XY}}{\sigma_X^2}$$

analysis of variance

- **Analysis of variance (ANOVA)** is a statistical procedure for analyzing the total variability of the dependent variable. Let’s define some terms before we move on to ANOVA tables:
- **Total sum of squares (SST)** measures the total variation in the dependent variable. SST is equal to the sum of the squared differences between the actual Y-values and the mean of Y.

$$SST = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

Analysis of variance

- **Sum of squares regression (SSR)** measures the variation in the dependent variable that is explained by the independent variable. SSR is the sum of the squared distances between the predicted Y-values and the mean of Y.

$$SSR = \sum_{i=1}^{n} \left(\hat{Y_i} - \bar{Y}\right)^2$$

- **Sum of squared errors (SSE)** measures the unexplained variation in the dependent variable. It's also known as the sum of squared residuals or the residual sum of squares. SSE is the sum of the squared vertical distances between the actual Y-values and the predicted Y-values on the regression line.

$$SSE = \sum_{i=1}^{n} \left(Y_i - \hat{Y}\right)^2$$

analysis of variance

- You probably will not be surprised to learn that:
- total variation = explained variation + unexplained variation
- or:

$$SST = SSR + SSE$$

Coefficient of determination (R^2)

- The coefficient of determination (R^2) is defined as the percentage of the total variation in the dependent variable explained by the independent variable. For example, an R^2 of 0.63 indicates that the variation of the independent variable explains 63% of the variation in the dependent variable.

$$R^2 = SSR / SST$$

Example: Using the ANOVA table

- Complete the ANOVA table for the ABC regression example and calculate the R^2 and the standard error of estimate (SEE).

Partial ANOVA Table for ABC Regression Example

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares
Regression (explained)	?	0.00756	?
Error (unexplained)	?	0.04064	?
Total	?	?	

Example: Using the ANOVA table

- Recall that the data included three years of monthly return observations, so the total number of observations (n) is 36.

Completed ANOVA Table for ABC Regression Example

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares
Regression (explained)	1	0.0076	0.0076
Error (unexplained)	34	0.0406	0.0012
Total	35	0.0482	

$$R^2 = \frac{\text{explained variation (SSR)}}{\text{total variation (SST)}} = \frac{0.0076}{0.0482} = 0.158 \text{ or } 15.8\%$$

$$SEE = \sqrt{MSE} = \sqrt{0.0012} = 0.035$$

The F-Statistic

- An F-test assesses how well a set of independent variables, as a group, explains the variation in the dependent variable.
- The F-statistic is calculated as:

$$F = \frac{MSR}{MSE} = \frac{SSR/k}{SSE/(n-k-1)}$$

where:

MSR = mean regression sum of squares

MSE = mean squared error

The F-Statistic

- For simple linear regression, there is only one independent variable, so the F-test is equivalent to a t-test for statistical significance of the slope coefficient:

$$H_0: b_1 = 0 \text{ versus } H_a: b_1 \neq 0$$

- calculated F-statistic is compared with the critical F-value, Fc, at the appropriate level of significance.
- The degrees of freedom for the numerator and denominator with one independent variable are:

$$df_{numerator} = k = 1$$

$$df_{denominator} = n - k - 1 = n - 2$$

where:

n = number of observations

Measures of investment risk

REVISION

- **a. calculate and interpret the sources of return from investing in a fixed-rate bond.**
- Sources of return from a bond investment include:
- Coupon and principal payments.
- Reinvestment of coupon payments.
- Capital gain or loss if bond is sold before maturity.
- Changes in yield to maturity produce market price risk (uncertainty about a bond’s price) and reinvestment risk (uncertainty about income from reinvesting coupon payments). An increase (a decrease) in YTM decreases (increases) a bond’s price but increases (decreases) its reinvestment income.

REVISION

- **b. define, calculate, and interpret Macaulay, modified, and effective durations.**
- Macaulay duration is the weighted average number of coupon periods until a bond's scheduled cash flows.

$$\text{approximate modified duration} = \frac{V_{-} - V_{+}}{2 \times V_0 \times \Delta YTM}$$

- Effective duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in the benchmark yield curve.

$$\text{effective duration} = \frac{V_{-} - V_{+}}{2 \times V_0 \times \Delta \text{curve}}$$

REVISION

- **c. explain how a bond's maturity, coupon, and yield level affect its interest rate risk.**
- Holding other factors constant:
- Duration increases when maturity increases.
- Duration decreases when the coupon rate increases.
- Duration decreases when YTM increases.

REVISION

- **d. calculate and interpret the money duration of a bond and price value of a basis point (PVBP).**
- Money duration is stated in currency units and is sometimes expressed per 100 of bond value.
- money duration = annual modified duration × full price of bond position
- money duration per 100 units of par value = annual modified duration × full bond price per 100 of par value
- The price value of a basis point is the change in the value of a bond, expressed in currency units, for a change in YTM of one basis point, or 0.01%.
- $PVBP = [(V_{-} - V_{+}) / 2] \times \text{par value} \times 0.01$

REVISION

- **h. calculate and interpret approximate convexity and compare approximate and effective convexity.**
- Convexity refers to the curvature of a bond's price-yield relationship.

$$\text{approximate convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta \text{YTM})^2 V_0}$$

- Effective co edded options:

$$\text{approximate effective convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta \text{curve})^2 V_0}$$

REVISION

- **i. calculate the percentage price change of a bond for a specified change in yield, given the bond's approximate duration and convexity.**
- Given values for approximate annual modified duration and approximate annual convexity, the percentage change in the full price of a bond can be estimated as:

$$\%\Delta \text{ full bond price} = -\text{annual modified duration}(\Delta \text{YTM}) + \frac{1}{2}\text{annual convexity}(\Delta \text{YTM})^2$$

REVISION

- **k. describe the relationships among a bond's holding period return, its duration, and the investment horizon.**
- Over a short investment horizon, a change in YTM affects market price more than it affects reinvestment income.
- Over a long investment horizon, a change in YTM affects reinvestment income more than it affects market price.
- Macaulay duration may be interpreted as the investment horizon for which a bond's market price risk and reinvestment risk just offset each other.
- duration gap = Macaulay duration – investment horizon

Modern Portfolio Theory

REVISION

- **a. explain risk aversion and its implications for portfolio selection.**
- A risk-averse investor is one that dislikes risk. Given two investments that have equal expected returns, a risk-averse investor will choose the one with less risk. However, a risk-averse investor will hold risky assets if he feels that the extra return he expects to earn is adequate compensation for the additional risk. Assets in the financial markets are priced according to the preferences of risk-averse investors.
- A risk-seeking (risk-loving) investor prefers more risk to less and, given investments with equal expected returns, will choose the more risky investment.
- A risk-neutral investor would be indifferent to risk and would be indifferent between two investments with the same expected return regardless of the investments' standard deviation of returns.

REVISION

- **b. explain the selection of an optimal portfolio, given an investor's utility (or risk aversion) and the capital allocation line.**
- An indifference curve plots combinations of risk and expected return that provide the same expected utility. Indifference curves for risk and return slope upward because risk-averse investors will only take on more risk if they are compensated with greater expected returns. A more risk-averse investor will have steeper indifference curves.
- Flatter indifference curves (less risk aversion) result in an optimal portfolio with higher risk and higher expected return. An investor who is less risk averse will optimally choose a portfolio with more invested in the risky asset portfolio and less invested in the risk-free asset, compared to a more risk-averse investor.

REVISION

- **c. calculate and interpret portfolio standard deviation.**
- The standard deviation of returns for a portfolio of two risky assets is calculated as follows:

$$\sigma_{\text{portfolio}} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{1,2}\sigma_1\sigma_2}$$

REVISION

- **d. explain the capital allocation line (CAL) and the capital market line (CML).**
- On a graph of return versus risk, the various combinations of a risky asset and the risk-free asset form the capital allocation line (CAL). In the specific case where the risky asset is the market portfolio, the combinations of the risky asset and the risk-free asset form the capital market line (CML).

REVISION

- **e. explain systematic and nonsystematic risk, including why an investor should not expect to receive additional return for bearing nonsystematic risk.**
- Systematic (market) risk is due to factors, such as GDP growth and interest rate changes, that affect the values of all risky securities. Systematic risk cannot be reduced by diversification.
- Unsystematic (firm-specific) risk can be reduced by portfolio diversification. Because one of the assumptions underlying the CAPM is that portfolio diversification to eliminate unsystematic risk is costless, investors cannot increase expected equilibrium portfolio returns by taking on unsystematic risk.

REVISION

- **f. explain return generating models (including the market model) and their uses.**
- A return generating model is an equation that estimates the expected return of an investment, based on a security's exposure to one or more macroeconomic, fundamental, or statistical factors.
- The simplest return generating model is the market model, which assumes the return on an asset is related to the return on the market portfolio in the following manner:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

REVISION

- **g. calculate and interpret beta.**
- Beta can be calculated using the following equation:

$$\beta_i = \frac{[Cov(R_i, R_m)]}{\sigma_m^2} = \rho_{im}\left(\frac{\sigma_i}{\sigma_m}\right)$$

- where $[Cov (R_i,R_m)]$ and $\rho_{i,m}$ are the covariance and correlation between the asset and the market, and σ_i and σ_m are the standard deviations of asset returns and market returns.
- The theoretical average beta of stocks in the market is 1. A beta of zero indicates that a security's return is uncorrelated with the returns of the market.

REVISION

- **h. explain the capital asset pricing model (CAPM), including its assumptions, and the security market line (SML).**
- The capital asset pricing model (CAPM) requires several assumptions:
  - Investors are risk averse, utility maximizing, and rational.
  - Markets are free of frictions like costs and taxes.
  - All investors plan using the same time period.
  - All investors have the same expectations of security returns.
  - Investments are infinitely divisible.
  - Prices are unaffected by an investor’s trades.
- The security market line (SML) is a graphical representation of the CAPM that plots expected return versus beta for any security.

REVISION

- **i. calculate and interpret the expected return of an asset using the CAPM.**
- The CAPM relates expected return to the market factor (beta) using the following formula:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f]$$

REVISION

- **j. describe and demonstrate applications of the CAPM and the SML.**
- The CAPM and the SML indicate what a security's equilibrium required rate of return should be based on the security's exposure to market risk. An analyst can compare his expected rate of return on a security to the required rate of return indicated by the SML to determine whether the security is overvalued, undervalued, or properly valued.

REVISION

- **k. calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen’s alpha**
- The Sharpe ratio measures excess return per unit of total risk and is useful for comparing portfolios on a risk-adjusted basis.

$$\text{Sharpe ratio} = \left(\frac{R_P - R_f}{\sigma_P}\right)$$