Sequences & Series: Exercise 2

Due date: 2024-12-17

1. Given the sequence $a_n = \frac{n}{n+1}$, determine whether the sequence converges. If it does, find its limit.

2. Given the sequence $a_n = \left(1 + \frac{1}{n}\right)^n$, determine whether the sequence converges. If it does, find its limit.

3. Given the sequence $a_n = \frac{\sin n}{n}$, determine whether the sequence converges. If it does, find its limit.

4. Given the sequence $a_n = \frac{(-1)^n n}{n+1}$, determine whether the sequence converges. If it does, whether it is absolutely convergent or conditionally convergent.

5. Given the sequence $a_n = \sqrt{n+1} - \sqrt{n}$, determine whether the sequence converges. If it does, find its limit.

6. Judge the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. If it converges, find its sum.

7. Judge the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (Hint: you can make use of the convergence conclusion of p-harmonic series).

8. Judge the convergence of the series$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (Hint: you can make use of the convergence conclusion of p-harmonic series).

9. Judge the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, including the situations of absolute convergence and conditional convergence.

10. Judge the convergence of the series using the ratio test: $\sum_{n=1}^{\infty} \frac{n}{2^n}$.