Sequences and series

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Learning objectives:
By the end of the term students should be able to:

(0) appreciate the definition of real numbers and elementary operations on them.

(1) establish the convergence/divergence of a given sequence or series either directly from the definitions, or by selecting an appropriate convergence/divergence theorem.

(2) construct proofs or provide counterexamples for statements involving the convergence of sequences and series.

Brief description:
More details would be provided later.

(i) **Real field** as the ordered filed with least upper bound property containing $\mathbb{Q}$ as a subfield.

(ii) **Sequences.** Convergence; theorems on limits; monotonic sequences; subsequences; the Bolzano-Weierstrass Theorem; Cauchy sequences.

(iii) **Series.** Convergence; series of positive terms; power series; the comparison test; absolute convergence; the ratio test; the root test; the integral test; the alternating series test; Taylor series; Maclaurin series.

Delivery
Lectures and seminars in person.

Assessment structure
Written exam (80%) in winter. Continuous assessment (20%): Handwritten Summative assessment (around the end of the 2nd week of this module); and Mobius test (around the end of the 4th week of this module)

Recommended textbook
Comprehensive lecture notes will be provided, so it will not be necessary for you to buy a book.
• Jiří Lebl, *Basic analysis: introduction to real analysis,* (Download: http://www.jirka.org/ra/)

Further reading
For those wishing to explore further.
• W. Rudin, *Principles of Mathematical Analysis,* McGraw-Hill.
• G. Shilov, *Elementary Real and Complex Analysis,* Dover

Syllabus:
Part 0: Foundations

1. Sets and sets of real numbers: sets of real numbers, standard set notations
2. Archimedean property, denseness of rational numbers, least upper bound, greatest lower bound,

Part I: Sequences

1. Definition of sequence
2. $\varepsilon - N$ definition of convergence
3. Uniqueness of limits and divergence to ∞
4. Boundedness properties of limits
5. Algebra of limits
6. Order properties of limits and squeeze theorem
7. Monotone sequence
8. Examples of commonly occurring sequences: geometric progression, sums of geometric progression, nth roots of n, $\frac{\ln n}{n^\varepsilon}$, $\frac{n!}{n^n}$, $(1+1/n)^n$
9. Compound interest
10. Subsequences and Bolzano-Weierstrass theorem
11. Cauchy convergence criterion

Part II: Series

12. Series and properties of convergent series.
13. Convergence criterion
14. Absolute convergence and conditional convergence
15. Good properties of absolutely convergent series and products of series
16. Convergence test I : null test, comparison test, ratio test
17. Convergence tests II: root test, integral test, alternating series test
18. Power series: radius of convergence
19. Convergence test for power series: ratio test, root test
20. Maclaurin and Taylor series
21. Taylor expansion of common functions
22. Manipulation of power series