

## RISK DIFFERENCE :

X = Exposed  $\begin{cases} \text{Yes, E} \\ \text{No, nE} \end{cases}$

Y = Disease  $\begin{cases} \text{Yes, D} \\ \text{No, nD} \end{cases}$

	D	nD	
E	30	70	100
nE	20	80	100
	50	150	

$$P(D|E) = \frac{30}{100} = 0.30$$

$$P(D|nE) = \frac{20}{100} = 0.20$$

On an Additive Scale : Risk Difference (RD) or Attributable Risk (AR) =  $P(D|E) - P(D|nE) = 0.30 - 0.20 = 0.10 = 10\%$   
 Prob. that an exposure leads to 10% inc. in D. (Extra Risk)

On Relative Scale :

$$\text{Relative Risk} = \frac{P(D|E)}{P(D|nE)} = \frac{0.30}{0.20} = 1.5 = 15\%$$

15% inc. on a relative scale  
10% inc. on an additive scale

$$\text{Odds (A)} = \frac{\text{Probability of A happening}}{\text{Prob. of A not happening}}$$

$$\text{Odds Ratio (OR)} = \frac{\text{Odds (D|E)}}{\text{Odds (D|nE)}} = \frac{P(D|E)/P(D|nE)}{P(D|nE)/P(nD|nE)} = \frac{0.30/0.70}{0.20/0.80} = 1.71\%$$

Odds of D for E is 1.71 times of some one nE

and  $1 - 1.71 = 0.71 \Rightarrow$  odds of D inc by 71%.

Odds Ratio for Case-control Study Design:  $OR = \frac{\text{Odds (D|E)}}{\text{Odds (D|nE)}}$

	D	nD	
E	a	b	a+b
nE	c	d	c+d
	a+c	b+d	a+b+c+d

$$OR = \frac{P(D|E)/P(D|nE)}{P(nD|E)/P(nD|nE)} = \frac{(\frac{a}{a+b})/(\frac{b}{a+b})}{(\frac{c}{c+d})/(\frac{d}{c+d})} = \frac{ad}{bc}$$

In a disease study, we select people disease and no-disease and ask them the exposure and can't estimate prevalence i.e. These probabilities. We can still conveniently use OR :-

We can estimate prevalence of exposure

$$OR = \frac{\text{Odds (E|D)}}{\text{Odds (E|nD)}} = \frac{P(E|D)/P(nE|D)}{P(E|nD)/P(nE|nD)} = \frac{(a/a+c)/(c/a+c)}{(b/b+d)/(d/b+d)} = \frac{ad}{bc}$$

OR  $\approx$  Rate Ratio (RR) — For RARE DISEASE!  
(Prevalence 5% or less)

## Non-Linearity In REGRESSION:

- ① Transform Y eg.  $\ln(Y)$ 
  - Can make linear relation
  - can address non-constant variance
- Can cause Problem in interpretability
- ② Transform X  $\Rightarrow \sqrt{X}, \ln(X)$ . Also  
\* Ladder of transformations

Assumptions in Linear Reg:

- ① Linearity
- ② Constant Variance (Homoscedasticity)

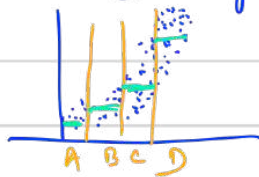
- ③ Polynomial / Quadratic fitting  $\hat{y} = b_0 + b_1X + b_2X^2$

Again loose interpretability

(Adding powers, more inflection points)

$$\begin{array}{l} X^2 \quad \text{U} \\ X^3 \quad \text{W} \end{array}$$

- ④ Categorizing X



Loss of Information

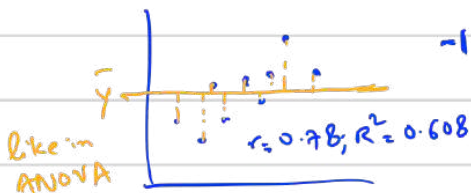
- ⑤ Non-Linear Regression Model eg. SPLINE

Disadv. No Interpretability  
like coeff for X feature imp

## $R^2$ = coeff of Determination:

In a simple linear reg.  $R^2$  is (Pearson Correlation coeff)<sup>2</sup>

$$-1 \leq r \leq +1 \quad 0 \leq R^2 \leq 1$$

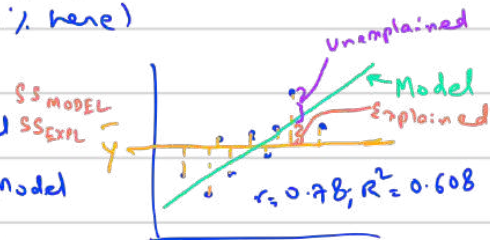


$R^2$  = %age of variability in Y explained by model.  
(61% here)

$$\text{Total Var: } SS_{\text{Total}} = \sum_{\text{all}} (Y_i - \bar{Y})^2$$

$SS_{\text{Total}}$  is split into:  
Explained by Model  
Unexplained by Model

$SS_{\text{UNEXPL}}$   
OR  $SS_{\text{ERROR}}$



$$R^2 = \frac{SS_{\text{Error}}}{SS_{\text{Total}}} = \frac{SS_{\text{Model}}}{SS_{\text{Model}} + SS_{\text{Error}}} = 1 - \frac{SS_{\text{Error}}}{SS_{\text{Total}}}$$

Adjusted  $R^2 = R^2$  - penalty for number of  $X$ 's in Model.

Used in multiple Linear Reg

to counteract some correl. bet  $X$ 's &  $Y$ .

$$Y = b_0 + b_1 X_1 + b_2 X_2$$

and Adj  $R^2$  doesn't any more explains % of variation as  $R^2$ .

NOTE:  $R^2$  is generally calculated on the same data.  
Use validation

MEASURES OF VARIABILITY: — RANGE (Max - Min.)

— IQR  $Q_3 - Q_1$

Range middle 50% of data ordered  
insensitive to outliers ✓

— Sample variance ( $s^2$ )

$$\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad \text{if } x \text{ in kg} \quad \text{Unit} = \text{kg}^2$$

\* sensitive to outliers ✓

— sample s.d ( $s$ ) =  $\sqrt{s^2}$

Monty hall PROBLEM:

	Door		
	1	2	3
Possibility 1	\$	x	—
2	x	\$	x
3	x	x	\$

You choose	Host	Outcome if you switch
1	2 or 3	Lose
1	has only 1 choice	Win
1	— or —	Win

if you don't switch = prob. of winning =  $1/3$