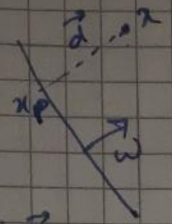


$\vec{d} \parallel \vec{w}$

$\therefore \vec{d} = \alpha \vec{w}$

$\vec{x}_p = \vec{x} - \vec{d}$

x_p lies on w

$\therefore w^T x_p + b = 0$

$\Rightarrow w^T(\vec{x} - \vec{d}) + b = 0$

$\Rightarrow w^T(\vec{x} - \alpha\vec{w}) + b = 0$

$\Rightarrow \alpha = \dfrac{w^T x + b}{w^T w}$

or, $\vec{d} = \underbrace{\dfrac{w^T x + b}{w^T w}}_{\alpha} \cdot \vec{w}$

$\|d\|_2 = \sqrt{d^T d}$

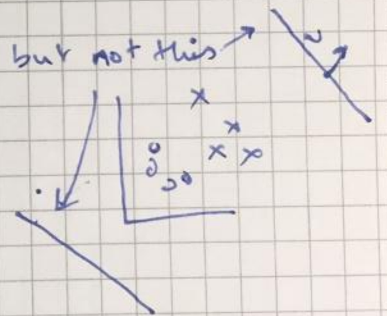
$= \sqrt{\alpha^2 w^T w} = \alpha\sqrt{w^T w} = \dfrac{w^T x + b}{w^T w}\sqrt{w^T w} = \dfrac{w^T x + b}{\sqrt{w^T w}}$

$$\boxed{\|d\|_2 = \frac{w^T x + b}{\|w\|_2}}$$

Margin of classifier $\gamma(w, b) = \min\limits_{x \in D} \dfrac{w^T x + b}{\|w\|_2}$

Aim find w which maximizes $\gamma(w, b)$, but not this

so need a constraint $\forall_i \; y_i(w^T x_i + b) \geq 0$



$\max\limits_{w,b} \left[\min\limits_{x \in D} \dfrac{w^T x + b}{w^T w} \right]$ with constraint

$\max\limits_{w,b} \dfrac{1}{w^T w} \left[\min\limits_{x \in D} w^T x + b \right]$

Our hypothesis $\mathcal{H} = \{x : w^T x + b = 0\}$, Rescale it such that

$\min\limits_{x \in D} w^T x + b = 1$

So now, we are left with $\max\limits_{w,b} \dfrac{1}{w^T w}$

Putting the two constraint together

$\left.\begin{array}{l} \forall_i \; y_i(w^T x_i + b) \geq 0 \\ \min\limits_{x \in D} w^T x + b = 1 \end{array}\right\} \Longleftrightarrow$

$\boxed{\begin{array}{l} \Rightarrow \min\limits_{w,b} w^T w \\ \\ y_i(w^T x_i + b) \geq 1 \end{array}}$ parabola with linear constraints

Use: Quadratic problem solver