

MATH 173 PROBLEM SET 7

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Problem 1. TODO

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Solution. For a harmonic function, we know the average over the surface of any ball is equal to the value at the center. We also know that the average over the inside of the ball is equal to the average in the center because the average on the inside is simply an integral of the averages over many concentric balls.

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Let x and y be two arbitrary points. Now, consider $B_x := B(x, R)$ and $B_y := B(y, R + d)$ where $d = |x - y|$, as the hint suggests. Then, we see that $B_x \subseteq B_y$.

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Let $\text{Avg}(\Omega)$ denote the average of u over the region Ω . Let $|\Omega|$ denote the area of Ω . We see that

$$\begin{aligned}\text{Avg}(B_y) &= \text{Avg}(B_x) \frac{|B_x|}{|B_y|} + \text{Avg}(B_y \setminus B_x) \frac{|B_y \setminus B_x|}{|B_y|} \\ &\geq \text{Avg}(B_x) \frac{|B_x|}{|B_y|}.\end{aligned}$$

In the second step, we used that u is non-negative. Note that $\lim_{R \rightarrow \infty} \frac{|B_x|}{|B_y|} = 1$, so for any ε , we can find R large enough that

$$u(y) = \text{Avg}(B_y) \geq \text{Avg}(B_x) \frac{|B_x|}{|B_y|} \geq \text{Avg}(B_x)(1 - \varepsilon) = u(x)(1 - \varepsilon).$$

Thus, we can conclude that $u(y) \geq u(x)$. By an identical argument, we also know $u(y) \leq u(x)$. Since x and y were arbitrary, u must be the constant function. \square

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