MATH 173 Problem Set 9

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Problem 1. ◁

Solution.

Problem 2. ◁

Solution.

(a) We need $\int_0^1 |x^\alpha|^2 = \int_0^1 x^{2\alpha}$ to converge. This converges for $\alpha > -1/2$ and diverges for $\alpha \leq -1/2$, so $\phi_\alpha \in L^2((0,1))$ for $\alpha > -1/2$. □

(b) We need $\phi_\alpha \in L^2((0,1))$, so $\alpha > -1/2$. But, since ϕ_α are smooth, we also need $\int_0^1 |\phi'_\alpha|^2$ to converge. We see $\phi'_\alpha = \alpha x^{\alpha-1}$. and $\int_0^1 |\alpha x^{\alpha-1}|^2 = |\alpha|^2 \int_0^1 x^{2(\alpha-1)}$ converges for $\alpha > 1/2$ and diverges for $\alpha \leq 1/2$. So, $\phi_\alpha \in H^1((0,1))$ for $\alpha > 1/2$. □

Problem 3. ◁

Solution.

Problem 4. ◁

Solution.

Problem 5. ◁

Solution.

Problem 6. ◁

Solution.

Problem 7. ◁

Solution.