MATH 173 Problem Set 3

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Problem 1. TODO ◁

Solution. As the hint suggests, $u' = 0$ means by definition $u(\phi) = 0$ for any $\phi \in C_c^\infty(\mathbb{R})$. For any $\psi \in C_c^\infty(\mathbb{R})$, let $\phi_0 \in C_c^\infty(\mathbb{R})$ be a bump function such that $\int_{\mathbb{R}} \phi_0(x)dx = 1$. Let $\hat{\psi} = \psi - \phi_0 \int_{\mathbb{R}} \psi(x)dx$. We see that

$$\int_{\mathbb{R}} \hat{\psi}(x)dx = \int_{\mathbb{R}} \psi(x)dx - \int_R \psi(x)dx \cdot \int_{\mathbb{R}} \phi_0(x)dx = \int_{\mathbb{R}} \psi(x)dx - \int_{\mathbb{R}} \psi(x)dx = 0.$$

So, we can let

$$\phi(x) = \int_0^x \hat{\psi}(x)dx.$$

We see $\hat{psi}$ has compact support (since it is the sum of two compact support functions). Since $\int_{\mathbb{R}} \hat{\psi}(x)dx = 0$, we know ϕ must have compact support as well. Now, let $c = u(\phi_0)$. We see that by linearity of u,

$$u(\psi) = u(\hat{\psi}) + u(\phi_0) \cdot \int_{\mathbb{R}} \psi(x)dx = u(\phi') + c \int_{\mathbb{R}} \psi(x)dx = 0 + c \int_{\mathbb{R}} \psi(x)dx = c \int_{\mathbb{R}} \psi(x)dx,$$

which is exactly what we wanted to prove. □

Problem 2. TODO $\triangleleft$

Solution. TODO $\square$

Problem 3. TODO $\triangleleft$

Solution. TODO $\square$

Problem 4. TODO ◁

Solution. TODO □

Problem 5. TODO $\triangleleft$

Solution. TODO $\square$

Problem 6. TODO $\triangleleft$

Solution. TODO $\square$

Problem 7. TODO ◁

Solution. TODO □