

MATH 173 PROBLEM SET 9

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June 1, 2022

Problem 1.

Solution.

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Problem 2.

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Solution.

- (a) We need $\int_0^1 |x^\alpha|^2 = \int_0^1 x^{2\alpha}$ to converge. This converges for $\alpha > -1/2$ and diverges for $\alpha \leq -1/2$, so $\phi_\alpha \in L^2((0, 1))$ for $\alpha > -1/2$. \square
- (b) We need $\phi_\alpha \in L^2((0, 1))$, so $\alpha > -1/2$. But, since ϕ_α are smooth, we also need $\int_0^1 |\phi'_\alpha|^2$ to converge. We see $\phi'_\alpha = \alpha x^{\alpha-1}$. and $\int_0^1 |\alpha x^{\alpha-1}|^2 = |\alpha|^2 \int_0^1 x^{2(\alpha-1)}$ converges for $\alpha > 1/2$ and diverges for $\alpha \leq 1/2$. So, $\phi_\alpha \in H^1((0, 1))$ for $\alpha > 1/2$. \square

Problem 3.

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Solution.

- (a) We know the statement is true for $f \in C^1((a, b))$ by FTC. Now, let $f_n \rightarrow f$ where $f_n \in C^1((a, b))$. By the continuity of the trace operator,

$$f(x) - f(y) = \lim_{n \rightarrow \infty} (f_n(x) - f_n(y)) = \lim_{n \rightarrow \infty} \int_x^y f'_n(t) dt$$

TODO: finish this.

- (b) TODO

Problem 4.

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Solution.