

# MATH 173 PROBLEM SET 9

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**Problem 1.**

*Solution.*

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**Problem 2.**

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***Solution.***

- (a) We need  $\int_0^1 |x^\alpha|^2 = \int_0^1 x^{2\alpha}$  to converge. This converges for  $\alpha > -1/2$  and diverges for  $\alpha \leq -1/2$ , so  $\phi_\alpha \in L^2((0, 1))$  for  $\alpha > -1/2$ .  $\square$
- (b) We need  $\phi_\alpha \in L^2((0, 1))$ , so  $\alpha > -1/2$ . But, since  $\phi_\alpha$  are smooth, we also need  $\int_0^1 |\phi'_\alpha|^2$  to converge. We see  $\phi'_\alpha = \alpha x^{\alpha-1}$ . and  $\int_0^1 |\alpha x^{\alpha-1}|^2 = |\alpha|^2 \int_0^1 x^{2(\alpha-1)}$  converges for  $\alpha > 1/2$  and diverges for  $\alpha \leq 1/2$ . So,  $\phi_\alpha \in H^1((0, 1))$  for  $\alpha > 1/2$ .  $\square$

**Problem 3.**

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***Solution.***

- (a) We know the statement is true for  $f \in C^1((a, b))$  by FTC. Now, let  $f_n \rightarrow f$  where  $f_n \in C^1((a, b))$ .  
By the continuity of the trace operator,

$$f(x) - f(y) = \lim_{n \rightarrow \infty} (f_n(x) - f_n(y)) = \lim_{n \rightarrow \infty} \int_x^y f'_n(t) dt$$

TODO: finish this.

- (b) TODO

**Problem 4.**

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*Solution.*