MATH 173 Problem Set 1

Stepan (Styopa) Zharkov

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Problem 1. TODO ◁

Solution. First, we see that

$$\nabla \times F = \nabla \times \nabla f = (\delta_2\delta_3 f - \delta_3\delta_2 f, \delta_3\delta_1 f - \delta_3\delta_1 f, \delta_1\delta_2 f - \delta_1\delta_2 f) = (0, 0, 0)$$

because order of differentiation does not matter. Also,

$$\nabla \cdot F = \nabla \cdot \nabla f = \delta_1\delta_1 f + \delta_2\delta_2 f + \delta_3\delta_3 f = \Delta f,$$

as we wanted. □

Problem 2. TODO ◁

Solution.

(a) Consider $u(x_1, x_2) = x_1 - x_2$. We see that $\delta_1 u + \delta_2 u = 1 - 1 = 0$ and $u(x, x) = 0$, but u is nonzero. □

(b) Suppose $u(\hat{x}_1, \hat{x}_2) \neq 0$ for some $\hat{x}_1, \hat{x}_2$. Consider the function $f(s) = u(\hat{x}_1 + s, \hat{x}_2 + s)$. We see that $f(0) \neq 0$ and $f((-\hat{x}_1 - \hat{x}_2)/2) = u((\hat{x}_1 - \hat{x}_2)/2, (-\hat{x}_1 + \hat{x}_2)/2) = 0$. By the mean value theorem, there is some point where $f' \neq 0$.

However, we see that $f'(s) = \delta_1 u(\hat{x}_1 + s, \hat{x}_2 + s) + \delta_2 u(\hat{x}_1 + s, \hat{x}_2 + s) = 0$. So, we have a contradiction and thus there is no such $\hat{x}_1, \hat{x}_2$ and $u = 0$. □

(c) Let $f_r(s) = u(r + s, -r + s)$. We $f'(s) = \delta_1 u(r + s, -r + s) + \delta_2 u(r + s, -r + s) = 0$, so f_r is constant. Thus, $u(r, -r)$ defines all of f_r. Note that any point (x_1, x_2) is expressed uniquely as $(r + s, -r + s)$, so the f_r cover the entire plane with no overlap. In other words, Any solution can be described as $u(x_1, x_2) = g(x_1 - x_2)$

Problem 3. TODO ◁

Solution.

(a) TODO

(b) TODO

Problem 4. TODO ◁

Solution. TODO

Problem 5. TODO $\triangleleft$

Solution.

(a) TODO

(b) TODO

Problem 6. TODO $\triangleleft$

Solution. TODO

Problem 7. TODO ◁

Solution.

(a) TODO

(b) TODO