

MATH 173 PROBLEM SET 7

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Problem 1.

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Solution. This problem is straightforward. Since $\overline{\Omega}$ is closed, and $c > 0$, there exists constants $C_1, C_2 > 0$ such that $C_1 < c(x) < C_2$. Assume $C_1 < 1$ and $C_2 > 1$. If not, we can always choose smaller C_1 and larger C_2 . So, for any $u \in C^1(\overline{\Omega})$,

$$\begin{aligned} \|u\|_{H_c^1(\Omega)}^2 &= \int_{\Omega} |u(x)|^2 dx + \int_{\Omega} c(x) |\nabla u(x)|^2 dx \\ &\geq C_1^2 \int_{\Omega} |u(x)|^2 dx + \int_{\Omega} C_1^2 |\nabla u(x)|^2 dx \\ &= C_1^2 \|u\|_{H^1(\Omega)}^2 \end{aligned}$$

Problem 2.

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Solution. TODO

Problem 3.

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Solution. TODO

Problem 4.

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Solution. TODO

Problem 5.

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Solution. TODO

Problem 6.

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Solution. TODO