Factor Analysis using EM

Wednesday, October 12, 2022 10:15 AM

$$\underline{Y} \sim \mathcal{N}(\underbrace{\underline{\underline{W}}\,\underline{z}}_{\text{signal}}, \overbrace{\underline{\underline{\Psi}}^{-1}}^{\text{noise}}) \qquad \underline{\underline{\Psi}} = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_p \end{pmatrix}$$

$$\underline{z} \sim \mathcal{N}(\underline{0}, \mathbb{I})$$

$\underline{\underline{\Psi}}$ noise covariance

$\underline{\underline{\Psi}}^{-1}$ noise precision

$\underline{Y} \in \mathbb{R}^d$, $\underline{z} \in \mathbb{R}^p$, $\underline{\underline{W}} \in \mathbb{R}^{d \times p}$

N trials

$d > p$

$p = 3$

$d = 7, 11, 13$

$N = 10^3$

$\underline{\underline{Y}}$ $d \times N$, $\underline{\underline{z}}$ $p \times N$ $\leftarrow \mathcal{N}(0, \mathbb{I})$, $\underline{\lambda}$ $d \times 1$, $\underline{\underline{W}}$ $d \times p$ $\leftarrow \mathcal{N}(0, 1)$

$\underline{\underline{Y}} \leftarrow \mathcal{N}(\underline{\underline{W}}\,\underline{\underline{z}}, \underline{\underline{\Psi}}^{-1})$

$$\underline{\underline{W}}\,\underline{z} \rightarrow \underline{w}^T \underline{z} = \sum_{i=1}^{p} w_i z_i$$

$SNR = \gamma$ (e.g. $\gamma = 100$)

$\mathrm{Var}\left[\sum^{p} w_i z_i\right]$ $\mathrm{Var}(\mathbb{E}[w_i])(\mathbb{E}[z]$

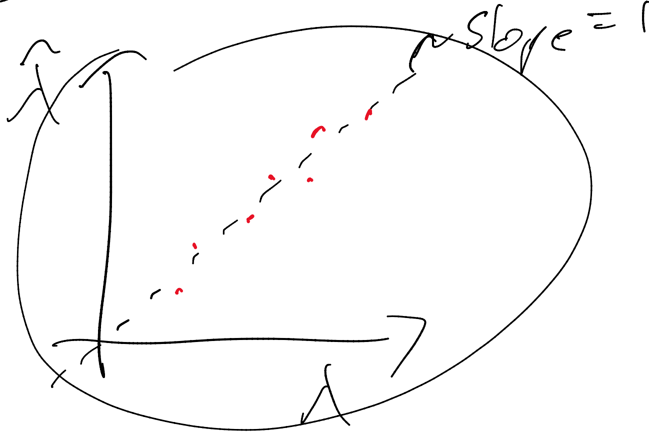
$SNR = \gamma$ (eg. ...)

$\Rightarrow$ set $\lambda \approx \frac{\gamma}{\rho}$

$\mathrm{Var}\left[\sum_{i=1}^{\rho} W_i z_i\right] = \rho \, \mathrm{Var}[W_i] \, \mathrm{Var}[z_i]$

Signal Var $= \rho$

do EM $\rightarrow \hat{\underline{W}}, \hat{\underline{\Psi}} \rightarrow (\lambda_1 \ldots \lambda_d)$



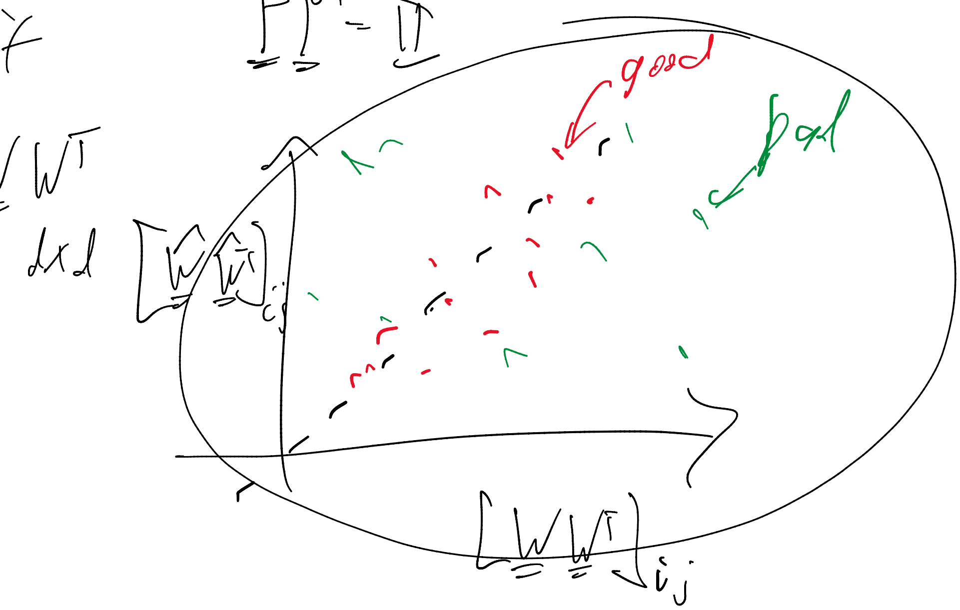
$\underline{W}\underline{z} = y$

$\underline{W}' = \underline{W}\underline{P}, \quad \underline{z}' = \underline{P}^T \underline{z}$

$y' = \underline{W}'\underline{z}' = \underline{W}\underline{P}\underline{P}^T\underline{z} = \underline{W}\underline{z} = y$

$\underline{P}\underline{P}^T = \underline{I}$

$\underline{W}'\underline{W}'^T = \underline{W}\underline{P}\underline{P}^T\underline{W}^T = \underline{W}\underline{W}^T$



$\sum_{n=1}^{N} \mathbb{E}[\underline{z}_n]$

$y_1 = \underline{W} z_1 + \epsilon_1$

trial 1

$\underline{y}_1 = \underline{\underline{W}}\, \underline{z}_1 + \underline{\epsilon}_1$ trial 1

$\underline{y}_2 = \underline{\underline{W}}\, \underline{z}_2 + \underline{\epsilon}_2$ trial 2

$\vdots$

$\mathbb{E}[\underline{z}_n] = \underline{m}_n$ ~ poster menu

$\mathbb{E}[\underline{z}_n \underline{z}_n^T] + 0$

$+ \underline{m}_n \underline{m}_n^T - \underline{m}_n \underline{m}_n^T$

$$\underbrace{\mathbb{E}[\underline{z}_n \underline{z}_n^T] - \mathbb{E}[\underline{z}_n]\mathbb{E}[\underline{z}_n]^T}_{\mathrm{Cov}[\underline{z}_n]} + \mathbb{E}[\underline{z}_n]\,\mathbb{E}[\underline{z}_n^T]$$

Look for intermediate steps.