# On the Insolvability of the Quintic

A more accessible approach (subtitle wip)

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# I Introduction

## 1 Motivation

Often in math do we come across and work with polynomials. They're simple, easy to understand, and widely applicable across many areas of study. For instance, we've seen polynomials a number of times within our course:

- Linear Diophantine equations are polynomials of one degree in two variables:
$$ax + by - c = 0$$
- Pythagorean triples are integer solutions to a polynomial of two degrees in three variables:
$$x^2 + y^2 - z^2 = 0$$
- The problem of the sum of two squares is similar:
$$a^2 + b^2 - n = 0$$
- Fermat's Last Theorem famously generalizes the Pyhthagorean triples:
$$x^n + y^n - z^n = 0$$
- Fermat's *Little* Theorem, and more generally Euler's Theorem, is a statement about polynomials of a particular degree, modulo some value (ok, this is pushing it):
$$g^{p-1} \equiv 1 \bmod p \qquad g^{\varphi(n)} \equiv 1 \bmod n$$

When we discuss these topics, we're really discussing these polynomials. But more specifically, we care about the *solutions* to these polynomials, moreso than the polynomials themselves. Thus, it is only natural to ask - when, in general, do we have solutions to any given polynomial, and if they exist - what are they? some mention of solns in $\mathbb{Z}$ vs $\mathbb{R}$ here?

# II History

# III Prerequisite Algebra

# IV Proof of the Statement

# V Bibliography