MAT367
Intro to Differential Geometry

Class Lecture Notes

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January 6, 2026

Last Updated:

January 6, 2026

Primary source material:

Class Lectures

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Contents

Preface

These notes were created during class lectures. As such, they may be incomplete or lacking in some detail at parts, and may contain confusing typos due to time-sensitivity. Additionally, these notes may not be comprehensive. Most statements in this document which are not Theorems, Problems, Lemmas, Corollaries, or similar, are likely paraphrased to a certain degree. Please do not treat any material in this document as the exact words of the original lecturer.

If you are viewing this document in Obsidian, you may notice that the links in the pdf document do not work. This is intentional behaviour, as I currently do not have or know of a decent solution which allows them to behave well with the setup in Obsidian. However, below certain pages, there may be links to other documents - these are usually context-relevant links between notes of different areas of study. I created these links to point out potential similarities, or in case one area of study is borrowing a concept, definition, or theorem from another area of study, and you wish to see the full, original definition/derivation/proof or whatever it may be.

I Introduction and Manifolds

1 Preliminaries and Basics

Lec 1 - Jan 6 (Week 1)

we know what manifolds are. recall implicit thm:

Theorem 1.1: Implicit Function Thm

given eqn $f(x_1, \ldots, x_{n+1}) = 0$ for smooth f and a soln $p \in \mathbb{R}^{n+1}$ s.t. $\nabla f \neq 0$ at p, then $f(x_1, \ldots, x_{n+1})$ is the soln set near p.

furthermore, we can represent solns as $(x_1, \ldots, x_n, g(x_1, \ldots, x_n))$, where g also smooth.

note $f(x_1, \ldots, x_{n+1}) = 0$ is locally the graph of a fn. if 0 a reg pt of f, then we can cvr $\{x : f(x) = 0\}$ by graphs, i.e. charts. oh example spam sure

Theorem 1.2: Whitney Embedding Thm

every n-mfld has an embedding in $\mathbb{R}^{2n}$.

we wont get to this thm in this course, but its neat. or sth

Example 1.3

Source: Primary Source Material

let M be the set of all rots of a ball. this is a *configuration space* - pts represent ways to configure another object/space.

how do we put coords on (a piece of) this space? one way: identify w SO(3). another way: where N goes has 2 degs of freedom. where the vector $e_1 = (1, 0, 0)$ goes @ N is 1 deg of freedom. thus, M is 3-dim; this also gives a way of defining a chart near the id.

Example 1.4

Source: Primary Source Material

consider M as the config space of a *linkage* (picture a graph, but physical, where edges can rotate around vertices - saw a vid abt this i think). in this ex, sps it "looks like" a quadrilateral, and wlog one edge is fixed in space; all edges coplanar. what dim is M?

each vec is in $\mathbb{R}^2$, so we start w 8 variables. but since its a closed shape, $n_1 + n_2 + n_3 + n_4 = 0$, so -1 dim. furthermore, $\|n_i\|$ fixed, so -4 dim. since n_1 fixed, -1 dim; this leaves 2 dims, and indeed, M is 2-dim'l. it can in fact be parameterized by two adj angles θ, φ.

Fact 1.5

"closed" (ie cpt) surfaces are easy to enumerate (classify).

- orientable: sphere or conn sum of torii
- non-orientable: $\mathbb{R}P^2$, klein bottle, conn sum of those w torii (handles)

ok now for our proper defns.

Definition 1.6

a **coordinate chart** is an inj $\varphi : U \longrightarrow \mathbb{R}^n$ w open img for some $U \subseteq M$.

two charts $\varphi : U \longrightarrow \mathbb{R}^n$ and $\psi : V \longrightarrow \mathbb{R}^n$ are **compatible** if the transition function given by:

$$\psi \circ \varphi^{-1} : \varphi(U \cap V) \longrightarrow \psi(U \cap V)$$

is a diffeo. we also enforce that if $U \cap V = \varnothing$, then they are compatible.