Chapter 3 Signals & Systems

Signal: Signal is a function of one or more independent variables which contains some information. e.g. Radio Signal, T.V Signal. The signals can be one-dimensional or multi-dimensional.

classification of signals
Based upon their nature & characteristics in the time domain, the signals may be broadly classified as under:
a) Continuous-time signals
b) Discrete-time signals

(a) Continuous-time Signals
- may be defined as a mathematical continuous func This funcⁿ is defined continuously in the time domain. For continuous-time signals, the independent variable is time t. e.g sinewave, cosinewave etc.

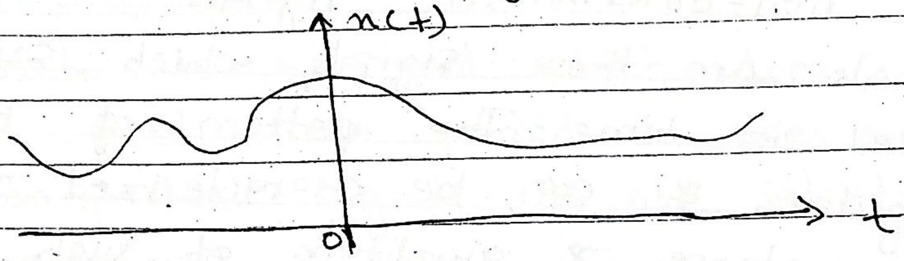


Fig: A continuous-time signal.

(b) Discrete-time signals
- may be defined only at certain time-instants. For discrete-time signal, the amplitude betⁿ two time instants is just not defined. For these signals,

the independent variable is time n.

e.g If we take the blood pressure readings of a pat ient after every one hour & plot the graph, then the resultant signal will be discrete time signal.

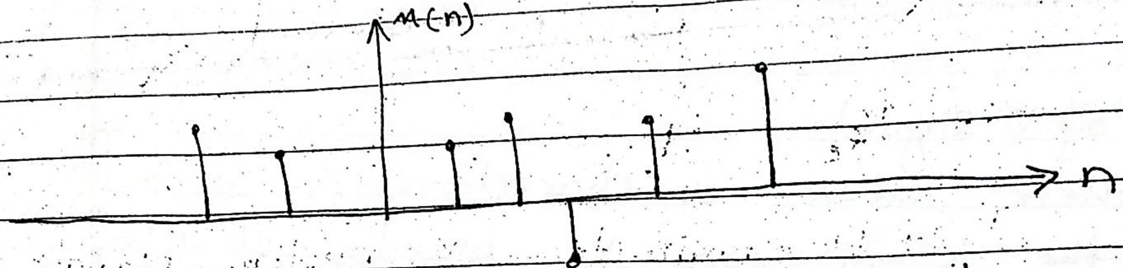


Fig: A discrete-time signal.

We may further classify both continuous-time & discrete-time signals as under.

(i) Deterministic & non-deterministic Signals.
(ii) Periodic & non-periodic | aperiodic Signals.
(iii) Even and odd Signals.
(iv) Energy & Power signals.

i) Deterministic & non-deterministic signals

:- Deterministic signals are those signals which can be completely specified in time. The pattern of this type of signal is regular & can be characterized mathematically. Also, the nature & amplitude of such a signal at any time can be predicted.

e.g $x(t) = bt$, $x(t) = a \sin \omega t$.

:- On the other hand, a non-deterministic signals is thermal noise generated in an electric circuit. Such a noise signal has probabilistic behaviour.

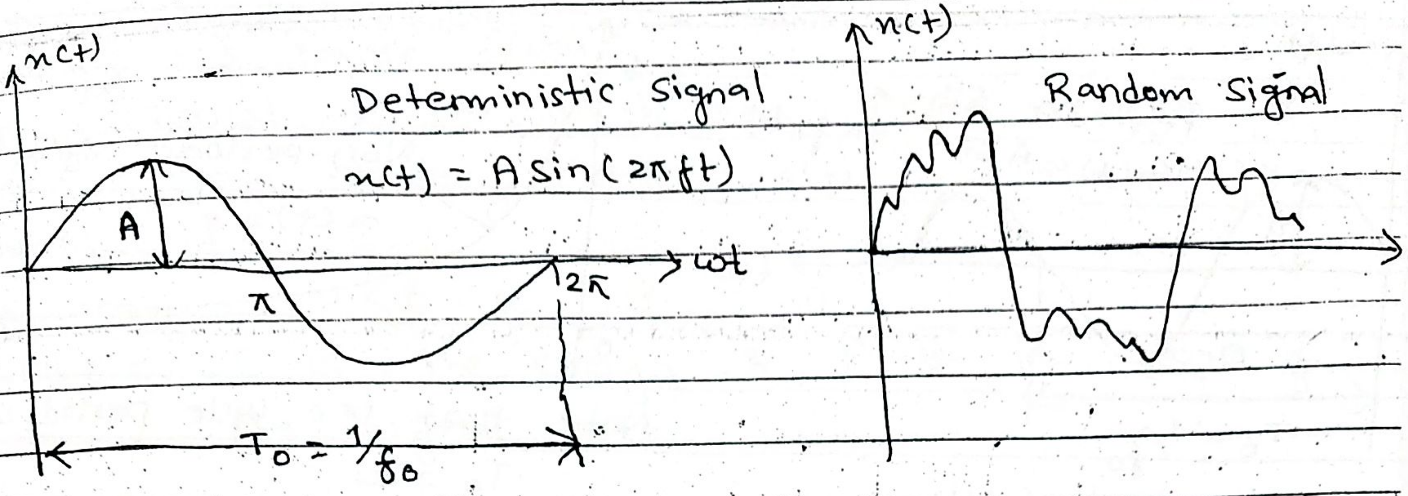


Fig: Deterministic & random signals.

(ii) Periodic & Non-periodic Signals.

Periodic signal is a signal which repeats itself af[ter] a fixed time period. The periodicity of a signal c[an] be defined mathematically as under:

$$x(t) = x(t + T_0) \quad : \text{cond}^n \text{ of periodicity}$$

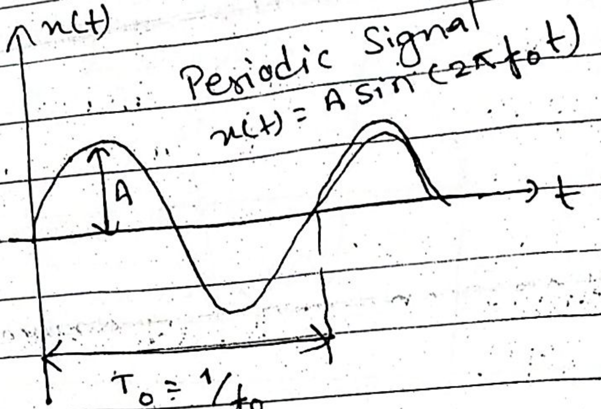
where T_0 is called as the period of signal $x(t)$. here signal $x(t)$ repeats itself af[ter] a period of T_0 sec. e.g sine wave, cosine wa[ve]

Non-periodic is a signal which does not repeat itse[lf] after a fixed time period or does not repeat at a[ll]. The non-periodic signals do not satisfy the cond^n of periodicity.

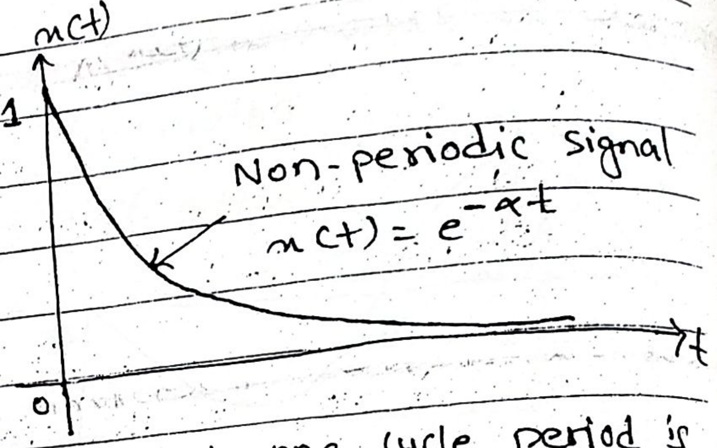
For a non-periodic signal, $x(t) \neq x(t + T_0)$

e.g exponential signal.

Aperiodic signal has a period $T_0 = \infty$. $x(t) =$ is non-periodic but it is deterministic



period of repetition.



Note that one cycle period is $T_0 = \infty$

iii) Even & odd signals (Symmetrical or antisymmetrical)

→ A signal $x(t)$ is said to be symmetrical or even if it satisfies following condn.

Condition for symmetry : $x(t) = x(-t)$

where $x(t)$ → value of signal for positive

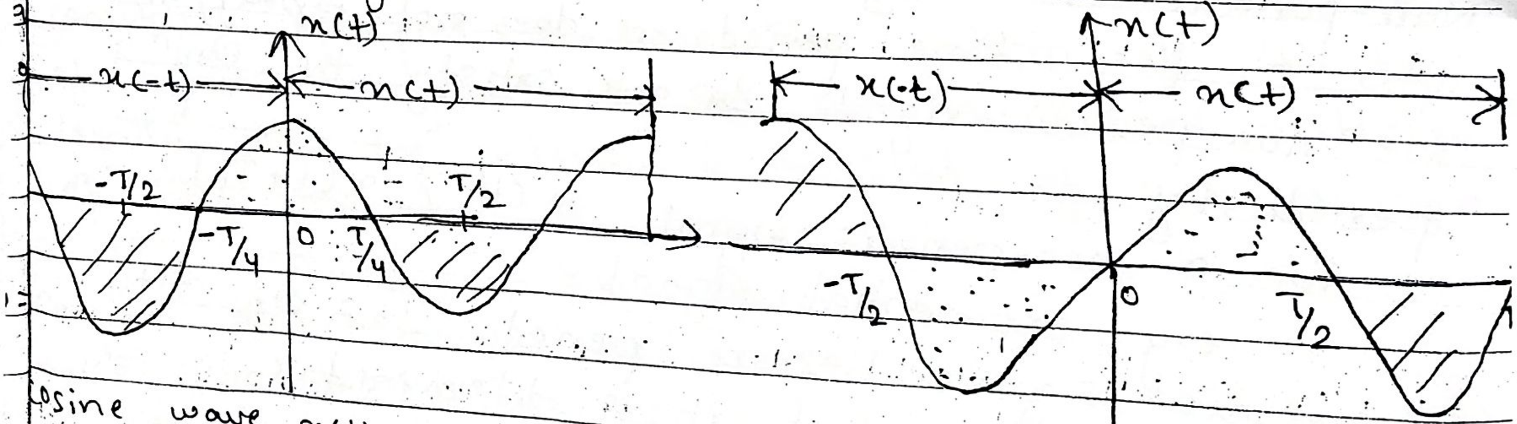
$x(-t)$ → " " " negative

e.g cosine wave.

→ A signal $x(t)$ is said to be antisymmetrical or odd if it satisfies the following condition.

Condition for antisymmetry : $x(t) = -x(-t)$

e.g. sine wave.



cosine wave $x(t) = x(-t)$
symmetrical or even signal.

sine wave $x(t) = x(-t)$
asymmetrical or odd signal

Fig : Symmetrical & antisymmetrical signals.

(iv) Energy & Power Signals

→ Energy signal is one which has finite energy & zero average power. Hence, $x(t)$ is an energy signal, if:

$$0 < E < \infty \quad \& \quad P = 0$$

where, E is the energy & P is the power of the signal $x(t)$.

- Power signal is one which has finite average power & infinite energy. Hence, $x(t)$ is a power signal, if:

$$0 < P < \infty \quad \& \quad E = \infty$$

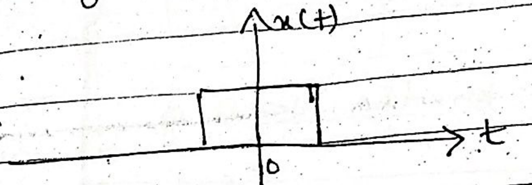
However, if the signal does not satisfy any of the above two conditions, then it is neither an energy signal nor a power signal.

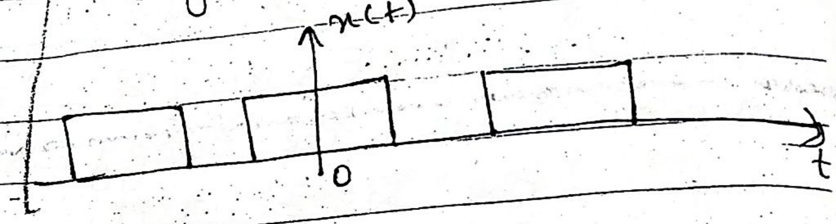
Comparision of Energy & Power Signal

Energy Signal	Power Signal
1. Total normalized energy is finite & nonzero.	1. The normalized average power is finite & nonzero.
2. The energy is obtained by, $E = \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt$.	2. The average power is obtained by, $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t) \rvert^2 dt$
3. Nonperiodic signals are energy signals.	3. Practical periodic signals are power signals.
4. These signals are time limited.	4. These signals can exist over infinite time.

5. Power of energy signal is zero. | 5. Energy of the power signal is infinite.

e.g. single rectangular pulse | e.g. periodic pulse train





Q. Prove that the sine wave ~~shown in fig~~ is a periodic signal.

→ Mathematically sine wave can be represented as,

$$x(t) = A \sin \omega_0 t \quad \text{——— (i)}$$

Now, let us test if it satisfies the condition for periodicity i.e if,

$$x(t) = x(t + T_0)$$

So, let us find the expression for $x(t+T_0)$ i.e

$$x(t+T_0) = A \sin \omega_0 (t + T_0)$$

$$= A \sin (\omega_0 t + \omega_0 T_0) \quad \text{——— (ii)}$$

$$\omega_0 = 2\pi f_0$$

$$\& \quad T_0 = 1/f_0$$

$$\therefore \omega_0 T_0 = 2\pi f_0 \times \frac{1}{f_0} = 2\pi$$

Substituting this in eq^n (iii), we obtain,

$$x(t+T_0) = A \sin[\omega_0 t + 2\pi]$$

$$= A[\sin\omega_0 t \cos 2\pi + \cos(\omega_0 t) \sin 2\pi]$$

$$= A \sin \omega_0 t = x(t)$$

Hence the sine wave is a periodic signal.

Prove that the exponential Signal is non-periodic.

→ Mathematically exponential signal is expressed as,

$$x(t) = e^{-at}$$

Substituting, $t = (t + T_0)$, we get,

$$x(t+T_0) = e^{-a(t+T_0)}$$

$$= e^{-at} \cdot e^{-aT_0}$$

But $T_0 = \infty$

$$\therefore e^{-aT_0} = e^{-\infty} = 0$$

$$x(t+T_0) = e^{-at} \cdot 0 = 0.$$

$$x(t) \neq x(t+T_0)$$

Hence exponential signal is non-periodic Signal.

System

→ is a set of elements or functional blocks which are connected together & produces an output in response to an input signal.

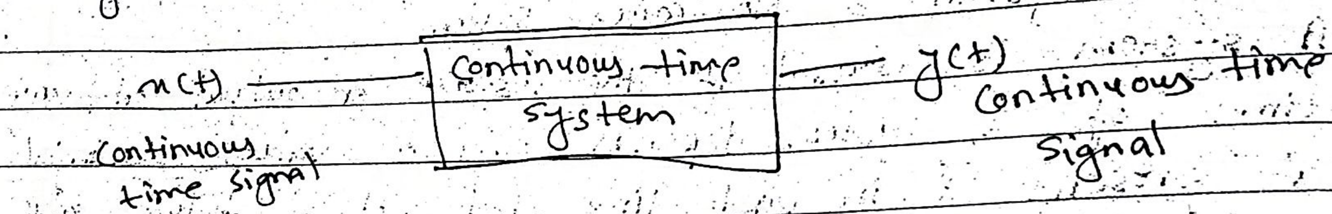
→ The response or output of the system depends upon transfer function of the system.

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

e.g. Filters, amplifiers, communication channel etc

Types of Systems
(a) Continuous time System
(b) Discrete time System

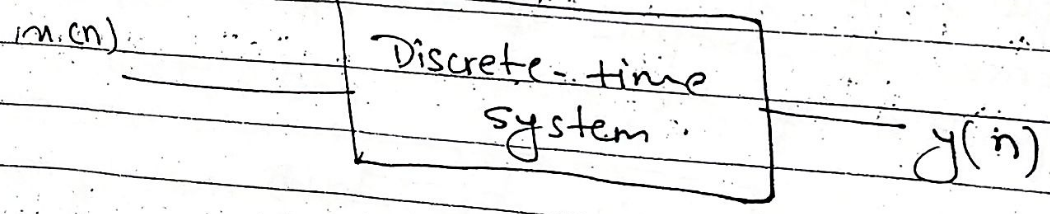
(a) Continuous time System
- may be defined as those system in which the associated signals are continuous, i.e. input & output of continuous time systems are both continuous-time signal.



$$y(t) = f[x(t)]$$

e.g power supply, audio/video amplifier

(b) Discrete time System
- may be defined as those system in which the associated signals are discrete time signal, i.e input & output of discrete time system are both discrete signal.



$$y(n) = f[x(n)]$$

e.g. microprocessor, flip flops, Shift-registers.

Basic properties of Systems

i) Causal systems & Non-causal Systems
ii) Time-invariant & Time-variant Systems
iii) Stable Systems & Unstable Systems
iv) linear Systems & Non-linear Systems
v) Static Systems & Dynamic Systems

i) Causal system & Non-causal system

- A system is causal if the response or output at any time of a system depends only on values of input at present & in past time, but not in future value that means if input is applied at $t = t_0$, then for causal system output will depend on values of input $x(t)$ for $t \leq t_0$.

Mathematically,

$$y(t_0) = f\,[x(t),\ t \leq t_0]$$

e.g. Real time systems, Resistors etc.

* $y(t) = 0.2\,x(t) - x(t-1)$
* $y(t) = 0.8\,x(t-1)$
* $y[n] = \sum_{k=-\infty}^{n} x(k)$
* $y[n] = x(n-1)$

- If the output or response of the system to input depends on the future values of that input then the system is called non-causal system.

e.g Image processing signal.

- $y(t) = x(t+1)$
- $y[n-2] = x(n)$
- $y[n] = x[2n] - x[n]$

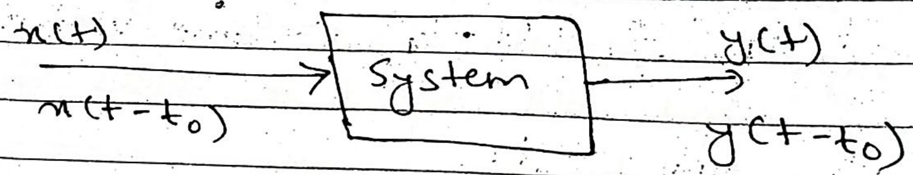
Note

Causal system is also called Non-anticipatory System.

- Non-causal " " " " Anticipatory "

(iii) Time-invariant & Time-variant Systems

- A system is called time invariant if the input, output relationship doesn't vary with time i.e the behaviour and input-output (i/o) characteristics do not change with time.



Mathematically,

Let a system defined by eq^n

$$y(t) = f[x(t)]$$

Now, if $x(t)$ is delayed by time t_0 then o/p $y(t)$ will also delayed by the same time t_0.

i.e $$f[x(t-t_0)] = y(t-t_0)$$

Similarly for Discrete-time System,

$$f[x(n-k)] = y[n-k]$$, for all $x(n)$ & any values of k.

Note : To check time-variance & time-invariance of a system, we have to show,

1) For a Continuous-time System:

$$y(t, t_1) = y(t - t_1) \leftarrow \text{Time-invariance}$$

where,

$$y(t, t_1) = f[x(t - t_1)]$$

i.e o/p of the system for delayed input.

$y(t - t_1) \Rightarrow$ delay the o/p by t_1 i.e simply place $(t-t_1)$ at the place of t.

$$y(t, t_1) \neq y(t - t_1) \leftarrow \text{Time-variance.}$$

(II) For Discrete time System

$$y[n, k] = y(n-k)$$

where

$y[n, k] = f[x[n-k]]$ i.e o/p of the system for delayed input by k unit.

& $y[n-k] \rightarrow$ delay the o/p by k i.e simply place $(n-k)$ at the place of n.

Q Check whether the following systems are time invariance or time-variance.

(i) $y(t) = \sin x(t)$ (ii) $y[n] = n\, x[n]$

$\Rightarrow$ we have,

$$y(t) = \sin x(t)$$

— (I)

i.e $y(t) = f[x(t)] = \sin x(t)$

Now, if the input to the system i.e $x(t)$ is delayed by t_1, then the o/p of the system is given by,

$$y[t, t_1] = f[x(t-t_1)]$$

$$\therefore y[t, t_1] = \sin x(t-t_1) \quad \text{— (II)}$$

Again, delay the o/p, $y(t)$ by t_1 then we get,

$$y(t-t_1) = \sin x(t-t_1) \quad \text{— (III)}$$

$\therefore$ from eqn (II) & (III) we get

$y(t, t_1) = y(t-t_1)$ so the given system is time-invariance

(ii) $y[n] = n\,x[n]$

we have,

$$y[n] = f[x[n]] = n\,x[n]$$

Now, if the input to the system i.e $x[n]$ is delayed by k, then the o/p of the system is given by,

$$y[n, k] = f[x(n-k)]$$

$$\therefore y[n, k] = n\,x(n-k) \quad \text{— (II)}$$

Again, delay the o/p, $y[n]$ by k then we get,

$$y[n-k] = (n-k)\,x(n-k) \quad \text{— (III)}$$

$\therefore$ from eqn (II) & (III)

$y(n, k) \neq y(n-k)$ hence time-variance

) Stable System & Unstable System

A system is called a bounded-input, bounded-output (BIBO) stable if & only if every bounded input results in a bounded output.

If the signal $g(t)$ is bounded, then its magnitude is always a finite value.

Mathematically,

$$|g(t)| \leq M$$

Let us consider a continuous-time system whose input-output relationship is given as.

$$y(t) = f(x(t))$$

For this system to be stable, the output signal $y(t)$ must satisfy the following condition:

$$|y(t)| \leq M_y < \infty \quad \text{for all values of } t.$$

whereas the input signal $x(t)$ must satisfy the following condition:

$$|x(t)| \leq M_x < \infty \quad \text{for all values of } t$$

Here, both M_x & M_y represent some finite positive numbers.

→ The systems not satisfying the above condition are unstable.

iv) Linear & Non linear Systems.

→ A system is called linear, if superposition principle applies to that system. This means that a linear system may be defined as one whose response to the sum of the weighted inputs is same as the sum of the weighted responses.

let us consider two systems defined as follows:

$$y_1(t) = f(x_1(t))$$

Here $x_1(t)$ is the input or excitation & $y_1(t)$ is its o/p or response,

$$y_2(t) = f(x_2(t))$$

Then for a linear system,

$$f(a_1 x_1(t) + a_2 x_2(t)) = a_1 y_1(t) + a_2 y_2(t)$$

where a_1 & a_2 are constants.

Symbolically,

for continuous-time system,

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t)$$

for discrete-time system,

$$a_1 x_2(n) + a_2 x_2(n) \longrightarrow a_1 y_1(n) + a_2 y_2(n)$$

(u) static & dynamic System

→ A system is called static or memory less, if its output signal value depends only upon the present value of the input signal, e.g $y(n) = x^2(n)$

→ A system is dynamic or with memory, if its output signal value depends upon the past values of the input signal.

e.g. $y(n) = x(n-1) + x(n)$

Q. Determine whether the following systems are linear or non-linear

(i) $y(n) = x(n^2)$ or $y(t) = x(t^2)$ (ii) $y(n) = x^2(n)$ or $y(t) = x$

(i) $y(n) = x(n^2)$

→ Here, the outputs of the system to two inputs $x_1(n)$ & $x_2(n)$ will be,

$$y_1(n) = x_1(n^2)$$

&, $$y_2(n) = x_2(n^2)$$

Thus, the linear combination of the two outputs will be,

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad \text{———} (i)$$

Also,

the response to the linear combination of inputs will be,

$$y_4(n) = f\{a_1 x_1(n) + a_2 x_2(n)\}$$

Since the linear system satisfy the additive property;

$y_4(n) = f\{a_1 x_1(n)\} + f\{a_2 x_2(n)\}$

$= a_1 f\{x_1(n)\} + a_2 f\{x_2(n)\}$ ——— (ii)

$= a_1 x_1(n^2) + a_2 \cdot x_2(n^2)$

comparing eq^n (i) & (ii),

$y_3(n) = y_4(n)$

Hence, the given system is linear.

(ii) $y(n) = x^2(n)$

→ Here, the o/ps of the system to two inputs $x_1(n)$ & $x_2(n)$ will be,

$y_1(n) = x_1^2(n)$

$y_2(n) = x_2^2(n)$

Thus, linear combination of outputs will be,

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

$= a_1 x_1^2(n) + a_2 x_2^2(n)$ ——— (i)

Also,

the linear combination of inputs will be,

$y_4(n) = f\{a_1 x_1(n) + a_2 x_2(n)\}$

$= \{a_1 x_1(n) + a_2 x_2(n)\}^2$ $[\because y(n) = x^2]$

$= a_1^2 x_1^2(n) + 2a_1 a_2 x_1(n) x_2(n) + a_2^2 x_2^2(n)$ ——— (ii)

comparing eq^n (i) & (ii)

$y_3(n) \neq y_4(n)$

Hence, the given system is non-linear.

A discrete time system is described as,

$$y(n) = y^2(n-1) + x(n)$$

Now, a bounded input of $x(n) = 2\delta(n)$ is applied to this system. Assume that the system is initially relaxed, check whether the system is stable or unstable.

→ we know, $\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$

$\therefore$ $x(n) = \begin{cases} 2 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$

Again, we have

$$y(0) = y^2(0-1) + x(0)$$
$$= y^2(-1) + x(0) \qquad [\because y^2(-1) = 0$$ initially relaxed$]$
$$= 2$$

$$y(1) = y^2(1-1) + x(1)$$
$$= y^2(0) + x(1)$$
$$= 2^2 \qquad [\because x(1) = 0]$$

Similarly,

$$y(2) = y^2(2-1) + x(2) = y^2(1) = (2^2)^2 = 2^4$$
$$y(3) = y^2(3-1) + x(3) = y^2(2) = (2^4)^2 = 2^8$$
$$y(4) = y^2(4-1) + x(4) = y^2(3) = (2^8)^2 = 2^{16}$$

$\therefore$ $y(n) = 2^{2^n}$

Here as $n \to \infty$, $y(n) \to \infty$. Hence, the input $x(n) = 2\delta(n)$ is bounded for all 'n'.

$\therefore$ System is unstable.

Elementary Signals

There are several elementry signals which serves as basic building blocks for construction of more complex signals.

Elementry signals are

1) Exponential Signals
2) Sinusoidal Signals
3) Unit-step function
4) Unit-impulse function (Direct funcn or Direct delta funcn)
5) Ramp function.

1) Exponential signals * for a continuous time Signal

General expression for a real time exponential signal may be written as,

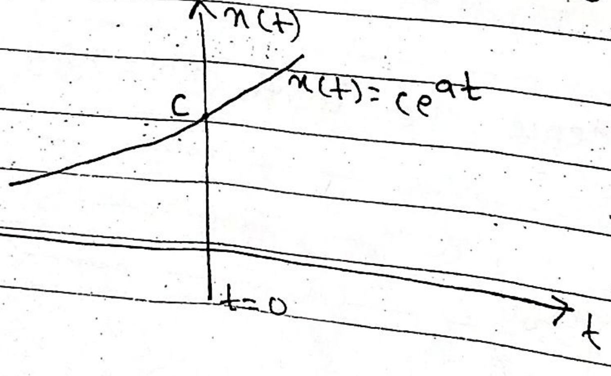
$$x(t) = Ce^{at}$$

where C & a are real parameters.

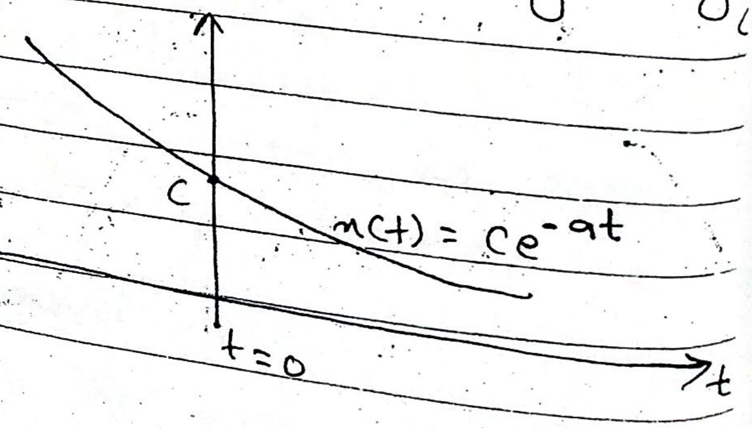
C is the amplitude of the exponential signal measured at t=0.

we can define two cases depending on whether the parameter a is positive or negative.

Case I : when $a > 0$
(Exponentially rising)



Case II : when $a < 0$
(Exponentially decaying)



for a discrete time signal

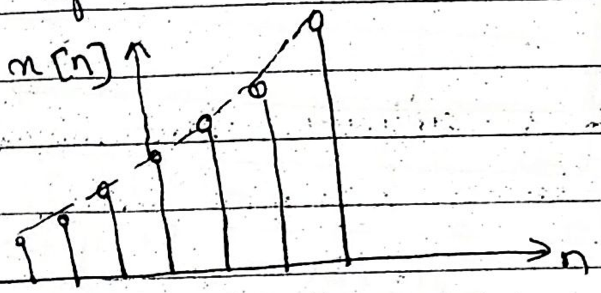
The general expression for a real Discrete-time exponential Signal may be expressed as

$$x[n] = Ce^{\alpha n}$$

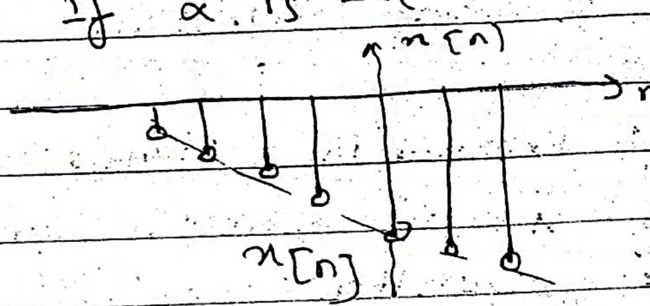
where C & α are real parameters

Case I : If α & C are real $|\alpha| > 1$

(i) If α is +ve

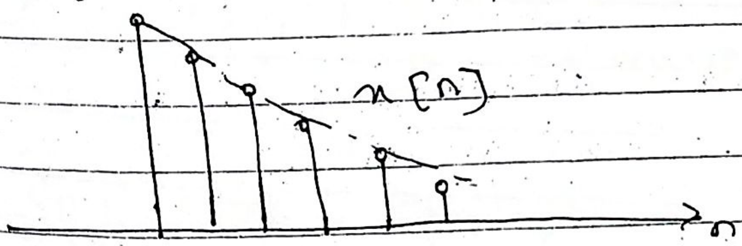


(ii) If α is -ve

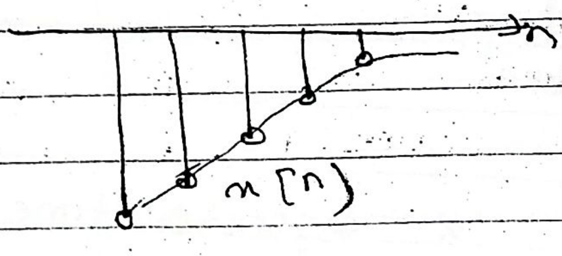


case II : If α & C are real $|\alpha| < 1$

(i) If α is +ve

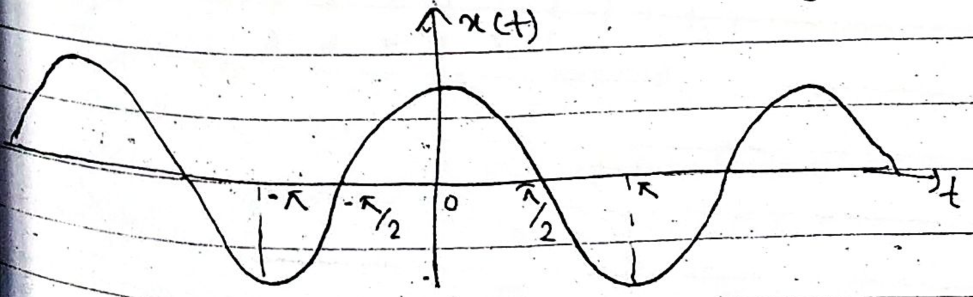


(ii) If α is -ve



(2) Sinusoidal Signal

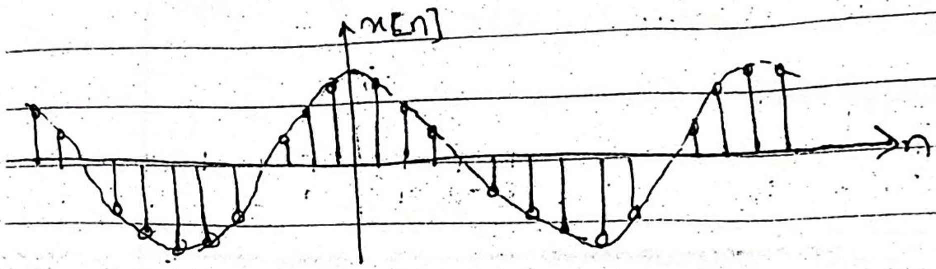
* For a continuous time signal



$$x(t) = A\cos(\omega t + \phi)$$

$$T = 2\pi/\omega$$

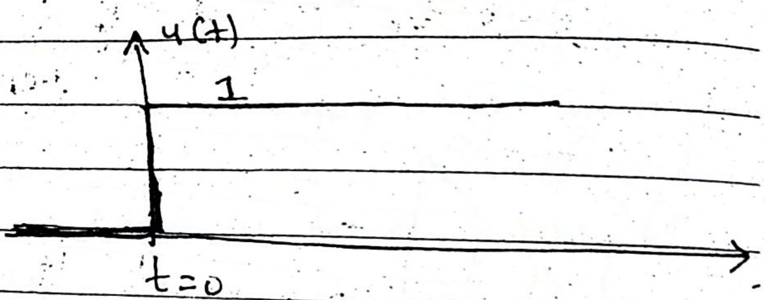
* For a discrete time Signal $x[n] = A\cos(\omega n + \phi)$

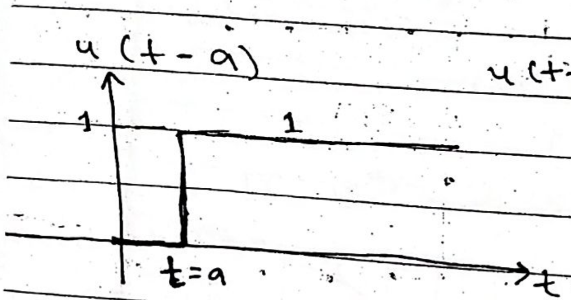


Unit step function :

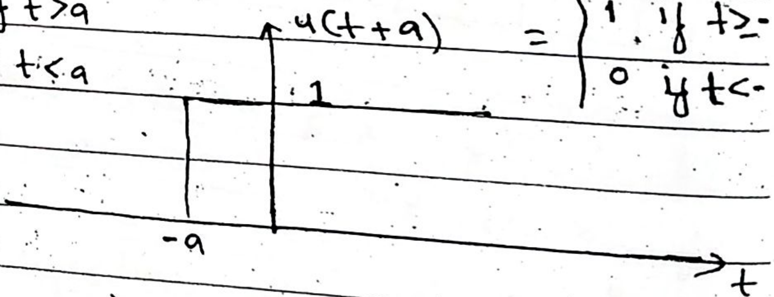
* For a continuous time Signal

$$u(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$$





$$u(t-a) = \begin{cases} 1 & \text{if } t > a \\ 0 & \text{if } t < a \end{cases}$$

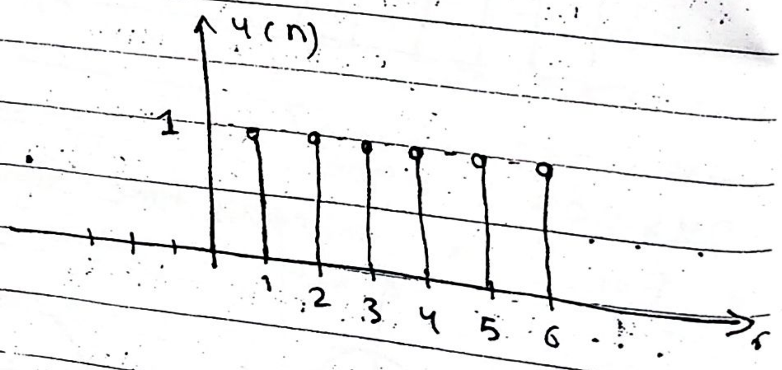


$$= \begin{cases} 1 & \text{if } t \geq - \\ 0 & \text{if } t < - \end{cases}$$

* For a discrete time Signal

Unit step sequence

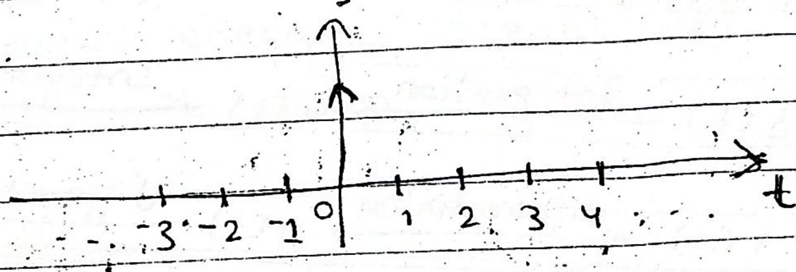
$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$



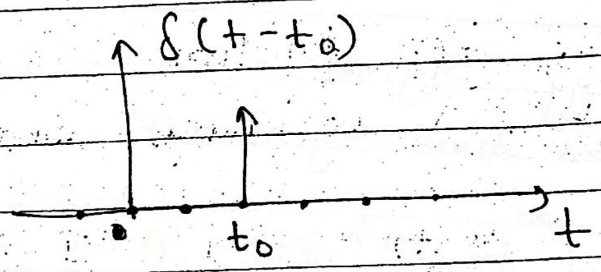
- Unit impulse function

* For Continuous time signal.

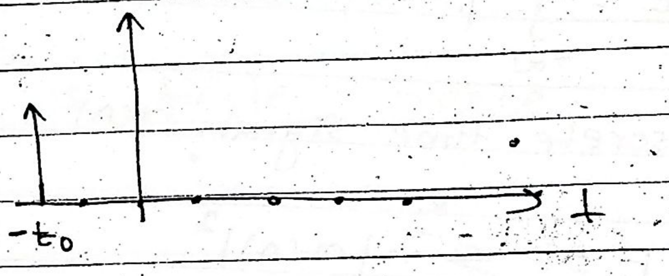
$$\delta(t) = \begin{cases} 1 & \text{if } t=0 \\ 0 & \text{otherwise} \end{cases}$$



$$\delta(t-t_0) = \begin{cases} 1 & \text{if } t=t_0 \\ 0 & \text{otherwise} \end{cases}$$

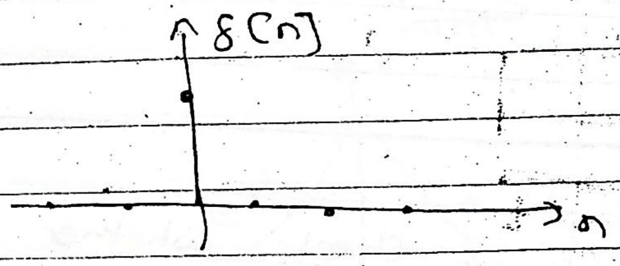


$$\delta(t+t_0) = \begin{cases} 1 & \text{if } t=-t_0 \\ 0 & \text{otherwise} \end{cases}$$



* for a discrete time

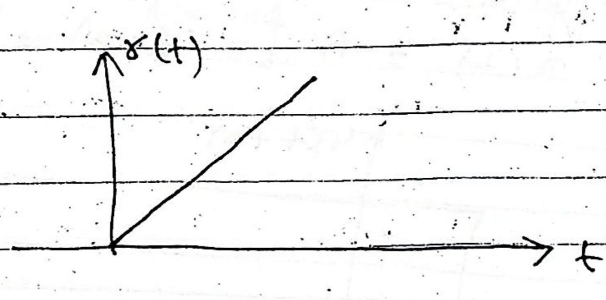
$$\delta(n) = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases}$$



(5) Unit ramp function

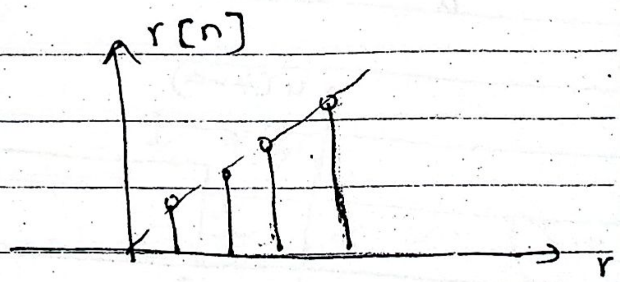
* For Continuous time

$$r(t) = \begin{cases} t & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases}$$



* For Discrete time

$$r[n] = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



Relationship bet^n unit step, unit^n impulse & Ramp func^n

* Relationship bet^n unit step, unit impulse & Ramp func^n

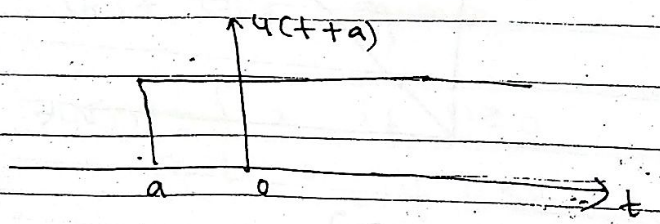
$$\delta(t) \xrightarrow{\text{Integration}} u(t) \xrightarrow{\text{Integration}} r(t)$$

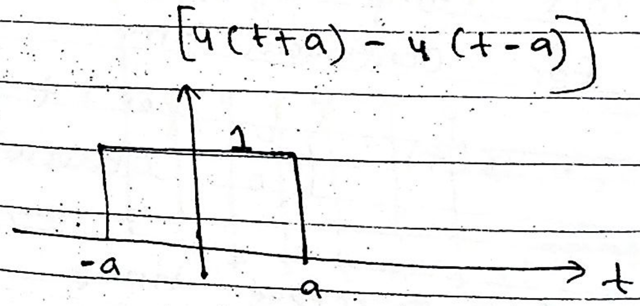
$$r(t) \xrightarrow{\text{differentiation}} u(t) \xrightarrow{\text{differentiation}} \delta(t)$$

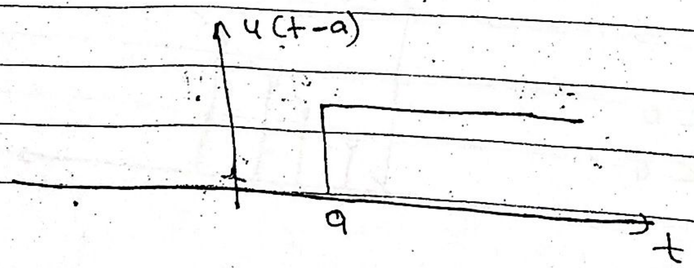
Energy Signal	Power Signal
Continuous time signal $x(t)$	Continuous time Signal
$E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \, dt$	$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t)\rvert^2 \, dt$
Discrete time signal $x(n)$	Discrete time signal
$E = \sum_{-\infty}^{\infty} \lvert x(n)\rvert^2$	$P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} \lvert x(n)\rvert^2$

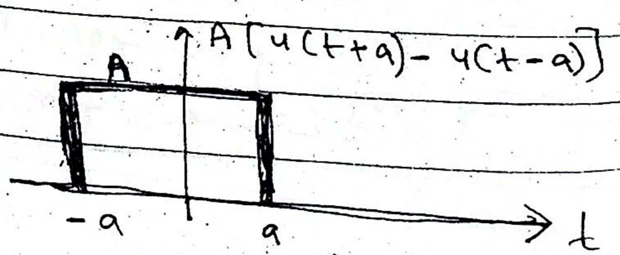
Q. Sketch & check whether the given signals are energy signal or a power signal or neither.

(i) $x(t) = A\,[u(t+a) - u(t-a)]$ for $a > 0$









Aperiodic Signal.

Here, the signal is aperiodic hence it is not a power signal. To check energy signal let us calculate the Energy of given signal,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

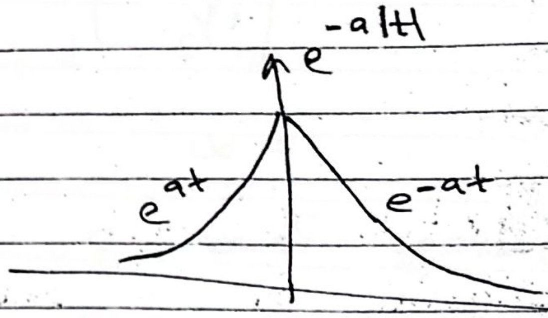
$$= \int_{-a}^{a} A^2 \, dt$$

$$= A^2 [t]_{-a}^{a} = 2aA^2 < \infty$$

~~Here~~, energy of the given signal is finite hence energy signal.

(ii) $x(t) = e^{-a|t|}$ for $a > 0$

$$x(t) = \begin{cases} e^{at} & \text{for } t < 0 \\ e^{-at} & \text{for } t > 0 \end{cases}$$



Energy of the given signal is,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{0} e^{2at} \, dt + \int_{0}^{\infty} e^{-2at} \, dt$$

$$= \left[\frac{e^{2at}}{a}\right]_{-\infty}^{0} + \left[\frac{e^{-2at}}{-a}\right]_{0}^{\infty} = \frac{2}{a} + \frac{2}{a}$$

$$= \frac{1}{a} < \infty \text{ (fin}$$

Hence, energy signal.

(iii) $x(t) = t^{-1/4} \cdot u(t-1)$

⇒ We know that Normalised Energy is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| t^{-1/4} \cdot u(t-1) \right|^2 dt$$

$$= \int_{1}^{\infty} t^{+1/2} \, dt \qquad \left[\because u(t-1) = 1, \; t>1 \atop = 0, \; t<1 \right]$$

$$= \left[\frac{t^{+1/2}}{+1/2} \right]_1^{\infty}$$

$$= \infty$$

Also, Average Power $P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| t^{-1/4} \cdot u(t-1) \right|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{1}^{T} t^{-1/2} \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{t^{1/2}}{1/2} \right]_1^{T}$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[T^{1/2} - 1 \right]$$

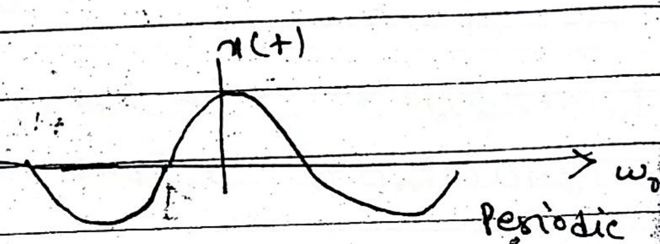
$$P = \lim_{T \to \infty} \frac{\left(1 - \frac{1}{\sqrt{T}}\right)}{\sqrt{T}} = 0$$

Here $P \to 0$, where $E \to \infty$ which means that the condⁿ $0 < E < \infty$ is not satisfied. Hence, $x(t)$ is not an energy signal.

when $E \to \infty$, the value of $P \to 0$, which implies the condⁿ $0 < P < \infty$ is not satisfied. Therefore, $x(t)$ is not a power signal.

Therefore ~~the~~ $x(t)$ is neither energy nor power signal.

v) $x(t) = A\cos(\omega_0 t)$



Here,

Normalize energy of given signal is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |A\cos(\omega_0 t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} A^2 \cos^2 \omega_0 t \, dt$$

$$= A^2 \int_{-\infty}^{\infty} \frac{1 + \cos 2\omega_0 t}{2} \, dt$$

$$= A^2 \left[\frac{1}{2} t + \frac{\sin 2\omega_0}{4\omega_0} \right]_{-\infty}^{\infty}$$

$$= \infty$$

$$\text{Average power } (P) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} A^2 \cos^2 \omega_0 t \, dt$$

$$= \frac{A^2}{2\pi} \lim_{T \to \infty} \int_{-\pi}^{\pi} \frac{1 + \cos 2\omega_0 t}{2} \, dt$$

$$= \frac{A^2}{2\pi} \lim_{T \to \infty} \left[\frac{1}{2} t + \frac{\sin 2\omega_0 t}{4\omega_0} \right]_{-\pi}^{\pi}$$

$$= \frac{A^2}{2\pi} \lim_{T \to \infty} \left(\pi + \frac{\sin 2\pi \omega_0}{4\pi} + \frac{\sin 2\pi}{4\pi} \right.$$

$$= \frac{A^2}{2} < \infty$$

Hence, given signal is a power signal. finite.