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1 Manski (1993)

Linear model

$$y = \alpha + \beta E(y|x) + E(z|x)'\gamma + z'\eta = u, \tag{1}$$

$$E(u|x,z) = x'\delta$$

$$E(y|x,z) = \alpha + \beta E(y|x) + E(z|x)'\gamma + x'\delta + z'\eta \tag{2}$$

y is a scalar outcome, x are attributes characterizing an individual's reference group, and (z, u) are attributes that directly affect y. (y, x, z) are observable and u are not observable.

(i) IF $\beta \neq 0$ the linear regression express an endogenous effect. A person's outcome y varies with $E(y|x)$, the mean of y among those persons in the reference group defined by x.

(ii) $\gamma \neq 0$ the model expresses an exogenous effect. y varies with $E(z|x)$, the mean of the exogenous variables z among these persons in the reference group.

(iii) $\delta \neq 0$ the model expresses correlated effect. Persons in reference group x tend to behave similarly because they have similar unobserved individual characteristics u or face similar institutional enviroments.

(iv) The parameter η expresses the direct effect of z on y.

2 Hsieh, C.-S. and L. F. Lee (2016)

$$Y_g = \lambda W_g Y_g + l_{m_g}\beta_1 + X_g\beta_2 + W_g X_g \beta_3 + l_{m_g}\alpha_g + \epsilon_g, g = 1, \ldots, G \tag{3}$$

(i) $Y_g = (y_{1,g}, \ldots, y_{m_g}, g)'$ where $y_{i,g}$ is the observed outcome of the ith member in the group g and m_g is the number of members in the gth group.

(ii) X_g is a $m_g \times k$ matrix of exogenous variables.

(iii) l_{m_g} is the m_g dimensional vectors of ones and α_g represents unobserved group specific effect.

(iv) W_g is an $m_g \times m_g$ spatial weight matrix which summarizes the network structure of the group g

3 A typical spatial autoregressive (SAR) model

$$Y_n = \lambda_0 W_n Y_n + X_n \beta_0 + \varepsilon_n \tag{4}$$

where ε_n is a n-dimensional vector consisting of *i.i.d.* disturbances with zero mean and a variance σ_0^2. In this model, n is the total number of spatial units, X_n is an $n \times k$ matrix of regressors, and W_n is a specified constant spatial weights matrix.

$$Y_n = S_n^{-1}(\lambda_0) X_n \beta_0 + V_n \tag{5}$$

Where $S_n(\lambda_0) = I_n - \lambda_0 W_n$ and $V_n = S_n^{-1}(\lambda_0)\varepsilon_n$

4 2-Stage-Least-Square

$$Y_1 = \alpha_1 y_2 + \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u \tag{6}$$

(i) Suppose there are m instrumental variables. $Z = (1, x_1, \ldots, x_k, z_1, \ldots, z_k)$ are correlated with y_2

(ii) $y_2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \cdots + \delta_k x_k + \delta_{k+1} z_1 + \cdots + \delta_{k+m} z_m + \varepsilon$

(iii) $y_2 = \hat{y}_2 + \varepsilon$

(iv) $\hat{y}_2 = Z\hat{\delta} = Z(Z'Z)^{-1}Z'y_2$

(v) $\hat{X} = Z(Z'Z)^{-1}Z'X = P_Z X$

$$\begin{aligned}
\hat{\beta}_{2SLS} &= (\hat{X}'\hat{X})^{-1}\hat{X}'Y \\
&= (X'P_Z X)^{-1} X' P_Z Y \\
&= (X'Z(Z'Z)^{-1}Z'X)^{-1} X'Z(Z'Z)^{-1}Z'Y
\end{aligned} \tag{7}$$