Variation Inference Linear Regression

May 16, 2019

First import required modules

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import scipy.stats as stats
        from scipy.special import digamma
```

1 Regression Spline

Assume that the range of x is $[a, b]$. Let the point

$$a < \xi_1 < \cdots < \xi_K < b$$

be a partion of the interval $[a, b]$
$\{\xi_1, \ldots, \xi_K\}$ are called knots.
Then make the function which return the knot points

```
In [2]: def defineKnot(X,K=10):
            upper = max(X)
            lower = min(X)
            out = np.linspace(start=lower,stop=upper,num=K+2)[1:K+1]
            return(out)
```

2 Radial Basis Function

A RBF φ is a real valued function whose value depends only on the distance from origin. A real function $\varphi : [0, \infty) \to \mathbb{R}$ with a metric on space $\|\cdot\| : V \to [0, \infty)$ a function $\varphi_c = \varphi(\|\mathbf{x} - \mathbf{c}\|)$ is said to be a radial kernel centered at c. A radial function and the associated radial kernels are said to be radial basis function

we use radial basis functions defined by

$$\mathbf{b}(u) = \left\{ u, \left| \frac{u - \tau_1}{c} \right|^3, \cdots, \left| \frac{u - \tau_K}{c} \right|^3 \right\}$$

where c is sample standard deviation

Then we can make the function which retrun the basis

```
In [3]: def b(u,tau,sd):
            lst = []
            lst.append(u)
            for i in tau:
                lst.append(abs((u-i)/sd)**3)
            out = np.array(lst)
            return(out)
```

Nonparametric linear model can be represented as

$$Y = \mathbf{b}(X)\boldsymbol{\beta} + \varepsilon$$

where $Y \in \mathbb{R}^{n\times 1}, X \in \mathbb{R}^{n\times 1}$ and $\varepsilon \sim N(0, \tau^{-1})$

3 Make toy data

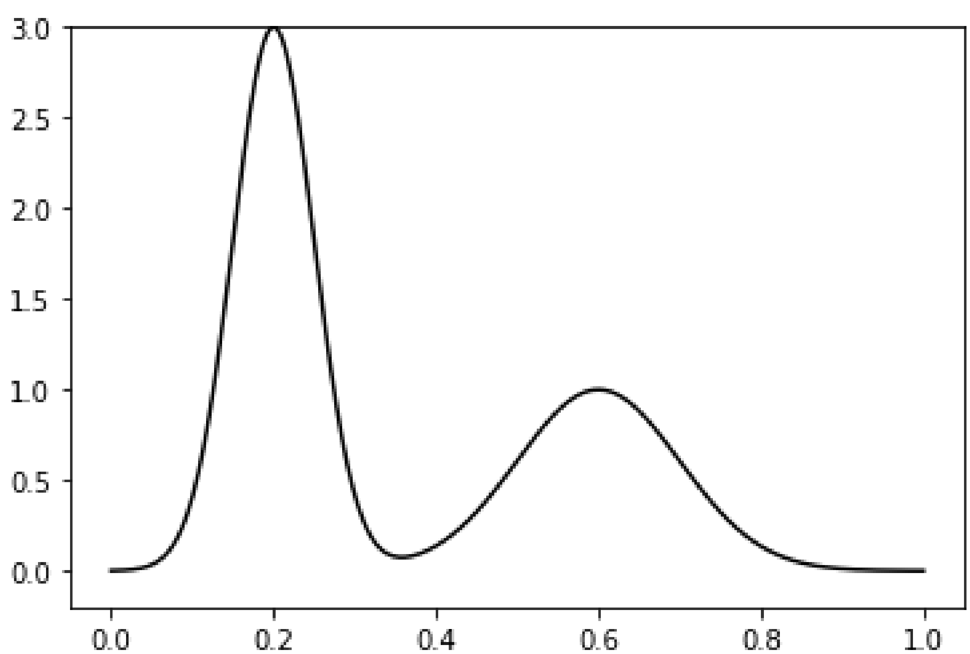
Let

$$y = 3\exp\left(-200(x-0.2)^2\right) + \exp\left(-50(x-0.6)^2\right)$$

Plotting true distribution of Y is

```
In [4]: x = np.linspace(0,1,300)
        y = 3*np.exp(-200*(x-0.2)**2) + np.exp(-50*(x-0.6)**2)
```

```
In [5]: plt.plot(x, y, 'k')
        plt.ylim(-0.2, 3)
        plt.show()
```



make the simulation function which make the obs with error $N(0, 0.5)$

```
In [6]: def mkToy(n=300,tau = 0.5):
            x = np.random.uniform(size = n)
            e = np.random.normal(0,np.sqrt(0.5), size= n)
            y = 3*np.exp(-200*(x-0.2)**2) + np.exp(-50*(x-0.6)**2) + e
            #out = np.column_stack([x,y])
            return(x,y)
```

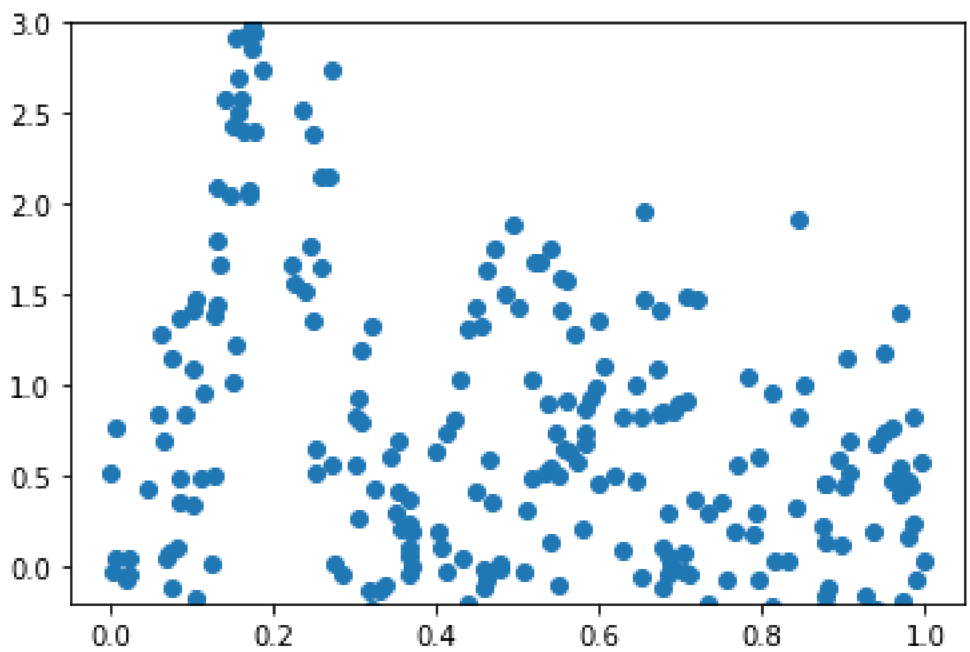
Plotting the distribution of simulated data

$$y = 3\exp\left(-200(x-0.2)^2\right) + \exp\left(-50(x-0.6)^2\right) + \varepsilon$$

where $\varepsilon \sim N(0, 0.5)$

```
In [7]: x,y = mkToy()
```

```
In [8]: plt.plot(x,y,'o')
        plt.ylim(-0.2, 3)
        plt.show()
```



Calculate the standard deviation of observed data

```
In [9]: sd = np.std(x)
        print(sd)
```

```
0.29923910225377626
```

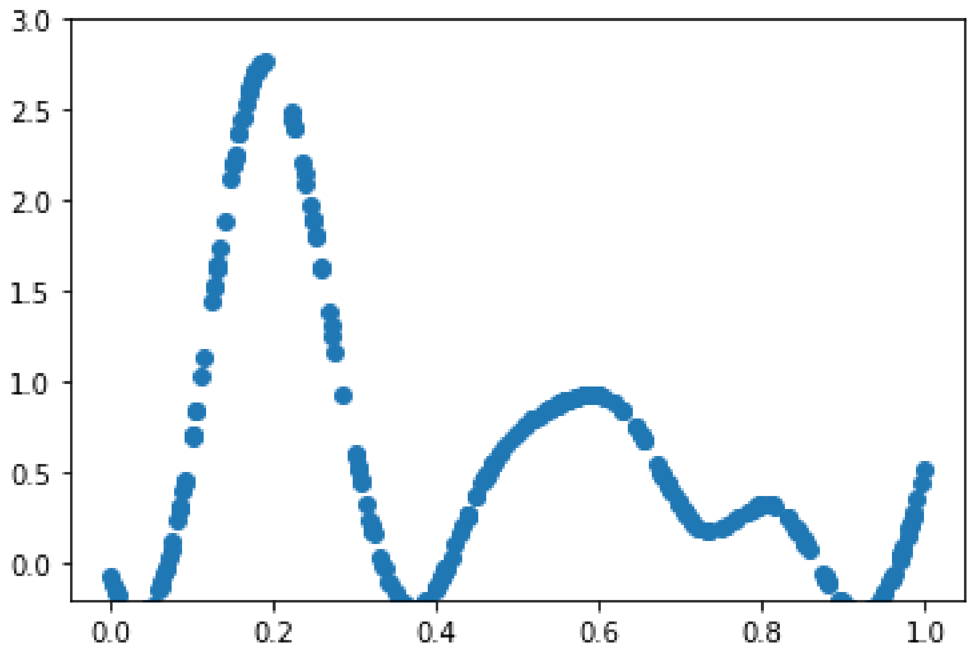
Define the knot and design matrix

```
In [10]: knot = defineKnot(x)
         d_x = b(x,knot,sd).T
```

plotting the fitted value

```
In [11]: fitted  = d_x.dot(np.linalg.inv(d_x.T.dot(d_x))).dot(d_x.T).dot(y)
```

```
In [12]: plt.plot(x,fitted,'o')
         plt.ylim(-0.2, 3)
         plt.show()
```

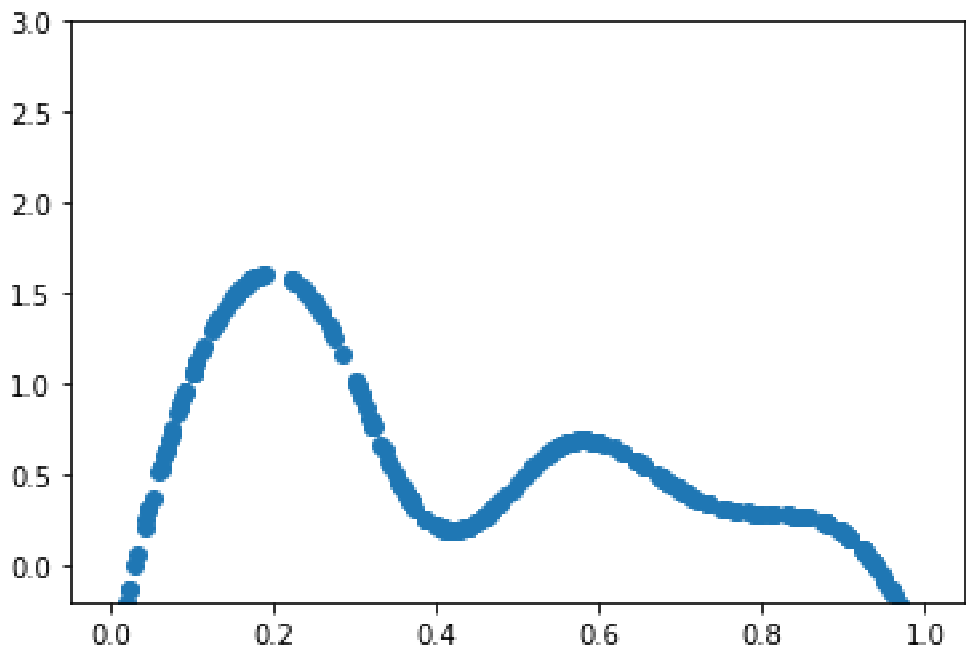



```
In [13]: def mfvb(X,y,max_iter):
             N,p = X.shape
             a ,b, c, d = [10**(-7)]*4

             a_tilde = np.repeat(a + 0.5, p)
             b_tilde = np.repeat(b,p)
             c_tilde = c + (N+1)/2
             d_tilde = d

             mu_coeffs = np.repeat(0,p)
             sigma_coeffs = np.diag(np.repeat(1,p))


             for i in range(max_iter):
                 expected_coeffs = mu_coeffs
```

```
            double_expected_coeffs = sigma_coeffs + mu_coeffs.dot(mu_coeffs.T)
            diagonal_sigma = np.diag(sigma_coeffs)
            expected_alpha = np.array(list(map(lambda x : a_tilde[x]/b_tilde[x] , np.arange
            log_expected_alpha = np.array(list(map(lambda x : digamma(a_tilde[x])-np.log(b_
            expected_tau = c_tilde / d_tilde
            log_expected_tau = digamma(c_tilde)-np.log(d_tilde)
            sigma_coeffs = np.linalg.inv(np.diag(expected_alpha)+expected_tau*(X.T.dot(X)))
            mu_coeffs =  expected_tau*sigma_coeffs.dot(X.T.dot(y))
            b_tilde = np.array(list(map(lambda x : (diagonal_sigma[x]+mu_coeffs[x]**2)/2 +
            d_tilde = d+0.5*(y.T.dot(y)) - expected_coeffs.T.dot((X.T.dot(y))+ 0.5*sum(np.d
        return mu_coeffs,sigma_coeffs
```

```
In [19]: m,c = mfvb(d_x,y, max_iter= 1000)
         plt.plot(x,d_x.dot(m),'o')
         plt.ylim(-0.2, 3)
         plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:21: RuntimeWarning: invalid val
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:19: RuntimeWarning: invalid val
```




```
In [16]: m,c = mfvb(d_x,y, max_iter= 100000)
         plt.plot(x,d_x.dot(m),'o')
         plt.ylim(-0.2, 3)
         plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:21: RuntimeWarning: invalid val
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:19: RuntimeWarning: invalid val
```