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1 The Bayesian Lasso

Linear regression model can be expressed as

$$\boldsymbol{y} = \mu \mathbf{1}_n + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{1}$$

and Lasso estimates can be describe as solutions to optimizations

$$\min_{\boldsymbol{\beta}} \quad (\tilde{\boldsymbol{y}} - \boldsymbol{X}\boldsymbol{\beta})^T(\tilde{\boldsymbol{y}} - \boldsymbol{X}\boldsymbol{\beta}) \quad + \quad \lambda \sum_{j=1}^{p} |\beta_j| \tag{2}$$

as $\tilde{\boldsymbol{y}} = \boldsymbol{y} - \bar{y}\mathbf{1}_n$ Lasso expression can be interpreted as posterior mode estimates when the regression parameters have independent and identical Laplace priors. and an independent prior $\pi(\sigma^2)$ on $\sigma^2 > 0$

$$\pi(\boldsymbol{\beta}) = \prod_{j=1}^{p} \frac{\lambda}{2} e^{-\lambda|\beta_j|} \tag{3}$$

$$\tilde{\boldsymbol{y}}|\sigma^2, \boldsymbol{\beta} \sim Normal(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2\boldsymbol{I}) \tag{4}$$

$$\pi(\boldsymbol{\beta}, \sigma^2|\tilde{\boldsymbol{y}}) \quad \propto \quad \pi(\tilde{\boldsymbol{y}}|\boldsymbol{\beta}, \sigma^2)\pi(\boldsymbol{\beta}, \sigma^2) = \quad \pi(\tilde{\boldsymbol{y}}|\boldsymbol{\beta}, \sigma^2)\pi(\boldsymbol{\beta})\pi(\sigma^2) \tag{5}$$

$$\propto \pi(\sigma^2)(\sigma^2)^{-(n-1)/2} \exp\left\{-\frac{1}{2\sigma^2}(\tilde{\boldsymbol{y}} - \boldsymbol{X}\boldsymbol{\beta})^T(\tilde{\boldsymbol{y}} - \boldsymbol{X}\boldsymbol{\beta}) - \lambda \sum_{j=1}^{p} |\beta_j|\right\} \tag{6}$$