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1 Changing point model

1.1 likelihood and prior

$$\beta_t \overset{\text{iid}}{\sim} Poisson(\lambda_j) \quad \text{for } t = k_{j-1}+1, \dots, k_j$$
$$\lambda_j \sim Gamma(a_j, b_j) \quad \text{for } j = 1, \dots, m$$
$$k_j \sim unif\{k_{j-1}+1, k_j\} \quad \text{for } j = 1, \dots, m$$

Where m is number of change point, $k_0 = 1$ and $k_{m+1} = T$, Then

$$p(\boldsymbol{\lambda}, \boldsymbol{k}|\boldsymbol{\beta}) \propto \prod_{j=1}^{m} \left[\exp\left(-(k_{j+1} - k_j)\lambda_j \left[\prod_{t=k_{j-1}}^{k_j} \lambda_j^{\beta_t} \right] \lambda^{a_j - 1} \exp(-b_j \lambda_j) \right) \right]$$

1.2 Gibbs sampler

$$\lambda_j | \boldsymbol{\lambda_{-j}}, \boldsymbol{k}, \boldsymbol{\beta} \sim Gamma(a_j + \sum_{t=k_{j-1}+1}^{k_j} \beta_t, k_j - k_{j-1} + b_j)$$
$$p(k_j | \boldsymbol{\lambda}, \boldsymbol{k_{-j}}, \boldsymbol{\beta}) = \frac{\exp(k_j(\lambda_{j+1} - \lambda_j) + \log(\lambda_j/\lambda_{j+1}) \sum_{t=k_{j-1}+1}^{k_j} \beta_t)}{\sum_{t=k_{j-1}}^{k_j} \exp(k_j(\lambda_{j+1} - \lambda_j) + \log(\lambda_j/\lambda_{j+1}) \sum_{t=k_{j-1}+1}^{k_j} \beta_t)}$$

1.3 Variational Bayes

$$q^*(\lambda_j) \sim Gamma(a_j + \sum_{t=E_{q^*}[k_{j-1}]+1}^{E_{q^*}[k_j]} \beta_t, E_{q^*}[k_j] - E_{q^*}[k_{j-1}] + b_j)$$
$$q^*(k_j) = \frac{\exp(k_j(E_{q^*}[\lambda_{j+1}] - E_{q^*}[\lambda_j]) + \log(E_{q^*}[\lambda_j]/E_{q^*}[\lambda_{j+1}]) \sum_{t=E_{q^*}[k_{j-1}]+1}^{k_j} \beta_t)}{\sum_{t=E_{q^*}[k_{j-1}]}^{k_j} \exp(k_j(E_{q^*}[\lambda_{j+1}] - E_{q^*}[\lambda_j]) + \log(E_{q^*}[\lambda_j]/E_{q^*}[\lambda_{j+1}]) \sum_{t=E_{q^*}[k_{j-1}]+1}^{k_j} \beta_t)}$$

We can use

$$X \sim Gamma(\alpha, \beta)$$
$$E[\log X] = -\log \beta + \psi(\alpha)$$

where ψ means digamma function

2 Simulation

Make simulation data from

$$\beta_t \overset{\text{iid}}{\sim} \begin{cases} Poisson(1) & t = 1, \ldots, 30 \\ Poisson(3) & t = 31, \ldots, 100 \end{cases}$$

2.1 Gibbs

Prior and initial value are

$$a = 4;\ b = 1;\ c = 1;\ d = 2$$

$$\phi = 1$$