September 19, 2019

1 Empirical Bayes

We need to calculate the likelihood $p(y|\rho)$ and select the hyper parameter ρ which maximize the likelihood

by varational bayes approach we can find lower bound of $p(y|\rho)$

$$\log p(y|\rho) \geq \sigma_\gamma \int q(\beta, \sigma^2, \gamma) \log \left\{ \frac{p(y, \beta, \sigma^2, \gamma)}{q(\beta, \sigma^2, \gamma)} \right\} d\beta d\sigma^2 = ELBO(\rho)$$

$$ELBO(\rho) = c - (A + \frac{n}{2}) \log(s) + \frac{1}{2}|\Sigma| - \frac{1}{\sigma_\beta^2} tr(\mu\mu' + \Sigma)$$
$$+ \sum_{j=1}^{p} \left[w_j \log \left(\frac{\rho}{w_j} + (1 - w_j) \log \left(\frac{1 - \rho}{1 - w_j} \right) \right) \right]$$

when variation distribution is

$$q(\beta) \sim N(\mu, \Sigma)$$
$$q(\sigma^2) \sim Inverse - Gamma(A + n/2, s)$$
$$q(\gamma_j) \sim Bern(w_j) \quad for \ \ j = 1, \ldots, p$$