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1 Changing point model

1.1 likelihood and prior

$$
\begin{aligned}
\beta_t &\overset{\text{iid}}{\sim} \begin{cases} Poisson(\lambda) & t = 1, \dots, k \\ Poisson(\phi) & t = k+1, \dots, T \end{cases} \\
\lambda &\sim Gamma(a, b) \\
\phi &\sim Gamma(c, d) \\
k &\sim unif\{1, T\}
\end{aligned}
$$

1.2 Gibbs sampler

$$
\begin{aligned}
\lambda|\phi, k, \beta &\sim Gamma(a + \sum_{t=1}^{k} \beta_t, k + b) \\
\phi|\lambda, k, \beta &\sim Gamma(c + \sum_{t=k+1}^{T} \beta_t, T - k + d) \\
p(k|\lambda, \phi, \beta) &= \frac{\exp(k(\phi - \lambda) + \log(\lambda/\phi) \sum_{i=1}^{k} \beta_t)}{\sum_{t=1}^{T} \exp(k(\phi - \lambda) + \log(\lambda/\phi) \sum_{t=1}^{k} \beta_t)}
\end{aligned}
$$

1.3 Variational Bayes

$$
\begin{aligned}
q_1^*(\lambda) &\sim Gamma(a + \sum_{t=1}^{E_{q_3^*}[k]} \beta_t, E_{q_3^*}[k] + b) \\
q_2^*(\phi) &\sim Gamma(c + \sum_{t=E_{q_3^*}[k]+1}^{T} \beta_t, T - E_{q_3^*}[k] + d) \\
q^*(k) &= \frac{\exp(k(E_{q_2^*}[\phi] - E_{q_1^*}[\lambda]) + \log(E_{q_1^*}[\log(\lambda)] - E_{q_2^*}[\log(\phi)]) \sum_{t=1}^{k} \beta_t)}{\sum_{k=1}^{T} \exp(k(E_{q_2^*}[\phi] - E_{q_1^*}[\lambda]) + \log(E_{q_1^*}[\log(\lambda)] - E_{q_2^*}[\log(\phi)]) \sum_{t=1}^{k} \beta_t)}
\end{aligned}
$$

We can use

$$
\begin{aligned}
X &\sim Gamma(\alpha, \beta) \\
E[\log X] &= -\log \beta + \psi(\alpha)
\end{aligned}
$$

where ψ means digamma function

2 Simulation

Make simulation data from

$$\beta_t \overset{\text{iid}}{\sim} \begin{cases} Poisson(1) & t = 1, \dots, 30 \\ Poisson(3) & t = 31, \dots, 100 \end{cases}$$

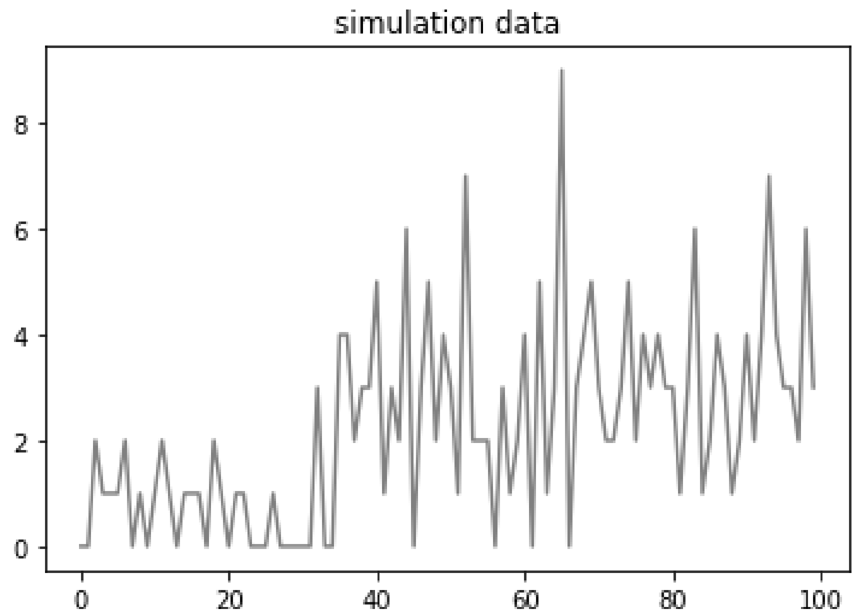

Figure 1: Simulated data time series plot

2.1 Gibbs

Prior and initial value are

$$a = 4;\ b = 1;\ c = 1;\ d = 2$$
$$\phi = 1$$

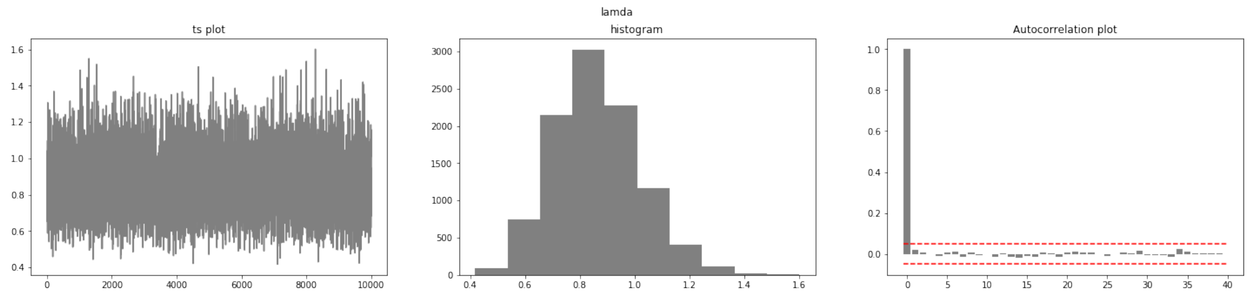

Figure 2: Gibbs sampling for λ

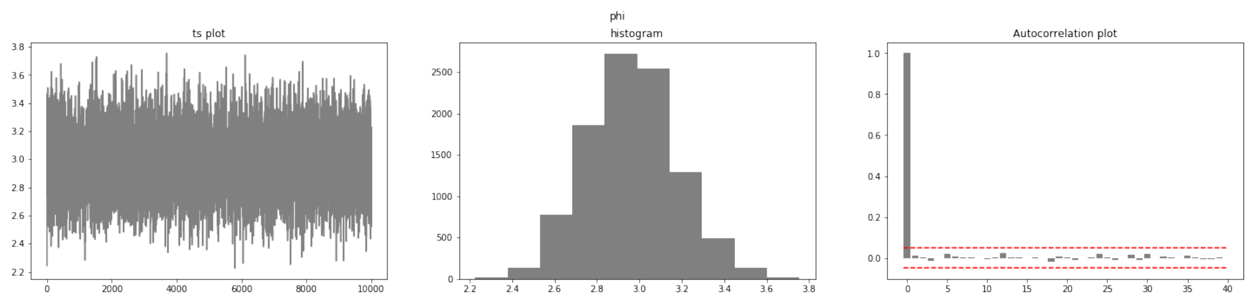

Figure 3: Gibbs sampling for ϕ

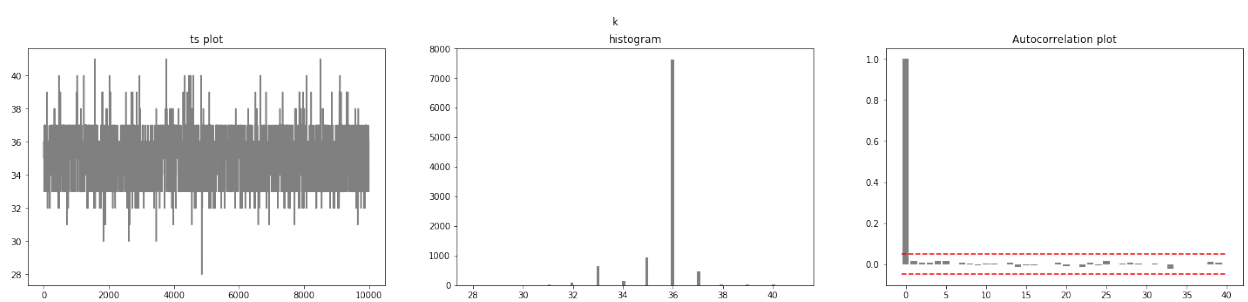

Figure 4: Gibbs sampling for k

2.2 VI

Prior and initial value are

$$a = 4;\ b = 1;\ c = 1;\ d = 2$$
$$k = 1$$

Variational distribution is

$$q_1^*(\lambda) \sim Gamma(32.0, 36.72)$$
$$q_2^*(\phi) \sim Gamma(196.0, 66.28)$$

and q_3^* is

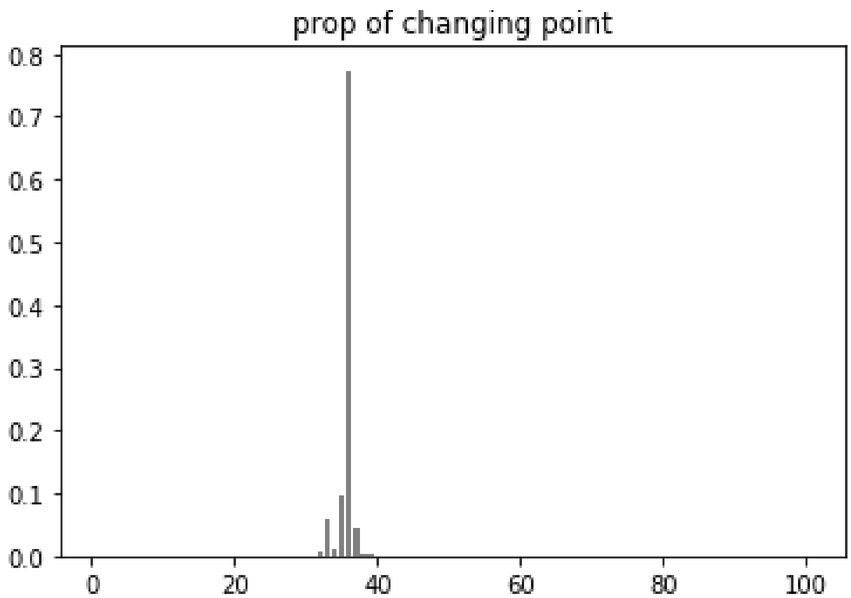

Figure 5: Variational distribution of q_3^*

$$E_{q_3}^*[k] = 35.72$$