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1 Spatial Autoregressive (SAR) Model

According to the type of response variable in the data, cross sectional spatial data can be divide into two types: Gaussian and non-Gaussian

- Spatial autoregressive(SAR) model : imply a geometric decay of spatial dependence.
- Matrix Exponential Spatial Specification(MESS) model : imply an exponential decay

are often used for modeling Gaussian spatial data. There is an close correspondence between SAR and MESS models if the same row-standarized spatial weight matrix is used in both models.

2 Intergrated Nested Laplace Approximation (INLA)

- To overcome the computational bottleneck of MCMC. Model parameters are divided into two subset - hyperpaprameters and latent effects parameters.
- Key assumption is that the conditional distribution of latent effcects given hyperparameters follows a multivariate Gaussian distribution
- Adversely affected by an increase in the number of hyperparameters and grid points chosen for them

3 Variational Bayes

Unlike MCMC Variational Bayes methods use optimization to fullfill the same inference purpose.

- Mean-Field Variational Bayes
  Relies on the assumption that the variational distribution can be factorized into a product form.
  $q(\mathbf{\Theta}) = \prod_{l=1}^{M} q_l(\theta_l)$
- Fixed-Form Variational Bayes
  avoids the product form assumption but restricts each variational factor to a parametic form applies to conjugate models.

- Integrated Nonfactorized Variational Bayes

  can provide more accurate and robust estimates by taking posterior dependence into account and INFVB algorithm can be parallelized.

  $q_{INFVB}(\mathbf{\Theta}) = q(\mathbf{\Theta}_c|\mathbf{\Theta}_d)q(\mathbf{\Theta}_d)$

4 Spatial Econometric Models for Gaussian Data

4.1 Spatial Autoregressive Confused (SAC) Model

$$\begin{aligned} \boldsymbol{y} &= \rho \mathbf{W}_1 \boldsymbol{y} + \mathbf{X}\beta + \boldsymbol{u} \\ \boldsymbol{u} &= \lambda \mathbf{W}_2 \boldsymbol{u} + \boldsymbol{\epsilon} \end{aligned} \tag{1}$$

- $\mathbf{X} = \mathbf{Z}\ \ or\ \ [\mathbf{Z}\ \ \mathbf{W}_1\mathbf{Z}]$
- $\boldsymbol{\epsilon} \sim N(0, \sigma^2 \mathbf{I}_n)$
- restriction on ρ and λ is

  $\frac{1}{\omega_{min}} < \rho < \frac{1}{\omega_{max}}$, $\frac{1}{\gamma_{min}} < \lambda < \frac{1}{\gamma_{max}}$ where ρ and λ are eigen values of $\mathbf{W}_1$ and $\mathbf{W}_2$

likelihood function of SAC model is

$$p(\boldsymbol{y}|\beta,\rho,\sigma^2,\lambda) = (2\pi\sigma^2)^{-\frac{n}{2}}|\mathbf{A}||\mathbf{B}|exp\{-\frac{[\mathbf{B}(\mathbf{A}\boldsymbol{y}-\mathbf{X}\beta)]'[\mathbf{B}(\mathbf{A}\boldsymbol{y}-\mathbf{X}\beta)]}{2\sigma^2}\} \tag{2}$$

4.2 MESS model

$$\begin{aligned} \mathbf{S}_1 \boldsymbol{y} &= \mathbf{X}\beta \boldsymbol{u} \\ \mathbf{S}_2 \boldsymbol{u} &= \epsilon \end{aligned}$$

where $\mathbf{S}_1 = e^{\alpha \mathbf{W}_1},\ \mathbf{S}_2 = e^{\tau \mathbf{W}_2}$

- $\mathbf{X} = \mathbf{Z}\ \ or\ \ [\mathbf{Z}\ \ \mathbf{W}_1\mathbf{Z}]$
- $\boldsymbol{\epsilon} \sim N(0, \sigma^2 \mathbf{I}_n)$
- $\alpha,\ \tau$ are unconstrained

likelihood function of MESS model is

$$p(\boldsymbol{y}|\beta,\alpha,\sigma^2,\tau) = (2\pi\sigma^2)^{-\frac{n}{2}}exp\{-\frac{[\mathbf{S}_2(\mathbf{S}_1\boldsymbol{y}-\mathbf{X}\beta)]'[\mathbf{S}_2(\mathbf{S}_1\boldsymbol{y}-\mathbf{X}\beta)]}{2\sigma^2}\} \tag{3}$$