2019 Oct. 23

2018321084 Juyoung Ahn

1 Changing point model

1.1 likelihood and prior

$$
\begin{aligned}
&\beta_t \overset{\text{iid}}{\sim} Normal(\mu_j, \tau_j^{-1}) \quad \text{for } t = k_{j-1}+1, \ldots, k_j \\
&\mu_j \sim Normal(0, b_j^{-1}) \quad \text{for } j = 1, \ldots, m \\
&\tau_j \sim Gamma(c_j, d_j) \quad \text{for } j = 1, \ldots, m \\
&k_j \sim unif\{k_{j-1}+1, k_{j+1}\} \quad \text{for } j = 1, \ldots, m-1
\end{aligned}
$$

where $k_0 = 0$ and $k_m = T$

1.2 Gibbs sampler

$$
\begin{aligned}
\mu_j|\boldsymbol{k}, \boldsymbol{\tau} &\overset{\text{ind}}{\sim} N\left([k_j - k_{j-1}\tau_j + b_j]^{-1}\tau_j \sum_{t=k_{j-1}+1}^{k_j} \beta_t, [k_j - k_{j-1}\tau_j + b_j]^{-1}\right) \\
\tau_j|\boldsymbol{k}, \boldsymbol{\mu} &\overset{\text{ind}}{\sim} Gamma\left(c_j + \frac{k_j - k_{j-1}}{2}, \frac{1}{2}\sum_{t=k_{j-1}+1}^{k_j} (\beta_t - \mu_j)^2 + d_j\right) \\
p(k_j|\boldsymbol{k_{-j}}, \boldsymbol{\tau}, \boldsymbol{\mu}) &\propto \left(\frac{\tau_j}{\tau_{j+1}}\right)^{\frac{k_j}{2}} \exp\left(\frac{1}{2}\sum_{t=k_{j-1}+1}^{k_j} [\tau_{j+1}(\beta_t - \mu_{j+1})^2 - \tau_j(\beta_t - \mu_j)^2]\right)
\end{aligned}
$$