gibbs

July 11, 2019

```
In [1]: import numpy as np
        import pandas as pd
        from scipy import stats
        import matplotlib.pyplot as plt

        from scipy.special import digamma
```

0.1 Make the function

Make the required functions, which calculate XX', $expit$ and draw the autocorelation plot

```
In [2]: def bernoulliSample(p=0.5,n=1):
            lst = []
            for i in range(n):
                if p > np.random.uniform(low=0,high=1):
                    lst.append(1)
                else:
                    lst.append(0)
            if n==1:
                return(lst[0])
            else:
                return(np.array(lst))
```

```
In [3]: def product(a):
            n = len(a)
            out = np.zeros([n,n])
            for i in range(n):
                for j in range(n):
                    out[i,j] = a[i]*a[j]
            return(out)
```

```
In [4]: def expit(x):
            #if x < 100:
            return(np.exp(x)/(1+np.exp(x)))
            #else:
                #return(1)
```

```
In [5]: def acf(sampl,lag =40):
            sampl= np.array(sampl)
            base = sampl.dot(sampl)/len(sampl)
            acr = [1]
            for t in range(1,lag):
                acr.append((sampl[t:].dot(sampl[:-t])/(len(sampl)-t))/base)
            x = list(range(lag))
            plt.bar(x,acr,color='gray')
            plt.title('Autocorrelation plot')
            plt.hlines(0.05,xmin=-0.5 ,xmax=lag,colors='r',linestyles='dashed')
            plt.hlines(-0.05,xmin=-0.5 ,xmax=lag,colors='r',linestyles='dashed')
            #plt.show()
```

```
In [6]: def hist(sampled,tlt,nbin =19):
            grid =np.linspace(-4,4,1000)
            plt.hist(sampled, bins=nbin,color='gray',density=True)
            plt.plot(grid , stats.norm.pdf(grid), 'black')
            plt.title(tlt)
            plt.xlabel('x')
            #plt.xlim(-4,4)
            plt.ylabel('density')
            #plt.show()
```

1 Gibbs Sampling

1.1 Prior

$$\begin{aligned}
Y|\beta,\sigma^2,\Gamma &\sim N(Z\Gamma\beta,\sigma^2 \cdot I)\\
\sigma^2 &\sim Inverse-Gamma(a,b)\\
\beta_j|\sigma^2_{\beta_j} &\sim^{ind} N(0,\sigma^2_{\beta_j})\\
\sigma^2_{\beta_j} &\sim^{iid} Inverse-Gamma(c,d)\\
\gamma_j &\sim^{iid} Bernoulli(\rho)\\
\rho &\sim Beta(u,v)
\end{aligned}$$

where
Z is n x p design matrix
$\Gamma = diag(\gamma_j)$ for $j = 1,\ldots,p$
a,b,c,d,u,v is flat prior

1.2 Posterior

$$
\begin{aligned}
p(\beta,\sigma^2,\sigma^2_\beta,\Gamma|Y) \propto & p(\beta,\sigma^2,\sigma^2_\beta,\Gamma,Y) \\
\propto & p(Y|\beta,\sigma^2,\Gamma)p(\beta|\sigma^2_\beta)p(\Gamma)p(\sigma^2)p(\sigma^2_\beta)p(\rho) \\
\propto & \left(\sigma^2\right)^{-n/2}\exp\left(-\frac{1}{2\sigma^2}\left(Y-Z\Gamma\beta\right)'\left(Y-Z\Gamma\beta\right)\right) \\
& \times\prod_{j=1}^{p}\left(\sigma^2_{\beta_j}\right)^{-1/2}\exp\left(-\frac{1}{2}\sum_{j=1}^{p}\frac{\beta_j^2}{\sigma^2_{\beta_j}}\right) \\
& \times\prod_{j=1}^{p}\rho^{\gamma_j}\left(1-\rho\right)^{1-\gamma_j} \\
& \times\left(\sigma^2\right)^{-a-1}\exp\left(-\frac{b}{\sigma^2}\right) \\
& \times\prod_{j=1}^{p}\left(\sigma^2_{\beta_j}\right)^{-c-1}\exp\left(-\sum_{j=1}^{p}\frac{d}{\sigma^2_{\beta_j}}\right) \\
& \times\rho^{u-1}\left(1-\rho\right)^{v-1}
\end{aligned}
$$

1.3 Sampling the β from

$$N\left(\mu,\Sigma\right)$$

where

$$\Sigma=\left(diag(\sigma^2_{\beta_j})+\frac{1}{\sigma^2}\Gamma'Z'Z\Gamma\right)^{-1},\quad \mu=\frac{1}{\sigma^2}\Sigma\Gamma'Z'y$$

```
In [7]: def sampleBeta(gamma,s2,sb2):
            Gamma = np.diag(gamma)
            D = np.diag(1/sb2)
            sinv = D + (1/s2)*Gamma.T.dot(Z.T.dot(Z.dot(Gamma)))
            Sigma = np.linalg.inv(sinv)
            mu = (1/s2)*Sigma.dot(Gamma.T).dot(Z.T).dot(y)
            out = np.random.multivariate_normal(mu,Sigma)
            return(out)
```

1.4 Sampling σ^2 from

$$Inverse-Gamma\left(a+\frac{N}{2},b+\frac{1}{2}\left(y-\Gamma\beta\right)'\left(y-\Gamma\beta\right)\right)$$

```
In [8]: def sampleS2(beta,gamma):
            Gamma = np.diag(gamma)
            alpha = a + N/2
            igbeta = b + 0.5*(y-Z.dot(Gamma).dot(beta)).T.dot((y-Z.dot(Gamma).dot(beta)))
            out = stats.invgamma.rvs(a=alpha,scale=igbeta)
            return(out)
```

1.5 Sampling $\sigma^2_{\beta_j}$ from

$$Inverse - Gamma\left(c + \frac{1}{2}, d + \frac{1}{2}\beta_j^2\right)$$

```
In [9]: def sampleSb2(beta):
            lst = []
            for j in range(p):
                alpha = c + 1/2
                igbeta = d+0.5*(beta[j]**2)
                lst.append(stats.invgamma.rvs(a=alpha,scale=igbeta))
            out = np.array(lst)
            return(out)
```

1.6 Sampling γ_j from

$$Bernoulli\left(expit\left(logit(\rho) + \frac{\beta_j}{\sigma^2}Z_j'\left(y - Z_{-j}\Gamma_{-j}\beta_{-j}\right) - \frac{\beta_j^2}{2\sigma^2}Z_j'Z_j\right)\right)$$

$expit$ is inverse function of $logit$

```
In [10]: def samplegamma(rho,s2,beta,gamma):
             gam = gamma.copy()
             wlst = []
             for j in range(p):
                 Gamma= np.diag(gam)
                 eta = np.log(rho/(1-rho))-((beta[j]**2)/s2)*Z[:,j].T.dot(Z[:,j])\
                 +(beta[j]/s2)*Z[:,j].T.dot(y-np.delete(Z,j,1).dot(np.diag(np.delete(gamma,j)).d
                 w = expit(eta)
                 wlst.append(w)
                 gam[j] = w
             outlst = []
             for k in gam:
                 outlst.append(stats.bernoulli.rvs(k))
             out = np.array(outlst)
             return(out)
```

1.7 Sampling ρ from

$$Beta\left(\sum_{j=1}^{p}\gamma_j + u, p - \sum_{j=1}^{p}\gamma_j + v\right)$$

```
In [11]: def sampleRho(gamma):
             alpha = sum(gamma) +u
             bbeta = p-sum(gamma) + v
             out = np.random.beta(alpha,bbeta)
             return(out)
```

1.8 Gibbs sampling function

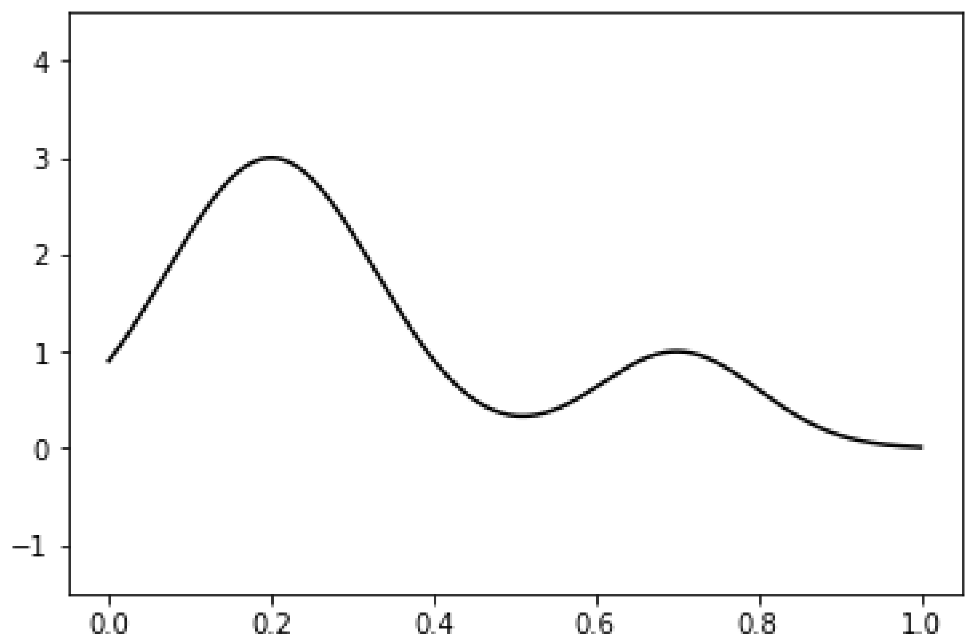
```
In [12]: def gibbs():
             lst = []
             for i in range(1000):
                 beta = sampleBeta(gamma,s2,sb2)
                 s2 = sampleS2(beta,gamma)
                 sb2 = sampleSb2(beta)
                 gamma = samplegamma(rho,s2,beta,gamma)
                 rho = sampleRho(gamma)

                 params = np.array([beta,s2,sb2,gamma,rho])
                 lst.append(params)
             return(pd.DataFrame(lst))
```

1.9 Simulation Data

$$f(x) = 3\exp(-30(x-0.2)^2) + \exp(-50(x-0.7)^2)$$

```
In [13]: def f(x):
             out = 3*np.exp(-30*((x-0.2)**2))+np.exp(-50*((x-0.7)**2))
             return(out)
```

```
In [14]: x = np.linspace(0,1,300)
         y = f(x)
         plt.plot(x, y, 'k')
         plt.ylim(-1.5, 4.5)
         plt.show()
```

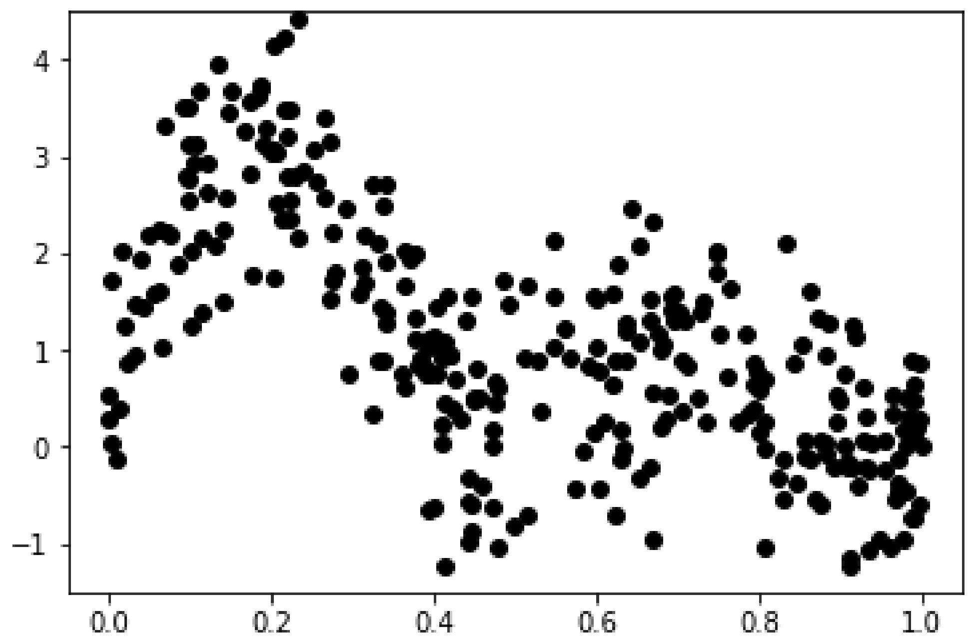


1.10 Make data wiht N(0,0.5) error

```
In [15]: def mkToy(n=300,tau = 0.5):
             x = np.random.uniform(size = n)
             e = np.random.normal(0,np.sqrt(0.5), size= n)
             y = f(x)+e
             #out = np.column_stack([x,y])
             return(x,y)
```

```
In [16]: x,y = mkToy()
```

```
In [17]: plt.plot(x,y,'ko')
         plt.ylim(-1.5, 4.5)
         plt.show()
```



1.11 Radial basis

we use radial basis functions defined by

$$\mathbf{b}(u) = \left\{ u, \left| \frac{u - \tau_1}{c} \right|^3, \cdots, \left| \frac{u - \tau_K}{c} \right|^3 \right\}$$

where c is sample standard deviation

```
In [18]: def defineKnot(X,K=10):
             upper = max(X)
```

```
        lower = min(X)
        out = np.linspace(start=lower,stop=upper,num=K+2)[1:K+1]
        return(out)
    def radialbasis(u,tau,sd):
        lst = []
        lst.append(u)
        for i in tau:
            lst.append(abs((u-i)/sd)**3)
        out = np.array(lst)
        return(out)
```

In [19]:
```
sd = np.std(x)
```

In [20]:
```
knot = defineKnot(x)
d_x = radialbasis(x,knot,sd).T
```

In [21]:
```
#initial value
Z = d_x
N, p = Z.shape
a,b,c,d = [10**-7]*4
e,f = [1,1]
gamma = bernoulliSample(0.5,p)
sb2 = np.repeat(0.5,p)
s2 = 0.5
rho=0.5
u,v = 1,1
print(gamma)
```

```
[1 0 1 1 1 0 0 0 1 0 0]
```

In [22]:
```
lst = []
blst = []
for i in range(2000):
    beta = sampleBeta(gamma,s2,sb2)
    s2 = sampleS2(beta,gamma)
    sb2 = sampleSb2(beta)
    gamma = samplegamma(rho,s2,beta,gamma)
    rho = sampleRho(gamma)

    params = np.array([beta,s2,sb2,gamma,rho])
    lst.append(params)
    blst.append(beta)
```

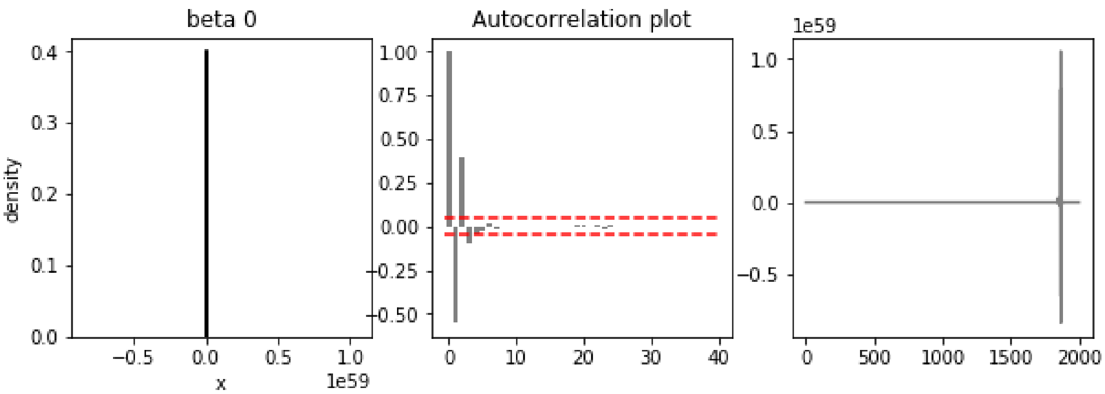
In [23]:
```
df = pd.DataFrame(lst)
```

In [24]:
```
bdf = pd.DataFrame(blst)
```

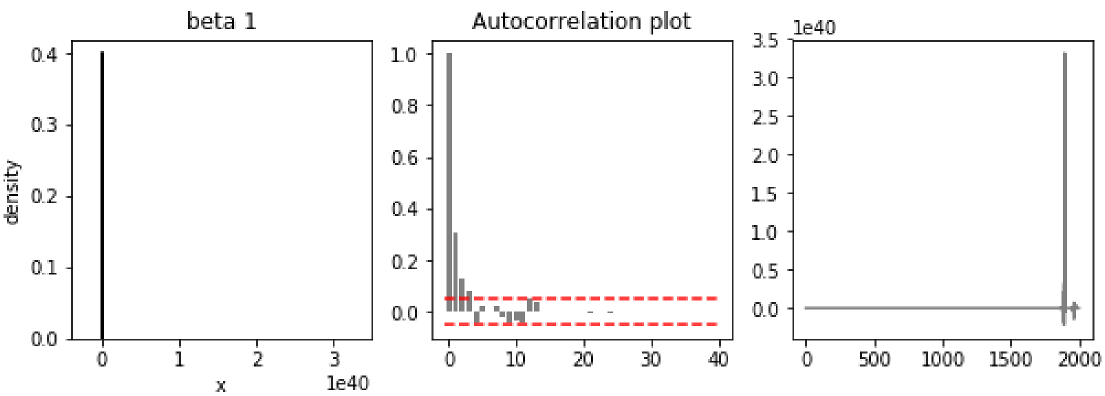
In [25]:
```
def betaplt(j):
    plt.figure(figsize=(10, 3))
```

```
        plt.subplot(1,3,1)
        hist(bdf[j],tlt='beta %d'%j)
        plt.subplot(1,3,2)
        acf(bdf[j])
        plt.subplot(1,3,3)
        plt.plot(bdf[j],color='gray')
        plt.show()
```

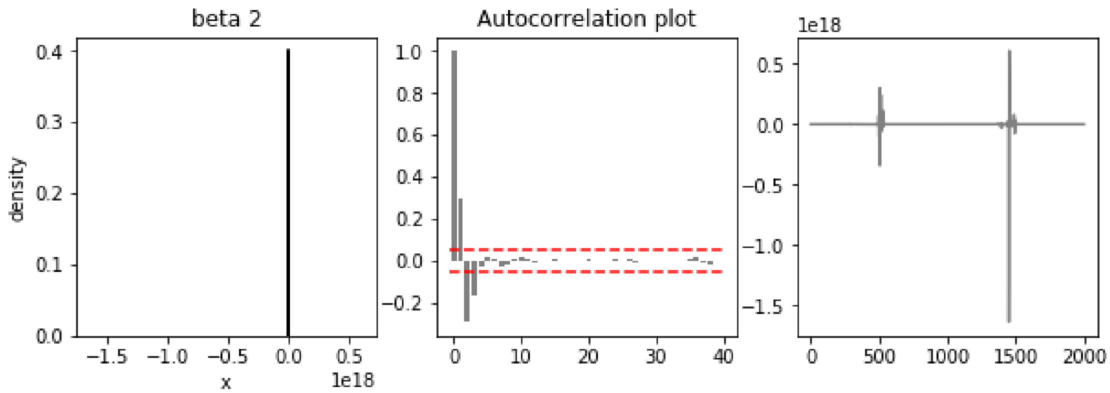
```
In [26]: betaplt(0)
```



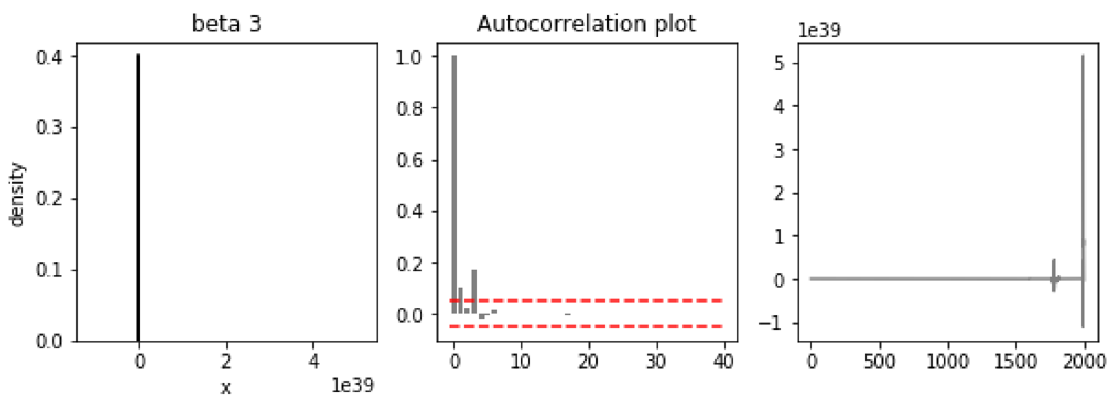

```
In [27]: betaplt(1)
```



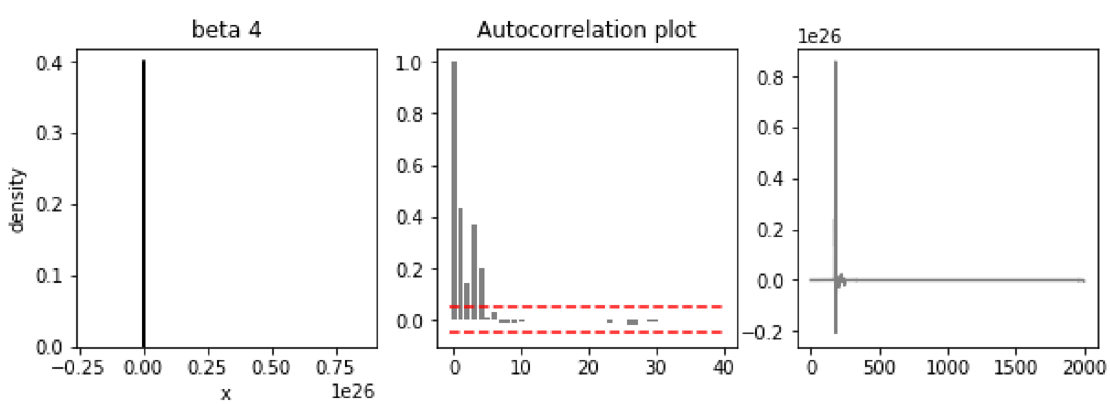

```
In [28]: betaplt(2)
```



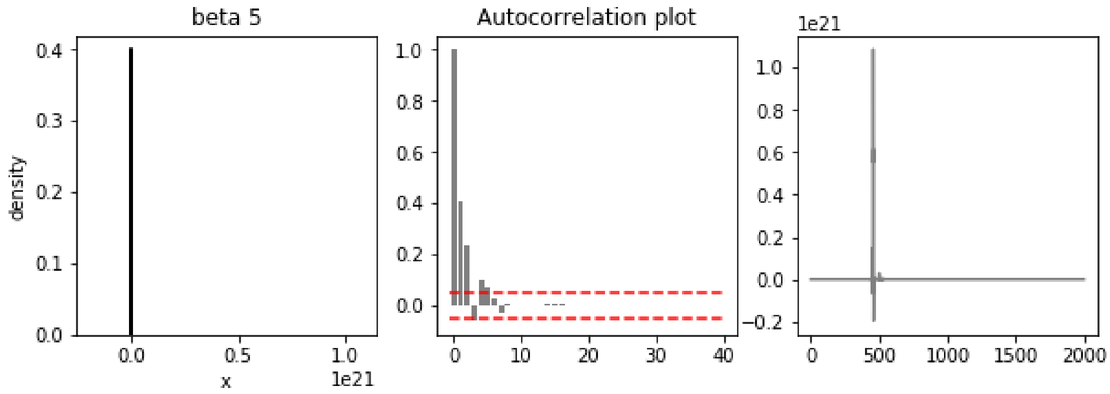

```
In [29]: betaplt(3)
```



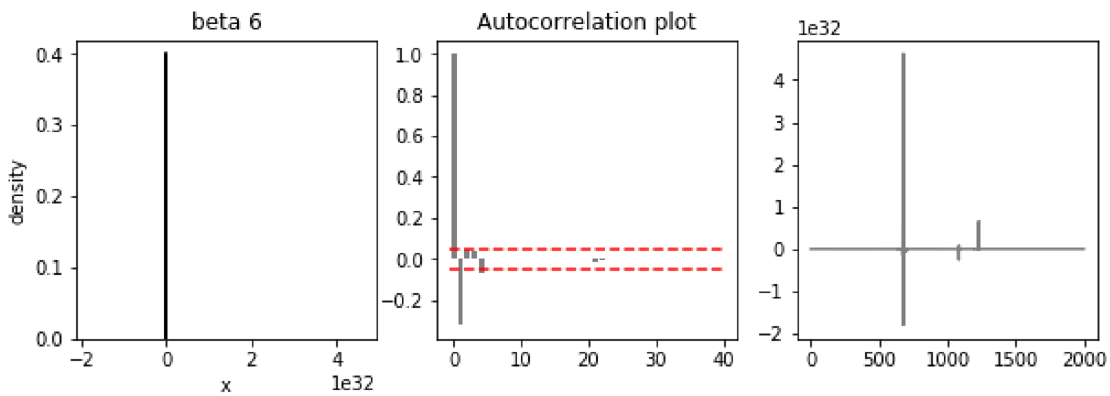

```
In [30]: betaplt(4)
```



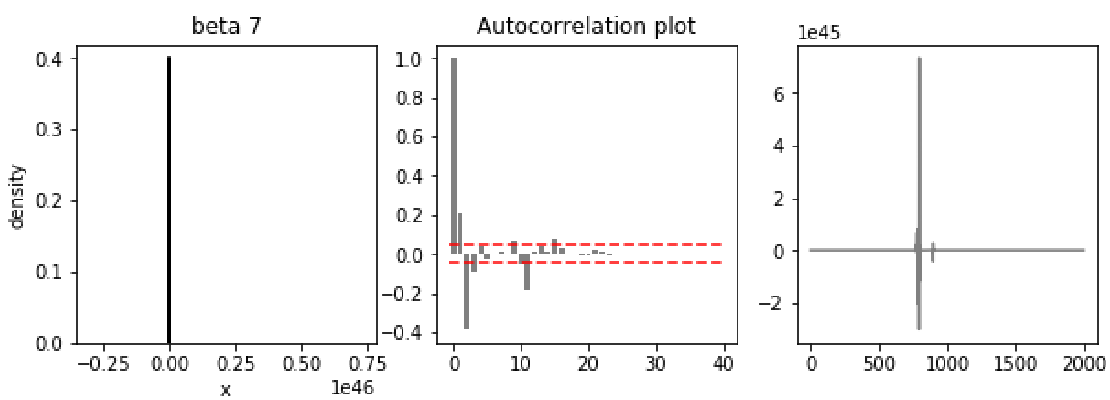
```
In [31]: betaplt(5)
```



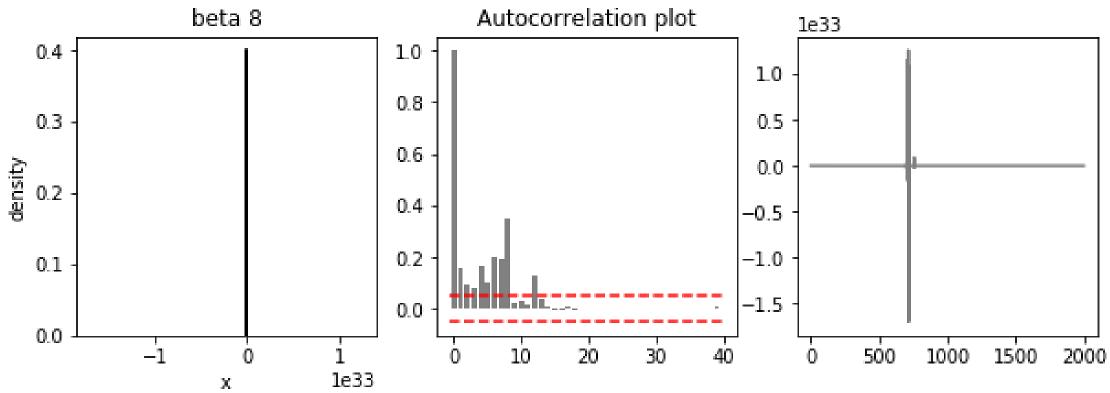

```
In [32]: betaplt(6)
```



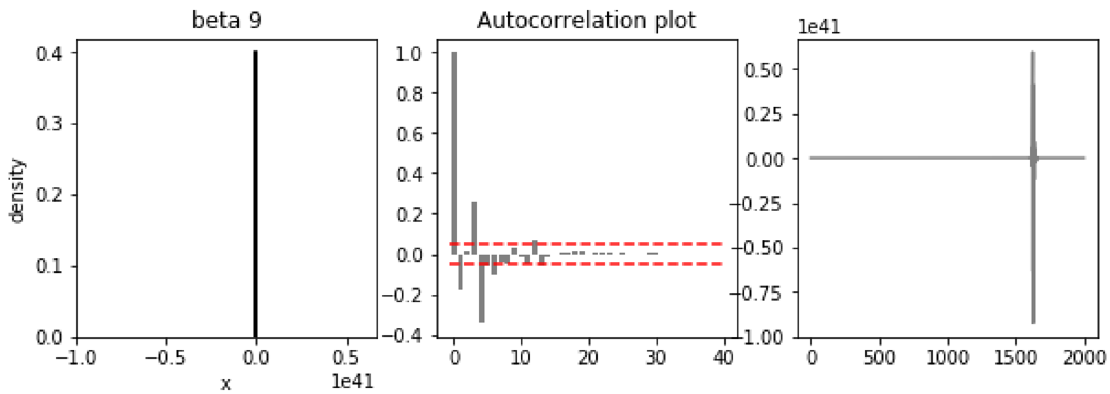

```
In [33]: betaplt(7)
```



```
In [34]: betaplt(8)
```




```
In [35]: betaplt(9)
```




```
In [36]:
```

Out[36]:

	0	1	2	3	4 \
0	1.139095e+00	6.062467e-01	-0.908350	3.198316e+00	-3.055753e+00
1	-3.263962e-01	-3.321794e-01	-0.120596	5.245981e+00	3.463103e-01
2	-9.435735e+00	1.273235e+00	-0.545270	-5.305497e+01	-1.783001e-01
3	-2.962979e-01	1.374975e+01	0.431212	6.596886e+01	9.805456e-02
4	4.273458e-01	3.036300e+01	2.666872	-5.708397e+00	2.720368e-01
5	1.869573e-01	2.048724e+01	251.239169	6.741929e+00	2.718878e-02
6	-6.515157e-02	-4.841269e+02	574.581445	2.943022e+01	-3.262062e-04
7	-1.024364e-01	4.899996e+02	-2426.766166	7.766651e+01	5.707119e-05

8	1.628380e-01	-1.160665e+03	28177.882356	1.165293e+02	9.144373e-04
9	9.546488e-03	2.234438e+03	21742.461764	-6.184575e+01	2.329543e-04
10	-1.055637e-01	-5.368920e+04	9164.817249	-3.278659e+01	-2.467904e-03
11	4.761234e-02	-4.132356e+05	29982.719875	-4.099793e+01	-5.253026e-03
12	5.923455e-02	-3.521920e+05	-88188.947050	-5.985200e+01	1.310610e-02
13	-5.605044e-02	-3.623607e+06	43701.257530	6.480355e+01	5.500439e-02
14	-2.318838e-01	1.068879e+05	44195.597818	2.900248e+03	1.500373e-03
15	6.578422e+00	6.112276e+03	-22131.641527	-6.916903e+03	-2.218799e-02
16	-1.393416e+01	4.257448e+03	-68095.420179	-7.650684e+03	4.104357e-02
17	8.329027e+01	-2.182972e+03	32468.873080	8.608190e+02	4.779419e-02
18	-4.501226e+00	-9.913195e+02	-38622.393610	6.197715e+01	-1.665490e-01
19	1.186571e+01	1.151631e+03	-2043.916390	-2.587637e+02	1.023415e-01
20	1.816777e+01	-1.046031e+03	-9327.434928	1.152831e+02	8.473970e-02
21	-6.081589e+00	-1.013275e+03	-6333.268895	3.570314e+01	-3.770613e+00
22	1.318502e+00	-2.534184e+04	148148.178850	-1.332696e+02	1.494851e+01
23	3.358448e-01	-1.004044e+04	-672460.346042	-5.069318e+02	-1.150367e+01
24	5.201989e-01	3.740227e+05	-671077.506707	-5.354940e+01	-6.231634e+00
25	-1.759775e+00	5.421251e+03	906637.036342	1.298935e+01	1.022491e+01
26	1.907910e-01	-6.199648e+03	267846.829067	1.164787e+02	-9.742005e+01
27	-1.772973e+00	2.082657e+03	143557.394035	4.162249e+01	8.739820e+01
28	2.100719e+00	1.605960e+03	-45052.716846	1.380316e+01	-8.006555e+01
29	-3.039894e+00	2.442477e+03	-107146.249811	-7.415219e+01	-6.614561e+01
...	...	...	...	...	...
1970	6.109597e+50	1.182300e+38	-0.002340	4.392516e+35	9.321523e+19
1971	-5.888000e+50	4.550653e+37	0.017706	-4.239631e+35	-7.182908e+18
1972	6.952781e+50	1.200672e+38	-0.014713	-1.537239e+35	-1.728596e+18
1973	7.466817e+50	1.269170e+38	0.020358	6.535743e+33	-2.479627e+18
1974	2.084842e+51	4.091952e+38	0.009272	2.230493e+34	5.456804e+17
1975	3.452582e+50	-7.284935e+37	0.066294	-6.565040e+34	1.134316e+17
1976	2.395265e+51	-9.118877e+36	0.039977	1.740198e+35	1.336737e+18
1977	-6.054078e+51	8.347356e+36	-0.039563	5.393477e+34	-4.529094e+18
1978	-9.185452e+50	-1.965727e+36	0.082367	1.269738e+36	-4.151426e+18
1979	5.874584e+49	-1.968171e+35	-0.848725	2.940100e+36	2.743307e+18
1980	-2.477944e+49	2.009030e+35	-0.392466	1.135560e+36	-6.929798e+17
1981	-2.698783e+49	2.250674e+35	-6.286675	1.400940e+36	9.584964e+17
1982	-1.107412e+49	3.232574e+35	-3.325459	3.431163e+35	4.191252e+16
1983	8.912129e+48	9.207117e+34	-7.409092	2.141332e+36	-4.290841e+16
1984	5.391798e+48	2.316025e+35	10.053499	-1.102721e+35	-5.906759e+16
1985	3.879712e+48	-3.353850e+35	-43.221339	-1.528937e+36	3.338636e+16
1986	-2.262781e+49	-3.956555e+36	94.263787	3.149637e+36	3.166621e+18
1987	3.582130e+49	-7.375290e+35	-69.096953	8.977611e+35	-7.965003e+18
1988	-3.344724e+49	4.487225e+35	58.223217	2.188571e+36	4.945433e+18
1989	-9.701958e+48	-2.977411e+35	55.331671	2.365237e+37	3.888108e+20
1990	2.412446e+49	-1.108019e+35	-112.110700	4.141403e+37	4.483102e+21
1991	-2.961959e+48	7.033847e+34	157.056423	4.091323e+38	1.229992e+22
1992	1.023060e+48	-3.544247e+35	-190.861962	-4.356314e+38	-2.339146e+22
1993	6.621297e+48	-1.286861e+35	-20.179116	-1.113423e+39	-5.534445e+21
1994	-1.493394e+48	3.322577e+35	8.983859	5.152969e+39	-9.975143e+21

1995	2.174337e+48	-1.247759e+35	7.035345	1.364149e+39	2.243411e+22
1996	6.476285e+47	-6.207570e+34	-10.699460	7.566489e+38	-1.195893e+22
1997	2.439204e+48	-2.157986e+34	6.620205	9.043918e+38	7.889450e+21
1998	-6.848081e+47	3.411707e+33	12.046650	5.252795e+37	-5.026736e+22
1999	-8.469713e+47	1.980310e+34	-0.753223	-3.367371e+37	-4.177019e+23

	5	6	7	8	9 \
0	7.207321e-01	-0.221335	6.082986e-02	4.229664e-01	-4.545302e-01
1	-2.246838e+01	0.100542	-6.031759e-02	-1.432814e+00	-1.218925e+00
2	1.005146e+01	0.157224	-5.704674e-03	-1.006667e+00	-5.065335e+00
3	2.469361e+00	-0.177110	1.260834e-01	-4.270541e-01	2.616418e+00
4	-1.291181e+00	0.021520	1.957543e-02	6.682673e-02	1.549576e+01
5	3.405116e+00	-0.047424	-8.651191e-03	-2.336197e-02	1.213972e+01
6	3.293084e+00	-0.179259	2.297092e-03	-9.971632e-03	4.302054e+01
7	-5.401263e-01	0.324669	-9.787528e-03	1.729763e-01	3.454007e+01
8	-3.586411e-01	0.197100	1.499073e-02	2.612774e-01	-3.236459e+01
9	2.931870e-01	-0.167951	1.650710e-03	-1.626444e-01	2.819256e+01
10	-2.446471e+00	-0.033909	-9.313114e-04	-5.073220e-02	-4.462530e+01
11	2.922611e+00	-0.028454	-1.416411e-03	-9.762048e-03	3.156897e+01
12	1.110851e+01	0.004567	-5.668865e-03	-7.113757e-03	-2.767804e+01
13	1.120516e+01	0.006512	5.034747e-03	-3.869255e-04	-2.840346e-01
14	-4.700879e+02	-0.001204	-1.926575e-03	-4.124094e-05	-7.429782e-01
15	1.313907e+02	-0.001994	7.156719e-03	1.712380e-03	1.385513e+00
16	-5.934723e+01	-0.031493	4.228623e-02	-1.659511e-03	2.764583e-01
17	1.940657e+02	0.043871	5.031004e-03	6.380092e-04	1.824279e-01
18	-2.109473e+02	-0.127174	-3.419944e-03	5.188163e-05	-1.115210e-01
19	1.332490e+02	-0.117683	2.961781e-04	6.129505e-04	-2.872272e-02
20	-2.553453e+02	-0.034033	9.833681e-04	-9.992523e-04	-3.629564e-02
21	-6.522698e+03	0.008231	9.988953e-03	-1.389921e-03	7.292547e-02
22	-7.449405e+03	-0.000573	-4.317794e-03	-3.964421e-03	-1.576878e-01
23	3.426598e+03	-0.000164	2.280714e-03	5.329738e-02	6.255927e-01
24	4.733477e+03	0.000455	1.296762e-03	1.483592e-01	1.212670e+00
25	1.076644e+02	0.001047	5.917112e-03	1.952397e-01	2.939961e-01
26	-1.305296e+02	0.000607	-4.887850e-04	-2.529120e-01	1.154174e+00
27	2.675422e+02	0.000878	9.825110e-04	5.943270e-03	-8.586443e-01
28	-3.634862e+02	-0.001634	-1.987391e-02	-4.324581e-03	-7.920879e-03
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1972	-5.093532e+03	0.445017	-5.719949e+10	-1.691647e+10	4.944704e+22
1973	1.272361e+04	-0.026987	4.028396e+09	3.580919e+09	-4.567865e+22
1974	5.497811e+04	-0.040957	-7.503998e+09	1.261609e+09	-1.272409e+23
1975	-1.228176e+04	-0.059318	1.469566e+10	-2.710023e+08	-9.336388e+22
1976	3.833719e+02	0.022106	2.022198e+10	-4.288279e+08	-2.596554e+23
1977	1.066234e+03	0.025199	-3.417007e+10	7.034555e+08	-1.804876e+23
1978	1.748963e+03	0.015024	6.275471e+10	2.543657e+08	6.547585e+24
1979	-2.685697e+04	-0.140447	1.754253e+10	-2.595781e+08	1.953365e+23

1980	-9.944318e+04	-0.060203	-1.046867e+10	1.414679e+08	-2.085020e+23
1981	2.716257e+05	-0.025990	3.280442e+08	1.102987e+08	-1.799315e+23
1982	3.709081e+04	-0.189248	-1.604604e+09	-1.699824e+08	3.803592e+22
1983	-1.282203e+05	-0.116875	-3.064321e+09	-5.376712e+08	1.439928e+22
1984	-7.902088e+03	0.181136	2.869571e+09	4.413650e+07	-9.217504e+21
1985	3.553859e+05	-0.055107	2.740210e+09	-9.182480e+07	-4.117918e+21
1986	1.866141e+05	-0.302203	2.566834e+09	9.077665e+07	-9.395137e+21
1987	-1.123007e+05	-1.494530	1.620162e+09	1.527929e+08	2.874986e+22
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1992	2.792683e+05	-41205.036232	-1.631912e+07	1.422463e+09	-1.037735e+21
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21	3.755058e-01
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25	-1.166210e-01

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