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1 Linear Mixed Effect Model

$$y = X\beta + Zb + \epsilon$$

- y is the $n \times 1$ response vector, and n is the number of observations.
- X is an $n \times p$ fixed-effects design matrix.
- β is a $p \times 1$ fixed-effects vector.
- Z is an $n \times q$ random-effects design matrix.
- b is a $q \times 1$ random-effects vector. $b \sim N(0, \sigma^2 D(\theta))$
- ε is the $n \times 1$ observation error vector. $\epsilon \sim N(0, \sigma^2 I)$
- Random-effects vector, b, and the error vector, ϵ, are independent from each other.

2 Non-Linear Mixed Effect Model

$$y_{ij} = f(\phi_i, x_{ij}) + e_{ij}$$

- y_{ij} is the jth response on the ith individual
- x_{ij} is the predictor vector for the jth response on the ith individual
- f is a nonlinear function of the predictor vector
- e_{ij} is normally distributed

$$\phi_i = A_i\beta + B_i b_i, \;\; b_i \sim N(0, \sigma^2 D)$$

3 Partially linear varying-coefficient mixed model

$$Y_{ij} = w'_{ij}\alpha(u_{ij}) + x'_{ij}\beta + z'_{ij}\xi_i + \epsilon_{ij}$$

- w_{ij}, x_{ij} and z_{ij} are covariates vector
- u_{ij} is voctor of underlying effect modifier that change the effects of w_{ij} on Y_{ij} in a non parametric way
- $\alpha(u_{ij})$ is varying coefficient vector
- β is fixed effect
- ξ_i is random effect $\xi_i \sim^{iid} N(0, \Psi)$
- $\epsilon_i \sim^{ind} N(0, \Sigma_i)$
- $\xi_i \perp \epsilon_i$