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1 Spatial Autoregressive (SAR) Model

According to the type of response variable in the data, cross sectional spatial data can be divided into two types: Gaussian and non-Gaussian

- Spatial autoregressive(SAR) model : imply a geometric decay of spatial dependence.
- Matrix Exponential Spatial Specification(MESS) model : imply an exponential decay

are often used for modeling Gaussian spatial data. There is an close correspondence between SAR and MESS models if the same row-standardized spatial weight matrix is used in both models.

2 Intergrated Nested Laplace Approximation (INLA)

- To overcome the computational bottleneck of MCMC. Model parameters are divided into two subset - hyperparameters and latent effects parameters.
- Key assumption is that the conditional distribution of latent effects given hyperparameters follows a multivariate Gaussian distribution
- Adversely affected by an increase in the number of hyperparameters and grid points chosen for them

3 Variational Bayes

Unlike MCMC Variational Bayes methods use optimization to fulfill the same inference purpose.

- Mean-Field Variational Bayes

Relies on the assumption that the variational distribution can be factorized into a product form.

$$q(\Theta) = \prod_{l=1}^M q_l(\theta_l)$$

- Fixed-Form Variational Bayes

avoids the product form assumption but restricts each variational factor to a parametric form applies to conjugate models.

- Integrated Nonfactorized Variational Bayes

can provide more accurate and robust estimates by taking posterior dependence into account and INFVB algorithm can be parallelized.

$$q_{INFVB}(\Theta) = q(\Theta_c|\Theta_d)q(\Theta_d)$$

4 Spatial Econometric Models for Gaussian Data

4.1 Spatial Autoregressive Confused (SAC) Model

$$\begin{aligned} \mathbf{y} &= \rho \mathbf{W}_1 \mathbf{y} + \mathbf{X} \beta + \mathbf{u} \\ \mathbf{u} &= \lambda \mathbf{W}_2 \mathbf{u} + \epsilon \end{aligned} \tag{1}$$

- $\mathbf{X} = \mathbf{Z}$ or $[\mathbf{Z} \ \mathbf{W}_1 \mathbf{Z}]$

- $\epsilon \sim N(0, \sigma^2 \mathbf{I}_n)$

- restriction on ρ and λ is

$$\frac{1}{\omega_{min}} < \rho < \frac{1}{\omega_{max}}, \quad \frac{1}{\gamma_{min}} < \lambda < \frac{1}{\gamma_{max}} \text{ where } \rho \text{ and } \lambda \text{ are eigen values of } \mathbf{W}_1 \text{ and } \mathbf{W}_2$$

likelihood function of SAC model is

$$p(\mathbf{y}|\beta, \rho, \sigma^2, \lambda) = (2\pi\sigma^2)^{-\frac{n}{2}} |\mathbf{A}| |\mathbf{B}| \exp\left\{-\frac{[\mathbf{B}(\mathbf{A}\mathbf{y} - \mathbf{X}\beta)]' [\mathbf{B}(\mathbf{A}\mathbf{y} - \mathbf{X}\beta)]}{2\sigma^2}\right\} \tag{2}$$

4.2 MESS model

$$\mathbf{S}_1 \mathbf{y} = \mathbf{X} \beta + \mathbf{u}$$

$$\mathbf{S}_2 \mathbf{u} = \epsilon$$

where $\mathbf{S}_1 = e^{\alpha \mathbf{W}_1}$, $\mathbf{S}_2 = e^{\tau \mathbf{W}_2}$

- $\mathbf{X} = \mathbf{Z}$ or $[\mathbf{Z} \ \mathbf{W}_1 \mathbf{Z}]$

- $\epsilon \sim N(0, \sigma^2 \mathbf{I}_n)$

- α, τ are unconstrained

likelihood function of MESS model is

$$p(\mathbf{y}|\beta, \alpha, \sigma^2, \tau) = (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{[\mathbf{S}_2(\mathbf{S}_1 \mathbf{y} - \mathbf{X}\beta)]' [\mathbf{S}_2(\mathbf{S}_1 \mathbf{y} - \mathbf{X}\beta)]}{2\sigma^2}\right\} \tag{3}$$