Question 1

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1 Profile likelihood

Let θ, η be two parameters that we want to estimate. If $P_\theta(x)$ and $P_\eta(y)$ are independent, we have

$$L_{(\theta,\eta)} = L_1(\theta) \cdot L_2(\eta)$$

And we can call θ and η orthogonal parameters.

If orthogonality of parameters holds, we do not have information loss by taking the profile likelihood of the parameter we are interested in. In this case, we can treat one of the parameters as a nuisance parameter.

For example, let η be a nuisance parameter. Then, by fixing θ, we can get $\hat{\eta}_{mle}$ which is maximum likelihood estimator of η.

After finding $\hat{\eta}_{mle}$, we can use Profile likelihood L(θ, $\hat{\eta}_{mle}$) to get $\hat{\theta}$. Then, where profile likelihood $L(\theta, \hat{\eta}_{mle})$ is maximized, we can find $\hat{\theta}$. Finally, we can estimate two parameters by using profile likelihood.

2 Marginal/Conditional likelihood

$$\begin{aligned} L_{(\theta,\eta)} &= P_\theta(v) \cdot P_{\theta,\eta}(w|v) \\ &= P_\theta(v|m) \cdot P_{\theta,\eta}(w) \end{aligned}$$

However, when θ and η are nonothogonal case like above, you may have information loss by taking profile likelihood. In this case, we can consider Marginal likelihood or Maximum likelihood.

Marginal likelihood

$$\text{(Marginal likelihood)} = p_\theta(v)$$

Conditional likelihood

$$\text{(Conditional likelihood)} = p_\theta(v|w)$$

If w is sufficient for η, this decomposition is available.

3 Testing

To test H_0: θ=θ_0 vs H_1: not H_0, we can use wald test, score test. We can also use likelihood ratio test. Test statistic of LRT(Likelihood Ratio Test) is

$$L = 2(\log L(\hat{\theta}, \hat{\eta}(\hat{\theta})) - \log L(\theta_0, \hat{\eta}(\theta_0))) \sim \chi^2_{2-1} = \chi^2_1$$

Then, reject H_0 if $L > \chi^2_{1,\alpha}$.