

Basics of modeling

Today's topics

- Procedures of analyzing data
- Some cautious facts
- Bias-Variance trade off
- Parametric model vs Nonparametric model

Procedure of analyzing data

- Specifying a goal of analysis
- Collecting data with respect to the goal
- while(Fail to satisfy the goal)
 - Do EDA(visualization, simple modeling, clustering...)
 - Do Modeling
- Conclusion

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Data type

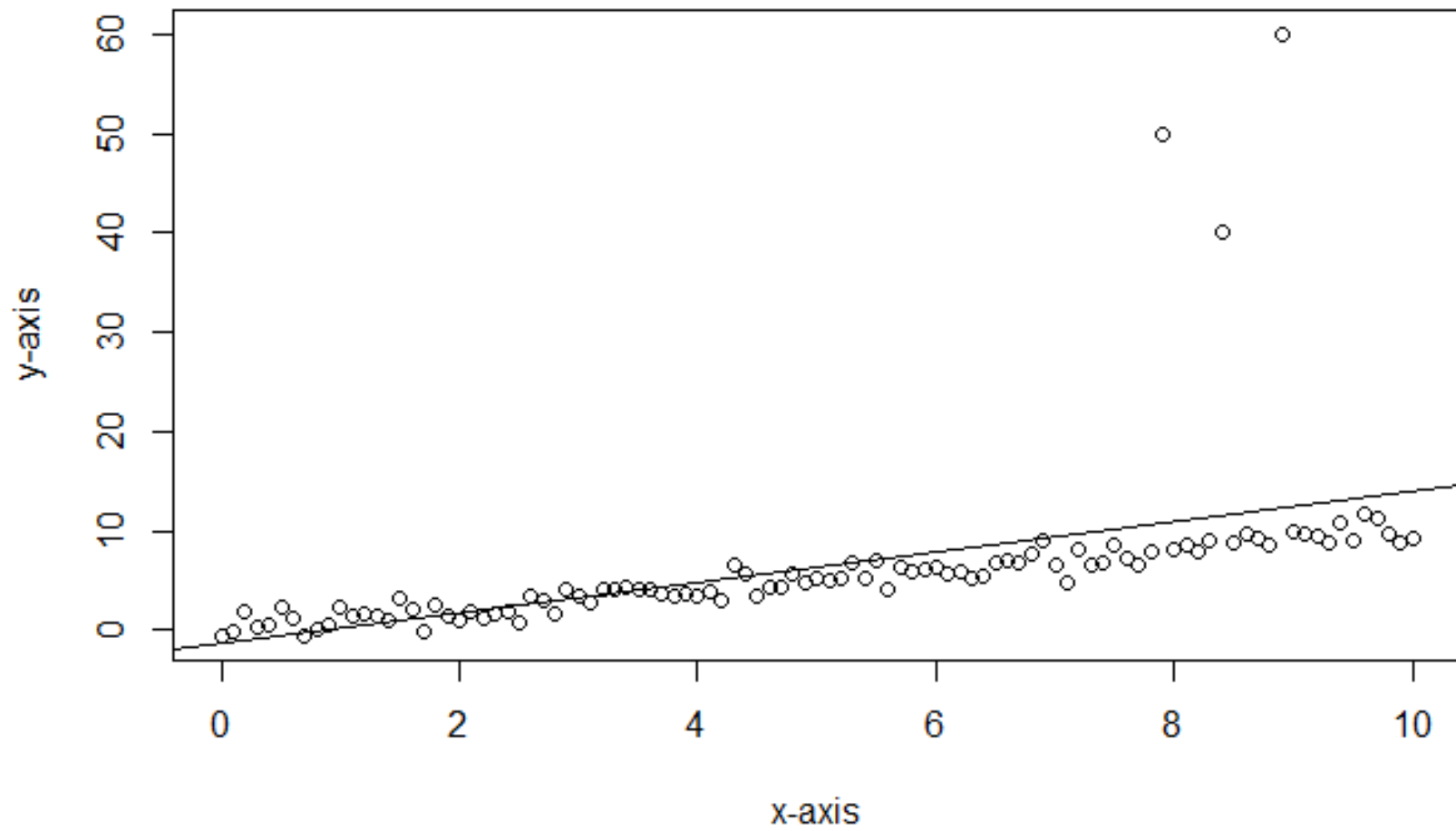
Given mixed data

- Many models have been developed for continuous predictors and responses.
- PCA, FA, K-means are only available for continuous data.
- If nominal type categorical predictors are in data, you should transform the categorical predictors into plausibly continuous data, or ordinal data.
- Or, you can search through google!(highly recommended)

Outliers

- Models based on ordinary least square can not be free from outliers.
- K-means, PCA, FA can be examples.
- In those cases, you have to find more robust models than the OLS based model., such as Robust K-means, Robust PCA.

Outliers



Sparse data

- 'Sparsity' denotes that too many zeros are included in data.
- You should try to decrease undesired effects caused by sparse data.
- Most of the models to handle sparsity data are developed already like Sparse logistic regression, sparse PCA.

Sparse data

Index	Y	X1	X2	X3
1	-0.68	0	0	1
2	0.95	0.05	0	0
3	0.82	0	0	0
...
199	18.77	0	0	20
200	17.68	9.95	0	0
201	18.53	10.00	1	0

Too many zeros

Imbalanced data

Zero-inflated data

- When too many zeros exist in data, it is better to use models which are developed to deal with zero-inflated structure.
- For example, zero-inflated poisson regression, zero-inflated factor analysis, zero-inflated logistic regression exist.

Imbalanced class

- Let's consider a binary classification problem.
- While doing EDA, we see that 95% of subjects belongs to 'Innocent', and only 5% to 'Fraud'.
- In this case, we call this 'imbalanced class problem'.
- If a model you consider ignores this imbalanced class, the model might become to have poor performance.

Imbalanced class

[Remedy]

- In perspective of data : up sampling, down sampling, SMOTE, ...
- In perspective of model : imposing more weight on minority class. Adjusting a threshold.
- In perspective of measure : Using proper metric to measure performance, such as f1-score.

Imbalanced class

A / P	Innocent	Faud
Innocent	800	100
Fraud	50	50

- Accuracy : 85%
- Precision : 33%
- Recall : 50%
- F1 score : 20%
 - > harmonic mean of precision and recall

Bias and Variance trade off

Bias-Variance trade off

- This is the basic concept of modeling.
- You MUST understand OVERFITTING and UNDERFITTING during this class.
- as well as BIAS and VARIANCE.

Bias-Variance trade off

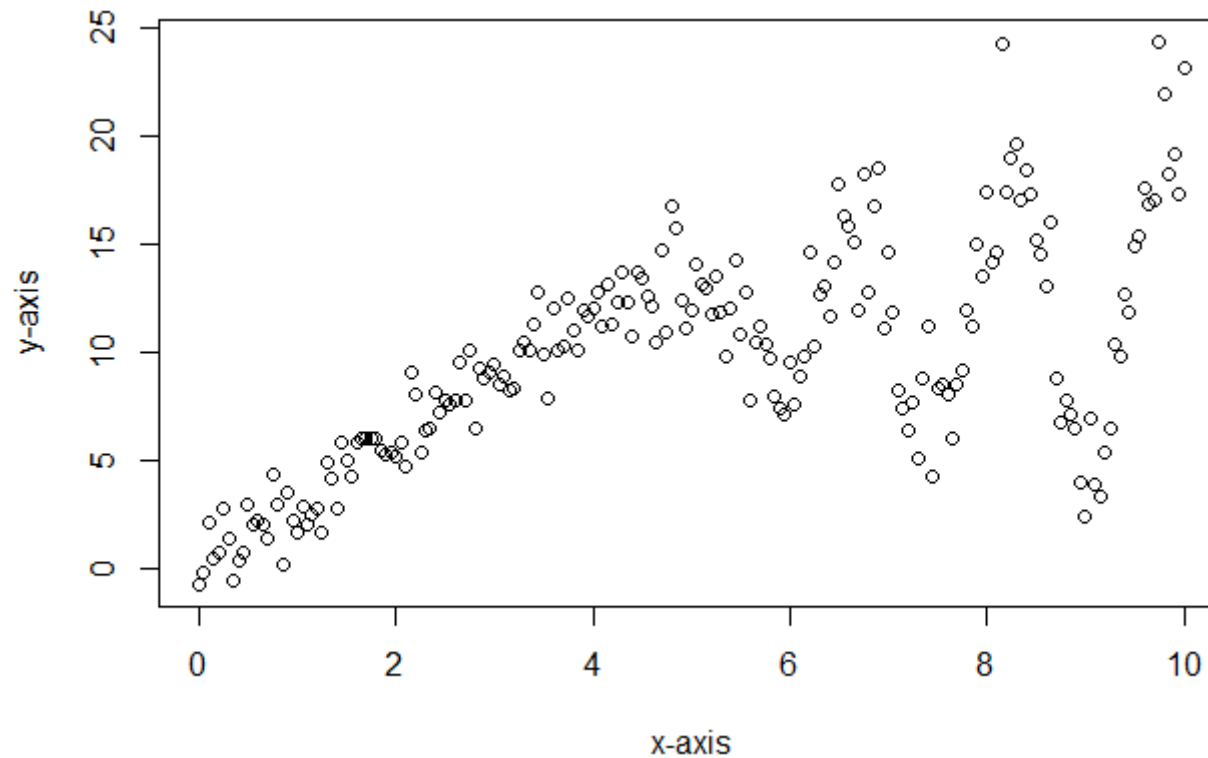
- sohnenn.tistory.com

<https://sohnenn.tistory.com/entry/Bias-and-Variance-tradeoff-1?category=783791>

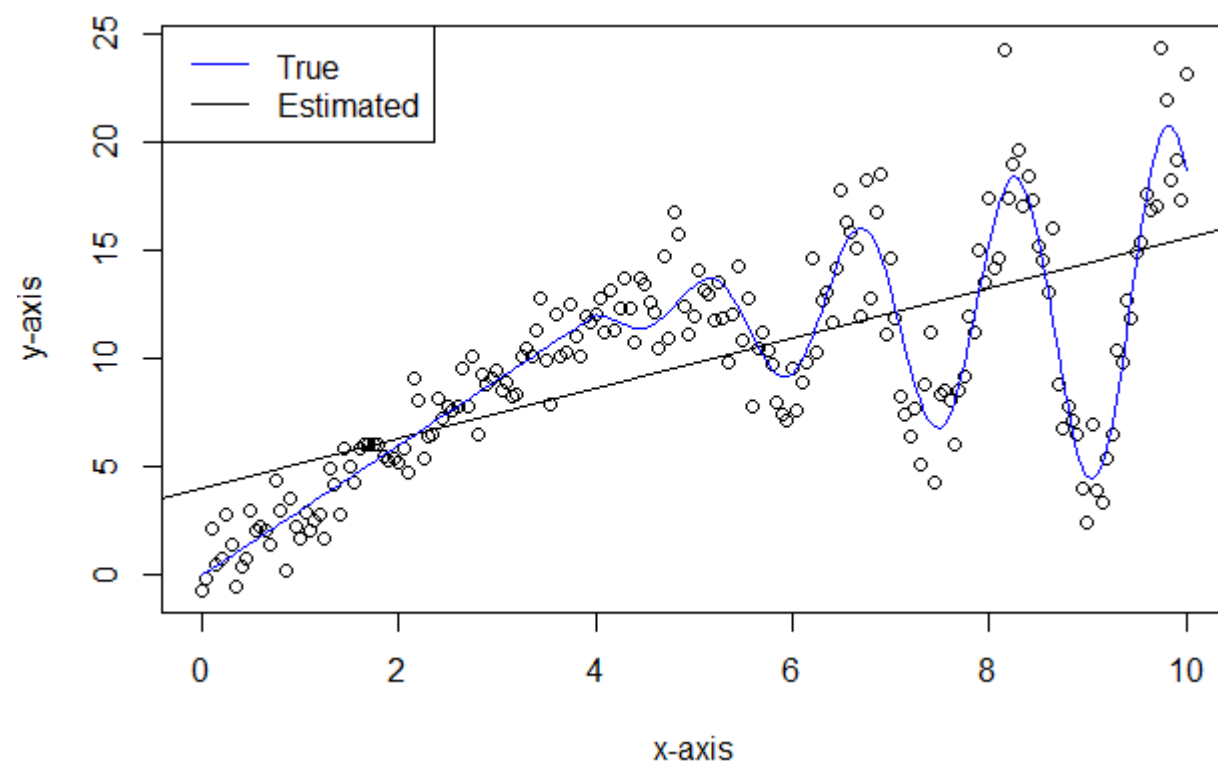
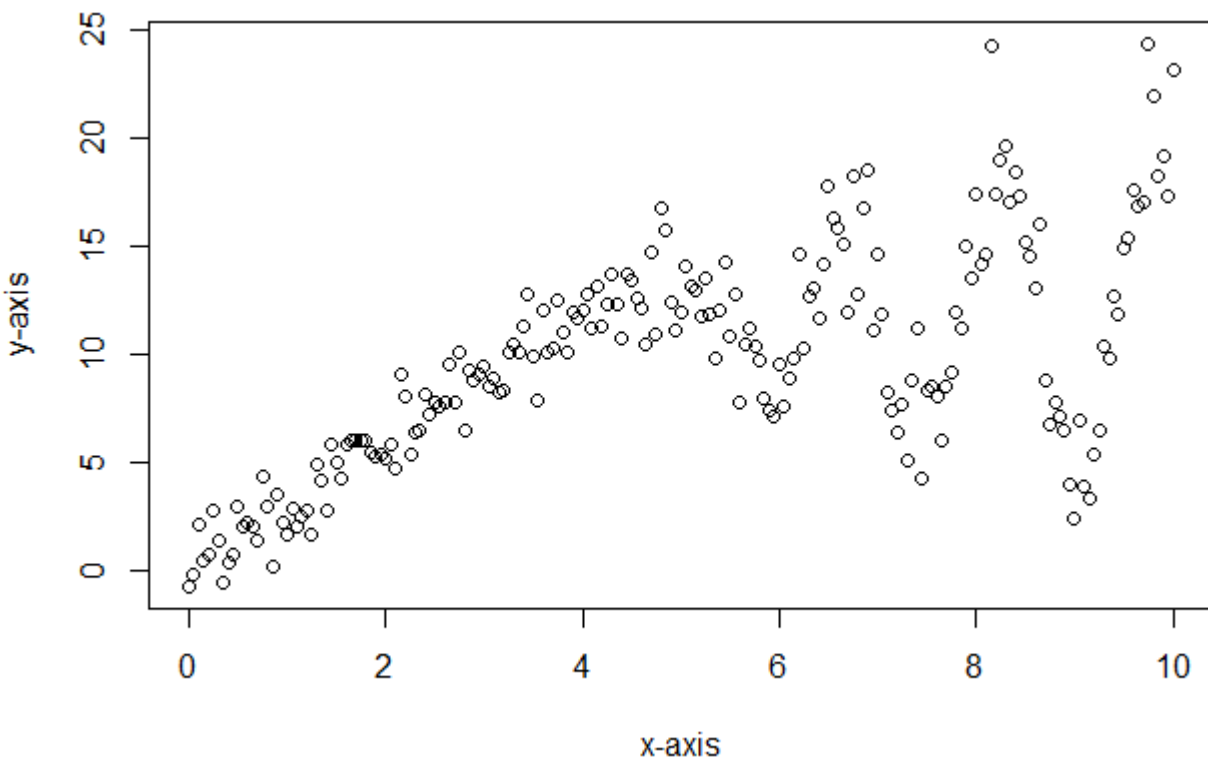
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Overfitting and Underfitting

- Considering following example

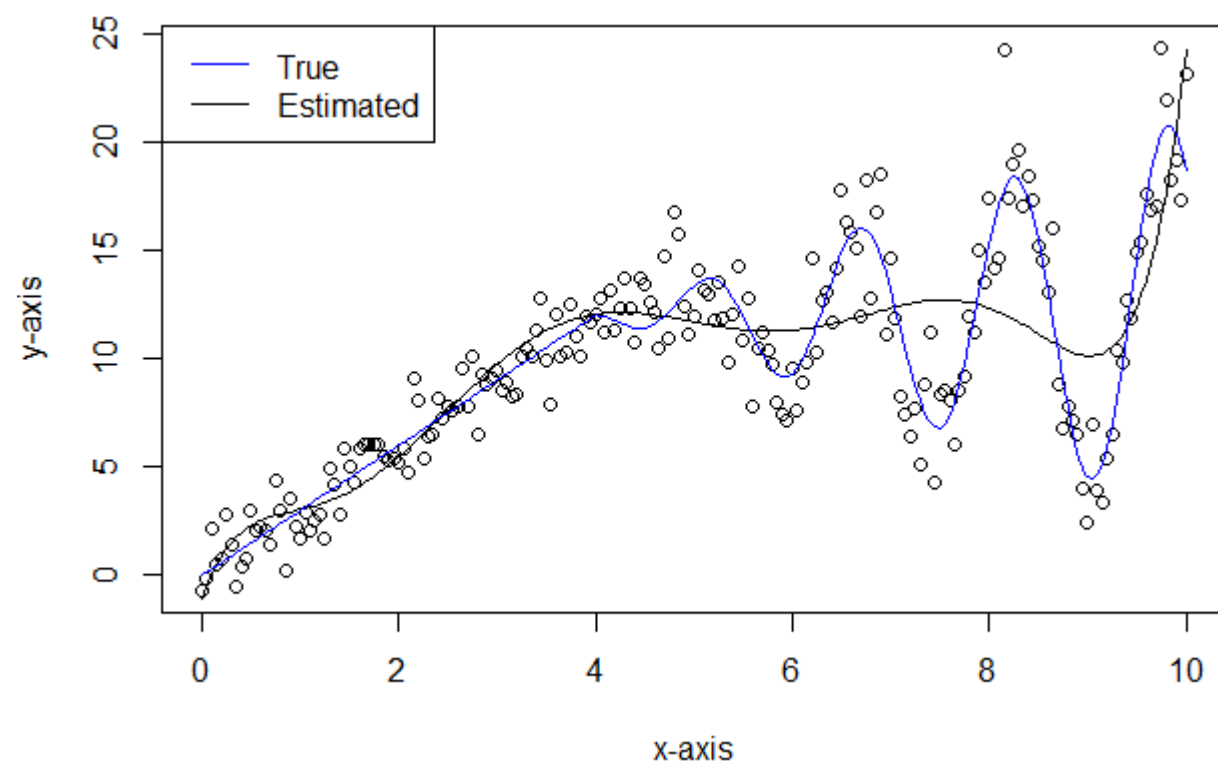
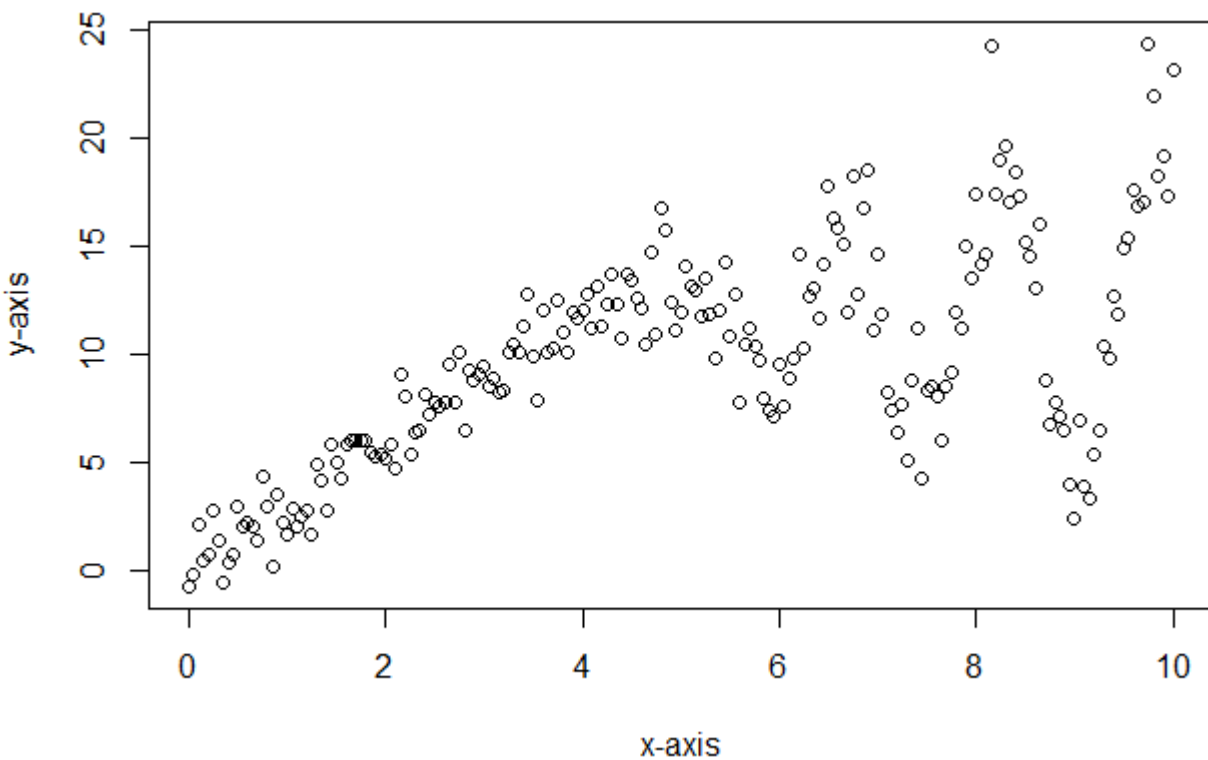


$$y = \beta_0 + \beta_1 x + \varepsilon$$



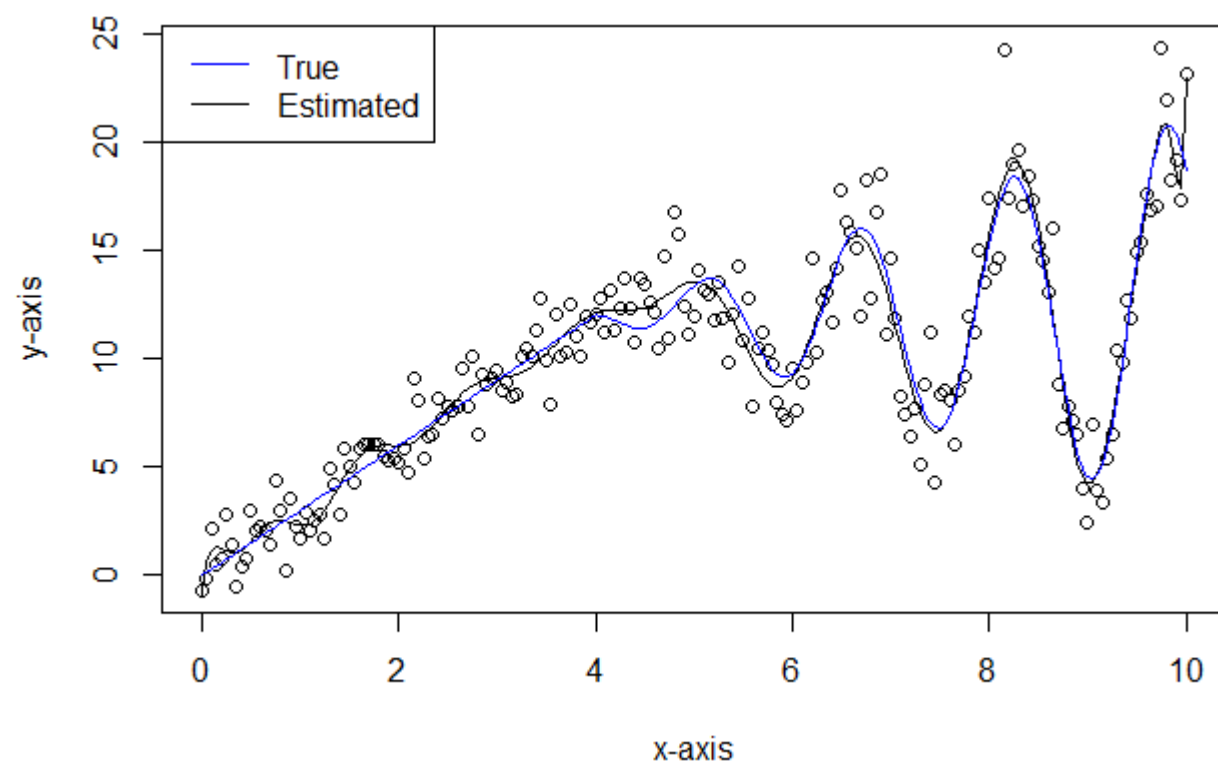
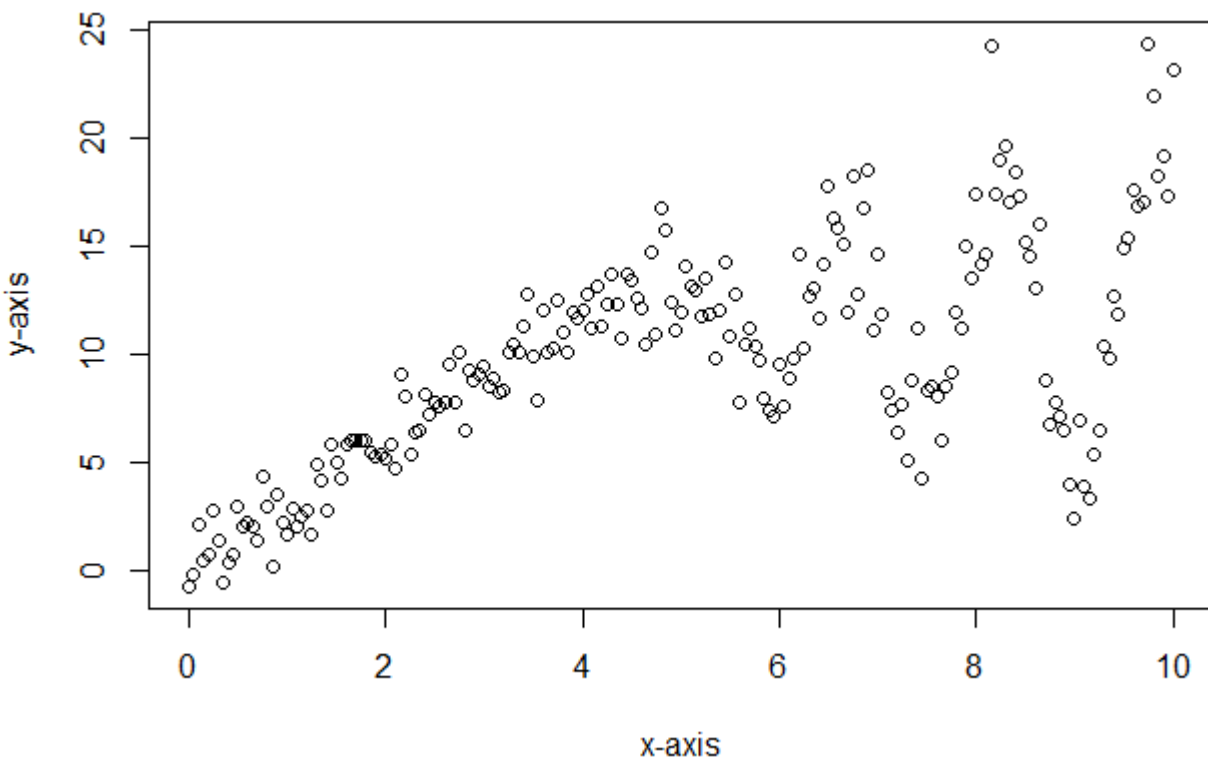
Too Biased!

$$y = \beta_0 + \beta_1 x + \dots + \beta_7 x^7 + \varepsilon$$



Still Biased...

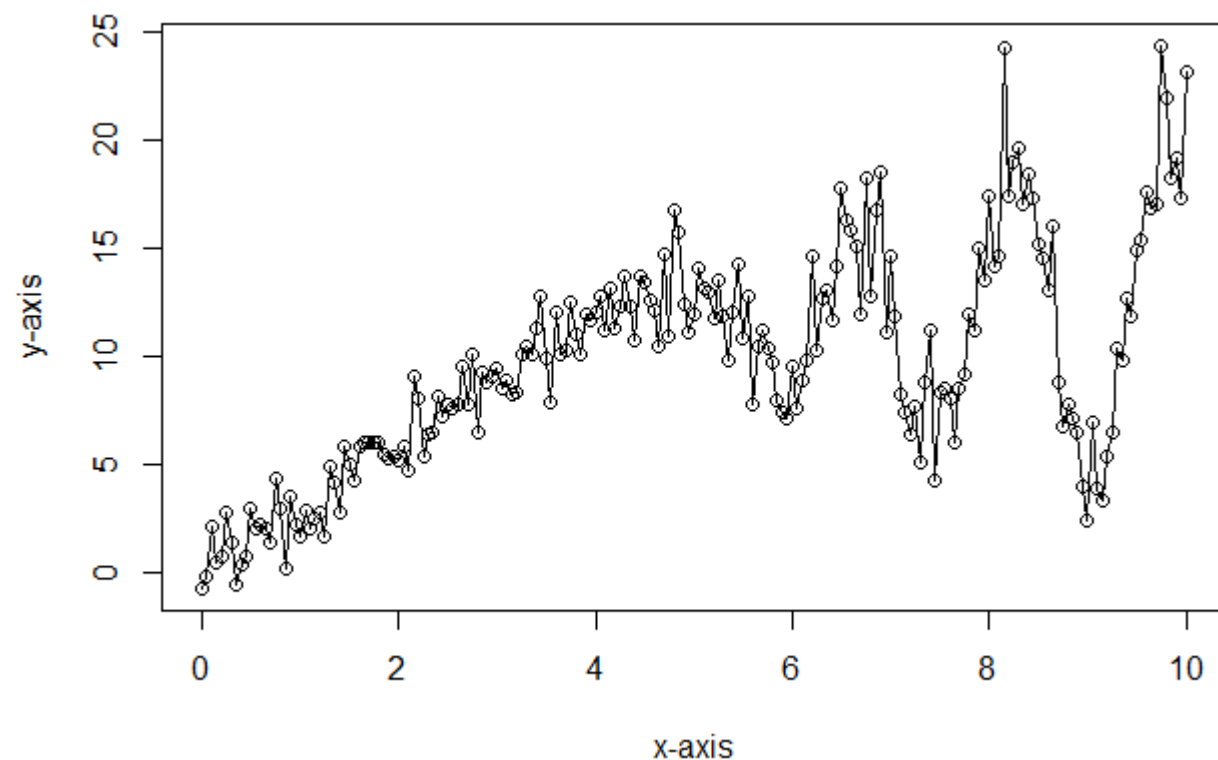
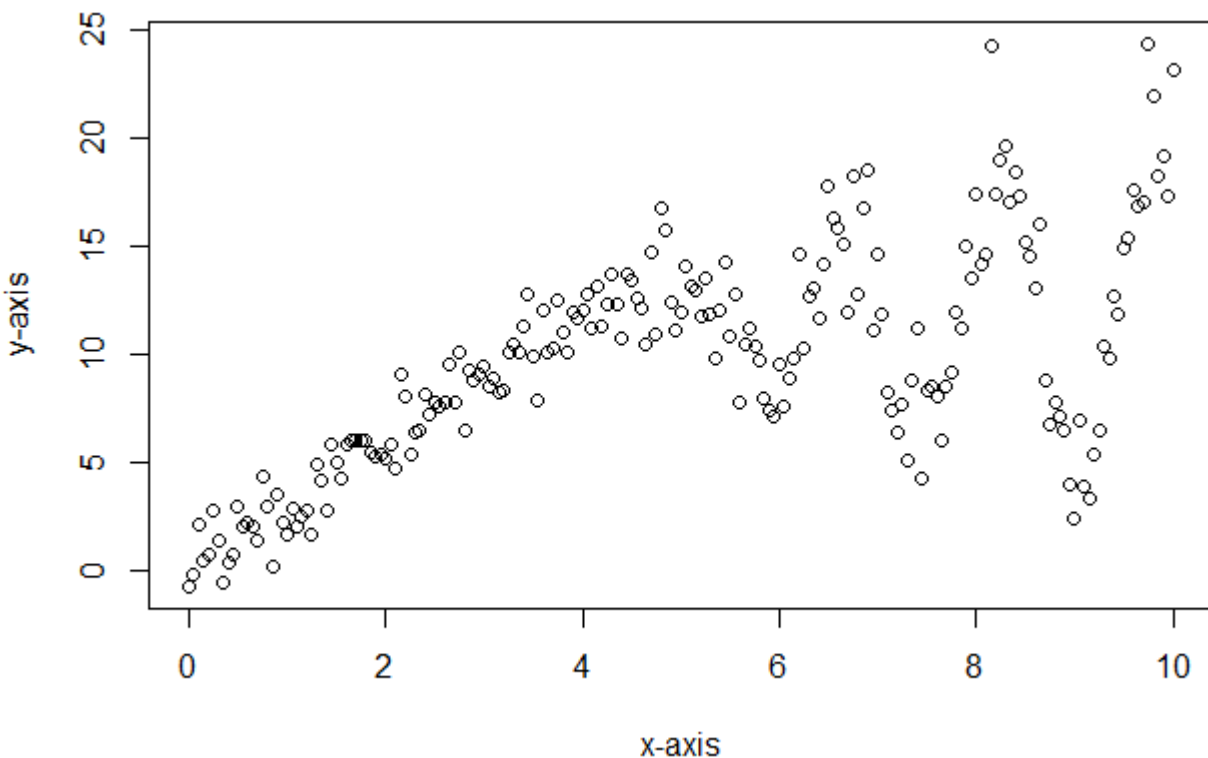
$$y = \beta_0 + \beta_1 x + \cdots + \beta_{25} x^{25} + \varepsilon$$



A bit Overfitted...

$$y = f_1(x) + \cdots + f_{200}(x)$$

$$f_i(x) = y_i I(x = x_i)$$



Fully Overfitted!!

A (non)parametric model

A Parametric Model

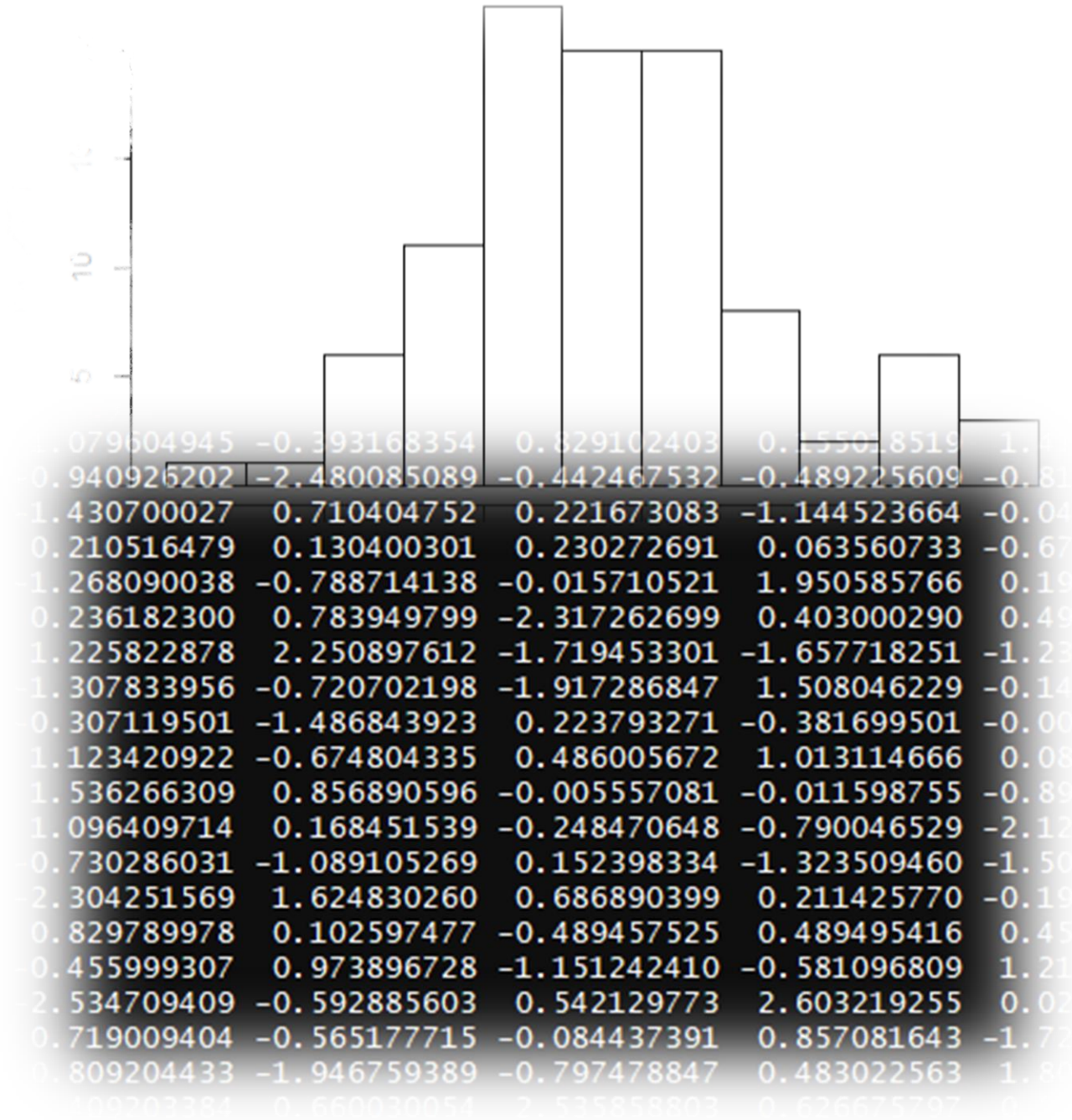
- Imagine data is given to you!

Index	Y	X
1	-0.68	0.00
2	0.95	0.05
3	0.82	0.10
...
199	18.77	9.90
200	17.68	9.95
201	18.53	10.00

“Your goal is to predict response Y with pre-specified finite parameters.”

What is meaning of 'parametric'?

- Imagine normal distribution.

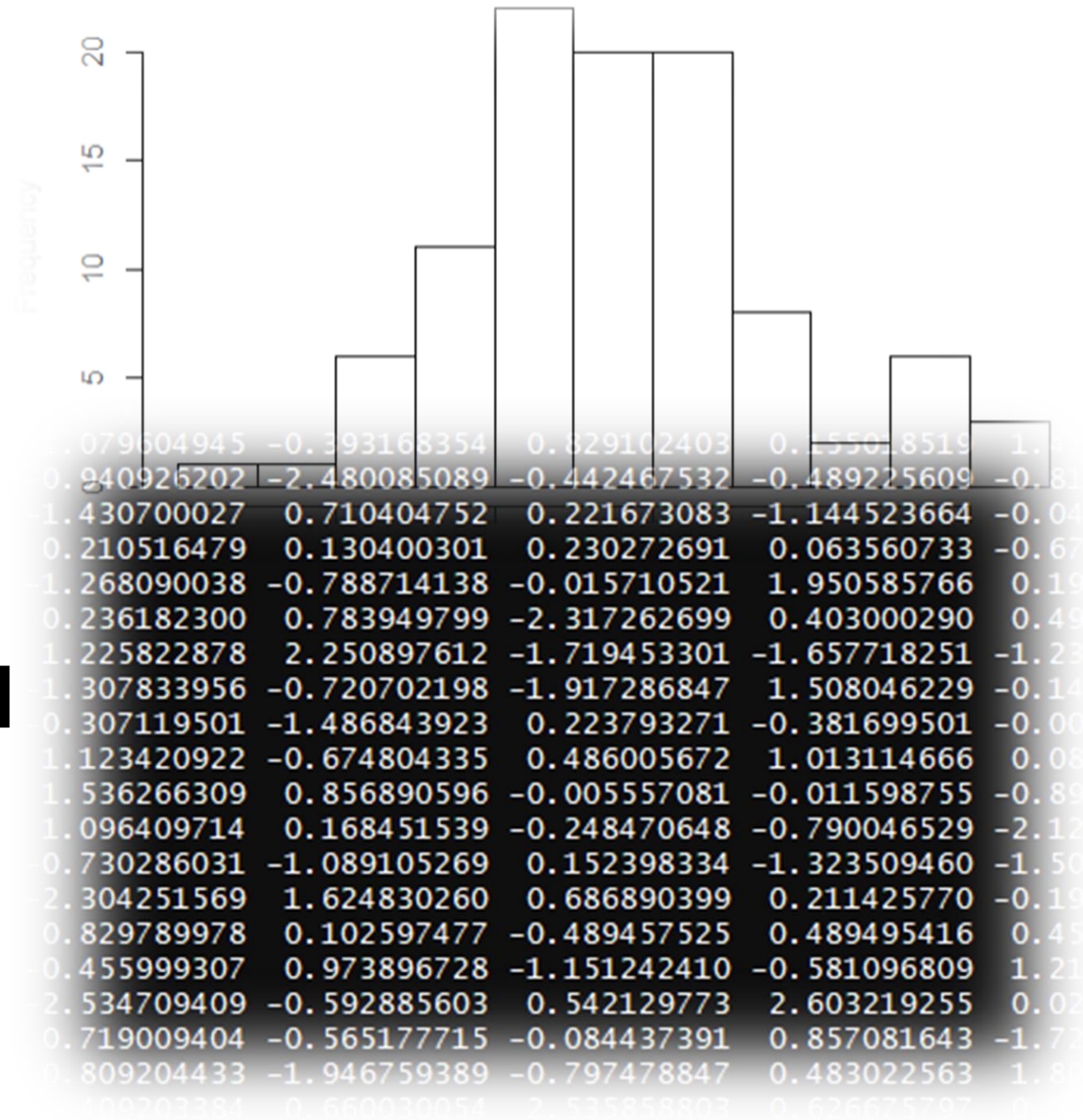


What is meaning of 'parametric'?

- Imagine normal distribution.

μ σ^2

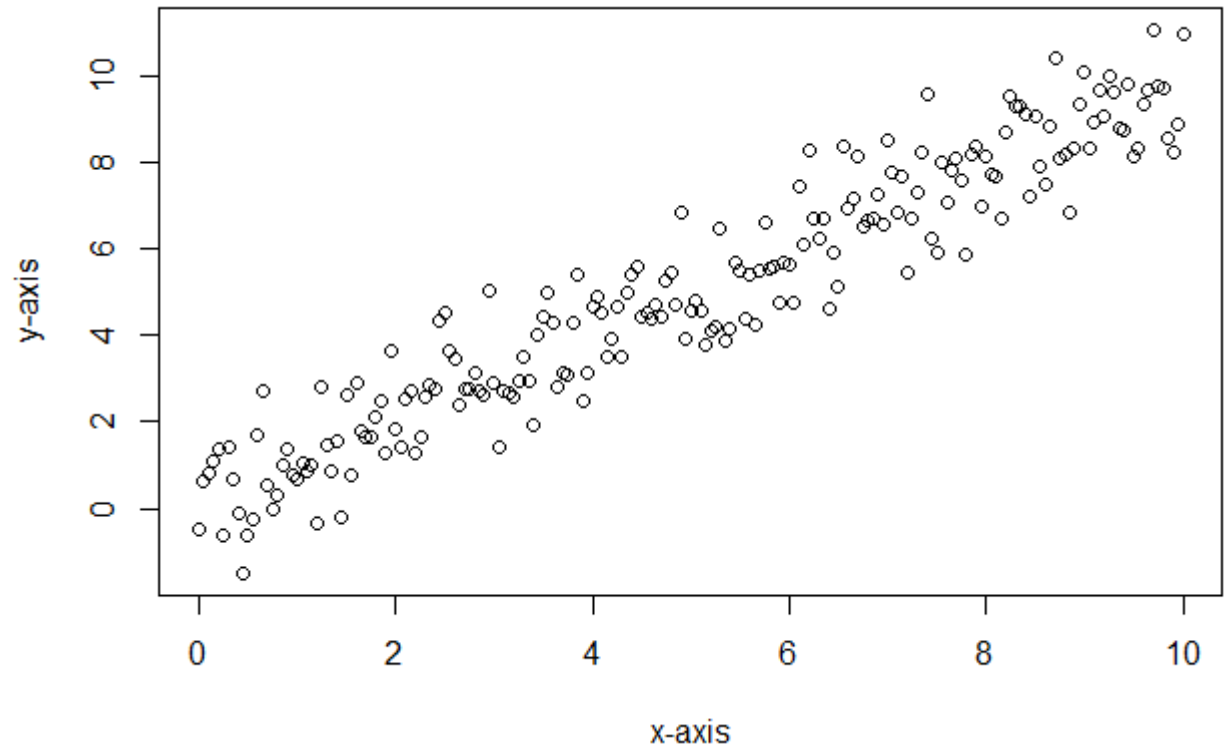
- Without loss of information
- Sufficient Statistics



What is meaning of 'parametric'?

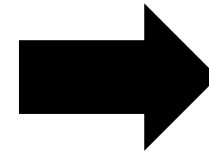
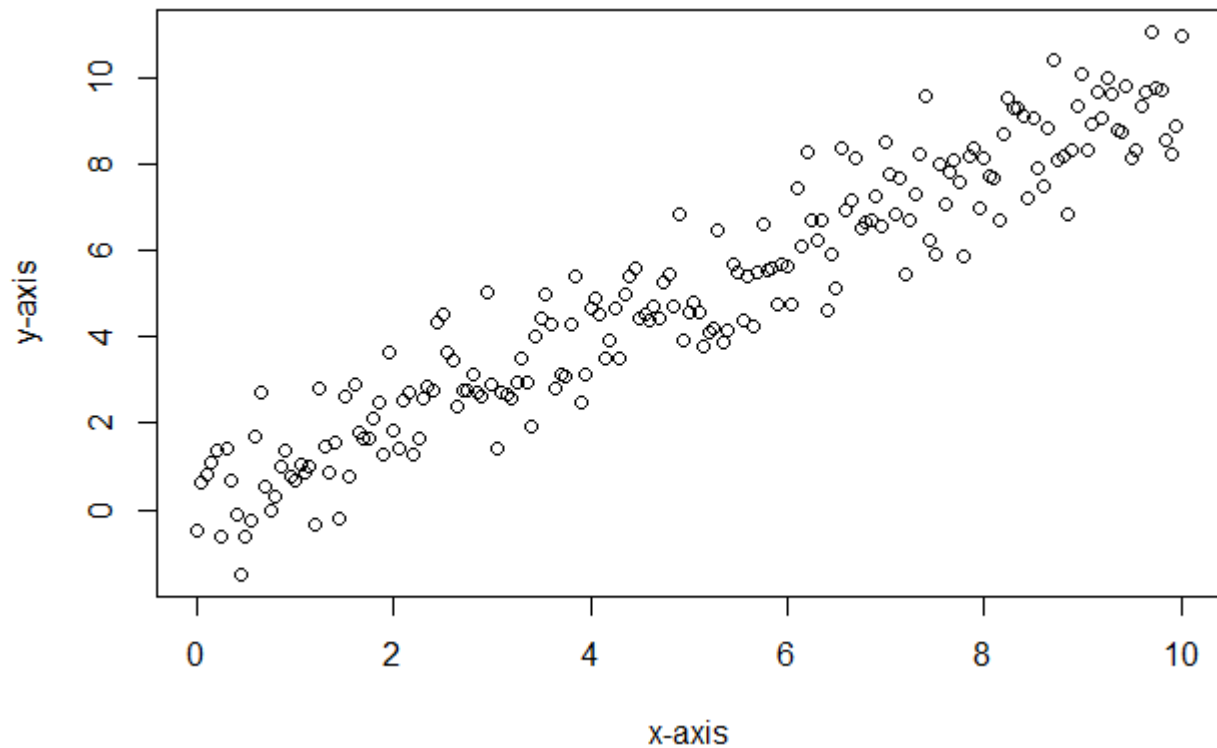
- Imagine simple linear model.

```
x sapply.x..function.x..f.x..  
1 0.00 0.3564862  
2 0.05 -1.7162925  
3 0.10 -0.2283640  
4 0.15 -1.0949264  
5 0.20 0.9651375  
6 0.25 0.5103146  
...  
196 9.75 9.727495  
197 9.80 10.201826  
198 9.85 10.863627  
199 9.90 10.937694  
200 9.95 9.810087  
201 10.00 9.668022
```



What is meaning of 'parametric'?

- Imagine simple linear model.

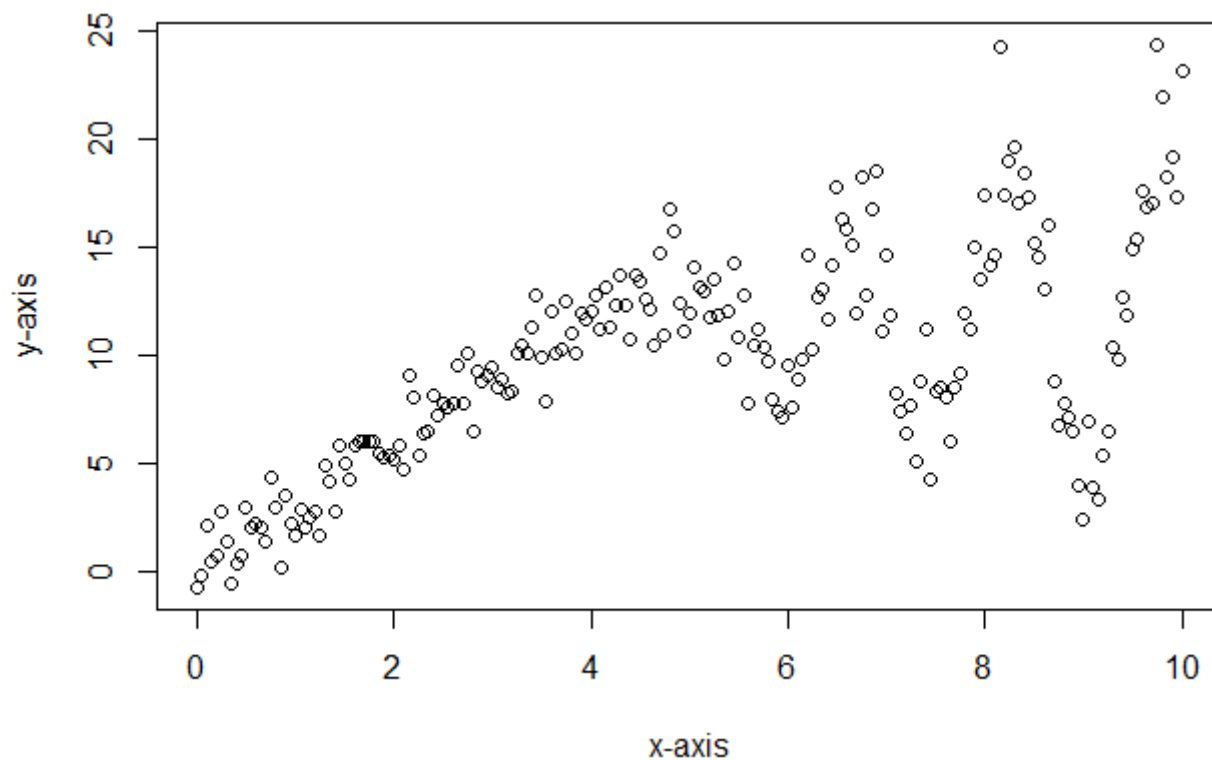


$$y = \beta_0 + \beta_1 x + \varepsilon$$

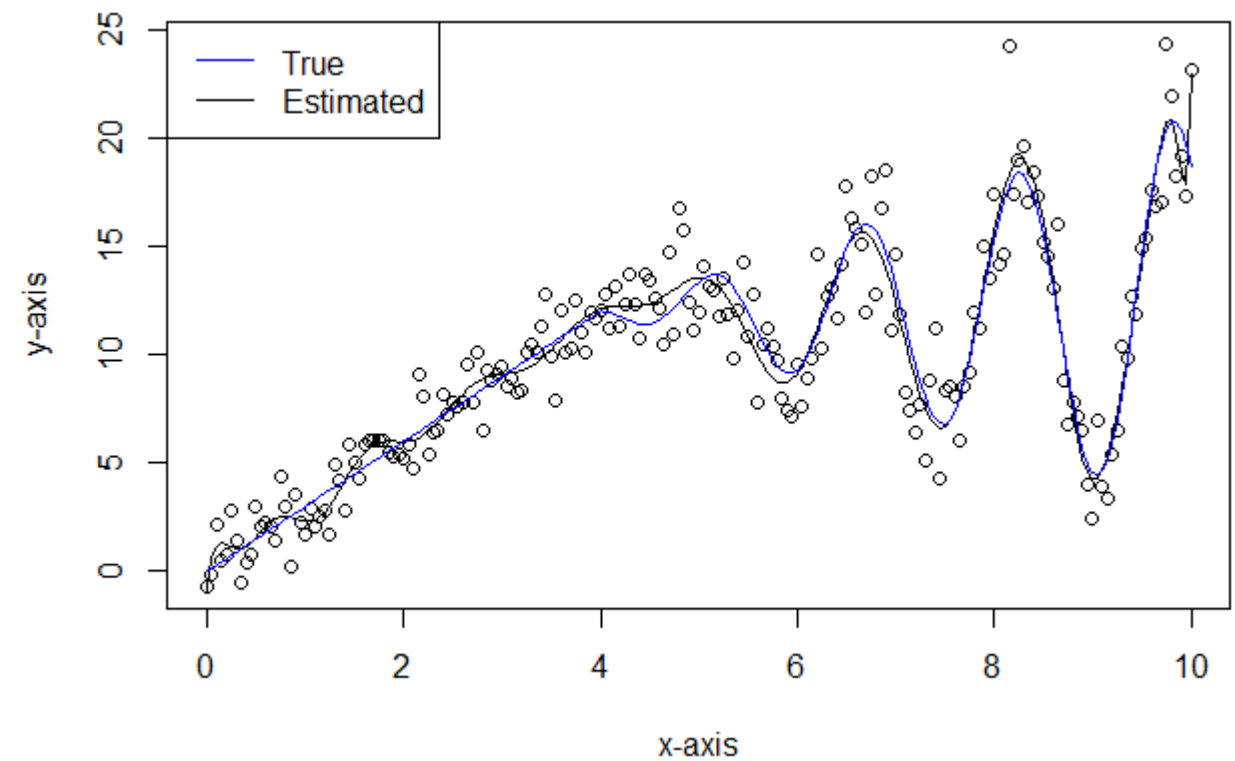
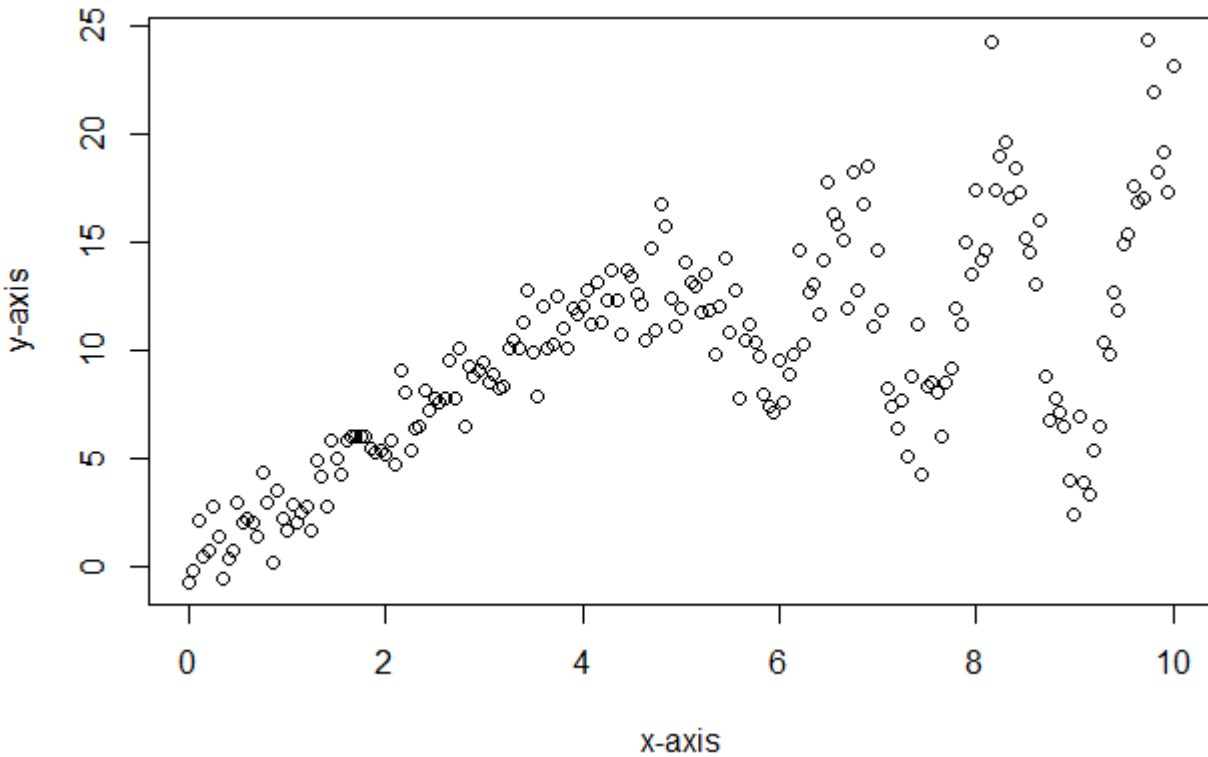
$$\beta_0 \quad \beta_1$$

Capturing trend with parameters

- Considering previous example



$$y = \beta_0 + \beta_1 x + \cdots + \beta_{25} x^{25} + \varepsilon$$



The model seems to be plausible, but

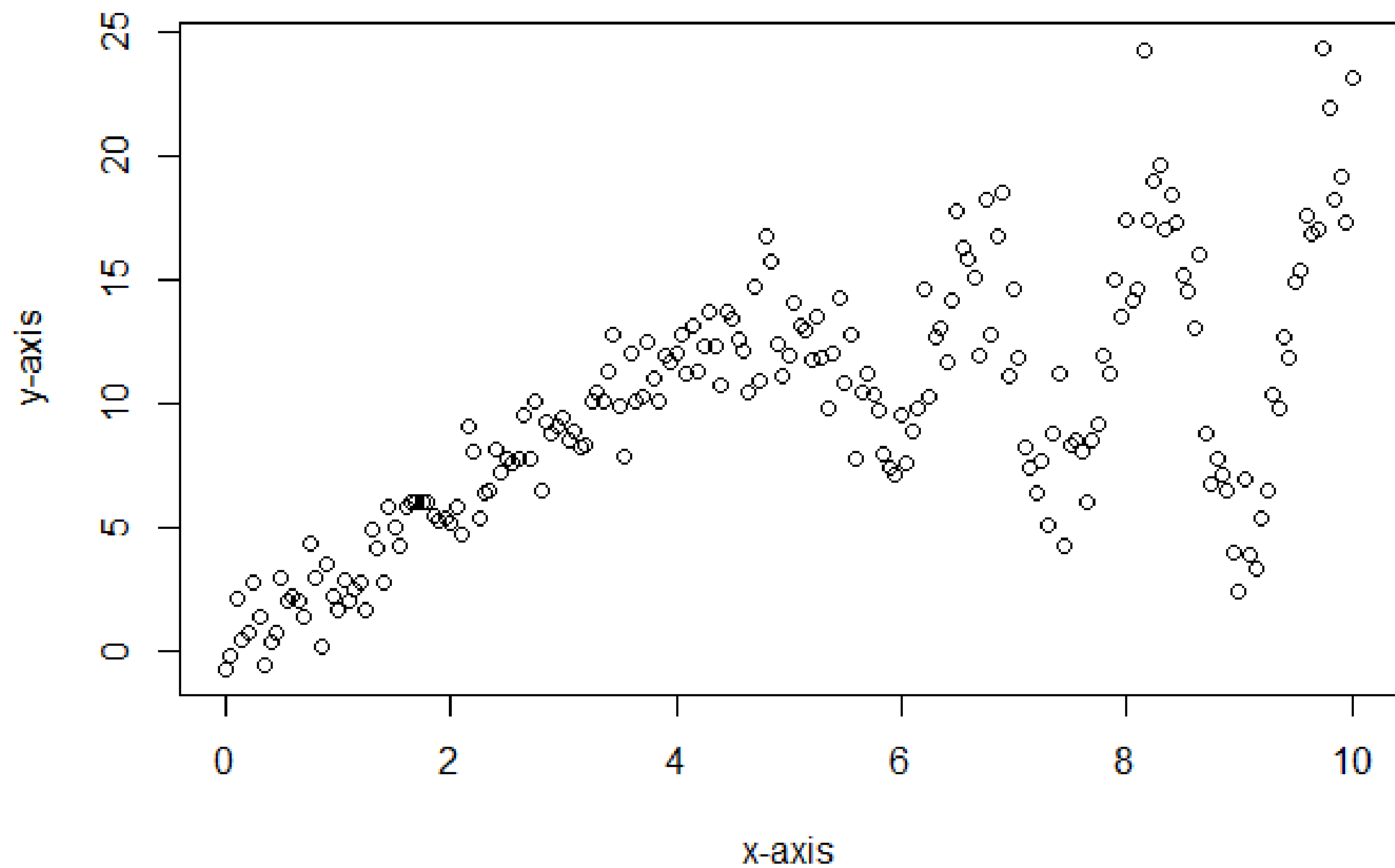
A Parametric Model

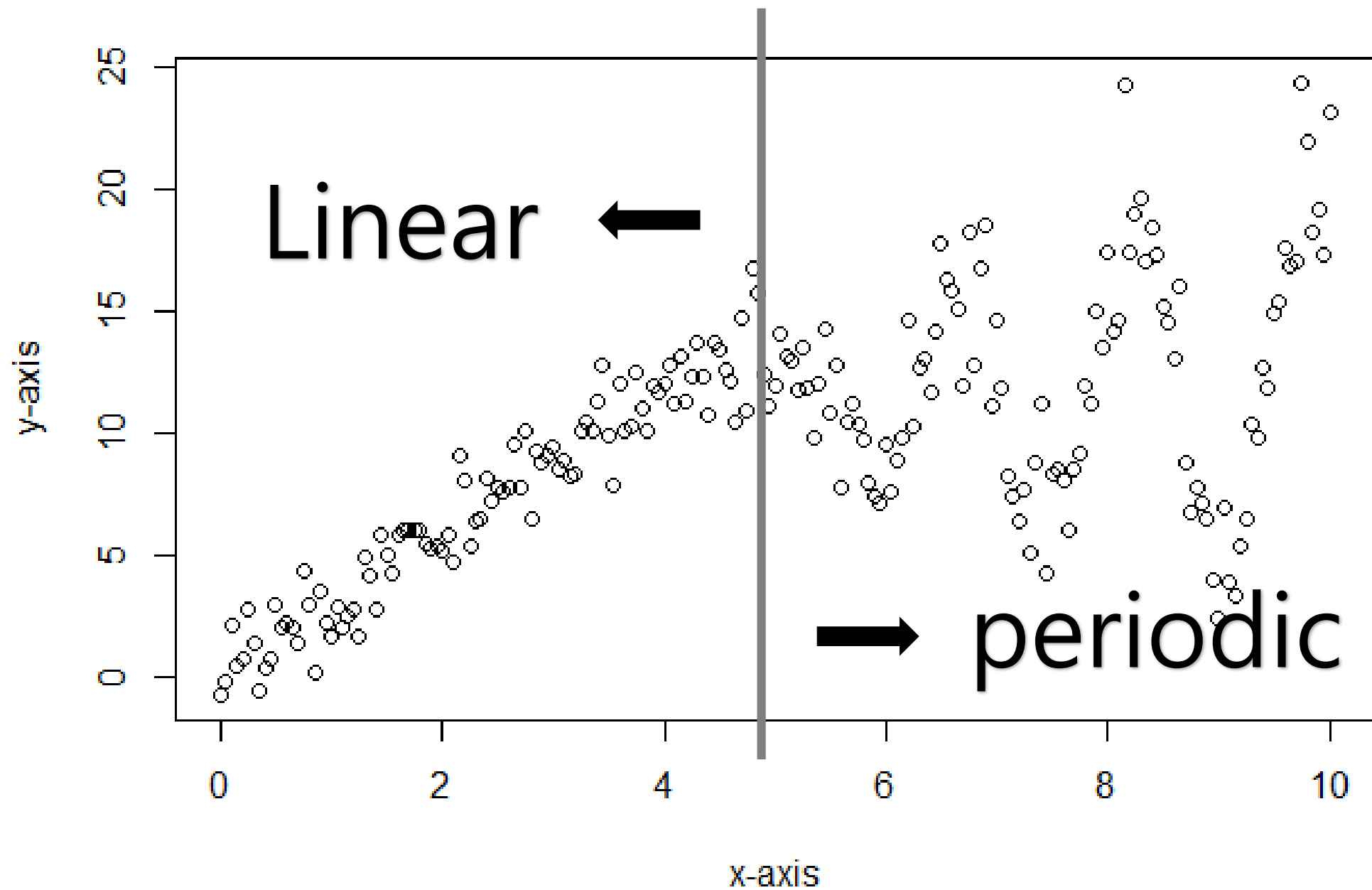
- can give us puzzled interpretations when a number of parameters are used although it has great prediction performance.
- can have the large variance of prediction by means of increasing the number of parameters.

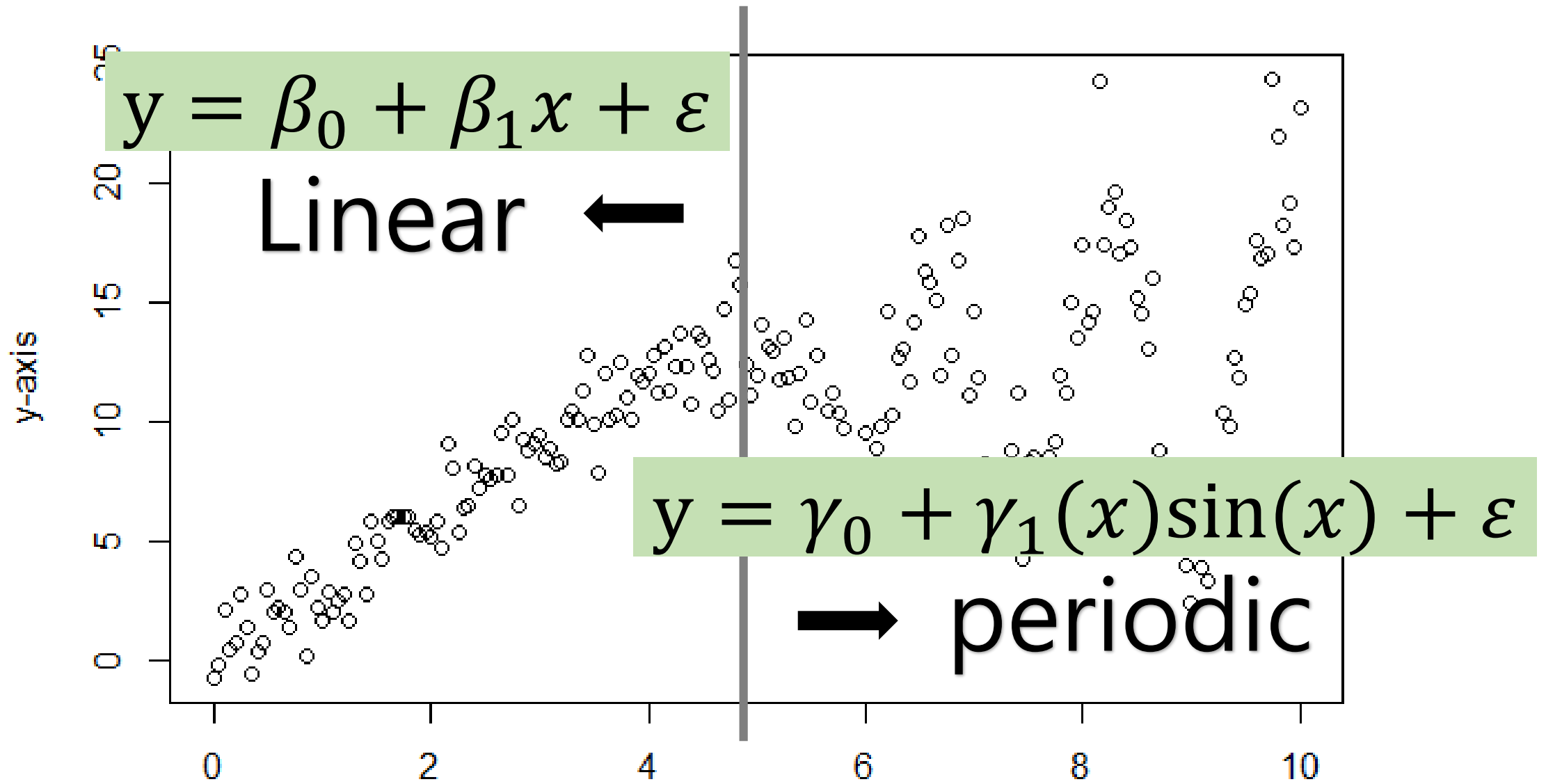
A Parametric Model

[Remedy]

- Find a parsimonious model.
- It can be partially achieved by finding proper features reducing the bias and variance.
- or by using more sophisticated parametric models.







Only four parameters are needed!

Raw data

Index	Y	X
1	-0.68	0.00
2	0.95	0.05
3	0.82	0.10
...
199	18.77	9.90
200	17.68	9.95
201	18.53	10.00

Feature engineering

Index	Y	X	$I(X > 4)$	$1.5(X - 4)\sin(X)$
1	-0.68	0.00	0	0.000
2	0.95	0.05	0	-1.17
3	0.82	0.10	0	-2.27
...
199	18.77	9.90	1	8.37
200	17.68	9.95	1	7.70
201	18.53	10.00	1	6.70

A Parametric model

- So, basically, our aim is to estimate pre-specified and finite model parameters to capture a signal avoiding overfitting and underfitting.
- via a parsimonious model as much as possible.
 - > Occam's razor.
- This can be substantially and normally achieved through **newly introduced features**, such as nonlinearity and interaction between variables. "Heart-of-DataScience"

A Parametric model

- If you succeed in finding those features and fitting to a proper model, not only to obtain great prediction accuracy but also to make inference about parameters can be achieved.
- However, this is so time-consuming that I think data science is a kind of 3D jobs. Think about when there are many parameters more than a few thousand!

A Nonparametric Model

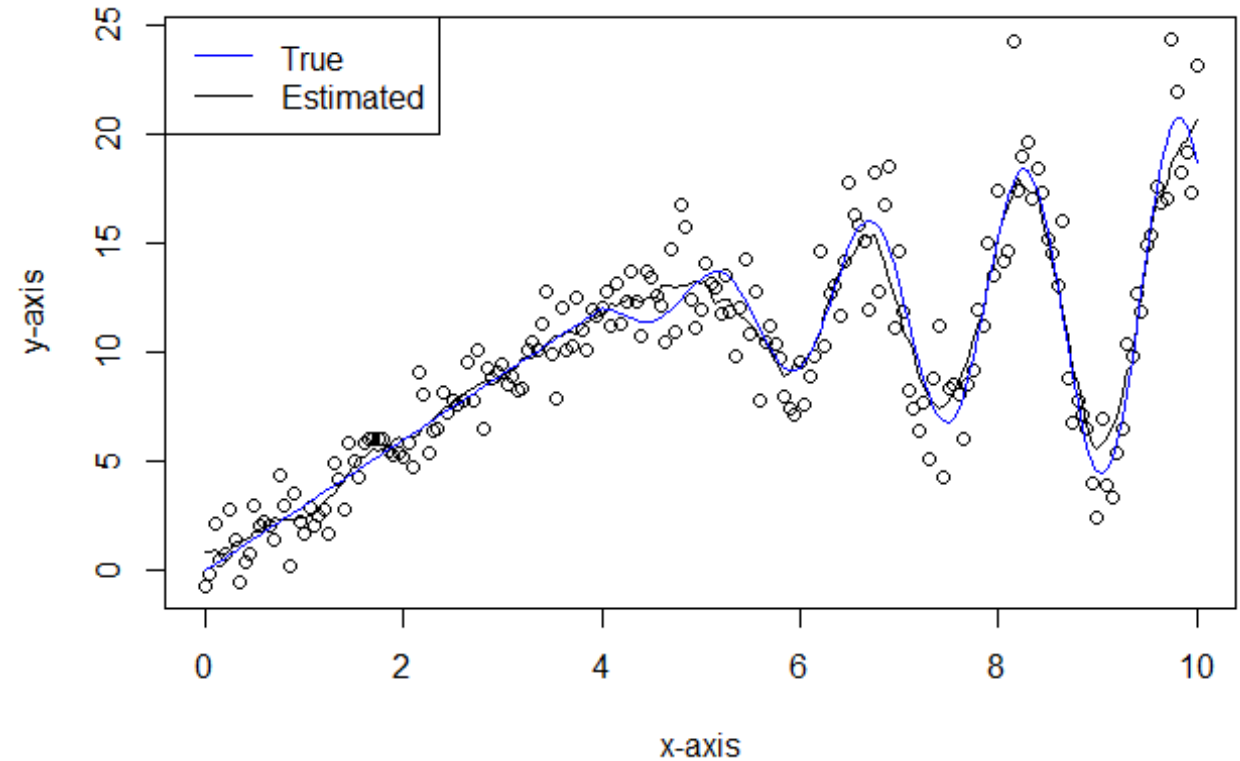
- When the data is too complex to be captured with finite number of parameters.
- When to find desired features is almost impossible.
 - > ex) $1.5(X-4)\sin(X)$.

A Nonparametric Model

- Unlike the parametric model, a nonparametric model assumes that there are infinite number of parameters, which implies that
- the model complexity grows with respect to the size of data,
- and a structure of the model generating data is not fixed a priori.
- but, the structure will be determined from the data instead.

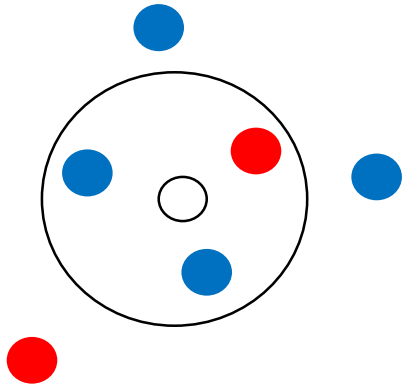
Kernel smoothing

$$K_h(x_{new}, x_i) = e^{-\left(\frac{x_{new} - x_i}{\sqrt{2}h}\right)^2}$$
$$\hat{y}_{new} = \sum_{i=1}^N \frac{K_h(x_{new}, x_i)}{\sum_{i=1}^N K_h(x_{new}, x_i)} y_i$$



- Kernel function represents the distance between two points.
- Imagine when h goes up or down.

K nearest neighborhood



- The empty circle will be filled with either blue or red color along with K.
- In this case, $K=3$.
- Euclidian distance, Mahalanobis distance can be chosen.

A Nonparametric Model

- Highly flexible model that better performances can be achieved.
- Requires too much computation along with the size of data.
- Generally, more difficult to distinguish the effects of the variables than the parametric model.
- Need to tune hyperparameter in some models, such as KNN.

Tuning parameters

Parameters

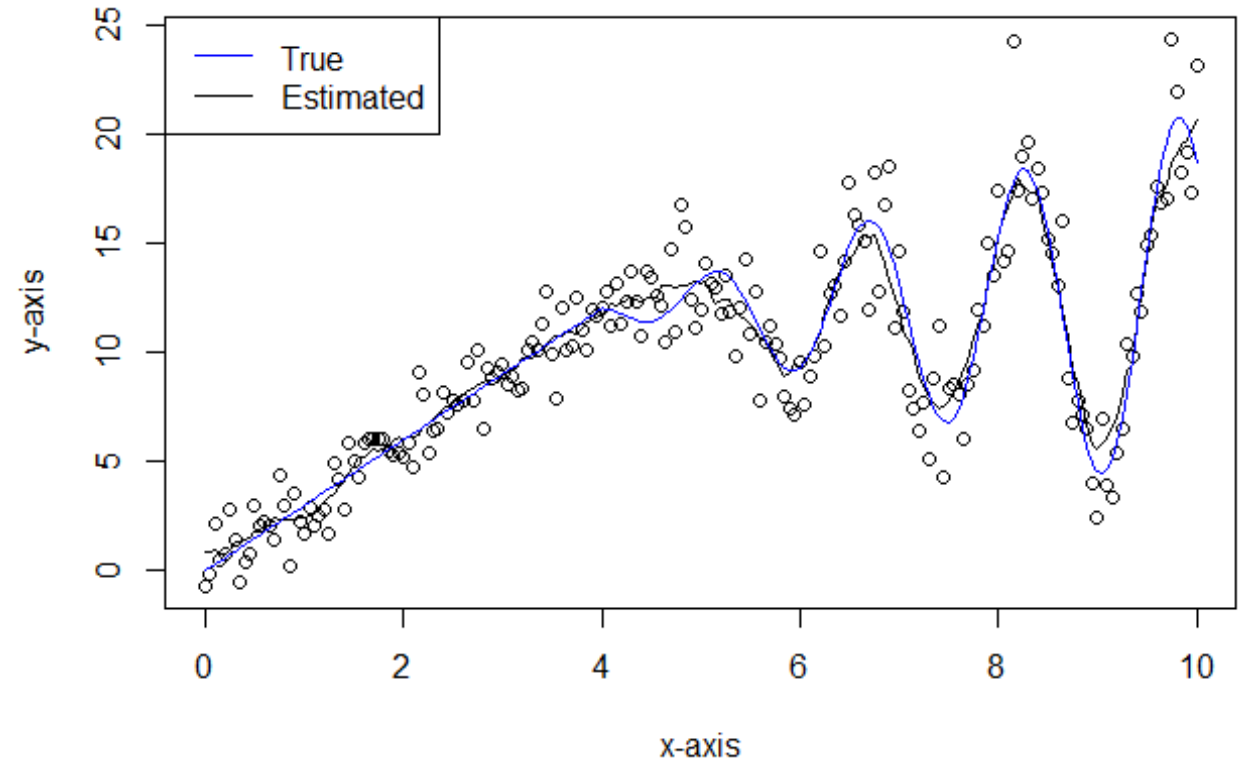
- Parameters : to be estimated (almost uniquely) with data, such as coefficients of regression, split points in tree, etc.
- Hyperparameters : can not be estimated with data. We have to specify proper values before constructing a model. It is also called as tuning parameters.

Tuning parameters

- Consider the previous kernel smoothing example.
- There is one tuning parameter, the bandwidth h .
- Simply, we can find the best tuning parameter, **using cross validation**.

Kernel smoothing

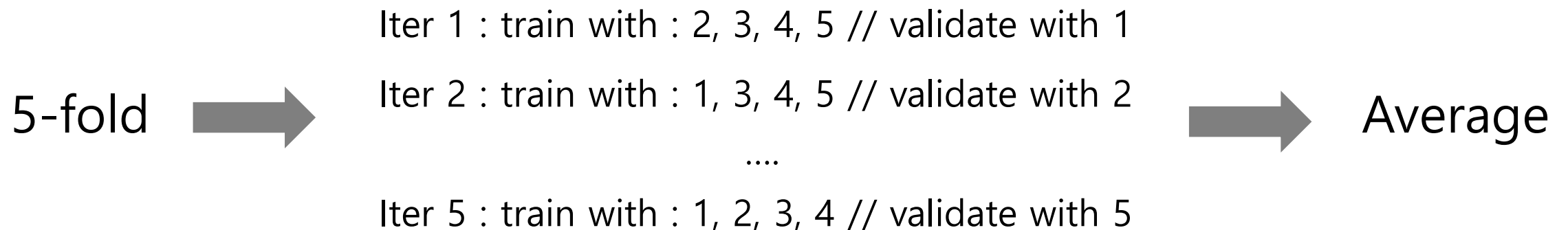
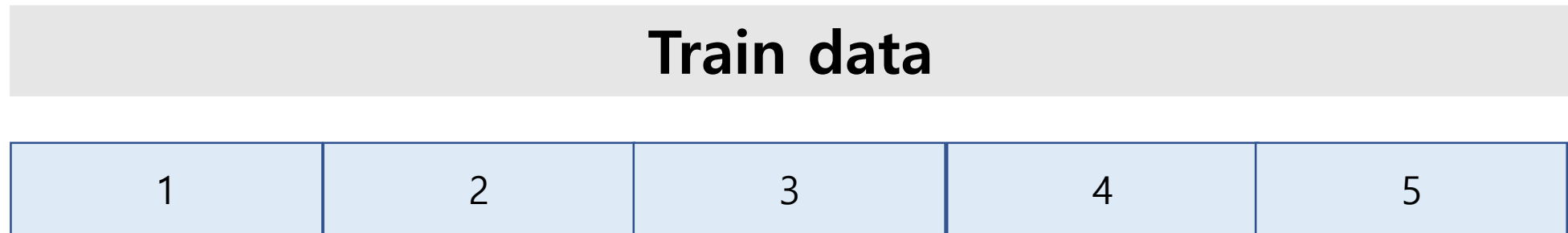
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$$\hat{y}_{new} = \sum_{i=1}^N \frac{K_h(x_{new}, x_i)}{\sum_{i=1}^N K_h(x_{new}, x_i)} y_i$$



- Kernel function represents the distance between two points.
- Imagine when h goes up or down.

K-fold cross-validation(K-CV)

- In cross-validation, we trust. <Kaggler>.
- Measuring performance



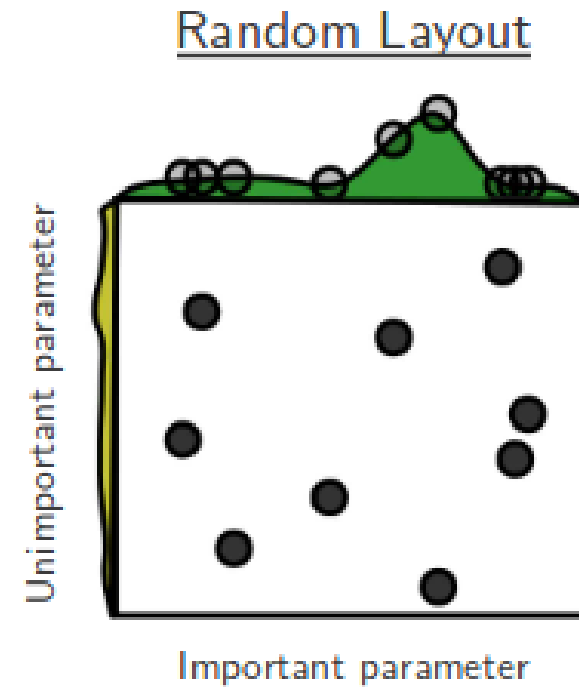
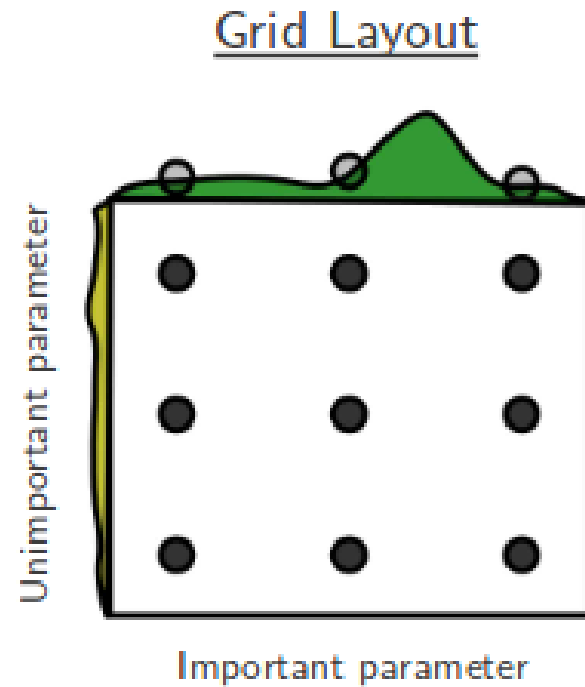
Leave One Out Cross Validation(LOOCV)

- When the size of data is big, it is computationally beneficial to make use of K-fold cross-validation.
- But, if you have small data, or the model requires only trivial computations, LOOCV might give you a plausible value.
- If you specify K as N, the number of observations, then N-fold, LOOCV, will be made.

Grid search vs Random search

- There are many tuning parameters. For examples, learning rate of neural network, sub-sampling ratio and the depth of tree of the tree based boosting, and bandwidth of kernel smoothing need to be adjusted.
- Unfortunately, performances of models heavily depend on the tuning parameters; therefore, we have to find the optimal parameter among a list of the parameters.
- K-fold cross-validation is a major gadget for comparing the parameters in the list.

Grid search vs Random search



Summary

- First of all, by doing EDA, you have to understand data thoroughly.
- Finding a proper models to catch a structure what data shows. Almost all of models you want can be found in Google.

Summary

- Most of the problems you will encounter are either causal inference or prediction.
- You must look for the best model to avoid overfitting and underfitting with appropriate feature engineering.
- If you are able to estimate pre-specified parameters well, the parametric model would outperform the nonparametric model in view of prediction.

Questions...

(Q) How does a Deep Neural Network model reach great prediction accuracy?

(Q) Why overfitting is a principal concern in DNN based models

(Q) Isn't it unnecessary to consider new features when we decide to use a nonparametric model?

(Q) What is the difference between multiple linear regression and regression tree in terms of interaction?

Questions...

(Q) How to decide the number of fold in K-fold cross-validation when data is too big or has highly imbalanced label?

(Q) How to measure performances when the computation is so burdensome for fitting a model actually? For instance, imagine that fitting a model with a set of parameters require the day.