2019 Spring Mathematical Statistics I: Home Work 2

1. (Casella and Berger) Exercise 5.3, 5.8, 5.18, 5.35

2. (Casella and Berger) Exercise 6.3, 6.9(c,d,e only), 6.16, 6.22, 6.30

3. (Casella and Berger) Exercise 7.3, 7.6, 7.9, 7.12, 7.37, 7.59, 7.60

4. Suppose that $X_1, X_2, \ldots, X_n$ are iid with a common pdf

$$f(x \mid \theta) = \frac{\gamma(x)\theta^x}{c(\theta)}, x = 0, 1, 2, \ldots,$$

   where $\gamma(x) \geq 0$, $\theta > 0$, and $c(\theta) = \sum_{x=0}^{\infty} \gamma(x)\theta^x$. The above is known as a power series distribution.

   (a) Show that $\{f(x \mid \theta) : \theta > 0\}$ is an exponential family.

   (b) Let $T = \sum_{i=1}^{n} X_i$. Show that $T$ has a power series distribution with pmf

$$f_n(t \mid \theta) = \frac{\gamma_n(t)\theta^t}{\{c(\theta)\}^n}, t = 0, 1, 2, \ldots,$$

   where $\gamma_n(t)$ is the coefficient of $\theta^t$ in the expansion $\{c(\theta)\}^n = \sum_{t=0}^{\infty} \gamma_n(t)\theta^t$.

   (c) For a fixed $r \geq 0$, show that the UMVUE of $\theta^r$ is

$$h(T) = \begin{cases} 0 & \text{if} \quad T < r \\ \frac{\gamma_n(T-r)}{\gamma_n(T)} & \text{Otherwise.} \end{cases}$$

5. Assume that $X_1, X_2, \ldots, X_n$ are iid with a pdf

$$f(x \mid \mu, \sigma) = \begin{cases} \frac{1}{\sigma} \exp\left(-\frac{(x-\mu)}{\sigma}\right) & \text{if} \quad x \geq \mu \\ 0 & \text{Otherwise,} \end{cases}$$

   where $\mu \in \mathcal{R}$ and $\sigma > 0$.

   (a) Find the MLE of $(\mu, \sigma)$.

   (b) Show that the MLEs of $\mu$ and $\sigma$ are consistent estimators.