March 14, 2019

1 Introduction

The Metropilis-Hastings(MH) algorithm simulates from a probability distribution by making use of the full joint density function and proposal distributions

- find $p(x|y)$ where $\pi(x)p(y|x) = \pi(y)p(x|y)$
- under some condition $p(x|y)$ converge to $\pi(x)$

$$p(x|y) = \alpha(x|y)q(x|y)$$

- $q(x|y)$ is proposal distribution.
- $\alpha(x|y)$ is acceptance probability

$$\pi(x)p(y|x) = \pi(y)q(x|y)$$
$$\pi(x)\alpha(y|x)q(y|x) = \pi(y)\alpha(x|y)q(x|y)$$

if $\pi(x)q(y|x) > \pi(y)q(x|y)$

$$\alpha(x|y) = 1$$
$$\alpha(y|x) = \frac{\pi(y)q(x|y)}{\pi(x)q(y|x)}$$

2 MH

Initailize $x^{(0)} \sim q(x)$ and repeat the Process during the iteration

- Propose $x^{cand} \sim q(x^{(i)}|x^{(i-1)})$
- Aceeptance Prob $\alpha(x^{(cand)}|x^{(i-1)}) = min(1, \frac{\pi(x^{(cand)})q(x^{(i-1)}|x^{(cand)})}{\pi(x^{(i-1)})q(x^{(cand)}|x^{(i-1)})})$
- $u \sim Unif(u; 0, 1)$
- if $u > \alpha$ Accept the Prob $x^{(i)} = x^{(cand)}$
- else reject the Prob $x^{(i)} = x^{(i-1)}$