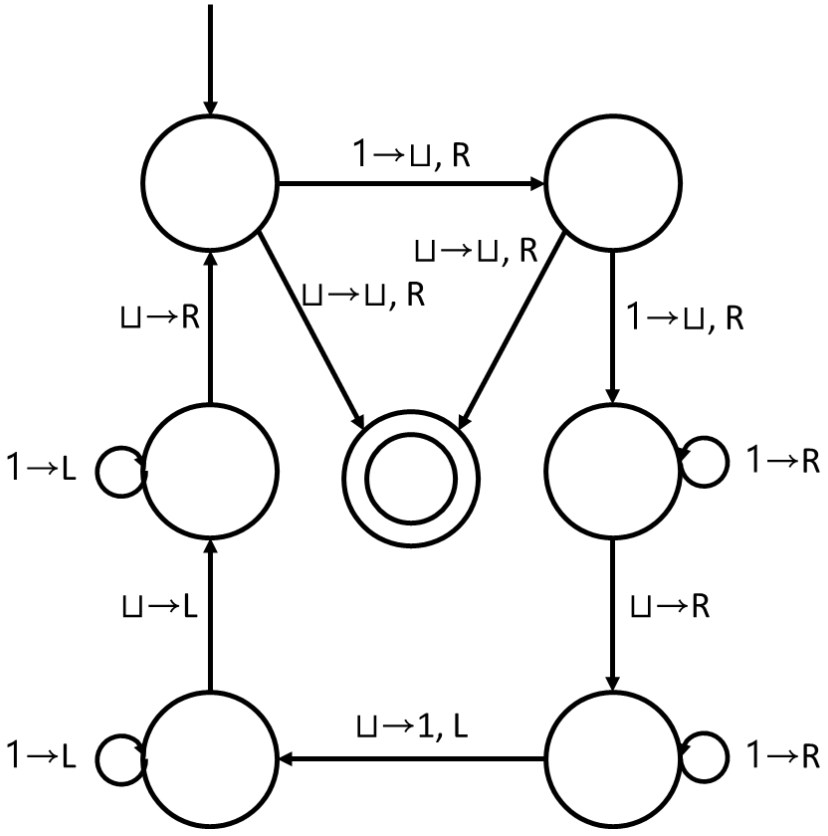
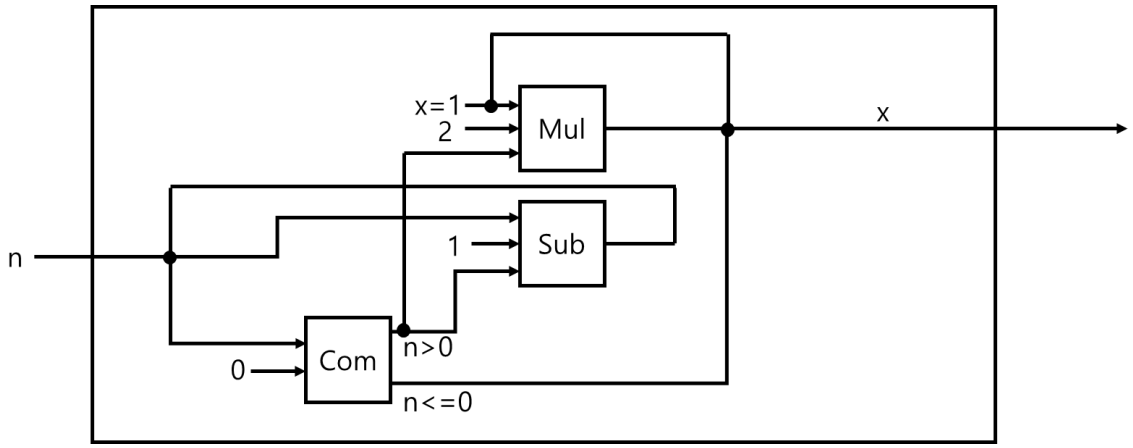
Assignment 6

1. Let's say $w = a^q b^0$, where $q \geq p$(p is pumping length), q is prime. $w \in L$ since q is prime.
   - Since $b^0 = \lambda$ and $|vy| \geq 1$, vxy should consists of only a. let's say $|vy| = k$. Then $|uv^i xy^i z| = q - k + ik = q + k(i-1)$
   - If we choose $i = q + 1$, $|uv^{q+1} xy^{q+1} z| = q + qk = q(k+1)$. Since q is prime, $q > 1$. Since $k = |vy| \geq 1$, $k + 1 > 1$. Therefore, $q(k+1)$ is composite number.
   - By proof by contradiction, L is not satisfying pumping lemma for CFL. Therefore, L is not context-free.
2. We should reduce the number of 1s to half. That means output has one 1 per two input 1. When x is odd, we can just drop one 1.



3. For simplicity, I assume that $n \in \mathbb{N}$. Initially, it compare n with 0. If $n \leq 0$, it returns x(the initial value of $x = 1$). Else, it subtracts 1 from n and multiply x by

2. Looping this will lead to 2^n.



4. Two-tape Turing machine is Turing machine that has $\delta : Q \times \Gamma \times \Gamma \rightarrow Q \times \Gamma \times \Gamma \times \{L, R\} \times \{L, R\}$.
   - We can define Two-tape Turing machine with stay option for convience.
     - $\delta : Q \times \Gamma \times \Gamma \rightarrow Q \times \Gamma \times \Gamma \times \{L, R, S\} \times \{L, R, S\}$