

NCDL

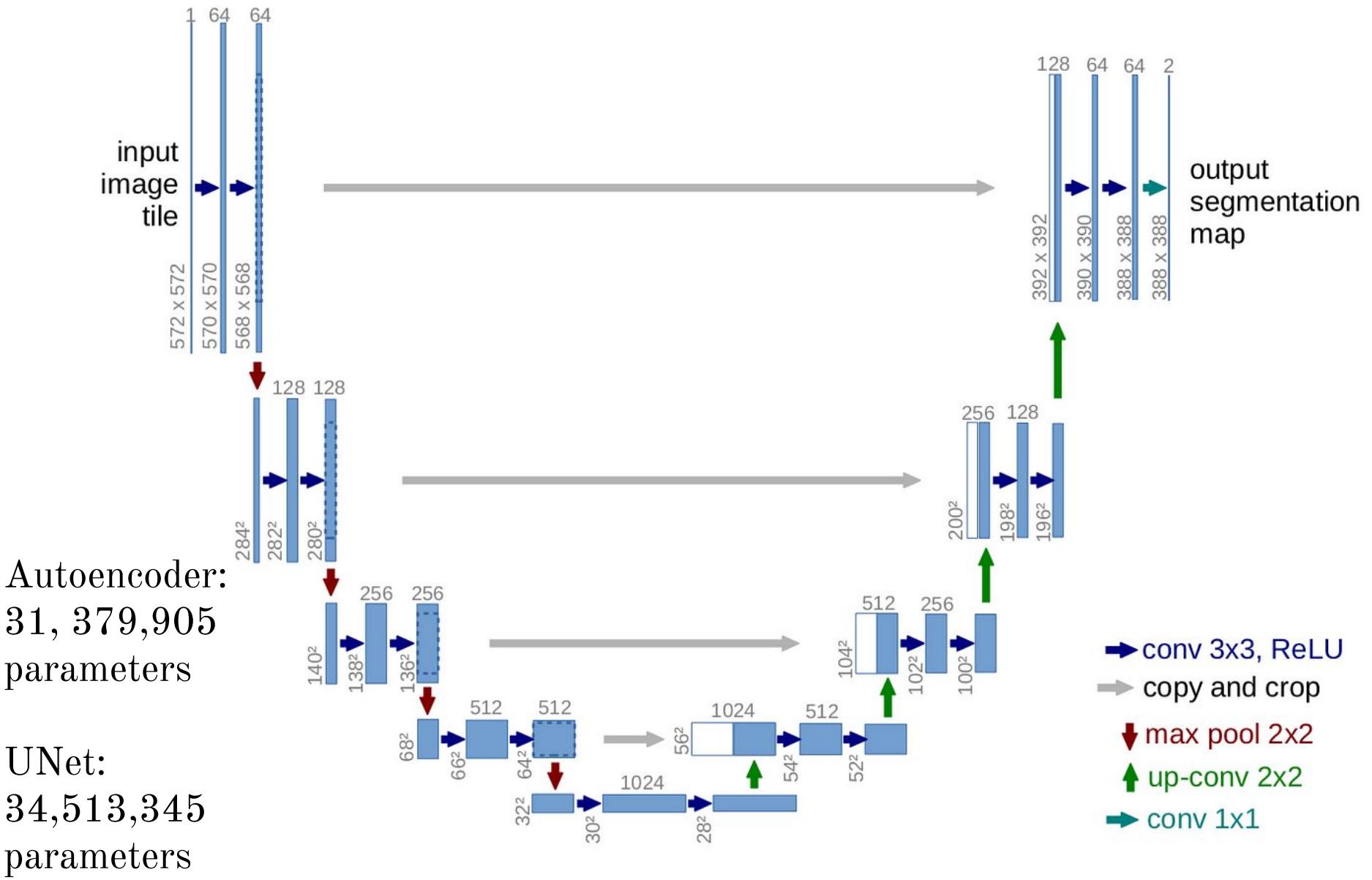
NCDL: A Framework for Deep Learning on non-Cartesian Lattices

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Sua Lim

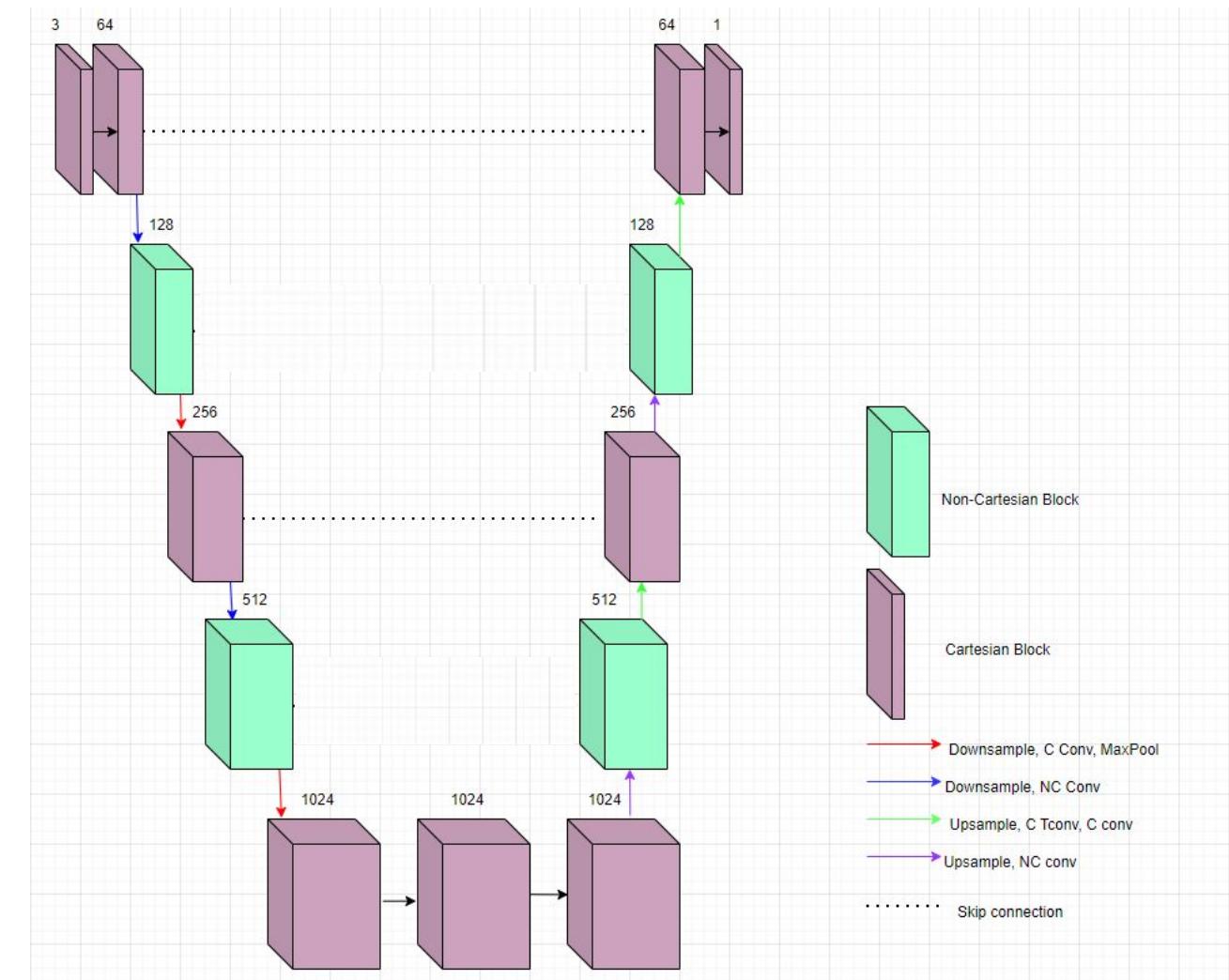
sua.lim@ucalgary.ca

Cartesian UNet

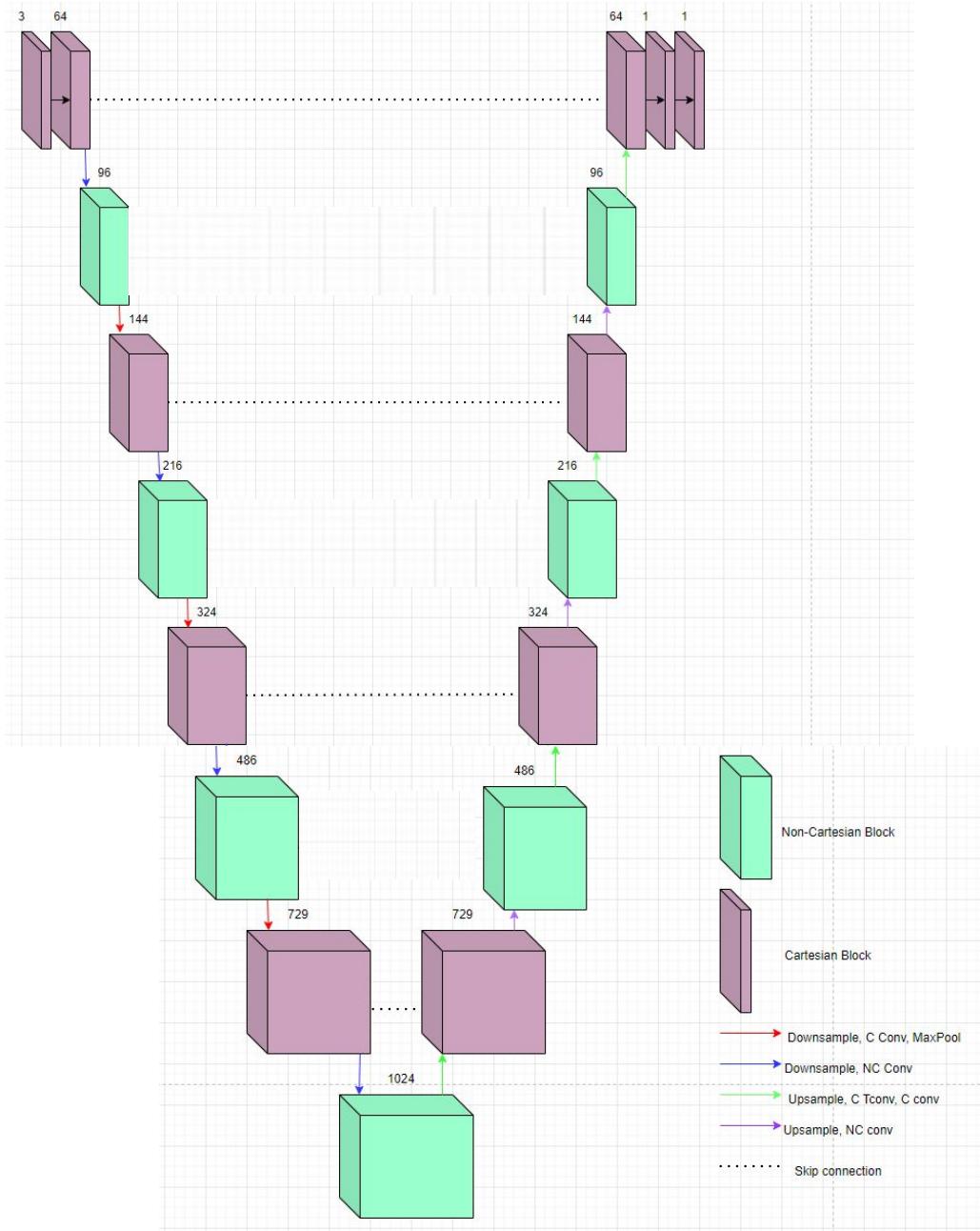


NC Case1

Autoencoder: 32,051,161
UNet: 32,677,849

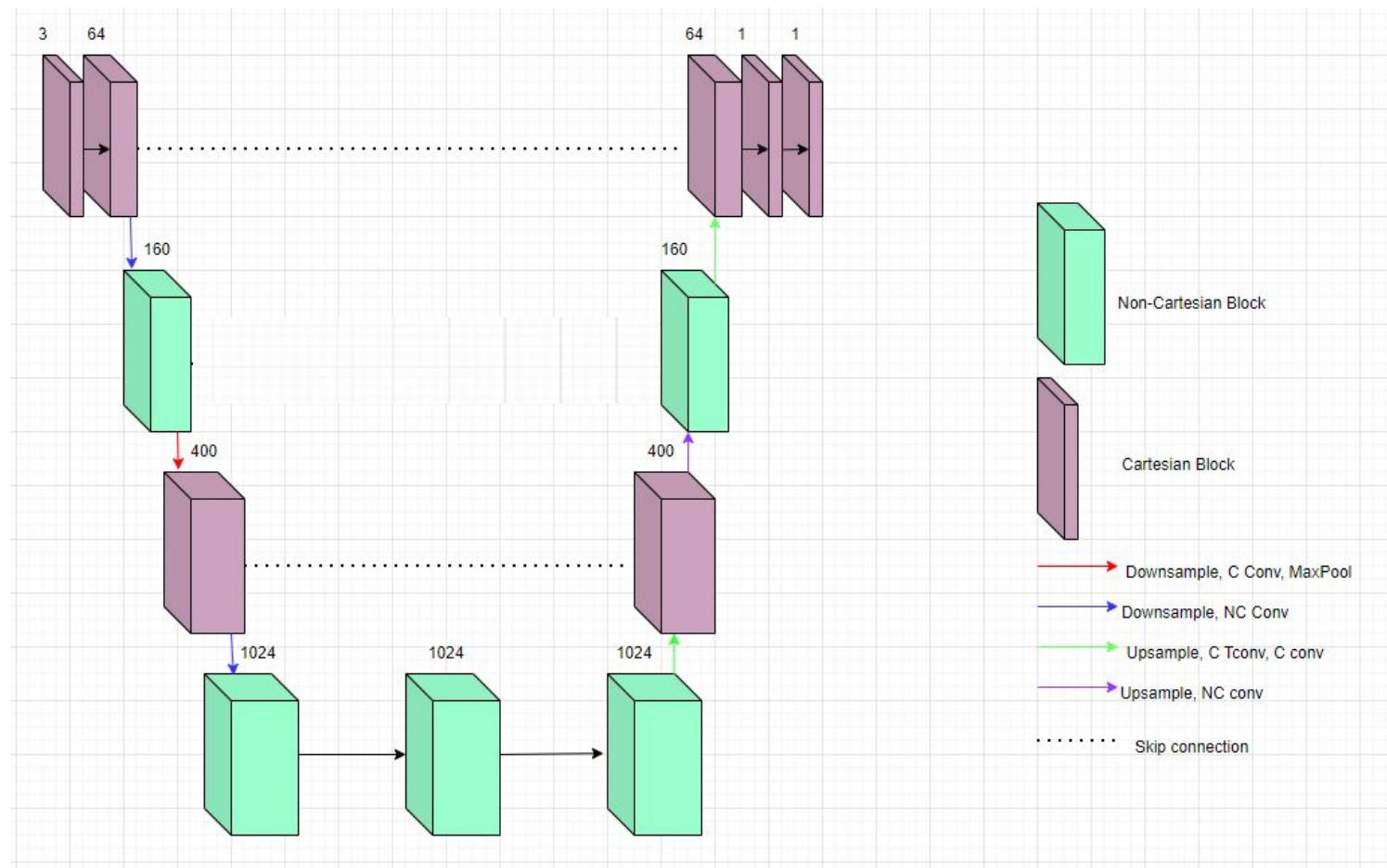


NC Case2



Autoencoder: 37,383,662
UNet: 43,334,903

NC Case3



Autoencoder:
32,011,582

UNet: 33,488,446

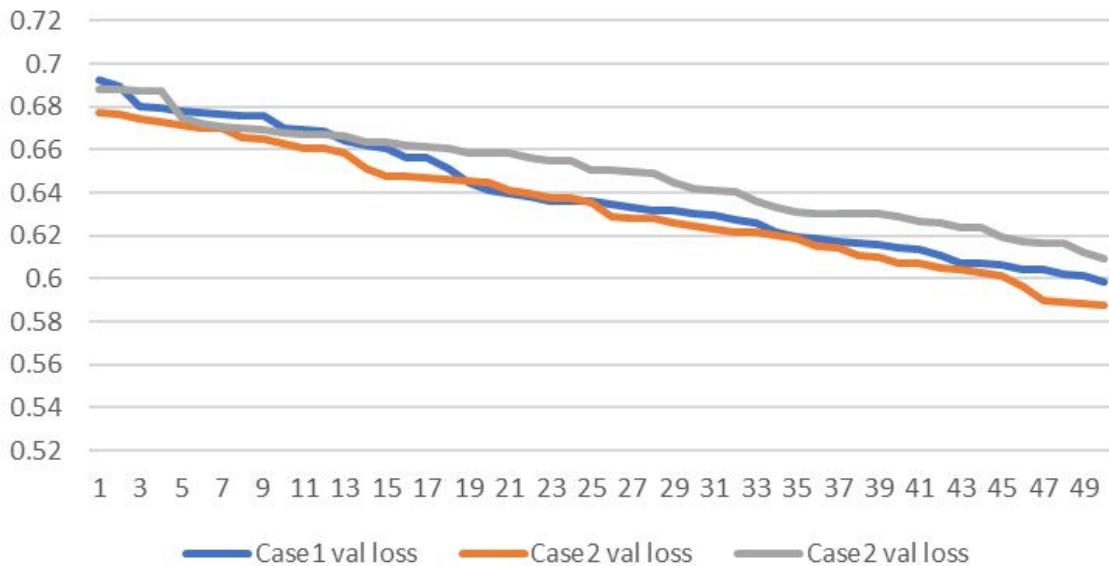
DUTS dataset

DUTS is a saliency detection dataset containing 10,553 training images and 5,019 test images. All training images are collected from the ImageNet DET training/val sets, while test images are collected from the ImageNet DET test set and the SUN data set.

All experiments that follow are performed using the DUTS dataset unless specified otherwise.

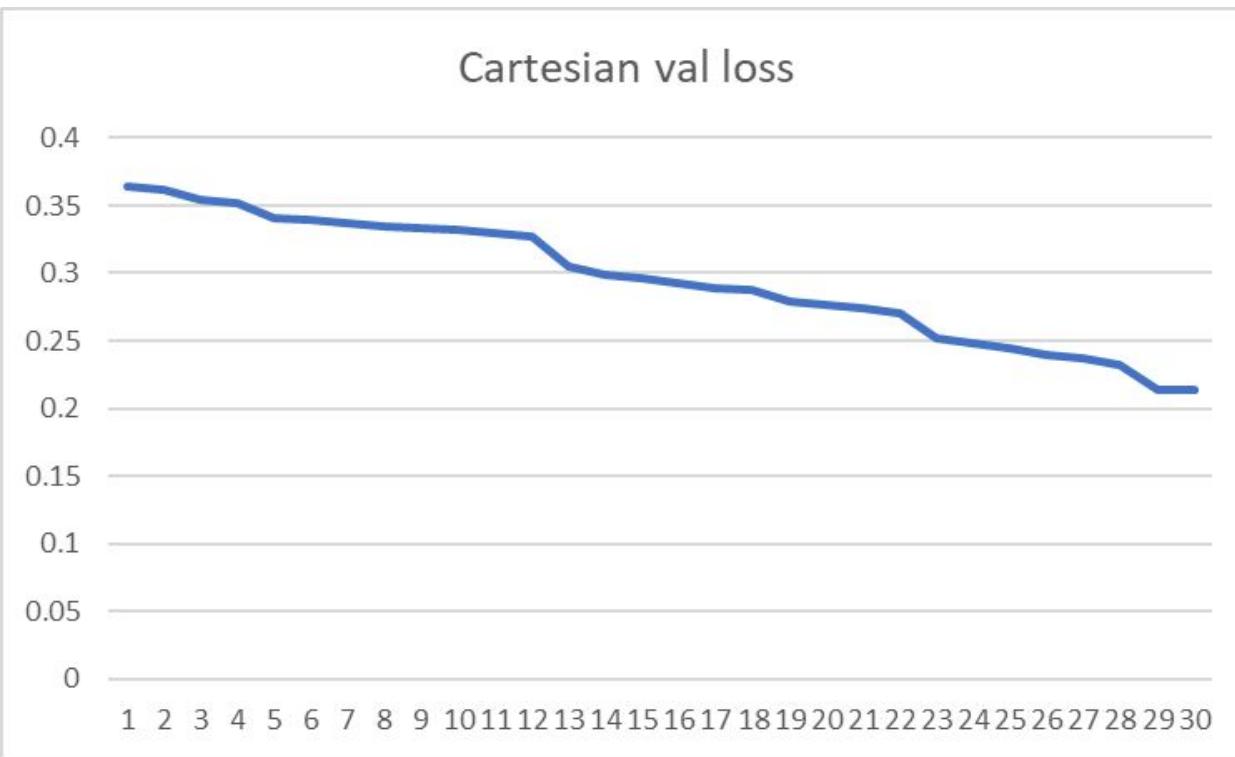
NC Autoencoder (BCE)

NC Autoencoder (BCE)



Epoch	Case 1 val loss	Case 2 val loss	Case 3 val loss
1	0.692891442323748	0.6781190718519744	0.6886611459944485
2	0.6922797485550433	0.6680757712413914	0.6877761977543763
3	0.6913338044243401	0.6550593582939762	0.6896446849497574
4	0.6898330893561738	0.6518252187995549	0.6777101601469573
5	0.6886192338161559	0.6465902863520582	0.6724633028721922
6	0.6856388229329439	0.6404902555366263	0.6723027311230158
7	0.683784945259727	0.6407582219178077	0.6769218658948961
8	0.6812743138927984	0.6390087971755114	0.6652207074572125
9	0.6782407219941017	0.6310979992857476	0.6614297773035782
10	0.6759951538384243	0.635627847542695	0.6643176481622091
40	0.605245274277095	0.5978248525004817	0.616586031167993
41	0.601907763944179	0.5932154798959669	0.6151689945239026
42	0.5984780654523045	0.594272436575867	0.6128663325761732
43	0.6003159915666445	0.5949065751939023	0.6105988507022225
44	0.5845931005703895	0.589532930195614	0.6095818414507319
45	0.5957667992013326	0.5866500829068405	0.6187313059494959
46	0.5945065446374541	0.5850211736715235	0.6094397872545143
47	0.6199039260358042	0.5858791342278793	0.6134838202553338
48	0.5919327182227402	0.582452431052782	0.6126026303282281
49	0.5908534090665845	0.5831370559913852	0.6173925968143047
50	0.5987716884409647	0.5801957235517095	0.608599875267083

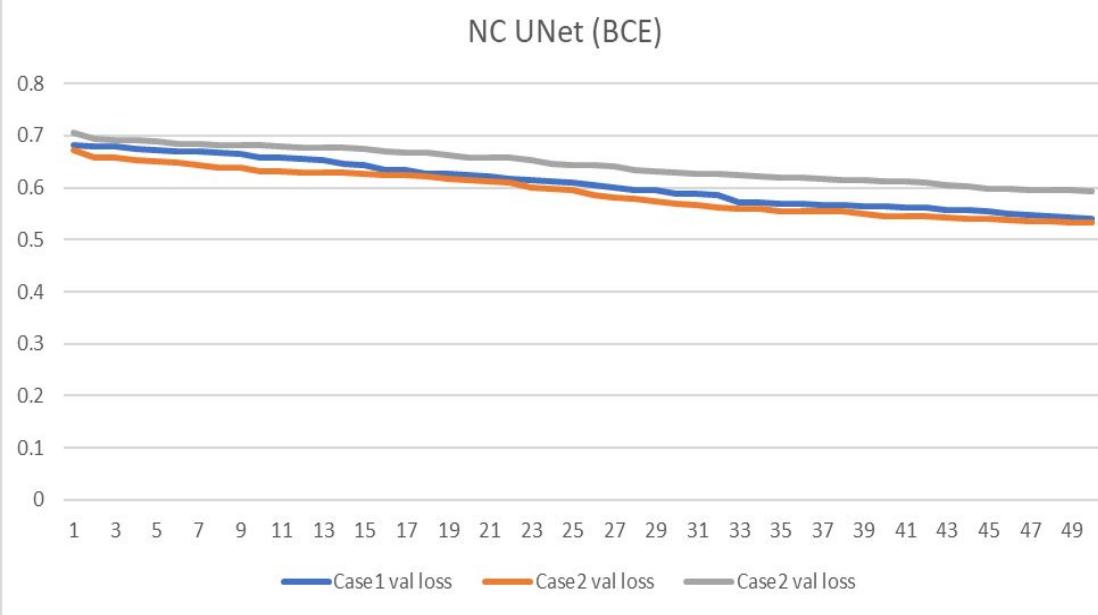
Cartesian Autoencoder (BCE)



* Only 30 epochs for Cartesian model as overfitting was observed with more epochs.

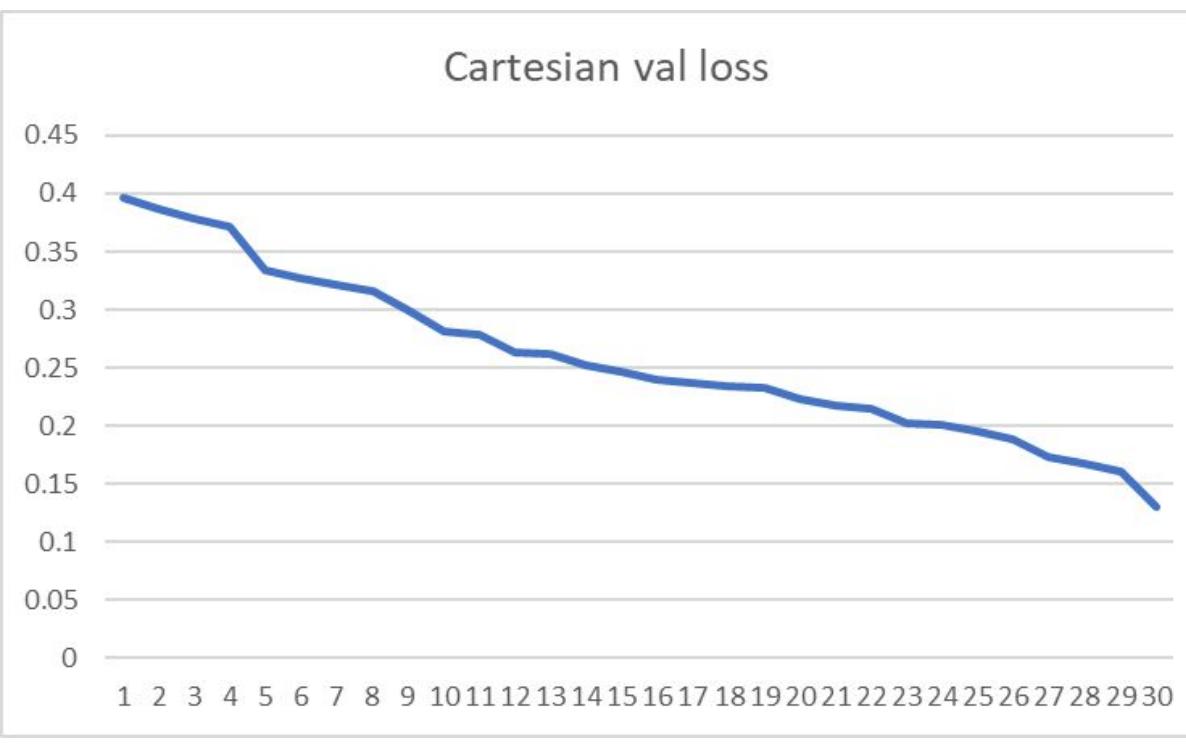
Epoch 1/30 - Train loss: 0.4818 - val_loss: 0.3648
Epoch 2/30 - Train loss: 0.3530 - val_loss: 0.2891
Epoch 3/30 - Train loss: 0.3043 - val_loss: 0.2885
Epoch 4/30 - Train loss: 0.2900 - val_loss: 0.2576
Epoch 5/30 - Train loss: 0.2777 - val_loss: 0.2460
Epoch 6/30 - Train loss: 0.2610 - val_loss: 0.2607
Epoch 7/30 - Train loss: 0.2628 - val_loss: 0.2406
Epoch 8/30 - Train loss: 0.2764 - val_loss: 0.2377
Epoch 9/30 - Train loss: 0.2524 - val_loss: 0.2610
Epoch 10/30 - Train loss: 0.2696 - val_loss: 0.2763
Epoch 11/30 - Train loss: 0.2721 - val_loss: 0.2377
Epoch 12/30 - Train loss: 0.2496 - val_loss: 0.2299
Epoch 13/30 - Train loss: 0.2477 - val_loss: 0.2274
Epoch 14/30 - Train loss: 0.2586 - val_loss: 0.2220
Epoch 15/30 - Train loss: 0.2414 - val_loss: 0.2256
Epoch 16/30 - Train loss: 0.2401 - val_loss: 0.2250
Epoch 17/30 - Train loss: 0.2332 - val_loss: 0.2197
Epoch 18/30 - Train loss: 0.2296 - val_loss: 0.2185
Epoch 19/30 - Train loss: 0.2295 - val_loss: 0.2087
Epoch 20/30 - Train loss: 0.2346 - val_loss: 0.2458
Epoch 21/30 - Train loss: 0.2670 - val_loss: 0.2158
Epoch 22/30 - Train loss: 0.2341 - val_loss: 0.2079
Epoch 23/30 - Train loss: 0.2337 - val_loss: 0.2144
Epoch 24/30 - Train loss: 0.2334 - val_loss: 0.2061
Epoch 25/30 - Train loss: 0.2292 - val_loss: 0.2603
Epoch 26/30 - Train loss: 0.2621 - val_loss: 0.2190
Epoch 27/30 - Train loss: 0.2282 - val_loss: 0.2031
Epoch 28/30 - Train loss: 0.2346 - val_loss: 0.2526
Epoch 29/30 - Train loss: 0.2666 - val_loss: 0.2129
Epoch 30/30 - Train loss: 0.2516 - val_loss: 0.2035

NC UNet (BCE)



Epoch	Case 1 val loss	Case 2 val loss	Case 3 val loss
1	0.6924034379104868	0.6736010493825397	0.706948513206892
2	0.6919348846679615	0.671023808836372	0.6951754524244517
3	0.6913488582412214	0.6598892893836397	0.6864141084006612
4	0.6904411724958375	0.6578154653169532	0.676285960775981
5	0.6755208277589337	0.6479755990878101	0.6631962621381498
6	0.6502093516254877	0.636999129006083	0.6585267634753368
7	0.641464809953319	0.63218382212788	0.6534872777654096
8	0.6361966625209103	0.6271389691750585	0.6478626238226326
9	0.6236901278179404	0.6236722893624509	0.6459828351346237
10	0.6190246945873821	0.620361716600391	0.6364435931517615
40	0.549245764519931	0.5478039884454267	0.6024030343616178
41	0.5515387091591459	0.5467016193539045	0.5948767558658292
42	0.5564489532420985	0.5421510582851573	0.5883153380375903
43	0.5501125772417439	0.5445190364715612	0.6059186610000393
44	0.5530069298653806	0.5414515405469597	0.6024739432108911
45	0.5500479024733412	0.5388207753687674	0.5926474006255091
46	0.5482052255015802	0.5376719341458868	0.6035217638264335
47	0.5490854248616368	0.5357156414556278	0.6000345565696463
48	0.5421522650673491	0.5333915553386742	0.589533561496373
49	0.5389244341737286	0.5317166313175906	0.5931143404748203
50	0.5396245856420688	0.5294875269817515	0.5903497633775828

Cartesian UNet (BCE)



* Only 30 epochs for Cartesian model as overfitting was observed with more epochs.

Epoch 1/30 - Train loss: 0.7719 - val_loss: 0.4036
Epoch 2/30 - Train loss: 0.3815 - val_loss: 0.3309
Epoch 3/30 - Train loss: 0.3228 - val_loss: 0.2659
Epoch 4/30 - Train loss: 0.2645 - val_loss: 0.2443
Epoch 5/30 - Train loss: 0.2435 - val_loss: 0.2174
Epoch 6/30 - Train loss: 0.2292 - val_loss: 0.2222
Epoch 7/30 - Train loss: 0.2219 - val_loss: 0.2002
Epoch 8/30 - Train loss: 0.2123 - val_loss: 0.1969
Epoch 9/30 - Train loss: 0.2054 - val_loss: 0.1887
Epoch 10/30 - Train loss: 0.1951 - val_loss: 0.1729
Epoch 11/30 - Train loss: 0.1878 - val_loss: 0.1776
Epoch 12/30 - Train loss: 0.1844 - val_loss: 0.1684
Epoch 13/30 - Train loss: 0.1834 - val_loss: 0.1747
Epoch 14/30 - Train loss: 0.1807 - val_loss: 0.1794
Epoch 15/30 - Train loss: 0.1755 - val_loss: 0.1591
Epoch 16/30 - Train loss: 0.1748 - val_loss: 0.1608
Epoch 17/30 - Train loss: 0.1728 - val_loss: 0.1467
Epoch 18/30 - Train loss: 0.1669 - val_loss: 0.1585
Epoch 19/30 - Train loss: 0.1671 - val_loss: 0.1500
Epoch 20/30 - Train loss: 0.1585 - val_loss: 0.1455
Epoch 21/30 - Train loss: 0.1635 - val_loss: 0.1554
Epoch 22/30 - Train loss: 0.1591 - val_loss: 0.1540
Epoch 23/30 - Train loss: 0.1569 - val_loss: 0.1474
Epoch 24/30 - Train loss: 0.1539 - val_loss: 0.1449
Epoch 25/30 - Train loss: 0.1572 - val_loss: 0.1518
Epoch 26/30 - Train loss: 0.1535 - val_loss: 0.1372
Epoch 27/30 - Train loss: 0.1474 - val_loss: 0.1404
Epoch 28/30 - Train loss: 0.1457 - val_loss: 0.1366
Epoch 29/30 - Train loss: 0.1486 - val_loss: 0.1384
Epoch 30/30 - Train loss: 0.1457 - val_loss: 0.1306

BCE & L1 Loss function

$$\text{BCE}(y, \hat{y}) = -[y \log(\hat{y}) + (1 - y) \log(1 - \hat{y})]$$

where:

- y is the true label (0 or 1).
- \hat{y} is the predicted probability (between 0 and 1).

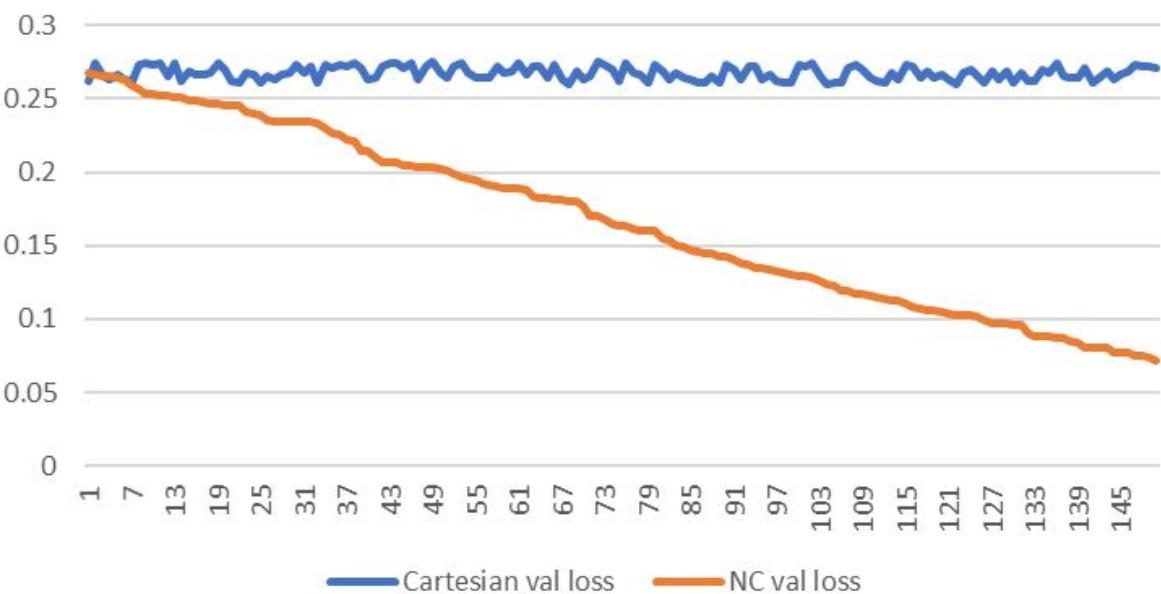
$$\text{MAE}(y, \hat{y}) = |y - \hat{y}|$$

where:

- y is the true continuous value.
- \hat{y} is the predicted continuous value.

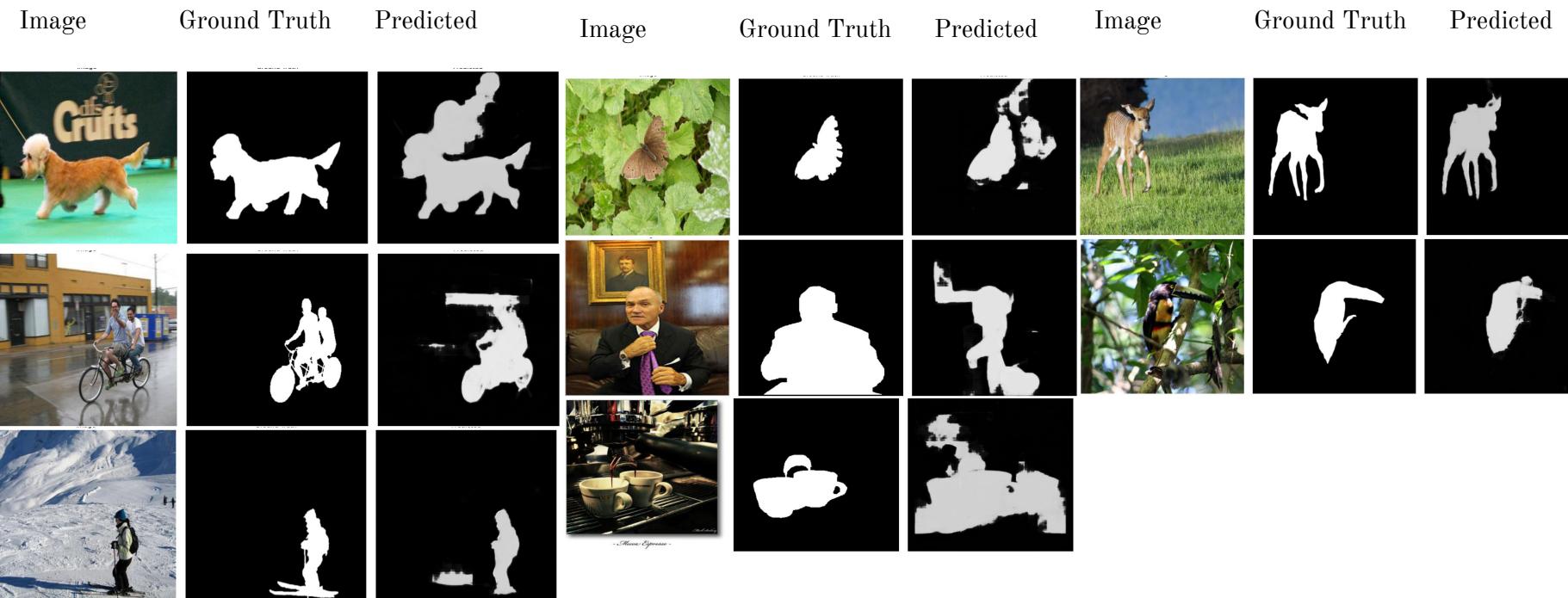
Comparison: NC UNet Case1 & C UNet (L1)

NC UNet Case 1 & C UNet (L1)

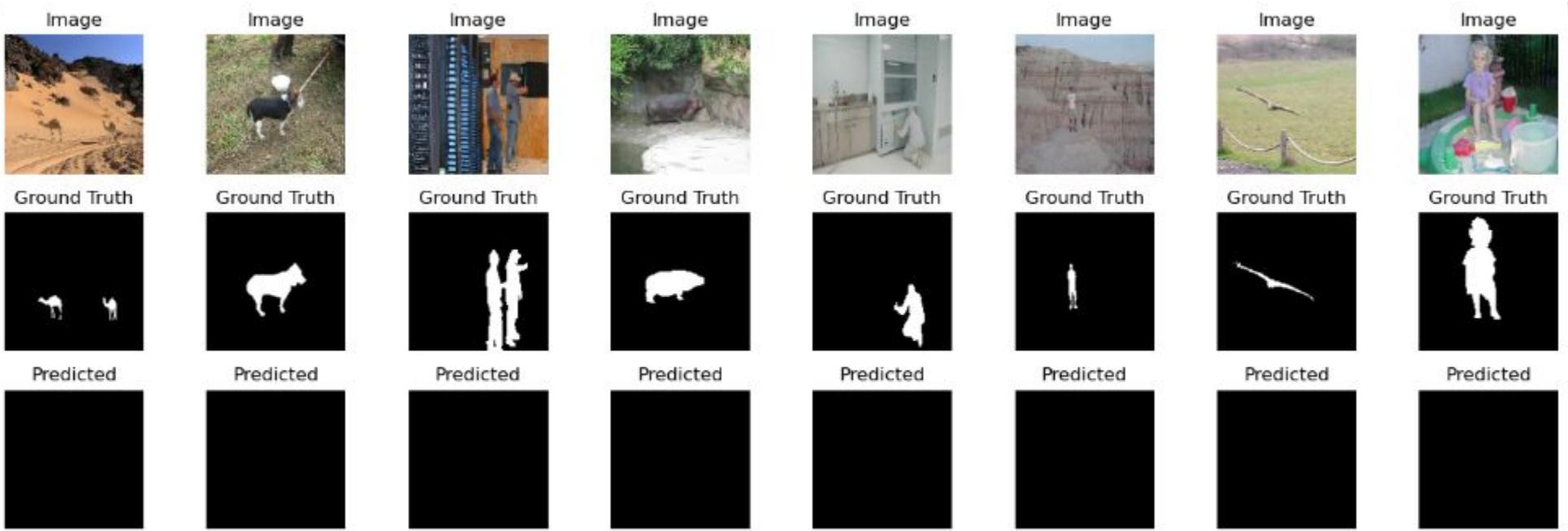


Epoch	Cartesian val loss	Non Cartesian val loss
1	0.2701	0.2709226734547818
2	0.2747	0.24550624879615568
3	0.2741	0.22517447248454342
4	0.2718	0.21266183291001342
5	0.2669	0.2052020972232683
6	0.2688	0.20846722876008653
7	0.2658	0.20274830839080268
8	0.2724	0.1374975763508494
9	0.2731	0.1374159695836605
10	0.2706	0.12477593825185468
140	0.2748	0.10226884307030817
141	0.2746	0.08545122301974002
142	0.2736	0.08429155084900382
143	0.2726	0.0687431732413328
144	0.2746	0.10112319220016353
145	0.2707	0.08279649357518878
146	0.2709	0.09553218410478384
147	0.2736	0.08970080347140254
148	0.2720	0.09607961886160747
149	0.2669	0.08582641906789129
150	0.2722	0.06878729564318725

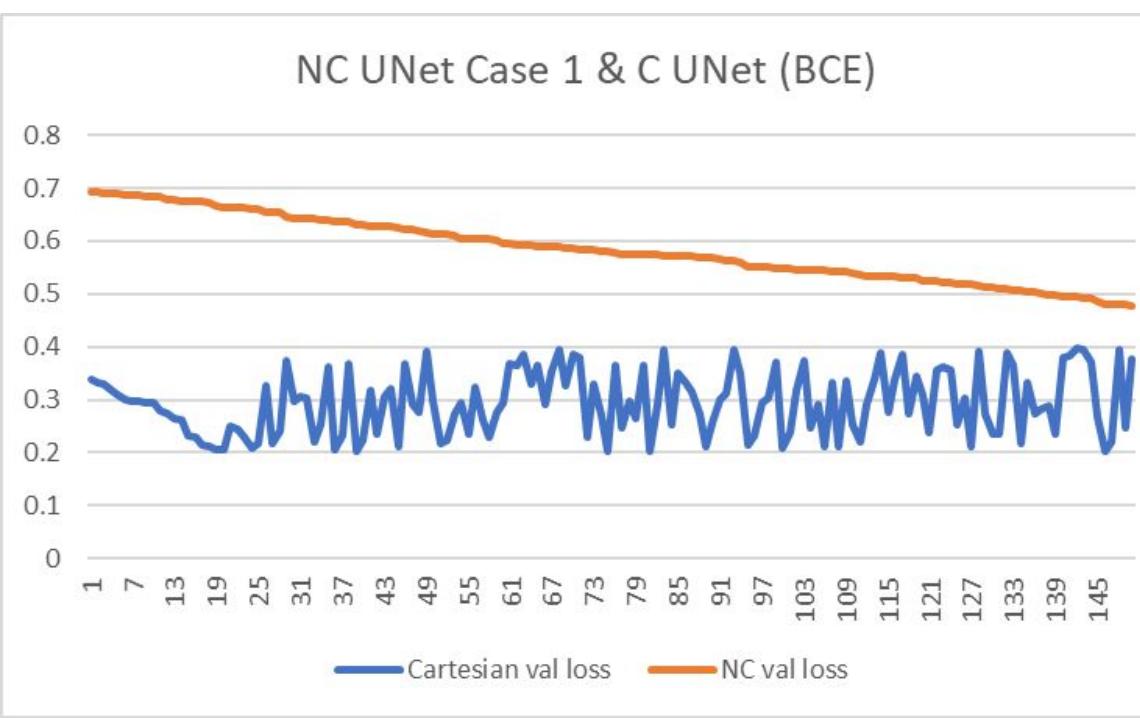
NC



C



Comparison: NC UNet Case1 & C UNet (BCE)



Epoch	Cartesian val loss	Non Cartesian val loss
1	0.3430	0.6929827467525175
2	0.2776	0.6921544803262322
3	0.2628	0.6914361481417977
4	0.2325	0.6905764557739005
5	0.2060	0.6893903330604046
6	0.2072	0.6637993883747625
7	0.1988	0.6450202204604849
8	0.1895	0.6349669966652495
9	0.1954	0.6245135477933839
10	0.1975	0.6185734069742863
140	0.3147	0.4793914595486429
141	0.2829	0.4469630130376861
142	0.2804	0.4627713339588653
143	0.2828	0.475282656016508
144	0.2867	0.436676598301431
145	0.2899	0.46168086076799725
146	0.2865	0.46852675724368525
147	0.2954	0.4522063015196561
148	0.2853	0.45015228820073094
149	0.2873	0.475948765831536
150	0.2894	0.5038547122083004

NC

Image

Ground Truth

Predicted

Image

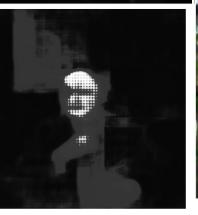
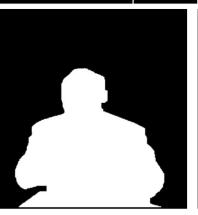
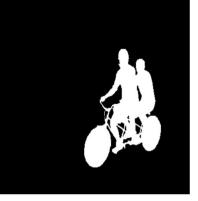
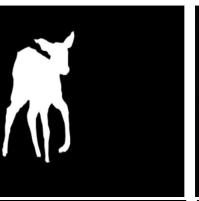
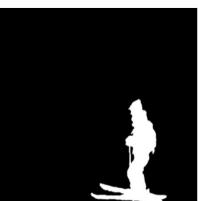
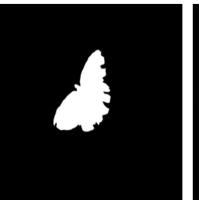
Ground Truth

Predicted

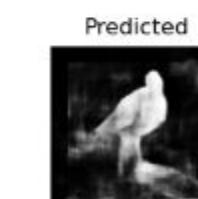
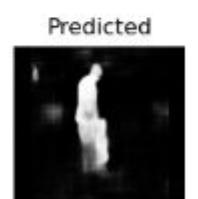
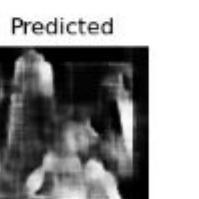
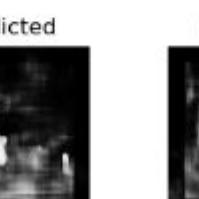
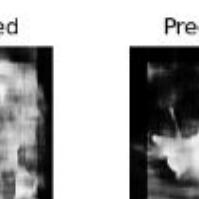
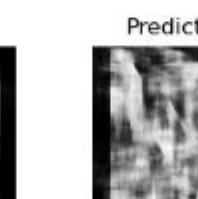
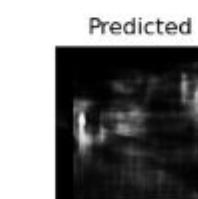
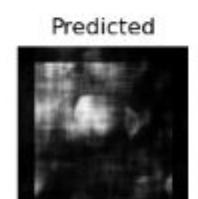
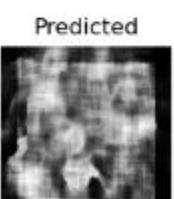
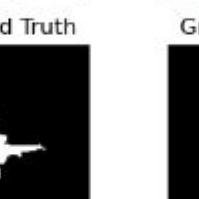
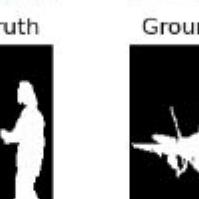
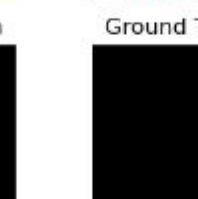
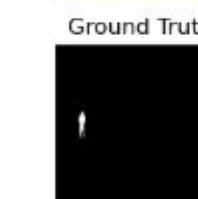
Image

Ground Truth

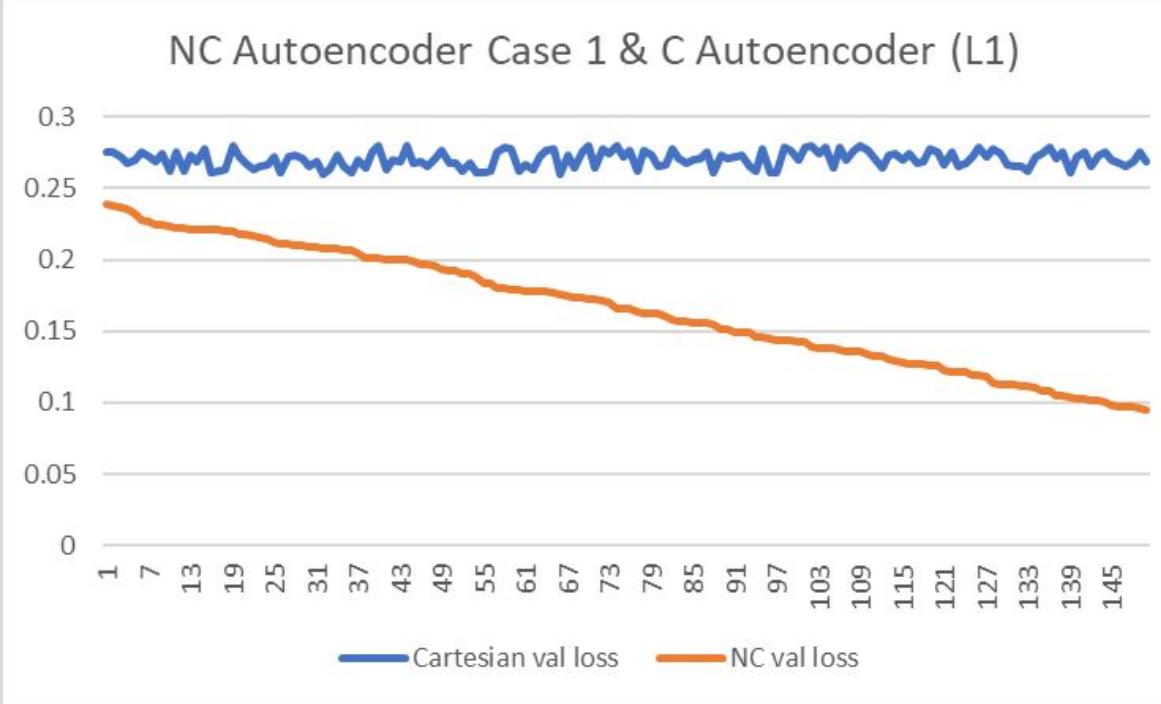
Predicted



C

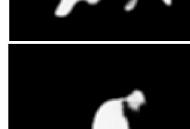
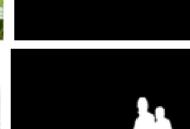


Comparison: NC Autoencoder Case1 & C Autoencoder (L1)

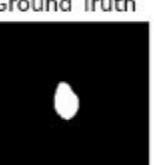
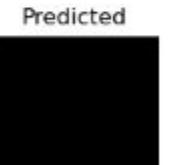
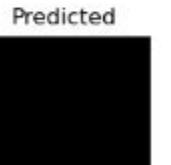
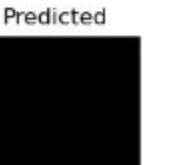


Epoch	Cartesian val loss	Non Cartesian val loss
1	0.2718	0.23971352263649492
2	0.2748	0.2144138769798369
3	0.2695	0.21632120801374247
4	0.2768	0.21748123877986347
5	0.2704	0.20572913568731732
6	0.2754	0.13059509087230356
7	0.2719	0.11966938206934816
8	0.2712	0.12088223142081528
9	0.2741	0.12262639359557798
10	0.2771	0.10736326082340349
140	0.2707	0.09489152365103717
141	0.2694	0.08789003408068165
142	0.2745	0.08118978988785314
143	0.2683	0.09657713253916157
144	0.2752	0.09727504079658274
145	0.2711	0.07439496756588679
146	0.2707	0.0811809312393315
147	0.2677	0.09802564372666074
148	0.2712	0.07360067883091516
149	0.2740	0.0796916049379873
150	0.2741	0.09416599246845427

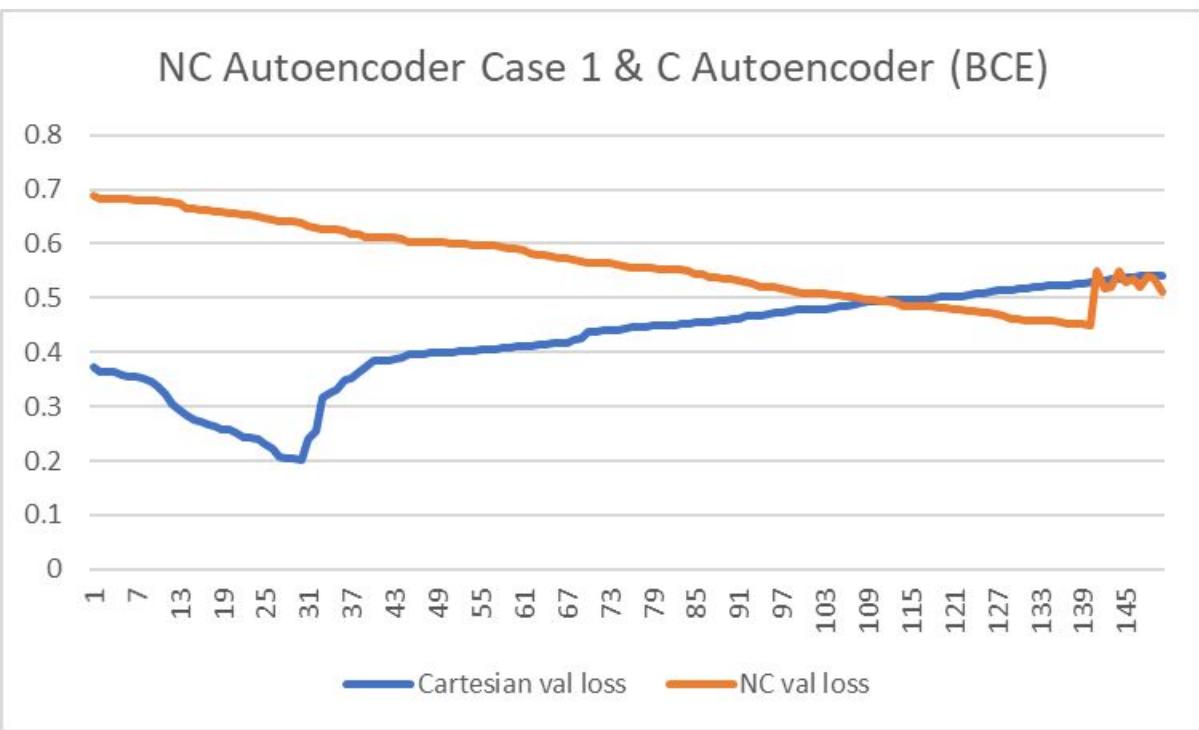
NC

Image	Ground Truth	Predicted	Image	Ground Truth	Predicted	Image	Ground Truth	Predicted
								
								
								

C

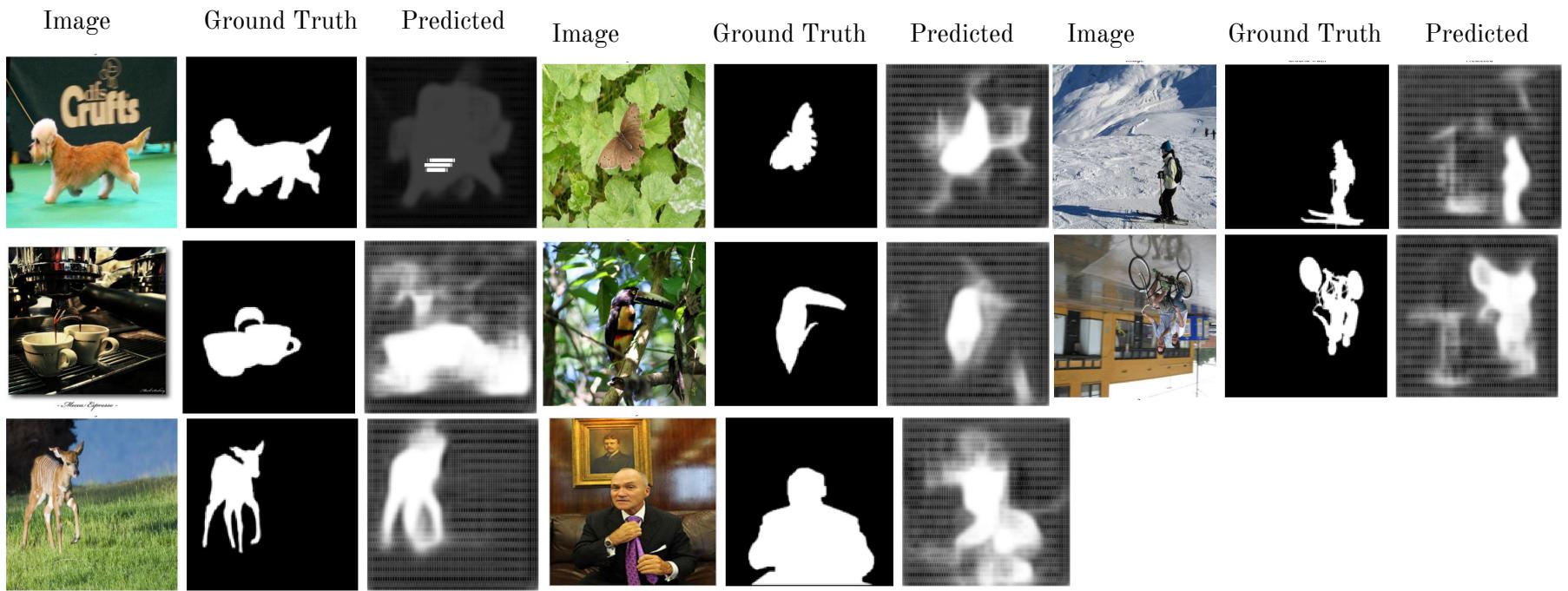
Image	Image	Image	Image	Image	Image	Image	Image
							
Ground Truth	Ground Truth	Ground Truth	Ground Truth	Ground Truth	Ground Truth	Ground Truth	Ground Truth
							
Predicted	Predicted	Predicted	Predicted	Predicted	Predicted	Predicted	Predicted
							

Comparison: NC Autoencoder Case1 & C Autoencoder (BCE)

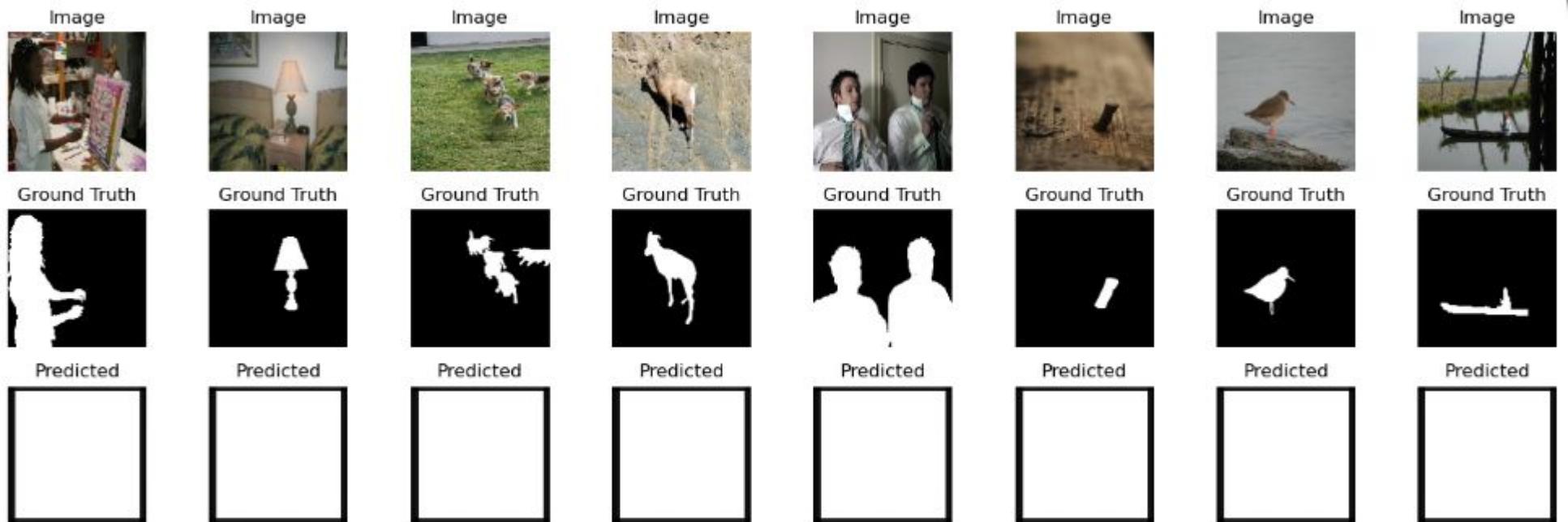


Epoch	Cartesian val loss	Non Cartesian val loss
1	0.3730	0.6881432845129221
2	0.3123	0.6756491058245654
3	0.2859	0.6415428192129632
4	0.2678	0.6331117988762698
5	0.2456	0.62253637183899
6	0.2309	0.6178294805553852
7	0.2393	0.6109071231566334
8	0.2458	0.6066718628621215
9	0.2859	0.6045886809227026
10	0.2247	0.6014458752921408
140	0.5412	0.44762983401239764
141	0.5436	0.542254879011362
142	0.5362	0.5279368183624122
143	0.5407	0.5050058098483424
144	0.5407	0.537318468658845
145	0.5350	0.5321410423206492
146	0.5383	0.5413146572655411
147	0.5414	0.5267133272654637
148	0.5385	0.5279565184037267
149	0.5371	0.5419383810594749
150	0.5418	0.5314477321095942

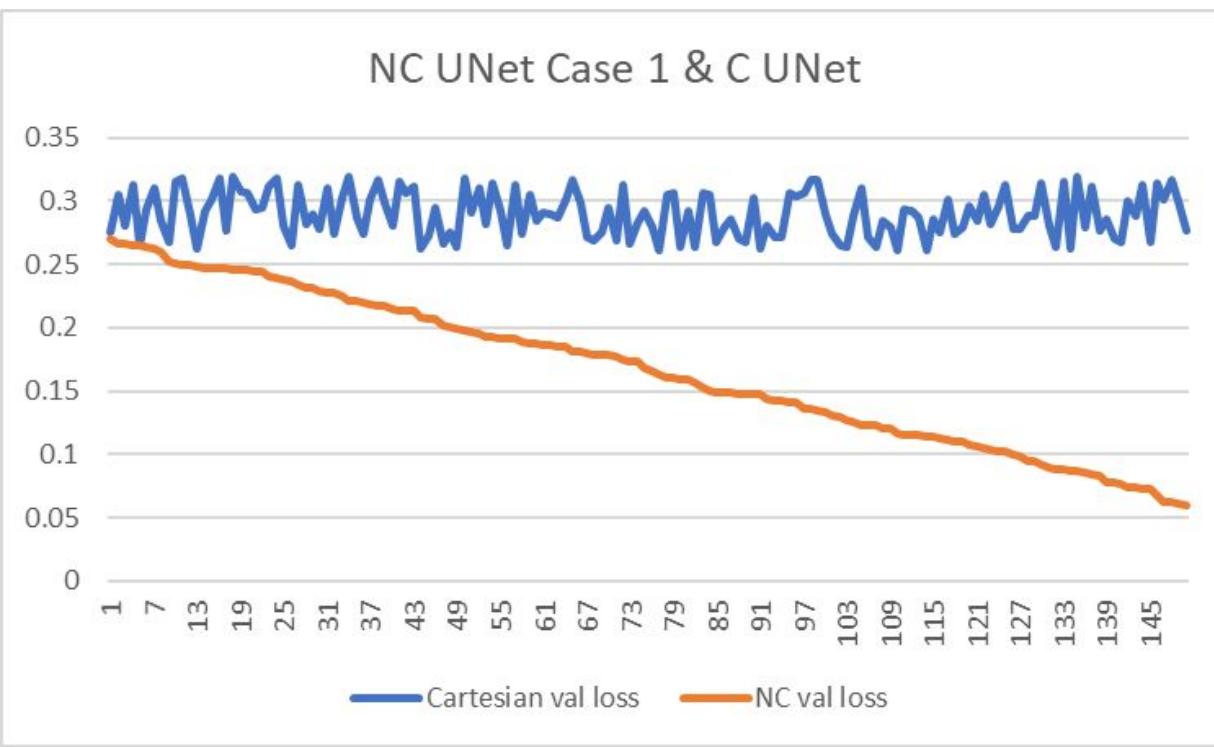
NC



C



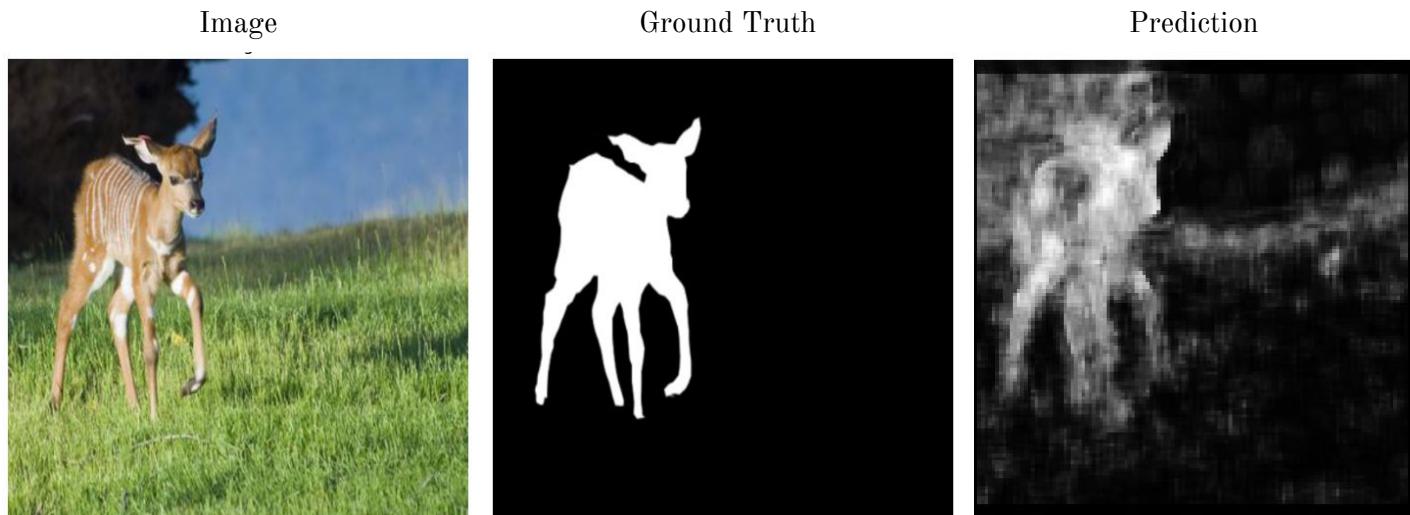
Best: Cartesian (UNet BCE but L1 for validation) & NC (UNet Case 1 L1)



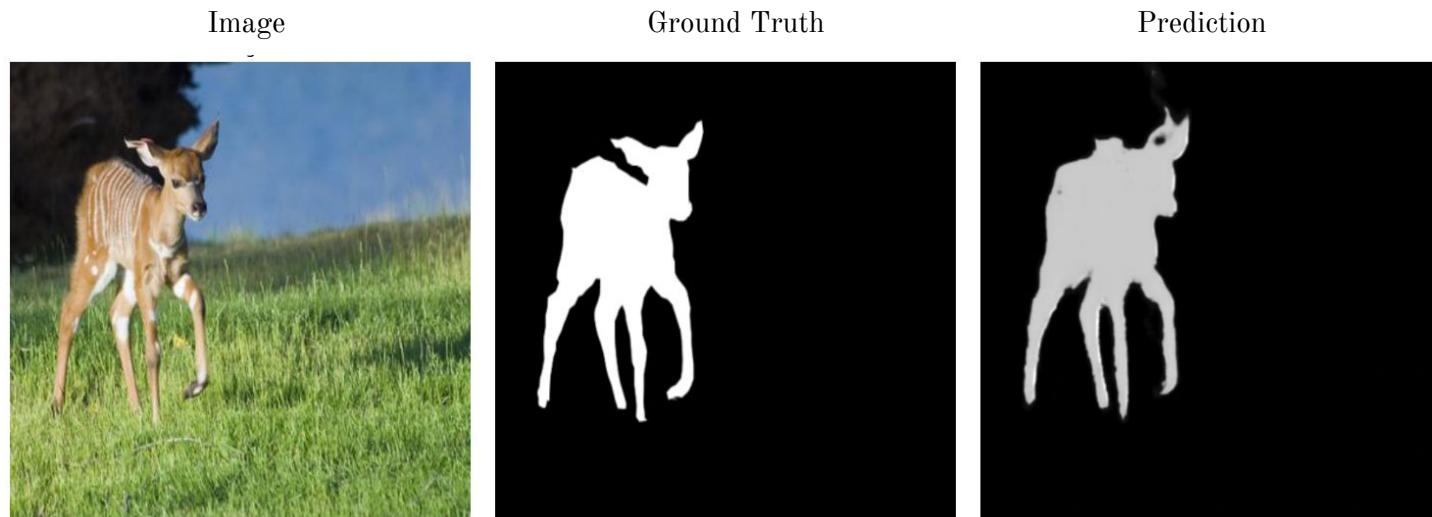
Epoch	Cartesian val loss	Non Cartesian val loss
1	0.2799566984176636	0.2709226734547818
2	0.27722216129303	0.24550624879615568
3	0.3258756697177887	0.22517447248454342
4	0.3287608325481415	0.21266183291001342
5	0.3173062205314636	0.2052020972232683
6	0.3111141324043274	0.20846722876008653
7	0.3292132616043091	0.20274830839080268
8	0.3085390627384186	0.1374975763508494
9	0.29495909810066223	0.1374159695836605
10	0.307666152715683	0.12477593825185468
140	0.2835249602794647	0.10226884307030817
141	0.27340608835220337	0.08545122301974002
142	0.2793143093585968	0.08429155084900382
143	0.2814796566963196	0.0687431732413328
144	0.2832799553871155	0.10112319220016353
145	0.2757195830345154	0.08279649357518878
146	0.282814621925354	0.09553218410478384
147	0.2765105366706848	0.08970080347140254
148	0.28162825107574463	0.09607961886160747
149	0.2801268398761749	0.08582641906789129
150	0.2780151963233948	0.06878729564318725

Best: Cartesian (UNet BCE) & NC (UNet L1)

Cartesian UNet BCE - 150 epochs



NC UNet L1 - 150 epochs



Image



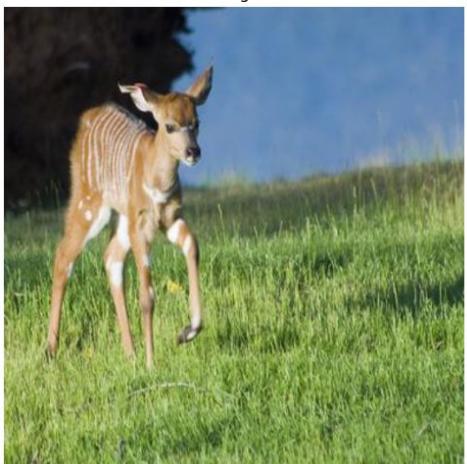
Ground Truth



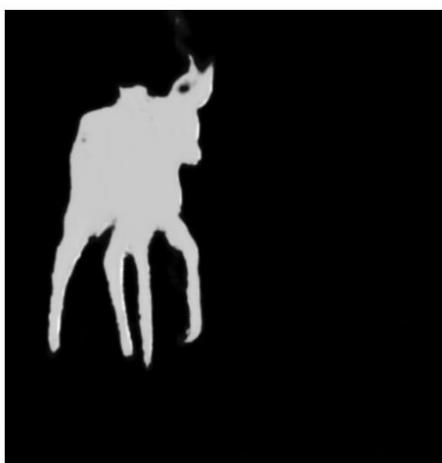
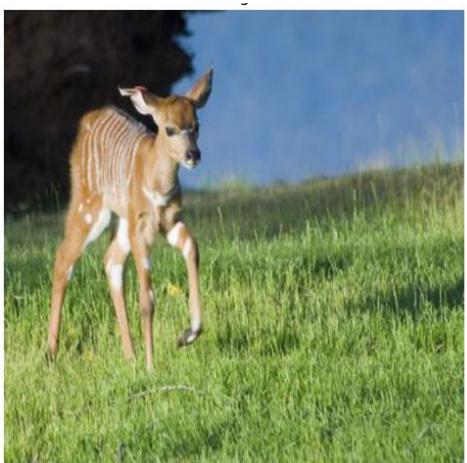
Prediction



Cartesian Unet
BCE 30 epochs



Cartesian Unet
BCE 150
epochs

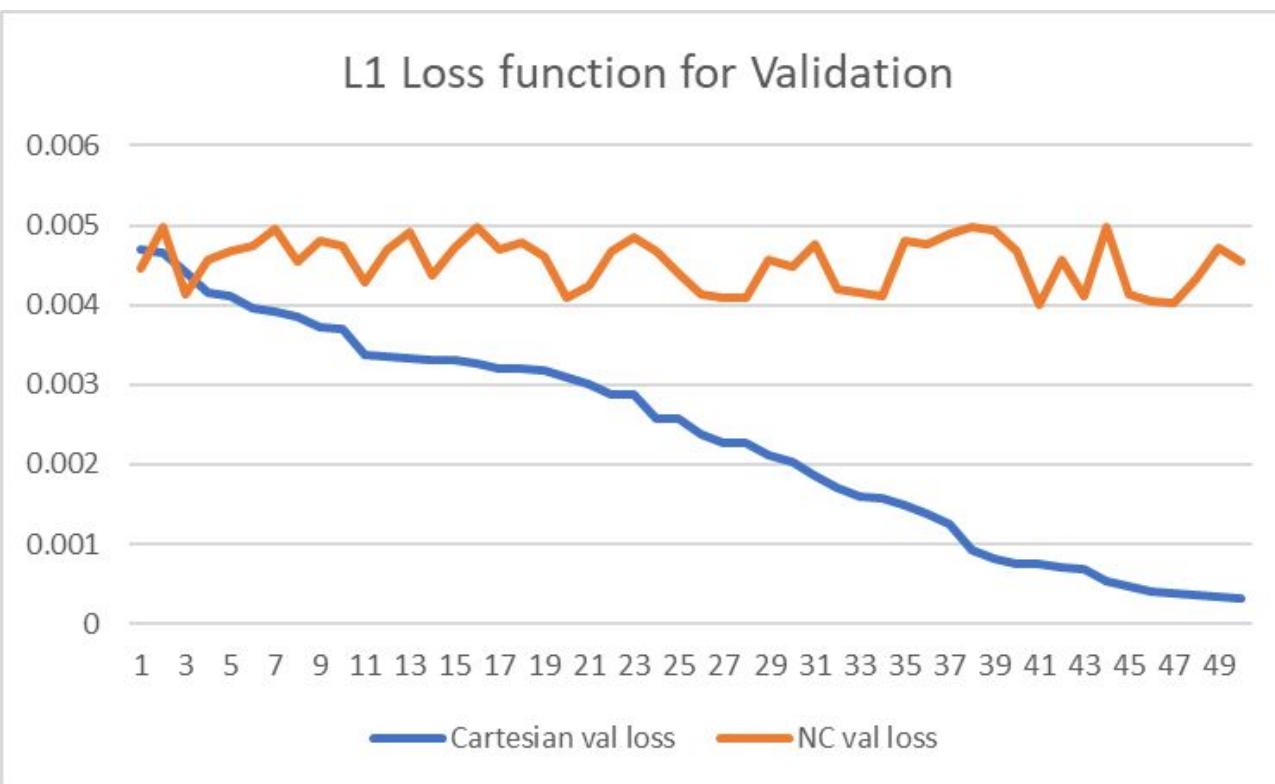


NC Unet L1
150 epochs

Medical Segmentation Decathlon dataset

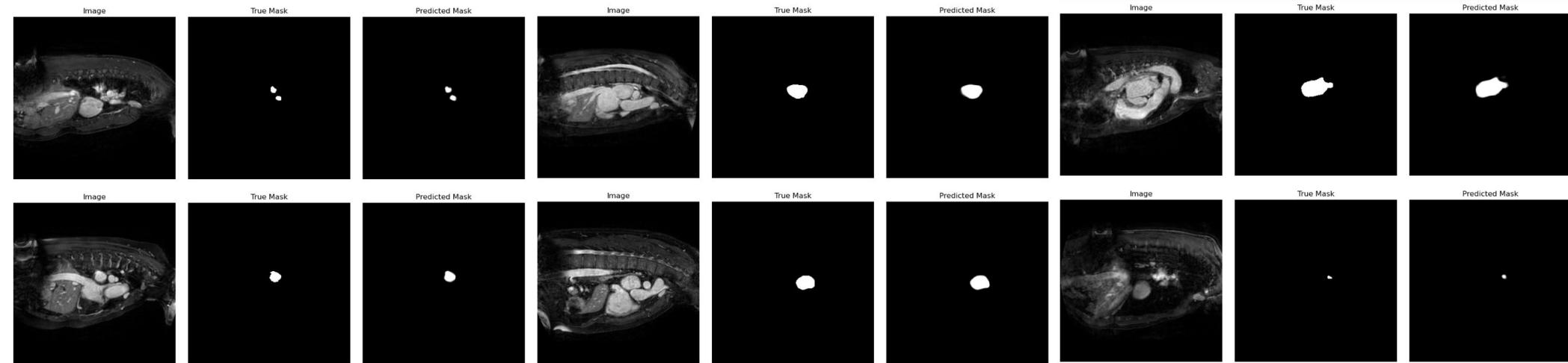
All experiments that follow are performed using the MSD dataset unless specified otherwise.

Comparison using: Medical Segmentation Decathlon L1 Loss Function

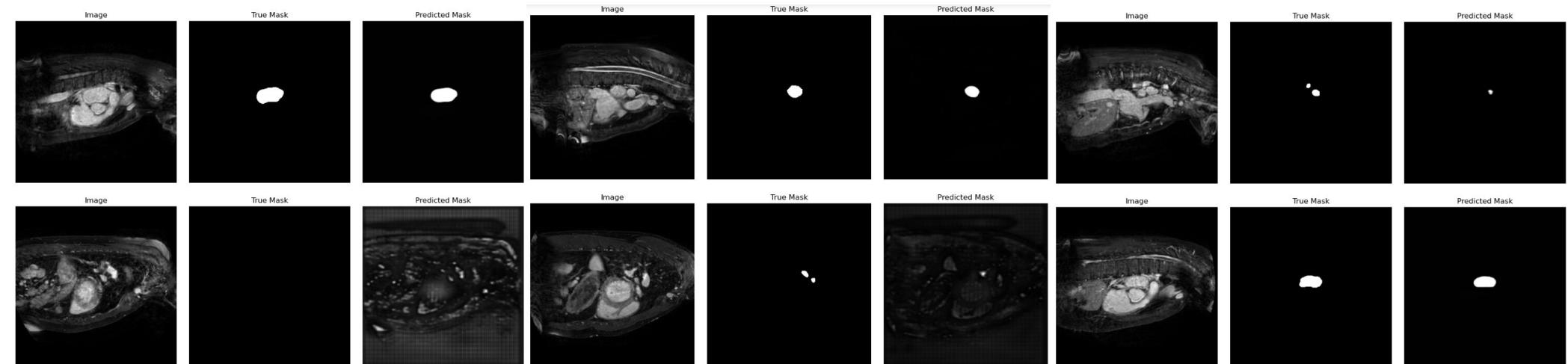


Epoch	Cartesian val loss	Non-Cartesian val loss
1	0.004024724349599753	0.0047311252617705
2	0.0021017561287715518	0.0041131738256049025
3	0.00174254469096367	0.004173506082004898
4	0.0010637946669104065	0.004064345694464314
5	0.0008627063770012316	0.00406185600108334
6	0.0009512434334590518	0.004047475875979596
7	0.0007636009178725369	0.004057327385705251
8	0.0007044779960745074	0.0038126501525307054
9	0.0006633078288562191	0.0040555459109472706
10	0.000639828836976601	0.003959435012107101
40	0.0003909016125307882	0.003621308297266836
41	0.000340395772398399	0.002139401566195634
42	0.00035139713381311574	0.0040555459109472706
43	0.00032648396609451973	0.004048720704248318
44	0.00031817055688115756	0.0029874870514274565
45	0.0003188650948660714	0.0040555459109472706
46	0.00031610798953201984	0.0040555459109472706
47	0.0003012581059498152	0.003970338162968631
48	0.0003054193205434113	0.0040555459109472706
49	0.0003256962217133621	0.0040555459109472706
50	0.000325189501321895	0.004962697340166554

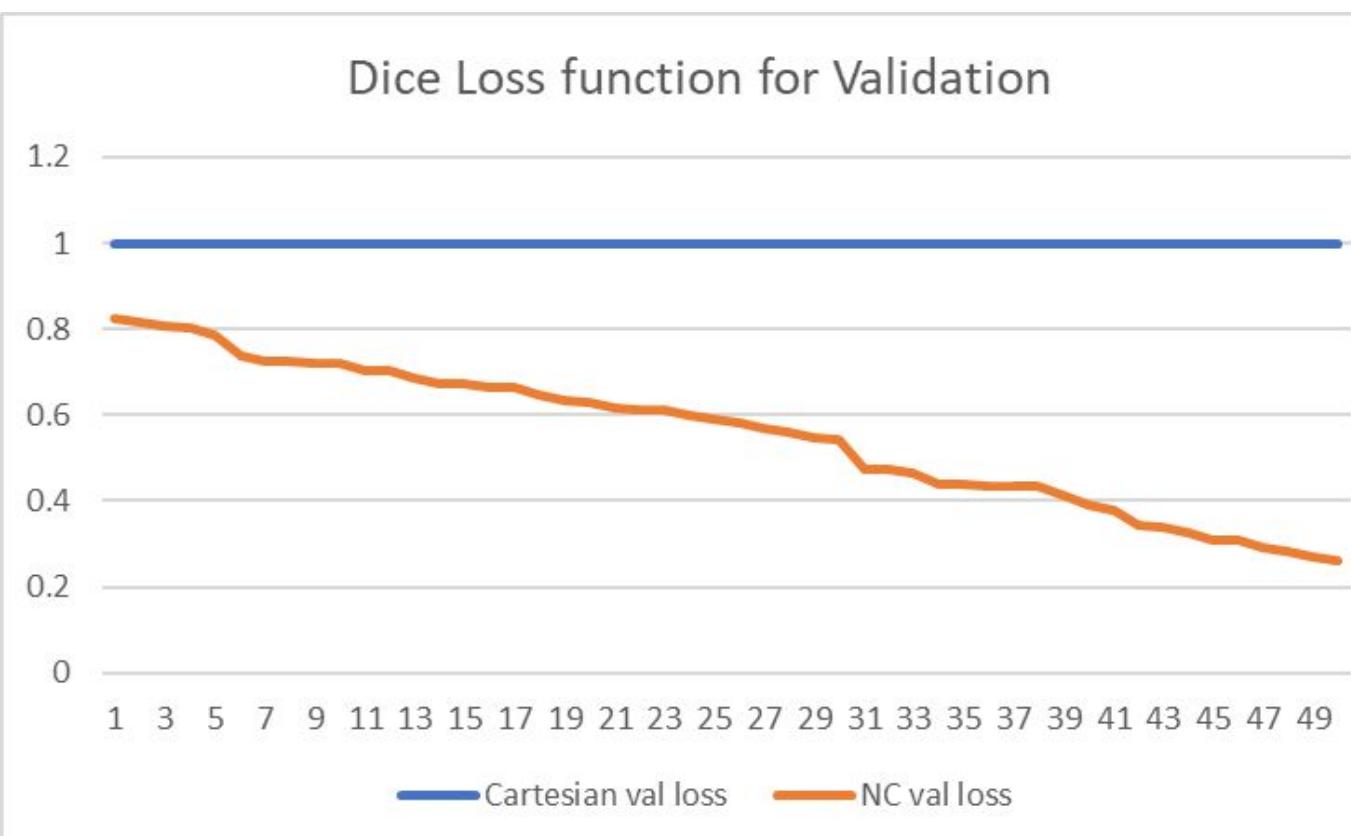
Cartesian



Non-Cartesian

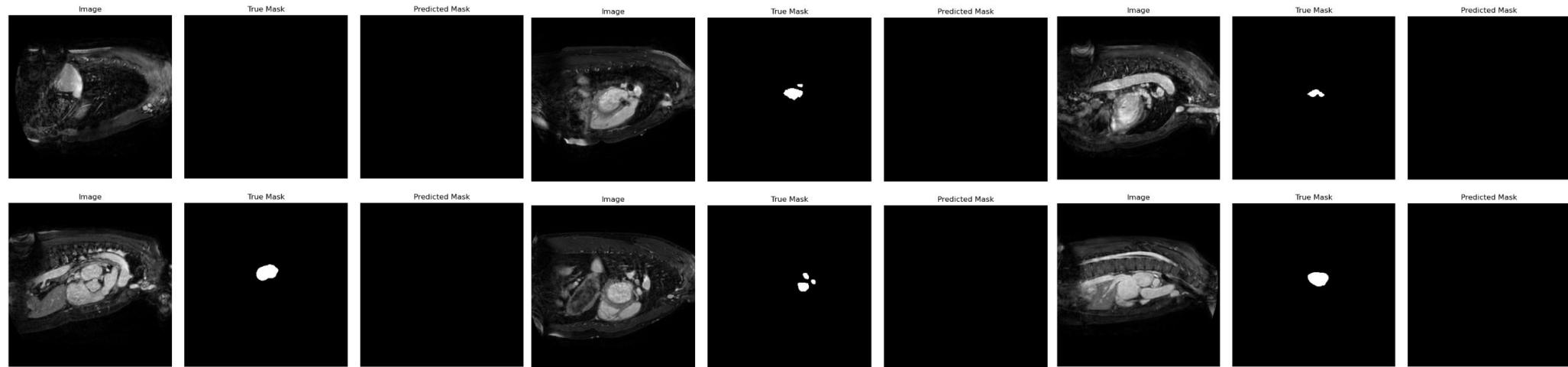


Dice Loss Function

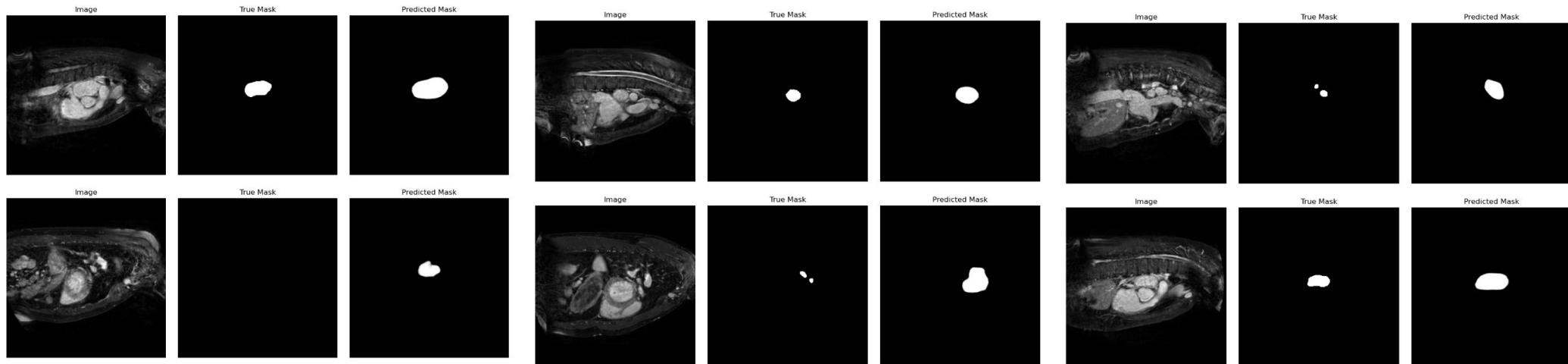


Epoch	Cartesian val loss	Non-Cartesian val loss
1	1.0000	0.8512002719627632
2	1.0000	0.7944844090021573
3	1.0000	0.6345047086149782
4	1.0000	0.5581716644894946
5	1.0000	0.5608106239811405
6	1.0000	0.5055772172225701
7	1.0000	0.4633482457517268
8	1.0000	0.5361993176596506
9	1.0000	0.43568157258924545
10	1.0000	0.38229073456355506
40	1.0000	0.29597100202853865
41	1.0000	0.25096011751300684
42	1.0000	0.2894125088230594
43	1.0000	0.25299381017684935
44	1.0000	0.3033186667568081
45	1.0000	0.2624119427177932
46	1.0000	0.2638828480636681
47	1.0000	0.25802101840029706
48	1.0000	0.2658518822638543
49	1.0000	0.3097275499459151
50	1.0000	0.24632325735720959

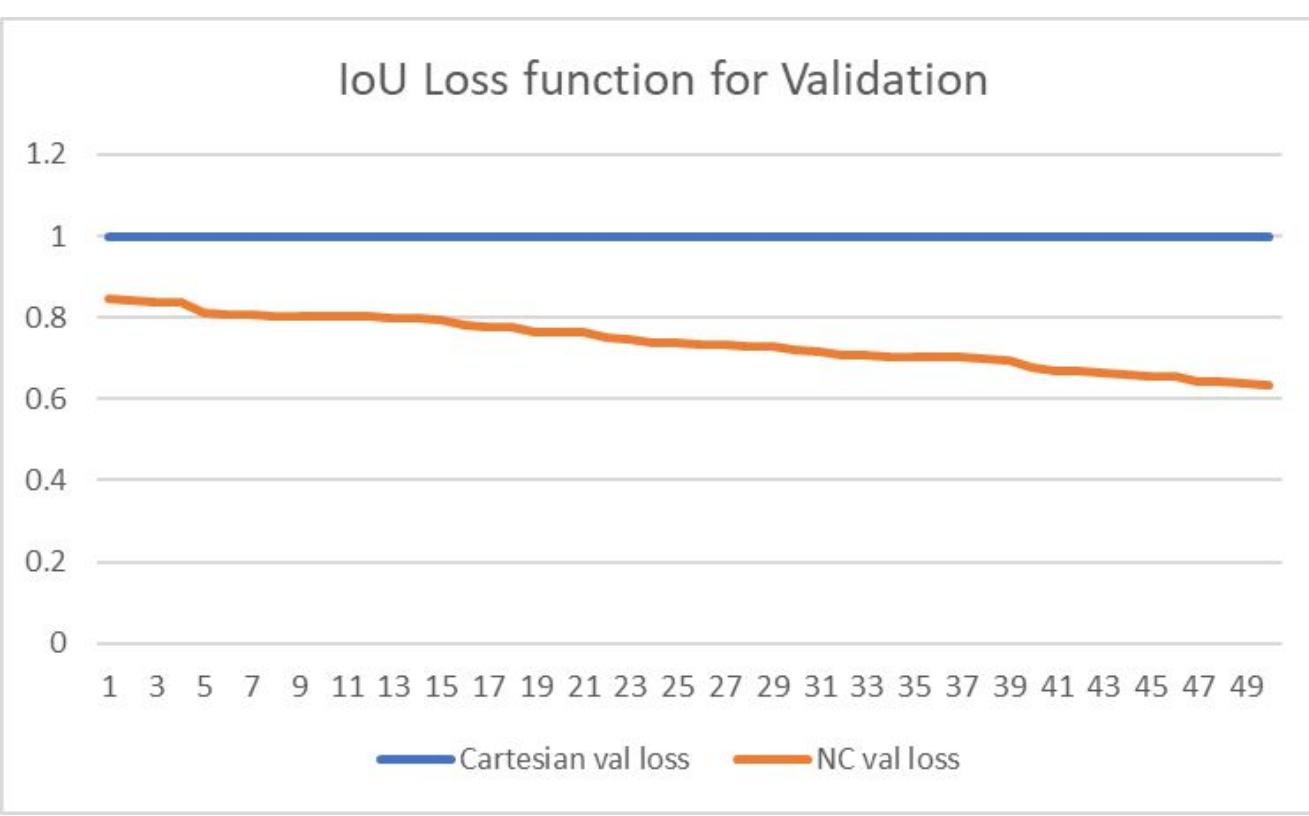
Cartesian



Non-Cartesian

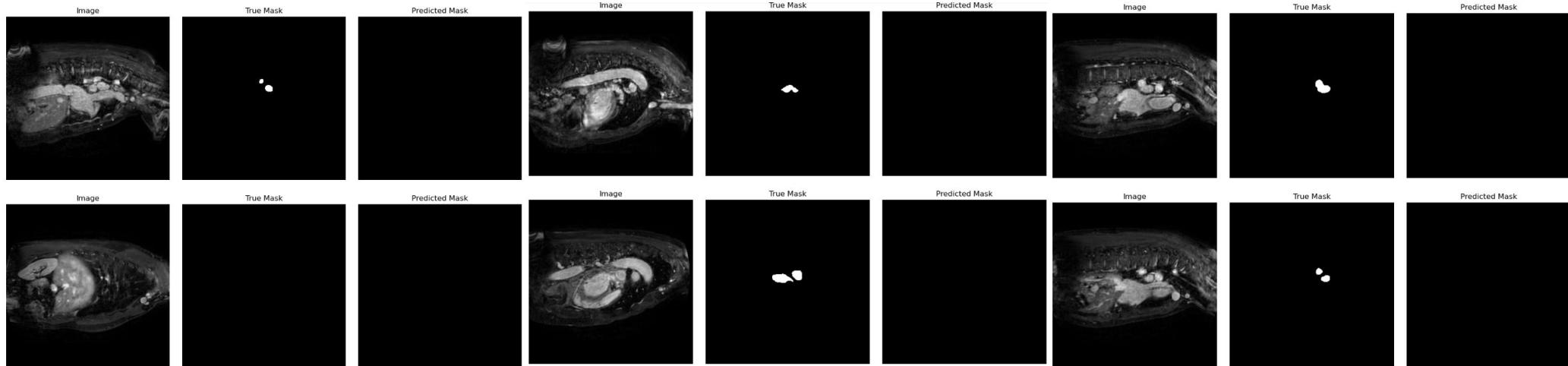


IoU Loss Function

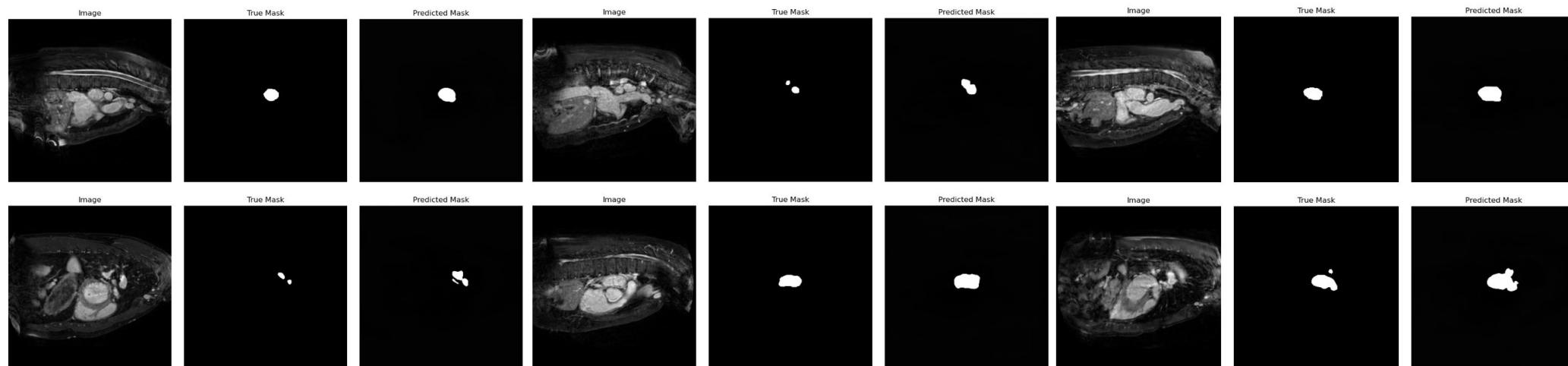


Epoch	Cartesian val loss	Non-Cartesian val loss
1	1.0000	0.8661046062197004
2	1.0000	0.863579175891457
3	1.0000	0.9280289800612481
4	1.0000	0.8377099058130285
5	1.0000	0.7809594185797722
6	1.0000	0.7707278429806887
7	1.0000	0.7495289815651192
8	1.0000	0.8282680825872736
9	1.0000	0.7366513193308653
10	1.0000	0.7456355694886092
40	1.0000	0.6572893219989735
41	1.0000	0.633866431424906
42	1.0000	0.6474843175856622
43	1.0000	0.6361067226954868
44	1.0000	0.6464375798518841
45	1.0000	0.6537662113105858
46	1.0000	0.633388574044783
47	1.0000	0.6268770482513931
48	1.0000	0.6186763877397056
49	1.0000	0.6370430187864617
50	1.0000	0.6353920155829126

Cartesian



Non-Cartesian



Thank You