1D Slow oscillations (A>>C0) (Analytical)

1) Failure of kink dynamics

Extending kink dynamics

Kink dynamics is well defined for strictly positive oscillations $C(t)$: such that a kink-like stationary state always exists.

A naive extension

For two isolated kinks, kink dynamics says

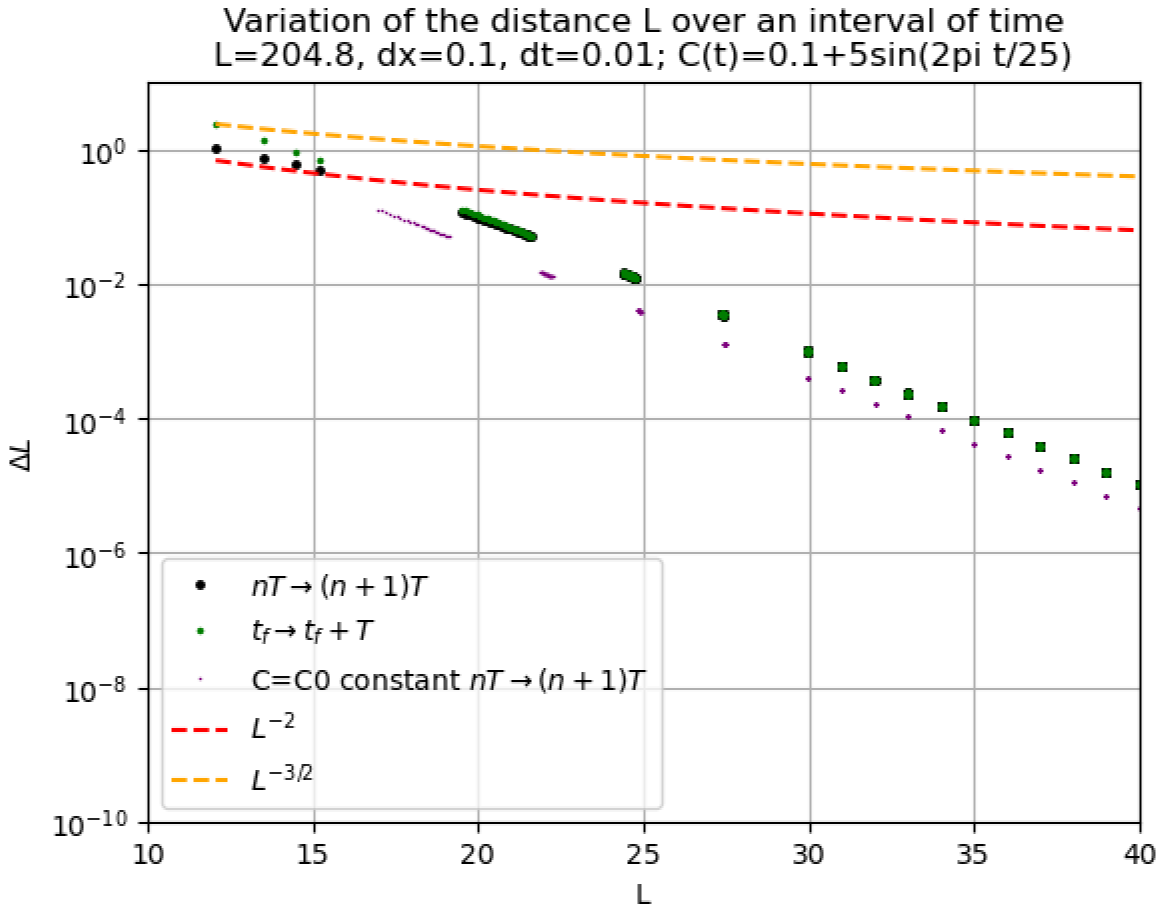
$$\dot{L} \simeq -24\sqrt{2}C(t)^{\frac{1}{2}}e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}L}$$

we can adopt this law when $C(t) > 0$ and **assume the distance does not change when $C(t) \leq 0$.**

Failure of this model

Failure of kink dynamics for negative oscillations
If we consider the variation of the distance over a period $\Delta L = \int_0^T \partial_t L$, the model predicts a

power law decay $\Delta L \sim L^{-3/2}$, while it is exponential experimentally (**much slower**).



2) Linear dynamics and erf-shaped kinks

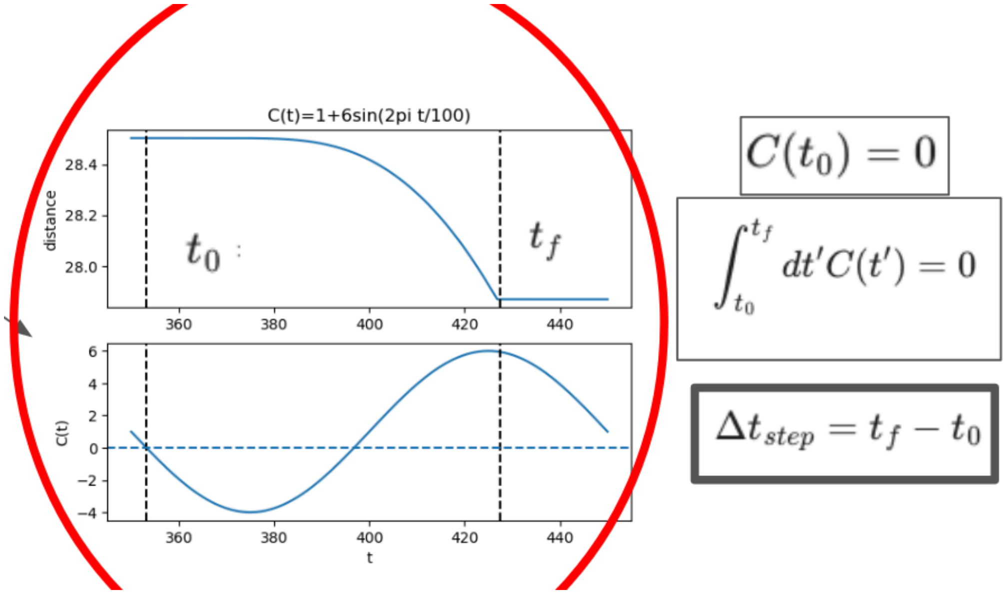
[Explaining the decay of the distance of two kinks with linear dynamics and a sum of Gaussians]

Idea

- We expect that, if $C(t)$ remains **negative for a long time**, there will be a time interval where $u(x)$ is very small and so the **non-linearity is negligible**.
- Then **we approximate the shape** of the kinks to erf-shaped to calculate **analytically** the evolution of the profile.

The steps can be described by linear dynamics only

The decay of the distance is a step-like profile. From empirical evidence, a step lasts from the moment when $C(t)$ crosses zero to become negative and until the moment when the non-linearity becomes non-negligible again (if we assume it become negligible when C become negative).



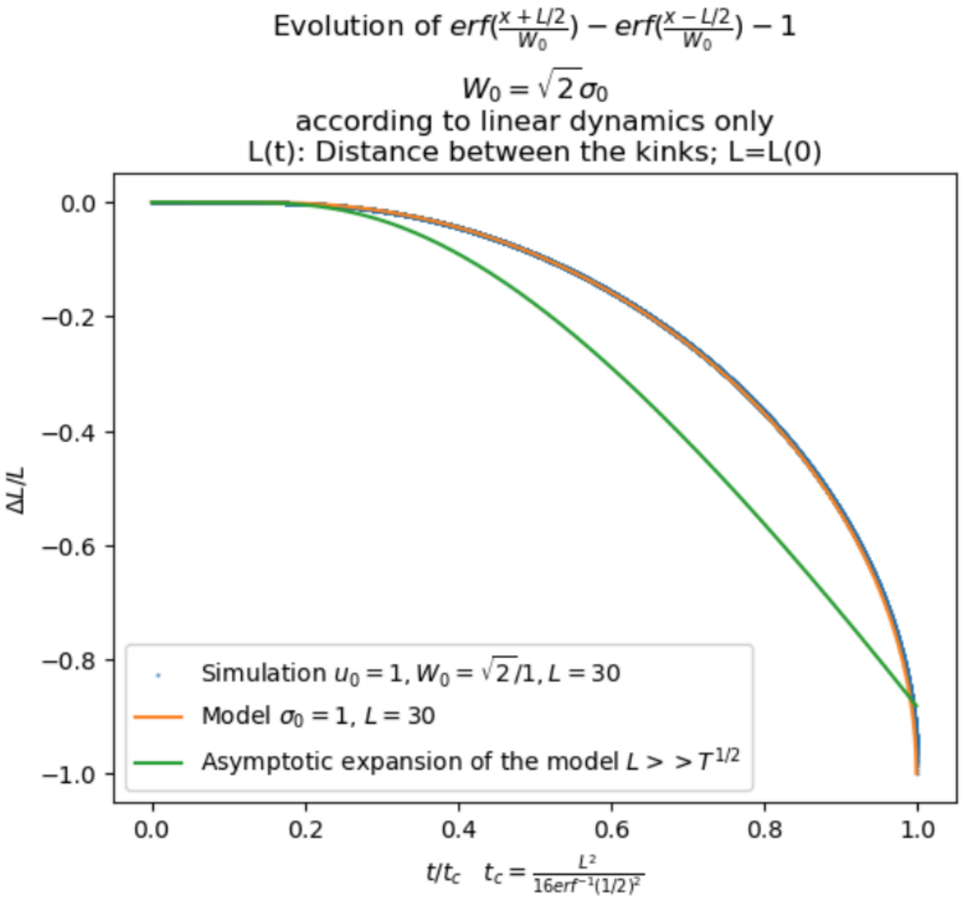
Practical ansatz for the shape of the kinks

Building a two-kinks profile with a sum of an infinite amount of gaussians, enables to know analytically the evolution of the state in the linear dynamics.

$$f(x) = \lim_{N\to\infty} \frac{2L}{N}\left(\sum_{n=1}^{N} g_n(x) - \frac{1}{2}\right) = \frac{1}{2}\left(\mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{x-L}{\sqrt{2}\sigma}\right) - 1\right)$$

This profile has **erf**-shaped kinks, those are **different from tanh**-shaped kinks, because their

tail decays as $\frac{e^{-x^2}}{x}$ instead of e^{-x}.



Two coarsening laws

At each step, we can **predict its depth** by considering the evolution of the distance in the linear dynamics, for a time interval equal to the step duration.
If we define a **macroscopic derivative** $\partial_t L = \frac{\Delta L_{step}}{T}$ we find a **new coarsening law**

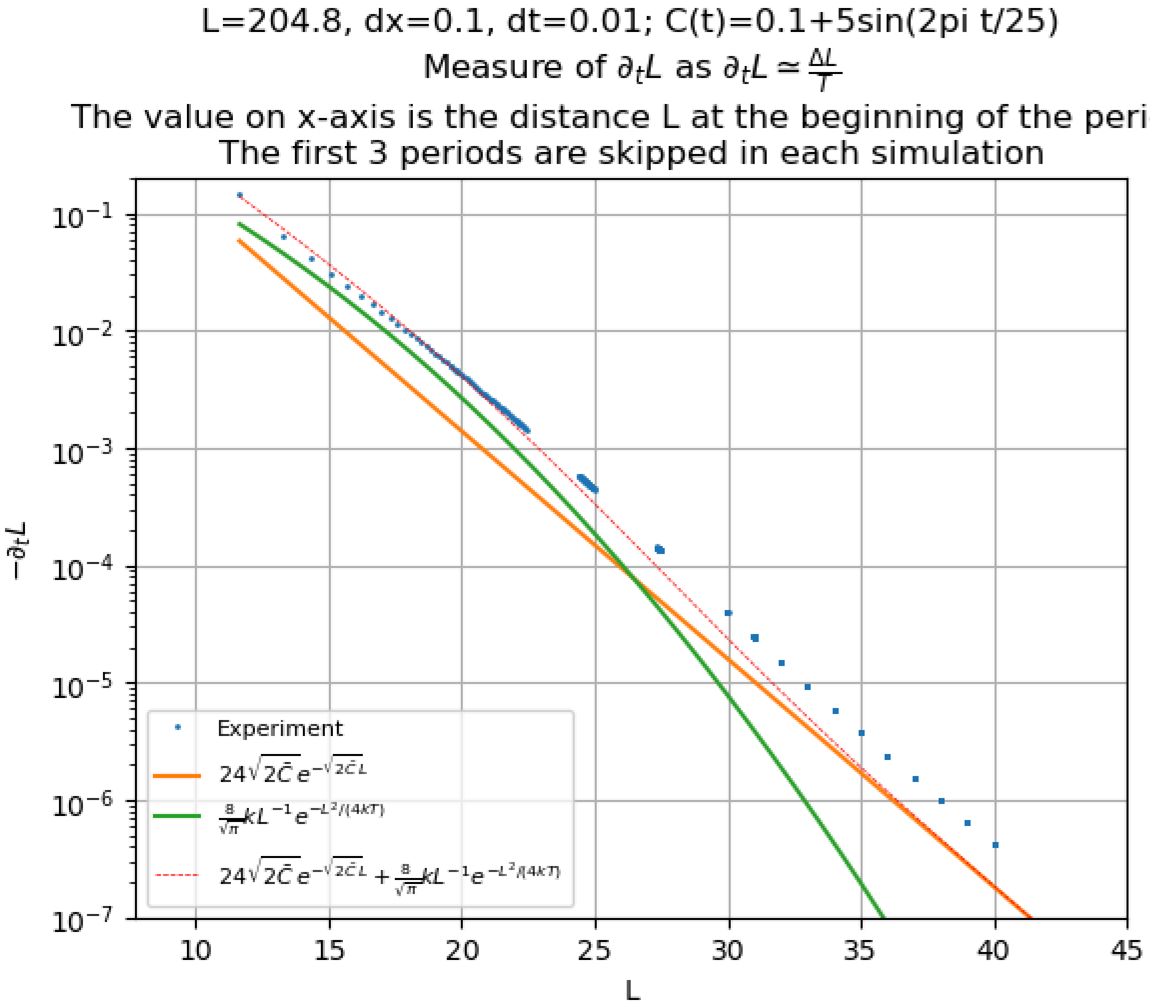
$$\partial_t L = -\frac{8}{\sqrt{\pi}} k L^{-1} e^{-L^2/4kT} \qquad k = 1 - \frac{1}{\sqrt{\pi}} (\frac{\bar{C}}{A})^{1/2}$$

this law is **different** from the one we expect when $C = \bar{C}$ is constant, due to **kink dynamics**

$$\partial_t L = -24\sqrt{2\bar{C}} e^{-\sqrt{2\bar{C}}L}$$

Crossover

If we consider a simulation and we look close to the point where the two laws cross (crossover point) we see that the data is not described by neither of the two points. Instead is well described **by their sum**.



Explaination:
It makes sense that the data at low L is described by the **sum** of the two curves and not just by the green line because

> "during the interval when the linear approximation is correct, the tail of the kinks is still e^{-x} and NOT $\frac{e^{-x^2}}{x}$, as in the analytical calculations. As kink dynamics is due to the shape e^{-x} of the kink's tail, our analytical calculations cannot predict kink dynamics and this contribution **must be added**"

Although, it is not clear why we should add the kink dynamics contribution with $C = \bar{C}$ constant. It is natural to make this choice, but I cannot justify it (we should monitor the shape of the tails during a period).

Asymptotics

Asymptotically, the sum of the two laws does not match the data. Here the depth of the steps gets smaller and smaller and it is kink dynamics the dominant effect. Althought, kink dynamics with constant $C = \bar{C}$ cannot describe the decay.

Coarsening

From a dimensional analysis (that works well to predict the coarsening law of 1D and 2D with constant C) the new law for $\partial_t L$ implies a coarsening that is $\sim \sqrt{\log(t)}$ instead of $\log(t)$. So the coarsening will proceed as a square root of a logarithm at the beginning, and asymptotically it will be logarithmic. We can see this square root behaviour, which starts around $t \simeq 50$, that is around the first time $C(t)$ becomes negative

$$C(t) = 1 + A\sin\left(\frac{2\pi t}{100}\right)$$