

# Kinks effective dynamics under slow POSITIVE oscillations

#twokinks

#1D

#constantC

If  $C(t)$  is

- **Strictly positive**  $C(t) > 0 \forall t$
- And its **oscillations are slow** ( $T \gg \tau_c$ )

following the idea presented in [Kink effective dynamics.pdf](#) (and generalized for  $C$  constant  $\rightarrow$  slow oscillations limit in [Kink effective dynamics under slow POSITIVE oscillations \(theory\)](#)) it is possible to describe the evolution dictated by the TDGL with an **effective law** for the velocity of each kink. If  $x_n$  is the position of the  $n$ -th kink (the  $n$ -th zero of  $u(x)$ ) and  $l_n \equiv x_{n+1} - x_n$  is the length of the  $n$ -th domain:

$$\dot{x}_n(t) = 16C^{\frac{1}{2}}(t) \frac{[e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}l_n} - e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}l_{n+1}}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_{\chi} u_p(\chi)}$$

where  $u_p(\chi)$  is the periodic stationary state with period  $(\chi_{n+1} - \chi_{n+1})$  and  $\chi_n = C(t)^{\frac{1}{2}}x_n$ . If there are only two kinks and PBC boundaries are adopted (so if the distance from the right is  $d$  then the distance from the left is  $L - d$ ) the distance  $d(t)$  will decrease in this way ( $L > d$ )

$$\dot{d}(t) = -2 * 16C^{\frac{1}{2}}(t) \frac{[e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d} - e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}(L-d)}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_{\chi} u_p(\chi)}$$

Where the integral at the denominator can be approximated by the integral of the single-kink stationary state and the integration is carried on the whole real axis:

$$\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_{\chi} u_p(\chi) \simeq \int_{-\infty}^{+\infty} d\chi u_k(\chi) = I_1$$

where  $I_1$  has been calculated in the [Master Report.pdf](#) and  $I_1 = \frac{2\sqrt{2}}{3}$ .

If also the smaller exponential is neglected, in the limit  $L \gg d$

$$\dot{d}(t) \simeq -24\sqrt{2}C^{\frac{1}{2}}(t)e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d}$$

## Annihilation time

To estimate the time-scale of the annihilation process, we consider the case where  $C$  is constant. The solution to the differential equation for  $\dot{d}$  is

$$d(t) = A + \log(\alpha(t_c - t))$$

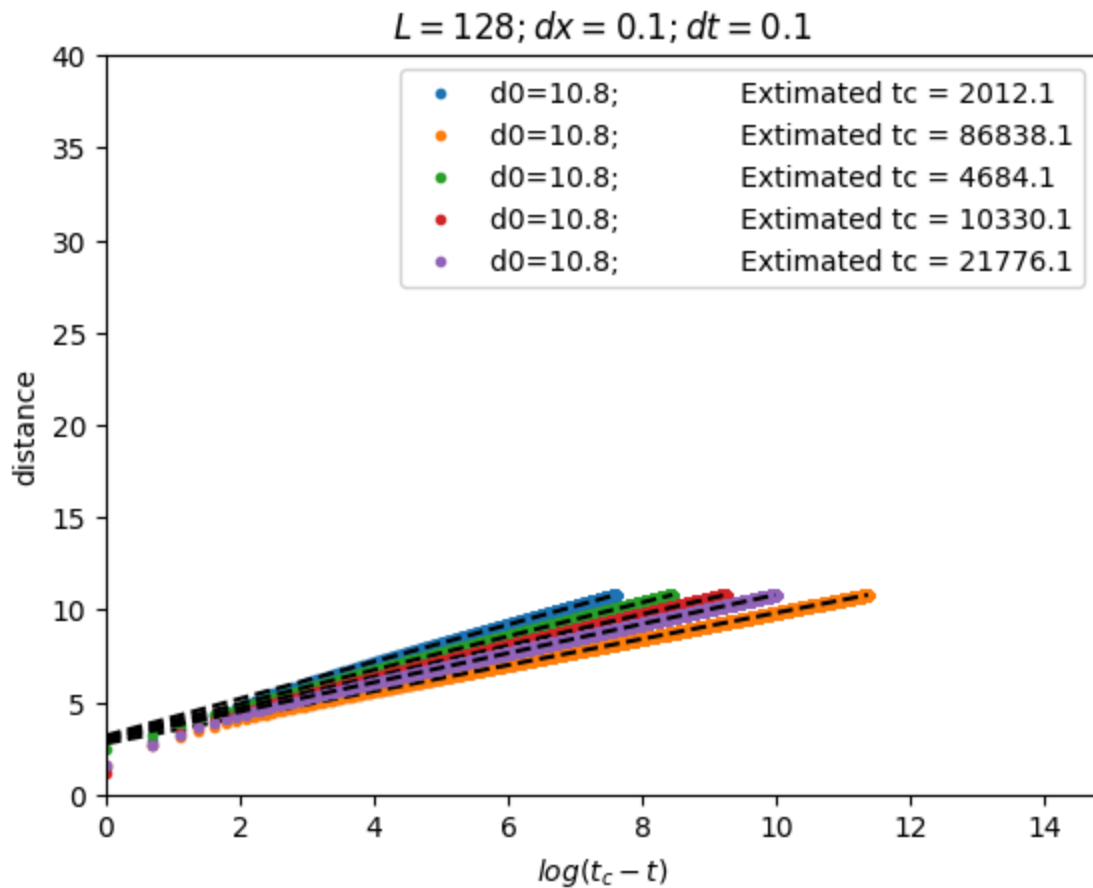
$A = (2C)^{-0.5}$ ;  $\alpha = 48C$  and the annihilation time is

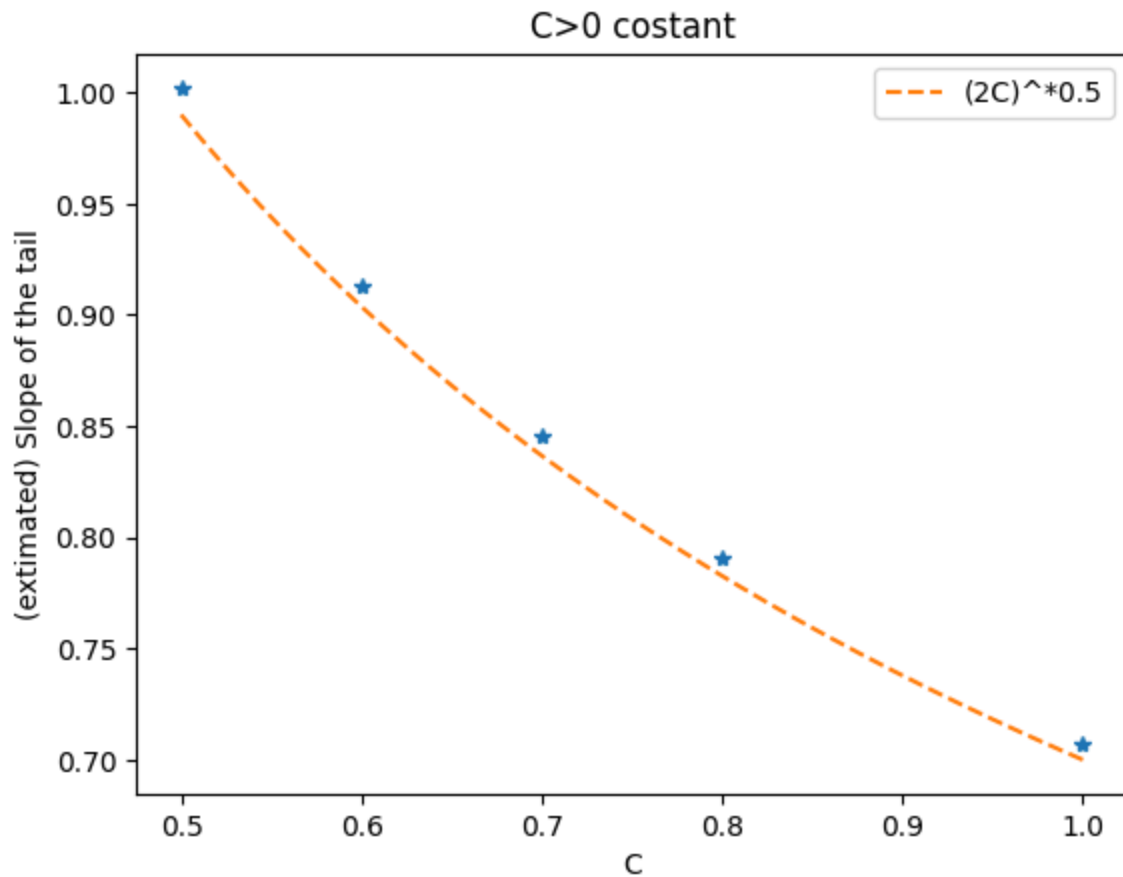
$$t_c = \frac{e^{d_0/A}}{\alpha} = \frac{e^{d_0(2C)^{0.5}}}{48C}$$

## Simulations

### C is constant

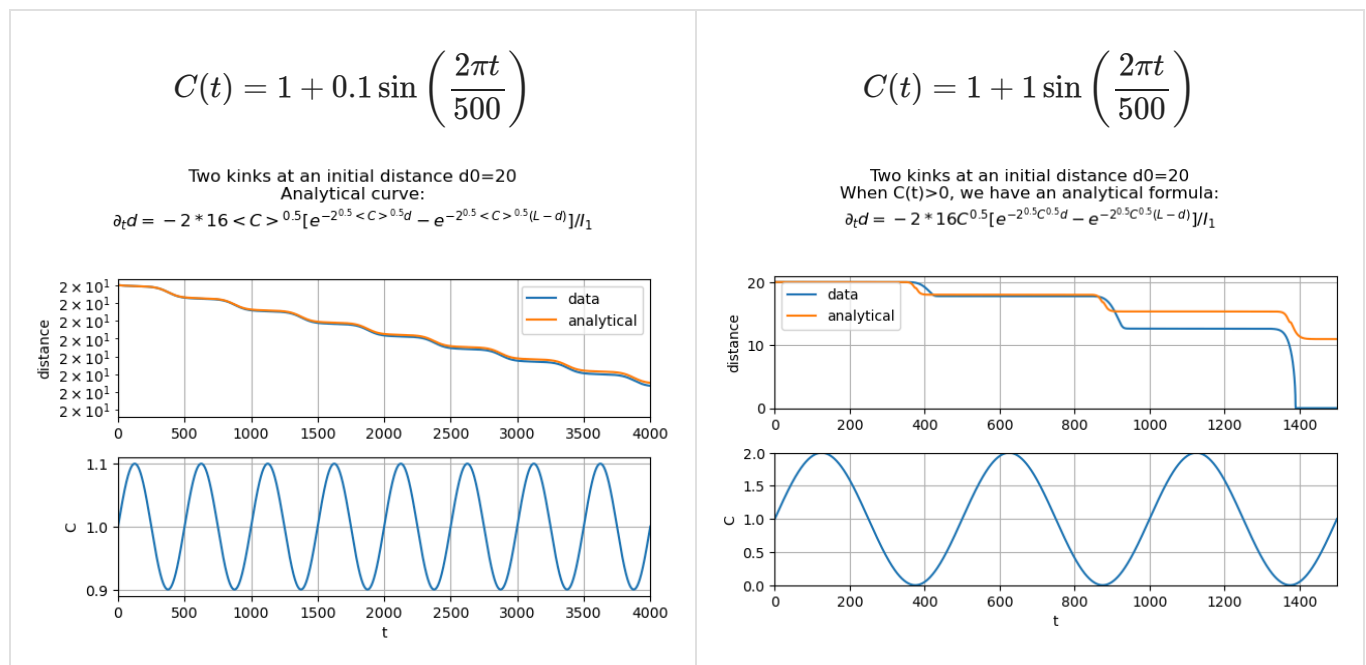
In this case, we can verify the predicted result that prefactor of the logarithm, is  $(2C)^{-\frac{1}{2}}$ .





## C(t) is a slow and positive oscillation

Here we can compare the expected law for  $\dot{d}$  with a numerical simulation. Here the equation for  $\dot{d}$  is integrated with Explicit Euler.



There isn't a good match when  $A = 1$ , probably because the distance decays when  $C$  is very close to zero and there **the intrinsic timescale of the problem**  $\tau_C \sim C^{-1}$  **diverges**, so we are no more in the limit of slow oscillations.

## Comparing $\partial_t d$

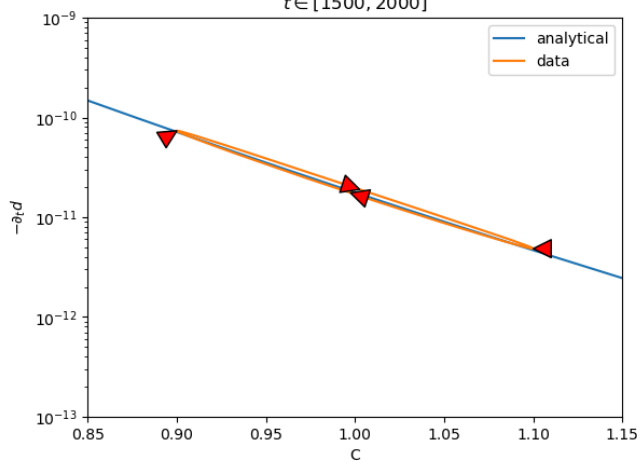
Below, if the measured value of  $\partial_t d$  is less than 1e-15, then it is put to 1e-17.

$$\bar{C} = 1; T = 500$$

**A=1/10**

Analytical result:  $\partial_t d = -2 * 16C^{0.5}[e^{-2^{0.5}C^{0.5}d} - e^{-2^{0.5}C^{0.5}(L-d)}]/I_1$

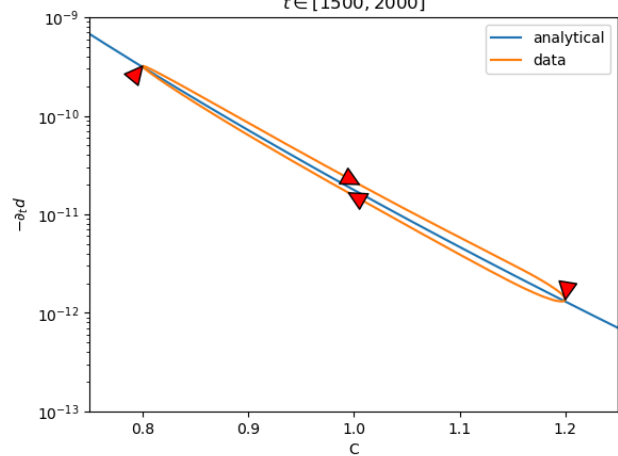
$d = 20, L = 102.4, I_1 = 2\sqrt{2}/3$   
Data:  $C(t) = 1 + 0.1\sin(2\pi t/500)$   
 $t \in [1500, 2000]$



**A=1/5**

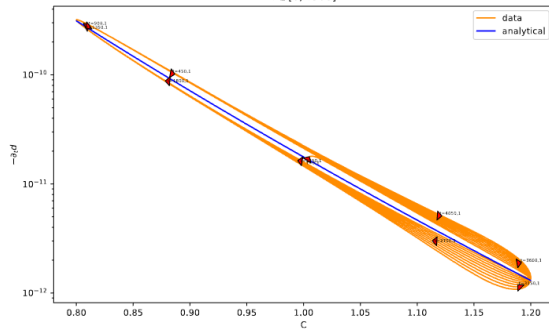
Analytical result:  $\partial_t d = -2 * 16C^{0.5}[e^{-2^{0.5}C^{0.5}d} - e^{-2^{0.5}C^{0.5}(L-d)}]/I_1$

$d = 20, L = 102.4, I_1 = 2\sqrt{2}/3$   
Data:  $C(t) = 1 + 0.2\sin(2\pi t/500)$   
 $t \in [1500, 2000]$



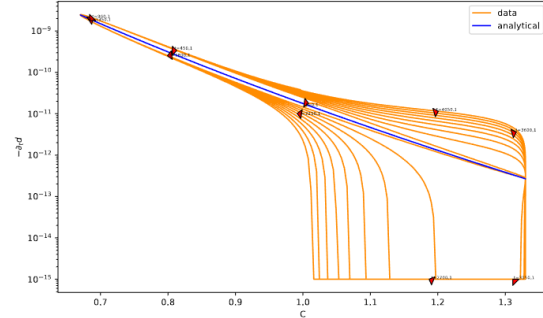
**A=0.2**

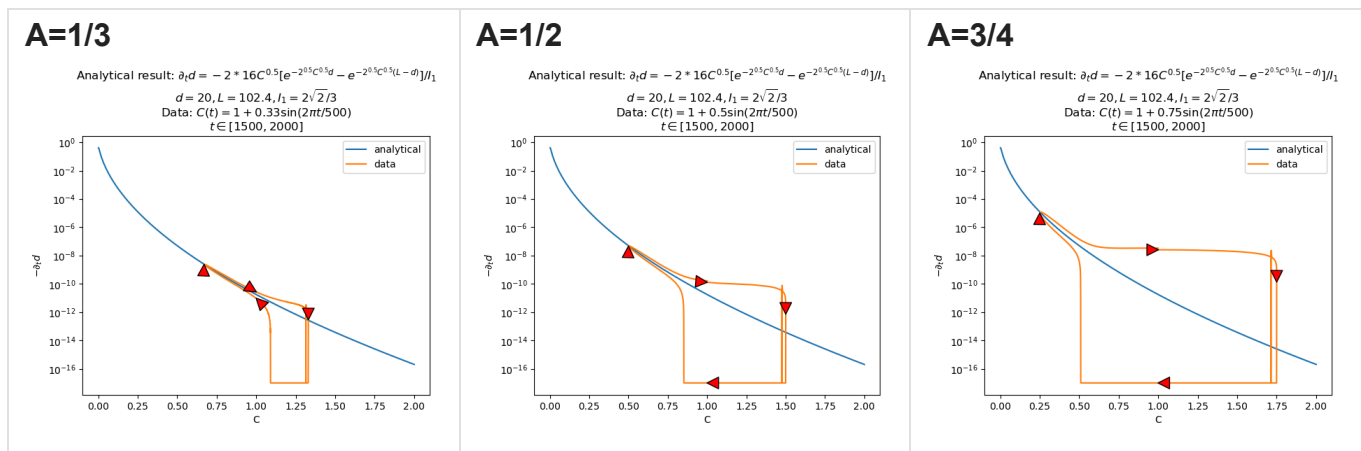
$d = 20, L = 102.4, I_1 = 2\sqrt{2}/3$   
Data:  $C(t) = 1 + 0.2\sin(2\pi t/500)$   
 $t \in [0, 4500]$



**A=0.33**

$d = 20, L = 102.4, I_1 = 2\sqrt{2}/3$   
Data:  $C(t) = 1 + 0.33\sin(2\pi t/500)$   
 $t \in [0, 4500]$



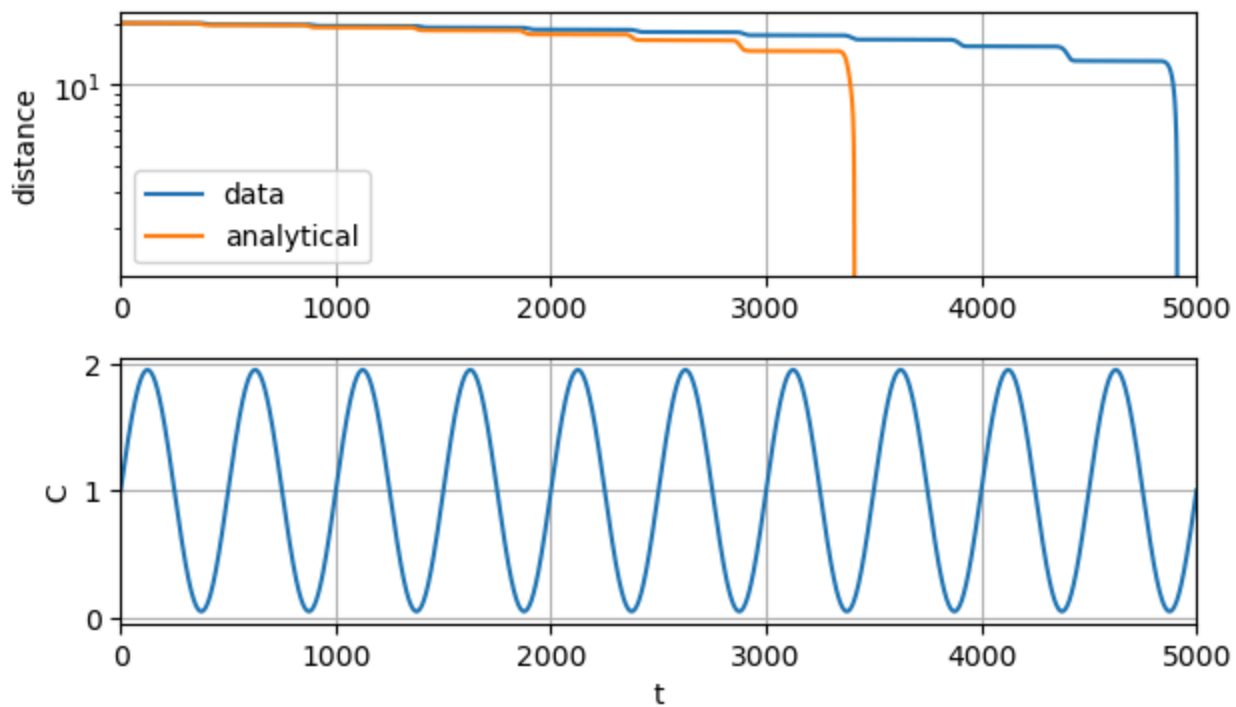


An if  $C(t)$  gets too close to zero (**A=0.95**)

Two kinks at an initial distance  $d_0=20$

Analytical curve:

$$\partial_t d = -2 * 16 < C >^{0.5} [e^{-2^{0.5} < C >^{0.5} d} - e^{-2^{0.5} < C >^{0.5} (L-d)}] / l_1$$

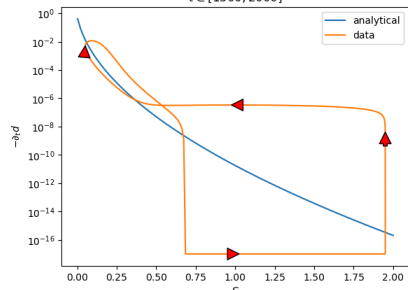


Analytical result:  $\partial_t d = -2 * 16C^{0.5}[e^{-2^{5.5}C^{0.5}d} - e^{-2^{5.5}C^{0.5}(L-d)}]/l_1$

$d = 20, L = 102.4, l_1 = 2\sqrt{2}/3$

Data:  $C(t) = 1 + 0.95\sin(2\pi t/500)$

$t \in [1500, 2000]$

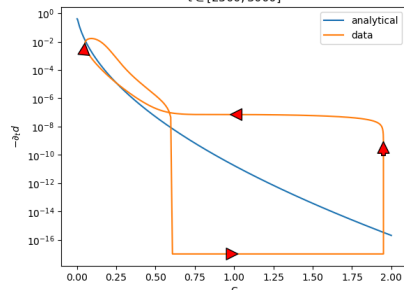


Analytical result:  $\partial_t d = -2 * 16C^{0.5}[e^{-2^{5.5}C^{0.5}d} - e^{-2^{5.5}C^{0.5}(L-d)}]/l_1$

$d = 20, L = 102.4, l_1 = 2\sqrt{2}/3$

Data:  $C(t) = 1 + 0.95\sin(2\pi t/500)$

$t \in [2500, 3000]$



Analytical result:  $\partial_t d = -2 * 16C^{0.5}[e^{-2^{5.5}C^{0.5}d} - e^{-2^{5.5}C^{0.5}(L-d)}]/l_1$

$d = 20, L = 102.4, l_1 = 2\sqrt{2}/3$

Data:  $C(t) = 1 + 0.95\sin(2\pi t/500)$

$t \in [3500, 4000]$

