Kinks effective dynamics under slow POSITIVE oscillations

#twokinks #1D #costantC

If C(t) is

- **Strictly positive** $C(t) > 0 \forall t$
- And its **oscillations are slow** $(T \gg \tau_c)$
  following the idea presented in Kink effective dynamics.pdf (and generalized for C constant $\rightarrow$ slow oscillations limit in Kink effective dynamics under slow POSITIVE oscillations (theory)) it is possible to describe the evolution dictated by the TDGL with an **effective law** for the velocity of each kink. If x_n is the position of the n-th kink (the n-th zero of $u(x)$) and $l_n \equiv x_{n+1} - x_n$ is the lenght of the n-th domain:

  $$\dot{x_n}(t) = 16C^{\frac{1}{2}}(t)\frac{[e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}l_n} - e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}l_{n+1}}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi)}$$

  where $u_p(\chi)$ is the periodic stationary state with period $(\chi_{n+1} - \chi_{n_+1})$ and $\chi_n = C(t)^{\frac{1}{2}x_n}$.
  If there are only two kinks and PBC boundaries are adopted (so if the distance from the right is d then the distance from the left is $L-d$) the distance $d(t)$ will decrease in this way (L>d)

  $$\dot{d}(t) = -2 * 16C^{\frac{1}{2}}(t)\frac{[e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d} - e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}(L-d)}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi)}$$

  Where the integral at the denominator can be approximated by the integral of the single-kink stationary state and the integration is carried on the whole real axis:
  $\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi) \simeq \int_{-\infty}^{+\infty} d\chi u_k(\chi) = I_1$
  where I_1 has been calculated in the Master Report.pdf and $I_1 = \frac{2\sqrt{2}}{3}$.

If also the smaller exponential is neglected, in the limit $L \gg d$

$$\dot{d}(t) \simeq -24\sqrt{2}C^{\frac{1}{2}}(t)e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d}$$

Annihilation time

To estimate the time-scale of the annihilation process, we consider the case where C is constant. The solution to the differential equation for $\dot{d}$ is

$$d(t) = A + \log(\alpha(t_c - t))$$

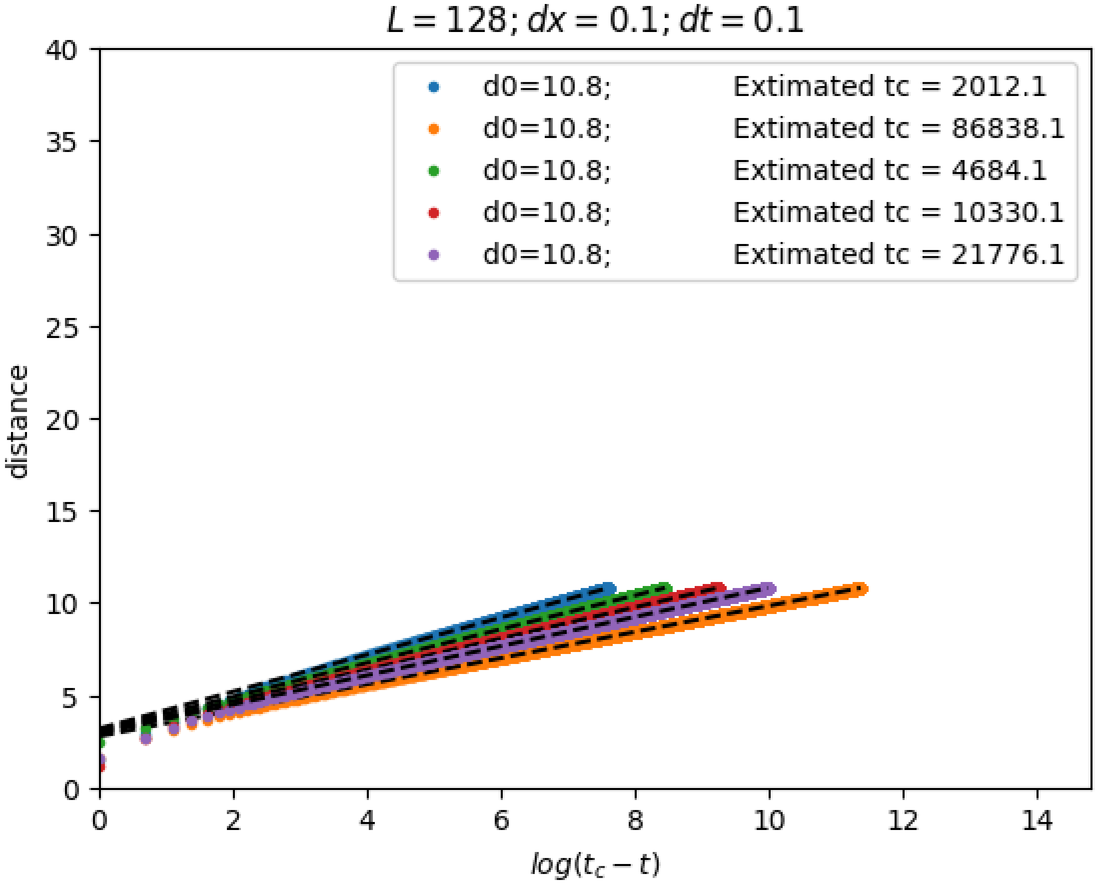
$A = (2C)^{-0.5}$; $\alpha = 48C$ and the annhilation time is

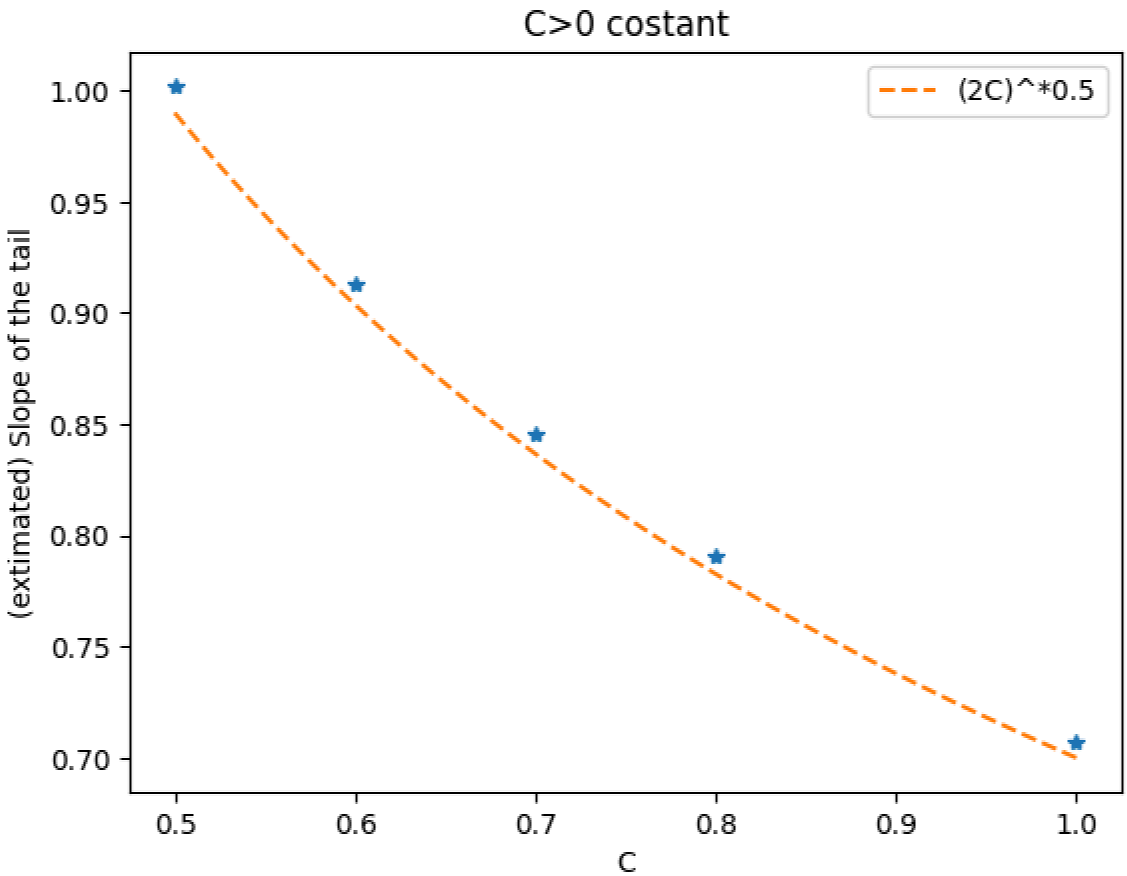
$$t_c = \frac{e^{d_0/A}}{\alpha} = \frac{e^{d_0(2C)^{0.5}}}{48C}$$

Simulations

C is constant

In this case, we can verify the predicted result that prefactor of the logarithm, is $(2C)^{-\frac{1}{2}}$.

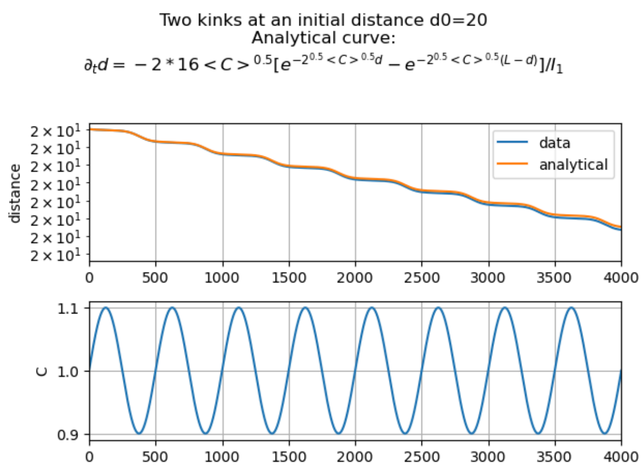




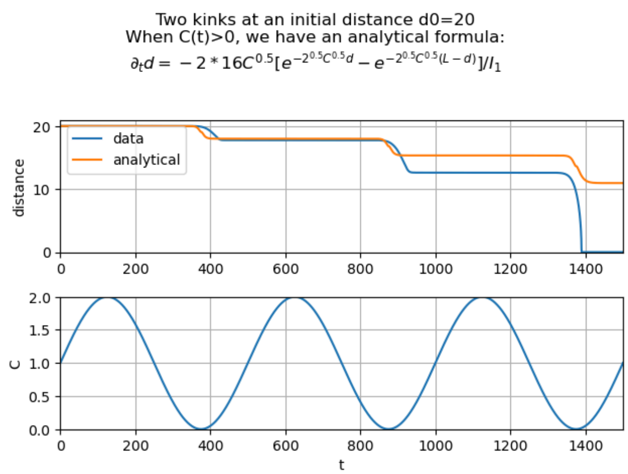
C(t) is a slow and positive oscillation

Here we can compare the expected law for $\dot{d}$ with a numerical simulation. Here the equation for $\dot{d}$ is integrated with Explicit Euler.

$$C(t) = 1 + 0.1 \sin\left(\frac{2\pi t}{500}\right)$$



$$C(t) = 1 + 1 \sin\left(\frac{2\pi t}{500}\right)$$



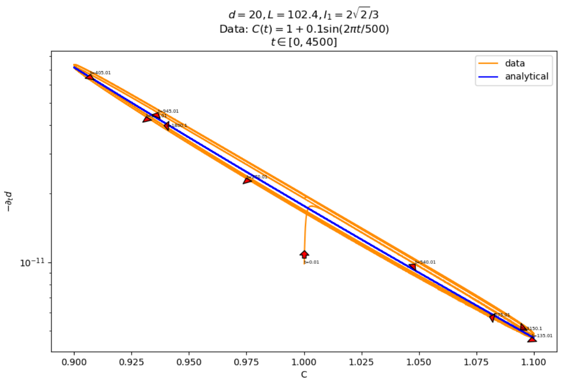
There isn't a good match when $A = 1$, probably because the distance decays when C is very close to zero and there **the intrinsic timescale of the problem $\tau_C \sim C^{-1}$ diverges**, so we are no more in the limit of slow oscillations.

Comparing $\partial_t d$

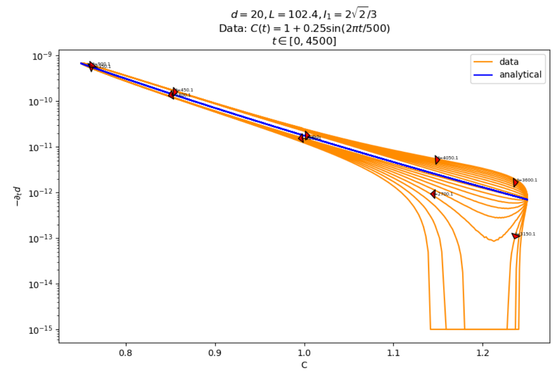
Below, if the measured value of $\partial_t d$ is less than 1e-15, then it is put to 1e-17.

$$\bar{C} = 1; T = 500$$

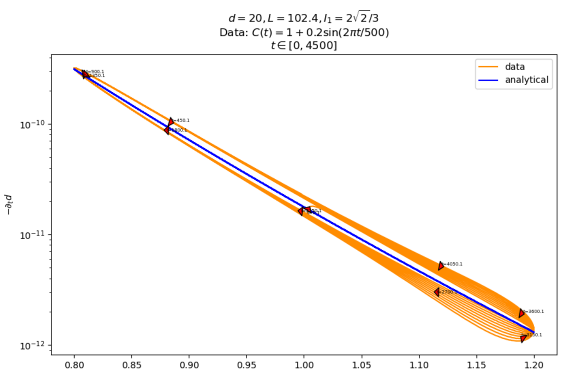
A=1/10



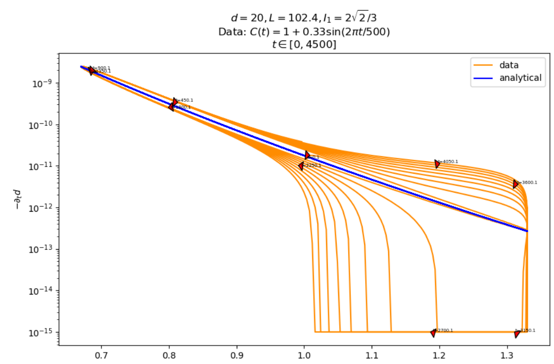
A=1/4



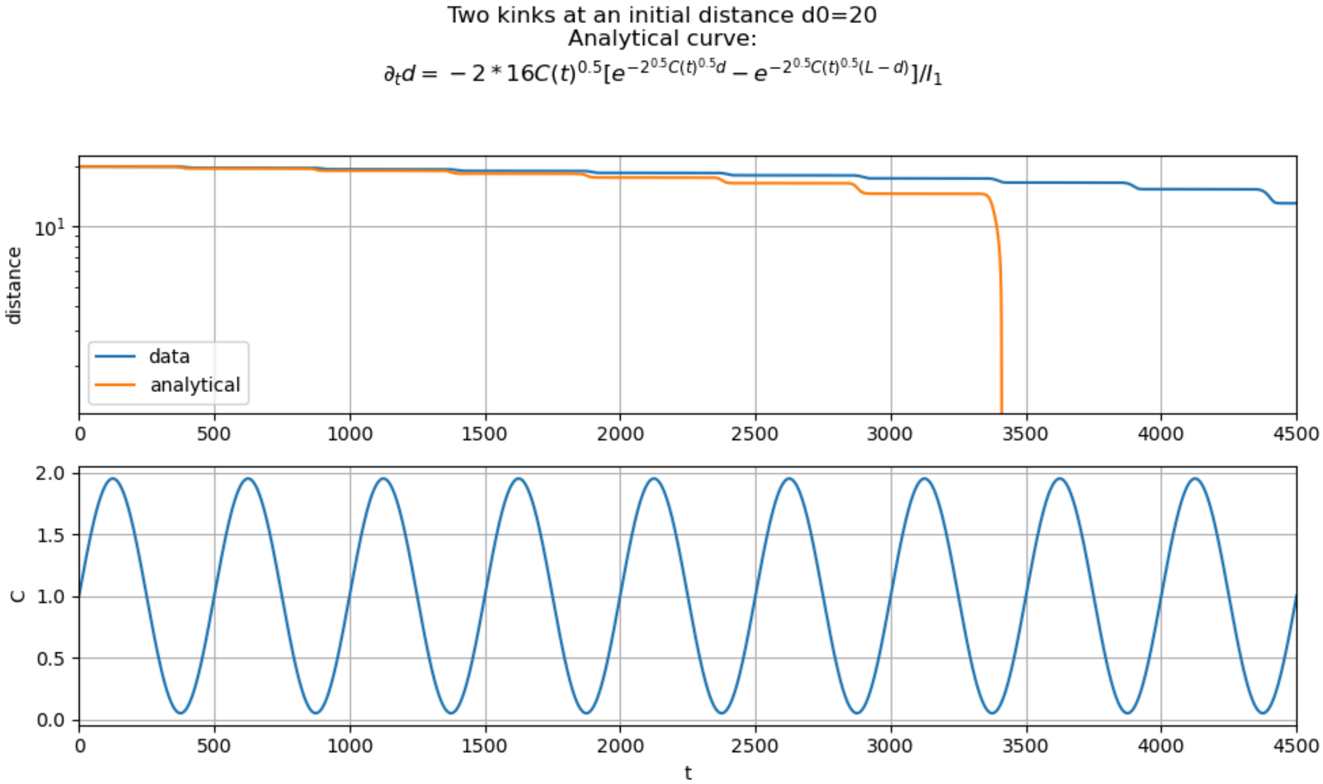
A=1/5

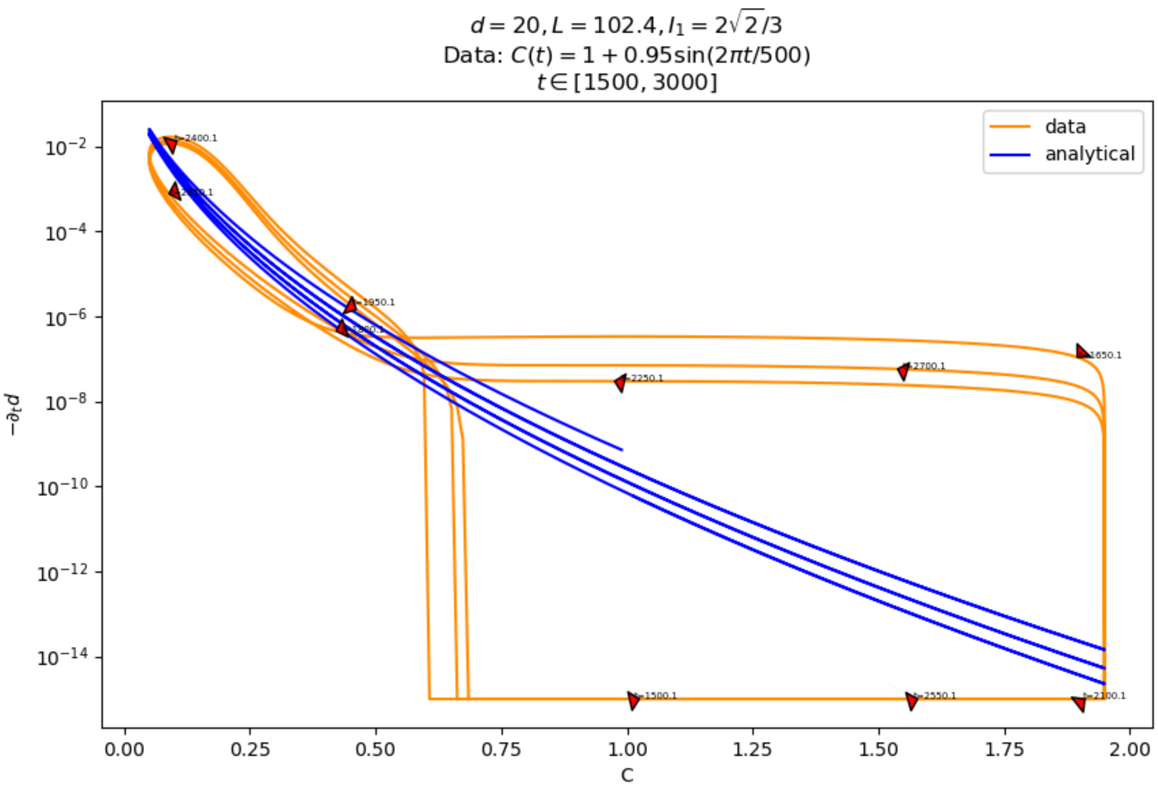


A=1/3



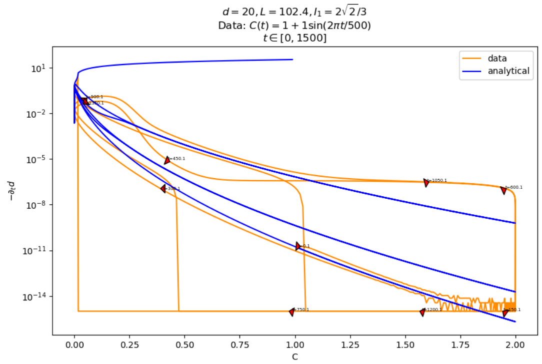
An if $C(t)$ gets too close to zero (**A=0.95**)



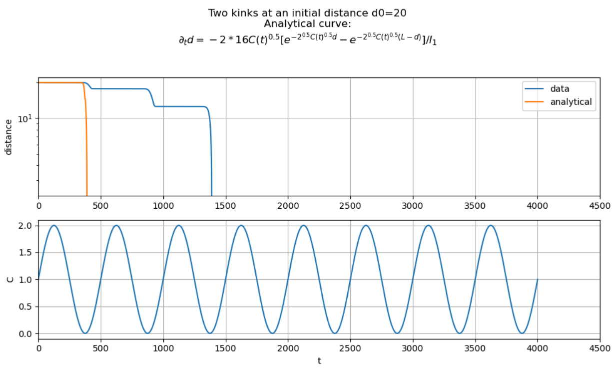


An if $C(t)$ gets negative

A=1

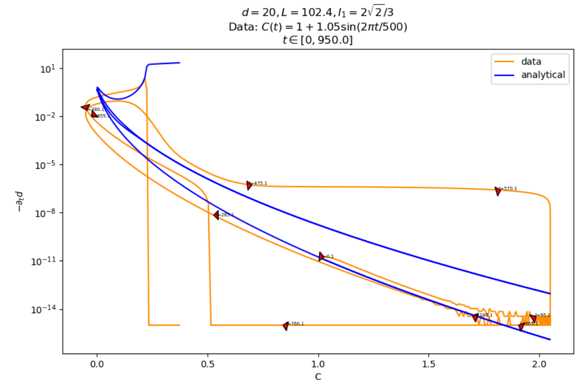


$d = 20, L = 102.4, l_1 = 2\sqrt{2}/3$
Data: $C(t) = 1 + 1\sin(2\pi t/500)$
$t \in [0, 1500]$
data
analytical
$-\partial_t d$
C

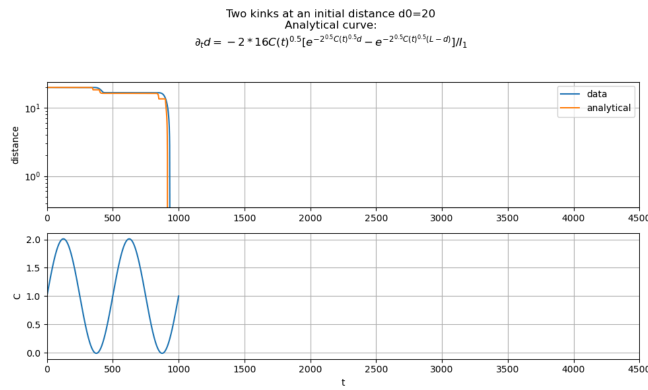


Two kinks at an initial distance d0=20
Analytical curve:
$\partial_t d = -2*16C(t)^{0.5}[e^{-2^{0.5}C(t)^{0.5}d} - e^{-2^{0.5}C(t)^{0.5}(L-d)}]/l_1$
data
analytical
distance
C
t

A=1.05



$d = 20, L = 102.4, l_1 = 2\sqrt{2}/3$
Data: $C(t) = 1 + 1.05\sin(2\pi t/500)$
$t \in [0, 950.0]$
data
analytical
$-\partial_t d$
C



Two kinks at an initial distance d0=20
Analytical curve:
$\partial_t d = -2*16C(t)^{0.5}[e^{-2^{0.5}C(t)^{0.5}d} - e^{-2^{0.5}C(t)^{0.5}(L-d)}]/l_1$
data
analytical
distance
C
t