Explaining the decay of the distance of two kinks with linear dynamics

#twokinks #1D #linear_regime

Motivation

If $C(t) < 0$ for a long time (as it happens in the cases above) eventually $u(x) \ll C_0$ and we expect the non-linearity to play a negligible role in the dynamics. So it is **natural** to expect that **the LINEAR dynamics is SUFFICIENT**.
Even if the initial state is, in principle, relevant for the decay $d(t)$, we build the initial state by summing Gaussian functions, for simplicity in the calculations.

Goal

Here we see if we can explain **how the distance** between two kinks **decays** in the cases of

- Quench to $C < 0$ constant;
- Slow oscillating $C(t)$ with $A \gg C_0$
  by calculating **analytically** how the distance of **two gaussian-shaped kinks** decays **according to LINEAR dynamics.**

- Motivation
- Goal
- Model
  - Parameters of the model
  - Decay of the distance
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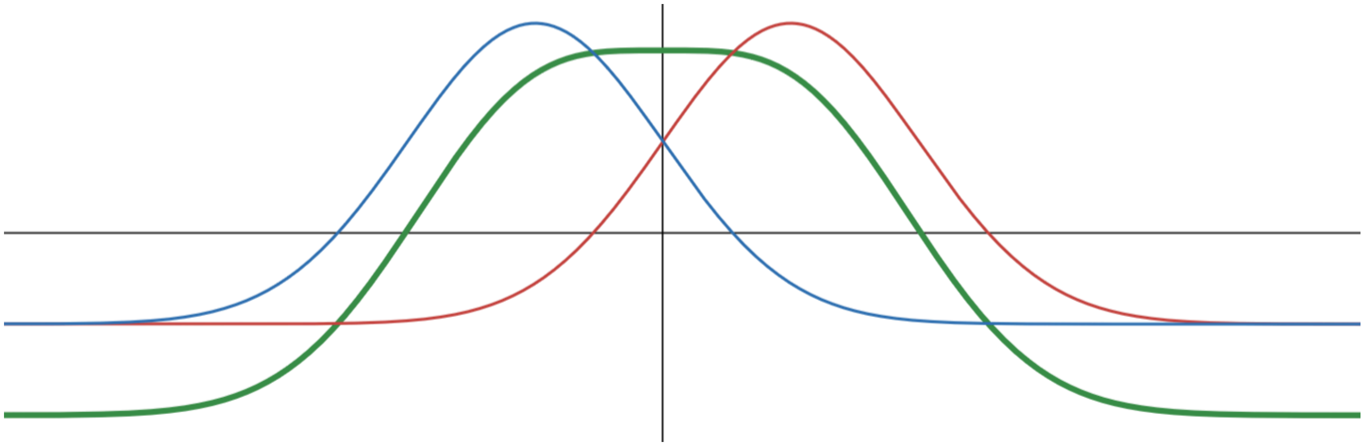
- Experiments
  - 1) Quench to $C<0$
- 2) Slow oscillations $A\gg C{0}$

Model

We compute analytically the decay of the distance of this initial state, according to linear dynamics.

$$u(x, t_0) = u_0 \mathcal{N} \left[g_+(x) + g_-(x) - \frac{e^{-1/2}}{\sqrt{2\pi}\sigma} \right]$$



$$g_{\pm}(x) = \frac{e^{-(x-x_{\pm})^2/2\sigma^2}}{\sqrt{2\pi}\sigma}$$

As $\partial_{xx} g_{\pm}(x_{\pm}) = \frac{3e^{-2}}{\sigma^2}$, then we choose $L = 2\sigma$ (where $L = x_+ - x_-$) such that the second derivative is zero in the midpoint (**plateau**).

Parameters of the model

There is **only one parameter** to be set in order to compute the prediction of $d(t)$, that is **the initial width** σ of the kinks. That's because $x_+ - x_- = 2\sigma$ is the condition required to have a plateau between the gaussians (a property that is kept with time as the width of the two gaussian is the same).
The initial distance between kinks d_0, is related to the distance L between the centers of the Gaussians ($x_{\pm}$) and their initial width σ as

$$d_0 \simeq L + 2\sigma$$

then

$$\sigma \simeq \frac{d_0}{4}$$

When **approximating** the shape of $u(x)$ to a sum of two Gaussians, this is the **natural** way of determining the (only) parameter σ, by measuring the distance between the kinks.

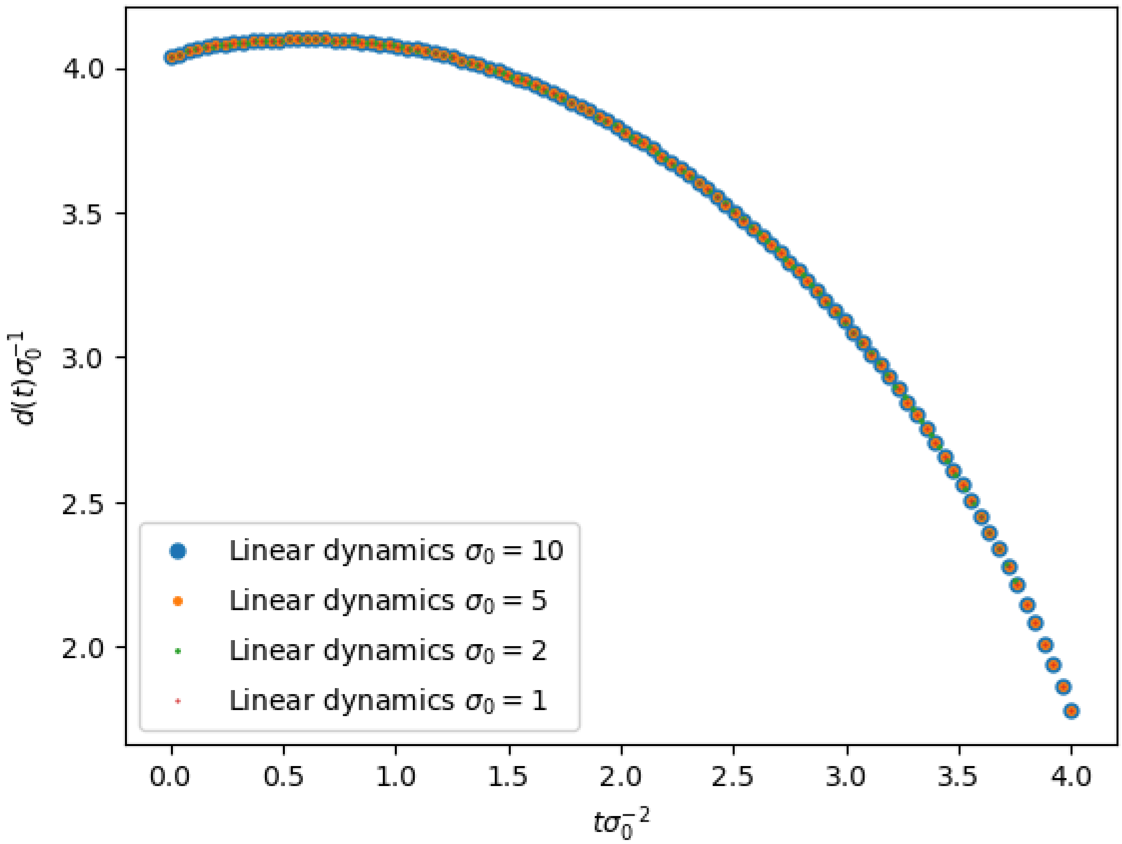
Decay of the distance

In Linear dynamics twokinks with Gaussian profile is calculated analytically the evolution of the above profile, under the linear dynamics

$$u(x,t) = e^{B(t)} u_0 \mathcal{N} \left[\frac{e^{-(x-x_+)^2/2\sigma(t)^2}}{\sqrt{2\pi}\sigma(t)} + \frac{e^{-(x-x_-)^2/2\sigma(t)^2}}{\sqrt{2\pi}\sigma(t)} - \frac{e^{-1/2}}{\sqrt{2\pi}\sigma} \right]$$

I cannot write a formula for $d(t)$, but I can find it numerically with the **Newton's algorithm**.

Properly rescaling the axis, I find a profile whose shape is not dependent on the only parameter σ.



Comparison with experiments

In below experiments

- **Blue** represents a simulations
- **Red** represent the model, where the parameter σ is determined by measuring the distance between kinks at the **beginning of the decay** d_0, as $\sigma = \frac{d_0}{4}$.
  The **beginning of the decay** seems to be (empirically) the moment t_0 **when** $C(t_0) = 0; \dot{C}(t_0) < 0$ (when $C(t)$ becomes negative). In a quench experiment, instead, $t_0 = 0$.

NOTICE: In experiments, $d(t)$ is stricly **decreasing**, while in the model it increases at the beginning. I guess this is due to the fact that, when $t \simeq t_0$ (beginning of the decay) the shape of the kinks matters, while then (I hope) it does not anymore.

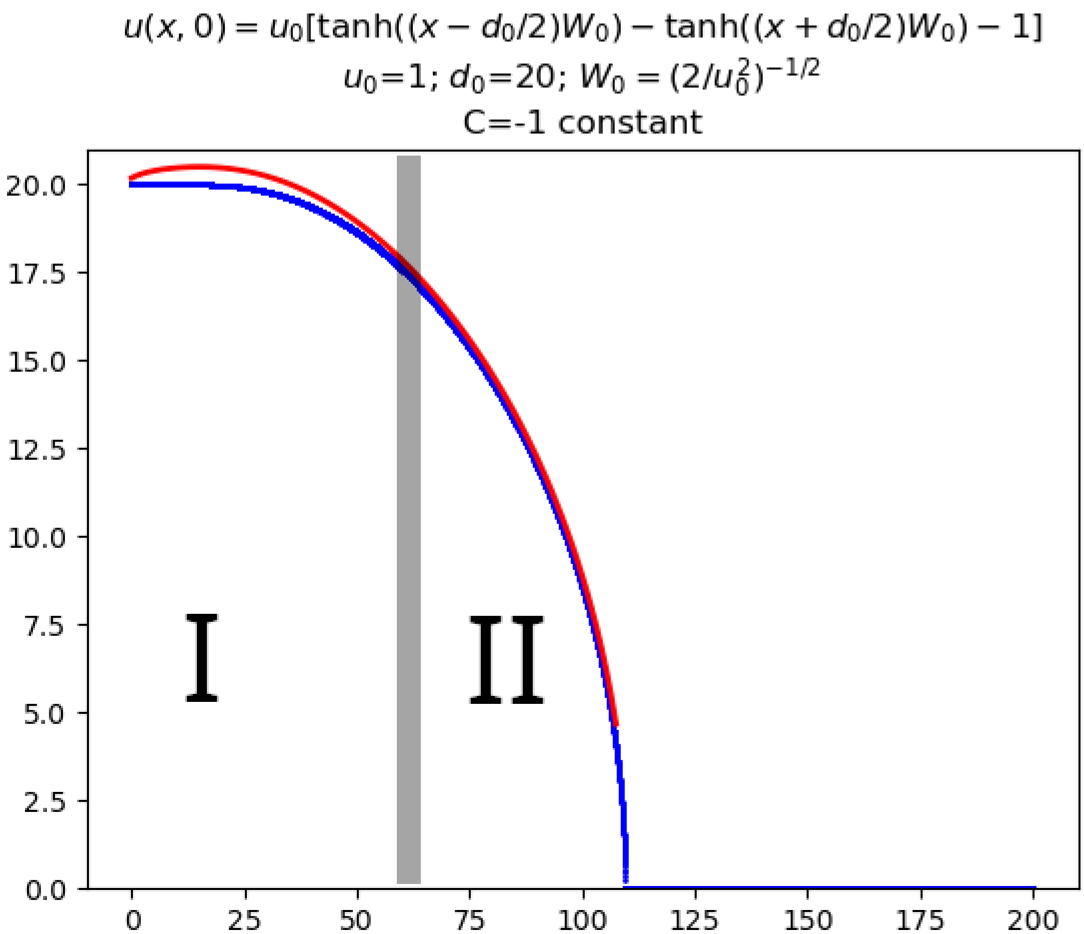
Experiments

We **plot** $d(t)$ as a function of t.

1) Quench to $C < 0$

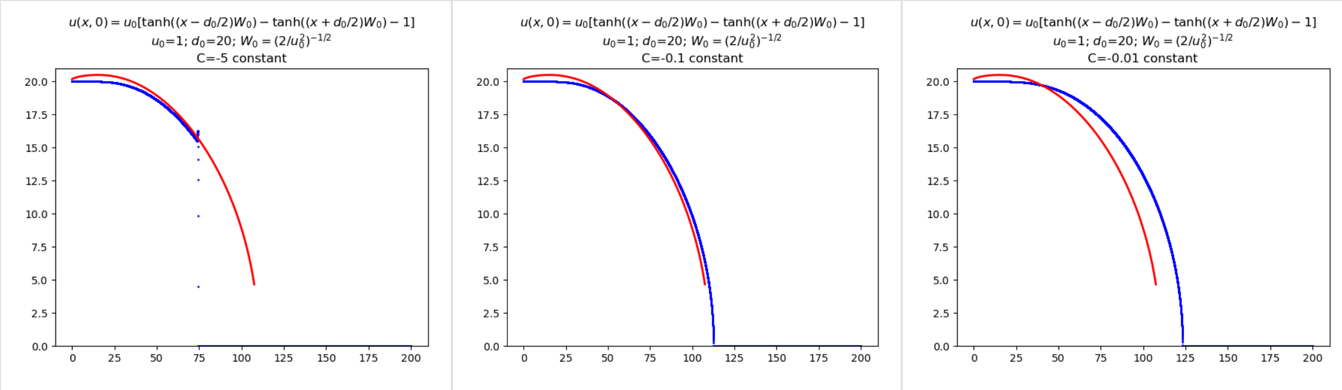
Paramters (simulation):

- u_0: amplitude of the initial state **and** $w = d_0^{-1/2}$ initial width of the kinks
- d_0: initial distance beetween kinks
- $C < 0$: constant value of C

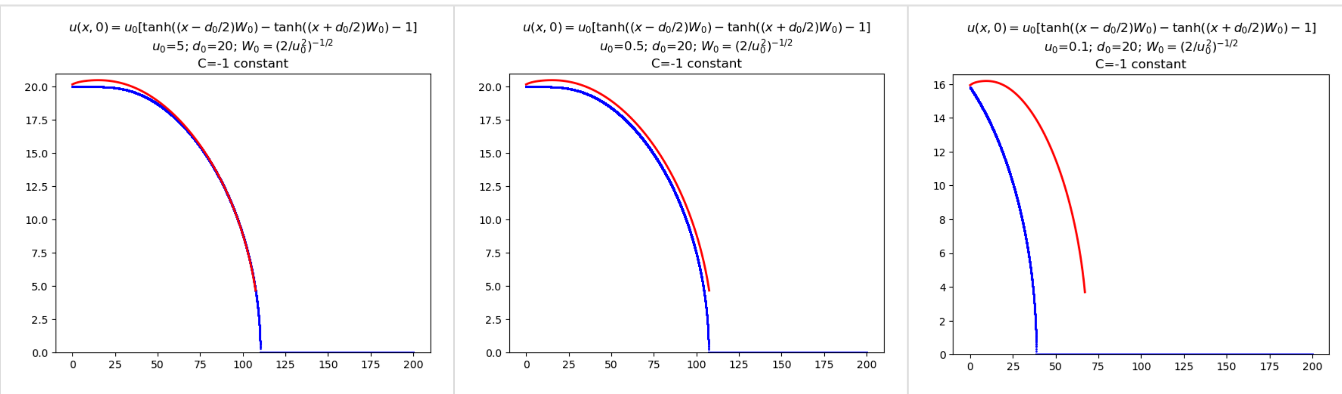


- There is a **good fit** in the region **II**, **without any shift!** I was expecting a shift, because I expect the deviation in region **I** to be an effect of the non-linearity, that is non negligible at the beginning: when $u \sim 1$. Then I was expecting this deviation to have a consequence in the 'linear regime' like a shift, while having the same shape of the decay.

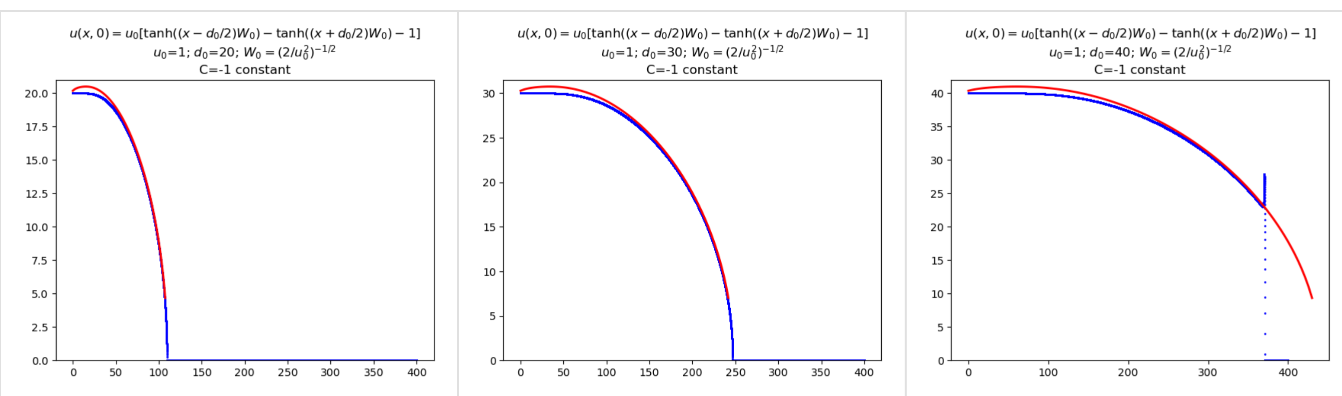
When $|C| \sim 0.1$ the fit is not good, not even in the **II** region. I guess this is due to the relevance of the non linearity up to longer times, as u decays slower to zero in this case.



- **Surprisingly** if the initial amplitude is small $u_0 \sim 0.1$, the fit is bad also in the **II** region. I would expect it to be better, as the non-linearity is less important!



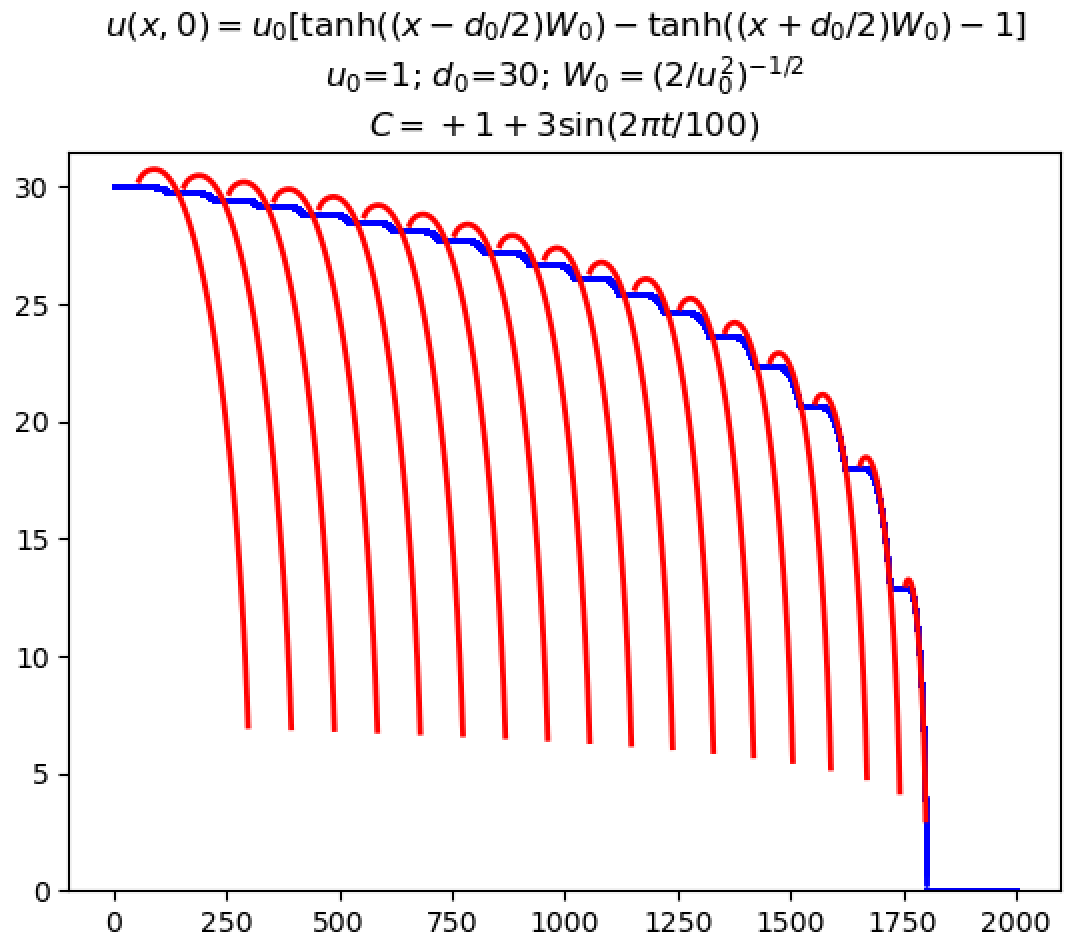
While the goodness of the fit does not depend on the initial distance d_0. For distance too large, it is not possible to sample the whole decay, as u gets too close to zero before the decay happens.



2) Slow oscillations $A \gg C_0$

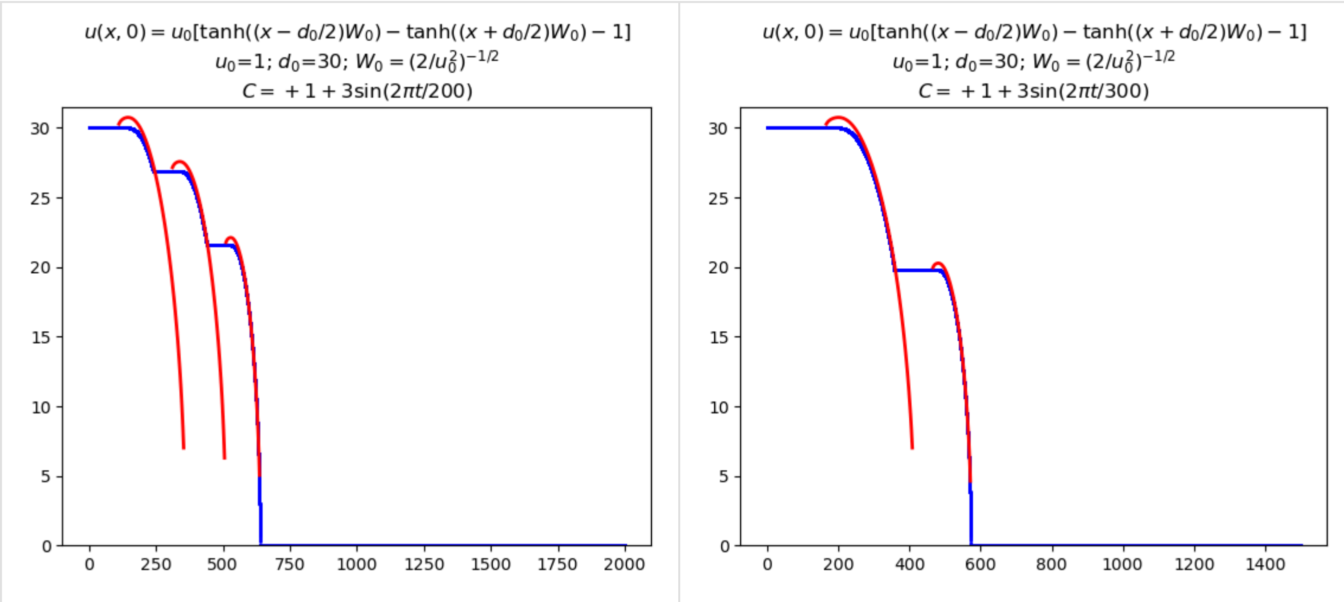
$$C(t) = C_0 + A \sin\left(\frac{2\pi t}{T}\right); \quad C_0 = 1$$

Here we consider the beginning of the decay t_0 as the moment, within a period, when $C(t)$ starts to take negative values: $C(t_0) = 0; \dot{C}(t_0) < 0$.



The fit is good when the **depth** (variation of d) of the decay is **large**.
To have deeper decays, you can

- Increase the time $C(t) < 0$: increase T (**below**);
- Decrease the distance at the beginning of the decay $d0$ (**above**, the decay is deeper in the last periods, when d_0 is smaller).



I was expecting the model to fit the data when there is **overlap** beetween the kinks. That's because in the model, $d_0 = 4\sigma$ that, interpreting σ as the kink's width is

$$\frac{4W_0}{d_0} = 1$$

so I was expecting the model to work when the ratio is of order 1 or more. But it is not the case as you can see here. Even during the first periods, where the fit is poor, the ratio is higher than

1 for a long time (the time when $C(t) < 0$).