1D Fast oscillations (Analytical)

Multiple scale analysis

> Fast oscillations 1D
>
> We say that the oscillations of C(t) are **slow**, if the time scale τ_C associated with the time variation of $C(t)$ (that, as $C(t)$ is periodic, can be identified as its **period** T) is **SMALL compared** to the **intrinsic time-scale** of the system τ_{linear} (see here).
> So we can define a **small parameter** ϵ as:
>
> $$\epsilon = \frac{\tau_C}{\tau_{linear}}$$
>
> ## Idea
>
> Then, assuming that $\tau_{linear} \sim 1$ (that means $\epsilon \sim \tau_C$), it is natural to make a Multiple scales expansion by introducing the new time-variables
>
> $$t_0 = t, t_{-1} = \epsilon^{-1} t$$
>
> where the dependence of m(x,t) on t_0 will capture processes occurring at the intrinsic time-scale, and t_{-1} the ones occurring at the time-scale of C(t)'s oscillations.
> This means that
>
> $$\partial_t = \partial_{t_0} + \epsilon^{-1} \partial_{t_{-1}}$$
>
> where
>
> $$C(t) = \tilde{C}(t_{-1}) \implies \partial_{t_0} \tilde{C}(t_{-1}) = 0$$
>
> From the relation above and the **ansatz**
>
> $$m(x,t) = m_0(x,t) + \epsilon m_1(x,t) + O(\epsilon^2)$$
>
> It follows that
>
> $$m_0(x,t) = \sqrt{\bar{C}} \tanh(x \sqrt{\frac{\bar{C}}{2}})$$
>
> But we could not find an equation for the first order correction $\epsilon m_1(x,t)$

Kink dynamics

> Kink dynamics (Fast oscillations)
>
> In Kink dynamics (Constant C) we obtained, for constant C
>
> $$\dot{d}(t) \simeq -24\sqrt{2}C^{\frac{1}{2}}e^{-2^{\frac{1}{2}}C^{\frac{1}{2}}d}$$
>
> If, instead, $C(t)$ is oscillating
>
> $$C(t) = \bar{C} + A\sin\left(\frac{2\pi t}{T}\right)$$
>
> and the period is **small** compared to the timescale of the system
>
> $$T \ll \tau_{linear} \sim \bar{C}^{-1}$$
>
> then the system **has not enough time to adapt** to the variations of C.
> The profile of an isolated kink, to leading order, does **NOT change in time** and it is
>
> $$u_k(x) = \bar{C}^{1/2}\tanh(2^{-1/2}\bar{C}^{1/2}x)$$
>
> This is the profile of an isolated kink if C is constant and equal to $\bar{C}$, and as the interaction between kinks is determined by the kink's shape (particularly the shape of their tails), we expect
>
> $$\dot{d}(t) \simeq -24\sqrt{2}\bar{C}^{\frac{1}{2}}e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}}d}$$
>
> that is the law valid for constant C, but $C \to \bar{C}$.