Kinks dynamics with Mass

#1D #twokinks #timedependingC

Simulations suggest that the model for kink dynamics developed for **slow oscillations** should be corrected by adding an **inertial term** $+M\partial_{tt}d$ in the differential equation for the distance between two isolated kinks.

$$(M\partial_{tt}d) + \partial_t d = f(C(t))$$

$$f(C(t)) = -24\sqrt{2}C^{\frac{1}{2}}(t)[e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d} - e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}(L-d)}]$$

Now we try to **tune by HAND** the mass **M** to fit the experimental data.

Resumee

- (1) Adding **mass** to the kink dynamics model **fits well** the data **when C is close to its minimum** value (and so when $-\partial_t d$ is large).
- (2) Adding mass does not extend the model to cases when C is very close to zero or negative.
- (3) As time passes, the shape of the trajectory in the region of large C continues to change. An **asymmetry** in the trajectory **never stops to grow** with time. And it is not a numerical effect!

Simulations

We simulated a system with only two kinks, under an oscillatory $C(t)$.
We measured, as a function of time, the distance $d(t)$ and the value o $C(t)$. So we can plot the trajectory $(d(t), C(t))$ and compare it with the trajectory obtained by **numerically solving** the model

Numerical integration of the model

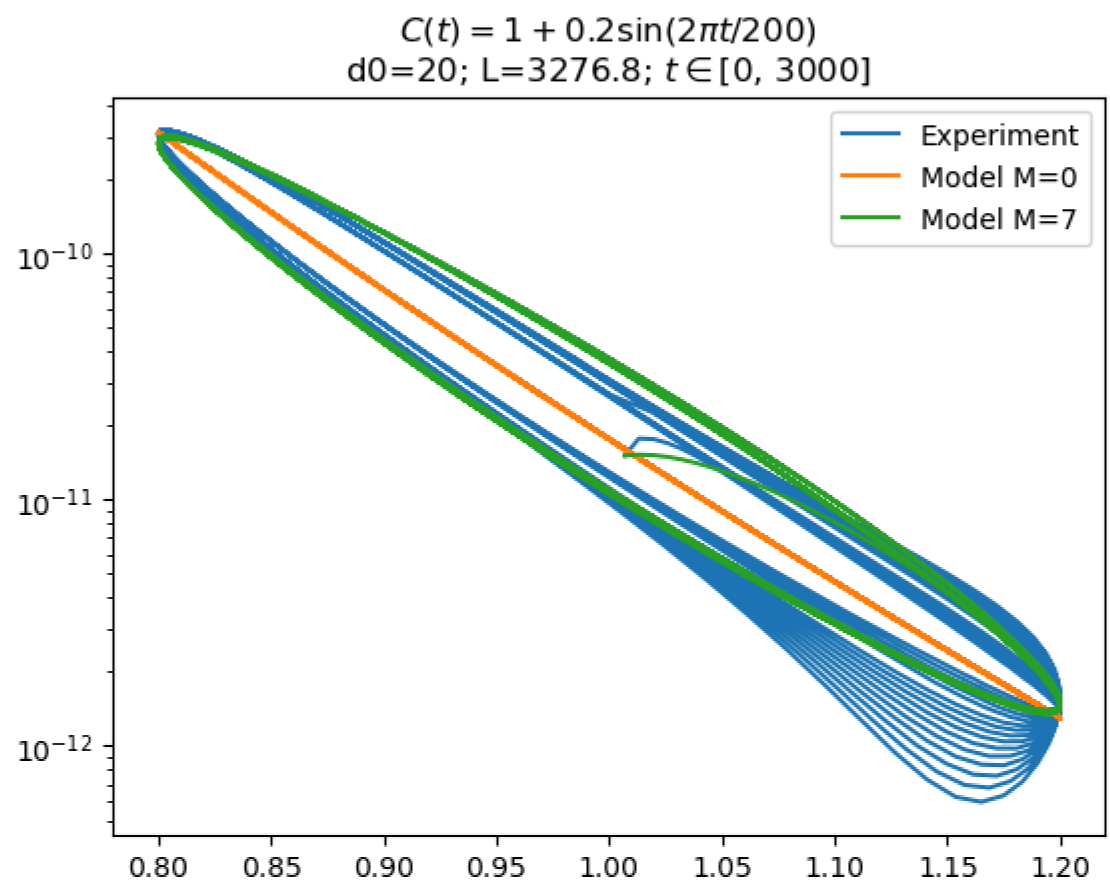
We can define $y = \partial_t d$ so the model can be re-written as

$$\partial_t y = \frac{f(C(t)) - y}{M}$$

$$\partial_t d = y$$

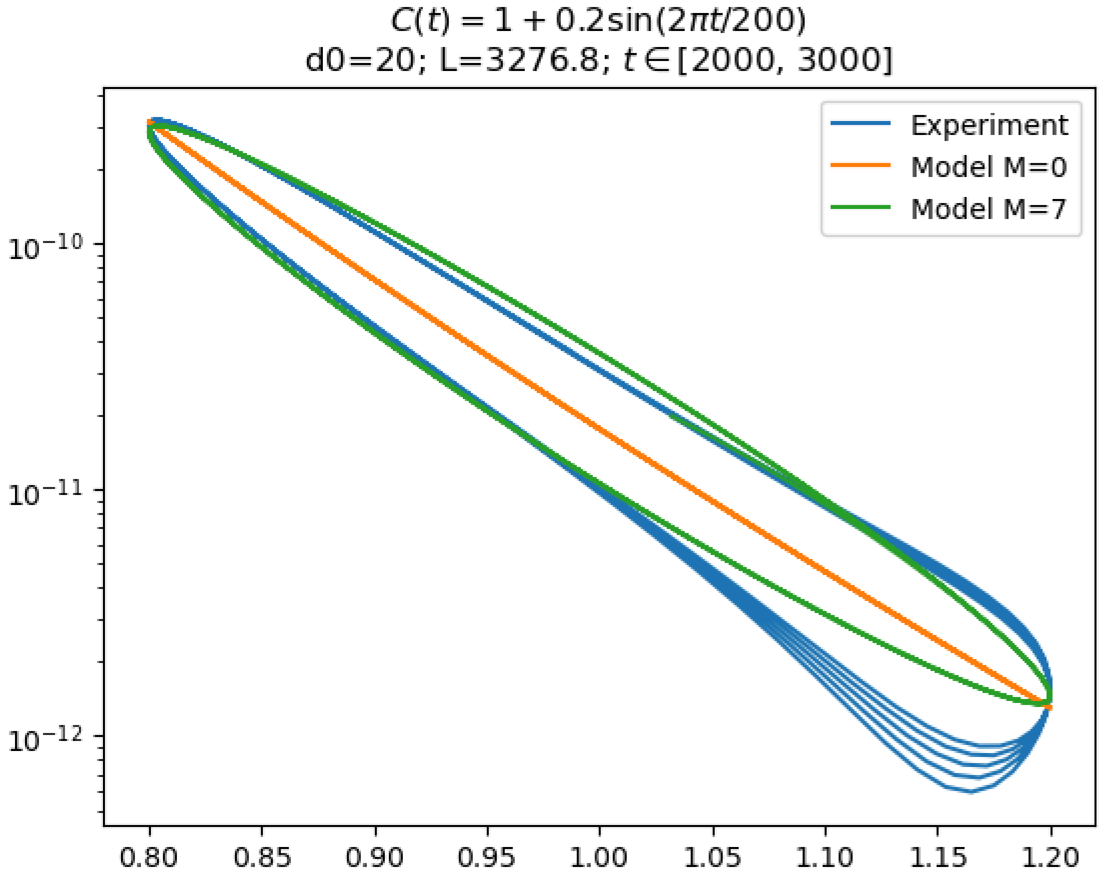
then we can integrate numerically these two coupled equations by using the **Explicit Euler scheme**, to find a solution $(d(t), y(t))$. The initial values d_0, y_0 are taken from the simulation.

We plot $-\partial_t d$ v.s. $C(t)$



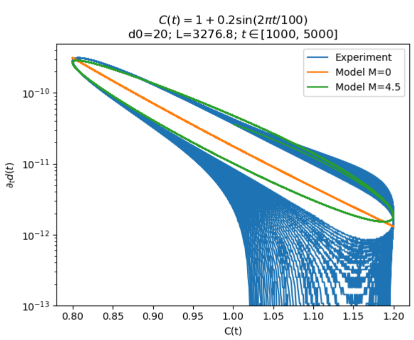
Eliminate initial dynamics

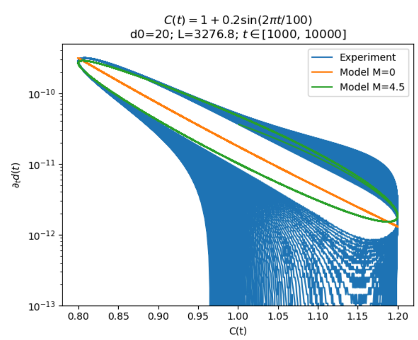
To understand what happens as time passes, here we plot the experimental curve **after some periods from the beginning** of the simulation

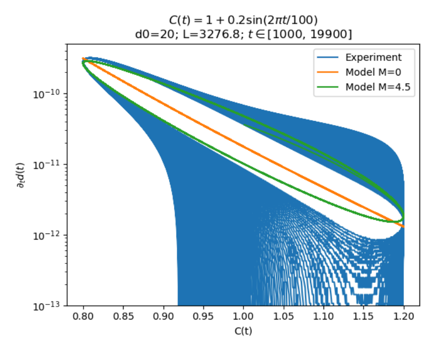


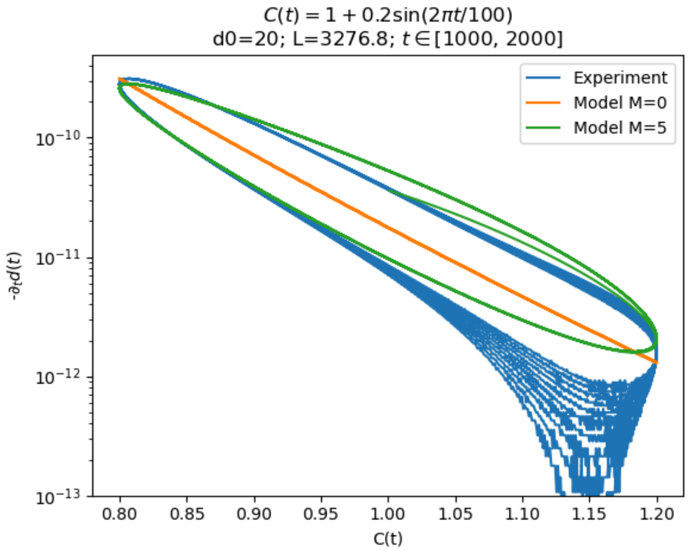
Asymmetry growing with time

This **asymmetry** that we see at large values of C **increases as time passes.**

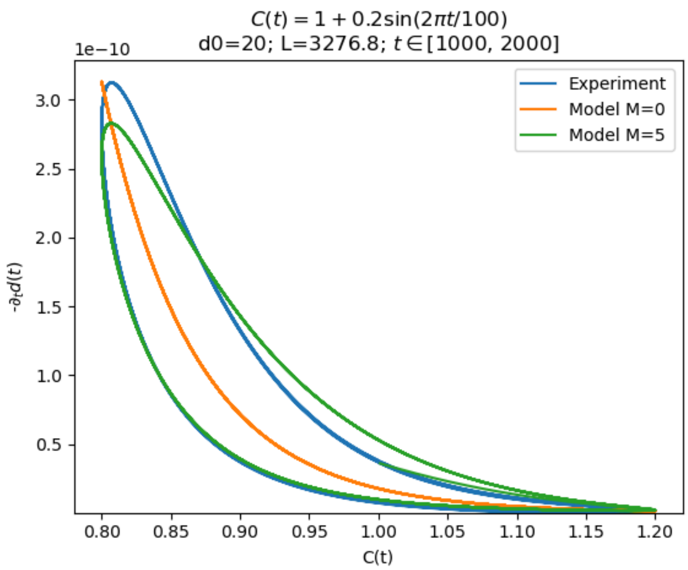




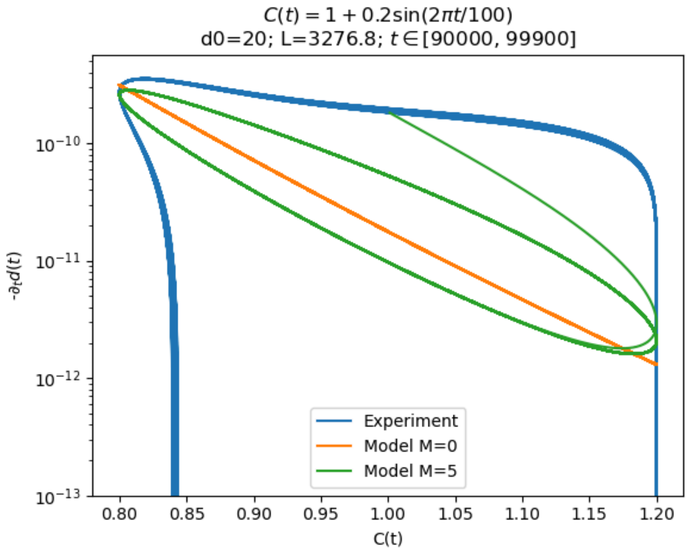


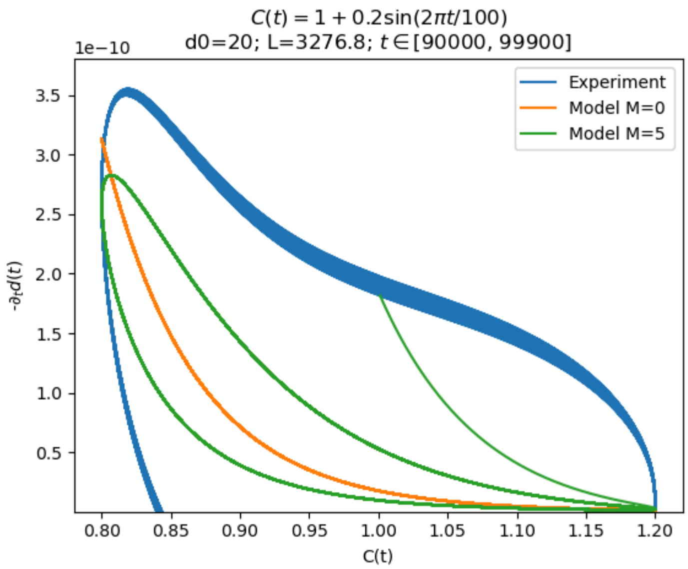


$C(t) = 1 + 0.2\sin(2\pi t/100)$
d0=20; L=3276.8; $t \in [1000, 2000]$
Experiment
Model M=0
Model M=5
$-\partial_t d(t)$
C(t)



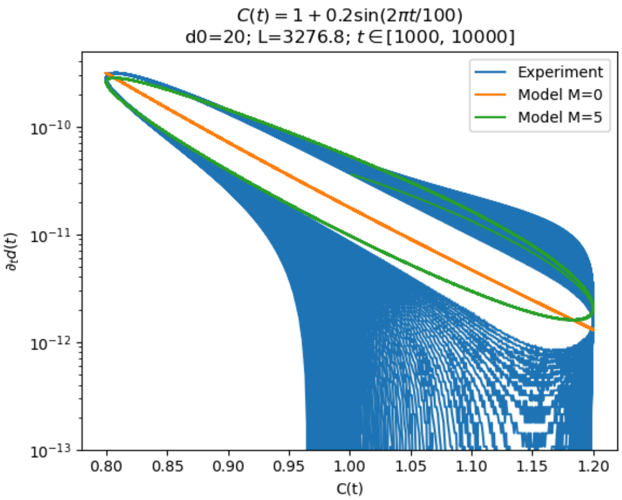
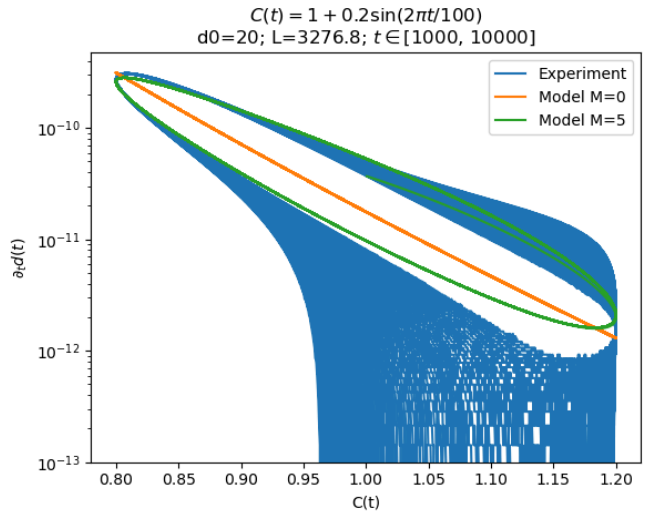
$C(t) = 1 + 0.2\sin(2\pi t/100)$
d0=20; L=3276.8; $t \in [1000, 2000]$
Experiment
Model M=0
Model M=5
1e−10
$-\partial_t d(t)$
C(t)





But, as the distance changes significatively when C is small (and not large), this asymmetry is responsible only for **higher order effects**.

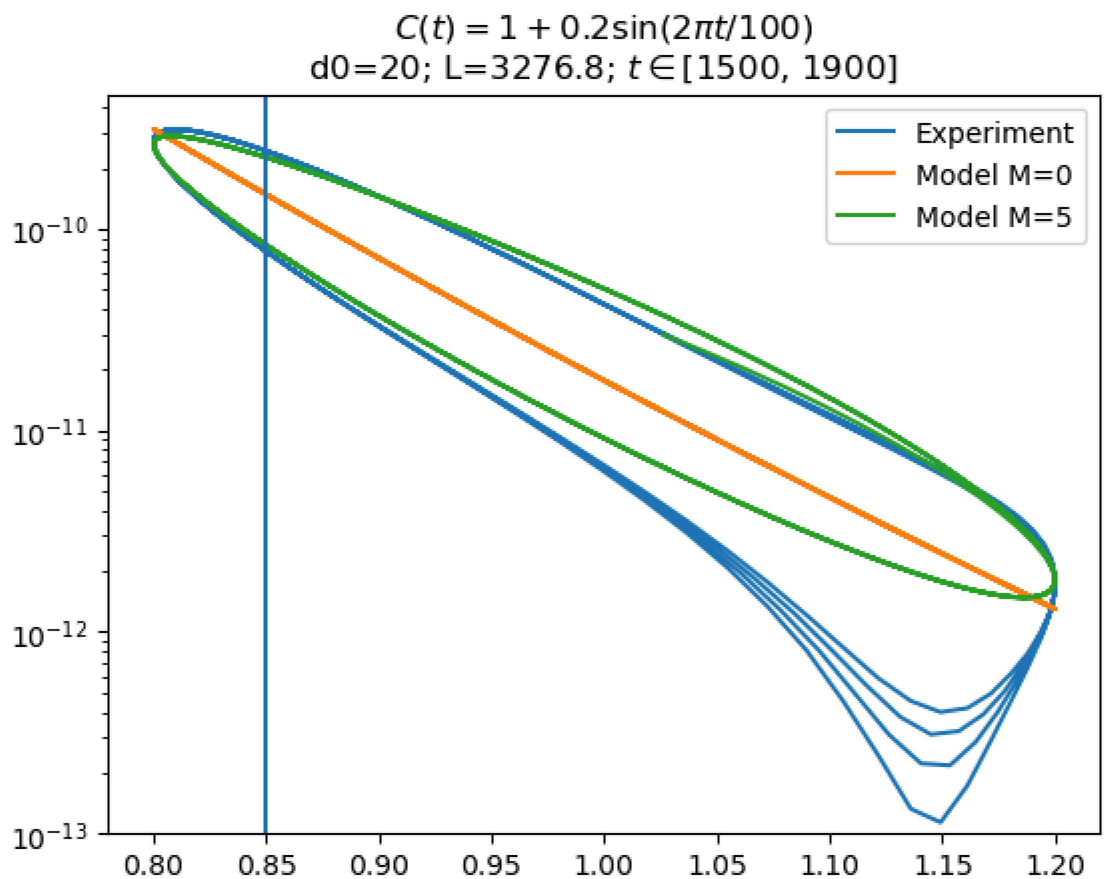
By running a simulation with a lower value of dt (0.05 instead of 0.1) the **asymmetry does NOT decrease**, so it is not a numerical error!

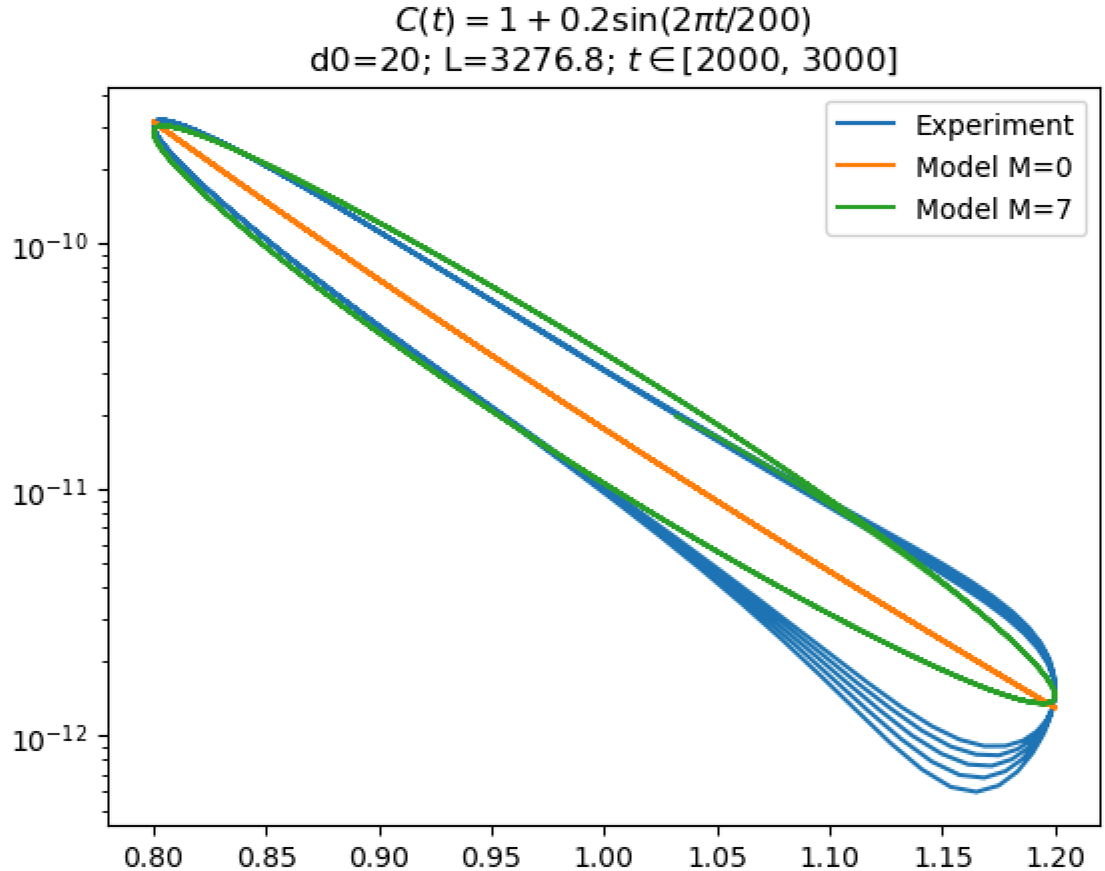
dt=0.1	dt=0.05
	

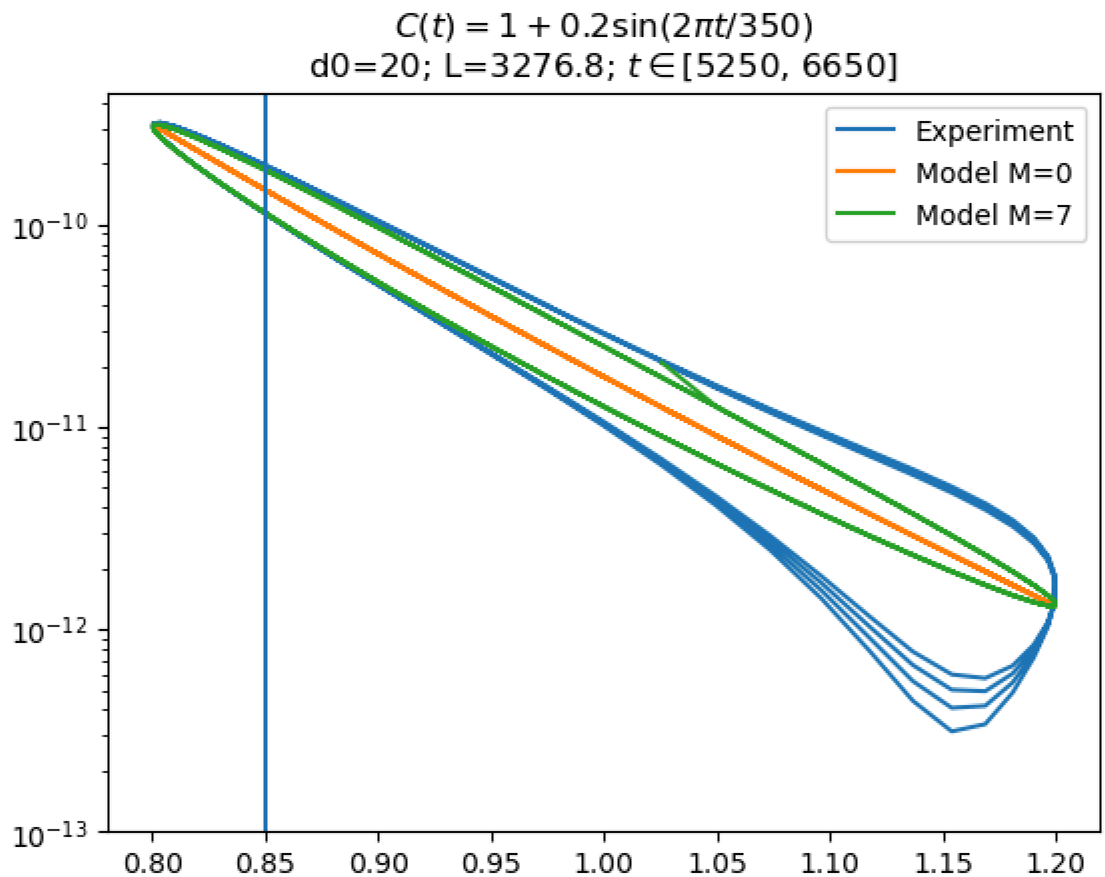
Mass as a function of the period T

The value of the mass **M** has been tuned **by HAND**, resulting

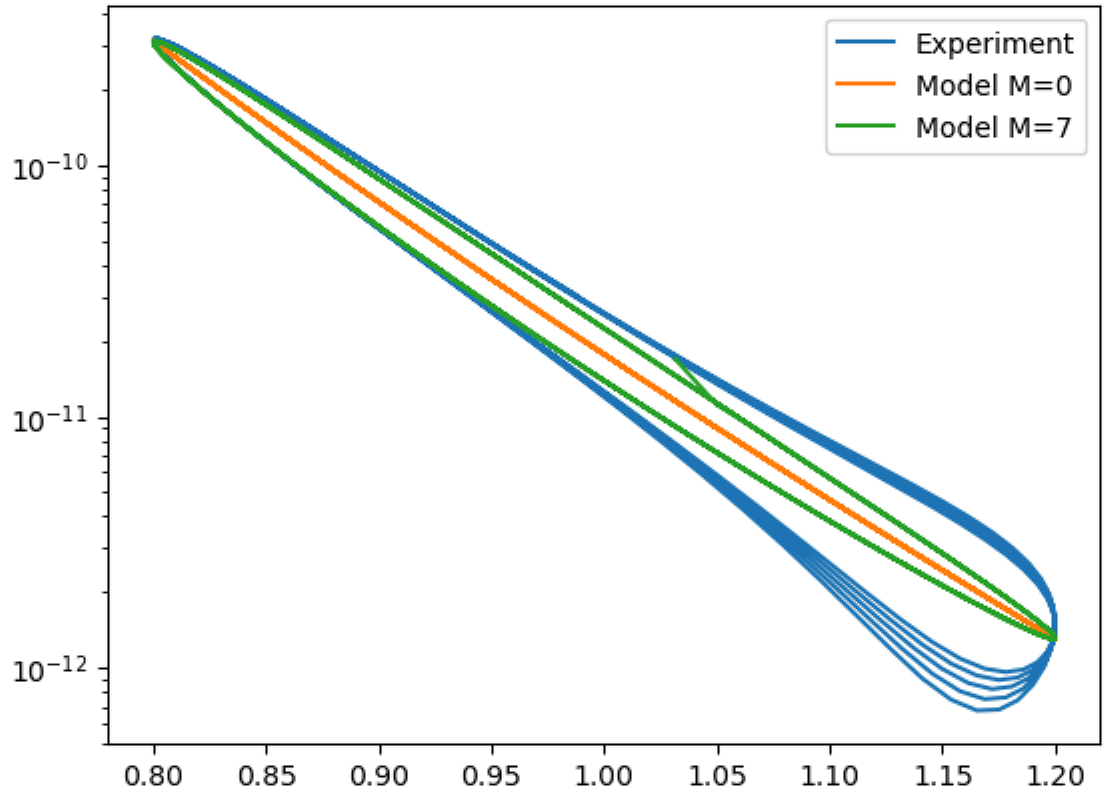
- The mass increases with the period
- Even if the period changes of orders of magnitude, the mass remains of the same order



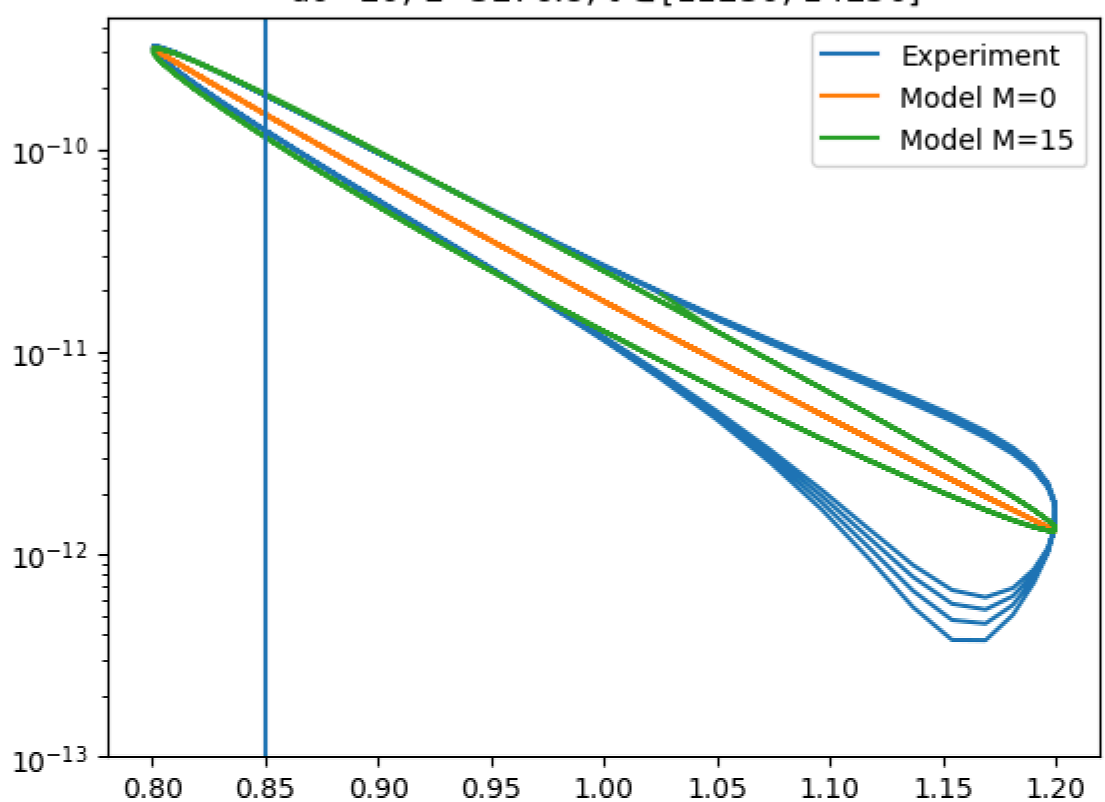

$C(t) = 1 + 0.2\sin(2\pi t/200)$
d0=20; L=3276.8; $t \in [2000, 3000]$
Experiment
Model M=0
Model M=7
10^{-10}
10^{-11}
10^{-12}
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20

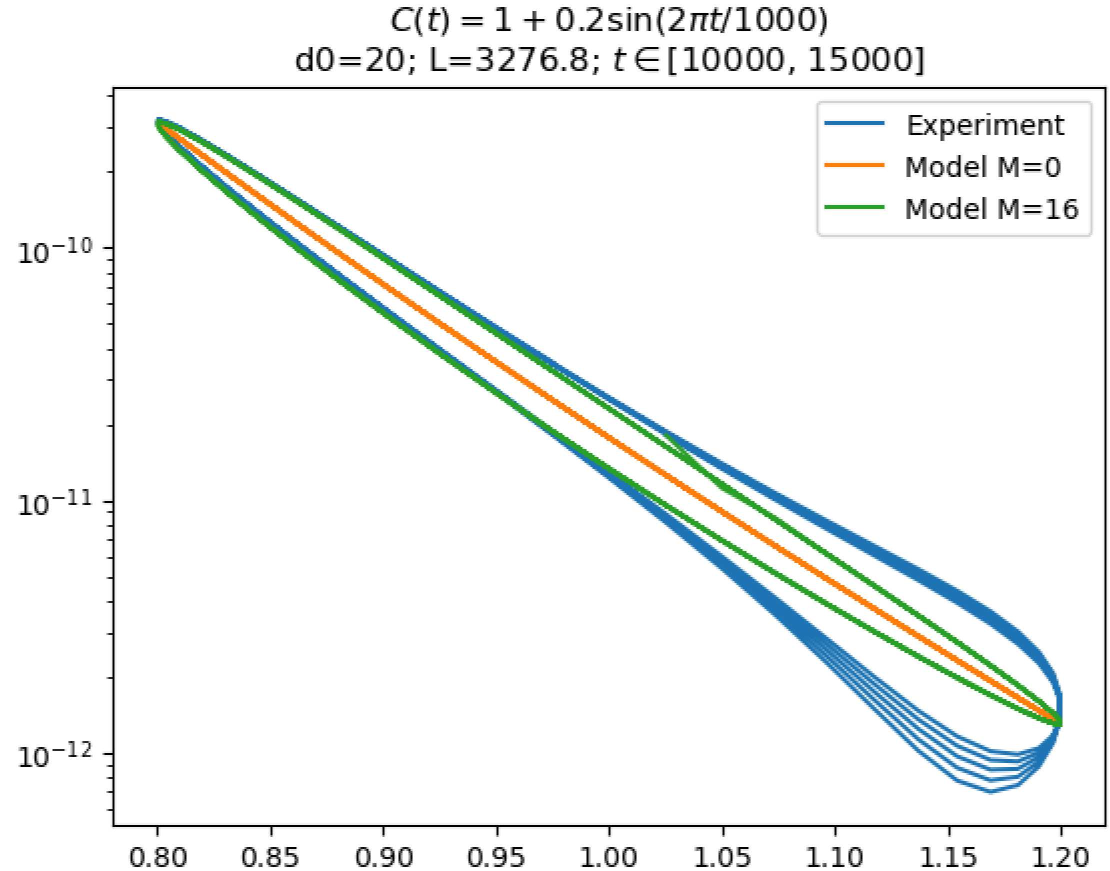

$C(t) = 1 + 0.2\sin(2\pi t/350)$
d0=20; L=3276.8; $t \in [5250, 6650]$
Experiment
Model M=0
Model M=7
10^{-10}
10^{-11}
10^{-12}
10^{-13}
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20

$C(t) = 1 + 0.2\sin(2\pi t/500)$
d0=20; L=3276.8; $t \in [5000, 7500]$



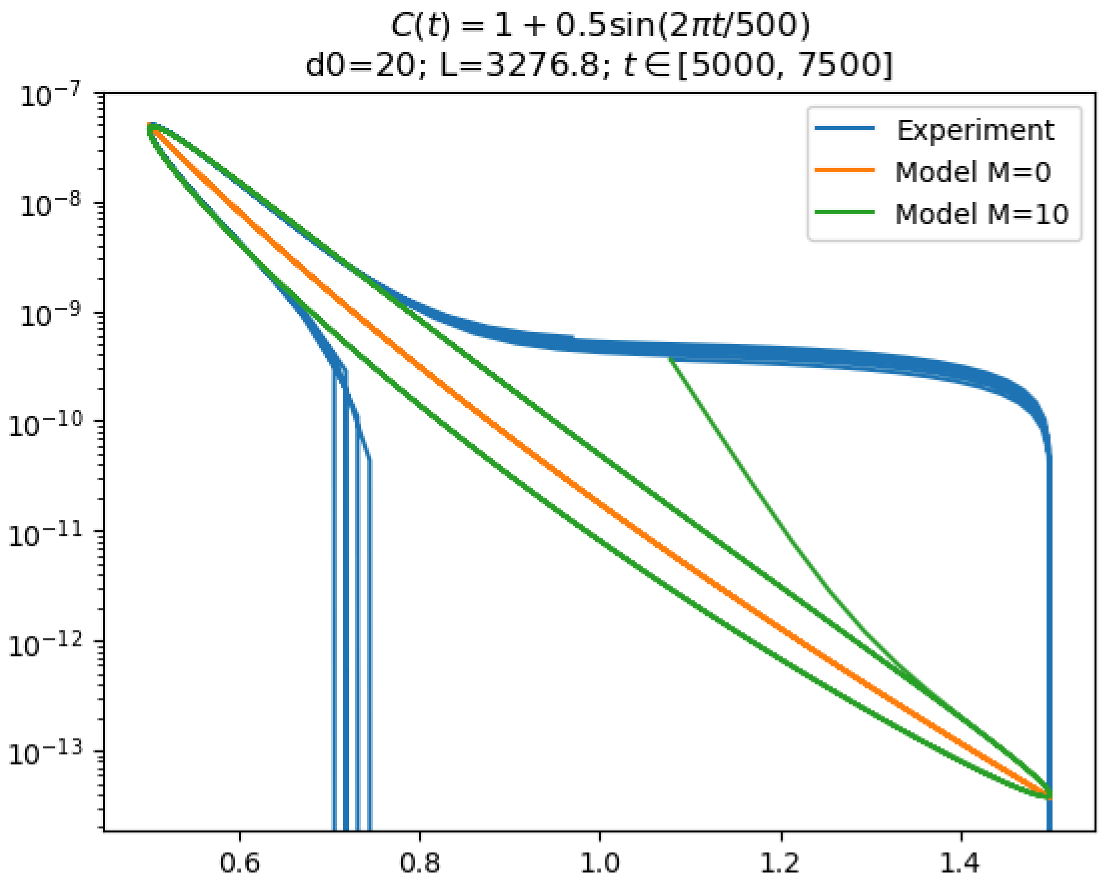
$C(t) = 1 + 0.2\sin(2\pi t/750)$
d0=20; L=3276.8; $t \in [11250, 14250]$

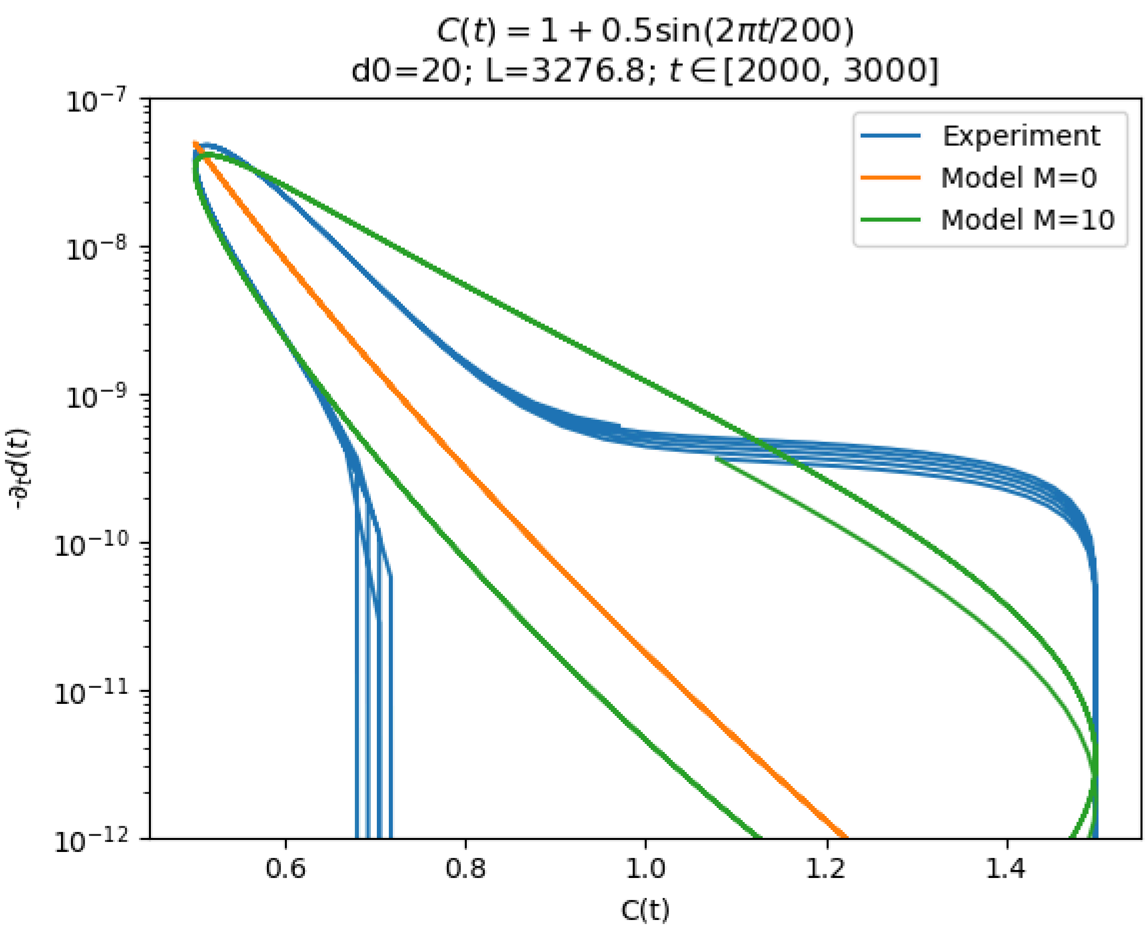




Mass as a function of the amplitude A

Also changing the amplitude, the order of magnitude of M is the same.



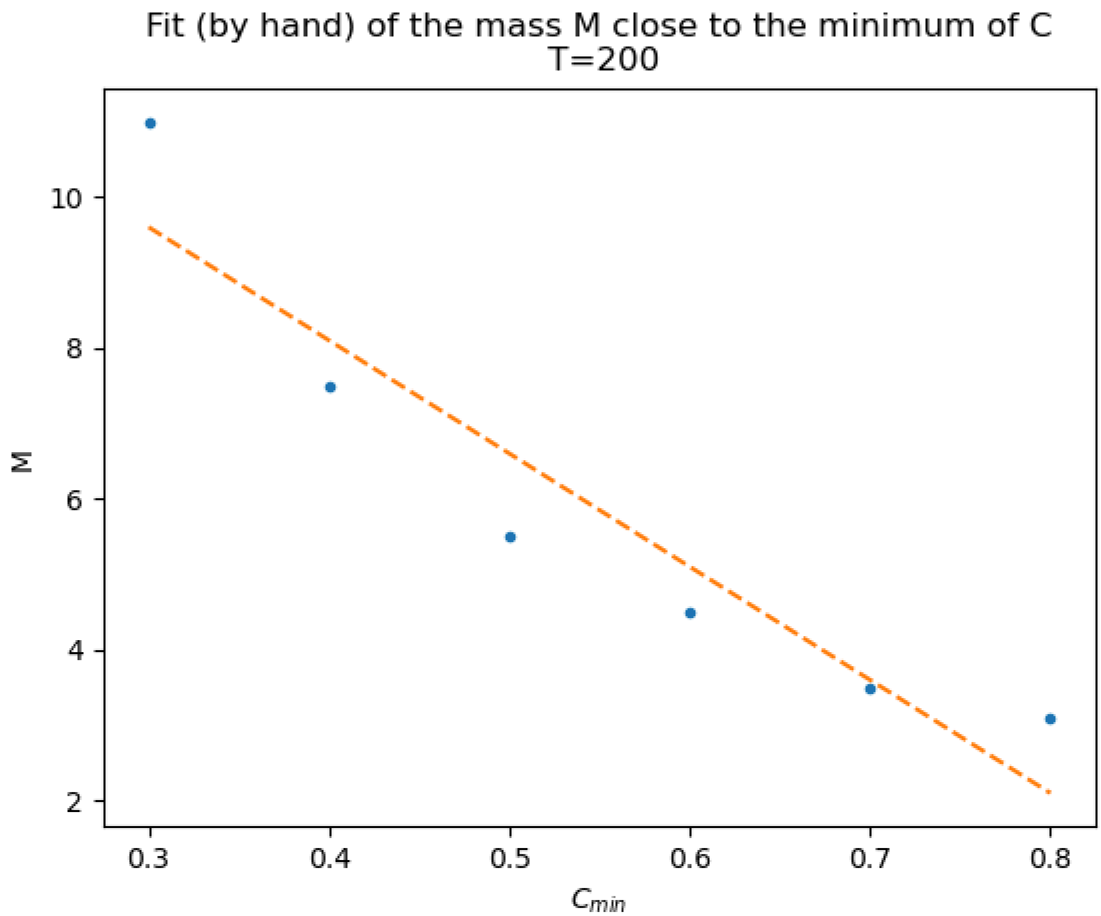


A Mass dependent on C and Cdot

As the order $\sim \epsilon^2$ correction depends, in principle, by C and $\dot{C}$, then we can try to fit the data with a mass

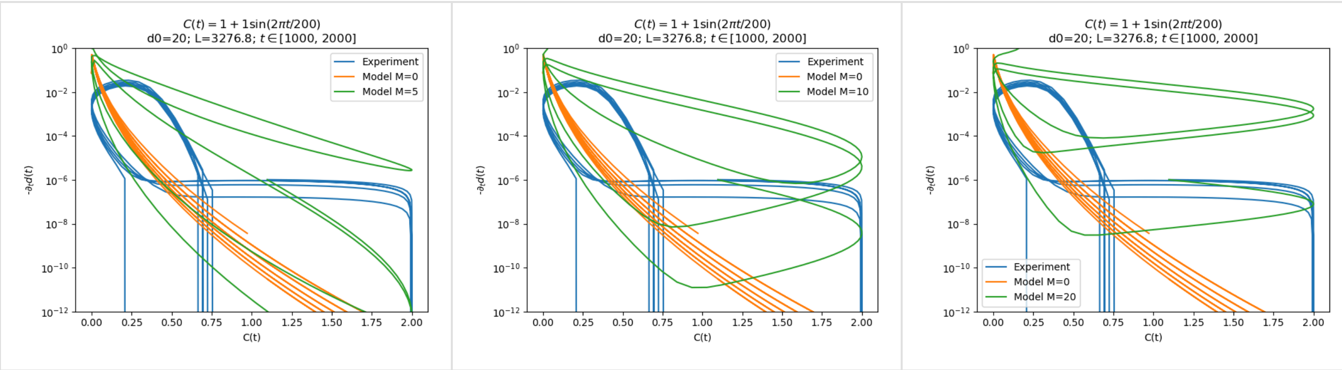
$$M(C, \dot{C}) = \alpha C + \beta \dot{C}$$

In order to find α, we make a fit **by hand** by tuning M such that the fit is good close to the minimum value of C. We do this for many different values of C_{min} , while the period T is the same.
What we see below is that the data **is not distributed along a line**. Instead it fits better a power law decay $M \sim C^{1/3}$.



Mass model when $A \geq \bar{C}$

In this case there is no suitable value of the mass **M** able to describe the deviation from the kink dyanmics model.



Conclusions

- Adding the mass **M** to the model for "kink dynamics under slow oscillations", we have a good prediction of $\partial_t d(C)$ **when C is small**. Althought it does not work when C gets close to zero or negative.
- The order of magnitude of the fitted **M** seems to not change with amplitude or period.
- The **asymmetry** that we see for large values of C is not predicted and it increases as time passes. Although, this asymmetry would be responsible **only for higher order** correction, as $\partial_t d$ is order of magnitude higher when C is close to its smallest value.