1D Fast oscillations (Analytical)

Multiple scale analysis

> Fast oscillations 1D
>
> We say that the oscillations of C(t) are **slow**, if the time scale τ_C associated with the time variation of $C(t)$ (that, as $C(t)$ is periodic, can be identified as its **period** T) is **SMALL compared** to the **intrinsic time-scale** of the system τ_{linear} (see here).
> So we can define a **small parameter** ϵ as:
>
> $$\epsilon = \frac{\tau_C}{\tau_{linear}}$$
>
> ## Idea
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> Then, assuming that $\tau_{linear} \sim 1$ (that means $\epsilon \sim \tau_C$), it is natural to make a Multiple scales expansion by introducing the new time-variables
>
> $$t_0 = t, t_{-1} = \epsilon^{-1} t$$
>
> where the dependence of m(x,t) on t_0 will capture processes occurring at the intrinsic time-scale, and t_{-1} the ones occurring at the time-scale of C(t)'s oscillations.
> This means that
>
> $$\partial_t = \partial_{t_0} + \epsilon^{-1} \partial_{t_{-1}}$$
>
> where
>
> $$C(t) = \tilde{C}(t_{-1}) \implies \partial_{t_0} \tilde{C}(t_{-1}) = 0$$
>
> From the relation above and the **ansatz**
>
> $$m(x,t) = m_0(x,t) + \epsilon m_1(x,t) + O(\epsilon^2)$$
>
> It follows that
>
> $$m_0(x,t) = \sqrt{\bar{C}} \tanh(x \sqrt{\frac{\bar{C}}{2}})$$
>
> But we could not find an equation for the first order correction $\epsilon m_1(x,t)$

Kink effective dynamics

If C(t)'s oscillations are fast respect to the intrinsic time-scale of the system, we know from Fast oscillations 1D that the zeroth-order shape of an isolated kink is

$$m_0(x,t) = \sqrt{\bar{C}}\tanh(x\sqrt{\frac{\bar{C}}{2}})$$

As a consequence, we expect that (but I didn't check this on paper) the kink's dynamics, to leading order, is the same that you have if C was constant, but with $C \to \bar{C}$ (see here for a proof)

$$\dot{x_n}(t) = 16\bar{C}^{\frac{1}{2}} \frac{[e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}}l_n} - e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}}l_{n+1}}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi)}$$

For two isolated kinks, at a distance $d \ll L$

$$\dot{d}(t) \simeq -24\sqrt{2}\bar{C}^{\frac{1}{2}}(t)e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}}d}$$