1D Fast oscillations (Analytical)

- $C(t)$ is a **periodic** function:
  - **Average** $\bar{C}$
  - **Period** T
  - **Amplitude** A.
- **Fast** oscillations: the period T is **small** compared to the other timescale(s) of the system.
- The **average** $\bar{C}$ is **positive** but $C(t)$ **can get negative values** during the period.

If the average is negative, from the 0D analysis we expect all domains to disappear (exponentially fast in time). So we're not interested in this case.

Main results

- Fast oscillations

$$T \ll \tau_{linear} \sim \bar{C}^{-1}$$

$$\epsilon \sim \frac{T}{\tau_{linear}} \ll 1$$

- We couldn't find the leading order correction to the isolated kink's shape
- Kinks dynamics is not affected by a time depending $C(t)$ to leading order (neglecting terms of order higher than $\epsilon \delta u_{k_0}$, $\delta u_{k_0}^2$)

$$\dot{x_n}(t) = 16\bar{C}^{\frac{1}{2}} \frac{\left[e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}} l_n} - e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}} l_{n+1}}\right]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi)}$$

  that for two isolated kinks leads to a decay of the distance

$$\dot{d}(t) \simeq -24\sqrt{2}\bar{C}^{\frac{1}{2}} e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}} d}$$

  that is the formula found for constant C, with $C \to \bar{C}$.

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Multiple scale analysis

If $C(t)$ is a periodic function of time, there are **at least two time-scales** in the system

- τ_{linear} it is the time-scale associated to the growth of unstable modes in the linear regime.
- T is the period of $C(t)$.

Timescale of the system

As the oscillation is fast, the system hasn't time to adapt to the changes of $C(t)$ so

$$\tau_{linear} \sim \bar{C}^{-1}$$

This argument does not make sense, because to state that the oscillation is fast, we need to see the value of the timescale. Here we're assuming before that is fast!

Introducing new time variables

We introduce a variable t_{-1} to describe changes occurring at the (fast) time-scale T and the variable t_0 for the changes occurring at the time-scale τ_{linear}. As

$$T \ll \tau_{linear} \implies \epsilon = \frac{T}{\tau_{linear}} \ll 1$$

$$t_0 = t; \quad t_{-1} = \epsilon^{-1} t$$

$$\partial_t = \partial_{t_0} + \epsilon^{-1} \partial_{t_{-1}}$$

$$\partial_{t_0} C(t) = 0$$

where ϵ **is a small parameter**.

Kink shape correction

Here we look for a correction to the kink's shape (deviation from the conventional shape $u_{k_0}(\chi) \sim \tanh(2^{1/2}\chi)$) **to leading order** in ϵ.

Anstatz

$$u(x,t) = u_0(x,t) + \epsilon u_1(x,t) + O(\epsilon^2)$$

inside the TDGL eq. ($\partial_t u = \partial_{xx} u + Cu - u^3$) and

- considering only terms of order zero
- using $\partial_t = \partial_{t_0} + \epsilon^{-1}\partial_{t_{-1}}$

  $$\partial_{t_{-1}} u_1 + \partial_{t_0} u_0 = C(t) u_0 - u_0^3 + \partial_{xx} u_0$$

  this time, we cannot find an equation for u_0 by considering only terms of order zero. Although it's **reasonable** to expect that, in the limit where the **period is infinitely small** (so in the limit $\epsilon \to 0$), the system **hasn't time to adapt** to the changes in C(t), so it f**eels an average effect**. So we expect

  $$u_0(x,t) = \beta(t) u_{k_0}(\chi) \quad \chi = \bar{C}^{1/2} x \quad \beta(t) = \bar{C}^{1/2}$$

  $$u_{k_0}(\chi) = \tanh(2^{1/2}\chi)$$

  Using the last result inside the equation for order ϵ^0

  $$\partial_{t_{-1}} m_1 = (C(t) - \bar{C}) m_0$$

  but the information contained in this equation is not enough to determine $m_1(x,t)$ and if we consider the equation for the next order ϵ^1 it would contain terms $\sim m_2$ so it won't be useful to determine the leading order correction $m_1(x,t)$.

Kink dynamics

In the analysis when C is constant, we found that the (approximated) interaction between kinks is determined by the shape of their tails.
As here the shape of an isolated kink is, to leading order (ϵ^0), the shape of an isolated kink in a system where C is constant and equal to $\bar{C}$, we expect that, to leading order, the formula for the interaction to be

$$\dot{x_n}(t) = 16\bar{C}^{\frac{1}{2}} \frac{[e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}} l_n} - e^{-2^{\frac{1}{2}}\bar{C}^{\frac{1}{2}} l_{n+1}}]}{\int_{\chi_{n-0.5}}^{\chi_{n+0.5}} d\chi \partial_\chi u_p(\chi)}$$

that is the formula found for a system where C is constant, but here we substituted $C \to \bar{C}$.