Variation of kink distance over a period

#twokinks

Definition

If two kinks are far from each other and $C(t)$ is a **slow and strictly positive oscillation** we know (Kinks effective dynamics under slow POSITIVE oscillations) the kink effective dynamics model states

$$\dot{d}(t) \simeq -24\sqrt{2}C^{\frac{1}{2}}(t)e^{-2^{\frac{1}{2}}C(t)^{\frac{1}{2}}d}$$

If we consider the variation of the distance over one period $\Delta d(d)$

$$\Delta d(d) = \int_0^T dt\partial_t d = -24\sqrt{2}\int_0^T dtC^{1/2}(t)e^{-\sqrt{2}dC^{1/2}(t)}$$

where this is a function of d, because you can **assume d to be CONSTANT in the integrand** as it does not change significatively over a period.

By changing variable $t \to \tau = \frac{t}{T}$

$$\Delta d(d) = -24\sqrt{2}T\int_0^1 d\tau C^{1/2}(\tau)e^{-\sqrt{2}dC^{1/2}(\tau)}$$

$$C(\tau) = \bar{C} + A\sin(2\pi\tau)$$

If we define

$$I(d) = \int_0^1 d\tau e^{\sqrt{2}dC^{1/2}(\tau)}$$

then

$$\Delta d(d) = +24T\frac{dI(d)}{d(d)}$$

Parabola approximation

As simulations show that the distance changes significatively when $C(t)$ is close to its minimum value $\bar{C} - A$, then it is natural to approximate, in the integrand:

$$C^{1/2}(t) \simeq C^{1/2}(t = 3/4) + \alpha\left(t - \frac{3}{4}T\right)^2$$

(as a **parabola** and for every $t \in [0, T]$), then

$$I(d) \simeq \int_{-\infty}^{\infty} d\tau e^{-\sqrt{2}d(\bar{C}-A)^{1/2}} e^{-\sqrt{2}d\alpha(\tau-3/4)^2}$$

where I changed the extreme of integration as the (new) integrand is peaked at $\tau = \frac{3}{4}$ and decays exponentially fast getting far from that value.
Now, if you change variable

$$z = (\sqrt{2}d\alpha)^{1/2}\left(\tau - \frac{3}{4}\right)$$

$$I(d) = e^{-\sqrt{2}d(\bar{C}-A)^{1/2}}(\sqrt{2}d\alpha)^{-1/2}\int_{-\infty}^{\infty} e^{-z^2} dz$$

where the integral here is just a finite number. So

$$I(d) \sim e^{-d} d^{-1/2}$$

Derivating the last expression respect to d, leads to

$$\Delta d \sim e^{\sqrt{2}d(\bar{C}-A)^{1/2}} d^{-1/2}\left[\sqrt{2}(\bar{C} - A) + \frac{1}{2}d^{-1}\right]$$

That is similar to the expression you have for constant C

$$\Delta d \sim e^{\sqrt{2}dC^{1/2}}$$

but now $C \to (\bar{C} - A)$ and a **power-law** term is multiplying the exponential decay.

- If $\bar{C} > A$
  The exponential dominates the behaviour of $\Delta d(d)$. So the variation of d is ruled by an exponential.

  $$\Delta d \sim e^{-d(\bar{C}-A)}$$

- If $\bar{C} = A$
  The exponential term vanishes, but also the term in $[\ldots]$ is affected, such that

  $$\Delta d \sim d^{-3/2}$$

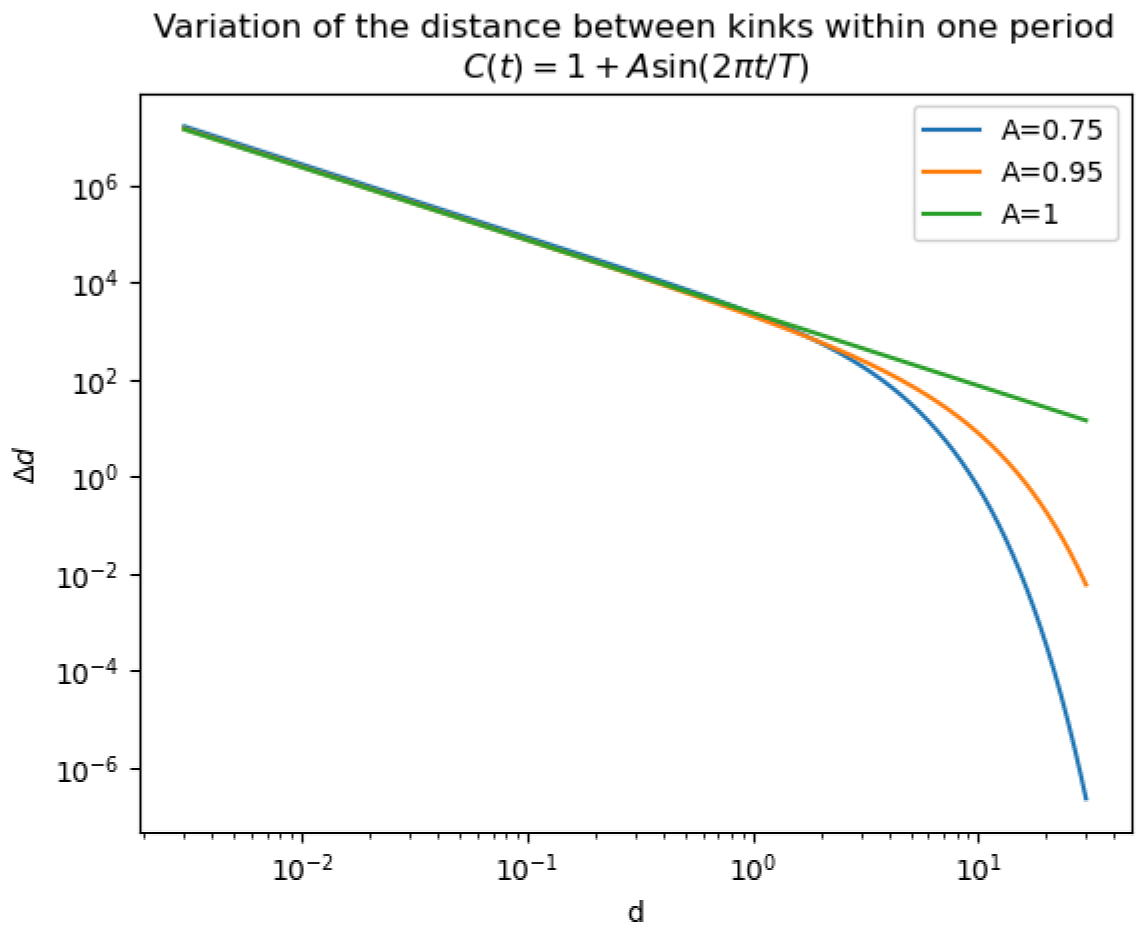
- If $\bar{C} < A$
  Here, it is necessary to extend the kink effective dynamics model to cases when $C(t) < 0$ sometimes. Indeed, the model is developed considering $C(t)$ as a strictly positive oscillation. We do this **by assuming** $\partial_t d = 0$ **when** $C(t) < 0$. Then it is not possible to approximate $C^{1/2}$ to a parabola as it is not possible to calculate the square root. We should approximate $C(\tau)$ to a parabola, then take its square root. But then if you use the

approximation $(1+\epsilon)^{1/2} \simeq 1 + \frac{1}{2}\epsilon$, this approximation is not good, so it is not clear what to expect.

Summary

From the considerations above, we **expect** $\Delta d(d)$ to decay as

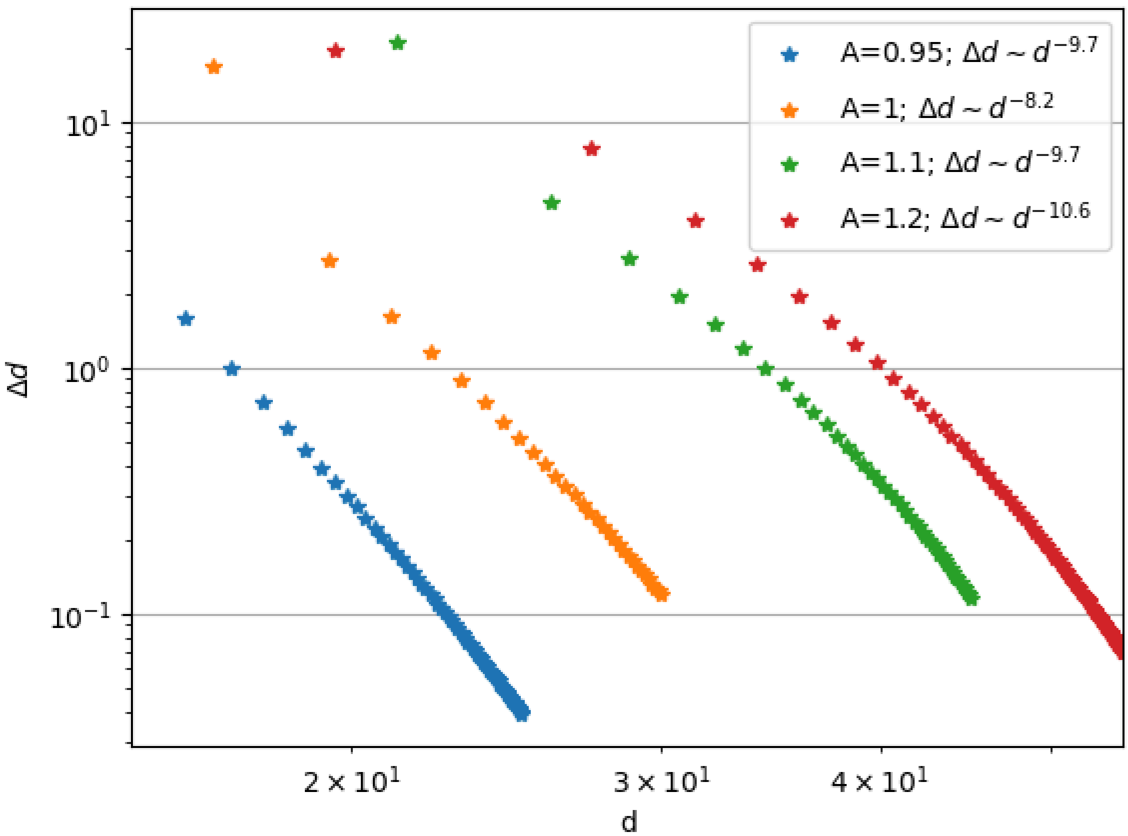
- A power-law when d is small
- Exponentially when d is large. **Unless $A = \bar{C}$ so the decay is power-law for any d.** And we're interested in the behaviour far from annihilation, so at large d.



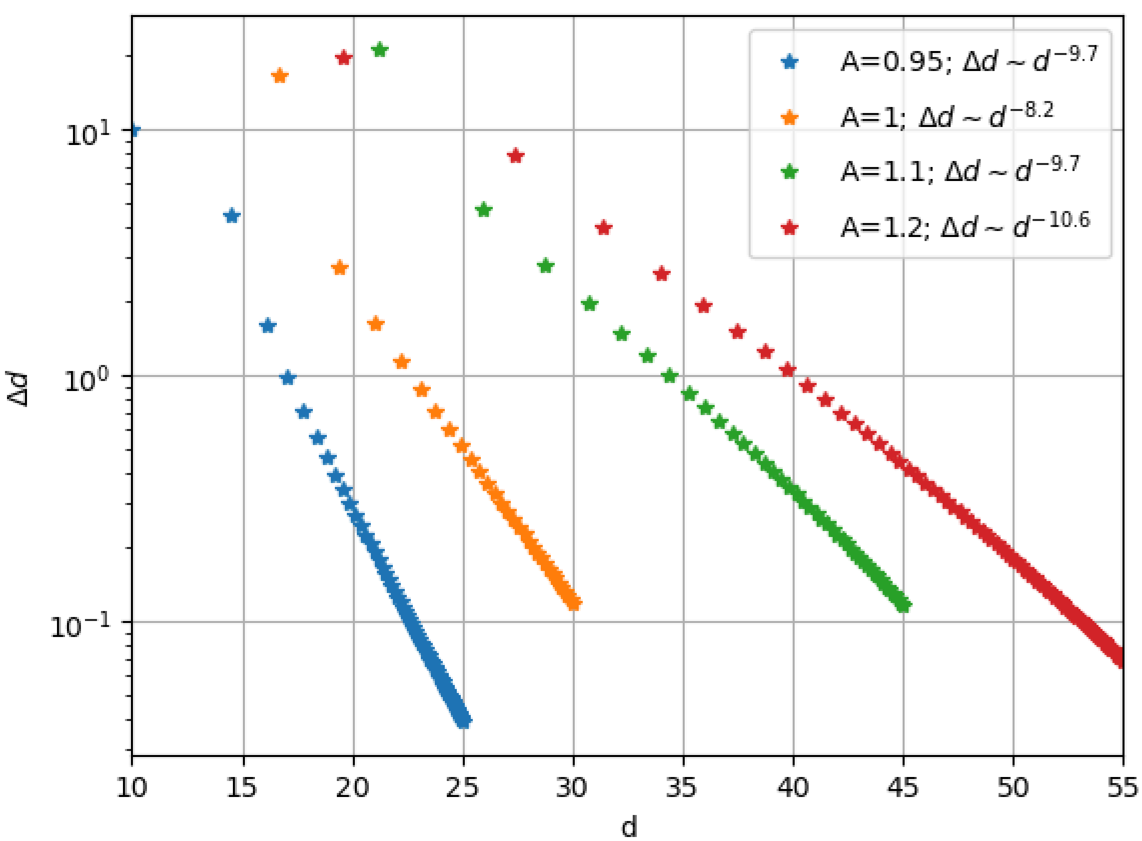
Simulations

In the simulations below, the tail is fitted with a line and the slope of the line is reported in the legend. This value is way far from the expected one (**-3/2=-1.5**) and the next plot enhances that the decay is exponential (and not power-law) also when $A \geq \bar{C}$.

$$C(t) = 1 + A\sin\left(\frac{2\pi t}{500}\right)$$



A=0.95; $\Delta d \sim d^{-9.7}$
A=1; $\Delta d \sim d^{-8.2}$
A=1.1; $\Delta d \sim d^{-9.7}$
A=1.2; $\Delta d \sim d^{-10.6}$
Δd
10^{1}
10^{0}
10^{-1}
2×10^{1}
3×10^{1}
4×10^{1}
d



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Δd
10^{1}
10^{0}
10^{-1}
10 15 20 25 30 35 40 45 50 55
d