Linear regime

#early_dynamics #linear_regime

In Linear analysis.pdf we **calculate analytically** how the characteristic wavelenght

$$\langle q^2 \rangle = \frac{1}{D} \frac{\int q^2 |u_q|^2 dq}{\int |u_q|^2 dq}$$

evolves in time **if**

- the non-linearity $-u^3$ can be neglected
- AND the **power is uniformly distributed** among the Fourier modes in the initial state

  $$|u_q|(t=0) = \bar{u}\Theta(q < q_{min})$$

  where q_{min} is a cutoff frequency, that we expect to have since the simulation box is finite. The calculations, predict that, **until the non-linearity becames relevant** (and eventually it happens because large-wavelenght modes grow in time if $C > 0$)

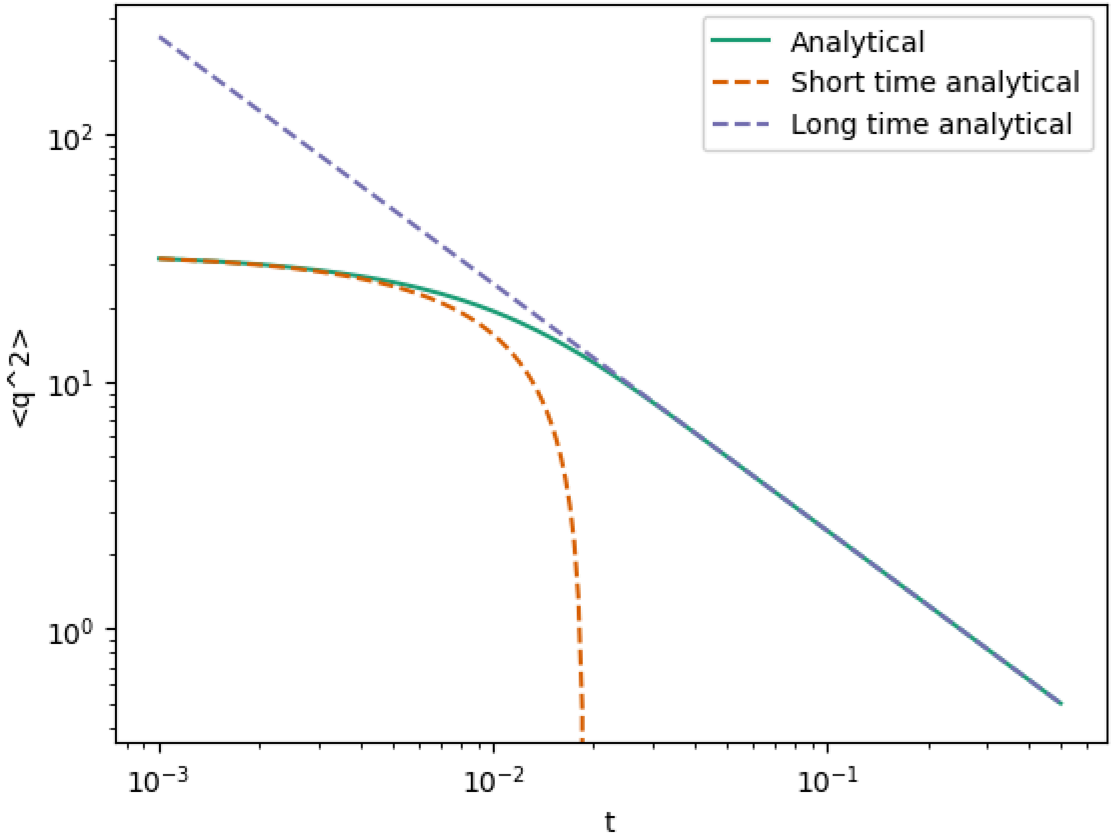
  $$\langle q^2 \rangle(t) = \frac{1}{4t}\{1 - q_{min}\frac{2^{\frac{3}{2}}}{\pi^{\frac{1}{2}}} t^{\frac{1}{2}} \frac{e^{-2tq_{min}^2}}{erf[q_{min}(2t)^{\frac{1}{2}}]}\}$$

  That gives the asymptotic behaviours:
- Short times:

  $$\langle q^2 \rangle(t) \simeq \frac{q_{min}^2}{3}(1 - \frac{8}{15}q_{min}^2 t)$$

- Large times (but not so large such that the non-linearity becomes non-negligible):

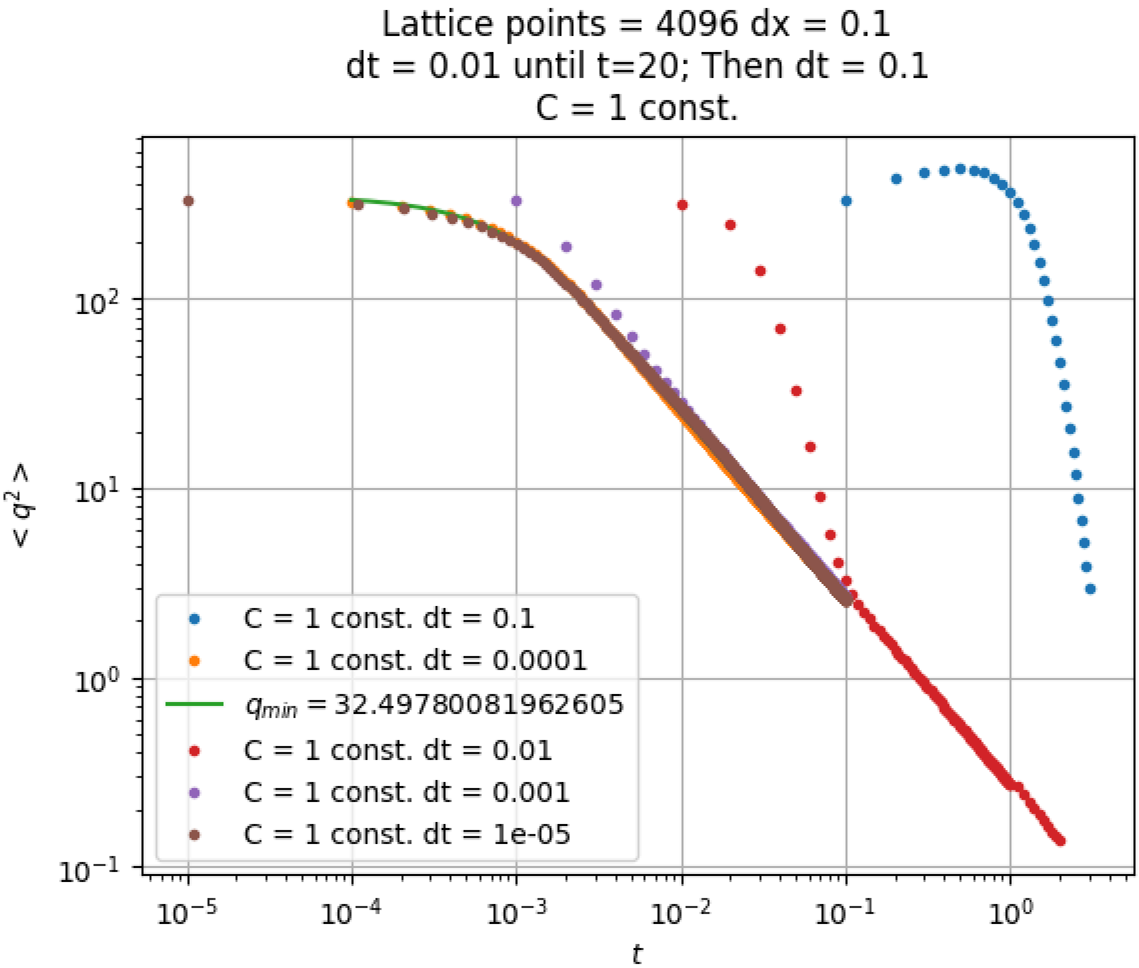
  $$\langle q^2 \rangle(t) \simeq \frac{1}{4t}$$



Simulations $\langle q^2 \rangle$

1D

Here I show a simulation of a 1D system.
The initial state is prepared starting from random initial conditions, without any bias. While $C = 1$ is kept constant during the whole simulation. *"1D/.saves/24_11_6_A"*.
The data (in orange) is fitted with the formula presented above (the long one) (green line) where q_{min} is a free parameter.



There is agreement between the model and the data, but the estimated value of q_{min} is way far from the smallest q-vector due to the discreteness of the lattice $2\pi/L \simeq 0.15$ as $L = N * dx = 409.6$.

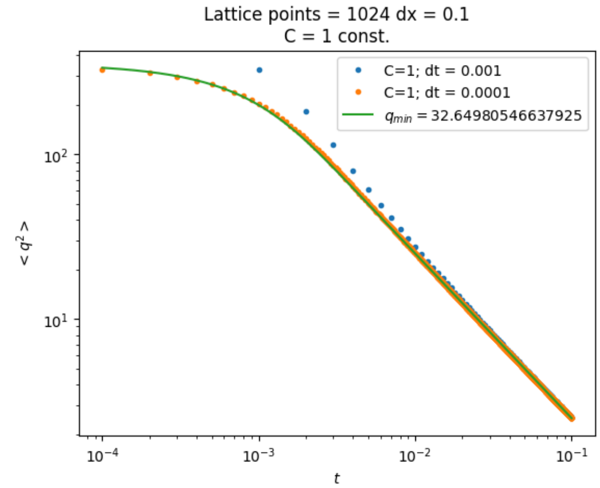
Notice that we are able to see **correct very early dynamics** only if dt is small enough!

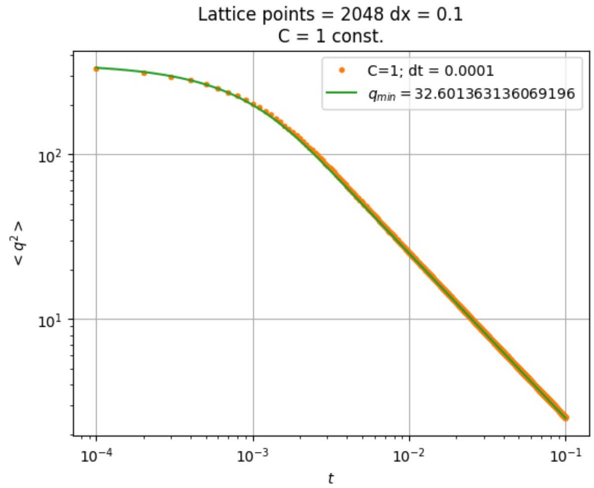
2D

The expected very early behavior of $\langle q^2 \rangle$ is verified also in 2D.

Also in 2D, we see that we need to go to small values of dt to capture the **non-power law initial decay** of $\langle q^2 \rangle$.

We also see that the fitted q_{min} does not depend on the size of the lattice L, while the smallest q-vector, due to the finiteness of the lattice depends over L: $q_{min} = 2\pi/L$.

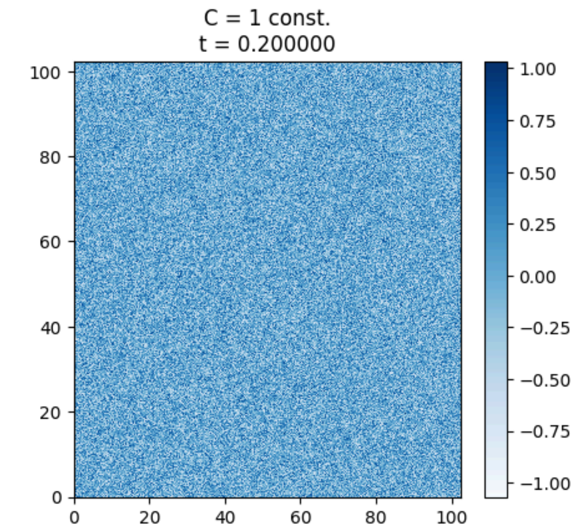


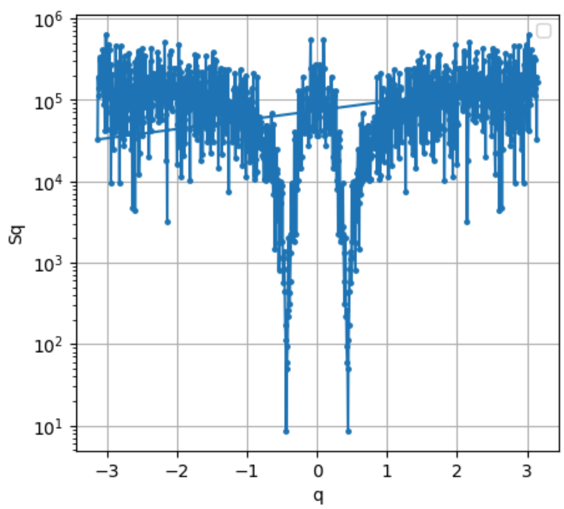


Simulations (the state (2D))

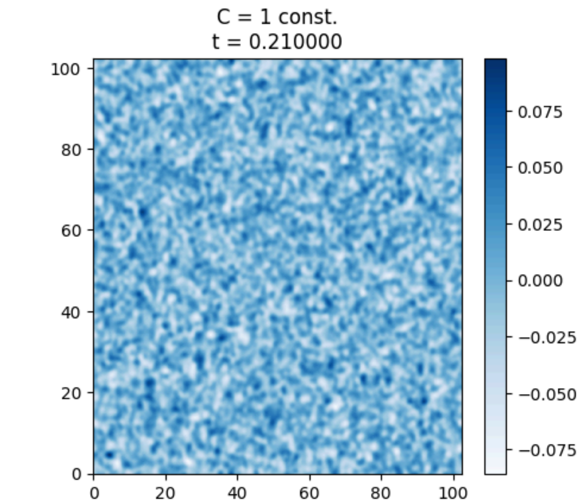
Above we showed how the time discretization dt affects the time evolution of $\langle q^2 \rangle$. Now I will show actual pictures of the state (and its structure factor) at time $t = 0.2$, for different values of dt. Here L=1024; dx=0.1; C=1 constant, as in the previous plot.

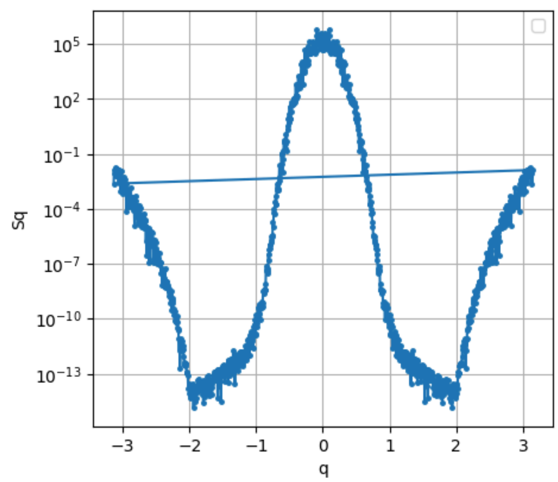
dt=1e-1

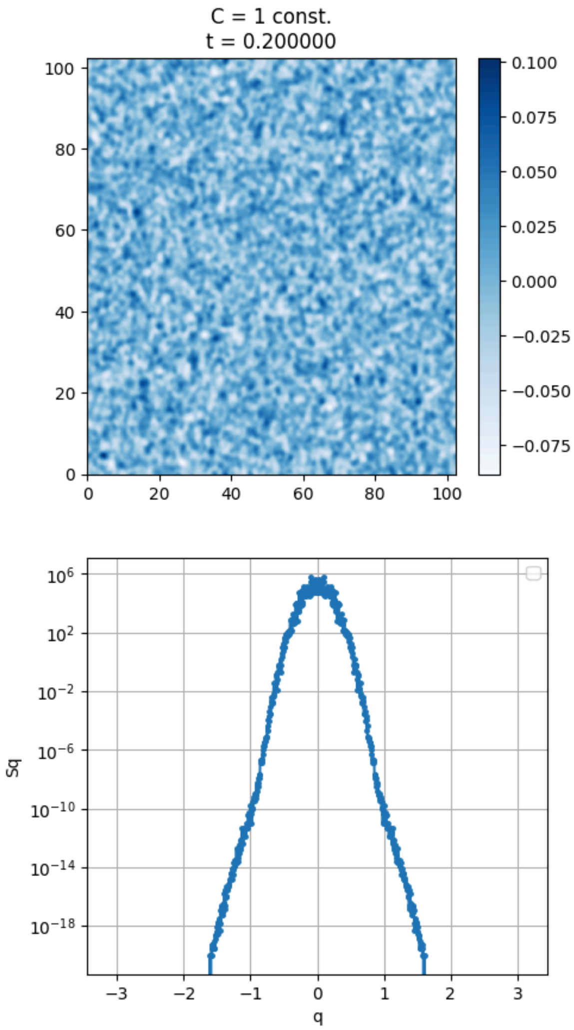
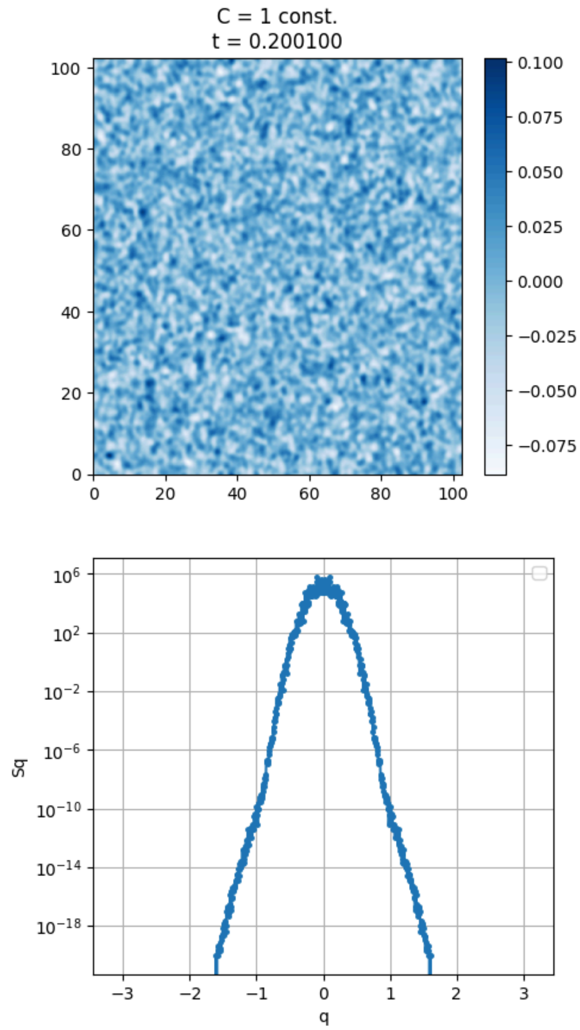




dt=1e-2





dt=1e-3	dt=1e-4	
		

Keeping the system in the linear regime

If we keep $C = -0.1$, then the non-linearity will never became relevant. We have an analytical formula for the evolution of ℓ in this regime, but what happens to the length of the zero level-set $\mathcal{L} = L^2/\ell_{CC}$ in the linear regime?
This question makes sense, because once the domains emerge, there is coarsening and we expect $\ell_{CC} \sim t^{\frac{1}{2}}$. But what about the linear regime, where the lenght of the interfaces $\mathcal{L}$ seems

to make no sense?