2D Structure factor

#structurefactor #2D #randinit #liquids

Like in the 1D case (1D Structure factor) I define:

I define the (2D) structure factor

$$S(q_x,q_y) \equiv <|\int e^{-i{f q}\cdot{f x}}u({f x})dxdy|^2>$$

where:

- $|...|^2 = (...)^*(...)$ is the norm.
- <...> is an average over multiple simulations, all starting from a random initial condition (different each time).

There is a problem - Need a function of ONE variable!

In 2D the structure factor is a function of two variables: q_x, q_y . While in all the articles it is presented a function of only one variable! So how do we compute this quantity $\hat{S}(q)$ from $S(q_x, q_y)$?

My idea

As there are **no preferential directions**, I expect that the structure factor is a function only of $q = |\mathbf{q}|$ and not of the direction. As a consequence, it is possible to choose a direction in q-space (passing through the origin) and evaluate $S(\mathbf{q})$ along that direction.

So the most simple way of extimating $\hat{S}(q)$ is to evaluate $S(q_x,q_y)$ along the direction $q_x=0$ and along $q_y=0$ and then take the average.

$$\hat{S}(q) = rac{1}{2}[S(q,0) + S(0,q)]$$

(considering other directions is not an available option, as in the other directions the points of the lattice are separated by a different distance, so we cannot compare the functions associated to those lines with the horizontal and vertical lines).

This approach of evaluating $S(\mathbf{q})$ along one direction is **motivated** by what happens experimentally with 3D liquids, where **to sample one point of the spectrum** $\hat{S}(q)$ the structure factor is **sampled at only one vector** \mathbf{q} with that norm. So they do not measure multiple vectors to take then an average, as they fix the angle φ of the detector's position.

See Structure factor in liquids for a motivation of the present approach.





