Comparing all wavelenghts

#characteristic_lenghts #ell #ellDW #ellCC

We can measure three characteristic wavelengths in the system:

ℓ

$$\langle q^2\rangle \equiv \frac{\int q^2 |u_q|^2 dq}{\int |u_q|^2 dq}$$

$$\ell \equiv \frac{2\pi}{\sqrt{\langle q^2\rangle}}$$

We know from Linear regime that, during the linear regime

$$\ell \sim t^{\frac{1}{2}}$$

Then, in appendix G of the Master Report.pdf we show that, asymptotically this wavelength is related to ℓ_{DW} that asymptotically shows the coarsening exponent $\frac{1}{2}$: $\ell_{DW} \sim t^{\frac{1}{2}}$ so

$$\ell \sim t^{\frac{1}{4}}$$

ℓ_{DW}

$$\ell_{DW} = \frac{L^2}{\int |\nabla u|^2 dx dy} * W$$

Where W is the width of the interface, defined as the integral of the derivative of the field $u(x, y)$ along a direction perpendicular to the interface. We estimate this integral by considering the stationary state with $C = \bar{C}$:

$$u_k(\xi) = \sqrt{C}\tanh(\sqrt{\frac{C}{2}}\xi)$$

$$W = \int_{-\infty}^{\infty} (\partial_\xi u_k(\xi))^2 d\xi = \frac{2}{3}\sqrt{2}C^{\frac{3}{2}}$$

(the last integral is calculated in Appendix G of the Master Report.pdf).
As the gradient is peaked in the regions close to the interfaces, ℓ_{DW} should estimate the ratio between the size of the system L^2 and the (total) length of the interfaces $\mathcal{L}$.

ℓ_{CC}

We estimate the (total) length of the interfaces $\mathcal{L}$ by using the Cauchy-Crofton theorem, using 4 families of parallel lines: horizontal, vertical, $\pi/4$, $3\pi/4$. Then we calculate

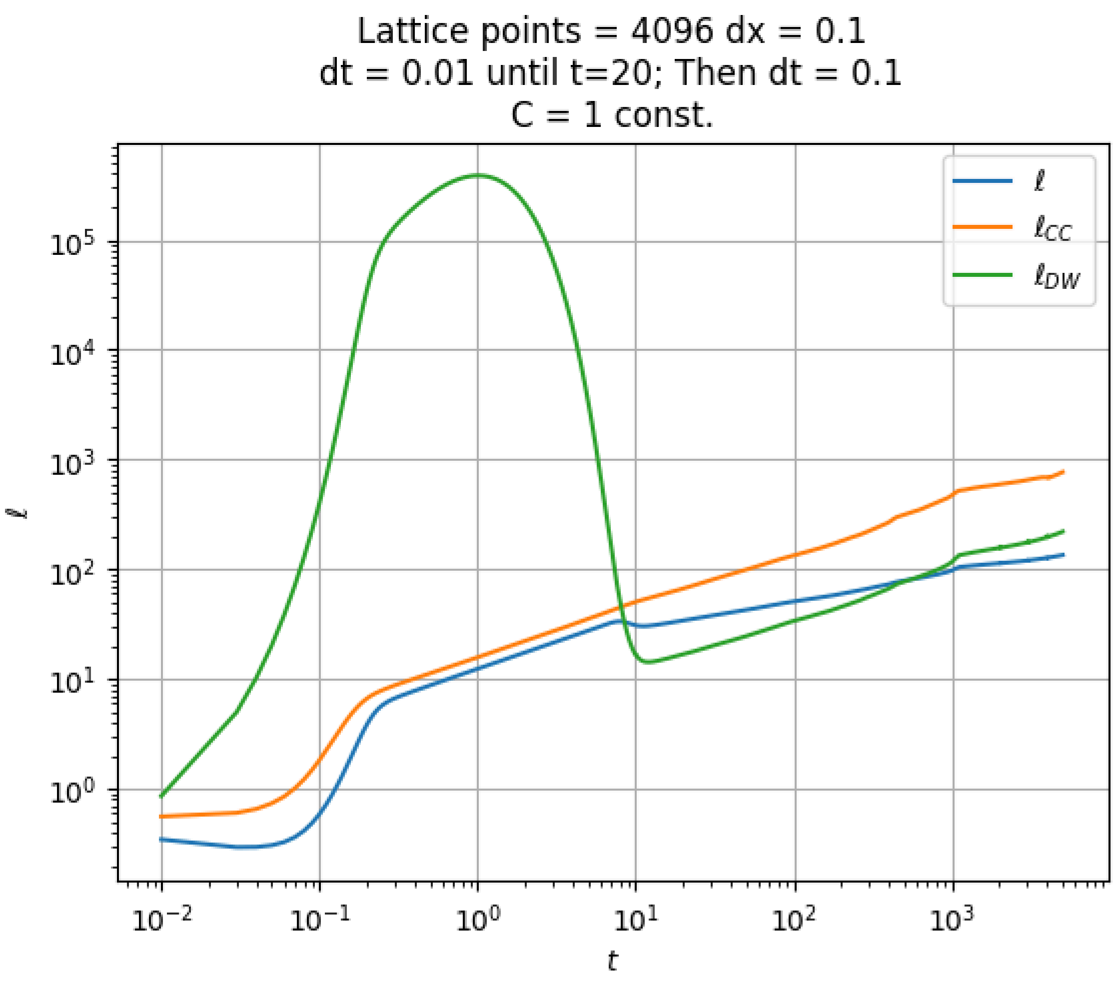
$$\ell_{CC} = \frac{L^2}{\mathcal{L}}$$

The Cauchy-Croft formula is

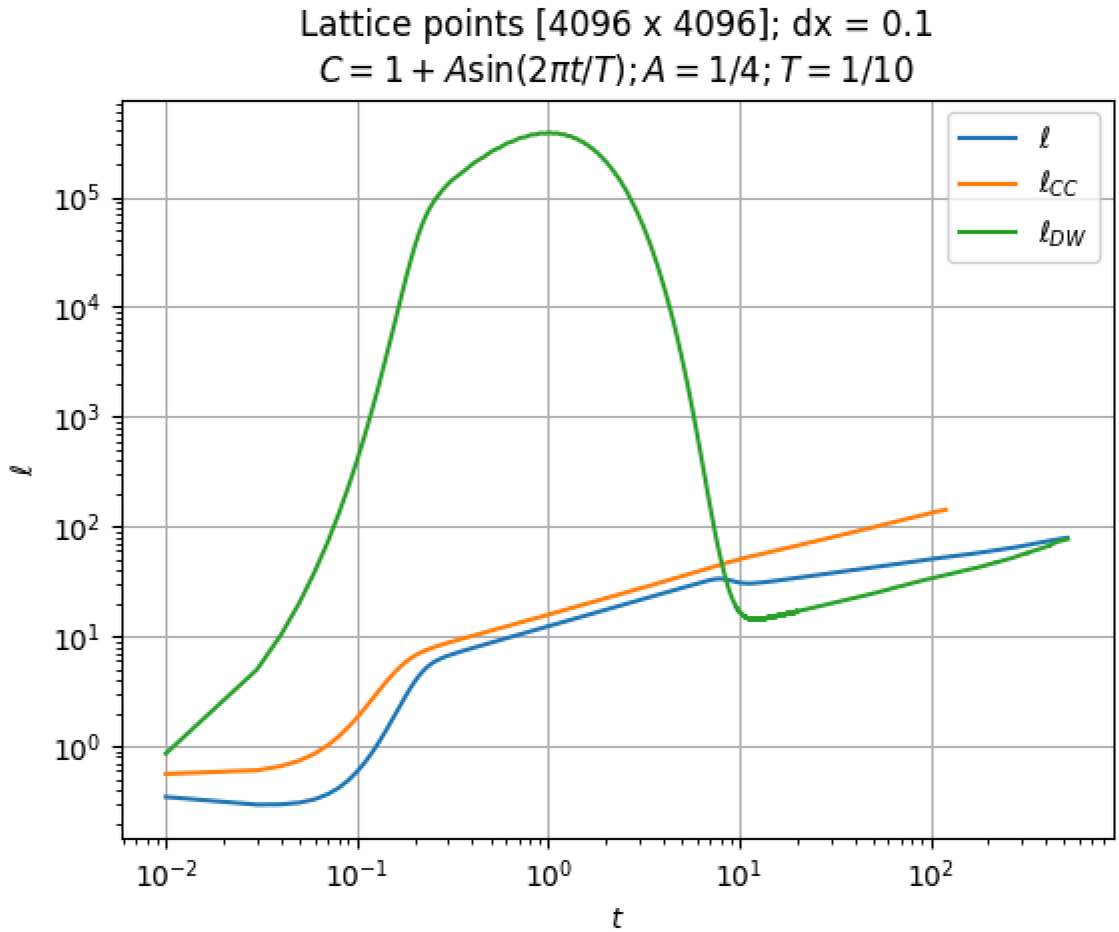
$$\mathcal{L} \simeq \frac{1}{2} * n * dx * (\pi/4)$$

- n is the number of times the interface crosses the parallel lines
- dx is the spacing between the lines

Comparison (in the same canvas)



In the following plot, the thickness of the interface is estimated as the one of the stationary state associated with constant $C = \bar{C}$.



Comparison (Asymptotically)

Asymptotically, precisely when ℓ_{DW} well estimates the ratio $L^2/\mathcal{L}$:

$$\ell^{-2} \simeq (2\pi)^{-2} D^{-1} I_1 (C\ell_{DW} - I_2)^{-1}$$

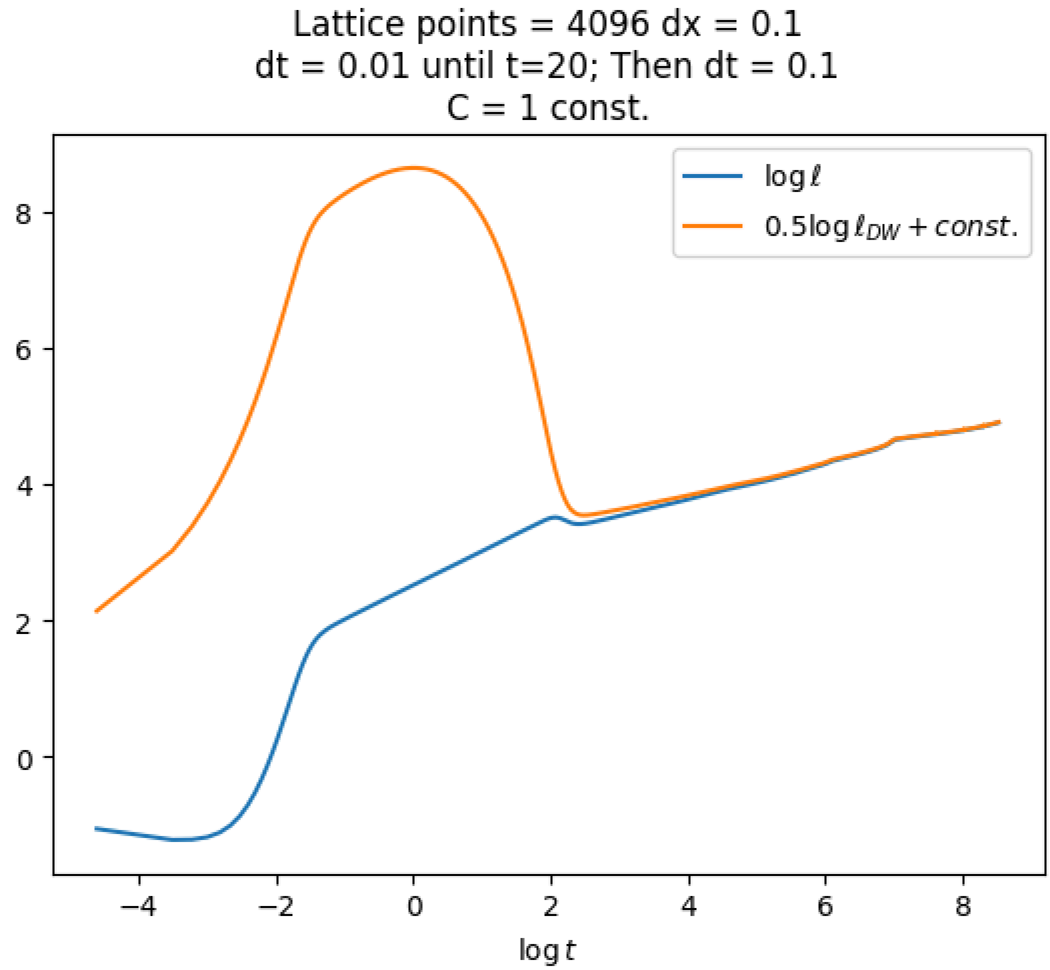
Then, at sufficiently large times: ℓ_{DW} grows, so $(C\ell_{DW} - I_2) \simeq C\ell_{DW}$ that means

$$\ell^{-2} = (2\pi)^{-2} (CD\ell_{DW})^{-1} I_1$$

Calculating the logarithm:

$$\log \ell = \frac{1}{2} \log \ell_{DW} + const.$$

$$const. = \log 2\pi + \frac{1}{2} \log(CD) - \frac{1}{2} \log I_1$$



Lattice points = 4096 dx = 0.1
dt = 0.01 until t=20; Then dt = 0.1
C = 1 const.
$\log \ell$
$0.5 \log \ell_{DW} + const.$
8
6
4
2
0
−4
−2
0
2
4
6
8
$\log t$