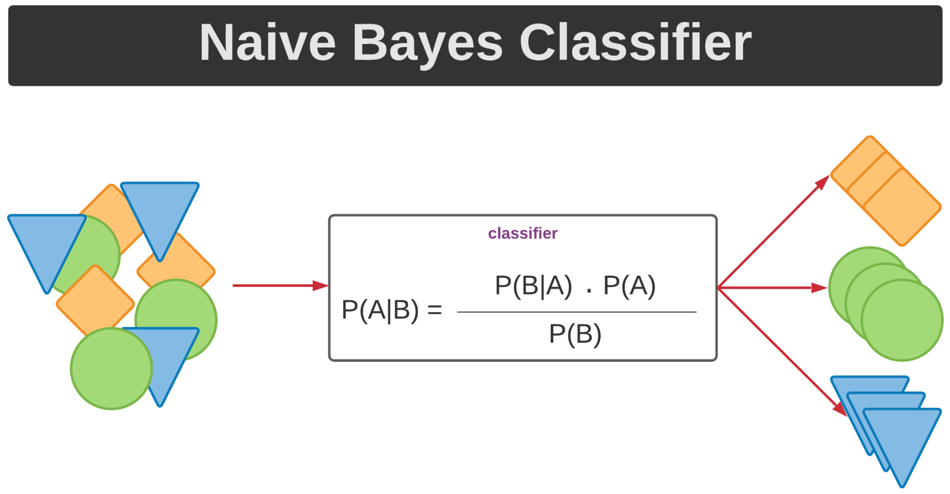
LESSON 14: NAIVE BAYES CLASSIFICATION



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1. Naive bayes introduction

1.1. Bayes formular

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$

1.2. Naive bayes classifier

We have the classification problem with C classes $1, 2, 3, \dots, C$, and we have to predict the probability of sample x belong to class c.

$$p(y = c|x) \quad \text{or} \quad p(c|x)$$

and we can choose the class for sample x by

$$c = \arg \max_{c \in \{1, \dots, C\}} p(c|x)$$

using Bayes formular and because $p(x)$ doesn't belong to c, we have

$$\begin{aligned} c &= \arg \max_{c \in \{1, \dots, C\}} p(c|x) \\ &= \arg \max_{c \in \{1, \dots, C\}} \frac{p(x|c)p(c)}{p(x)} \\ &= \arg \max_{c \in \{1, \dots, C\}} p(x|c)p(c) \end{aligned}$$

Analyze each elements in $\arg\max_{c \in \{1, \dots, C\}} p(x|c)p(c)$, we have:

- $p(c)$ is the probability of a random data sample belong to class c:
  - We can calculate this value by Maximum Likelihood Estimation or Maximum a Posteriori estimation.

- MLE is the more popular way.
- MLE calculates $p(c)$ from the training data by calculate the ratio of number data samples in class c and number data samples in the whole dataset.

- $p(x|c)$ is the distribution of data sample in class c:
  - It's hard to calculate $p(x|c)$ because x is a multi-dimension data point.
  - To simplify the calculation, we assume that each element in x is independent. That's why we call ***NAIVE BAYES***.
  - Specifically, with $x = [x_1, x_2, \dots, x_d]$, we have

$$p(x|c) = p(x_1, x_2, \dots, x_d|c) = \prod_{i=1}^{d} p(x_i|c)$$

In the training phase, we calculate $p(c)$ and each $p(x_i|c)$ from the training data. In the prediction phase, with calculated $p(c)$ and each $p(x_i|c)$, we can easily find $\arg\max_{c \in \{1,\dots,C\}} p(x|c)p(c)$.

The product of multiple probability values $\prod_{i=1}^{d} p(x_i|c)$ is really small and it can cause numeric error.

To solve this problem, we can use logarit function (logarit function is a covariate function)

$$\begin{aligned} c &= \arg \max_{c \in \{1,\dots,C\}} p(c|x) \\ &= \arg \max_{c \in \{1,\dots,C\}} p(x|c)p(c) \\ &= \arg \max_{c \in \{1,\dots,C\}} p(c) \prod_{i=1}^{d} p(x_i|c) \\ &= \arg \max_{c \in \{1,\dots,C\}} (log(p(c)) + \sum_{i=1}^{d} log(p(x_i|c))) \end{aligned}$$

2. Multinomial Naive Bayes

This Naive Bayes model is often used in text classification problem.

In the text classification problem, we have the vocabulary which contains d words.

$p(x_i|c)$ is frequency of the i^{th} word appear in the text of class c in the training dataset.

$$p(x_i|c) = \frac{N_{ci}}{N_c} = \lambda_{ci}$$

with

- N_{ci} is total number of the i^{th} word appear in the text of class c
- N_c is total number of words in the text of class c

One problem is that if one word doesn't appear in the text of class c in the training dataset, $p(x_i|c) = 0$, this makes the results wrong.

To solve this problem, we modify λ_{ci} by the following formular

$$\hat{\lambda}_{ci} = \frac{N_{ci} + \alpha}{N_c + d\alpha}$$

α is often chosen to be 1.

3. Example

	Document	Content	Class
Training	d1	hanoi pho chaolong hanoi	B
	d2	hanoi buncha pho omai	B
	d3	pho banhgio omai	B
	d4	saigon hutiu banhbo pho	N
Test	d5	hanoi hanoi buncha hutiu	?

Step 1: We have vocabulary $V = \{\text{hanoi, pho, chaolong, buncha, omai, banhgio, saigon, hutiu, banhbo}\}$ and $d = |V| = 9$.

Step 2: We calculate $p(c)$. Specifically, $p(\text{B}) = \frac{3}{4}, p(\text{N}) = \frac{1}{4}$

Step 3: We calculate $p(x_i|c)$ with $\alpha = 1$

On class $c = B$,

- $p(\text{'hanoi'}|B) = 3/11$ and $\hat{\lambda}_{B-hanoi} = 4/20$
- $p(\text{'pho'}|B) = 3/11$ and $\hat{\lambda}_{B-pho} = 4/20$
- $p(\text{'chaolong'}|B) = 1/11$ and $\hat{\lambda}_{B-chaolong} = 2/20$
- $p(\text{'buncha'}|B) = 1/11$ and $\hat{\lambda}_{B-buncha} = 2/20$
- $p(\text{'omai'}|B) = 2/11$ and $\hat{\lambda}_{B-omai} = 3/20$
- $p(\text{'banhgio'}|B) = 1/11$ and $\hat{\lambda}_{B-banhgio} = 2/20$
- $p(\text{'saigon'}|B) = 0/11$ and $\hat{\lambda}_{B-saigon} = 1/20$
- $p(\text{'hutiu'}|B) = 0/11$ and $\hat{\lambda}_{B-hutiu} = 1/20$
- $p(\text{'banhbo'}|B) = 0/11$ and $\hat{\lambda}_{B-banhbo} = 1/20$

On class $c = N$,

- $p(\text{'hanoi'}|N) = 0/4$ and $\hat{\lambda}_{N-hanoi} = 1/13$
- $p(\text{'pho'}|N) = 1/4$ and $\hat{\lambda}_{N-pho} = 2/13$
- $p(\text{'chaolong'}|N) = 0/4$ and $\hat{\lambda}_{N-chaolong} = 1/13$
- $p(\text{'buncha'}|N) = 0/4$ and $\hat{\lambda}_{N-buncha} = 1/13$
- $p(\text{'omai'}|N) = 0/4$ and $\hat{\lambda}_{N-omai} = 1/13$
- $p(\text{'banhgio'}|N) = 0/4$ and $\hat{\lambda}_{N-banhgio} = 1/13$
- $p(\text{'saigon'}|N) = 1/4$ and $\hat{\lambda}_{N-saigon} = 2/13$
- $p(\text{'hutiu'}|N) = 1/4$ and $\hat{\lambda}_{N-hutiu} = 2/13$
- $p(\text{'banhbo'}|N) = 1/4$ and $\hat{\lambda}_{N-banhbo} = 2/13$

Step 4: We calculate the prediction with `d5 = "hanoi hanoi buncha hutiu"`,

$$p(B|d5) = log(p(B)) + \sum_{i=1}^{d} log(p(x_i|B))$$
$$= log(p(B)) + log(\hat{\lambda}_{B-hanoi}) + log(\hat{\lambda}_{B-hanoi}) + log(\hat{\lambda}_{B-buncha}) + log(\hat{\lambda}_{B-hutiu})$$
$$= log(\frac{3}{4}) + log(\frac{4}{20}) + log(\frac{4}{20}) + log(\frac{2}{20}) + log(\frac{1}{20})$$
$$= -3.82$$

$$p(N|d5) = log(p(N)) + \sum_{i=1}^{d} log(p(x_i|N))$$
$$= log(p(N)) + log(\hat{\lambda}_{N-hanoi}) + log(\hat{\lambda}_{N-hanoi}) + log(\hat{\lambda}_{N-buncha}) + log(\hat{\lambda}_{N-hutiu})$$
$$= log(\frac{1}{4}) + log(\frac{1}{13}) + log(\frac{1}{13}) + log(\frac{1}{13}) + log(\frac{1}{13})$$
$$= -5.06$$

=> d5 belong to class B

4. Implementation example

4.1. Prepare library and data

In [1]:
```
import sys

import numpy as np
np.set_printoptions(threshold=sys.maxsize)
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.naive_bayes import MultinomialNB
from sklearn.metrics import classification_report

from scipy.sparse import coo_matrix

sns.set()
```

In [2]:
```
train_data_path = '../data/ling_spam_dataset/train-features.txt'
test_data_path = '../data/ling_spam_dataset/test-features.txt'
train_label_path = '../data/ling_spam_dataset/train-labels.txt'
test_label_path = '../data/ling_spam_dataset/test-labels.txt'
```

In [3]:
```
n_words = 2500
```

In [4]:
```
def read_data(data_fn, label_fn, n_words):
    ## read label_fn
    with open(label_fn) as f:
        content = f.readlines()
    label = [int(x.strip()) for x in content]

    ## read data_fn
    with open(data_fn) as f:
        content = f.readlines()
    # remove '\n' at the end of each line
    content = [x.strip() for x in content]

    dat = np.zeros((len(content), 3), dtype = int)

    for i, line in enumerate(content):
        a = line.split(' ')
        dat[i, :] = np.array([int(a[0]), int(a[1]), int(a[2])])

    # remember to -1 at coordinate since we're in Python check this:
    # https://docs.scipy.org/doc/scipy/reference/generated/scipy.sparse.coo_matrix.html
    # for more information about coo_matrix function
    data = coo_matrix(
        (dat[:, 2], (dat[:, 0] - 1, dat[:, 1] - 1)),
        shape=(len(label), n_words)
```

```python
    )
    return data, label
```

In [5]:
```python
train_data, train_label = read_data(train_data_path, train_label_path, n_words)
train_data
```

Out[5]:
```
<700x2500 sparse matrix of type '<class 'numpy.int64'>'
	with 80248 stored elements in COOrdinate format>
```

In [6]:
```python
train_data.getrow(2).toarray()
```

Out[6]:
```
array([[ 0,  0,  0,  1,  0,  0,  0,  0,  0,  2,  1,  0,  1,  0,  0,  1,
         0,  0,  2,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
         0,  0, 15,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
         0,  1,  0,  0,  0,  0,  0,  1,  0,  0,  0,  0,  0,  0,  0,  0,
         0,  0,  0,  1,  1,  0,  0,  0,  0,  0,  0,  2,  0,  0,  0,  0,
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         0,  0,  0,  0,  0,  1,  0,  0,  0,  0,  0,  0,  0,  2,  6,  0,
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```

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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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        0,   2,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
        0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
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In [7]:

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train_label
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In [8]:

```python
test_data, test_label = read_data(test_data_path, test_label_path, n_words)
test_data
```

Out[8]:

```
<260x2500 sparse matrix of type '<class 'numpy.int64'>'
	with 27979 stored elements in COOrdinate format>
```

In [9]:

```python
test_label
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Out[9]:

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```

4.2. Use `sklearn`

In [10]:
```
sklearn_naive_bayes = MultinomialNB()
```

In [11]:
```
sklearn_naive_bayes.fit(train_data, train_label)
```

Out[11]:
```
MultinomialNB()
```

In [12]:
```
y_pred = sklearn_naive_bayes.predict(test_data)
y_pred
```

Out[12]:
```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
       0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
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       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])
```

In [13]:
```
print(classification_report(y_pred, test_label))
```

```
              precision    recall  f1-score   support
```

```
           0       0.97      0.99      0.98       127
           1       0.99      0.97      0.98       133

    accuracy                           0.98       260
   macro avg       0.98      0.98      0.98       260
weighted avg       0.98      0.98      0.98       260
```

In [ ]: