Week 1 - Kinematics

Goals

- Describe geometric motion of an object in 1D and 2D
- Discuss why an object's motion changes when a force is applied

Definitions

- Kinematics
  - **How** does an object move?
  - What does the geometry of motion look like?
  - Described using $m\vec{a}$ from Newton's Second Law of Motion
- Dynamics
  - **Why** does an object move?
  - What are the causes of motion?
  - Described using $\vec{F}$ from Newton's Second Law of Motion
- Coordinate Systems
  - Defined with the following properties
    - Origin $(0, 0)$
    - An axis should be logically chosen w.r.t the problem
    - Define unit vector(s) for each point in the coordinate space
- Distance
  - The length of the path travelled by an object between two objects in space
  - Distance is a **scalar** quantity
- Displacement
  - Describes a change in position after some time Δt

Equations

- The **position** function is described as $\vec{r}(t) = x(t)\hat{\imath}$
  - Note that it is a vector function
  - We can also define it in terms of an initial time plus some offset: $\vec{r}(t + \Delta t) = x(t + \Delta t)\hat{\imath}$
- The **displacement** vector is $\Delta\hat{r} = \hat{r}(t + \Delta t) - \vec{t}$ or $(x(t + \Delta t) - x(t))\hat{\imath}$
  - This is the difference between the new position an old position
  - Δx is called the component of the displacement vector
- Average velocity depends on A) the time interval $[t, t + \Delta t]$ and the change in position

$$\vec{V}_{\text{ave}} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath}$$

  - $\frac{\Delta x}{\Delta t}$ is called the component of the average velocity
- The instantaneous velocity (at some specific time t_i is defined as the derivative at the position function $\vec{r}$ at time t_i

$$\vec{v}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}(t)$$

  - graphically, this is the slope of the tangent line at t_i

Reference

Here is a list of useful derivatives. Hints are 1) power rule, 2) product rule, and 3) quotient rule

1. Polynomial function

$$\text{if } x(t) = At^n \rightarrow \frac{\mathrm{d}}{\mathrm{d}t}x = nAt^{n-1}$$

- the derivative of t^n is nt^{n-1}

2. Exponential function

$$\text{if } x(t) = Ae^{bt} \rightarrow \frac{\mathrm{d}}{\mathrm{d}t}x = Abe^{bt}$$

- the derivative of e^x is e^x

3. Logarithmic function

$$\text{if } x(t) = A\ln(b + ct) \rightarrow \frac{\mathrm{d}}{\mathrm{d}t}x = \frac{Ac}{b + ct}$$

- the derivative of $\ln(t)$ is $\frac{1}{t}$

4. Sine function

$$\text{if } x(t) = A\sin(b + ct) \rightarrow \frac{\mathrm{d}}{\mathrm{d}t}x = Ac\cos(b + ct)$$

- the derivative of $\sin(t)$ is $\cos(t)$

5. Cosine function

$$\text{if } x(t) = A\cos(b + ct) \rightarrow \frac{\mathrm{d}}{\mathrm{d}t}x = -Ac\sin(b + ct)$$

- the derivative of $\cos(t)$ is $-\sin(t)$