CONTENTS

I. CHECKING BZO NANORODS IN YBCO FILM SUBSTRATE CALCULATIONS OF STRAIN ENERGY DENSITY

The calculations, which will involve finite element analysis simulations to check Dr Shi's analytical calculations, will begin from the start, including rudimentary concepts, as a learn-and-record-as-I-go document, á là Wade's suggestion. This section will need to be reworded / rewritten.

II. ELASTICITY

A. Mathematical preliminaries

1. Conveniences

Elasticity theory is formulated in terms of variables that are given or are desired for spatial points in a body. Some may be scalar, such as material density (ρ), Poisson's ratio (ν), shear modulus (μ), or Young's modulus (E). Some others may be *vector* quantities; displacement of material points in the elastic continuum, or rotation of material points are common examples. However, elasticity theory requires also *matrix variables*, which require, generally, ≥ 3 components. Examples of matrix variables are stress and strain (6 components (due to antisymmetry) are required to specify stress or strain at a point).

A reminder of the "Einstein" summation notation used:

$$a_{ij}b_j = \sum_{i=1}^{3} a_{ij}b_j = a_{i1}b_1 + a_{i2}b_2 + a_{i3}b_3. \tag{1}$$

List of symbol properties: NB: we use three-index symbols for convenience; more can and are used. 1. Symmetric symbol: A symbol is *symmetric* with respect to index pair ij if

$$a_{ijk} = a_{jik}.$$

2. Antisymmetric symbol: A symbol is *antisymmetric*, or *skew symmetric* w.r.t. index pair ij if

$$a_{ijk} = -a_{jik}.$$

Identity (used in some Landau relations):

$$\begin{aligned} a_{ij} &= \underbrace{\frac{1}{2}(a_{ij} + a_{ji})}_{\text{symmetric matrix}} + \underbrace{\frac{1}{2}(a_{ij} - a_{ji})}_{\text{antisymmetric matrix}} \\ &= a_{(ij)} + a_{[ij]} \end{aligned}$$

In other words, and this is useful in many areas, any matrix may be written as a symmetric and antisymmetric matrix (where the word "matrix" shows, feel free to replace it with "rank-n tensor").

2. Coordinate transformations

To write the primed frame in terms of the unprimed frame,

$$\begin{aligned} \vec{e_1}' &= Q_{11}\vec{e_1} + Q_{12}\vec{e_2} + Q_{13}\vec{e_3} \\ \vec{e_2}' &= Q_{21}\vec{e_1} + Q_{22}\vec{e_2} + Q_{23}\vec{e_3} \\ \vec{e_3}' &= Q_{31}\vec{e_1} + Q_{32}\vec{e_2} + Q_{33}\vec{e_3}, \end{aligned} \tag{2}$$

or, in index notation,

$$\vec{e_i}' = Q_{ij}\vec{e_j}.$$

The inverse transform is simply (in index notation, anyway; computation may be involved)

$$\vec{e_i} = Q_{ji}\vec{e_j}'$$

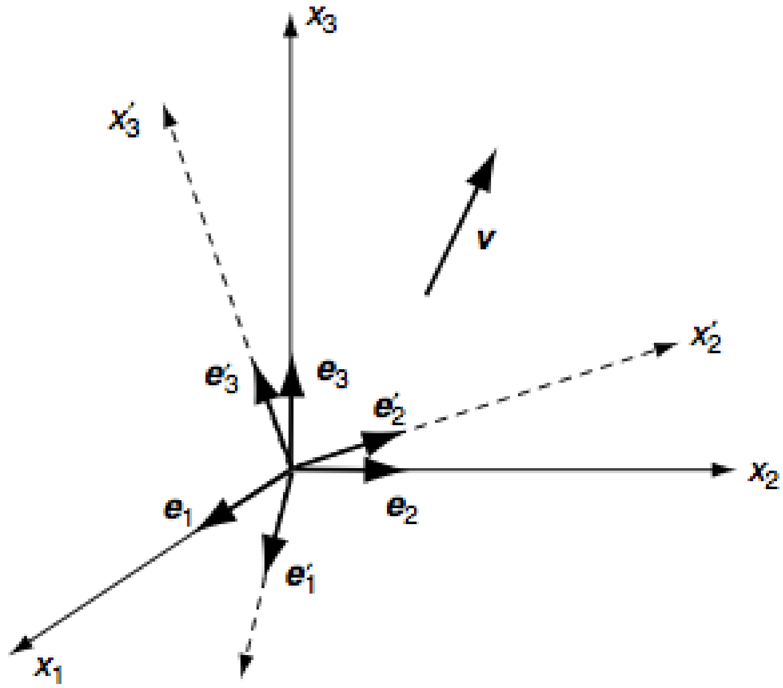

FIG. 1. Change of coordinate frames in Cartesian coordinates.

Note that instead of the Cartesian unit vectors and coordinates $\vec{e_i}$, one can use rectilinear coordinates $\vec{v_i}$ with no loss of generality.

Due to the coordinate systems being orthogonal, some constraints are put on the transforms, the most obvious being the *orthogonality condition* on the transformation matrices:

$$Q_{ji}Q_{jk} = \delta_{ik} = Q_{ji}Q_{kj} \tag{3}$$

$$\Longrightarrow detQ_{ij} = \pm 1. \tag{4}$$

(Rem: this is the *definition* of an *orthogonal matrix*.)

3. Properties of tensors

4. Isotropic tensors

A tensor which has the property that its components take the *same* value in all coordinate systems is called an *isotropic* tensor. There are, of course, n of these, for n-rank tensors; only the first, second, and third are of use to us in elasticity and fluid mechanics.

Proposition 1.

The most general second-order isotropic tensor a_{ij} is, as described above, defined such that

$$a'_{ij} = \mathcal{R}_{ip}\mathcal{R}_{jq}a_{pq} = a_{ij}, \tag{5}$$

for arbitrary rotations of the coordinate axes. Knowing that an infinitesimal rotation may be represented as $a'_i = a_i + \epsilon_{ijk}\delta\theta_j a_k$, to first order in $\delta\theta_i$,

$$\delta\theta_m \left(\epsilon_{mis}a_{sj} + \epsilon_{mjs}a_{is}\right) = 0.$$

Because the $\delta\theta_i$ are arbitrary, we may write

$$\epsilon_{mis}a_{sj} + \epsilon_{mjs}a_{is} = 0.$$

Using the notorious Kronecker-Levi-Čivita identity ($\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$),

$$\left(\delta_{ii}\delta_{ks} - \delta_{is}\delta_{ki}\right)a_{sj} + \left(\delta_{ij}\delta_{ks} - \delta_{is}\delta_{kj}\right)a_{is} = 0 \tag{6}$$

$$2a_{ij} + a_{ji} = a_{ss}\delta_{ij}. \tag{7}$$

The absolute starting point for considerations of elasticity is the displacement, taken within the realm of solid bodies. The most basic (and trivial) displacement is defined as the displacement of a body coordinate, x_i, by some distance x: $u_x = x_i - x$.

B. Rigid bodies