□ Class 10 Maths – Chapter 2: Polynomials

Important PYQs and Practice Questions with Step-by-Step Solutions

---

□ 1-MARK QUESTIONS

Q1. Is $3x - 5$ a polynomial? Give reason.
Answer:
Yes, $3x - 5$ is a polynomial.
Explanation:
A polynomial is an algebraic expression where the exponents of the variable are whole numbers (0, 1, 2, ...).
Here, the variable x has exponent 1, which is a whole number.
Hence, **it is a polynomial of degree 1**.

Q2. Write the degree of the polynomial: $2x^4 - 3x^2 + 5x - 7$.
Answer:
The degree of a polynomial is the highest power of the variable.
Here, the highest power of x is 4.
∴ **Degree = 4**

Q3. What is the maximum number of zeroes a cubic polynomial can have?
Answer:
The maximum number of zeroes of a polynomial = its degree.
Degree of a cubic polynomial = 3
∴ **Maximum number of zeroes = 3**

Q4. If one of the zeroes of a quadratic polynomial $x^2 - 3x + k$ is 2, find the value of k.
Solution:
Let the zeroes be $\alpha = 2$ and β.
Sum of zeroes = $\alpha + \beta = -(\text{coefficient of } x)/\text{coefficient of } x^2 = -(-3)/1 = 3$
So, $2 + \beta = 3 \Longrightarrow \beta = 1$
Product of zeroes = $\alpha \times \beta = 2 \times 1 = 2$
Now, Product of zeroes = $c/a = k/1 = k$
So, **$k = 2$**

Q5. Write a quadratic polynomial whose sum and product of zeroes are 7 and 12 respectively.
Solution:
Let the quadratic polynomial be:
$x^2 - (\text{sum of zeroes})\cdot x + (\text{product of zeroes})$
$= x^2 - 7x + 12$
Answer: $x^2 - 7x + 12$

□ 2-MARK QUESTIONS

Q6. Find the zeroes of the polynomial $x^2 - 6x + 8$ and verify the relationship between zeroes and coefficients.
Solution:
Factor the polynomial:
$x^2 - 6x + 8 = (x - 2)(x - 4)$
Zeroes are: $x = 2$, $x = 4$
Sum $= 2 + 4 = 6$, Product $= 2 \times 4 = 8$
Compare with:
Sum $= -(b/a) = -(-6)/1 = 6$
Product $= c/a = 8/1 = 8$
Verified.

Q7. If one zero of $x^2 - 5x + 6$ is 3, find the other.
Solution:
Sum of zeroes $= -(-5)/1 = 5$
One zero $= 3$
So, other zero $= 5 - 3 =$ **2**

Q8. If α and β are zeroes of $x^2 - 2x - 8$, find the value of $\alpha^2 + \beta^2$.
Solution:
Sum $= \alpha + \beta = 2$, Product $= \alpha\beta = -8$
Formula:
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2(-8) = 4 + 16 =$ **20**

Q9. Find a quadratic polynomial whose zeroes are 3 and –2.
Solution:
Let the polynomial be:
$(x - 3)(x + 2) = x^2 - x - 6$
Answer: $x^2 - x - 6$

Q10. Find the value of k if $x^2 + kx + 16$ has equal zeroes.
Solution:
For equal zeroes: Discriminant (D) = 0
$D = b^2 - 4ac = k^2 - 4\times1\times16 = k^2 - 64 = 0$
$\Rightarrow k^2 = 64$
$\Rightarrow k = \pm8$
Answer: $k = 8$ or -8

□ 3-MARK QUESTIONS

Q11. Divide $p(x) = x^3 - 3x^2 + 5x - 3$ by $d(x) = x - 1$.
Solution (Using Long Division):

Divide $x^3 - 3x^2 + 5x - 3$ by $x - 1$:

1. $x^3 \div x = x^2 \rightarrow$ Multiply: $x^2(x - 1) = x^3 - x^2$
   Subtract: $(x^3 - 3x^2) - (x^3 - x^2) = -2x^2$
2. $-2x^2 \div x = -2x \rightarrow$ Multiply: $-2x(x - 1) = -2x^2 + 2x$
   Subtract: $-2x^2 + 5x - (-2x^2 + 2x) = 3x$
3. $3x \div x = 3 \rightarrow$ Multiply: $3(x - 1) = 3x - 3$
   Subtract: $3x - 3 - (3x - 3) = 0$

Quotient = $x^2 - 2x + 3$, Remainder = 0

Q12. If $2x^2 + kx + 3$ has zeroes 1 and –3, find the value of k.
Solution:
Sum of zeroes = $1 + (-3) = -2$
Now, Sum of zeroes = $-b/a = -k/2$
$\Rightarrow -2 = -k/2 \Longrightarrow k = 4$
Answer: $k = 4$

Q13. Find a quadratic polynomial whose zeroes are the reciprocals of the zeroes of $2x^2 - 5x + 3$.
Solution:
Let original zeroes be α and β
Sum = $\alpha + \beta = 5/2$, Product = $\alpha\beta = 3/2$
New zeroes: $1/\alpha$, $1/\beta$
Sum of reciprocals = $(1/\alpha + 1/\beta) = (\alpha + \beta) / (\alpha\beta) = (5/2) / (3/2) = 5/3$
Product = $1/\alpha \times 1/\beta = 1/(\alpha\beta) = 2/3$
Required polynomial:

x^2 – (sum of zeroes)x + product
= $x^2 - (5/3)x + 2/3$
To avoid fractions, multiply by 3:
Polynomial = $3x^2 - 5x + 2$

Q14. Find the zeroes of $6x^2 - x - 2$ and verify the relationship.
Solution:
Use factorisation:
$6x^2 - x - 2 = (3x + 2)(2x - 1)$
Zeroes: $x = -2/3$ and $x = 1/2$
Sum = $-2/3 + 1/2 = (-4 + 3)/6 = -1/6$
Product = $-2/3 \times 1/2 = -1/3$
Now verify with:
Sum = $-b/a = -(-1)/6 = 1/6$ □ (So check calculation again...)
Actually: $3x + 2$ and $2x - 1$ → Multiply:
$(3x + 2)(2x - 1) = 6x^2 - 3x + 4x - 2 = 6x^2 + x - 2$ □
So correct factorisation of $6x^2 - x - 2$ is:
$(3x - 2)(2x + 1) \rightarrow 6x^2 + 3x - 4x - 2 = 6x^2 - x - 2$ □
Zeroes: $2/3, -1/2$
Sum = $2/3 + (-1/2) = (4 - 3)/6 = 1/6 = -(-1)/6$
Product = $2/3 \times (-1/2) = -1/3 = -2/6 = c/a$
Verified.

Q15. Find a quadratic polynomial whose zeroes are 4 and –7. Verify the relationship.
Solution:
Sum = $4 + (-7) = -3$
Product = $4 \times (-7) = -28$
Polynomial = $x^2 - (-3)x + (-28) = x^2 + 3x - 28$
Verification:
Sum = $-b/a = -3/1 = -3$
Product = $c/a = -28/1 = -28$
Verified

□ 4-MARK QUESTIONS

Q1. Divide the polynomial p(x)=x3+2x2–5x–6p(x) = x^3 + 2x^2 – 5x – 6p(x)=x3+2x2–5x–6 by d(x)=x+2d(x) = x + 2d(x)=x+2 and find the quotient and remainder.

Solution:

We will use **polynomial long division** to divide p(x)=x3+2x2–5x–6p(x) = x^3 + 2x^2 – 5x – 6p(x)=x3+2x2–5x–6 by d(x)=x+2d(x) = x + 2d(x)=x+2.

1. **Step 1: Divide the first term**:
   Divide the leading term of the dividend x3x^3x3 by the leading term of the divisor xxx:
   x3x=x2\frac{x^3}{x} = x^2xx3=x2
   Multiply the entire divisor x+2x + 2x+2 by x2x^2x2:
   x2(x+2)=x3+2x2x^2(x + 2) = x^3 + 2x^2x2(x+2)=x3+2x2
   Subtract this from the original polynomial:
   (x3+2x2–5x–6)−(x3+2x2)=−5x−6(x^3 + 2x^2 – 5x – 6) - (x^3 + 2x^2) = -5x - 6(x3+2x2–5x–6)−(x3+2x2)=−5x−6
2. **Step 2: Divide the next term**:
   Divide the leading term −5x-5x−5x by xxx:
   −5xx=−5\frac{-5x}{x} = -5x−5x=−5
   Multiply the entire divisor x+2x + 2x+2 by −5-5−5:
   −5(x+2)=−5x−10-5(x + 2) = -5x - 10−5(x+2)=−5x−10
   Subtract:
   (−5x–6)−(−5x−10)=4(-5x – 6) - (-5x - 10) = 4(−5x–6)−(−5x−10)=4
3. **Step 3: Conclude the division**:
   Since the degree of the remainder (4) is less than the degree of the divisor x+2x + 2x+2, we stop here.

Result:
The quotient is x2−5x^2 - 5x2−5 and the remainder is 444.

Thus,

x3+2x2−5x−6x+2=x2−5+4x+2\frac{x^3 + 2x^2 - 5x - 6}{x + 2} = x^2 - 5 + \frac{4}{x + 2}x+2x3+2x2−5x−6=x2−5+x+24

Q2. If the zeroes of the polynomial 2x2−5x+32x^2 - 5x + 32x2−5x+3 are α\alphaα and β\betaβ, find a quadratic polynomial whose zeroes are α+1\alpha + 1α+1 and β+1\beta + 1β+1.

Solution:

1. **Sum and product of the original zeroes**:
   The sum of the zeroes α+β\alpha + \betaα+β and the product αβ\alpha \betaαβ can be found using the relationships from the given polynomial.
   For the polynomial 2x2−5x+32x^2 - 5x + 32x2−5x+3,
   - Sum of zeroes:
     α+β=−ba=52\alpha + \beta = -\frac{b}{a} = \frac{5}{2}α+β=−ab=25
   - Product of zeroes:
     αβ=ca=32\alpha \beta = \frac{c}{a} = \frac{3}{2}αβ=ac=23
2. **New sum and product**:
   The new zeroes are α+1\alpha + 1α+1 and β+1\beta + 1β+1.
   - New sum:
     (α+1)+(β+1)=(α+β)+2=52+2=92(\alpha + 1) + (\beta + 1) = (\alpha + \beta) + 2 = \frac{5}{2} + 2 = \frac{9}{2}(α+1)+(β+1)=(α+β)+2=25+2=29
   - New product:
     (α+1)(β+1)=αβ+(α+β)+1=32+52+1=4(\alpha + 1)(\beta + 1) = \alpha \beta +

(\alpha + \beta) + 1 = \frac{3}{2} + \frac{5}{2} + 1 = 4(α+1)(β+1)=αβ+(α+β)+1=23+25+1=4

3. **Form the new quadratic polynomial**:
   Using the new sum and product, the quadratic polynomial is:

   x2−(sum of zeroes)·x+product of zeroesx^2 - (\text{sum of zeroes}) \cdot x + \text{product of zeroes}x2−(sum of zeroes)·x+product of zeroes =x2−92x+4= x^2 - \frac{9}{2}x + 4=x2−29x+4

   To avoid fractions, multiply the entire equation by 2:

   2x2−9x+82x^2 - 9x + 82x2−9x+8

Answer: The required quadratic polynomial is 2x2−9x+82x^2 - 9x + 82x2−9x+8.

Q3. Given that the polynomial x3−4x2+5x−2x^3 - 4x^2 + 5x - 2x3−4x2+5x−2 is divided by x−1x - 1x−1, use synthetic division to find the quotient and remainder.

Solution (Using Synthetic Division):

1. Write down the coefficients of the polynomial x3−4x2+5x−2x^3 - 4x^2 + 5x - 2x3−4x2+5x−2:
   Coefficients: [1,−4,5,−2][1, -4, 5, -2][1,−4,5,−2]
2. Set the divisor x−1x - 1x−1 equal to 1 for synthetic division.
   Set up the synthetic division as follows:

11−45−21−321−320\begin{array}{r|rrrr} 1 & 1 & -4 & 5 & -2 \\ & & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & 0 \\ \end{array}111−41−35−32−220

3. **Result**:
   The quotient is x2−3x+2x^2 - 3x + 2x2−3x+2 and the remainder is 000.

Thus, we have:

x3−4x2+5x−2x−1=x2−3x+2\frac{x^3 - 4x^2 + 5x - 2}{x - 1} = x^2 - 3x + 2x−1x3−4x2+5x−2=x2−3x+2

Answer: The quotient is x2−3x+2x^2 - 3x + 2x2−3x+2 and the remainder is 000.