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PRACTICE PAPER 09 (2024-25)

CHAPTER 08 INTRODUCTION TO TRIGONOMETRY (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : X | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$ is equal to:

(a) –1 (b) 1 (c) 0 (d) 2

Ans. (b) 1

$(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta$

$= \tan^2\theta \times 1/\tan^2\theta = 1$

2. In Δ ABC right angled at B, sin A = $\frac{7}{25}$, then the value of cos C is …………

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{7}{24}$ (d) $\frac{24}{7}$

Ans: (a) $\frac{7}{25}$

3. If 5 tan θ = 4, then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is

(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

Ans: (a) 1/6

4. If cosec A = 13/12, then the value of $\frac{2\sin A - 3\cos A}{4\sin A - 9\cos A}$

(a) 4 (b) 5 (c) 6 (d) 3

Ans: (d) 3

Given cosec A = 13/12,

$\sin A = \frac{12}{13}, \cos A = \frac{5}{13}$

$$Now, \frac{2\sin A - 3\cos A}{4\sin A - 9\cos A} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)} = \frac{24-15}{48-45} = \frac{9}{3} = 3$$

5. Given that sin α = 1/2 and cos β = 1/2, then the value of (β – α) is

(a) 0° (b) 30° (c) 60° (d) 90°

Ans: (b) 30°

6. If tan θ = 1, then the value of sec θ + cosec θ is:
(a) 3√2 (b) 4√2 (c) 2√2 (d) √2
Ans: (c) 2√2
Given, tan θ = 1, we have θ = 45°
So, sec θ + cosec θ = √2 + √2 = 2√2.

7. If sin 2A = $\frac{1}{2}$ tan² 45° where A is an acute angle, then the value of A is
(a) 60° (b) 45° (c) 30° (d) 15°
Ans: (d) 15°

$$\sin 2A = \frac{1}{2}\tan^2 45^0 = \frac{1}{2}\times 1^2 = \frac{1}{2} = \sin 30^0 \Rightarrow 2A = 30^0 \Rightarrow A = 15^0$$

8. If θ is an acute angle and tan θ + cot θ = 2, then the value of $\sin^3\theta + \cos^3\theta$ is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

Ans: (c) $\frac{\sqrt{2}}{2}$

$$\tan\theta + \cot\theta = 2 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2 \Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$$

$$\Rightarrow (\tan\theta - 1)^2 = 0 \Rightarrow \tan\theta = 1 = \tan 45^0 \Rightarrow \theta = 45^0$$

$$Now, \sin^3\theta + \cos^3\theta = \sin^3 45^0 + \cos^3 45^0 = \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

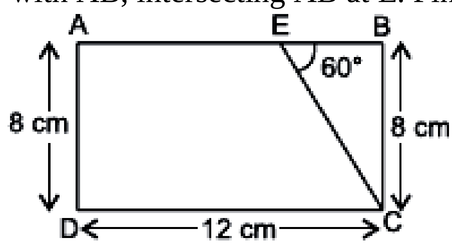
9. Assertion (A): In a right ΔABC, right angled at B, if tan A = 1, then 2 sin A. cos A = 1.
Reason (R): tan 45° = 1 and sin 45° = cos 45° = 1/√2
Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. Assertion (A): sin (A + B) = sin A + sin B
Reason (R): For any value of θ, 1 + tan²θ = sec²θ
Ans. (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of 60° with AB, intersecting AB at E. Find the length of CE and BE.



Ans: In ΔCBE, we have $\tan 60^0 = \frac{CB}{BE}$

$$\Rightarrow \sqrt{3} = \frac{8}{BE} \Rightarrow BE = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} cm$$

and $\sin 60^0 = \frac{CB}{CE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{CE} \Rightarrow CE = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3} cm$

12. If sin (A + B) = √3/2 and sin (A – B) = $\frac{1}{2}$, $0 \le A + B \le 90°$ and A > B, then find A and B.

Ans: sin(A + B) = √3/2 = sin 60°
⇒ A + B = 60° ....(i)
sin (A - B) = 1/2 = sin 30°
⇒ A - B = 30° ....(ii)
Solving eq. (i) and (ii), A = 45° and B = 15°

13. Evaluate: $3\cos^2 60° \sec^2 30° – 2\sin^2 30° \tan^2 60°$.
Ans: $3\cos^2 60° \sec^2 30° – 2\sin^2 30° \tan^2 60°$

$$= 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2 - 2\left(\frac{1}{2}\right)^2\left(\sqrt{3}\right)^2 = \frac{3}{4}\times\frac{4}{3} - 2\times\frac{1}{4}\times 3 = 1 - \frac{3}{2} = -\frac{1}{2}$$

14. Simplify: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

Ans: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$

$$= \frac{\sin^2\theta}{\cos^2\theta}\times\frac{\cos^2\theta}{1} + \frac{\cos^2\theta}{\sin^2\theta}\times\frac{\sin^2\theta}{1} = \sin^2\theta + \cos^2\theta = 1$$

OR

If $7\sin^2 A + 3\cos^2 A = 4$, then find tan A
Ans: Given, $7\sin^2 A + 3\cos^2 A = 4$
Dividing both sides by $\cos^2 A$, we get
7 tan2A + 3 = 4 $\sec^2 A$ [∵ $\sec^2\theta = 1 + \tan^2\theta$]
⇒ $7\tan^2 A + 3 = 4(1 + \tan^2 A)$
⇒ $7\tan^2 A + 3 = 4 + 4\tan^2 A$
⇒ $3\tan^2 A = 1 \Rightarrow \tan^2 A = 1/3 \Rightarrow \tan A = 1/\sqrt{3}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If cosecθ + cotθ = p, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$

Ans: Given cosecθ + cotθ = p ….. (1)
$\Rightarrow (cosec\theta - \cot\theta)(cosec\theta + \cot\theta) = 1 \Rightarrow (cosec\theta - \cot\theta)\,p = 1$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = \frac{1}{p}$ ……. (2)

Adding (1) and (2), we get

$$cosec\theta = \frac{p+\frac{1}{p}}{2} = \frac{p^2+1}{2p}; \cot\theta = \frac{p-\frac{1}{p}}{2} = \frac{p^2-1}{2p}$$

Now, $\cos\theta = \dfrac{\cot\theta}{\cos ec\theta} = \dfrac{\frac{p^2-1}{2p}}{\frac{p^2+1}{2p}} = \dfrac{p^2-1}{p^2+1}$

16. Prove that $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \sec\theta+\tan\theta$

Ans: LHS = $\dfrac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$ (Dividing numerator and denominator by cos□)

$= \dfrac{\tan\theta+\sec\theta-1}{\tan\theta+1-\sec\theta}$

$= \dfrac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta+1-\sec\theta}$

$= \dfrac{(\sec\theta+\tan\theta)(1-\sec\theta+\tan\theta)}{\tan\theta+1-\sec\theta}$ $= \sec\theta+\tan\theta$ = RHS

OR

If sin θ + cos θ = √3 , then prove that tan θ + cot θ = 1.

Ans: $\sin\theta+\cos\theta=\sqrt{3} \Rightarrow (\sin\theta+\cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = 3$

$\Rightarrow 1+2\sin\theta\cos\theta = 3 \Rightarrow 2\sin\theta\cos\theta = 2$

$\Rightarrow \sin\theta\cos\theta = 1 = \sin^2\theta+\cos^2\theta$

$\Rightarrow 1 = \dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta} = \tan\theta+\cot\theta$ $\Rightarrow \tan\theta+\cot\theta = 1$

17. Prove that: $\dfrac{\cos^2\theta}{1-\tan\theta}+\dfrac{\sin^2\theta}{1-\cot\theta} = 1+\sin\theta\cos\theta$

Ans: $LHS = \dfrac{\cos^2\theta}{1-\tan\theta}+\dfrac{\sin^2\theta}{1-\cot\theta}$

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta}-\dfrac{\sin^3\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta} = \dfrac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{\cos\theta-\sin\theta}$

$= \cos^2\theta+\sin^2\theta+\cos\theta\sin\theta$ $= 1+\sin\theta\cos\theta = RHS$

OR

If cos θ + sin θ = √2 cos θ, show that cos θ – sin θ = √2 sin θ.

Ans: Given, cos θ + sin θ = √2 cos θ

Squaring both sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta)$

$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ [3]

(b) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\sin\theta$ then find $x^2 + y^2$. [2]

Ans: (a) L.H.S = $(\sin A + \text{cosec} A)^2 + (\cos A + \sec A)^2$

$=\sin^2 A + \text{cosec}^2 A + 2\sin A \text{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 2\sin A\times 1/\sin A + 2\cos A\times 1/\cos A$

Since, $(\sin^2 A + \cos^2 A = 1)$

$(\sec^2 A = 1 + \tan^2 A, \text{cosec}^2 A = 1 + \cot^2 A)$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A$ = RHS

(b)

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

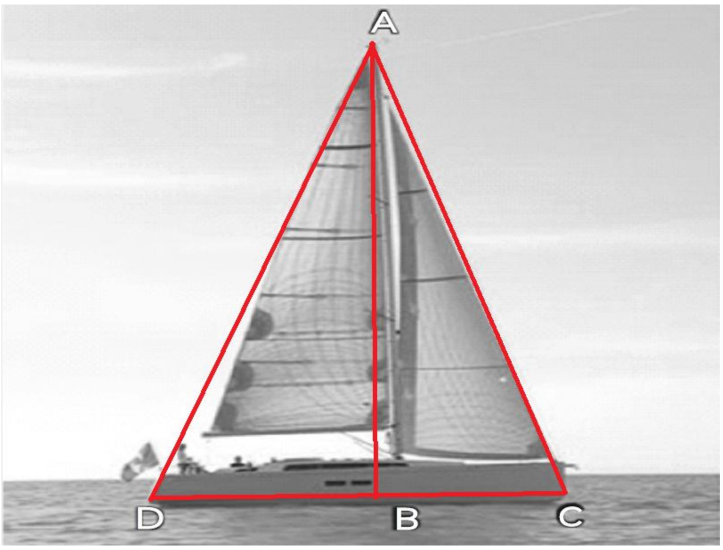
Now, $x\sin\theta = y\cos\theta \Rightarrow \cos\theta\sin\theta = y\cos\theta \Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance BC = 1 m and BD = 2 m and height AB = 4 m.



Based on the given in formation, answer the following questions:

(a) Find the value of sec D. [1]

(b) Find the value of cosec C. [1]

(c) Find the value of tan D + cot C. [1]

(d) Find the value of $\sin^2 C + \cos^2 D$ [1]

Ans. (a) In ΔABD, sec D = AD/BD

by using Pythagoras theorem in right triangle ABD.

$AD^2 = BD^2 + AB^2 = 2^2 + 4^2 = 20$

$\Rightarrow AD = \sqrt{20} = 2\sqrt{5}$m

$\therefore$ sec D = AD/BD = $2\sqrt{5}/2 = \sqrt{5}$

(b) In ΔABC, cosec C = AC/AB

by using Pythagoras theorem in right triangle ABC.

$AC^2 = AB^2 + BC^2 = 4^2 + 1^2 = 17$

$\Rightarrow AC = \sqrt{17}$ m

∴ cosec C = AC/AB = √17/ 4
(c) In ΔABD, tanD = AB/BD = 4/2 = 2
In ΔABC, cotC = BC/AB = 1/4
∴ tanD + cotC = 2 + 1/4 = 9/4
(d) In ΔABC, sinC = AB/AC = 4/√17
In ΔABD, cosD = BD/AD = 1/√5
∴ $\sin^2 C + \cos^2 D$ = 16/17 + 1/5 = 97/85

20. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river “Ganges” or “Ganga”. A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.

Based on the above information, answer the following questions. (Take √3 =1.732)
(a) Find the value of sin A. [1]
(b) Find the value of sin C. [1]
(c) Find the value of tan A – cot C. [1]
(d) Find the value of $\text{cosec}^2 C$. [1]
Ans. (a) In ΔABC, sinA = BC/AC
by using Pythagoras theorem in right triangle ABC.
$AC^2 = AB^2 + BC^2 = 12^2 + 5^2 = 144 + 25 = 169$
⇒ AD = 13 m
∴ sinA = BC/AC = 5/13
(b) In ΔABC, sinC = AB/AC
⇒ sinC = AB/AC = 12/13
(c) In ΔABC, tanA = BC/AB = 5/12
⇒ cotC = BC/AB = 5/12
Therefore, tan A – cot C = 0
(d) In ΔABC, sinC = AB/AC = 12/13
cosecC = 1/sinC = 13/12
Therefore, $\text{cosec}^2 C$ = 169/144