2 1. What is the HCF of the smallest prime number and the smallest composite number?

Year: 2018 Solution:

- Smallest prime number = 2
- Smallest composite number = 4

HCF(2, 4) = 2

$\ \square$ 2. Find after how many places of decimal the decimal form of the number will terminate.

Year: 2019 Solution:

For a rational number to have a terminating decimal, its denominator (in the lowest terms) must have only 2 and/or 5 as prime factors.

Example:

- 1/8 = 0.125 (terminates after 3 decimal places)
- 1/20 = 0.05 (terminates after 2 decimal places)

\square 3. Express 429 as a product of its prime factors.

Year: 2019 **Solution:** 429 ÷ 3 = 143 143 ÷ 11 = 13

Prime factorization of $429 = 3 \times 11 \times 13$

 \square 4. Given that HCF(135, 225) = 45, find LCM(135, 225).

Year: 2020 Solution:

Using the relation:

 $LCM \times HCF = Product of the numbers$

 $LCM = (135 \times 225) / 45 = 675$

LCM(135, 225) = 675

 \Box 5. After how many decimal places will the decimal representation of the rational number terminate?

Year: 2020 Solution:

For a rational number to have a terminating decimal, its denominator (in the lowest terms) must have only 2 and/or 5 as prime factors.

Example:

- 1/8 = 0.125 (terminates after 3 decimal places)
- 1/20 = 0.05 (terminates after 2 decimal places)
- \Box 6. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number.

Year: 2018 Solution:

Assume $(5 + 3\sqrt{2})$ is rational.

Then, $5 + 3\sqrt{2} = p/q$, where p and q are integers.

 $3\sqrt{2} = (p/q) - 5$ $\sqrt{2} = [(p - 5q) / 3q]$

Since $\sqrt{2}$ is irrational, this leads to a contradiction.

Therefore, $(5 + 3\sqrt{2})$ is irrational.

 \Box 7. If HCF of 65 and 117 is expressible in the form 65n – 117, then find the value of n.

Year: 2019 Solution:

Using Euclid's division algorithm:

 $117 = 65 \times 1 + 52$ $65 = 52 \times 1 + 13$ $52 = 13 \times 4 + 0$ HCF = 13

Express 13 as a linear combination of 65 and 117:

 $13 = 65 - 1 \times 52$

Substitute 52 = 117 - 65:

 $13 = 65 - 1 \times (117 - 65) = 65 - 117 + 65 = 2 \times 65 - 117$

n = 2

 \square 8. On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm, and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Year: 2019 Solution:

Find the LCM of 30, 36, and 40:

- $30 = 2 \times 3 \times 5$
- $36 = 2^2 \times 3^2$
- $\bullet \quad 40 = 2^3 \times 5$

$$LCM = 2^3 \times 3^2 \times 5 = 180$$

Minimum distance = 180 cm

\square 9. Find HCF and LCM of 404 and 96 and verify that HCF \times LCM = Product of the two numbers.

Year: 2018 Solution:

- $404 = 2^2 \times 101$
- $96 = 2^5 \times 3$

$$HCF = 2^2 = 4$$

$$LCM = 2^5 \times 3 \times 101 = 4832$$

Verification:

$$HCF \times LCM = 4 \times 4832 = 19328$$

Product of the numbers = $404 \times 96 = 19328$

Verification is correct.

\square 10. Prove that $\sqrt{3}$ is an irrational number.

Year: 2019 Solution:

Assume $\sqrt{3} = p/q$, where p and q are coprime integers.

Squaring both sides:

$$3 = p^2/q^2 \longrightarrow p^2 = 3q^2$$

This implies p^2 is divisible by 3, so p is divisible by 3. Let p = 3k.

Substitute p in the equation:

$$(3k)^2 = 3q^2 \rightarrow 9k^2 = 3q^2 \rightarrow q^2 = 3k^2$$

This implies q^2 is divisible by 3, so q is divisible by 3.

Thus, p and q have a common factor of 3, contradicting the assumption that they are coprime.

Therefore, $\sqrt{3}$ is irrational.

\Box 11. Find the largest number which on dividing 1251, 9377, and 15628 leaves remainders 1, 2, and 3 respectively.

Year: 2019 Solution:

Let the required number be x.

Then, $1251 = x \times q_1 + 1$, $9377 = x \times q_2 + 2$, and $15628 = x \times q_3 + 3$.

Subtracting the equations:

9377 - 1251 = $x(q_2 - q_1) + 1 \rightarrow 8126 = x(q_2 - q_1) + 1$ 15628 - 9377 = $x(q_3 - q_2) + 1 \rightarrow 6251 = x(q_3 - q_2) + 1$ Thus, x divides 8125 and 6250. The HCF of 8125 and 6250 is 125. **Therefore, x = 125.**