

SUBJECT: MATHEMATICS

MAX. MARKS : 40

CLASS : X

DURATION : 1½ hrs

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

SECTION – A**Questions 1 to 10 carry 1 mark each.**

1. If 300 m high pole makes an angle of elevation at a point on ground which is 300 m away from its foot, then the angle of elevation is:
(a) 60° (b) 90° (c) 30° (d) 45°
Ans. (d) 45°
2. The angle of depression of a bike parked on the road from the top of a 90 m high pole is 60 degrees. The distance of the bike from the pole is:
(a) $20\sqrt{3}$ m (b) 90 m (c) $15\sqrt{3}$ m (d) $30\sqrt{3}$ m
Ans. (d) $30\sqrt{3}$ m
3. A stone is $15\sqrt{3}$ m away from a tower 15 m high, then the angle of elevation of the top of the tower from the stone is:
(a) 45° (b) 60° (c) 30° (d) 90°
Ans. (c) 30°
4. The ratio of the length of a tower and its shadow is $\sqrt{3} : 1$. The altitude of the sun is:
(a) 0° (b) 60° (c) 30° (d) 45°
Ans. (b) 60°
5. The tops of the poles of height 16 m and 10 m are connected by a wire of length l meters. If the wire makes an angle of 30° with the horizontal, then l =
(a) 26 m (b) 16 m (c) 12 m (d) 10 m
Ans. (c) 12 m
6. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is
(a) 8 m (b) 10 m (c) 12 m (d) 14 m
Ans. (c) 12 m
7. If the angle of depression of an object from a temple is 30° , and the distance of the object from the temple is 45 m, then the height of the temple is:
(a) $45\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) 20 m (d) $20\sqrt{3}$ m
Ans. (b) $15\sqrt{3}$ m

8. If two towers of heights h_1 and h_2 subtend angles of 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2 =$

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

Ans. 3 : 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
 (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
 (c) Assertion (A) is true but reason (R) is false.
 (d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45° .

Reason (R): According to Pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base.

Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. **Assertion (A):** The ladder 20 m long makes an angle 60° with the wall, then the height of the point where the ladder touches the wall is 15 m.

Reason (R): For an angle θ , $\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$

Ans. (d) Assertion (A) is false but reason (R) is true.

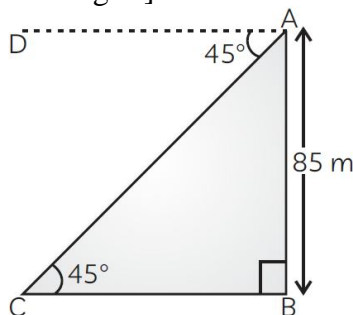
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The angle of depression of a car standing on the ground, from the top of a 85 m high tower is 45° . Find the distance of the car from the base of the tower.

Ans. Let $AB = 85$ m be the height of the tower and angle of depression is $\angle DAC = 45^\circ$.

Then, $\angle ACB = \angle DAC = 45^\circ$ [alternate angles]



Now, in right-angled $\triangle ABC$, $\tan 45^\circ = AB/BC$

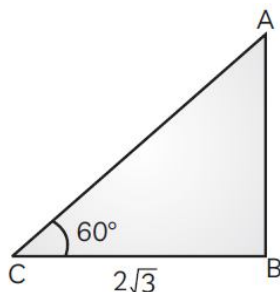
$$\Rightarrow 1 = 85/BC$$

$$\Rightarrow BC = 85 \text{ m}$$

Hence, the distance of the car from the base of the tower is 85 m.

12. A pole casts a shadow of length $2\sqrt{3}$ m on ground, when the sun's elevation is 60° . Find the height of the pole.

Ans. Let AB be the pole and BC be its shadow.



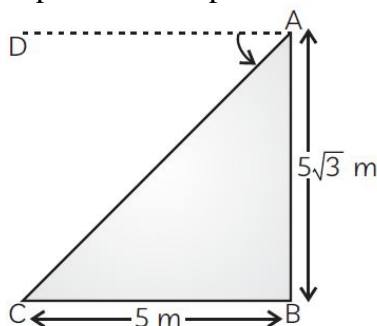
\therefore In $\triangle ABC$, $\tan 60^\circ = AB/BC$

$$\Rightarrow \sqrt{3} = AB/2\sqrt{3}$$

$$\Rightarrow AB = 6 \text{ m}$$

Hence, the height of the pole is 6 m.

- 13.** The figure shows the observation of point C from point A. Find the angle of depression from A.



Ans. In right-angled $\triangle ABC$, $\angle B = 90^\circ$

Let $\angle DAC = \theta$

Then $\angle DAC = \angle ACB = \theta$ [alternate angles]

Now, $\tan \theta = AB/BC = 5\sqrt{3}/5$

$$\Rightarrow \tan \theta = \sqrt{3}$$

$$\Rightarrow \tan \theta = \tan 60^\circ$$

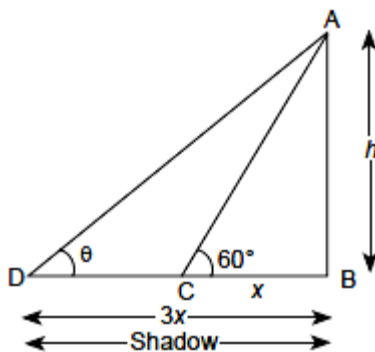
$$\Rightarrow \theta = 60^\circ$$

Hence, the angle of depression from A is 60° .

- 14.** The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sunrays meet the ground at an angle of 60° . Find the angle between the sunrays and the ground at the time of longer shadow.

Ans. In $\triangle ABC$, $\tan 60^\circ = \frac{AB}{BC} = \frac{h}{x}$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$$



In $\triangle ABD$, $\tan \theta = \frac{AB}{BD} \Rightarrow \tan \theta = \frac{h}{3x}$

$$\Rightarrow \tan \theta = \frac{\sqrt{3}x}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

SECTION – C

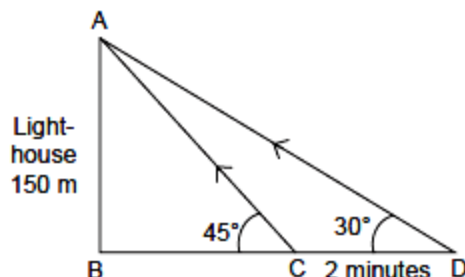
Questions 15 to 17 carry 3 marks each.

- 15.** A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30° . Find the speed of the boat. (Use $\sqrt{3} = 1.732$)

Ans. Let AB is lighthouse.

$\therefore AB = 150$ m

Initially boat is at C and after 2 minutes it reaches at D.



In right $\triangle ABC$, $\frac{AB}{BC} = \tan 45^\circ$

$$\Rightarrow \frac{150}{BC} = 1 \Rightarrow BC = 150 \text{ m}$$

In right $\triangle ABD$, $\frac{AB}{BD} = \tan 30^\circ$

$$\Rightarrow \frac{150}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 150\sqrt{3} \text{ m}$$

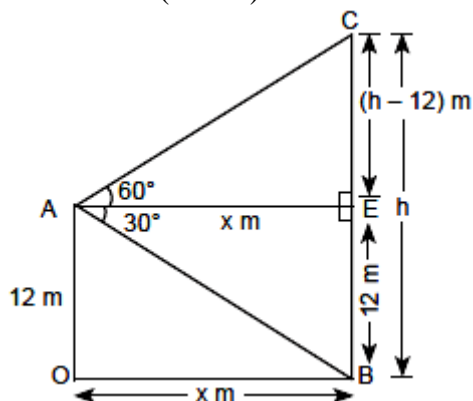
Distance covered in 2 minutes = $BD - BC = 150\sqrt{3} - 150 = 150(\sqrt{3} - 1)$ m

$$\begin{aligned} \therefore \text{Speed} &= \frac{\text{Distance covered}}{\text{time taken}} = \frac{150(\sqrt{3} - 1)}{2} \\ &= 75 \times (1.732 - 1) = 54.9 \text{ m/minutes} \end{aligned}$$

- 16.** A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30° . Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3} = 1.732$]

Ans. A is the position of the man, OA = 12m, BC is cliff.

Let height of the cliff BC = h m and CE = $(h - 12)$ m.



Let $AE = OB = x$ m

In right angled triangle AEB, $\frac{AE}{BE} = \cot 30^\circ \Rightarrow AE = 12 \times \sqrt{3}$

$$= 12 \times 1.732 \text{ m} = 20.78 \text{ m.}$$

\therefore Distance of ship from cliff = 20.78 m.

In right angled triangle AEC, $\frac{CE}{AE} = \tan 60^\circ \Rightarrow \frac{h-12}{12\sqrt{3}} = \sqrt{3} \Rightarrow h - 12 = 36 \Rightarrow h = 48 \text{ m}$

\therefore Height of the cliff = 48 m

17. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Ans: Let AB be the tower and ships are at points C and D. As per question statement we have shown diagram below.

Now in $\triangle ABC$ we have $\tan 45^\circ = \frac{AB}{AC}$

$$\Rightarrow \frac{AB}{AC} = 1 \Rightarrow AB = BC$$

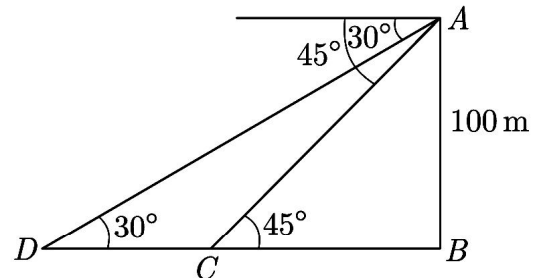
Now in $\triangle ABD$ we have $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$\Rightarrow AB + CD = \sqrt{3} AB$$

$$\Rightarrow CD = AB(\sqrt{3} - 1) = 100 \times (1.732 - 1) = 73.2 \text{ m}$$

Distance between two ships is 73.2 m.



SECTION – D

Questions 18 carry 5 marks.

18. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30° . The angle of depression of the reflection of the cloud in the lake, at A is 60° . Find the distance of the cloud from A.

Ans: Let DE be the level of water and cloud be at position B which is h m above the level of water and reflection of cloud be at F and AC = DE = x m.

$$\therefore BC = (h - 20)\text{m}, CF = (h + 20)\text{m}$$

In $\triangle ABC$, $\tan 30^\circ = \frac{BC}{AC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-20}{x} \Rightarrow x = \sqrt{3}(h-20) \dots(i)$$

In $\triangle ACF$,

$$\tan 60^\circ = \frac{CF}{AC} \Rightarrow \sqrt{3} = \frac{h+20}{x}$$

$$\Rightarrow x = \frac{h+20}{\sqrt{3}} \dots(ii)$$

From (i) and (ii), we get $\sqrt{3}(h-20) = \frac{h+20}{\sqrt{3}}$

$$\Rightarrow 3h - 60 = h + 20 \Rightarrow 2h = 80 \Rightarrow h = 40$$

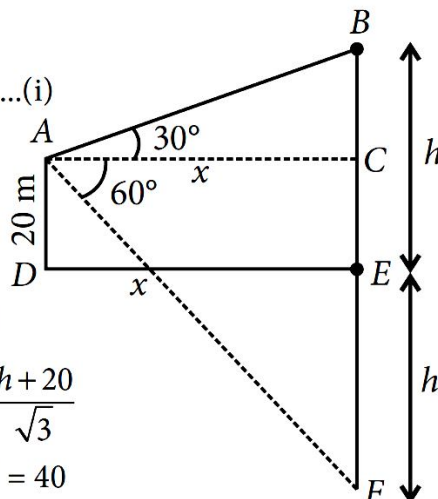
From (i), we have $x = \sqrt{3}(40 - 20) = 20\sqrt{3}$

Applying Pythagoras theorem in $\triangle ABC$,

$$AB^2 = BC^2 + AC^2 = (20)^2 + (20\sqrt{3})^2$$

$$= 400 + 1200 = 1600 \Rightarrow AB = \sqrt{1600} = 40 \text{ m}$$

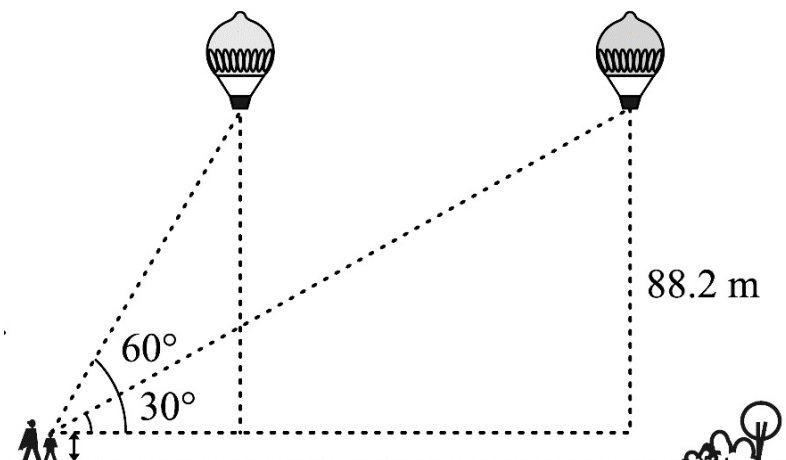
\therefore Distance of the cloud from point A = 40 m



SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After 30 seconds, the angle of elevation reduces to 30° (see the below figure).



Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.732$)

- (i) Find the distance travelled by the balloon during the interval. (2)
(ii) Find the speed of the balloon. (2)

OR

- (ii) If the elevation of the sun at a given time is 30° , then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

Ans: (i) In the figure, let C be the position of the observer (the girl).

A and P are two positions of the balloon.

CD is the horizontal line from the eyes of the (observer) girl.

Here $PD = AB = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

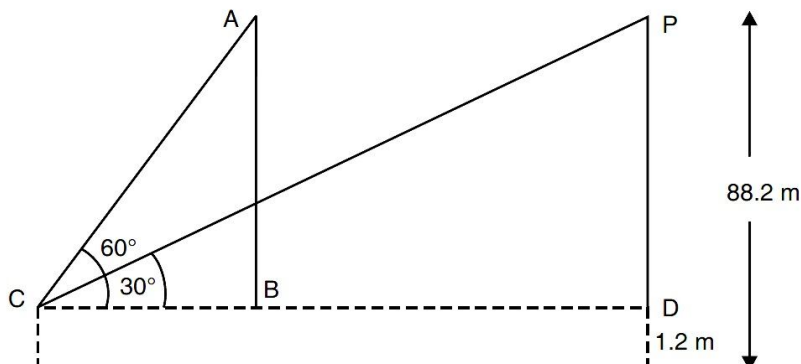
In right $\triangle ABC$, we have $\frac{AB}{BC} = \tan 60^\circ$

$$\Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} \text{ m}$$

In right $\triangle PDC$, we have $\frac{PD}{CD} = \tan 30^\circ$

$$\Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3}$$

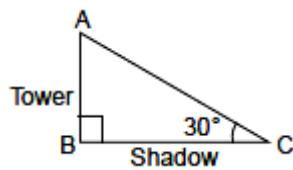
$$\text{Now, } BD = CD - BC = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 58\sqrt{3} \text{ m}$$



Thus, the required distance between the two positions of the balloon = $58\sqrt{3}$ m
 $= 58 \times 1.732 = 100.46$ m (approx.)

(ii) Speed of the balloon = Distance/time = $100.46/30 = 3.35$ m/s (approx.)

OR



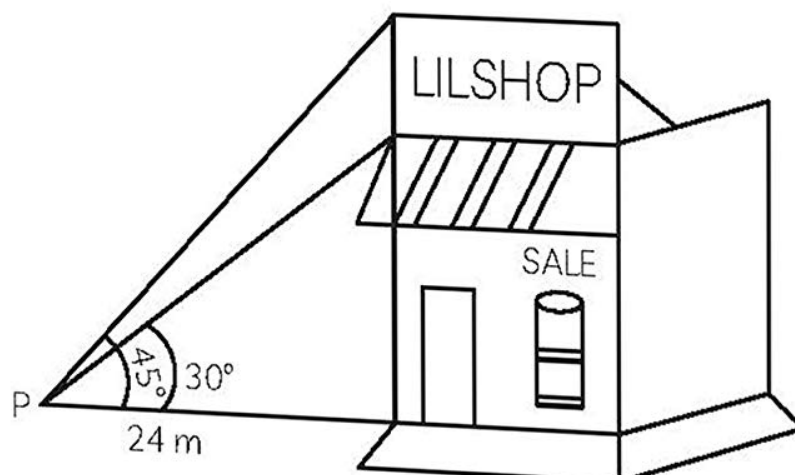
In right $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ \Rightarrow \frac{150}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 150\sqrt{3} \text{ feet}$$

20. Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.



From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45° . The point P is at a distance of 24 m from the base of the building.



On the basis of the above information, answer the following questions:

(i) Find the height of the building (without the sign board). (2)

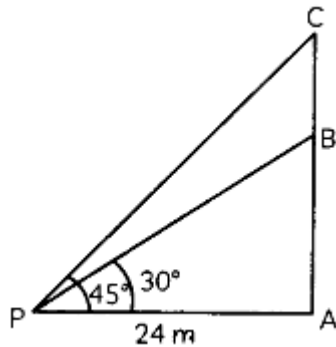
OR

Find the height of the building (with the sign board) (2)

(ii) Find the height of the sign board. (1)

(iii) Find the distance of the point P from the top of the sign board. (1)

Ans: (i) In $\triangle APC$,



$$\tan 30^\circ = AB/AP$$

$$\Rightarrow 1/\sqrt{3} = AB/24$$

$$\Rightarrow AB = 24/\sqrt{3} \text{ m} = 13.85 \text{ m} = 14 \text{ m (approx)}$$

OR

Considering, the diagram in the above question, AC as the new height of the shop including the sign-board.

In $\triangle APC$,

$$\tan 45^\circ = AC/AP$$

$$\Rightarrow 1 = AC/24$$

$$\Rightarrow AC = 24 \text{ m}$$

(ii) From Q (i) and Q (ii).

$$\text{Length of sign board, } BC = AC - AB$$

$$= 24 - 14$$

$$= 10 \text{ m}$$

(iii) In $\triangle APC$,

$$\cos 45^\circ = AP/AC$$

$$\Rightarrow 1/\sqrt{2} = 24/AC$$

$$\Rightarrow PC = 24\sqrt{2} \text{ m}$$

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