Fact: $\lg \frac{\pi}{1-\pi} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Interpretation: After adjusting for the effect of other predictors (x_2), as x_1 increases 1 unit, then

① the $\boxed{\text{log-odds}}$ will increase $\boxed{\beta_1}$

when $x_1 = a$,

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = \cancel{\lg \pi_0}\ \lg \underbrace{\frac{\pi_0}{1-\pi_0}}_{\text{odds}_0} = \text{log-odds}_0$$

when $x_1 = a+1$

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = \lg \frac{\pi_0}{1-\pi_0} + \beta_1 = \text{log-odds}_1$$

change: $\text{log-odds}_1 - \text{log-odds}_0 = \beta_1$

② the $\boxed{\text{log-odds-ratio}}$ will increase $\boxed{\beta_1}$

when $x_1 = a$

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = \text{log-odds}_0 = \lg(\text{odds}_0)$$

$$\text{log-odds-ratio}_0 = \lg(\text{odds}_0) - \lg(\text{odds}_0) = \lg \frac{\text{odds}_0}{\text{odds}_0} = 0$$

when $x_1 = a+1$

$$\beta_0 + \beta_1 x_1 + \beta x_2 = \text{log-odds}_1 = \lg(\text{odds}_1)$$

$$\text{log-odds-ratio}_1 = \text{log-odds}_1 - \text{log-odd}_0 = \lg \frac{\text{odds}_1}{\text{odds}_0} = \beta_1 \quad (\text{from ①})$$

change: $(\text{log-odds-ratio}_1) - (\text{log-odds-ratio}_0) = \beta_1 - 0 = \beta_1$

③ the $\boxed{\text{odds}}$ will change by a factor (or multiplier) of $\boxed{e^{\beta_1}}$

when $x_1 = a$,

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = \lg(\text{odds}_0)$$

$$\text{odds}_0 = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2}$$

when $x_1 = a+1$,

$$\text{odds}_1 = e^{\beta_0 + \beta x_1 + \beta x_2 + \beta_1}$$

change: $\text{odds}_1 / \text{odds}_0 = \frac{\text{odds}_1}{\text{odds}_0} = e^{\beta_1}$

④ the $\boxed{\text{odds}}$ will increase by $\boxed{e^{\beta_1} - 1} \times 100\%$

change: $\text{odds}_1 - \text{odds}_0 = (\text{odds}_0 \cdot e^{\beta_1}) - \text{odds}_0 = \text{odds}_0 \cdot (e^{\beta_1} - 1)$