

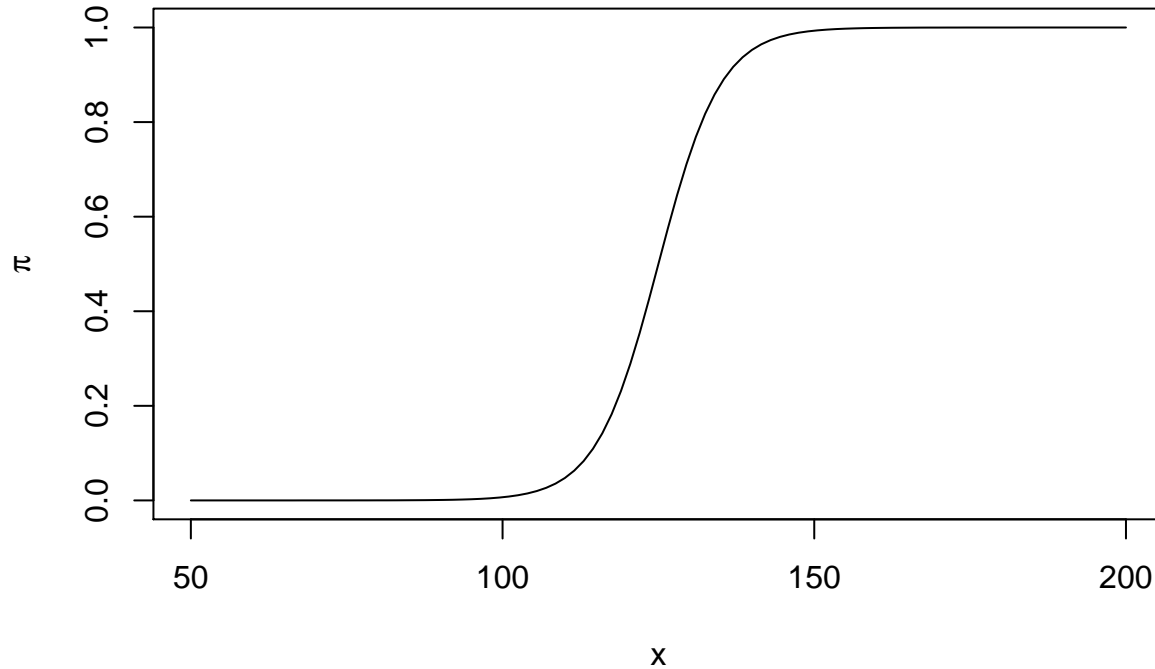
HW02

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1. (Kutner et al, Problem 14.4)

a.

Logistic mean response function: $E(Y_i) = \pi_i = F_L(\beta_0 + \beta_1 X_i) = \frac{\exp(\beta_0 + \beta_1 X_i)}{1 + \exp(\beta_0 + \beta_1 X_i)}$



b.

```
logit <- function(p){log(p/(1-p))}
linear.component <- function(x){beta.0+beta.1*x}
GoFKernel::inverse(linear.component)(logit(0.5)) #inverse function
```

```
## [1] 125
```

c.

```
odds.1 <- exp(linear.component(150))
odds.2 <- exp(linear.component(151))
ratio.odds <- odds.2/odds.1
ratio.odds
```

```
## [1] 1.221403
```

```
exp(beta.1)
```

```
## [1] 1.221403
```

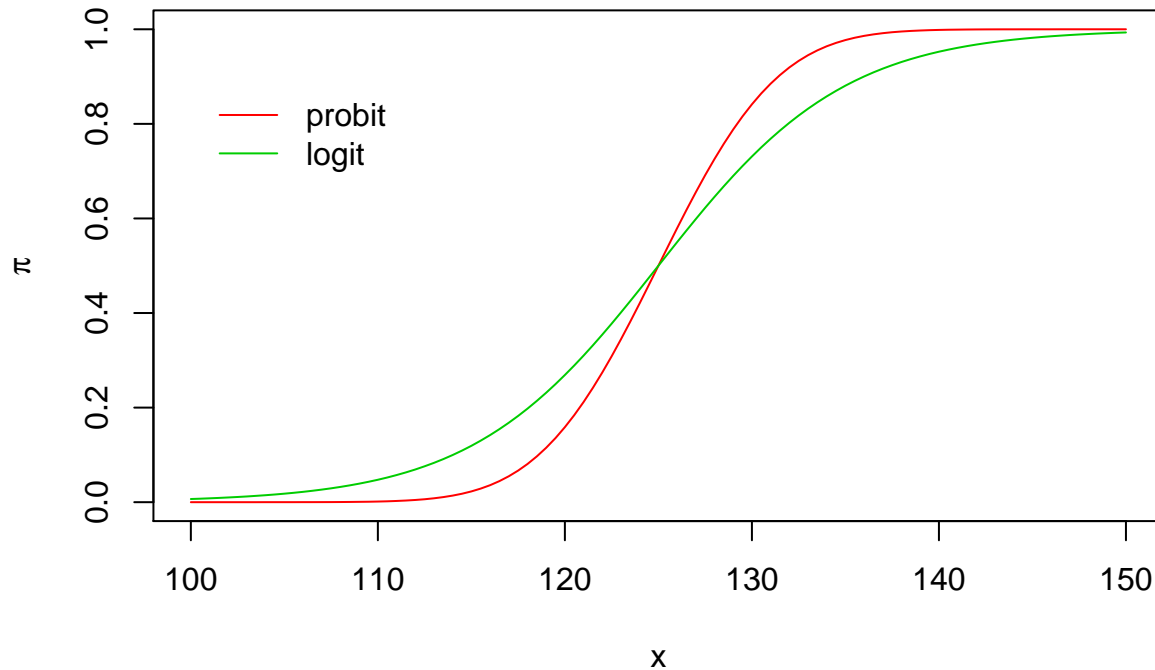
They are the same.

2. (Kutner et al, Problem 14.6)

a.

Probit mean response function: $E(Y_i) = \pi_i = \Phi(\beta_0 + \beta_1 X_i)$

$$\beta_0 = -25, \beta_1 = 0.2$$



The logistic curve is smoother and have a more constant marginal effect.

b.

```
GoFKernel::inverse(linear.component)(GoFKernel::inverse(pnorm)(0.5))
```

```
## [1] 125
```

```
#GoFKernel::inverse(linear.component)(qnorm(0.5))
```

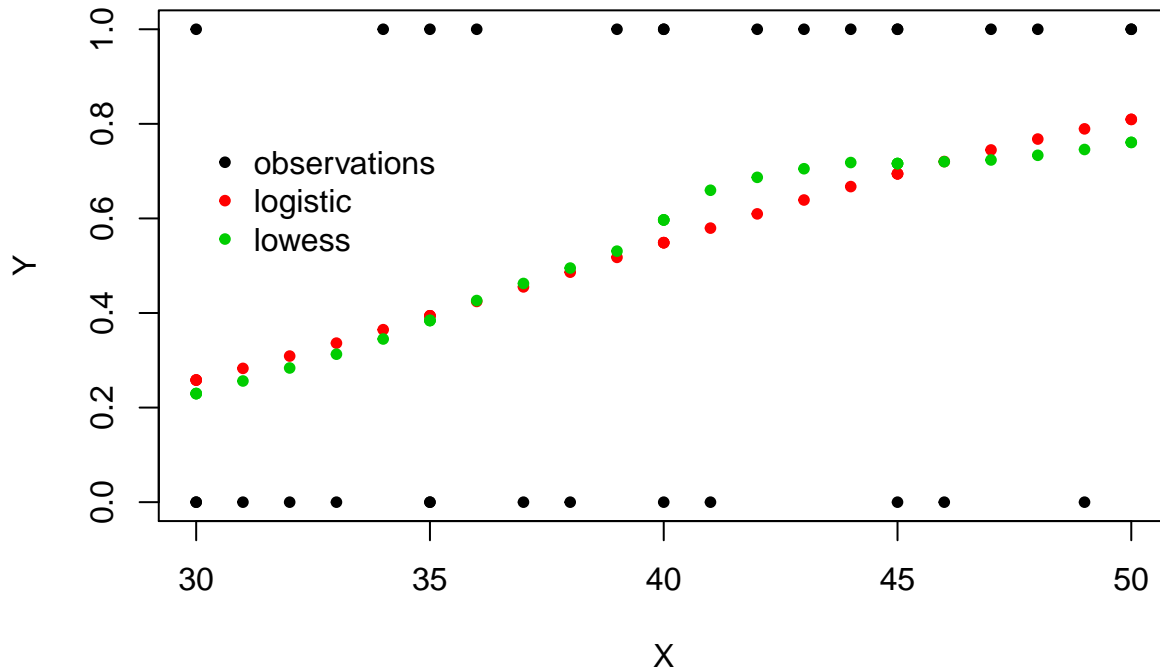
3. (Kutner et al, Problem 14.7)

a.

```
##  
## Call: glm(formula = Y ~ X, family = binomial(link = logit))  
##  
## Coefficients:  
## (Intercept)          X  
##    -4.8075         0.1251  
##  
## Degrees of Freedom: 29 Total (i.e. Null);  28 Residual  
## Null Deviance:      41.46  
## Residual Deviance: 37.46    AIC: 41.46
```

Fitted response function: $\hat{\pi} = \frac{\exp(-4.8+0.125X)}{1+\exp(-4.8+0.125X)}$

b.



It appears to be a good fit.

c.

```
data.frame("exp(b1)"=exp(coef(reg)[2]))
```

```
## exp.b1.
```

```
## X 1.133237
```

$e^{\beta_1} = 0.125$. Every unit of increase in dues would cause the odds of not renewing the membership will change by a multiplier of 0.125.

d.

```
## pi
```

```
## 0.5487487
```

If the dues are increased by \$40, the estimated probability that association members will not renew their membership is 0.5487.

e.

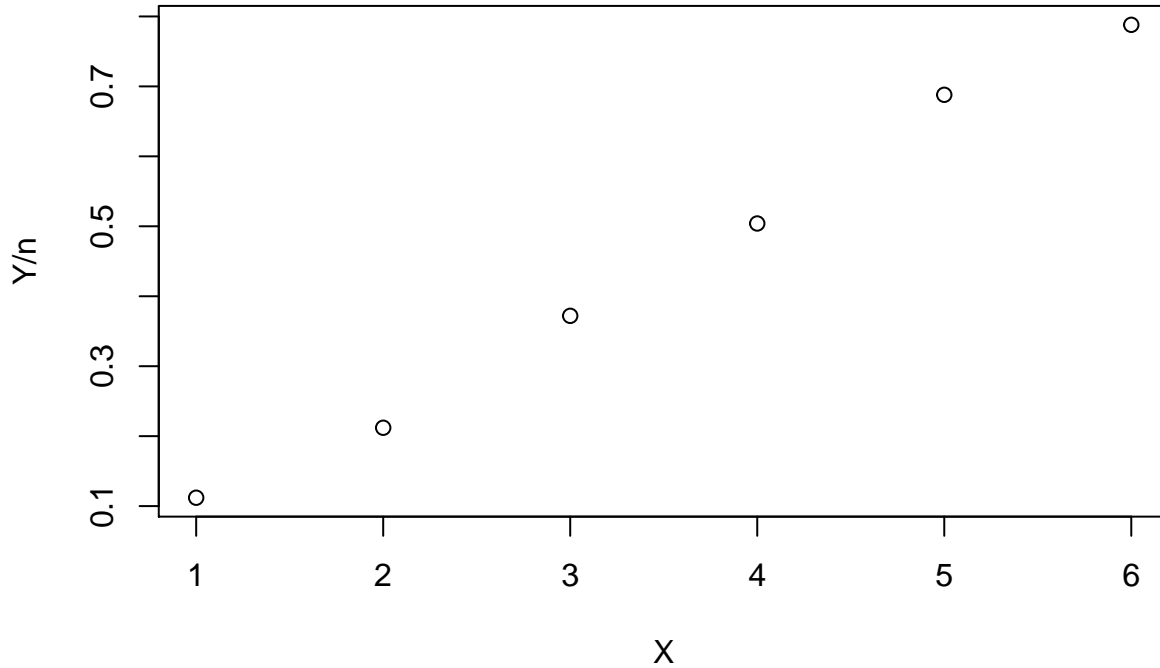
```
## X
```

```
## 47.21945
```

The amount of dues increase for which 75 percent of the members are expected not to renew their association membership is 47.219.

4. (Kutner et al, Problem 14.12)

a.



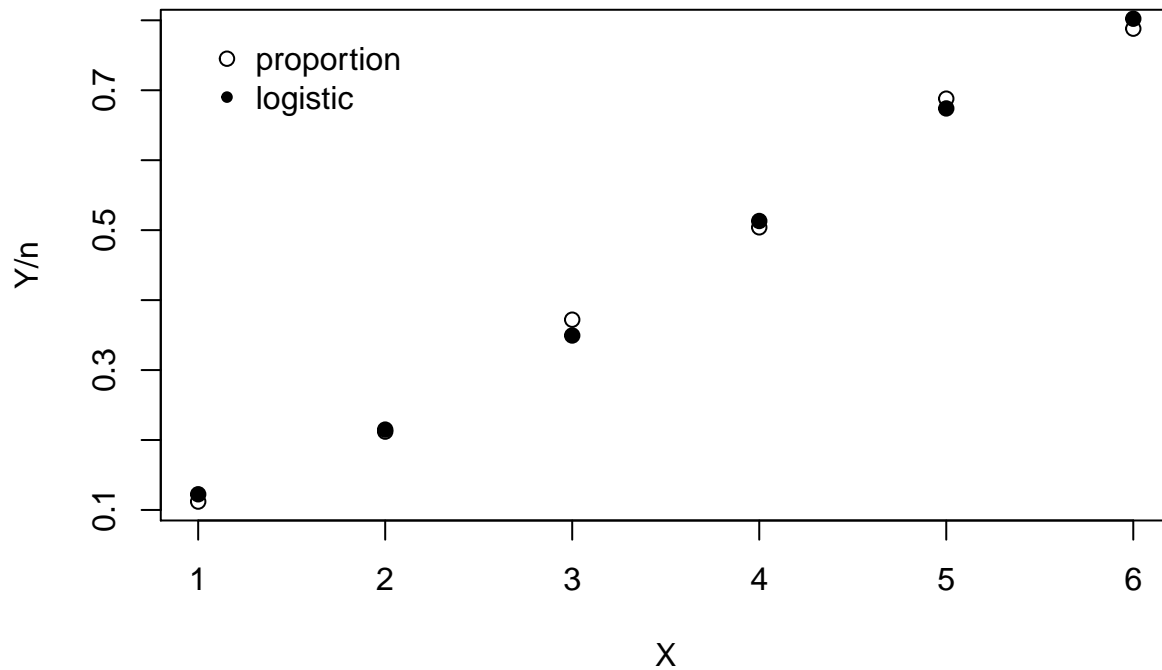
The plot to some extent follows a S shape, so the logistic response function can be appropriate.

b.

```
##
## Call: glm(formula = Y/n ~ X, family = binomial(link = logit), weights = n)
##
## Coefficients:
## (Intercept)          X
##      -2.644         0.674
##
## Degrees of Freedom: 5 Total (i.e. Null);  4 Residual
## Null Deviance:      383.1
## Residual Deviance: 1.449    AIC: 39.36
```

Fitted response function: $mortality\ rate = \frac{\exp(-2.644 + 0.674X)}{1 + \exp(-2.644 + 0.674X)}$

c.



The fitted logistic response function appear to fit well.

d.

```
##      exp.b1.
## X 1.962056
```

$e^{\beta_1} = 1.96$. Every unit of increase in the dose level would cause the odds of mortality will change by a multiplier of 1.96.

e.

```
##      pi
## 0.4293018
```

When the dose level is 3.5, the estimated probability that an insect dies is 0.429.

f.

```
##      X
## 3.922409
```

The estimated median lethal dose is 3.922.

5. (Kutner et al, Problem 14.14)

a.

```
##
## Call: glm(formula = Y ~ X1 + X2 + as.factor(X3), family = binomial(link = logit),
##      data = df)
##
## Coefficients:
```

```
##      (Intercept)          X1          X2  as.factor(X3)1
##      -1.17716        0.07279       -0.09899        0.43397
##
## Degrees of Freedom: 158 Total (i.e. Null);  155 Residual
## Null Deviance:      134.9
## Residual Deviance: 105.1    AIC: 113.1
```

Fitted response function: $\hat{\pi} = \frac{\exp(-1.177+0.0728X_1-0.099X_2+0.434I_{X_3=1})}{1+\exp(-1.177+0.0728X_1-0.099X_2+0.434I_{X_3=1})}$

b.

```
##      exp(b1)    exp(b2)    exp(b3)
## 1.0755025 0.9057549 1.5433801
```

- Keeping other factors constant, every unit of increase in age would cause the odds of receiving a flu shot will change by a multiplier of 1.0755.
- Keeping other factors constant, every unit of increase in health awareness would cause the odds of receiving a flu shot will change by a multiplier of 0.906.
- Keeping other factors constant, for a male the odds of receiving a flu shot will higher than that for a female by 0.0755.

c.

```
##      pi
## 0.06422197
```

The estimated probability that male clients aged 55 with a health awareness index of 60 will receive a flu shot is 0.064.