

# Lecture 3

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**Announcements:**

Four quizzes: Best three. No make up quizzes. 30%

Two assignments: Both 10% (total 20%) One

midterm: 25%

One final exam: 25%

## Verification of Security Protocols

Abstract the protocol into a **formal model** (automata, logic)

Assume perfect cryptography

Specify required guarantees as **mathematical properties** over these abstract models

*Prove* these properties hold,  
preferably by automated means



## **Our Formalism: Dolev-Yao Model**

*On the Security of Public Key Protocols (1983).*

$$A \rightarrow B : \{(A, \{m\}_B)\}_B$$

$$B \rightarrow A : \{(B, \{m\}_A)\}_A$$

$$A!B : \text{aenc} \left( A, \text{aenc} \left( (m, \text{pk}_B) \right), \text{pk}_B \right)$$

$$B? : \text{aenc} \left( X, \text{aenc} \left( (m', \text{pk}_B) \right), \text{pk}_B \right)$$

$$A? : \text{aenc} \left( B, \text{aenc} \left( (m, \text{pk}_A) \right), \text{pk}_A \right)$$

$$B!X : \text{aenc} \left( B, \text{aenc} \left( (m', \text{pk}_X) \right), \text{pk}_X \right)$$

$pk_x)$

## Our Formalism: Dolev-Yao Model Split

each communication into a send and a receive.

/

The intruder is essentially the network.

/

- Each send captured by

/

- Each receive assumed to come from

A send action need not have a corresponding receive action.

## Our Formalism: Dolev-Yao Model

/

Intruder cannot break encryption. It can:

- **See** any message sent on the public channel
- **Block** any message from reaching the intended recipient •

**Re-route** any message to any principal

- **Masquerade** as any principal and send messages in their name •

**Initiate** new communication according to the protocol • **Generate** messages according to some rules

## Messages as Term Algebra

Messages are **not** structured documents.

Ignore extraneous details (headers, metadata, formatting)

Formally modelled as symbolic terms

$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a))$  Atomic terms  $m$

(messages) and  $a$  (agent names).

## Proof Rules for Generating Terms

$$\begin{array}{c} \text{ax } (t \in X) \\ X \vdash a \end{array} \quad \text{pk} \quad \begin{array}{c} t_1) X \vdash t_i \\ \\ \text{split}_i \\ X \vdash \text{pair}(t_0, X \vdash t \quad X \vdash t' \end{array}$$
$$X \vdash \text{pk}(a)$$
$$X \vdash t$$



$$X \vdash \text{pair}(t, \quad t')$$

pair

$$\text{pk}(a) \mid X \vdash \text{sk}(a) \mid X \vdash m \mid X \vdash \text{pk}(a) \quad \text{aenc}$$

$$X \vdash \text{aenc}(m, \quad X \vdash m \quad X \vdash \text{aenc}(m,$$

$$\text{pk}(a))$$

# adec

# Proof Rules for Generating Terms

$$\text{pk}(B) \quad , \quad \text{pk}(I) \quad , \quad \text{pk}(I)$$

$$X_I \vdash \text{sk}(I)\text{ax} \ (\text{sk}_I \in X_I) \text{ a dec}$$

$$X_I \vdash \text{aenc pair } B, \text{aenc pair } A, \text{aenc } m, \quad \lambda I \vdash \text{sk}(I)^{\text{ax}} (\text{sk } I \in \lambda I) \text{ aenc}$$

$$X_I \vdash \text{pair } B, \text{aenc pair } A, \text{aenc } (m, \text{pk}(B)), \text{pk}(I)) \quad X_I \vdash \text{aenc pair } A, \text{aenc } m,$$

$$\text{pk}(B), \text{pk}(I)$$

$$X_I \vdash \text{sk}(I) \text{ ax } (\text{sk}_I \in X_I) \text{ adec}$$

$$\text{split}_1$$

$$\text{pk}(B)) \text{ split}_1 X_I \vdash \text{aenc}$$

$$X_I \vdash \text{pk}(B)$$

$$\text{ax } (B \in X_I) \text{ pk}$$

$$X_I \vdash \text{ax } (I \in X_I)$$

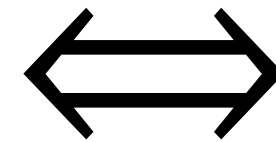
$$X_I \vdash \text{pair } (A, \text{aenc } (m, \text{pk}(B)))$$

$$X_I \vdash B$$

$$X_I \vdash \text{pair } (I, \text{aenc } (m, \text{pk}(B))) \text{ aenc } X_I \vdash \text{aenc } (\text{pair } (I, \text{aenc } (m, \text{pk}(B))), \text{pk}(B))$$

## Proof Rules for Generating Terms

Given  $I$ 's knowledge, can it derive a given term  $t$ ?



Given a deductive proof system for  
generating terms and a set of known terms  
 $X$ , does there exist a derivation for term  $t$ ?

That is,  $X \vdash t$ ? And can we  
automate this?

# Two Intruder Problems

*Passive intruder problem:* Can the intruder violate some property just by listening to the network?

*Active intruder problem:* Can the intruder violate some property by listening to network + allowable behaviours?

# Two Intruder Problems

*Passive intruder problem:* Can the intruder violate some property just by listening to the network?

Fixed  $X$ , fixed  $t$ . Check if  $X \vdash t$ .

*Active intruder problem:* Can the intruder violate some property by listening to network + allowable behaviours?

## Two Intruder Problems

*Passive intruder problem:* Can the intruder violate some

property just by listening to the network?

Fixed  $X$ , fixed  $t$ . Check if  $X \vdash t$ .

*Active intruder problem:* Can the intruder violate some property by listening to network + allowable behaviours?

$X$

Come up with an and a suitable mapping for  $X \vdash t$

$X \vdash t$

variables in and such that we have .

**Passive Intruder Problem**

Given  $X$  and  $t$ , check if  $X \vdash t$ .

check if  $X \vdash t$ .

$X = \{\text{aenc}(m, \text{pk}(A)),$   
 $\text{pair}(\text{sk}(B),$   
 $\text{aenc}(\text{pair}(n, \text{sk}(A)), \text{pk}(C)))$   
 $\text{pair}(n, \text{aenc}(\text{sk}(C),$   
 $\text{pk}(B))))\}$

Consider

**Passive Intruder**

**Problem** Given  $X$  and  $t$ ,

$X \vdash m?$

Consider

## Passive Intruder

**Problem** Given  $X$  and  $t$ ,

check if  $X \vdash t$ .



$$X = \{\text{aenc}(m, \text{pk}(A)), \\ \text{pair}(\text{sk}(A), \\ \text{aenc}(\text{pair}(n, \text{sk}(A)), \text{pk}(C))) \\ \text{pair}(n, \text{aenc}(\text{sk}(C), \\ \text{pk}(B)))\}$$
$$X \vdash m?$$

check if  $X \vdash t$ .

$X = \{\text{aenc}(m, \text{pk}(A)),$   
 $\text{pair}(\text{pk}(B), \text{aenc}(\text{pair}(m,$   
 $\text{pk}(B)), \text{pk}(C)))$   
 $\text{aenc}(\text{sk}(C), \text{pk}(B))\}$

Consider

**Passive Intruder**

**Problem** Given  $X$  and  $t$ ,

$X \vdash \text{aenc}(m, \text{pk}(C))?$

**Automated Proof Discovery**

Given  $X$  and  $t$ , check if  $X \vdash t$ .

$X$   
Rules do not change .

Some rules *construct*, that is give rise to bigger terms: encryption, pairing

Some rules *destruct*, that is give rise to smaller terms: decryption, split

## Automated Proof Discovery

What is the size of a term?

*X*

Rules do not change .

Some rules *construct*, that is give rise to **bigger terms**: encryption, pairing

Some rules *destruct*, that is give rise to **smaller terms**: decryption, split

**Size of Terms**

$\text{aenc } \text{pair } B, \text{aenc } \text{pair } A, \text{aenc } m, \text{pk}(B), \text{pk}(I)$   
 $( ( ( ( ( ) ) ) ) ) )$

$\text{pk}(I)$   
 ) Treat a term like a tree. Count the number of nodes!

Formally modelled as symbolic terms

$$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a))$$

## Size of Terms

$$\text{aenc} \left( \text{pair} \left( B, \text{aenc} \left( \text{pair} \left( A, \text{aenc} \left( m, \text{pk}(B) \right), \text{pk}(I) \right), \text{pk}(I) \right) \right) \right)$$

$$\text{size}(t) = \begin{cases} 1 & \text{if } t \text{ is atomic} \\ 1 + \sum \text{size}(t_i) & \text{if } t = f(t_1, t_2, \dots, t_k) \end{cases}$$

Formally modelled as symbolic terms

$$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a))$$

## Abnormal Proofs

$$X \vdash m$$

$$X \vdash \text{pk}(a)_{\text{aenc}}$$

$$X \vdash t_0 \quad X \vdash t_1^{\text{pair}}$$

$$X \vdash \text{pair}(t_0,$$

$$t_1)_{\text{split}_0} \quad X \vdash t_0$$

$$X \vdash \text{aenc}(m, \text{pk}(a)) \quad X \vdash m \quad \text{adec}$$

$$X \vdash \text{sk}(a)$$

One where a *construct rule* is immediately followed by *destruct rule*.

## **Normal Proofs**



One where no *construct rule* is immediately followed by *destruct rule*.

## Normal Proofs

A normal proof is one where the major premise of a *destructor rule* is not obtained by the application of a *constructor rule*.

One where no *construct rule* is immediately followed by *destruct rule*.

## Normal Proofs

destructor rule constructor rule

$$\begin{array}{c}
\text{pk} \\
\frac{X \vdash t}{ax \ (t \in X)} \quad \frac{X \vdash t \quad X \vdash t'}{\text{pair}} \\
\frac{X \vdash a \quad X \vdash \text{pair}(t_0, t_1)}{X \vdash \text{pair}(t, t')} \\
\frac{X \vdash \text{pk}(a) \quad X \vdash t_i}{\text{pk}(a)) \ X \vdash \text{sk}(a) \ X \vdash m \ X \vdash \text{pk}(a)} \\
\frac{X \vdash \text{aenc}(m, \text{pk}(a)) \quad X \vdash m}{X \vdash \text{aenc}(m, \text{pk}(a))} \text{aenc} \\
\frac{X \vdash \text{aenc}(m, \text{pk}(a))}{X \vdash m} \text{adec}
\end{array}$$

**Normal Proofs**

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.

## Normal Proofs

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.

Suppose not!

$$\xi X \vdash u \xi$$

Then there is a subproof of such that ends in a destructor rule, and  $\xi$  the major premise of is yielded by some constructor rule. We will show how  $\xi X \vdash u \pi$  to replace by a smaller proof of , thus contradicting the minimality of .

## Normal Proofs

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.

Case I:

$X \vdash t$       ax ( $t \in X$ ) major premise is empty.

## Normal Proofs

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.

Case II:

$\pi_0 \pi_1$

$$\begin{array}{c}
 \vdots \quad \vdots \\
 X \vdash t_0 \quad X \vdash t_1^{\text{pair}} \\
 X \vdash \text{pair}(t_0, t_1) \text{ split}_i \\
 X \vdash t_i
 \end{array}$$

## Normal Proofs

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.

Case II:

$$\begin{array}{c}
 \pi_0 \pi_1 \\
 \vdots \vdots \\
 X \vdash t_0 \quad X \vdash t_1^1 \quad \text{pair} \quad X \vdash \\
 \text{pair}(t_0, t_1^1) \text{split}_i \quad \pi_i \\
 \vdots \\
 X \vdash t_i
 \end{array}$$

## Normal Proofs

If there exists a proof of  $X \vdash t$ , then there exists a minimal proof.

The shortest proof  $\pi$  must be normal.



Case III:

$\pi' \pi''$

$\vdots \vdots$

$X \vdash m$

$X \vdash \text{pk}(a)$   
aenc

$\pi''' :$

$\pi' :$   
 $\vdots$

$X \vdash \text{aenc}(m, \text{pk}(a))$   $X \vdash m$

adec

$X \vdash \text{sk}(a)$

$X \vdash m$

normalisation theorem

## Normal Proofs

$X \vdash t$

There exists a proof of if and  
only if  $X \vdash t$   
there exists a normal proof of .

normalisation theorem

## Normal Proofs

$X \vdash t$

There exists a proof of if and only if  $X \vdash t$

there exists a normal proof of . What is the size of the shortest normal proof?

**Subterms**

Treat a term like a tree. Subterm is like a subtree.

Formally modelled as symbolic terms

$$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a))$$

**Subterms**

Treat a term like a tree. Subterm is like a subtree.

$$X \subseteq \text{st}(X)$$

$$\text{pair}(t_0, t_1) \in \text{st}(X) \Rightarrow \{t_0, t_1\} \subset \text{st}(X)$$

$$\text{aenc}(m, k) \in \text{st}(X) \Rightarrow \{m, k\} \subset \text{st}(X)$$

Formally modelled as symbolic terms

$$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a))$$

**Subterms**

Treat a term like a tree. Subterm is like a subtree.

$$\text{aenc } \text{pair } (B,$$

$$\text{aenc } \text{pair } (A, \text{aenc } (m, \text{pk}(B)), \text{pk}(I), \text{pk}(I))$$

## Subterms

Treat a term like a tree. Subterm is like a subtree.

$$\text{aenc } \text{pair } B, \text{aenc } \text{pair } A, \text{aenc } (m, \text{pk}(B)), \text{pk}(I),$$

$$\text{pk}(I), \text{pair } B, \text{aenc } \text{pair } A, \text{aenc } (m, \text{pk}(B)), \text{pk}(I), \text{pk}(I)$$

## Subterms

Treat a term like a tree. Subterm is like a subtree.

$$\text{aenc}_{\left(\left(\left(\text{pair } B, \text{aenc}_{\left(\left(\left(m, \text{pk}(B)\right)\right), \text{pk}(I)\right)\right)}\right.\right.\right.$$

$$\left.\text{pk}(I)\right) \text{pair } B, \text{aenc}_{\left(\left(\left(m, \text{pk}(B)\right)\right), \text{pk}(I)\right)} \text{pk}(I)$$

$$\text{aenc}_{\left(\left(m, \text{pk}(B)\right)\right), \text{pk}(I)} \text{pair } A, \text{aenc}_{\left(m, \text{pk}(B)\right), \text{pk}(I)} B$$

**Subterms**



Treat a term like a tree. Subterm is like a subtree.

aenc ( pair ( *B*, aenc ( pair ( *A*, aenc ( *m*, pk(*B*) ), pk(*I*) ),

pk(*I*) ) pair ( *B*, aenc ( pair ( *A*, aenc ( *m*, pk(*B*) ), pk(*I*) ) ) pk(*I*)

aenc ( pair ( *A*, aenc ( *m*, pk(*B*) ), pk(*I*) ) *B A*

pair (A, aenc (m, pk(B)))      aenc (m, pk(B)) m pk(B)

## Subterms

Treat a term like a tree. Subterm is like a subtree. Is the

number of subterms equal to the size of the term?

## Subterms

Treat a term like a tree. Subterm is like a subtree. Is the  
 number of subterms equal to the size of the term?

$$|\text{st}(X)| \stackrel{?}{=} \sum_{t \in X} |t|$$

**Normal Proofs**

$$X \vdash t$$

There exists a proof of if and only if

$$X \vdash t$$

there exists a normal proof of .

$$\pi \quad X \vdash t \quad X \vdash u$$

Claim: Let be a normal derivation of . If is an  $X \vdash t \quad u \in \text{st}(X \cup \{t\})$

intermediate step in , then .

If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

$$\pi \quad X \vdash t \quad X \vdash u$$

Let be a normal derivation of . If is an  $X \vdash t \quad u \in \text{st}(X \cup \{t\})$  intermediate step in , then . If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

By Induction:  
**Subterm Property**

then ...

If the last term is generated by  $\alpha X$ ,

## Subterm Property

$$\pi \quad X \vdash t \quad X \vdash u$$

Let  $\pi$  be a normal derivation of  $X \vdash t$ . If  $X \vdash u \in$

$$\text{st}(X \cup \{t\})$$

is an intermediate step in  $\pi$ , then  $X \vdash u$ .

If the last rule is a destruction rule, then  $u \in$

$\text{st}(X)$ . **By Induction:** If the last term is generated by pair, then ...

## Subterm Property

$$\pi \ X \vdash t \ X \vdash u$$

Let  $\pi$  be a normal derivation of  $u$ . If  $t$  is an  $X \vdash t \ u \in$

$$\text{st}(X \cup \{t\})$$

intermediate step in  $\pi$ , then  $u \in \text{st}(X \cup \{t\})$ .

If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

By Induction: If the last term is generated by pair, then ...

$$\begin{array}{c} \pi_0 \pi_1 \\ \vdots \quad \vdots \end{array}$$

$$X \vdash t_0 \quad X \vdash t_1 \quad \text{pair}$$

$$X \vdash \text{pair}(t_0, t_1)$$

**Subterm Property**

If  $X \vdash u$  is an intermediate step...



Either it is in  $\pi$  or  $\pi_0 \pi_1 u = \text{pair}(t_0, t_1)$

Therefore  $u \in \text{st}(X \cup \{t_0\}) \cup \text{st}(X \cup \{t_1\}) \cup \{\text{pair}(t_0, t_1)\}$

$$\begin{array}{c} \pi_0 \pi_1 \\ \vdots \end{array}$$

$$X \vdash t_0 \quad X \vdash t_1 \quad \text{pair}$$

$$X \vdash \text{pair}(t_0, t_1)$$

**Subterm Property**

If  $X \vdash u$  is an intermediate step...

Either it is in  $\pi$  or or  $\pi_0 \pi_1 u = \text{pair}(t_0,$   
 $t_1)$

Therefore  $u \in \text{st}(X \cup \text{pair}(t_0, t_1))$

$$\begin{array}{cc} \pi_0 & \pi_1 \\ \vdots & \vdots \end{array}$$

$$X \vdash t_0 \quad X \vdash t_1 \quad \text{pair}$$

$$X \vdash \text{pair}(t_0, t_1)$$

## Subterm Property

$$\pi \quad X \vdash t \quad X \vdash u$$

Let  $\pi$  be a normal derivation of  $X \vdash u$ . If  $X \vdash t$  is an intermediate step in  $\pi$ , then  $t \in \text{st}(X \cup \{u\})$ .

$$\text{st}(X \cup \{u\})$$

intermediate step in  $\pi$ , then  $t \in \text{st}(X \cup \{u\})$ .

If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

By Induction: If the last term is generated by encryption, then ...

$$\begin{array}{c}
 \pi' \pi'' \\
 \vdots \vdots \\
 X \vdash m \quad \text{pk}(a) \\
 X \vdash \quad \text{aenc}
 \end{array}$$

$$X \vdash \text{aenc}(m, \text{pk}(a))$$

**Subterm Property**

$$\pi \quad X \vdash t \quad X \vdash u$$

Let be a normal derivation of . If is an  $X \vdash t \quad u \in$

$$\text{st}(X \cup \{t\})$$

intermediate step in , then .

If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

**By Induction:** If the last term is generated by split, then ...  $\pi$ :

$$X \vdash \text{pair}(t_0, t_1) \text{ split}_i$$

$$X \vdash t_i$$

**Subterm Property**

$$\pi \ X \vdash t \ X \vdash u$$

Let  $\pi$  be a normal derivation of  $t$ . If  $u \in \text{st}(X \cup \{t\})$  is an intermediate step in  $\pi$ , then  $u \in \text{st}(X)$ .

If the last rule is a destruction rule, then  $u \in \text{st}(X)$ .

**By Induction:** If the last term is generated by decryption, then ...

$$\begin{array}{c} \pi' : \pi'' : \\ X \vdash \text{aenc}(m, \text{pk}(a))_{\text{adec}} \\ X \vdash \text{sk}(a) \end{array}$$

$$X \vdash m$$

## Normalisation + Subterm Property

$$X \vdash t$$

If there exists a proof of  $t$ , if and only if there exists a normal proof of  $t$ .

$$\pi \vdash t \vdash u$$

Let  $\pi$  be a normal derivation of  $t$ . If  $u$  is an  $X \vdash t \vdash u \in$

$$\text{st}(X \cup \{t\})$$

intermediate step in , then . If the last rule is a

destruction rule, then  $u \in \text{st}(X)$ .

## Normalisation + Subterm Property

$$X \vdash t$$

There exists a proof of if and only if there  $X \vdash t$   
exists a normal proof of , with each branch  $|\text{st}(X \cup \{t\})|$



bounded by .

## Normalisation + Subterm Property

$$X \vdash t$$

There exists a proof of if and only if there  $X \vdash t$   
exists a normal proof of , with each branch  $X \cup$

$$\{t\}$$

bounded by the size of , that is

$$|t| + \sum_{t' \in X} |t'|$$

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

If you have  $X$ , you can build

$X' = \{\text{terms generated from } X \text{ in one step}\}$  *How?*

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum$$

$$\sum_{t' \in X} |t'|$$

If you have  $X$ , you can build

$X' = \{\text{terms generated from } X \text{ in one step}\}$  *How?*  $X' = \{t :$

$$\exists \text{ pair}(t, \_) \in X \vee \text{ pair}(\_, t) \in X\}$$

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum \cdot$$

$$\sum_{t' \in X} |t'|$$

If you have  $X$ , you can build

$X' = \{\text{terms generated from } X \text{ in one step}\}$  <sup>How?</sup>  $X' = \{t :$

$\exists \text{ pair}(t, \_) \in X \vee \text{pair}(\_, t) \in X\} \cup \{\text{pair}(t, t') : t, t' \in X\}$

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum \cdot$$

$$\begin{array}{c} |t'| \\ t' \in X \end{array}$$

If you have  $X$ , you can build

$X' = \{\text{terms generated from } X \text{ in one step}\}$  How?

$$\begin{aligned} X' = \{t : \exists \text{ pair}(t, \_) \in X \} & \cup \{\text{pair}(\_, t) \in X\} \\ & \cup \{\text{pair}(t, t') : t, t' \in X\} \\ & \cup \{m : \text{sk}(A) \in X \wedge \text{enc}(m, \text{pk}(A)) \in X\} \end{aligned}$$

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

If you have  $X$ , you can build

$X' = \{\text{terms generated from } X \text{ in one step}\}$  How?

$$X' = \{t : \exists \text{pair}(t, \_) \in X \vee \text{pair}(\_, t) \in X\} \\ \cup \{\text{pair}(t, t') : t, t' \in X\} \\ \cup \{m : \text{sk}(A) \in X \wedge \text{enc}(m, \text{pk}(A)) \in X\}$$

$$\cup \{ \text{enc}(m, \text{pk}(A)) : m \in X, \text{pk}(A) \in X \} \cup \{ \text{pk}(A) : A \in X \}$$

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

If you have  $X$ , you can build

$$X' = \{ \text{terms generated from } X \text{ in one step} \}$$

$^2)$

In time  $O(|X|)$ .

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

If you have  $X$ , you can build



$X' = \{\text{terms generated from } X \text{ in one step}\}$

$^2)$

In time  $O(N)$ .

**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

If you have  $X$ , you can build  
 $X' = \{\text{terms generated from } X \text{ in one step}\}$

In time  $O(N^2)$ .

Repeat this  $N$  times.  
**Naive Algorithm:**

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

$$O(N^3)$$

In time , construct the set of all  $N$   
terms derivable in at most steps.  $t$

derivable in  $N$  steps  $\Leftrightarrow X \vdash t$ .

$$\text{Let } N = |t| + \sum_{t' \in X} |t'|.$$

Inputs:  $X, t$

Can you do it in linear time?

**Naive Algorithm:**

For  $i$  in range  $(0, N)$ ,

Construct  $X'$  in  $O(N^2)$

Check  $t \in X$ !

Set  $X \leftarrow X'$

## Passive Intruder Problem:

*Passive intruder problem:* Can the intruder violate some property just by listening to the network?

Fixed  $X$ , fixed  $t$ . Check if  $X \vdash t$ .

Decidable in polynomial time given certain operators and term algebra.

## Passive Intruder Problem:

*Passive intruder problem:* Can the intruder violate some property just by listening to the network?

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Decidable in polynomial time given certain operators and term algebra.

$t := m \mid a \mid \text{pk}(a) \mid \text{sk}(a) \mid \text{pair}(t_1, t_2) \mid \text{aenc}(t, \text{pk}(a)) \mid k \mid$   
 $\text{senc}(t, k) \mid \text{sign}(t, \text{sk}(a)) \mid \text{hash}(t)$

**Active Intruder Problem:**

*Active intruder problem:* Can the intruder violate some property by listening to network + allowable behaviours?

$$A \rightarrow : (A, \text{aenc}(m, \text{pk}_B))$$

$$I \rightarrow B : (I, \text{aenc}(m, \text{pk}_B))$$

$$B \rightarrow I : (B, \text{aenc}(m, \text{pk}_I))$$

$$\rightarrow A : (B, \text{aenc}(m, \text{pk}_A))$$

# Active Intruder Problem:

$M$

- Take an arbitrary Turing machine
- Encode its configurations (state, tape, head position) as symbolic messages
- Design a protocol such that:

$M$

- Each valid protocol step corresponds to one transition of  $M$ .
- The intruder can drive the protocol forward iff  $M$  makes a valid transition



- Define an insecure state iff  $M$  reaches a halting configuration.

**Formal Verification Tools** solver.

DY-based bounded Active  
Intruder detection.

Relational modelling with a SAT

alloy



ProVerif Tamarin Alloy 6

**What all can you verify?**

*As models:*

Dolev–Yao adversary

- Full network control (intercept, replay, modify, inject)
- Perfect cryptography (no guessing / no breaking primitives)
- Multiple concurrent protocol sessions

Compromised principals (key reveal, corruption models)

**What all can you verify?**

*As specifications:*

**Secrecy/Confidentiality:** Message secrecy, key secrecy

**Authentication:** Aliveness, weak agreement

**Protocol Correctness:** Message origin authenticity, session binding, freshness guarantees, replay resistance

**Equivalence / Privacy Properties:** anonymity, unlinkability, observational equivalence,

**What all can you NOT verify?**

**Cryptographic Strength & Computation**

**Implementation & Deployments**

**Real-time guarantees**

**Assumptions**

**What all can you NOT verify?**

**Cryptographic Strength & Computation**

**Implementation & Deployments**

**Real-time guarantees**

**Assumptions**

Polynomial-time/probabilistic adversaries

Quantitative Properties

User Intent or Semantic Meaning

Correctness of modelling or specifications.