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The application of reliability models in traffic accident frequency analysis

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ABSTRACT

Analysis of traffic accident frequency represents an important subject of research of many authors. From the aspect of temporal analysis of traffic accident occurrence, two approaches have been singled out in previous practice: the collective (analyzes traffic accidents over a longer period of time) and individual (analyzes traffic accidents in real time). The paper shows that the system reliability theory, with certain adjustments, can be largely used to analyze traffic accident frequency based on the individual approach. A certain similarity has been observed between the system reliability theory and the traffic safety theory, and conceptual adjustment of equivalent terms and states has been performed based on this. A model has been successfully tested on the basis of which, for the road and sections, we have determined the traffic accident frequency, the probability of the occurrence of a certain number of traffic accidents and the mean time between two consecutive traffic accidents.

1. Introduction

The frequency of traffic accidents traditionally represents the subject of a great deal of research and until now, many different methodological approaches to the modeling of the occurrence of traffic accidents have been developed (Lord and Mannering, 2010). From the aspect of temporal analysis of traffic accident occurrence, two approaches were singled out: (1) collective (Hauer, 1986; Persaud, 1991; Miaou and Lum, 1993; Milton and Mannering, 1998; Golob and Recker, 2003), which determines the frequency of traffic accidents over a longer period of time and (2) individual (Hughes and Council, 1999; Lee et al., 2003; Golob and Recker, 2004), which determines the probability of traffic accident occurrence in real time (Abdel-Aty and Pande, 2007).

In this paper, we went from the assumption that the traffic accident frequency, based on the individual approach, can be analyzed through the application of a reliability model. The reliability theory deals with the issues of calculation, experimental evaluations, provision and optimization of technical systems reliability (Ushakov and Harrison, 1994). The broad field of application of the reliability theory and technical servicing has had an impact on a number of approaches and levels of study of the reliability problem (Ushakov and Harrison, 1994; O'Connor et al., 2002; Rausand and Høyland, 2004; Dhillon, 2005). The mathematical approach to reliability analysis includes general problems, regardless of the nature of the system. Therefore, in engineering, general mathematical models are adapted to real problems and serve towards their solution.

At the state change analysis of the real system the basic assumption is that the time is uniform and absolute. In accordance with that hypothesis, the order of the analysis aspects is as follows: 1. temporal (*when* or *how often*), 2. spatial (*where* or *which part*). The reliability theory is based on the analysis of failure occurrence, or in other words the duration of the period without failure in the coordinate system time-state. The process of continuous road traffic, observed as a system, can be compared to the functioning without failure of a technical system. In other words, similar to models used for the analysis of traffic accident frequency, the theory of reliability of technical systems is used to determine the probability and possibility of predicting the occurrence of a failure. In this paper, the application of the reliability theory in the analysis of traffic accident frequency is limited to the temporal–spatial aspect. The occurrence of traffic accidents on road sections was analyzed from a temporal aspect, i.e. the occurrence of traffic accidents was analyzed in the coordinate system time-location.

2. Possibilities of associating the basic principles of the reliability theory with the analysis of traffic accident frequency

In the reliability theory, technical systems represent the formal objects of study. They can be represented as complex systems, at any level of complexity, composed of components. The system components can be complex (subsystems) or simple (indivisible components or units). Units are constituent components of a system and they are simple systems for which, within the given research, there is no need for further parsing (Ushakov and Harrison, 1994). Roads are complex systems consisting of a number of sections and they can be identified with complex technical systems.

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In the field of traffic safety, as a complex and heterogeneous system, there have been no attempts to consider the process of occurrence of traffic accidents from the aspect of system reliability. Analogous to defining the reliability of technical systems (Ushakov and Harrison, 1994; Dhillon, 2005), the term reliability of the traffic safety system can be viewed as the ability of the system to fulfil the given functions within a certain time interval and at the same time maintain determined projected characteristics within given limits, in appropriate conditions of exploitation and level of traffic regulation.

By looking at and comparing characteristic processes in traffic and technical systems, many similarities and dependencies studied by theory of reliability of technical systems have been noticed (e.g. traffic accident ≡ failure). A traffic accident as an unplanned and unwanted event causes an interruption in road traffic flow and requires an emergency response in order to ease and neutralize the congestion as quickly as possible.

Apart from the attempt to analyze the process of road traffic flow from the aspect of technical system reliability, the second task was conceptual definition. This procedure required an analysis of terms and conditions of the reliability theory, followed by finding potential similar occurrences and states in the traffic safety theory, i.e. the model for the analysis of traffic accident frequency. Thus the terms of the reliability theory received their equivalent terms in the model for traffic accident frequency analysis. Traffic safety covers a much wider area and contains a significantly greater number of disciplines and factors than the reliability of technical systems. In a technical system a failure causes an interruption in the functioning of the system and requires repair or replacement, i.e. recovery of the system, followed by putting the system back into operation. In the process of road traffic flow, the occurrence of a traffic accident causes a delay or significant slowing down of traffic, but following the investigation (departure of the police and other services from the scene of the accident) traffic flow spontaneously and relatively quickly returns to its standard regime. A traffic accident has the character of a sudden failure, or in other words it instantly switches the system from an operational (properly working) state to a state of failure. In accordance with this, basic concept, terminological mapping has been done from the technical system reliability theory to the theory of road traffic safety. Table 1 provides an overview of the terminological mapping of the basic terms.

3. Defining the model

3.1. Determining road reliability

In determining the reliability of a road a preliminary analysis is necessary that would determine its structure and level of complexity, or the defining of the system units – the sections. Each road as a complex system consists of n units – sections (Fig. 1), where the traffic flow interruption in any unit causes the traffic flow interruption in the whole road.

Table 1
Terminological mapping from the technical systems reliability theory into the theory of road traffic safety.

Reliability of technical systems	Road safety
System	Road
System unit	Road section
Failure	Traffic accident
Failure rate	Traffic accident frequency
Reliability	Reliability
Unreliability	Unreliability
Mean time between failures	Mean time between traffic accidents
Work without failure	Period without accident
Renewal	Traffic recovery

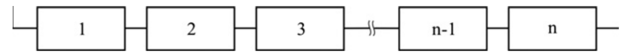

Fig. 1. A road observed as a series system.

In order to achieve an adequate mathematical approach with the real system, reliability theory applies different statistical distributions with the help of which real processes are approximated, e.g. distributions of time between two events. Traffic flow on the section of road begins at the time $t = 0$ and the time period without traffic accidents is the interval $[0, \tau_1]$, where τ_1 is the moment of occurrence of the first traffic accident. The next period without traffic accidents is the interval $[\tau_1, \tau_2]$, where τ_2 is the moment of occurrence of the second traffic accident.

Poisson's distribution of events in stochastic traffic processes has a significant role and represents a starting distribution for most of the analyses (Jovanis and Chang, 1986; Elvik and Vaa, 2004; Shankar et al., 2008). The basic form of Poisson process is a continuous-time counting process that possesses the following properties: independent increments, stationary increments and no counted occurrences are simultaneous. A homogeneous Poisson process is characterized by its rate parameter λ, which is the expected number of "events" or "arrivals" that occur per unit time. If the stream of events has Poisson's distribution, then the time between the occurrences of two events (T) can be described with an exponential distribution with parameter λ. Therefore, it has been adopted that all the empirical distributions of the duration of the period without accidents can be replaced with the corresponding exponential distributions.

The parameters of road reliability for Poisson' process are calculated from a temporal aspect according to the forms of the technical systems reliability theory (Eqs. (1)–(13)) (Ushakov and Harrison, 1994; O'Connor et al., 2002; Rausand and Høyland, 2004; Dhillon, 2005).

The density function of the distribution of the time between two accidents $f(t)$ ($f_S(t)$ – road, $f_i(t)$ – ith section) represents a measure of the speed of the accident's occurrence. The function of accident distribution density for the ith section is:

$$f_i(t) = \lambda_i e^{-\lambda_i t}, \quad \text{where is } \lambda_i > 0, t \geqslant 0 \tag{1}$$

Distribution function $F(t)$ ($F_S(t)$ – road, $F_i(t)$ – ith section) of the random variable T (the time between two accidents), is equal to the probability that an accident will occur before the moment t. This function is also called the function of unreliability. The distribution function of the time between two accidents on the ith section – unreliability function of the ith section is:

$$F_i(t) = \int_0^\infty f_i(t)\,dt = \int_0^\infty \lambda_i e^{-\lambda_i t}\,dt = 1 - e^{-\lambda_i t} \tag{2}$$

Using the unreliability function $F(t)$ we introduce the reliability function $R(t)$ ($R_S(t)$ – road, $R_i(t)$ – ith section), as the probability of a time period without accidents until the moment t. The reliability function of the ith section is:

$$R_i(t) = 1 - F_i(t) = e^{-\lambda_i t} \tag{3}$$

The mean time between two consecutive accidents T_0 ($T_{0S}(t)$ – road, $T_{0i}(t)$ – ith section) is obtained as a mathematical expectation of the random variable T. The mean time between two consecutive accidents on the ith section is:

$$T_{0i} = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-\lambda_i t}\,dt = \frac{1}{\lambda_i} \tag{4}$$

The accident frequency $a(t)$ ($a_S(t)$ – road, $a_i(t)$ – ith section) also represents an important and widely used reliability characteristic, and represents the measure of the current speed of the accident's occurrence. The $a(t)$ can be expressed in various time units and depends on the time unit which is used for calculation of T_0 (minute^{-1}, hour^{-1}, ..., year^{-1}). For Poisson's distribution, the accident frequency as the current speed of the accident's occurrence is constant at any moment in the observed time period. The accident frequency function on the ith section is:

$$a_i(t) = \lambda_i = const \tag{5}$$

The expected number of accidents into the certain period $A(t)$ ($A_S(t)$ – road, $A_i(t)$ – ith section) is calculated as:

$$A_i(t) = \lambda_i \cdot t \tag{6}$$

where is t – duration of the certain time period expressed in basic time unit as λ.

If the time unit is the same as duration of the observed period, the accident frequency $a(t)$ and the expected number of accidents $A(t)$ will be equal.

The road reliability is conditioned by the reliability of the sections that it consists of. The observed road represents a simple system, because of the serial connection of the sections. Since the sections are independent in terms of reliability, in the given case the reliability function of the road is equal to:

$$R_S(t) = \prod_{i=1}^{n} R_i(t) = e^{-(\lambda_1+\lambda_2+\ldots+\lambda_n)t} \tag{7}$$

If the reliability functions are expressed using the accident frequency, the reliability function of the road is equal to:

$$\lambda_S = \sum_{i=0}^{n} \lambda_i = \lambda_1 + \lambda_2 + \ldots + \lambda_n \tag{8}$$

Since it is a serial connection in the case of exponential distribution, the parameter λ_S of the road is equal to the sum of the parameters λ_i of the sections. The parameters of road reliability as a system, for Poisson' process, are calculated according to equations from (1)–(5).

3.2. The basic characteristics of the traffic recovery process

In the previous section we determined the basic parameters of the reliability of the road and sections until the first traffic accident. Traffic flow on the road and sections represents a process that is being recovered. The duration of traffic interruption on the section is negligibly small compared to the time between two consecutive traffic accidents, so the calculation of road and section reliability is performed with the assumption that the traffic recovery is executed instantaneously. It is also assumed that the moments of accident occurrences on the sections are independent, and that traffic flow, accidents, and the traffic recovery on a section do not affect the reliability of the other sections.

The moments of the occurrence of accidents on the section, i.e. the moments of recovery of traffic flow on the sections, $\tau_1, \tau_2, \ldots, \tau_n$ form the traffic recovery process (Fig. 2).

The process of traffic recovery can be considered simple, since the random variables $T_1, T_2, \ldots, T_n$ are independent and with the same probability distribution. It is assumed that the traffic recovery processes are Poisson's, from which it follows that the time intervals between two recoveries, or two consecutive accidents, have an exponential distribution.

From this it follows that the density of time until the nth traffic recovery is:

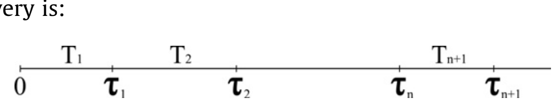

Fig. 2. The traffic recovery process.

$$r_{ni}(t) = \frac{\lambda_i(\lambda_i t)^{n-1}}{(n-1)!} e^{-\lambda_i t} \tag{9}$$

Then the average number of traffic recoveries H, as a linear time function, and the traffic recovery intensity h have the following forms:

$$H_i(t) = \lambda_i t \tag{10}$$

$$h_i(t) = a_i(t) = \lambda_i = const \tag{11}$$

The probability of the occurrence of n accidents or traffic flow recoveries, during a time interval of a length of t, does not depend on the position of that interval and is expressed with the following formula:

$$P_{ni}(t) = \frac{(h_i t)^n}{n!} e^{-h_i t} = \frac{(\lambda_i t)^n}{n!} e^{-\lambda_i t} \tag{12}$$

The probability that there will not be an accident, during the time interval of a length of t, is equal to:

$$P_{0i}(t) = e^{-h_i t} = e^{-\lambda_i t} \tag{13}$$

4. Testing the model

To test the model we selected a main road (the road M-22), which is made up of 11 sections of a total length of 115 km (Table 2). The sections represent parts of the road network between two consecutive traffic nodes and are used to provide for continuous and unobstructed traffic flows (Hauer et al., 2002). Each section is specific by the structure and volume of the traffic, road environment, units and road equipment.

During the analyzed period, from 2001 to 2008, 796 traffic accidents occurred on the observed road. All the traffic accidents have been allocated based on the time of occurrence (year/month/hour) and location on the road (kilometer/meter), which provides us with the temporal–spatial distribution of traffic accidents per section. Testing whether the empirical time distributions between successive accidents on the sections match the theoretical distributions was performed using the χ^2 test.

Due to a small sample, distributions of traffic accidents in sections 3 and 6 were approximated using Poisson process, i.e. it was assumed that in those sections as well, time between two traffic accidents has exponential distribution.

With probability of 1.0 we can point out that, on the road analyzed, in the period of 365 days, the number of accidents will be between 43 and 170, and with probability of 0.0397537246, that in the same time period, there will be exactly 100 accidents on the road.

By comparing the reliability, or the mean time between two consecutive accidents on the sections, we may conclude that section 10 is the least reliable or it has the shortest T_0, followed by sections 11, 7, 9, 8, 1, 4, 2 and 5, while the most reliable sections are 3 and 6. The reliability of a road during 1 week is 0.144869487, within one year 0, or unreliability is 0.855130513 and 1, respectively. On the observed road, on average, 1.932 accidents happen within 1 week, that is 100.736 accidents per year, i.e. a traffic accident happens every 86.96 h.

When analyzing the probability and the occurrence of accidents on the sections, and on the road as a whole, according to Eqs. (12) and (13), we also observed the periods of t = 7 (days) = 168 (h) and t = 365 (days) = 8760 (h). Figs. 3 and 4 show the probabilities of accident occurrence ("equal or more than a certain number", "a certain number" and "less or equal than a certain number") on the road as a whole, or in the Figs. 5 and 6 on the road sections.

In pictures 3 and 4, probability curve points to the probability of a certain number of accidents, which is greater, smaller or equal to

Table 2
Basic characteristics of observed road and appropriate main results of analysis.

Road section	Length (km)	Observed number of accidents	T_{0i} (h)	λ_i (h^{-1})	t = 7 (days) = 168 (h)		t = 365 (days) = 8 760 (h)	
					$A_i(t)$	$R_i(t)$	$A_i(t)$	$R_i(t)$
1	4.823	59	1171.47	0.000853625	0.143409002	0.866399637	7.477755111	0.000565526
2	12.600	55	1257.27	0.000795372	0.133622560	0.874920232	6.967462039	0.000942041
3	6.293	12	5487.50	0.000182232	0.030615034	0.969848860	1.596355353	0.202633702
4	14.541	57	1229.74	0.000813182	0.136614594	0.872306353	7.123475284	0.000805961
5	12.403	52	1346.42	0.000742709	0.124775045	0.882695447	6.506127346	0.001494255
6	4.236	31	2110.65	0.000473789	0.079596515	0.923488885	4.150389729	0.015758274
7	16.486	102	684.17	0.001461632	0.245554202	0.782270882	12.803897686	0.000002750
8	10.094	65	1062.03	0.000941592	0.158187507	0.853689694	8.248348592	0.000261690
9	12.735	85	817.64	0.001223039	0.205470582	0.814264048	10.713823220	0.000022235
10	18.434	165	420.04	0.002380712	0.399959600	0.670347127	20.855036288	0.000000001
11	2.486	113	612.88	0.001631651	0.274117392	0.760242823	14.293264024	0.000000620
Road	115.131	796	86.96	0.011499536	1.931922035	0.144869487	100.735934672	0.000000000

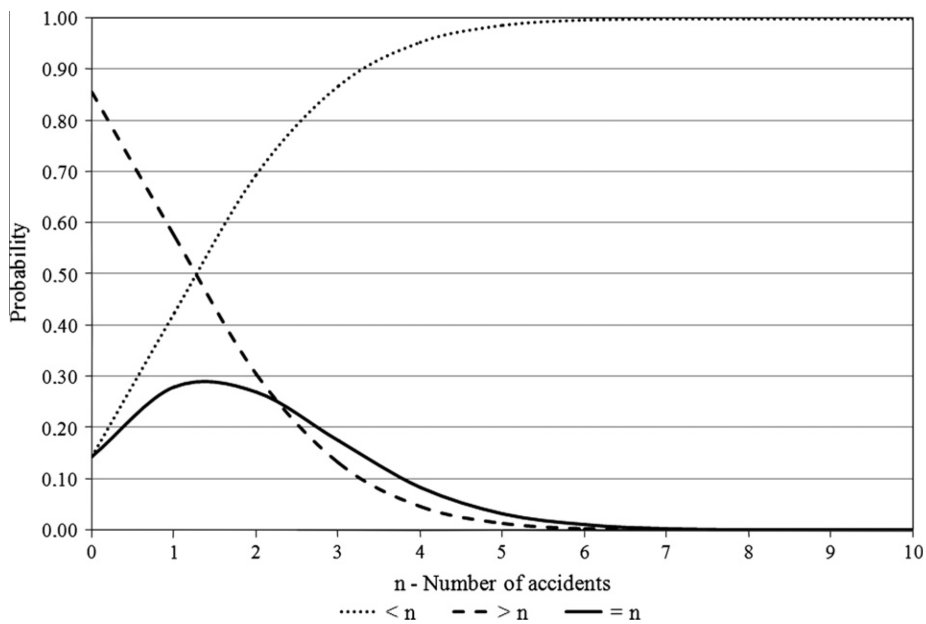

Fig. 3. The probabilities of accident occurrence ("equal or more than a certain number", "a certain number" and "less or equal than a certain number") on the road as a whole for the period t = 7 (days) = 168 (h).

n, in other words, it points to the probability of certain speed of traffic accident occurrence. Picture 3 shows that with probability of around 0.15 we can claim that there will not be any accidents on the road within a period of 7 days, i.e. the speed of traffic accident occurrence will be 0 in the observed period,. With probability of about 0.90 it can be asserted that the speed of traffic accident occurrence will be less than three accidents per week. Also, with probability of 1.0 it can be asserted that the speed of traffic accident occurrence, i.e. number of traffic accidents within a week, will be less than six accidents per week. It is important to point out, which is confirmed with probability of about 0.85, that the speed of traffic accident occurrence will be greater than 0 accidents per week. With probability of 0.28 it can be asserted that the speed of traffic accident occurrence will be 1.1 accidents per week. For the period of 365 days situation is different (picture 4). With probability of 1.0 it can be asserted that the speed of traffic accident occurrence will be greater than 75 and less than 130, i.e. with probability of about 0.9 it can be asserted that the speed of traffic accident occurrence will be between 85 and 115 traffic accidents in 365 days.

Taking into account conclusions of the analysis of pictures 3 and 4, during the analysis of pictures 5 and 6 there are obvious differences between shapes of probability curves. In picture 4 some curves are narrower and higher, which means that cumulants, too, are close to each other and they are steep. In other words, that points to narrow interval of expected speeds of accident occurrence, with high probability of happening. Besides that, section 11 corresponds to flattened probability curve of a certain speed of accident occurrence, which means that the interval of possible speeds of accident occurrence is wider. On the other hand, all curves have prominent peak, i.e. the speed of accident occurrence with greatest probability.

5. Conclusions and discussion

In recent years, traffic accident analysis models based on the individual approach are becoming more and more important. Defining and testing of the model confirmed the initial assumption that the traffic accident frequency, based on the individual approach, can be analyzed by applying the reliability theory model. To this end, an analysis of application possibilities has been performed as well as conceptual adaptation.

In spatial analysis, the starting point is the total number of accidents in a certain period and causes and solutions are sought. In the temporal analysis, from the point of reliability theory, the start-

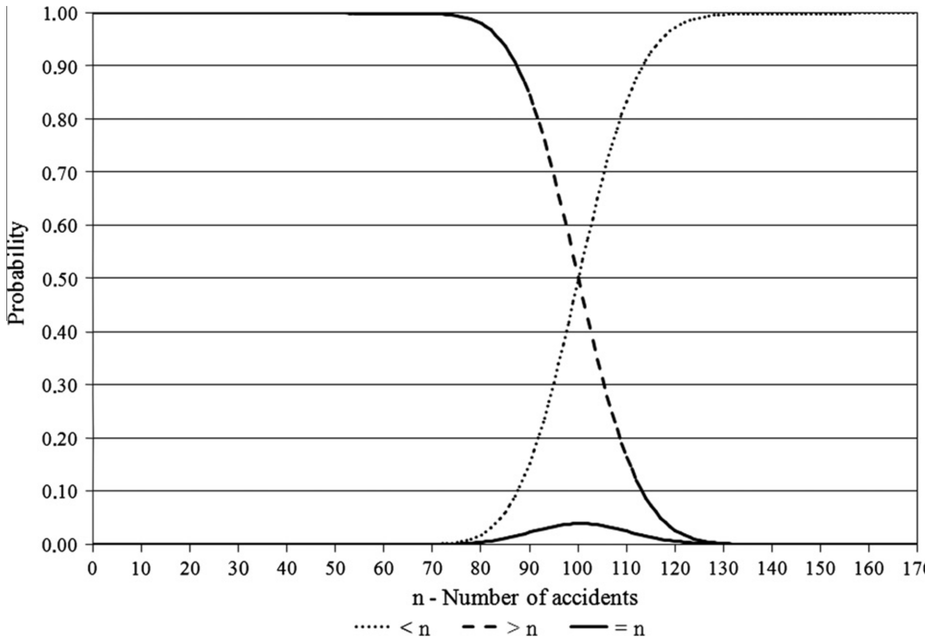

Fig. 4. The probabilities of accident occurrence ("equal or more than a certain number", "a certain number" and "less or equal than a certain number") on the road as a whole for the period $t = 365$ (days) = 8760 (h).

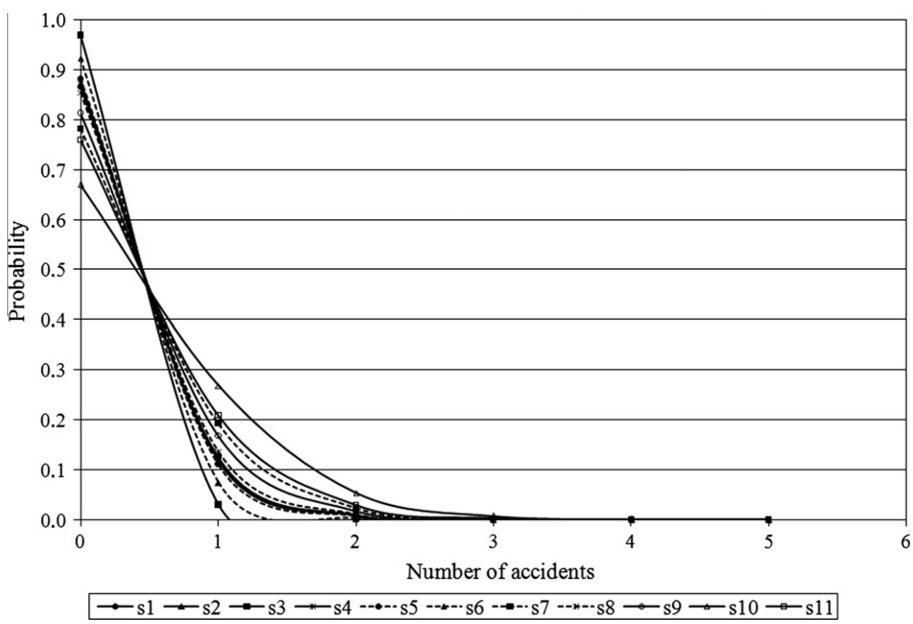

Fig. 5. The probabilities of the occurrence of a certain number of accidents on the sections as units for the period $t = 7$ (days) = 168 (h).

ing point is the time between two accidents on road sections, i.e. the speed of traffic accident occurrence. Determining the laws of traffic accident occurrence with the goal of reduction the total number of accidents is the ultimate goal of both analyses. In the spatial analysis, the only indicator is the number of traffic accidents. In the temporal analysis, the main indicator is average time between two accidents and the number of traffic accidents is the consequence of that. The temporal analysis from the point of view of reliability theory does not exclude the spatial analysis; on the contrary, it represents the starting analysis. Therefore, the spatial analysis should not exclude the temporal either.

The ranking of safety of two independent roads based on a simple comparison of number of accidents requires that the accidents occurred during the same, exactly defined period. The problem arises when it is necessary to compare the reliability of more roads, and to thereby vary the data collection period. In this case, the use of the proposed model which is based on the system approach will help to determine the mean time between accidents, i.e. the accident frequency. In that way, a better comparison will be allowed without reduction, extrapolation or averaging of heterogeneous data. In addition, reliability analysis based on the number of accidents does not contain a dynamic component and it is possible only after the data collection period. On the other hand, the proposed analysis of accidents frequency, which is based on the temporal aspect, allows monitoring in the real time and does not require defining the duration of the periods of observation and data collection. This possibility gives the represented model better application flexibility, i.e. much lower dependence on the length of observation

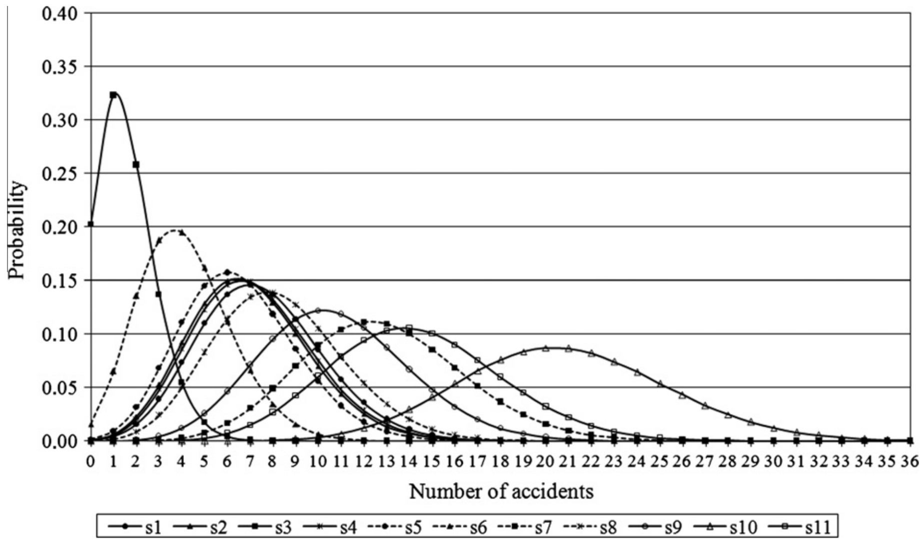

Fig. 6. The probabilities of the occurrence of a certain number of accidents on the sections as units for the period $t = 365$ (days) = 8760 (h).

period. While most models demand observation over a relatively long period of time (3–5 years), this model can be applied on relatively short time periods (a week, a month, a year). In other words, instead of passively registering the number of accidents in the defined period, the suggested model promotes permanent and active monitoring and measuring of time between two accidents.

Traffic safety analysis, i.e. traffic accident occurrence frequency in this model, as the output result gives reliability of the road and its sections. It can be seen that the model considers all recorded traffic accidents, regardless of the type of consequence, which contributes to greater reliability of the model.

The more unreliable section is, i.e. the shorter the time between two accidents, the harder it is to determine the probability of the accurate number of accidents. Because of accidents happen often, i.e. high speed of accident occurrence diminishes the possibility of precise prediction of the number of accidents. Therefore, the proposed model also points to the importance of directing attention to unreliable sections, i.e. sections with higher speed of traffic accident occurrence. Since the suggested model is based on system approach to monitoring of traffic accident occurence, it also enables determination of occurrence probability of a certain number of accidents on the sections, as well as the road as a whole. Also, it is possible to determine probability that there would not be any accidents in a certain period, i.e. reliability analyses of both the sections and the road by following the periods of functioning without accidents.

Through application of the given model, for the road and sections over a defined period of time, the following parameters are determined:

- the mean time between two consecutive traffic accidents;
- the traffic accident frequency;
- the probability of the occurrence of a certain number of traffic accidents on any section or whole road.

The proposed model may serve the road authority as a significant tool in decision making on the application of traffic safety measures on the roads. Namely, the road authority tends to make the time between two consecutive traffic accidents on a road to be as long as possible, or that the accident occurrence rate is as low as possible. The reliability of the road is increased in this way, as well as the traffic flow system. On the other hand, the proposed model has great potential for expansion and development.

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