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Evaluating alternate discrete outcome frameworks for modeling crash injury severity

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ABSTRACT

This paper focuses on the relevance of alternate discrete outcome frameworks for modeling driver injury severity. The study empirically compares the ordered response and unordered response models in the context of driver injury severity in traffic crashes. The alternative modeling approaches considered for the comparison exercise include: for the ordered response framework-ordered logit (OL), generalized ordered logit (GOL), mixed generalized ordered logit (MGOL) and for the unordered response framework-multinomial logit (MNL), nested logit (NL), ordered generalized extreme value logit (OGEV) and mixed multinomial logit (MMNL) model. A host of comparison metrics are computed to evaluate the performance of these alternative models. The study provides a comprehensive comparison exercise of the performance of ordered and unordered response models for examining the impact of exogenous factors on driver injury severity. The research also explores the effect of potential underreporting on alternative frameworks by artificially creating an underreported data sample from the driver injury severity sample. The empirical analysis is based on the 2010 General Estimates System (GES) data base—a nationally representative sample of road crashes collected and compiled from about 60 jurisdictions across the United States. The performance of the alternative frameworks are examined in the context of model estimation and validation (at the aggregate and disaggregate level). Further, the performance of the model frameworks in the presence of underreporting is explored, with and without corrections to the estimates. The results from these extensive analyses point toward the emergence of the GOL framework (MGOL) as a strong competitor to the MMNL model in modeling driver injury severity.

1. Introduction

The problem of morbidity and mortality from motor vehicle crashes is now acknowledged to be a global phenomenon. According to World Health Organization (WHO), more than one million people get killed in traffic accidents each year (WHO, 2004). These incidents affect the society as a whole both emotionally and economically (Subramanian, 2006; Blincoe et al., 2002). These road crashes not only result in loss of life, but also impact the quality of life and productivity of the motor vehicle crash survivors. Given the import of the consequences of motor vehicle crashes, the issue has received significant attention from researchers and practitioners. In particular, the emphasis is on examining the influence of several factors, comprising of driver characteristics, vehicle characteristics, roadway design and operational attributes, environmental factors and crash characteristics on motor vehicle crash related severity.

The commonly available traffic crash databases compile injury severity data, primarily, as an ordinal discrete variable (for example, no injury, minor injury, major injury, and fatal injury). Naturally, many earlier studies examining the influence of exogenous factors employ ordered discrete outcome modeling approaches to evaluate their influence on crash severity (for example O'Donnell and Connor, 1996; Renski et al., 1999; Eluru et al., 2008). However, researchers have also employed unordered discrete outcome frameworks to study the influence of exogenous variables (for instance Chang and Mannering, 1999; Khorashadi et al., 2005). The ordered response models represent the decision process under consideration using a single latent propensity. The outcome probabilities are determined by partitioning the unidimensional propensity into as many categories as the dependent variable alternatives through a set of thresholds. Unordered discrete outcome frameworks offer a potential alternative to the analysis of ordered discrete variables. These models are characterized, usually, by a latent variable per alternative and an associated decision rule. The unordered models, usually, allow for additional parameter specification because they are tied to alternatives as opposed to a single propensity in the ordered models.

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The applicability of the two frameworks for analyzing ordinal discrete variables has evoked considerable debate on using the appropriate model for analysis. There are many strengths and weaknesses for the ordered framework *vis-à-vis* the unordered framework (Eluru, 2013). The ordered response models explicitly recognize the inherent ordering within the decision variable whereas the unordered response models neglect the ordering or require artificial constructs to consider the ordering (for example the ordered generalized extreme value logit model). On the other hand, the traditional ordered response models restrict the impact of exogenous variables on the outcome process to be same across all alternatives while the unordered response models allow the model parameters to vary across alternatives (see Eluru et al., 2008 for a discussion). The restricted number of parameters ensures that ordered response models have a parsimonious specification. The unordered response models might not be as parsimonious but offer greater explanatory power because of the additional exogenous effects that can be explored. In fact, several studies highlight the advantages of multinomial logit model over the ordered response models (see for example Bhat and Pulugurta, 1998). Hence, an empirical examination of alternative approaches in the context of injury severity analysis will allow us to determine the appropriateness of the two frameworks. Further, the recent revival of generalized ordered logit model (proposed by Terza, 1985) offers an ordered framework that allows the analyst to estimate the same number of parameters as the multinomial logit for an ordinal discrete variable. Hence, an exercise comparing the alternative frameworks is incomplete without considering the generalized ordered logit.

The conventional police/hospital reported crash databases may not include precious behavioral, physiological and psychological characteristics of individual involved in collisions. Due the presence of such unobserved information, the effect of exogenous variables might not be the same across individuals in the event of a crash (see for example Srinivasan, 2002; Eluru et al., 2008; Morgan and Mannering, 2011; Kim et al., 2013). For example, careful driving on behalf of a safe driver might moderate the severity outcome of a crash during night-time and while less cautious driving of an aggressive driver might exacerbate the crash severity in the same situation. In non-linear models, neglecting the effect of such unobserved heterogeneity can result in inconsistent estimates (Chamberlain, 1980; Bhat, 2001). Our study incorporates the influence of unobserved heterogeneity in both the ordered and unordered response frameworks.

The comparison exercise is particularly relevant in the context of injury severity data. The estimation of injury severity models correspond to the assumption of random sampling of severities from a population, where the probability of occurring for each individual crash is equal (Savolainen et al., 2011). However, the unknown population shares of such outcome-based crash severity data make the estimation of parameters even more challenging. Moreover, most of the crash data are sampled from police reported crash database. Several previous studies (Elvik and Mysen, 1999; Yamamoto et al., 2008) have provided evidence of underreporting issues related to the police-reported crash database. In such cases, the application of traditional econometric frameworks may result in biased estimates (Yamamoto et al., 2008). In the presence of underreported data, the unordered response framework is considered to be more effective compared to the ordered response framework. In the case of an underreported decision variable, the traditional multinomial logit model provides estimates that are unbiased *i.e.* the elasticity effects of the variables are not affected by the underreported data. This is often considered as a strong reason for promoting the use of unordered models over ordered models in modeling injury severity. It is important to recognize that the potential advantage applies only to MNL models under the condition that the dataset under examination satisfies the Independence of Irrelevant Alternatives (IIA) property (Ben-Akiva and Lerman, 1985). Hence, the nested logit and other advanced logit models that relax the IIA property are unlikely to yield unbiased estimates in the presence of underreporting. Moreover, the comparison of these two frameworks has mostly been undertaken in the context of traditional ordered models. The generalized ordered logit framework with its improved flexibility will provide the true benchmark for a fair comparison. It is also essential to examine how alternative modeling frameworks are impacted by underreporting; thus allowing us to adopt frameworks that are least affected by underreporting.

In summary, an accurate estimation of the associated risk factors is critical to assist decision makers, transportation officials, insurance companies, and vehicle manufacturers to make informed decisions to improve road safety. Yet, there is little research on empirically examining the differences between the ordered and unordered frameworks. Further, the influence of underreporting on alternative model frameworks has also received little attention. The current study proposes a framework to compare and contrast the alternative frameworks available for modeling driver injury severity. Further, the study also incorporates the underreporting issue associated with traditional crash databases. Specifically, the current study examines the performance of alternative modeling frameworks in the context of estimation from an observed sample and also in the context of an artificially created underreported data sample. Further, the study generates elasticity measures for the true and underreported samples to illustrate the influence of underreporting. The parameters from these model estimations are also used on a validation hold-out sample to evaluate model predictions (in the true as well as underreported case). The alternative modeling approaches considered for the exercise include: for the ordered response framework-ordered logit (OL), generalized ordered logit (GOL), mixed generalized ordered logit (MGOL) and for the unordered response framework-multinomial logit (MNL), nested logit (NL), ordered generalized extreme value logit (OGEV) and mixed multinomial logit (MMNL) model. We generate a series of measures to evaluate model performance in estimation and prediction thus allowing us to draw conclusions on model applicability for injury severity analysis.

The rest of the paper is organized as follows. Section 2 provides a discussion of earlier research on driver injury severity modeling while positioning the current study. Section 3 provides details of the various econometric model frameworks used in the analysis. In Section 4, the data source and sample formation procedures are described. The model comparison results, elasticity effects and validation measures are presented in Section 5. Section 6 concludes the paper and presents directions for future research.

2. Earlier research

A number of research efforts have examined driver injury severity to gain a comprehensive understanding of the factors that affect injury severity. In our review of earlier research we focus on studies examining severity at a disaggregate accident or individual level models of driver injury severity. For a detailed review of modeling frameworks employed in transportation safety the reader is referred to review studies: for example Savolainen et al. (2011) and Eluru et al. (2008). More recently, Eluru (2013) examined the performance of the MNL and GOL models by examining the issue from the data generation perspective; the authors argued that it is not possible to conclude which of the MNL and GOL is the better model without considering the dataset structure. Also, notably, even in cases where MNL performs better than GOL, the difference in data fit measures was relatively small.

A summary of earlier research on driver injury severity analysis from the perspective of the various ordered and unordered

Table 1
Summary of existing driver injury severity studies.

Paper	Methodological approach	Driver injury severity representation	Accident characteristics considered				
			Driver characteristics	Vehicle characteristics	Roadway design and operational attributes	Environmental factors	Crash characteristics
Shibata and Fukuda (1994)	Logistic regression	Fatal; non-fatal	Yes	–	–	–	Yes
Krull et al. (2000)	Logistic regression	Fatal/incapacitating injury; non-incapacitating/possible/no injury	Yes	Yes	Yes	Yes	Yes
Toy and Hammitt (2003)	Logistic regression	Serious injury/death; non-fatal	Yes	Yes	–	–	Yes
Conroy et al. (2008)	Logistic regression	Severe injury	Yes	Yes	–	–	Yes
Fredette et al. (2008)	Logistic regression	Fatality, major injury (hospitalized)	Yes	Yes	Yes	–	Yes
Bédard et al. (2002)	Multivariate logistic regression	Fatal; non-fatal	Yes	Yes	–	–	Yes
Dissanayake and Lu (2002)	Sequential binary logistic regression	No injury; possible injury; non-incapacitating injury; incapacitating injury; fatality	Yes	–	Yes	Yes	Yes
Huang et al. (2008)	Bayesian hierarchical binomial logistic regression	Fatal/severe injury; slight/no injury	Yes	Yes	Yes	Yes	Yes
Khattak et al. (2002)	Ordered probit	Fatality; incapacitating injury; evident injury; possible injury	Yes	Yes	Yes	Yes	Yes
Kockelman and Kweon (2002)	Ordered probit	No injury; minor injury; severe injury; fatal injury	Yes	Yes	–	Yes	Yes
Abdel-Aty (2003)	Ordered probit, ordered logit, multinomial logit, nested logit	property damage only, possible injuries, evident injuries, severe/fatal injuries	Yes	Yes	Yes	Yes	Yes
Khattak and Rocha (2003)	Ordered logit	No injury; minor injury; moderate injury; serious injury; severe injury; critical injury; max injury	Yes	Yes	Yes	–	Yes
Kweon and Kockelman (2003)	Ordered probit and Poisson model	No injury; not severe injury; severe injury; fatal injury	Yes	Yes	–	–	–
Khattak et al. (1998)	Binary probit & ordered probit	Fatal; severe injury; moderate injury; minor injury	Yes	Yes	Yes	Yes	–
Yamamoto and Shankar (2004)	Bivariate ordered-response probit	property damage only, possible injury, evident injury, disabling injury, fatality	Yes	Yes	Yes	Yes	Yes
Yamamoto et al. (2008)	Sequential binary probit model; ordered-probit model	Property damage only; possible injury; evident injury; disabling injury; fatality	Yes	Yes	Yes	Yes	Yes
Xie et al. (2009)	Bayesian ordered probit	No injury, possible injury, non-incapacitated injury, capacitated injury, and fatal injury	Yes	Yes	Yes	Yes	Yes
Eluru and Bhat (2007)	Mixed joint binary logit-ordered logit	No injury; possible injury; non-incapacitating injury; incapacitating injury; fatal injury	Yes	Yes	Yes	Yes	Yes
Paleti et al. (2010)	Random coefficients heteroscedastic ordered-logit	No injury; possible injury; non-incapacitating injury; incapacitating/fatal injury	Yes	Yes	Yes	Yes	–
de Lapparent (2008)	Bivariate ordered probit	No injury; light injury; severe injury; fatal injury	Yes	–	Yes	Yes	Yes
Srinivasan (2002)	Ordered logit; ordered mixed logit	No injury/property damage; moderate injury; severe injury; fatal injury	Yes	Yes	–	Yes	Yes
Ulfarsson and Mannering (2004)	Multinomial logit	No injury; possible injury; evident injury; fatal/disabling injury	Yes	Yes	Yes	Yes	Yes
Rana et al. (2010)	Copula-based joint Ordered logit–ordered logit; copula-based joint multinomial logit-ordered logit	No injury; possible injury; non-incapacitating injury; incapacitating injury; fatal injury	Yes	Yes	Yes	Yes	Yes
Eluru et al. (2012)	Latent segmentation based ordered logit	No injury; injury; fatal injury	Yes	Yes	Yes	Yes	–
Eluru et al. (2010)	Copula based approach	No injury; possible injury; non-incapacitating injury; incapacitating/fatal injury	Yes	Yes	Yes	Yes	Yes
Khorashadi et al. (2005)	Multinomial logit	No injury; complaint of pain; visible injury; severe/fatal injury	Yes	Yes	Yes	Yes	Yes
Islam and Mannering (2006)	Multinomial logit	No injury; injury; fatality	Yes	Yes	Yes	Yes	Yes
Awadzi et al. (2008)	Multinomial logit	No injury; injury; fatality	Yes	Yes	Yes	Yes	Yes
Schneider et al. (2009)	Multinomial logit	Property damage only; possible injury; non-incapacitating injury; incapacitating injury; fatal	Yes	Yes	Yes	Yes	Yes
Morgan and Mannering (2011)	Mixed multinomial logit	Severe injury, minor injury, no injury	Yes	Yes	Yes	Yes	Yes
Kim et al. (2013)	Mixed multinomial logit	Fatal injury, severe injury, visible injury, complaint of pain/no injury	Yes	Yes	–	Yes	Yes
Xie et al. (2012)	Latent class logit	No injury; possible injury; non-incapacitating injury; incapacitating injury; fatal injury	Yes	Yes	Yes	Yes	Yes

response models is provided in Table 1. The information presented in the table includes model structure employed for the analysis and identifies the variable categories considered in the analysis from the five broad categories of variables identified earlier. The following observations may be made from the table. *First*, the most prevalent mechanisms to study driver injury severity are logistic regression[2] and ordered response models (24 out of 31). The number of studies employing unordered models has been steadily increasing in recent years. *Second*, the most prevalent unordered response structure considered is the multinomial logit model. *Third*, it is evident from the analysis that very few studies (except Abdel-Aty, 2003; Ye and Lord, 2011) have empirically examined the different frameworks for modeling injury severity[3]. Finally, the maturity of the transportation safety community in examining driver injury severity is highlighted by the fact that a majority of studies (seventeen out of thirty one) have considered exogenous variables from all broad categories of variables.

2.1. Current study in context

Given the significance of examining the influence of exogenous variables on injury severity it is important that we undertake a comparison based on the performance of alternative frameworks. The current study contributes to literature on driver injury severity in multiple ways. First, the study provides a comparison exercise of the performance of ordered and unordered response models for examining the impact of exogenous factors on driver injury severity. We consider multiple models from ordered (OL, GOL and MGOL) and unordered frameworks (MNL, NL, OGEV and MMNL) to undertake the comparison exercise. Second, a host of comparison metrics are computed to evaluate the performance of the alternative models. Third, we compare the performance of the various models in the presence of underreporting. Elasticity measures are generated for the "true" dataset and the "artificial" dataset to compare the predicted elasticities for different models. Finally, we undertake the examination of driver injury severity using a comprehensive set of exogenous variables.

3. Econometric framework

In this section, we provide a brief description of the methodology of all the models considered for examining driver injury severity in our research.

3.1. Standard ordered logit model

In the traditional ordered response model, the discrete injury severity levels (y_i) are assumed to be associated with an underlying continuous latent variable (y_i^*). This latent variable is typically specified as the following linear function:

$$y_i^* = \boldsymbol{X}_i\boldsymbol{\beta} + \varepsilon_i \quad \text{for} \quad i = 1, 2, \ldots, N \tag{1}$$

where $i(i = 1, 2, \ldots, N)$ represents the drivers, $\boldsymbol{X_i}$ is a vector of exogenous variables (excluding a constant), $\boldsymbol{\beta}$ is a vector of unknown parameters to be estimated, $\boldsymbol{\varepsilon}$ is the random disturbance term assumed to be standard logistic.

Let $j(j = 1, 2, \ldots, J)$ denote the injury severity levels and τ_j represents the thresholds associated with these severity levels. These unknown τ_js are assumed to partition the propensity into $J-1$ intervals. The unobservable latent variable y_i^* is related to the observable ordinal variable y_i by the τ_j with a response mechanism of the following form:

$$y_i = j, \quad \text{if} \quad \tau_{j-1} < y_i^* < \tau_j \quad \text{for} \quad j = 1, 2, \ldots, J \tag{2}$$

In order to ensure the well-defined intervals and natural ordering of observed severity, the thresholds are assumed to be ascending in order, such that $t_0 < t_1 < \cdots < t_J$ where $\tau_0 = -\infty$ and $\tau_J = +\infty$. Given these relationships across the different parameters, the resulting probability expressions for individual i and alternative j for the OL take the following form:

$$\pi_{ij} = Pr(y_i = j|\boldsymbol{X}_i) = \Lambda\left(\tau_j - \boldsymbol{X}_i\boldsymbol{\beta}\right) - \Lambda\left(\tau_{j-1} - \boldsymbol{X}_i\boldsymbol{\beta}\right) \tag{3}$$

where $\Lambda(.)$ represents the standard logistic cumulative distribution function.

3.2. Generalized ordered logit model

The GOL model relaxes the constant threshold across population restriction to provide a flexible form of the traditional OL model. The basic idea of the GOL is to represent the threshold parameters as a linear function of exogenous variables (Maddala, 1983; Terza, 1985; Srinivasan, 2002; Eluru et al., 2008). Thus the thresholds are expressed as

$$\tau_j = \text{fn}(\boldsymbol{Z}_{ij}) \tag{4}$$

where $\boldsymbol{Z}_{ij}$ is a set of exogenous variable (including a constant) associated with jth threshold. Further, to ensure the accepted ordering of observed discrete severity $\left(-\infty < \tau_1 < \tau_2 < \cdots < \tau_{J-1} < +\infty\right)$ we employ the following parametric form as employed by Eluru et al. (2008):

$$\tau_j = \tau_{j-1} + \exp(\boldsymbol{\delta}_j\boldsymbol{Z}_{ij}) \tag{5}$$

where, $\boldsymbol{\delta}_\mathrm{j}$ is a vector of parameters to be estimated. The remaining structure and probability expressions are similar to the OL model. For identification reasons, we need to restrict one of the $\boldsymbol{\delta}_\mathrm{j}$ vectors to zero.

3.3. Mixed generalized ordered logit model

The MGOL accommodates unobserved heterogeneity in the effect of exogenous variable on injury severity levels in both the latent injury risk propensity function and the threshold functions (Srinivasan, 2002; Eluru et al., 2008). Let us assume that $\boldsymbol{\alpha}_\mathrm{i}$ and $\boldsymbol{\gamma}_{ij}$ are two column vectors representing the unobserved factors specific to driver i and his/her trip environments in Eqs. (1) and (5), respectively. Thus the equation system for MGOL model can be expressed as:

$$y_i^* = (\boldsymbol{\beta} + \boldsymbol{\alpha}_i)\boldsymbol{X}_i + \varepsilon_i, \quad \text{for} \quad i = 1, 2, \ldots, N \tag{6}$$

and

$$\tau_{i,j} = \tau_{i,j-1} + \exp[(\boldsymbol{\delta}_j + \boldsymbol{\gamma}_{i,j})\boldsymbol{Z}_{i,j}] \tag{7}$$

In Eqs. (6) and (7), we assume that $\boldsymbol{\alpha}_i$ and $\boldsymbol{\gamma}_{ij}$ are independent realizations from normal distribution for this study. Thus, conditional on $\boldsymbol{\alpha}_i$ and $\boldsymbol{\gamma}_{ij}$, the probability expressions for individual i and alternative j in MGOL model take the following form:

$$\begin{aligned}\pi_{ij} &= Pr\left(y_i = j|\boldsymbol{\alpha}_i, \boldsymbol{\gamma}_{ij}\right)\\ &= \Lambda[(\boldsymbol{\delta}_j + \boldsymbol{\gamma}_{i,j})\boldsymbol{Z}_{i,j} - \left(\beta + \alpha_i\right)X_i] - \Lambda[(\boldsymbol{\delta}_{j-1} + \boldsymbol{\gamma}_{i,j-1})\boldsymbol{Z}_{i,j}\\ &\quad - \left(\boldsymbol{\beta} + \boldsymbol{\alpha}_i\right)\boldsymbol{X}_i]\end{aligned} \tag{8}$$

[2] To be sure, the logistic regression with two alternatives can be regarded as an ordered logit model with two alternatives.

[3] To be sure, Ye and Lord (2011) have compared the ordered probit, multinomial logit and mixed logit model in terms of underreported data. The authors conclude that all the three models considered in the study perform poorly in the presence of underreported data. The exact impact of underreporting on these model frameworks needs further investigation. The study employs data simulation; however, the models are estimated with just one parameter and for a particular aggregate sample share.

The unconditional probability can subsequently be obtained as

$$P_{ij} = \int_{\alpha_i,\gamma_{ij}} [Pr\left(y_i = j|\boldsymbol{\alpha}_i, \boldsymbol{\gamma}_{ij}\right)] \times dF\left(\boldsymbol{\alpha}_i, \boldsymbol{\gamma}_{ij}\right) d(\boldsymbol{\alpha}_i, \boldsymbol{\gamma}_{ij}) \quad (9)$$

In this study, we use a quasi-Monte Carlo (QMC) method proposed by Bhat (2001) for discrete outcome model to draw realization from its population multivariate distribution. Within the broad framework of QMC sequences, we specifically use the Halton sequence (200 Halton draws) in the current analysis (see Eluru et al., 2008 for a similar estimation process).

3.4. Multinomial logit model

Let us consider the probability of a driver i ending in a specific injury-severity level j. The alternative specific latent variables for MNL take the form of

$$U_{ij} = \boldsymbol{\beta_j X_{ij}} + \varepsilon_{ij} \quad (10)$$

where $\boldsymbol{\beta_j}$ is a vector of coefficients to be estimated for outcome j, $\boldsymbol{X_{ij}}$ is a vector of exogenous variables, U_{ij} is a function of covariates determining the severity, ε_{ij} is the random component assumed to follow a gumbel type 1 distribution.

Thus, the MNL probability expression is as follows

$$P_i(j) = \frac{\exp[\boldsymbol{\beta}_j \boldsymbol{X}_{ij}]}{\sum_{j=1}^{J} \exp[\boldsymbol{\beta}_j \boldsymbol{X}_{ij}]} \quad (11)$$

3.5. Nested logit model

The NL model allows the incorporation of correlation across alternatives and results in two kinds of alternatives: those that are part of a nest (*i.e.* alternatives that are correlated) and alternatives that are not part of nest. The crash severity probabilities for the nested alternatives in the NL are composed of the nest probability as well as the alternative probability (same structure as the MNL applies).

In the first step, the probability of choosing the nest is determined followed by the probability of choosing alternative within the nest

$$P_i(j) = \frac{\exp[\boldsymbol{\beta_j X_{ij}} + \boldsymbol{\theta_j L_{ij}}]}{\sum_{j \in J} \exp[\boldsymbol{\beta_j X_{ij}} + \boldsymbol{\theta_j L_{ij}}]} \quad (12)$$

$$P_i(k|j) = \frac{\exp[\boldsymbol{\beta_{k|j} X_{ij}}]}{\sum_{k \in K} \exp[\boldsymbol{\beta_{k|j} X_{ij}}]}$$

where $P_i(j)$ is the unconditional probability of ith crash falling in nest j, $P_i(k|j)$ is the conditional probability of ith crash having severity outcome k (lower level) conditioned on the nest j (higher level), J is the actual severity and K is the alternative represented by the nest, $\boldsymbol{L_{ij}}$ is the inclusive value (log sum) representing the expected value of the attributes from the nest j, $\boldsymbol{\theta_j}$ is the nesting coefficient.

The alternative probabilities for non-nested alternatives take a form similar to the MNL probabilities while considering the utility of the nested alternatives as a composite alternative. To be consistent with the NL derivation, the value of the $\boldsymbol{\theta_j}$ should be greater than 0 and less than 1 (McFadden, 1981). If the estimated value of $\boldsymbol{\theta_j}$ is not significantly different from 1, then the NL model collapses to a simple MNL model.

3.6. Ordered generalized extreme value model

Injury levels of a crash are typically progressive (ranging from non-injury to fatal). MNL and NL models do not account for any inherent ordering in the outcomes. Small (1987) proposed the OGEV model for such ordered discrete outcomes. The OGEV model allows for the correlations between the error terms of outcomes which are close to each other in the ordered scale.

We employ the structure proposed in Wen and Koppelman (2001) for the OGEV model with j alternatives as follows

$$P_i(j) = \sum_{m=i}^{i+L} P_{i|m}.P_m$$

$$= \sum_{m=1}^{i+L} \left[\frac{(w_{mi}.e^{U_{ij}})^{1/\mu_m}}{\sum_{k\varepsilon N_m} (w_{mj}.e^{U_{ik}})^{1/\mu_m}} \times \frac{\left\{\sum_{k\varepsilon N_m} (w_{mj}.e^{U_{ik}})^{1/\mu_m}\right\}}{\sum_{s=1}^{J+L} \left\{\sum_{k\varepsilon N_s} (w_{sj}.e^{U_{ik}})^{1/\mu_s}\right\}^{\mu_s}} \right] \quad (13)$$

The probability of alternative j in an accident for driver i is computed as the sum of probability computed from all nests to which i belongs. In the above notation, L is the number of contiguous alternatives considered in a nest, w_{mi} represents the allocation weight for each alternative i to nest m, The total number of nests is given as a combination ${}^{J}C_{L}$. The allocation parameter satisfies the property $\sum_i w_{mi} = 1$. μ_m represents the log-sum parameter for nest m. N_m represents the set of alternatives in nest m. In our analysis we set $L = 1$ *i.e.* we consider the following nests 1, 1 2, 2 3, 3 4, and 4 (where 1 = no injury, 2 = possible injury, 3 = Non-incapacitating Injury and 4 = incapacitating/fatal injury).

3.7. Mixed multinomial logit model

The MMNL is a generalized version of traditional MNL model. It allows the parameters for exogenous variables to vary across individual involved in the collision by accommodating unobserved heterogeneity on the utility functions for different injury severity levels. Let us assume that ω_{ij} is a column vectors representing the unobserved factors specific to driver i and his/her trip environments in Eq. (10). Thus the equation system for MMNL model can be expressed as:

$$U_{ij} = (\boldsymbol{\beta}_j + \boldsymbol{\omega}_{ij})\boldsymbol{X_{ij}} + \varepsilon_{ij} \quad (14)$$

In Eq. (14), we assume that $\boldsymbol{\omega}_{ij}$ is an independent realization from normal distribution for this study. Thus, conditional on $\boldsymbol{\omega}_{ij}$, the probability expression for individual i and alternative j in MMNL model take the following form:

$$P_{ij}|\boldsymbol{\omega}_{ij} = \frac{\exp[(\boldsymbol{\beta}_j + \boldsymbol{\omega}_{ij})\boldsymbol{X}_{ij}]}{\sum_{j=1}^{J} \exp[(\boldsymbol{\beta}_j + \boldsymbol{\omega}_{ij})\boldsymbol{X}_{ij}} \quad (15)$$

The unconditional probability can subsequently be obtained as

$$P_{ij} = \int_{\omega_{ij}} (P_{ij}|\boldsymbol{\omega}_{ij}) \times dF\left(\boldsymbol{\omega}_{ij}\right) d\boldsymbol{\omega}_{ij} \quad (16)$$

To estimate the MMNL model, we apply the QMC simulation techniques in a similar fashion as described in MGOL model section.

4. Data

4.1. Data source

The data for the current study is sourced from the “General Estimates System (GES)” database for the year 2010. The GES database is a nationally representative sample of road crashes collected and compiled from about 60 jurisdictions across the United States.

The data is obtained from the U. S. Department of Transportation, National Highway Traffic Safety Administration's National Center for Statistics and Analysis (ftp://ftp.nhtsa.dot.gov/GES/GES10/). The data includes information of reports compiled by police officers for crashes involving at least one motor vehicle traveling on a roadway and resulting in property damage, injury or death to the road users. The GES crash database has a record of 46,391 crashes involving 81,406 motor vehicles and 116,020 individuals for the year of 2010. A five point ordinal scale is used in the database to represent the injury severity of individuals involved in these crashes: (1) no injury; (2) possible injury; (3) non-incapacitating injury; (4) incapacitating injury and (5) fatal injury. Further, the dataset compiles information on a multitude of factors (driver characteristics, vehicle characteristics, roadway design and operational attributes, environmental factors and crash characteristics) representing the crash situations and events. Accordingly, a number of crash-related factors are extracted from this database in order to explore the variables that might influence the driver injury severity.

4.2. Sample formation and description

The main focus of this study is injury severity of drivers of passenger vehicles (passenger car, sport utility vehicle, pickup or van). Thus, the following criteria were employed for sample formation:

- The crashes that involve only non-commercial (private) passenger vehicle drivers are selected (to avoid the potential systematic differences between commercial and non-commercial driver groups).
- The passenger vehicle crashes that involve another passenger vehicle or a fixed object are examined.
- The crashes that involve more than two vehicles are excluded from the analysis.

The final dataset of non-commercial driver of passenger vehicles, after removing records with missing information for essential attributes consisted of about 30,371 records. In this final sample of accidents the percentage of fatal crashes sustained by drivers is extremely small (0.7%). Therefore, both the fatal and incapacitating injury categories are merged together to ensure a representative share for each alternative crash level. From this dataset, a sample of 12,170 records is sampled out for the purpose of analysis and 18,201 records are set aside for validation. In the final estimation sample, the distributions of driver injury severities are: No injury 65.9%, possible injury 15.1%, non-incapacitating injury 12.1% and incapacitating/fatal injury 6.9%.

5. Empirical analysis

5.1. Variables considered

In our analysis, we selected a host of variables from five broad categories: driver characteristics (including driver gender, driver age, restraint system use, alcohol consumption and drug use), vehicle characteristics (including vehicle type and vehicle age), roadway design and operational attributes (including roadway class, speed limit, types of intersection and traffic control device), environmental factors (including time of day and road surface condition) and crash characteristics (including driver ejection, vehicle rolled over, air bag deployment, manners of collision and collision location). It should be noted here that several variables such as presence of shoulder, shoulder width, point of impact, number of lanes, lighting condition could not be considered in our analysis because either the information was entirely unavailable or there was a large fraction of missing data for these attributes in the dataset. To be sure, we employ the manner of collision and time of day variables to act as surrogates for point of impact and lighting condition, respectively. In the final specification of the model, statistically insignificant variables were removed (95% confidence level). Further, in cases where the variable effects were not significantly different, the coefficients were restricted to be the same.

5.2. Overall measures of fit

In the research effort, we estimated seven different models: (1) OL, (2) GOL, (3) MGOL, (4) MNL, (5) OGEV, (6) NL and (7) MMNL model. After extensively testing for different nesting structures for NL and parametric assumptions for OGEV models we found that these models collapsed to the MNL model. Hence, the entire comparison exercise is focussed on five models: OL, GOL, MGOL, MNL and MMNL. Prior to discussing the estimation results, we compare the performance of these models in this section.

The log-likelihood values at convergence for the various frameworks are as follows: (1) OL (with 29 parameters) is −10617.51; (2) GOL (with 50 parameters) is −10517.83, (3) MGOL (with 55 parameters) is −10506.97, (4) MNL (with 57 parameters) is −10517.59 and (5) MMNL (with 61 parameters) is −10508.76. The corresponding value for the "constant only" model is −12164.58. The ordered models (OL, GOL and MGOL) are nested version of each other. Thus, we can compare the ordered models among those by using likelihood ratio (LR) test for selecting the preferred model. Similarly, the MNL and MMNL models can be compared using LR test. However, to compare the ordered approaches with the unordered approach, the LR test is not appropriate because these structures are not nested within one another. Hence, to undertake the comparison we employ a two-step process. In the first step, we use the LR test to determine the superior model within each framework. Subsequently, we compare the best model from each framework using the non-nested measures applicable for such comparison.

5.3. Comparison within ordered and unordered frameworks

The LR test statistic is computed as $2[LL_U - LL_R]$, where LL_U and LL_R are the log-likelihood of the unrestricted and the restricted models, respectively. The computed value of the LR test is compared with the $\aleph^2$ value for the corresponding degrees of freedom (*dof*). The resulting LR test values for the comparison of OL/GOL, OL/MGOL and GOL/MGOL models are 199.36 (21 *dof*), 221.08 (26 *dof*) and 21.72 (5 *dof*), respectively. The LR test values indicate that MGOL outperforms the OL model at any level of statistical significance. The MGOL outperforms the GOL model at the 0.001 significance level indicating that MGOL offers superior fit compared to both OL and GOL models. In the unordered context, the LR test value (17.66, 4 *dof*) for the comparison of MNL/MMNL indicates that MMNL offers superior fit over MNL model at the 0.001 significance level.

5.4. Comparison between ordered and unordered frameworks—Non-nested test

To evaluate the performance of the ordered and unordered models, we employ different measures that are routinely applied in comparing econometric models including: (1) Bayesian Information Criterion (BIC), (2) Akaike Information Criterion corrected (AICc)[4] and (3) Ben-Akiva and Lerman's adjusted likelihood ratio (BL) test. The BIC for a given empirical model is equal to $-2\ln(L)+K\ln(Q)$ and the AICc for an empirical model is given by AIC + [2

[4] AICc is a more stringent version of the AIC [AIC = $2K - 2\ln(L)$] in penalizing for additional parameters.

$K(K+1)/(Q-K-1)]$, where $\ln(L)$ is the log-likelihood value at convergence, K is the number of parameters, and Q is the number of observations. The model with the *lower* BIC and AICc values is the preferred model. The BIC (AICc) values for the final specifications of the MGOL and MMNL models are 21531.31 (21124.45) and 21591.33 (21140.14), respectively.

The BL test statistic (Ben-Akiva and Lerman 1985) is computed as: $\lambda = \Phi\left\{-\left[\sqrt{-2\left(\bar{\rho}_2^2-\bar{\rho}_1^2\right)L(C)+(M_2-M_1)}\right]\right\}$where $\bar{\rho}^2$ represents the McFadden's adjusted rho-square value for the model. It is defined as $\bar{\rho}_2^2 = 1-\frac{L_i(\beta)-M_i}{L(C)}$where $L_i\left(\beta\right)$ represents log-likelihood at convergence for the ith model, $L(C)$ represents log-likelihood at sample shares and M_i is the number of parameters in the model (Windmeijer, 1995). The $\Phi(.)$ represents the cumulative standard normal distribution function. The resulting λ value for the comparison of MGOL and MMNL is 0, clearly indicating that MGOL offers superior fit compared to MMNL model. The comparison exercise clearly highlights the superiority of the MGOL in terms of data fit compared to MMNL model. In the subsequent section, we discuss the results from MGOL and MMNL frameworks.

6. Estimation results

Table 2 presents the results of the MGOL and MMNL models. The reader would note that the interpretation of the MGOL is slightly different from the MMNL model. In MGOL, when the threshold parameter is positive (negative) the result implies that the threshold is bound to increase (decrease); the actual effect on the probability is quite non-linear and can only be judged in conjunction with the influence of the variable on propensity and other thresholds. MMNL represents the effect of exogenous variables on each injury category relative to the base category. In the following sections, the estimation results are discussed by variable groups.

6.1. Driver characteristics

In safety research, driver demographics, particularly driver's age and gender have always been considered to have a significant influence on injury severity. In the current research, the effects of these variables are found to be significant. In particular, MGOL estimates indicate that compared to the female drivers, the latent injury propensity is lower for male drivers, while the negative sign of threshold demarcating the possible and non-incapacitating injury indicates a higher likelihood of non-incapacitating and incapacitating/fatal injuries for the male drivers. It is important to note that the variable impacts in propensity and thresholds are counteracting one another and the exact impact realized is specific to every individual. Corresponding results from MMNL indicate that male drivers are more likely to evade injury relative to their counterparts. The estimates associated with driver age, from both the MGOL and MMNL, suggest a reduction in the likelihood of severe injuries for the young drivers (age <25) compared to middle-aged drivers (age 25 to 64). However, the parameter characterizing the effect of older age (age $\geq$65) on driver injury severity is found significant in the MMNL model only. The result suggests that the odds of suffering an incapacitating/fatal injury are significantly higher for the older drivers compared to the middle-aged drivers.

Seat belt use is found to have a significant influence on driver injury severity. Consistent with several previous studies (Preusser et al., 1991; Janssen, 1994; Eluru and Bhat, 2007), our analysis showed an unequivocal benefit for employing seat belts. MGOL model estimates for the driver not wearing safety belts results in a parameter that is normally distributed with a mean 1.528 and standard deviation 0.844, which indicates that almost 96% of the drivers involved in the collision cannot evade injury if they do not wear seat belts at the time of crash. MMNL model estimates indicate that the likelihood of suffering from possible, non-capacitating and incapacitating/fatal injuries is higher for the unrestrained driver and these effects are fixed.

As expected, drivers under the influence of alcohol are likely to have a higher injury risk propensity compared to the sober drivers. Positive sign of the latent propensity of MGOL model estimate indicates that the latent injury risk propensity is higher for drivers who are impaired by alcohol, while the negative sign of threshold demarcating the non-incapacitating and incapacitating/fatal injury indicates a higher likelihood of incapacitating/fatal injury for this group of drivers. MMNL model estimates also reveal that the odds of suffering incapacitating/fatal injury are higher for non-sober drivers. The effect of impairment by drugs is found significant in MMNL model only and the result shows that the drivers are more likely to suffer an incapacitating/fatal injury when they are impaired by drugs. The MGOL model is unable to pick such an effect of drugs involvement on driver injury severity and the reason might be attributed to a small share (0.9%) of drivers under the influence of drug in the dataset.

6.2. Vehicle characteristics

With respect to driver's vehicle type, the MGOL model results indicate that the latent injury propensity is higher for the driver of a passenger car compared to the driver of other passenger vehicles (sports utility vehicle (SUV), pickup and vans). This is expected because in collisions with other vehicles or fixed objects, the drivers in passenger cars are usually the most likely to be severely injured (Mayrose and Jehle, 2002; O'Neill and Kyrychenko, 2004; Fredette et al., 2008). The corresponding results from MMNL suggest that the likelihood of sustaining possible, non-capacitating and incapacitating/fatal injuries is higher for the drivers in a passenger car relative to drivers in other passenger vehicles.

The vehicle age result of MGOL model demonstrates that the drivers in older vehicles (6–10 years and above 10 years) have a higher injury risk propensity compared to the drivers in newer vehicles (vehicle age <6 years). The MMNL model estimates indicate that the drivers in older vehicles (6–10 years old and above 10 years old) have a higher likelihood of suffering from possible, non-capacitating and incapacitating/fatal injuries relative to the drivers in newer vehicles. The higher injury risk of older vehicle's driver might be attributed to the mechanical defect, lack of safety equipment, exposure of younger driver to these vehicles or the involvement of suspended and unlicensed drivers of these vehicles (Lécuyer and Chouinard, 2006). The lower injury risk for the driver of new vehicles may reflect the advancement in the vehicle-based safety equipments (such as airbag, antilock braking system, center high-mounted stoplight, crash cage, shatter resistant windshield).

6.3. Roadway design and operational attributes

With respect to the roadway functional class, the MGOL model estimates show that the injury risk propensity of drivers is higher when the crash occurs on an interstate highway. Again, the effect of "interstate highway" variable on the threshold demarcating non-incapacitating and incapacitating/fatal injuries shows a higher likelihood of incapacitating/fatal injuries of the drivers during crashes on an interstate highway. The MMNL model estimates show that the likelihood of both possible and incapacitating/fatal injury increases when crash occur on interstate highway. The MGOL results for speed limit indicate that latent injury propensities are higher for crashes occurring on roads with medium (26 to 50 mph) and higher (above 50 mph) speed limits relative to crashes on lower speed limit (less than 26 mph). The effect of speed limit variables on the threshold indicates increased likelihood of

Table 2
MGOL and MMNL estimates.

Variables	MGOL			MMNL			
	Latent propensity	Threshold between possible and non-incapacitating injury	Threshold between non-incapacitating and incapacitating/fatal injury	No injury	Possible injury	Non-incapacitating injury	Incapacitating/fatal injury
Constant	−1.819	0.208	0.624	–	−2.239	−2.989	−5.215
Driver characteristics							
Driver gender (base: female)							
Male	−0.565 (0.046)	−0.258 (0.046)	–	–	−0.656 (0.057)	−0.540 (0.064)	−0.500 (0.090)
Driver age (base: age 25 to 64)							
Age less than 25	−0.441 (0.050)	–	–	0.411 (0.051)	–	–	–
Age above 65+	–	–	–	–	–	–	0.403 (0.137)
Restraint system use (base: restrained)							
Unrestrained	1.528 (0.142)	–	–	–	1.303 (0.065)	1.695 (0.073)	2.127 (0.101)
SD Unrestrained	0.844 (0.223)	–	–	–	–	–	–
Under the influence of alcohol	0.489 (0.130)	–	−0.353 (0.122)	–	–	–	0.887 (0.166)
Under the influence of drug	–	–	–	–	–	–	0.776 (0.293)
Vehicle characteristics							
Vehicle type (base: SUV, pickup and vans)							
Passenger car	0.269 (0.046)	–	–	−0.262 (0.047)	–	–	–
Vehicle age (base: vehicle age less than 6)							
Vehicle Age 6 to 10	0.144 (0.052)	–	–	–	0.122 (0.057)	0.122 (0.057)	0.308 (0.111)
Vehicle age above 10	0.405 (0.055)	–	–	–	0.312 (0.067)	0.444 (0.073)	0.684 (0.111)
Roadway design and operational attributes							
Interstate highways	0.303 (0.088)	–	−0.246 (0.090)	–	0.224 (0.092)	0.224 (0.092)	0.672 (0.163)
Speed limit (base: speed limit less than 26 mph)							
Speed limit 26 to 50 mph	0.462 (0.072)	−0.127 (0.046)	–	–	0.268 (0.088)	0.541 (0.105)	0.985 (0.172)
Speed limit above 50 mph	0.715 (0.089)	–	–	–	0.616 (0.107)	0.767 (0.123)	1.122 (0.196)
Types of intersection							
Four way intersection	0.177 (0.062)	–	–	−0.172 (0.060)	–	–	–
Traffic control device (base: non traffic control device)							
Traffic signal/stop/yield sign	−0.119 (0.059)	–	–	–	–	−0.252 (0.073)	–
Other traffic control device	0.376 (0.142)	–	–	–	–	–	0.567 (0.239)
Environmental factor							
Time (base: 3 pm to 6 pm)							
6 pm to 6 am	–	−0.141 (0.048)	–	–	–	0.032 (0.091)	0.032 (0.091)
SD 6 pm to 6 am	–	–	–	–	–	0.772 (0.211)	0.772 (0.211)
6 am to 9 am	0.173 (0.069)	–	–	−0.214 (0.073)	–	–	–
9 am to 3 pm	0.195 (0.048)	–	–	−0.244 (0.052)	–	–	–
Surface condition (base: dry)							
Wet	–	–	–	–	–	−0.179 (0.087)	–
Snowy	−0.648 (0.120)	–	–	–	−0.592 (0.123)	−0.592 (0.123)	−1.041 (0.263)
Crash characteristics							
Driver ejected out of the vehicle	6.040 (2.655)	1.583 (0.751)	–	–	–	–	–
Vehicle rolled over	2.111 (0.209)	0.177 (0.220)	–	–	1.923 (0.224)	1.923 (0.224)	2.877 (0.286)
SD vehicle rolled over	–	0.989 (0.343)	–	–	–	–	–
Air bag deployment	1.595 (0.066)	0.270 (0.073)	–	–	1.303 (0.065)	1.695 (0.073)	2.127 (0.101)
SD air bag deployment	0.844 (0.223)	–	–	–	–	–	–
Collision object (base: another moving vehicle)							
Collision with stationary object	0.774 (0.081)	−0.283 (0.074)	−0.226 (0.087)	–	0.416 (0.097)	0.936 (0.098)	1.203 (0.257)
SD collision with stationary object	–	–	0.847 (0.233)	–	–	–	1.310 (0.379)
Collision with other object	−1.174 (0.189)	−1.162 (0.313)	–	–	−1.774 (0.329)	−0.647 (0.233)	–

Table 2 (*continued*)

Variables	MGOL			MMNL			
	Latent propensity	Threshold between possible and non-incapacitating injury	Threshold between non-incapacitating and incapacitating/fatal injury	No injury	Possible injury	Non-incapacitating injury	Incapacitating/fatal injury
Manner of collision							
Head on	0.966 (0.100)	–	−0.393 (0.100)	–	0.805 (0.109)	0.805 (0.109)	1.974 (0.175)
Angular	0.382 (0.063)	−0.150 (0.061)	−0.244 (0.067)	–	0.317 (0.068)	0.317 (0.068)	1.153 (0.155)
Side swipe-same direction	−0.534 (0.097)	–	0.316 (0.151)	–	−0.334 (0.122)	−0.512 (0.150)	−1.206 (0.330)
Rear to side collision	–	−3.683 (0.717)	2.309 (0.182)	–	–	–	–
Other manners of collision	−1.258 (0.627)	–	1.651 (0.178)	–	–	–	–
Collision location (base: Non-intersection)							
Intersection	–	0.227 (0.061)	–	–	–	–	−0.369 (0.141)
Intersection related	−0.255 (0.071)	–	–	–	−0.430 (0.155)	−0.430 (0.155)	−0.530 (0.170)
SD Intersection related	0.007 (0.002)	–	–	–	0.915 (0.323)	0.915 (0.323)	–
Driveway access	−0.477 (0.243)	–	–	–	–	–	–
Entrance and exit ramp	−0.323 (0.150)	–	–	–	–	–	–
Railway grade crossing	–	1.181 (0.421)	−3.981 (0.987)	–	–	–	–
Driveway access related	−0.427 (0.087)	–	–	–	−0.335 (0.090)	−0.335 (0.090)	−2.649 (1.210)
SD Driveway access related	–	–	–	–	–	–	2.332 (0.896)
Through roadway	–	–	–	–	0.913 (0.428)	0.913 (0.428)	–
Other location	–	–	–	–	−0.768 (0.375)	−0.768 (0.375)	–

non-incapacitating and incapacitating/fatal injuries at higher speed limits. The corresponding results from MMNL suggest that the likelihood of sustaining possible, non-incapacitating and incapacitating/fatal injuries is higher for crashes on both the medium and higher speed limit roads compared to the crashes on lower speed limit roads. As is expected, within the two speed categories considered the higher speed category has a larger impact relative to the medium speed category.

With respect to the types of intersection, only four way intersections are found to have significant influence on driver injury severity. The MGOL model estimates reflect the higher injury risk propensity to drivers on a four-way intersection. The MMNL results also indicate very similar impact of four-way intersection on injury severity. The four way intersection reduces the likelihood of no injury crashes and in turn increases the likelihood of a driver sustaining severe injury. The presence of traffic control device is also found to have significant effect on the severity of crashes. MGOL estimates reveal that the presence of a traffic signal/stop/yield sign reduces the likelihood of injury risk propensity of the drivers relative to the absence of a control measure. The MMNL estimates show that the likelihood of non-incapacitating injury reduces with the presence of a traffic signal/stop/yield sign. However, MGOL estimates also indicate that the injury risk propensity increases when there are other traffic control system or a warning sign present on the roadway. The corresponding result of MMNL specify that the odds of suffering an incapacitating/fatal injury increase significantly with the presence of these control measures relative to uncontrolled measure.

6.4. *Environmental factors*

Time-of-day and surface condition are two of the environmental factors that are found to significantly influence driver injury severity. Compared to the evening peak, the likelihood of injury risk propensities are found to be higher for both the morning peak and off-peak periods in the MGOL estimates. At the same time, the effect of night-time variable on the threshold demarcating possible and non-incapacitating injuries shows a higher likelihood of non-incapacitating and incapacitating/fatal injuries. The MMNL estimates reveal that the drivers are less likely to evade no injury during morning peak and off-peak period. However, the effect of night-time variable results in an estimate that is normally distributed with 0.032 and standard deviation 0.772. But, the mean coefficient for night-time is not significantly different from zero, while the standard deviation is highly significant. This result indicates that driver injury severity outcome varies widely during night-time crash and the exact nature of injury severity is determined by the unobserved factors specific to the crash.

The findings of MGOL estimates indicate that if collisions occur on a snowy road surface, the consequence is likely to be less injurious as compared to the accident on dry road surface. The MMNL results also indicate very similar impacts of snowy road surface on driver injury severity. On a snowy road the drivers are more likely to evade serious injury relative to crashes on a dry surface. The effect of wet road surface condition is found significant only in the MMNL model estimates and the result indicates a lower likelihood of non-incapacitating injury on wet roads. The reduced risk of injury on snowy/wet road can be attributed to more careful driving and reduced speeding possibility (Edwards, 1998; Mao et al., 1997; Eluru and Bhat, 2007).

6.5. *Crash characteristics*

Several crash characteristics considered are found to be significant determinants of driver injury severity. Among those, the injury risk propensities are observed to be higher in MGOL estimates when a driver is ejected out from his/her vehicle or when the vehicle rolled over. At the same time, the positive values of the first thresholds of driver ejection reflect an increase in possible injury probability. But, the first threshold of vehicle rolled over is found to be random with a statistically insignificant mean and a highly significant standard deviation. The result indicates that while injury risk propensity is likely to increase the impact on crash severity, the threshold is determined by unobserved factors specific to the crash.

The likelihood of injury risk propensity for the deployment of air bag is also found to be significant and normally distributed in the MGOL model estimate. The result implies that air bag deployment increases the probability of injury in almost 97% cases. At the same time, the positive values of the first thresholds of air bag deployment reflect an increase in possible injury probability. The corresponding results from the MMNL model estimates indicate that the drivers are less likely to avoid serious injury when the vehicle rolled over or an air bag deployed during a crash. However, none of the aforesaid two variable estimates are found to be random, while the effect of driver ejection is found to be insignificant both as fixed and random parameter in MMNL.

With respect to the collision object, MGOL and MMNL model estimates indicate very similar effects indicating that the odds of suffering serious injury is higher when a vehicle strikes a stationary object (such as: pole, guard rail, tree and post) compared to the crashes with a moving vehicle. However, the threshold demarcating non-incapacitating injury to incapacitating/fatal injury of MGOL is distributed normally. With the estimated parameter, 39.36% of the distribution is greater than zero and 60.64% of the distribution is less than zero. At the same time, MMNL model also results in a random parameter for incapacitating/fatal injury category, which indicates that 82.12% of the distribution is above zero and only 17.88% is less than zero. The parameters characterizing the effects of manner of collision in Table 2, for both MMNL and MGOL models, suggest that the drivers are less likely to evade serious injury in the event of head-on or angular collision relative to the rear-end collision. Side-swipe collisions with vehicles traveling in the same direction and rear to sideswipe collisions are less severe than rear end collision.

Finally, both the MGOL and MMNL model estimates indicate that collision location has a significant influence on injury severity profile. Specifically, collisions at an intersection or entry/exit ramp or driveway access or intersection related collisions are less likely to result in injuries to the drivers in the event of a crash relative to non-intersection location. At the same time, the latent propensity of MGOL and the possible/non-incapacitating injury coefficient of MMNL for intersection related collision indicate the presence of significant unobserved heterogeneity in those estimates. The driveway access related variable also results in a random parameter for incapacitating/fatal injury category in only MGOL model. Further, the MGOL estimates show that collision on driveway access or entrance/exit ramp has a reduced likelihood of severe injury, while railway grade crossing has a positive impact on possible injury outcome. In the MMNL model, the variable representing through roadway results in a higher likelihood of possible and non-incapacitating injuries, while the variable representing other location reduces the likelihood of possible and non-incapacitating injuries.

The broad characterization of exogenous variable effects across the MGOL and MMNL model systems is similar with some differences. These differences can be attributed to the different model structures and different outcome mechanism. The reader would note that in both systems, the impact of exogenous variables was moderated by unobserved effects resulting in statistically significant standard deviation parameters.

7. Model comparison

In the preceding section, we have presented a discussion of model results for the MGOL and the MMNL model. To investigate the comparison further, we examine the model performance under two contexts: (1) presence of underreporting and (2) validation on a hold-out sample.

7.1. Underreporting

In police reported crash database, many property damage and minor injury crashes might go underreported since lower crash severity levels make reporting to authorities less likely (Savolainen and Mannering, 2007). Researchers have argued that underreporting of data will have minimal impact on the model estimation result of standard MNL model (Kim et al., 2007; Shankar and Mannering, 1996; Savolainen and Mannering, 2007; Islam and Mannering, 2006). On the other hand, ordered response models are particularly susceptible to underreporting issue (Savolainen and Mannering, 2007; Ye and Lord, 2011) and can result in biased or inconsistent parameter estimates. However, recent evidence on examining underreporting suggests that none of the models (including unordered response systems) are immune to the underreporting issue (Ye and Lord, 2011). This is expected because the presence of underreporting would not affect the unordered systems only when the dataset under consideration satisfies the independence of irrelevant alternatives property. Hence, even the MNL model will yield biased estimates if the IIA property does not hold for the dataset. To reinforce this, we undertake a comparison in the context of underreported data. For this purpose, we generate an underreported data set by randomly removing 50% of no injury crash records from the estimation sample. This reduced dataset is used to re-estimate MGOL and MMNL models. To compare the differences between the estimates from "true" and underreported dataset we compute elasticity effects for a selected set of independent variables—male, age less than 25, passenger car, high speed limit, snowy road surface and head-on collision (see Eluru and Bhat, 2007 for a discussion on computing elasticities). The elasticity estimates are presented in Table 3. For the ease of presentation, we focus on the elasticity effects for the two severe injury categories. The results from the "true" sample and underreported sample indicate that the underreported sample consistently obtains the wrong elasticities, as expected. The percentage error in computing elasticity for the selected variables for the two injury severity categories has an average of (33.69, 19.11) and (31.81, 25.96) while the range of the errors is (2.97, 75.99) and (5.85, 57.83) for MGOL and MMNL models, respectively. From the estimated measures we can argue that neither of the models results in unbiased estimates in the underreporting context.

In addition to direct comparison in the context of underreporting, we also undertake a comparison of the elasticity effects with corrections to the MMNL and MGOL models. The correction exercise for altering constants estimated from an underreported sample is relatively straight forward. Specifically, all parameter estimates are kept the same and the constants are altered to match the population shares in the "true" sample. A trial and error approach to alter the constants is employed to generate "corrected" constants for the MMNL model. Further, we employ a similar approach to correct the threshold parameters for the MGOL model. In the MGOL model the population share can be influenced by altering the threshold constants thus achieving the same correction process as the MMNL model. In both correction exercises, adequate care is taken to ensure that the population shares match with the "true" shares after the parameters are corrected. Subsequent to the constant and threshold corrections, the elasticity values are recomputed for the

Table 3
Elasticity effects.

Variables	MGOL				MMNL			
	Non-incapacitating injury	Incapacitating/fatal injury	% Of error in non-incapacitating injury	% Of error in incapacitating/Fatal injury	Non-incapacitating injury	Incapacitating/fatal injury	% Of error in non-incapacitating injury	% Of error in incapacitating/fatal injury
Estimation sample								
Male	−17.28	−20.35	–	–	−25.26	−14.51	–	–
Age less than 25	−24.07	−29.69	–	–	−19.97	−14.72	–	–
Passenger car	15.23	18.76	–	–	13.02	9.50	–	–
High speed limit	43.77	57.44	–	–	38.41	63.82	–	–
Snowy surface	−32.69	−38.40	–	–	−24.20	−44.32	–	–
Head-on collision	27.54	153.04	–	–	20.27	173.52	–	–
Underreported sample without corrections								
Male	−11.33	−12.06	34.44	40.74	−16.42	−7.07	35.00	51.25
Age less than 25	−18.47	−25.14	23.26	15.31	−18.80	−13.62	5.85	7.49
Passenger car	12.25	16.76	19.60	10.65	11.08	6.03	14.89	36.47
High speed limit	36.23	55.73	17.23	2.97	28.37	47.52	26.15	25.54
Snowy surface	−22.36	−28.92	31.61	24.70	−11.83	−34.33	51.13	22.53
Head-on collision	6.61	121.98	75.99	20.30	8.55	151.89	57.83	12.46
Average error	–	33.69	–	19.11	–	31.81	–	25.96
Underreported sample with corrections								
Male	−15.57	−17.32	9.88	14.87	−23.19	−12.78	8.17	11.90
Age less than 25	−20.96	−26.24	12.93	11.62	−23.14	−17.43	15.88	18.39
Passenger car	13.95	17.49	8.43	6.78	17.69	10.42	35.89	9.70
High speed limit	43.44	58.88	0.74	2.51	38.87	57.10	1.20	10.53
Snowy surface	−24.85	−29.97	23.99	21.96	−16.83	−37.70	30.46	14.94
Head-on collision	16.96	130.13	38.41	14.96	24.56	177.21	21.19	2.13
Average error	–	–	15.73	12.12	–	–	18.80	11.27

Table 4
Disaggregate measures of fit in validation sample.

Summary statistic	MGOL predictions	MMNL predictions
Number of observations	3993.9900	3993.9900
Number of parameters	55	61
Log-likelihood at zero	−5536.8458	−5536.8458
Log-likelihood at sample shares	−3962.5600	−3962.5600
Predictive Log-likelihood	−3671.0702	−3643.0636
C.I.	−3685.6638/−3656.4766	−3657.3289/−3628.7984
AICc	7453.7050	7410.0514
C.I.	7424.5207/7482.8892	7381.5246/7438.5782
BIC	7798.2252	7791.9668
C.I.	7768.9357/7827.5147	7763.3179/7820.6156
Predictive adjusted likelihood ratio index	0.0597	0.0652
C.I.	0.0578/0.0615	0.0638/0.0667
Average probability of correct prediction	0.6649	0.6663
C.I.	0.6636/0.6662	0.6650/0.6677
Average probability for chosen probability >0.70	0.4787	0.4620
C.I.	0.4774/0.4799	0.4609/0.4632

Table 5
Aggregate measures of fit in validation sample.

Injury categories/measures of fit	Actual shares	MGOL predictions	MMNL predictions
No injury	66.4311	65.8805	65.9509
C.I.	–	65.8118/65.9492	65.8842/66.0174
Possible injury	15.0667	15.1281	15.0362
C.I.	–	15.1034/15.1528	15.0139/15.0583
Non-incapacitating injury	11.3647	12.0757	12.0754
C.I.	–	12.0449/12.1064	12.0476/12.1032
Incapacitating/fatal injury	7.1375	6.9157	6.9376
C.I.	–	6.8823/6.9492	6.9029/6.9722
RMSE	–	0.6319	0.6105
C.I.	–	0.5883/0.6756	0.5667/0.6544
MAPE	–	3.7679	3.6586
C.I.	–	3.7651/3.7706	3.6558/3.6613
	Driver age less than 25		
No injury	69.1630	67.9434	67.8363
C.I.	–	67.8059/68.0809	67.71094/67.9617
Possible injury	12.8669	14.1267	13.3549
C.I.	–	14.0783/14.1751	13.3131/13.3967
Non-incapacitating injury	11.2528	11.3173	11.7434
C.I.	–	11.2599/11.3747	11.6869/11.7999
Incapacitating/fatal injury	6.7173	6.6126	7.0653
C.I.	–	6.5453/6.6799	6.9988/7.1319
RMSE	–	1.1199	1.0354
C.I.	–	1.0377/1.2023	0.9641/1.1067
MAPE	–	6.6456	6.1554
C.I.	–	6.6408/6.6505	6.1509/6.1600
Predictive log-likelihood	–	−1028.3794	−1015.5878
C.I.	–	−1036.0795/−1020.6794	−1023.1219/−1008.0537
	Air bag deployed		
No injury	34.6793	34.8052	34.4638
C.I.	–	34.6797/34.9307	34.3658/34.5619
Possible injury	23.7389	23.7988	23.4669
C.I.	–	23.7434/23.8541	23.4176/23.5162
Non-incapacitating injury	23.1525	23.0632	24.1901
C.I.	–	22.9902/23.1361	24.1354/24.2449
Incapacitating/fatal injury	18.4293	18.3329	17.8792
C.I.	–	18.2296/18.4362	17.7821/17.9762
RMSE	–	1.2129	1.2902
C.I.	–	1.1276/1.2984	1.1869/1.3934
MAPE	–	4.2884	4.6403
C.I.	–	4.2852/4.2915	4.6364/4.6441
Predictive log-likelihood	–	−1385.1886	−1318.6118
C.I.	–	−1394.7695/−1375.6077	−1327.2064/−1310.0172
	Off-peak period		
No injury	66.9671	65.8187	65.6960
C.I.	–	65.7138/65.9236	65.5950/65.7969
Possible injury	15.8240	16.1584	16.4015
C.I.	–	16.1176/16.1993	16.3655/16.4375
Non-incapacitating injury	10.9846	11.9150	11.8761
C.I.	–	11.8676/11.9624	11.8398/11.9123
Incapacitating/fatal injury	6.2242	6.1078	6.0265
C.I.	–	6.0606/6.1550	5.9774/6.0755

Table 5 (*Continued*)

Injury categories/measures of fit	Actual shares	MGOL predictions	MMNL predictions
RMSE	–	0.9911	1.0427
C.I.	–	0.9119/1.0703	0.9637/1.1218
MAPE	–	5.7662	6.0102
C.I.	–	5.7612/5.7711	6.0054/6.0150
Predictive log-likelihood	–	−1226.6454	−1207.2053
C.I.	–	−1234.7771/−1218.5138	−1215.5970/−1198.8135
		Snowy surface	
No injury	73.0563	71.9579	71.7287
C.I.	–	71.6597/72.2560	71.4244/72.0330
Possible injury	10.7654	12.2862	11.4324
C.I.	–	12.1692/12.4032	11.3389/11.5259
Non-incapacitating injury	11.6573	9.9632	11.6253
C.I.	–	9.8255/10.1009	11.4894/11.7612
Incapacitating/fatal injury	4.5210	5.7927	5.2135
C.I.	–	5.6498/5.9356	5.0748/5.3523
RMSE	–	2.1626	1.8423
C.I.	–	1.9874/2.3379	1.6628/2.0217
MAPE	–	20.8766	16.7887
C.I.	–	20.8500/20.9033	16.7651/16.8122
Predictive log-likelihood	–	−150.5851	−149.2434
C.I.	–	−153.9116/−147.2586	−152.4695/−146.0173
		Passenger car	
No injury	63.3983	62.5658	62.6231
C.I.	–	62.4731/62.6584	62.5320/62.7141
Possible injury	16.4833	16.3340	16.5008
C.I.	–	16.3018/16.3661	16.4707/16.5309
Non-incapacitating injury	12.3735	13.2977	13.3121
C.I.	–	13.2583/13.3371	13.2753/13.3489
Incapacitating/fatal injury	7.7449	7.8026	7.5640
C.I.	–	7.7552/7.8499	7.5178/7.6102
RMSE	–	0.8573	0.8286
C.I.	–	0.7917/0.9229	0.7598/0.8974
MAPE	–	4.6446	4.5066
C.I.	–	4.6412/4.6479	4.5029/4.5102
Predictive log-likelihood	–	−2311.8055	−2301.2185
C.I.	–	−2322.8512/−2300.7599	−2313.0902/−2289.3468

updated estimates. The results are presented in the last block of rows in Table 3.

The elasticity errors reduce substantially for both MGOL and MMNL models as a result of the parameter corrections. The average percentage errors in computing elasticity for the selected variables ranges are (15.73, 12.12) and (18.80, 11.27) for MGOL and MMNL models with a range of (0.74, 38.41) and (1.2, 35.89), respectively. We can argue that both the unordered and ordered frameworks perform almost equivalently with underreported dataset and the performance for both of these structures can be improved with the correction measure if the true population share is available to the analyst.

7.2. Validation analysis

A validation experiment is also carried out in order to ensure that the statistical results obtained above are not a manifestation of over fitting to data. For testing the predictive performance of the models, 100 data samples, of about 4000 records each, are randomly generated from the hold out validation sample consisting of 18,201 records. We evaluate both the aggregate and disaggregate measure of predicted fit by using these 100 different validation samples. For these samples, we present the average measure from the comparison, and also the confidence interval (C.I.), of the fit measures at 95% level.

At the disaggregate level we computed predictive log-likelihood (computed by calculating the log-likelihood for the predicted probabilities of the sample), AICc, BIC, predictive adjusted likelihood ratio index, probability of correct prediction, and probability of correct prediction >0.7. The results are presented in Table 4. In terms of disaggregate validation measures, the MMNL model consistently outperforms the MGOL model (except for probability of correct prediction >0.7). At the aggregate level, root mean square error (RMSE) and mean absolute percentage error (MAPE) are computed by comparing the predicted and actual (observed) shares of injuries in each injury severity level. We compute these measures for each set of full validation sample and specific sub-samples within that validation population—driver age less than 25, air bag deployed, off-peak hour crash, snowy surface and passenger car. The results for aggregate measure computation are presented in Table 5.

The comparison of MGOL and MMNL model at the aggregate level is far from conclusive. However, it is clear that MGOL and MMNL models perform very well at the aggregate level. For the full sample, both the MAPE and RMSE values are very close for both models. The RMSE and MAPE values show that the predicted performance for the MGOL model is superior to that of the MMNL model for sub-samples air bag deployed and off-peak hour crash while the MMNL model is superior to that of the MGOL model for driver age less than 25, snowy surface and for passenger car. Thus, we can argue that the differences in the validation measures at aggregate level are not as conclusive as the measures at disaggregate level. Further, the differences in the aggregate level characteristics between the models are very small.

We extend the validation exercise to examine the performance of underreported sample estimates (uncorrected and corrected) as well on the 100 randomly selected validation samples. We compute these measures only for each of the full validation samples (results are presented in Table 6). Clearly, based on the underreported sample estimates, the overall errors at disaggregate and aggregate levels are much larger than previously for both systems. In the uncorrected system, MGOL has lower AICc and BIC values, but MMNL has lower RMSE and MAPE values. But in the corrected system, MGOL consistently outperforms the MMNL model (except for RMSE) and the aggregate predicted shares from MGOL model

Table 6
Measures of fit in validation for underreported sample.

Measure of fit in underreported sample			
Injury categories/measures of fit	Actual shares	MGOL predictions	MMNL predictions
No injury	66.4311	52.4731	52.6582
C.I.	–	52.4051/52.5411	52.5779/52.7386
Possible injury	15.0667	21.6642	21.5562
C.I.	–	21.6359/21.6925	21.5045/21.6079
Non-incapacitating injury	11.3647	17.0554	16.9202
C.I.	–	17.0207/17.0901	16.8876/16.9528
Incapacitating/fatal injury	7.1375	8.8073	8.8653
C.I.	–	8.7683/8.8463	8.8277/8.9029
RMSE	–	8.2760	8.1565
C.I.	–	8.2049/8.3470	8.0806/8.2324
MAPE	–	34.7376	34.3961
C.I.	–	34.7334/34.7418	34.3918/34.4005
Predictive log-likelihood	–	−4080.7320	−4089.1194
C.I.	–	−4096.0726/−4065.3915	−4104.0381/−4074.2008
AICc	–	8264.8098	8293.9191
C.I.	–	8234.1313/8295.4884	8264.0853/8323.7529
BIC	–	8584.3790	8650.9086
C.I.	–	8553.6005/8615.1576	8620.9523/8680.8649
Measure of fit in underreported sample with correction			
No injury	66.4311	69.4232	69.4094
C.I.	–	69.3574/69.4889	69.3349/69.4839
Possible injury	15.0667	13.7549	13.8957
C.I.	–	13.7262/13.7835	13.8526/13.9389
Non-incapacitating injury	11.3647	10.9293	10.8844
C.I.	–	10.8999/10.9586	10.8553/10.9135
Incapacitating/fatal injury	7.1375	5.8926	5.8105
C.I.	–	5.8599/5.9253	5.7786/5.8423
RMSE	–	1.7944	1.7827
C.I.	–	1.7256/1.8633	1.7119/1.8536
MAPE	–	8.6295	8.7599
C.I.	–	8.6266/8.6325	8.7569/8.7629
Predictive log-likelihood	–	−3853.4807	−3881.9877
C.I.	–	−3869.9209/−3837.0405	−3898.5934/−3865.3820
AICc	–	7810.3072	7879.6556
C.I.	–	7777.4290/7843.1853	7846.4471/7912.8641
BIC	–	8129.8764	8236.6451
C.I.	–	8096.9087/8162.8441	8203.3327/8269.9575

is closer to the actual shares for three out of four injury categories compared to those from MMNL model.

In summary, from the host of validation statistics we can argue that neither the ordered nor the unordered frameworks exclusively outperforms each other both at the aggregate and the disaggregate levels. The relatively close performance of the two model systems is further illustrated through the computation of the validation measures for various sub-samples of the population. Overall, the results indicate that MGOL and MMNL offer very similar prediction for the various sub-samples at the aggregate and disaggregate level. The results reinforce that MGOL model performs very close to the MMNL model in examining driver injury severity.

8. Conclusions and implications

This paper focuses on the relevance of alternate discrete outcome frameworks for modeling driver injury severity. The most prevalent framework employed to model injury severity is the ordered response mechanism. However, unordered response models were also employed in the past to model crash injury severity. The applicability of the two frameworks for analyzing ordinal discrete variables has evoked considerable debate on using the appropriate framework for analysis. An empirical examination of alternative approaches to modeling injury severity would enable us to determine the appropriateness of the two frameworks.

Further, the two frameworks are also influenced by the underreporting issue associated with crash data sample. Most of the crash data are sampled from police reported crash database, where many property damage and minor injury crashes might go underreported. In the case of an underreported decision variable, the application of traditional econometric frameworks may result in biased estimates. Unfortunately, the unknown population shares of such outcome-based crash severity data make the estimation of parameters even more challenging. In this context, it is essential to examine how alternative modeling frameworks are impacted by underreporting; thus allowing us to adopt frameworks that are least affected by underreporting.

The current paper addresses the aforementioned issues of identifying the more relevant framework to model crash injury severity by empirically comparing the ordered response and unordered response models. The performances of these models are also tested in the presence of underreported crash data by creating an artificial reduced dataset. Elasticity measures are generated for the "true" dataset and the artificial underreported dataset to compare the predicted elasticities for the different models. Thus, the current research contributes to the safety analysis literature from both the methodological and empirical standpoint.

The alternative modeling approaches considered for the exercise include: for the ordered response framework-ordered logit, generalized ordered logit, mixed generalized ordered logit and for the unordered response framework-multinomial logit, nested logit, ordered generalized extreme value logit and mixed multinomial logit model. The empirical analysis is based on the 2010 General Estimates System (GES) data base. The focus in the analysis is exclusively on non-commercial passenger vehicle driver crash-related injury severity. Several types of variables are considered in the empirical analysis, including driver characteristics, vehicle characteristics, roadway design and operational attributes, environmental factors and crash characteristics. The empirical results indicate the important effects of all of the above types of variables on injury severity. The model comparison for the estimation sample clearly indicates that the MGOL model outperforms the MMNL model.

To investigate the comparison further, we studied the model performance under two contexts: (1) presence of underreporting and (2) validation on a hold-out sample. We generated a series of measures to evaluate model performance in estimation and prediction thus allowing us to draw conclusions on model applicability for injury severity analysis. In the context of underreporting, the comparison between the elasticity estimates from "true" and "underreported" sample indicates that the underreported sample consistently obtains the wrong elasticities for both MGOL and MMNL models. The most striking finding is the fact that the MMNL model does not perform any better in the underreporting context than MGOL. Moreover, the correction measures for the thresholds/constants based on the true aggregate shares reduce the elasticity errors substantially for both MGOL and MMNL models. In the context of validation analysis at the aggregate and disaggregate level, we can argue that neither the ordered nor the unordered frameworks exclusively outperforms each other. The relatively close performance of the two model systems is further illustrated through the computation of the validation measures for various sub-samples of the population and in the presence of underreporting. Overall, the results of the empirical comparison provide credence to the belief that an ordered system that allows for exogenous variable effects to vary across alternatives and accommodates unobserved heterogeneity offer almost equivalent results to that of the corresponding unordered systems in the context of driver injury severity.

The results have significant implications for safety research. There is growing recognition in the safety community that modeling injury severity as exogenous to seat belt use, alcohol consumption, or collision type is not realistic. For instance, the common unobserved factors that influence seat belt usage might also influence injury severity (see Eluru and Bhat, 2007). Incorporating such interactions in a joint framework increases the complexity of the models involved. However, by allowing for injury severity to follow an ordered response structure we can reduce the complexity of the joint model because of the single error term of this structure. The unordered model would lead to a more cumbersome modeling approach because of the multiple error terms involved (Eluru, 2013). Recent research has demonstrated the advantages of such joint frameworks (see for example Castro et al., 2013; Narayanamoorthy et al., 2012).

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