

Accident Analysis and Prevention 37 (2005) 185-199



www.elsevier.com/locate/aap

Modeling crash-flow-density and crash-flow-V/C ratio relationships for rural and urban freeway segments[☆]

Dominique Lord^{a,*}, Abdelaziz Manar^{b,1}, Anna Vizioli^{b,2}

 Department of Civil Engineering, Center for Transportation Safety, Texas Transportation Institute, Texas A&M University System, 3135 TAMU, College Station, TX, 77843-3135, USA
 Ministère des Transports du Québec, Direction de l'Ouest de la Montérégie, 245, boul. Saint-Jean-Baptiste, Châteauguay Qué., J6K 3C3, Canada

Received 12 February 2004; received in revised form 7 July 2004; accepted 16 July 2004

Abstract

There has been considerable research conducted in recent years into establishing relationships between crashes and various traffic flow characteristics for freeway segments. Most of the research has focused on determining the relationship between crashes and highway traffic volumes, while little attention has been focused on the relationships of vehicle density, level of service (LOS), vehicle occupancy, V/C ratio and speed distribution. Despite overall progress, there is still no clear understanding about the effects of different traffic flow characteristics on safety. In fact, several studies reviewed in this work were found to have methodological limitations. These include using predictive models with a normal error structure, aggregated crash rates, and inadequate functional forms for the data at hand. The original research on which this paper is based is aimed to determine the statistical relationship using commonly applied predictive models (i.e., functional forms) between crashes and hourly traffic flow characteristics, such as traffic volume, vehicle density and V/C ratios, for rural and urban freeway segments respectively. To accomplish this objective, predictive models have been developed from data collected on freeway segments located in downtown and outside of Montreal, Quebec. Three different functional forms are evaluated. The results show that predictive models that use traffic volume as the only explanatory variable may not adequately characterize the accident process on freeway segments. Functional forms that incorporate density and V/C ratio offer a richer description of crashes occurring on these facilities, whether they are located in a rural or urban environment. Finally, separate predictive models for single- and multi-vehicle crashes should be developed rather than one common model for all crash types.

© 2004 Elsevier Ltd. All rights reserved.

Keywords: Predictive models; Crashes; Safety; Freeways; Density; Hourly volumes; Capacity; V/C ratio

1. Introduction

There has been considerable research conducted in recent years into establishing relationships between crashes and various traffic flow characteristics for freeway segments. Most of the research has focused on determining the relationship between crashes and highway traffic volumes, either at the aggregated (e.g., Annual Average Daily Traffic or AADT) or disaggregated (e.g., hourly volumes) levels (Gwynn, 1967; Cedar and Livneh, 1982; Turner and Thomas, 1986; Huang et al., 1992; Persaud and Dzbik, 1993; Hadi et al., 1995; Mensah and Hauer, 1998; Martin, 2002; Abbas, 2003). Other work has also examined the safety of freeway segments as a function of other traffic flow characteristics, such as V/C ratios (Frantzeskakis and Iordanis, 1987; Hall and Pendleton, 1989; Zhou and Sisiopiku, 1997; Vizioli and Manar, 2002), vehicle density or occupancy (Brodsky and Hakkert, 1983; Garber and Subramanyan, 2002), speed distribution (Garber

paper was presented at the 83rd Annual Meeting of the Transportation Research Board.

^{*} Corresponding author. Tel.: +1 979 458 1218; fax: +1 979 845 4872. E-mail addresses: d-lord@tamu.edu (D. Lord),

amanar@mtq.gouv.qc.ca (A. Manar), avizioli@mtq.gouv.qc.ca (A. Vizioli).

¹ Tel.: +1 450 698 3400x285; fax: +1 450 698 3452.

² Tel.: +1 450 698 3400x284; fax: +1 450 698 3452.

and Gadiraju, 1989; Davis, 2002) and level of service (LOS) (Persaud and Nguyen, 1998a).

Despite overall progress, there is still no clear understanding about how these different traffic flow characteristics affect the safety of both rural and urban freeway segments, which operate under different traffic flow conditions. Unfortunately, previously established statistical relationships and, to some extent, predictive models have been plagued by important limitations. For instance, crash rates (usually defined as crashes per year per 108 veh-km) have often been employed for determining these relationships. It has been shown that crash rates, used as the dependent variable in statistical relationships and modeled as a linear regression model with a normal error distribution (see Knuiman et al., 1993; Garber and Subramanyan, 2002), are inadequate for predicting the safety of highway facilities (Mahalel, 1986; Persaud and Dzbik, 1993; Hauer, 1997). For example, there is nothing that proves that crash rates follow a normal distribution (i.e., Poisson process divided by a round number). Predictive models that employ count data should be used for such purpose. In other instances, the data are often aggregated for sites with similar characteristics (e.g., sites with similar flows are grouped together to compute one crash rate or sites with no crashes are removed from the analysis), which masks the true effects of these traffic flow characteristics on safety. The selection of the functional form is also very critical for developing predictive models and very few researchers question the rationale or the logic behind their choice (Miaou and Lord, 2003). In fact, different model forms that employ the same data will lead to different estimations of safety. As a result, there is an important need for better understanding the statistical relationships between crashes and traffic flow characteristics, especially in the light of the recent work on how the use of crash risk can affect the decision-making process for transportation policy and ITS strategies (Mahalel, 1986; Maher et al., 1993; Lord, 2002).

The objectives of this study were twofold. The first objective sought to investigate how hourly traffic flow characteristics, such as traffic volume, vehicle density and V/C ratios, influence crashes on rural and urban freeway segments. The second objective consisted of evaluating different functional forms for modeling traffic flow characteristics. The intent was to investigate how traffic flow characteristics could be incorporated into commonly available predictive models that utilize the number of vehicles per hour as the primary explanatory variable or covariate. To accomplish these objectives, exploratory analyses are performed on data collected on freeway segments located in and outside of Montreal, Quebec. Furthermore, predictive models or safety performance functions (SPFs) are developed for determining these statistical relationships, using a modeling approach proposed by Heydecker and Wu (2001) and Miaou and Lord (2003). Finally, important methodological issues about modeling crashes for rural and urban freeway segments, using vehicle density and V/C ratio as explanatory variables, are discussed.

2. Previous work

It has been suggested by most studies that crash counts follow a nonlinear relationship with traffic volume, as dictated by the functional form $Y = \alpha F^{\beta}$ (Y: crashes per year, F: flow). This relationship has been investigated extensively and dates as far back as the 1950s (Tanner, 1953). It is usually assumed that the number of crashes increases at a decreasing rate as traffic volume increases. This relationship is characterized in predictive models with the coefficient for the traffic volume parameter (β) to be below 1. This characteristic has also been found in many studies that focused on freeway segments with a few exceptions (see Hadi et al., 1995; Mensah and Hauer, 1998). Recent work by Persaud (1992), Persaud and Dzbik (1993), and Persaud and Bahar (2000) found that crashes increase as traffic volume increases, particularly for segments with more than 3 lanes per direction. Persaud (1992) reported that 6-lane freeways have a greater likelihood of crash occurrences since lane changes happen more frequently on segments with 6 lanes than on segments having fewer lanes. Abbas (2003) developed more than 200 predictive models for rural roads in Egypt and evaluated 5 different function forms: linear, power, logarithmic, polynomial, and exponential. He reported that about half of the predictive models followed a power relationship.

A few researchers have examined the relationships between crashes and V/C ratios or LOS for freeway segments. The studies have found that crash rates typically follow a U-shaped relationship when plotted as a function of V/C ratio (Gwynn, 1967; Zhou and Sisiopiku, 1997; Vizioli and Manar, 2002). Cedar and Livneh (1982) and Martin (2002) also found a similar U-shaped relationship between crash rate and hourly traffic volumes. On the other hand, Hall and Pendleton (1989), and Frantzeskakis and Iordanis (1987) established that crash rates increased as a function of V/C ratio. Frantzeskakis and Iordanis (1987) and Persaud and Nguyen (2000) have examined the effects of LOS on safety. These authors have found that both the number of crashes and crash rates increase as the LOS decreases (from A to F)

The relationship between crashes and vehicle density or lane occupancy has seldom been investigated, since this type of data cannot be easily obtained. Although Brodsky and Hakkert (1983) did not use vehicle density as defined in this work for their analysis of freeway segments, they did examine the effect of travel densities defined as annual vehicle-miles per mile on the likelihood of crashes. Brodsky and Hakkert reported that crashes are invariant to travel densities for rural freeways. Garber and Subramanyan (2002) studied the relationship between crashes and lane occupancy for four rural freeway segments in Virginia. They developed SPFs for predicting crashes as a function of lane occupancy. The predictive models followed a polynomial of the third order, in which the probability of a crash occurrence increases, peaks and decreases as the percentage of lane occupancy increases.

3. Data collection

Two study sections, one located in a rural area and the other located in an urban environment, were used for this study. The 40.5-km rural section is located on Highway A-40 between the Ontario border and the west end of the Island of Montreal. This section was separated into eight segments varying from 1.5 to 9.4 km. Six of eight sections are 4-lane divided facilities, while two sections have six lanes. The 5-km 6-lane divided urban section is also located on A-40 between Décarie Blvd and Highway A-15 in the north end of Montreal. This section is the most heavily used freeway facility in Quebec. The data used for this analysis were the same data used by Vizioli and Manar (2002). The reader is referred to their publication for additional information about the exact location of these sections and other characteristics.

The study period covered 5 years from 1994 to 1998 inclusively. Crash data were obtained for the entire period and contained detailed information about the severity, the location, the crash type, the day of the week, the direction of travel, and the time of day among others. The data were grouped according to whether crashes occurred on a weekday, a Saturday or a Sunday, by direction and by the hour of day (3×24 -h period = 72 time periods). The data was adjusted for the time period (crashes per year) required for the analysis (e.g., 260 Saturdays in 5 years, etc.). In the end, 1152 observations (72 time periods \times 8 sections \times 2 directions) and 144 observations (72 time periods \times 2 directions) were analyzed for the rural and urban segments respectively.

Various traffic flow characteristics were obtained from permanent and temporary count stations. On urban sections, the permanent stations record, through loop detectors, vehicle speed, traffic volume, and vehicle occupancy for each lane at 5-min intervals on a 24-h basis. The count stations are also able to classify the traffic volume by vehicle type, which is essential for computing the freeway segment capacities. For urban sections, the data was aggregated for each hour and for each direction separately. On rural sections, only traffic volume is recorded, also with loop detectors, on an hourly basis through temporary counting stations. The traffic flow counts are recorded for a 24-h period, for up to 4 days, two or three times a year.

The vehicle density (defined as the number of vehicles per kilometer per lane) used for the exploratory analysis and the predictive models was computed with the methodology proposed in the HCM 2000 (TRB, 2001) for rural freeway sections. Fig. 13-3 in the Manual was employed to this effect (curve 65 mph). The vehicle density for the urban segments was estimated with the method proposed in the special report 165 on traffic flow theory (TRB, 1992). The vehicle density was estimated using the flow obtained from the loop detectors divided by the speed for the given section. This method is the one most commonly used by local or state transportation agencies. Furthermore, this method was selected as the most suitable estimation procedure based on data availability, costs and the accuracy of the prediction (TRB, 1992; Coifman,

2003). The reader is referred to Coifman (2001) and Coifman and Cassidy (2002) for additional information on different techniques for estimating vehicle density. The data were then grouped for each hour and averaged for the entire section and across all three lanes. The final average vehicle density was estimated with a C-Exponential ($r^2 = 80.5$) regression equation (see Fig. 1). For both urban and rural sections, the vehicle density was validated with the density estimated from the V/C analysis described below. The vehicle density varied from 0.16 to 13.1 veh/km/ln and from 1.1 to 50.0 veh/km/ln for the rural and urban sections, respectively.

As reported in the literature, speed-flow-density relationships can be separated into different traffic flow regimes (Roess et al., 1998). The most common relationships are usually classified under two traffic flow regimes: non-congested and congested. These two regimes can actually be seen in Fig. 1 (below and above 20 veh/km/ln). Recent development on this topic has shown that traffic flow could exist into a substantial high number of traffic flow regimes (Schoen et al., 1995). In fact, Golob and Recker (2004) noted up to 21 traffic flow regimes that were dependent upon weather patterns and time of day among others. Given the fact that no consensus currently exists about which traffic flow regime should be used for a particular set of conditions (Roess et al., 1998), it was decided to fit one common function for both regimes noted in Fig. 1. As discussed below, the use of a single function was deemed adequate for modeling crashes for this work.

The V/C ratios were also estimated with the method proposed in the HCM 2000 (TRB, 2001). The various adjustment factors, such as lane width, grade, and percentage of trucks were calculated for each section separately. The V/C ratios varied from 0.01 to 0.62 and from 0.07 to 0.95 for rural and urban sections respectively. It should be pointed out that no segments in the rural area are subjected to recurrent congestion.

The final database contained information on crashes (summed over 5 years), vehicular flows, vehicle density and V/C ratio for each hour of the day for a typical weekday, a Saturday or a Sunday respectively. The characteristics of this database (and the type of analysis) are very similar to previous work performed this subject (i.e., data grouped for each hour for multiple years) (see Frantzeskakis and Iordanis (1987); Hadi et al., 1995; Mensah and Hauer, 1998; Persaud and Nguyen, 1998a, b; Persaud and Look, 2000; Martin, 2002). It is important to point out that traffic flow, vehicle density and V/C ratio are in fact estimates and do not represent the exact traffic flow conditions at the time of the crash. However, these estimates are adequate, given the relationships detailed above, for the nature of the analysis performed within the scope of this work. Additional discussion on this topic is presented at the end of the paper.

4. Exploratory analysis

This section describes the exploratory analyses conducted on the data for different crash types and severities. An

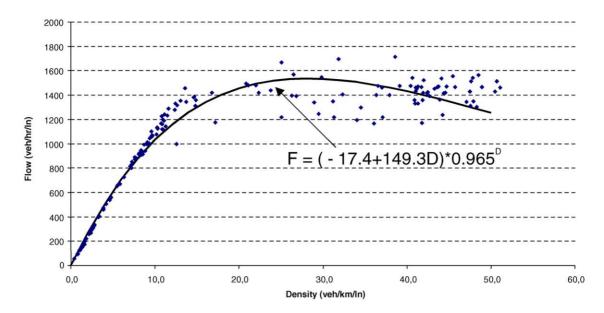


Fig. 1. Flow-density relationship for urban segments.

overview of the characteristics of the crash data is presented in Table 1. The results of the analyses are shown for single- and multi-vehicle crashes. They are presented separately for rural and urban sections. The intent of this section is not to present all possible characteristics of the data, but the ones that will be used in the development of SPFs. The reader is referred to Frantzeskakis and Iordanis (1987) and Martin (2002) for additional descriptive statistics on the safety effects of hourly volumes on freeway segments.

4.1. Rural segments

Fig. 2 illustrates the relationships between crashes and three traffic flow characteristics: hourly volume, vehicle density and V/C ratio respectively. The relationships are illustrated for single- and multi-vehicle crashes. Trend curves estimated using third degree polynomial equations were fitted with the data for single- and multi-vehicle crashes. Fig. 2 shows some trends for single-vehicle and multi-vehicle crashes. For instance, it can be seen that as density and V/C increase, the number of single-vehicle crashes decreases. On the other hand, the number of multi-vehicle crashes increases with vehicle density and the V/C ratio. The data shows that crashes become less severe with an increasing V/C ratio, but does not seem to be affected by the vehicle density. Although

Table 1 Characteristics of crashes on rural and urban segments

Crash type	Rural	Urban	
Fatal + severe	68	30	
Single-vehicle	1235	632	
Multi-vehicle	630	2185	
Total	1865	2817	

not shown in the figure, nighttime crashes occur much more frequently at low volumes and V/C ratios.

4.2. Urban segments

Fig. 3 shows the relationships between the same traffic flow characteristics illustrated above. Overall, as opposed to the results found for rural segments, the total number of crashes appears to slightly increase as vehicle density increases. Similarly, the total number of crashes also shows an increasing trend with a decrease in LOS. Single- and multivehicle crashes share similar characteristics as the ones found for rural segments. It should be pointed out that Fig. 1A and C are essentially the same, but with a different scale on the abscissa, since only one highway section (each direction was modeled separately) was used in the analysis.

5. Statistical modeling

Three series of predictive models were developed for urban and rural freeway segments. The probabilistic structure used for developing the models was the following: The number of crashes at the i-th segment and t-th time period, Y_{it} , when conditional on its mean μ_{it} , is assumed to be Poisson distributed and independent over all segments and time periods as:

$$Y_{it}|\mu_{it} \sim Po(\mu_{it})$$
 $i = 1, 2, ..., I$ and $t = 1, 2, ..., T$ (1)

The mean of the Poisson is structured as:

$$\mu_{it} = f(L, F, D, X; \beta) \exp(e_{it})$$
 (2)

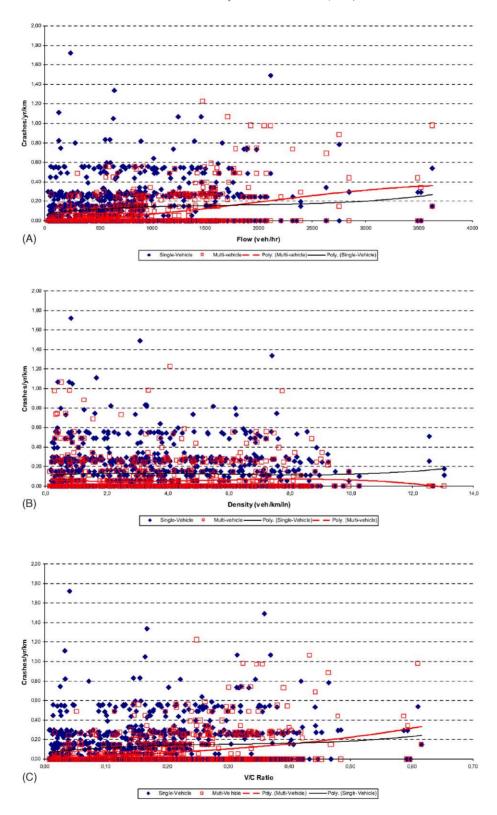


Fig. 2. Crash-traffic flow characteristics for rural segments. (A) Crash-flow relationship, (B) crash-density relationship, (C) crash-V/C ratio relationship.

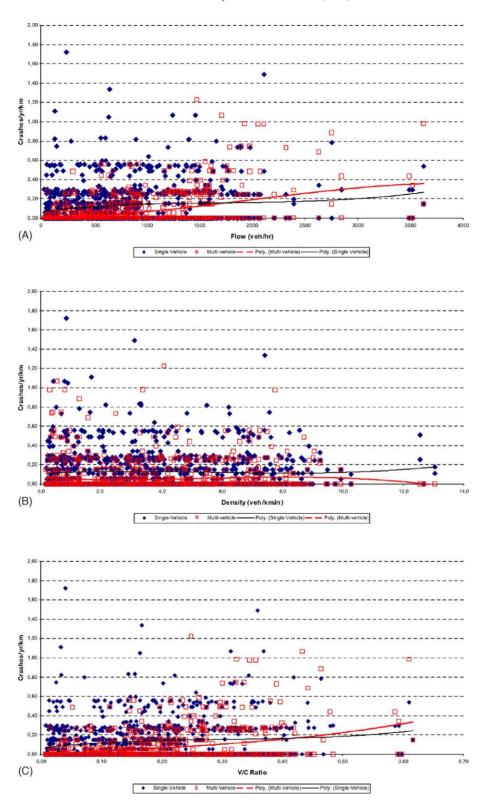


Fig. 3. Crash-traffic flow characteristics for urban segments. (A) Crash-flow relationship, (B) crash-density relationship, (C) crash-V/C ratio relationship.

where, $f(\cdot)$ is a function of traffic flow (F), segment length (L), vehicle density (D), and V/C ratio (X); β is a vector of unknown coefficients; and, e_{it} is a the model error dependent on the covariates (explained below) and with a serial correlation $COV(\alpha_t, \alpha_{t+1})$.

It is usually assumed that $\exp(e_{it})$ is independent and gamma distributed with a mean equal to 1 and a variance $1/\phi$ for all i and t (with $\phi > 0$). With this characteristic, it can be shown that Y_{it} , conditional on $f(\cdot)$ and ϕ , is distributed as a Negative Binomial random variable with a mean $f(\cdot)$ and a variance $f(\cdot)(1 + f(\cdot)/\phi)$ respectively. The term ϕ is usually defined as the "dispersion parameter" for the NB distribution.

In the present approach, it is proposed to estimate the variance of the mean (defined as σ_{it}^2) and dispersion parameter (defined as ϕ_{it}) for each site i and time t separately. Recent work has shown that ϕ , hence the variance, is not fixed across different sites and time periods (Heydecker and Wu, 2001; Miaou and Lord, 2003). This concept (known as generalized negative binomial) is not new and has been explored by others (e.g., see Taylor et al., 1978a, b; Famoye, 1997; Bebbington and Lai, 1998). The variance and dispersion parameter are estimated with the method proposed by Heydecker and Wu (2001). The variance of the mean is computed with Eq. (3).

$$\sigma_{it}^2 = \frac{\mu_{it}^2}{\phi_{it}} = f(L, F, D, X; \gamma) \exp(e'_{it})$$
 (3)

where, $f(\cdot)$ is a function of traffic flow (F), segment length (L), vehicle density (D), and V/C ratio (X); γ is a vector of unknown coefficients; and, is e'_{it} the model error that is normally distributed with a serial correlation $COV(\alpha_t, \alpha_{t+1})$.

The dispersion parameter is estimated by combining Eqs. (2) and (3) into the log-linear model shown in Eq. (4):

$$\phi_{it} = \exp((2\beta - \gamma)^T . x) \tag{4}$$

where, β is a vector of unknown coefficients (estimated with Eq. (2)); γ is a vector of unknown coefficients (estimated with Eq. (3)); and, x is a vector of explanatory covariates (L, F, D, X).

The modeling procedure shown above dictates that the mean μ_{it} , the variance σ_{it}^2 , and the dispersion between the mean crash frequencies ϕ_{it} vary systematically according to the covariates. This approach offers more flexibility since the variance is no longer dependent upon the mean, but on the covariates of the model. The procedure is estimated iteratively until Eqs. (2)–(4) converge (all coefficients remain unchanged). The equations are estimated sequentially in that order. This procedure offers the advantage that it can be easily used with commercially available statistical software packages. With the present choice of the Poisson–gamma relationship, it is important to point out that multiple local optima and unbounded solutions may exist.

An important characteristic associated with the development of statistical relationships is the choice of the functional form for the mean and the variance. For this work, three types of functional form were evaluated. A separate form was estimated for the crash-flow, crash-flow-density and crash-flow-V/C ratio relationships respectively. The three proposed functional forms are shown in Eqs. (5)–(7):

Crash-flow

$$\mu_{it} = \beta_0 L_i F_{it}^{\beta_1}, \quad \sigma_{it}^2 = \gamma_0 L_i F_{it}^{\gamma_1}$$
 (5)

Crash-flow-density

$$\mu_{it} = \beta_0 L_i F_{it} e^{(\beta_1 . D_{it})}, \quad \sigma_{it}^2 = \gamma_0 L_i F_{it} e^{(\gamma_1 . D_{it})}$$
 (6)

Crash-flow-V/C ratio

$$\mu_{it} = \beta_0 L_i F_{it} e^{(\beta_1 X_{it})}, \quad \sigma_{it}^2 = \gamma_0 L_i F_{it} e^{(\gamma_1 X_{it})}$$
 (7)

where, μ_{it} is the estimated number of crashes per year for site i and time t; σ_{it}^2 is the estimated variance for site i and time t; L_i is the length of section i in kilometers; F_{it} is the hourly volume for site i and time t; D_{it} is the density in vehicles per kilometer per lane for site i and time t; X_{it} is the V/C ratio for site i and time t; and, β_0 , β_1 , γ_0 , γ_1 are the coefficients to be estimated.

Eq. (5) represents the model form typically used for predicting crashes in the safety literature. This functional form shows that a non-linear relationship between crashes and traffic volume can exist since the coefficient for the parameter $F(\beta_1)$ is allowed to be different than one. In Eqs. (6) and (7), the volume parameter (or exposure) has been used as an offset rather than as a covariate. In order to establish the true effects of density and V/C ratio on the likelihood of a crash, the coefficient for the volume parameter has been fixed to one. This relationship indicates that the risk for each individual driver is solely a function of density and V/C respectively. In other words, for the same density and V/C ratio each vehicle is theoretically subjected to the same risk when the driver travels on a given freeway section. Predictive models that included parameters F, D and X simultaneously as covariates have been tested and are discussed in the next section. Finally, the segment length (L) has been used as an offset since the length is an exposure variable and cannot in itself be considered a cause-effect covariate (i.e., the length of a segment does not explain the risk of a crash).

The coefficients of the predictive models were estimated with Genstat (Payne, 2000). Since the data included repeated measurements (the observations are measured on the same site repeatedly, in this case 24 times), the Generalized Estimating Equation (GEE) procedure that handles temporal correlations (COV(α_t, α_{t+1})) has been used to this effect. A diagnosis, using the tools in Genstat, was conducted on the data and confirmed that a positive temporal correlation existed within variance-covariance matrix. It should be pointed out that since the data did not contain any missing values, the coefficients are actually the same as it they were estimated using a generalized linear modeling (GLM) framework. The only difference is related to the standard errors of the coefficients. The standard errors are usually underestimated when temporal effects are not included in the modeling framework (see Lord and Persaud, 2000; Hardin and Hilbe, 2003 for additional information).

Table 2 Safety performance functions for crash-flow relationship

	Rural				Urban		
	All crashes	Fatal + severe	Single-vehicle	Multi-vehicle	All crashes	Single-vehicle ^a	Multi-vehicle
Mean (μ)							
Intercept (β_0)	-5.133(0.405)	-7.170(0.723)	-4.647(0.371)	-10.330(0.964)	-2.035(0.999)	4.34 (0.750)	-8.264(0.527)
$Ln(Flow)(\beta_1)$	0.542 (0.071)	0.354 (0.111)	0.412 (0.061)	1.156 (0.158)	0.355 (0.125)	-0.660 (0.980)	1.085 (0.085)
Variance (σ^2)							
Intercept (γ_0)	-14.750(1.930)	-7.110(3.970)	-8.890(1.404)	-16.830(2.033)	-37.400(20.200)	-8.530(1.368)	-50.800 (10.300)
$Ln(Flow)(\gamma_1)$	2.075 (0.244)	0.073 (0.645)	1.055 (1.266)	2.157 (0.257)	4.750 (2.180)	-1.199(0.220)	6.310 (1.210)
Deviance	910.9	935.1	965.3	672.0	228.9	151.5	226.0
Pearson statistic	907.3	938.2	966.1	673.0	223.3	180.5	197.2
Deviance at in- tercept	575.3	1099.0	744.5	572.4	82.5	157.9	66.23
Observations	1152	1152	1152	144	144	144	144

^a This model is shown for demonstration purpose only. The model does not represent a logical relationship since the intercept does not pass through the origin.

The GEE procedure was modified to accommodate a modeling structure that incorporates a dispersion parameter that varies with the covariates. The coefficients were estimated iteratively until Eqs. (2)–(4) converged. The iterative process usually converged after three or four iterations. Different predictive models were estimated for the total number of crashes, severe + fatal, single-vehicle and multi-vehicle crashes respectively. There were not enough observations for developing a SPF for severe + fatal crashes for the urban section.

The statistical fit of models was measured through various tools, including the deviance and the Pearson statistic. However, since the predictive models are not nested models and the dispersion parameter is not constant for each model, traditional methods used to assess the fit of competitive statistical models cannot be used (i.e., *F*-test for sequential scaled-deviance, etc.) (McCullagh and Nelder, 1989; Payne, 2000; Myers et al., 2002). These limitations are also compounded with the low mean problem commonly found with crash data (Maycock and Hall, 1984; Maher and Summersgill, 1996; Wood, 2002); two data sets contained a sample mean below 0.5. Other methods, such as the cumulative residuals (CURE) (Hauer and Bamfo, 1997) and the deviance estimated with

the intercept (no covariate), were utilized in the evaluation process. In sum, given the limitations detailed above, the assessment of models had to be done subjectively using the combination of various tools.

Table 2 shows the estimated relationship between crashes and hourly traffic volume. All coefficients were significant at the 95% level, with the exception of the variance function (γ_1) for the rural fatal + injury model; there were not enough observations for this model to properly explain the variance relationship. The results of the modeling process indicate that the coefficient for the parameter F is usually below one for most models, with the exception of multi-vehicle crashes for both rural and urban sections. The predictive model for the single-vehicle crashes for the urban section is shown for demonstration purpose. This model does not represent a logical relationship since the intercept does not go through the origin, as it should (there was no observation with 0 crash at low volumes). Nonetheless, the relationship indicates that single-vehicle crashes strongly decrease as traffic volume increases, as seen in Fig. 3b.

Table 3 shows the results of the predictive models for the crash-flow-density relationships. Almost all coefficients in this table are significant at the 95% level. The coefficient

Table 3
Safety performance functions for crash-flow-density relationship

	Rural				Urban		
	All crashes	Fatal + severe	Single-vehicle	Multi-vehicle	All crashes	Single-vehicle	Multi-vehicle
Mean (μ)							
Intercept (β_0)	-7.435(0.129)	-10.416(0.258)	-7.552(0.141)	-9.465(0.149)	-6.875(0.188)	-7.209(0.642)	-7.726(0.055)
Density (β_1)	-0.209 (0.038)	-0.294 (0.061)	-0.294 (0.033)	0.056 (0.039)	-0.017 (0.007)	-0.077(0.067)	0.012 (0.002)
Variance (σ^2)							
Intercept (γ_0)	-8.194(0.295)	-12.150(1.530)	-9.079(0.442)	-9.237(0.185)	-6.344(0.990)	-4.972(0.336)	-24.77(8.080)
Density (γ_1)	0.264 (0.034)	-0.391(0.524)	0.163 (0.051)	0.152 (0.022)	0.023 (0.023)	-0.183(0.046)	0.414 (0.162)
Deviance	819.4	1112.0	1155.0	457.9	220.8	137.8	385.9
Pearson statistic	809.8	1114.0	1133.0	456.9	221.6	160.7	383.9
Deviance at in- tercept	968.5	1994.0	1595.0	219.2	127.4	363.9	71.6
Observations	1152	1152	1152	144	144	144	144

Table 4 Safety performance functions for crash-flow-V/C ratio relationship

	Rural				Urban		
	All	Fatal + severe	Single-vehicle	Multi-vehicle	All	Single-vehicle	Multi-vehicle
Mean (μ)							
Intercept (β_0)	-7.426(0.127)	-10.409(0.259)	-7.533(0.141)	-9.465(0.149)	-6.114(0.269)	-5.987(0.301)	-7.795(0.079)
V/C ratio (β_1)	-3.666 (0.808)	-5.116 (1.808)	-5.216 (0.713)	0.964 (0.680)	-1.957 (0.538)	-4.965 (0.690)	0.375 (0.134)
Variance (σ^2)							
Intercept (γ_0)	-8.164(0.272)	-12.140(2.076)	-9.054(0.412)	-9.235(0.185)	-8.720(3.120)	-4.602(0.440)	-14.220(1.820)
V/C ratio (γ_1)	4.532 (0.426)	-6.900(10.200)	2.824 (0.609)	2.623 (0.451)	4.490 (2.782)	-7.400(2.92)	10.120 (2.015)
Deviance	805.2	1111.0	1124.0	456.6	203.7	146.8	251.8
Pearson statistic	804.2	1110.4	1123.7	456.6	200.3	146.9	251.5
Deviance at in- tercept	968.5	1994.0	1595.0	219.2	127.4	363.9	71.6
Observations	1152	1152	1152	144	144	144	144

for vehicle density for the urban single-vehicle model was significant at 90% level (note: this coefficient is significant above 95% when the temporal effect is removed). Table 3 appears to indicate that as vehicle density increases the total number of crashes decreases (to be discussed below). The decrease is even more pronounced, as expected, for fatal + injury and single-vehicle crashes. However, the decrease is much less significant, if not almost non-existent, in the urban environment. There appears to be a linear relationship between density and multi-vehicle crashes particularly for the urban environment since the coefficient for the parameter *D* is almost zero in the exponential component of the model. In general, these results indicate that density affects the number of crashes on freeway segments.

Table 4 presents the characteristics of the crash-V/C ratio relationship. Similar to the results shown above, the total number of crashes, fatal + injury and single-vehicle crashes appear to decease as the V/C ratio increases, with the exception of multi-vehicle crashes. The decrease is less significant for the total number of crashes occurring on the urban sections

Overall, when one looks at the deviance and Pearson statistic, about half of predictive models for crash-flow-density and crash-flow-V/C ratio seem to offer a better fit than for crashflow relationships. On the other hand, when the predictive models are compared against the nominal model (intercept at zero), it can be seen that the inclusion of the covariate decreased the fit for most of the models that showed an increase in statistical fit as determined by the criteria discussed above (remember that one cannot use the F-Test to compare these models). The CURE method also showed similar characteristics as the other tools. Given the combined outcome of all the tools, it can be recognized that the models perform relatively equally. Thus, this indicates that the selection of the predictive models should not be solely based on the statistical fit (discussed below). In a practical matter, the use V/C ratio in the predictive model may be more useful since it is easier to compute than vehicle density.

The goodness of fit for each urban model was tested separately for both regimes (congested and non-congested) using

the tools described above. The analysis has shown that the predictive models performed equally well in both regimes. However, the data was subjected to a greater variation in the congested flow regime. In fact, most of the observations were found on either side of the predicted mean, rather than located near the mean. This may indicate that other traffic flow regimes could exist in the congested regime. It is possible that developing a distinct relationship (in Fig. 1) for congested and non-congested flow regimes separately may improve the fit of the predictive models. This analysis was unfortunately beyond the scope of the project. Additional discussion on this topic is presented in the next section.

The results shown in Tables 2–4 provide interesting characteristics that cannot be easily observed by only examining the coefficients of the predictive models. Thus, several plots were produced to explicitly examine the various relationships as estimated by the models. Figs. 4-6 illustrate the actual relationship for the total number of crashes (All), single- (SV) and multi-vehicle (MV), and the sum of single- and multivehicle crashes (SV + MV) as estimated separately by the predictive models respectively. It is important to point out that the curve for All, SV, and MV represents the average relationship between all the observations included in each category (i.e., observations lie on both sides of the curve). Thus, it is expected that the curve for SV + MV will not follow the same relationship as the one for the curve developed for all crashes since the former was not estimated from one single regression equation.

The crash-flow relationships indicate, as described previously, that the total number of crashes increases at a decreasing rate as traffic volume increases both for the rural and urban segments (Fig. 4). This relationship basically means that there are proportionally less crashes per passing vehicles (all types combined) as the traffic volume increases (β_1 < 1). Put simply, the crash risk diminishes when traffic volume increases. However, when one takes a closer look at the data and separates single- and multi-vehicle crashes, it can be seen that the likelihood of being involved in a multi-vehicle crash increases as the traffic volume increases. Based on these plots, the opposite is found for single-vehicle crashes, in which the

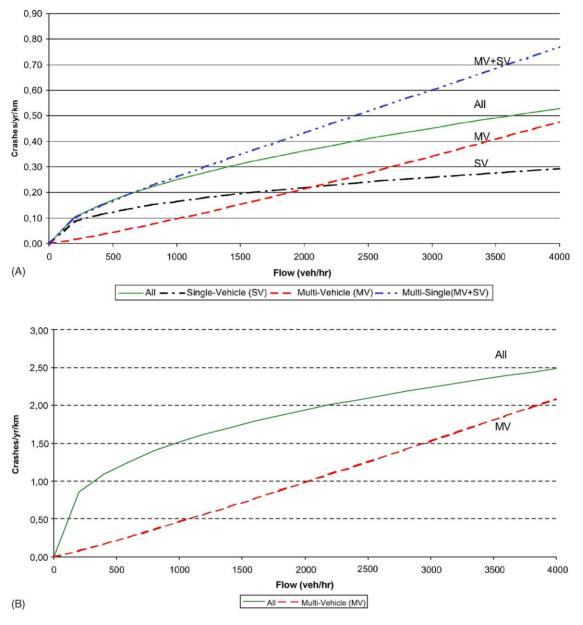


Fig. 4. Crash-flow relationships estimated from SPFs. (A) Rural segments, (B) urban segments.

accident risk decreases with traffic volume. It is worth mentioning that the curve representing the sum of single- and multi-vehicle crashes (MV + SV in the figure) for the rural section becomes almost linear between crashes and traffic flow.

The crash-flow-density relationships appear to exhibit similar characteristics to what was found for the crash-flow relationships (Fig. 5). However, when the traffic volume is incorporated into the predictive models, the number of crashes increases, peaks and decreases as vehicle density increases for all crashes and single-vehicle crashes. This relationship is more pronounced for rural segments than for urban segments. For the latter, the number of multi-vehicle crashes increases at a decreasing rate as vehicle density increases. This implies that, although more crashes occur with increased density,

there are proportionally more crashes per entering vehicle occurring at low density than at high density, similar to the characteristics exhibited in the crash-flow relationships.

The predictive models that estimate single-vehicle, multivehicle and the total number of crashes as a function of the V/C ratio share similar characteristics as the models developed for crash-flow-density relationship. For both rural and urban segments, the likelihood to be involved in multi-vehicle crash increases when the V/C ratio increases.

6. Discussion

The results of the analysis presented above raise a few important issues that merit further discussion. First, predic-

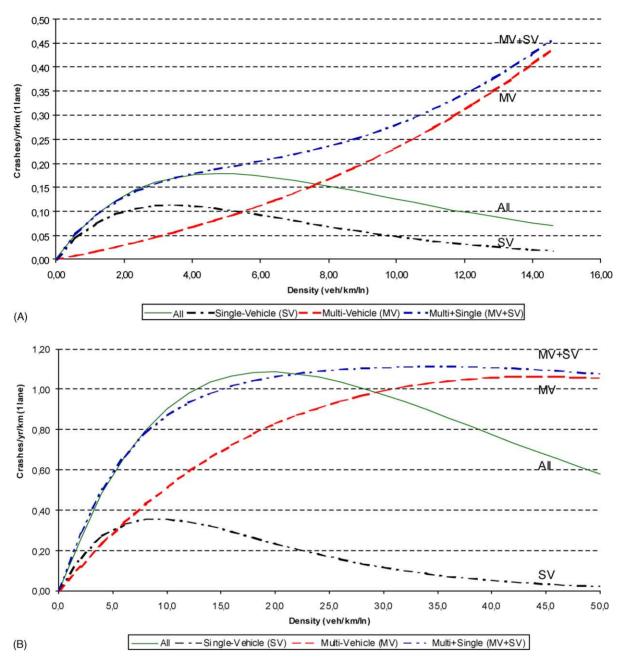


Fig. 5. Crash-density relationships estimated from SPFs. (A) Rural segments, (B) urban segments.

tive models developed solely from traffic volume may not adequately characterize the true nature of the accident process for a given freeway facility. If one looks at Fig. 4a, the relationship for single-vehicle crashes shows an increasing trend with increasing traffic volume (although at a decreasing rate), which implies that there are more single-vehicle crashes at higher than at lower traffic flow conditions. This contradicts the trend illustrated in Fig. 2a, which clearly shows a somewhat constant decreasing trend in crash count for single-vehicle crashes. This result is counterintuitive since there should be less single-vehicle crashes at high volumes. The same kind of unusual characteristic can be found for the predictive models estimated for the total num-

ber of crashes. In this case, individual crash risk (defined as crashes per vehicle) decreases with increasing traffic since $\beta_1 < 1$.

By incorporating vehicle density and the V/C ratio in the predictive models, one can see that a different picture emerges, as shown in Figs. 5 and 6 respectively. It can be seen that as vehicle density increases the probability of being involved in a single-vehicle crash increases, peaks and decreases, while at the same time, the probability of being involved in a multi-vehicle crash becomes more likely. These relationships, when an additional covariate is included in the model, are in accordance with what one would expect for single- and multi-vehicle crashes. However, this characteris-

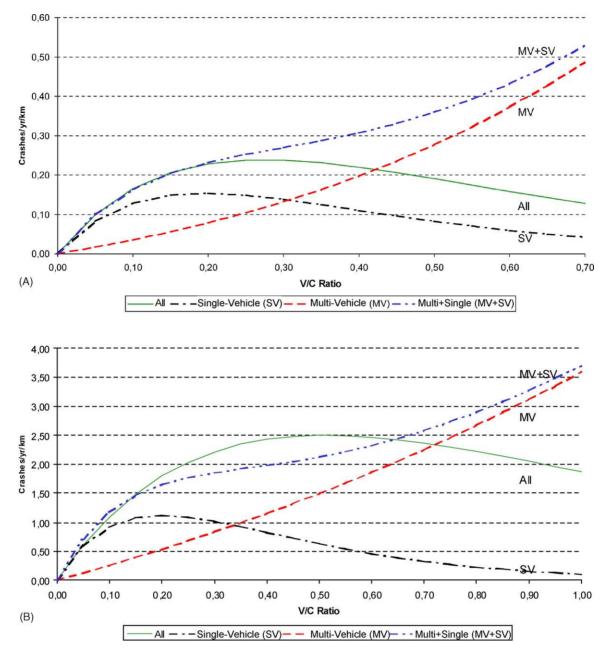


Fig. 6. Crash-V/C ratio relationships estimated from SPFs. (A) Rural segments, (B) urban segments.

tic cannot be seen for the predictive models for all crashes, which lead to the second issue described below.

The second issue is related to the development of one predictive model for estimating all crashes on freeway segments. The results indicate that developing a single predictive model is not adequate for predicting crashes on freeway segments. This argument is valid for the three types of predictive model. For instance, the predicted model used for all crashes shows the following characteristic: it initially increases, peaks and then decreases as the density or the V/C increases. Yet, it is clear that multi-vehicle crashes increase with increasing density or V/C ratio. The predictive models for all crashes show that the individual crash risk follows the same pattern.

As shown in the curve grouping single- and multi-vehicle crashes together (e.g., MV + SV), the crash risk is much higher at higher densities and V/C ratios. It would therefore be more appropriate to develop predictive models for single-and multi-vehicle crashes separately. This supports the modeling approach proposed by Mensah and Hauer (1998), in which different SPFs should be estimated where appropriate. However, the only divergence between their work and this one is related to the selection of the model form rather than the issue related to the bias caused by averaging traffic volume.

The third issue is associated with the selection of the appropriate functional form and the use of exposure. As dis-

cussed above, the functional form is a critical component for the development of any predictive models. Different predictive models will provide different estimations of safety and crash risk. Loader (1999) and Miaou and Lord (2003) reported that producing the best fit is no longer a challenge and that over-stretching the data can become problematic. The goal is therefore to find a balance between a logical relationship and predictive capabilities for the selected functional form. Recent work by Miaou and Lord (2003) for instance has shown that a selected functional form characterized by linear relationships between crashes and traffic volume provided a fit as good as the traditional functional form $Y = \alpha F_1^{\beta_1} F_2^{\beta_2}$ for predicting crashes at signalized intersections, but offered a more rationale description of the accident process at these locations.

To expand on the third issue, predictive models that simultaneously combined exposure, density, and V/C ratio as covariates (e.g., $\mu_{it} = \beta_0 L_i F_{it}^{\beta_1} e^{(\beta_2 D_{it})}$ or $\mu_{it} =$ $\beta_0 L_i F_{it}^{\beta_1} D_{it}^{\beta_2}$) were tested as part of this work. The results are not shown here, but although some predictive models provided a somewhat better statistical fit, many statistical relationships were found to be counterintuitive. For instance, a few models contained a negative coefficient for the traffic flow parameter when density or V/C ratios were used as covariates. In other instances, the traffic volume captured most of the variation, which indicates that density and V/C ratio did not play any role within the predictive model. Interestingly, some predictive models for multi-vehicle and the total number of crashes showed characteristics where crashes increased, peaked, and decreased as vehicle density or V/C ratio increased. In short, a better fit does not necessarily mean that the predictive model illustrates a better characterization of the data.

The last issue is related to the effects of different traffic flow regimes on modeling crash prediction models. Although different traffic flow regimes have an influence on the likelihood of a crash (Lee et al., 2002, 2003; Golob and Recker, 2003, 2004), it is unclear at this point how they would affect predictive models using the functional forms and the type of data presented herein. Most research performed on the relationship between traffic flow regimes and safety contained traffic flow data disaggregated into 30 s to 5 min time intervals (microscopic level). This kind of disaggregated data is obviously not compatible with the modeling framework used in this work. Nonetheless, further work is need to incorporated different traffic flow regimes into the predictive models illustrated in Eqs. (5)–(7).

In conclusion, the outcome of this analysis effort supports the work performed by Mahalel (1986) and Lord (2002) on the estimation of crash risk. Both have reported that the estimation of crash risk should not be solely based on traffic volume, but should incorporate other important variables such as vehicle density and the V/C ratio. Estimating the proper relationship between crashes and traffic flow characteristics is very important, as many ITS strategies and transportation policies often rely on predictive models for estimating vari-

ous safety effects. For instance, Maher et al. (1995) indicated that predictive models (solely based on traffic volume) used for minimizing the number of crashes on transportation networks often lead to empty links on the network. This would obviously be inapplicable in real life applications. The use of functional forms illustrated in Eqs. (6) and (7) combined with the use of predictive models for single- and multi-vehicle crashes would overcome this problem.

7. Further work

There are many avenues for further work. First, with the increased use of ITS for managing urban freeway segments, better and more accurate data on traffic flow characteristics should be used within the context of this work. It is suggested to monitor freeway segments on a permanent basis and record traffic flow characteristics, including the traffic flow regime, at the time the crash occurred, as opposed to use the average values; one should take another look the safety effects of speed distribution on freeways (Davis, 2002). Hughes and Council (1999), Lee et al. (2002, 2003) and Golob and Recker (2003, 2004) have already started examining such characteristics for predicting crash risk, on a microscopic level, for urban freeway segments. Second, serious thoughts should be given to determine new or improved functional forms. The model forms should incorporate the properties at the boundaries, as suggested by Miaou and Lord (2003); the boundary conditions are particularly important when a facility operates near capacity. Third, expand this work to other types of facilities such as arterial roads and intersections. Fourth, during the course of this work, it was noticed that some crashes occurred within the same time period (often within 15 min of each other) as a previous crash on the same segment. This characteristic implies that crashes may not be an independent process in every circumstance. Thus, additional work is suggested for determining the true independence between successive crash events.

8. Summary and conclusion

Traffic flow characteristics such as traffic volume, vehicle density, and the V/C ratio have a direct influence on the likelihood and severity of a crash. So far, the effects of these characteristics on safety have not been clearly established nor properly modeled for rural and urban freeway segments. The effort presented in this work attempted to shed additional light on these effects and, most importantly, how they are statistically related to the number of crashes using commonly existing predictive models or functional forms. A series of predictive models was developed for such purpose. In all, three functional forms were evaluated that incorporated traffic volume, vehicle density, and V/C ratio as explanatory covariates.

The results have shown that predictive models that use only traffic volume as a covariate may not capture adequately the characteristics of crashes on freeway segments. Hence, functional forms that include vehicle density and V/C ratio offer a better characterization of the crash process on these facilities. No matter which functional form is selected, a separate predictive model should be developed for single- and multi-vehicle crashes. The combination of both types of predictive models shows that accident risk and the number of crashes increases with higher vehicle density and V/C ratio. It is the hope that the effort put in this work will foster new approaches about how to establish statistical relationships between crashes and traffic flow characteristics.

Acknowledgements

The authors would like to express their gratitude to Prof. Ben Heydecker for providing additional information about his modeling approach. This paper benefited from the input of Prof. John N. Ivan, three TRB referees and two anonymous referees. Their comments were very well appreciated.

References

- Abbas, K.A., 2003. Traffic safety assessment and development of predictive models for accidents on rural roads in Egypt. Accident Analysis and Prevention, (in press).
- Bebbington, M.S., Lai, C.D., 1998. A generalized negative binomial and applications. Commun. Stat.: Theory Methods 27 (10), 2515–2533.
- Brodsky, H., Hakkert, A.S., 1983. Highway accident rates and rural travel densities. Accid. Anal. Prev. 15 (1), 73–84.
- Cedar, A., Livneh, M., 1982. Relationship between road accidents and hourly traffic flows-I and II. Accid. Anal. Prev. 14 (1), 19–44.
- Coifman, B., 2001. Traffic data measurement and validation UCB-ITS-PRR 40. In: California PATH Research Report. Berkley, California.
- Coifman, B., 2003. estimating density and lane inflow on a freeway segment. Submitted to Transport. Res.: Part A 37 (8), 689–701.
- Coifman, B., Cassidy, M., 2002. Vehicle reidentification and travel time measurement on congested freeways. Transport. Res.: Part A 36 (10), 899–917.
- Davis, G.A., 2002. Is the claim that "variance kills" an ecological fallacy. Accid. Anal. Prev. 34 (3), 343–346.
- Famoye, F., 1997. Parameter estimation for generalized negative binomial distribution. Commun. Stat.: Simulation. 26 (1), 269–279.
- Frantzeskakis, J.M., Iordanis, D.I., 1987. Volume-to-capacity ratio and traffic accidents on interurban four-lane highways in Greece. Transport. Res. Record 1112, 29–38.
- Garber, N.J., Gadiraju, R., 1989. Factors affecting speed variance and its influence on accidents. Transport. Res. Record 1213, 64–71.
- Garber, N.J., Subramanyan, S., 2002. Incorporating crash risk in selecting congestion-mitigation strategies: Hampton Roads area (Virginia) case study. Transport. Res. Record 1746, 1–5.
- Golob, T.F., Recker, W.W., 2003. Relationships among urban freeway accidents, traffic flow, weather, and lighting conditions. J. Transport. Eng. 129 (4), 342–353.
- Golob, T.F., Recker, W.W., 2004. A method for relating type of crash to traffic flow characteristics on urban freeways. Transport. Res. Part A: Policy Practice 38 (1), 53–80.
- Gwynn, D.W., 1967. Relationship of accident rates and accident involvements with hourly volumes. Traffic Quarter., 407–418.

- Hadi, M., Aruldhas, J., Chow, L.-F., Wattleworth, J.J.A., 1995. Estimating safety effects of cross-section design for various highway types using negative binomial regression. Transport. Res. Record 1500, 169–176.
- Hall, J.W., Pendleton, O.J., 1989. Relationship between V/C ratios and accident rates, FHWA Report HPR-NM-88-02, US Department of Transportation.
- Hardin, J.W., Hilbe, J.M., 2003. Generalized Estimating Equations. CRC Press Company, Boca Raton, Florida.
- Hauer, E., 1997. Observational Before-After Studies in Road Safety: Estimating the Effect of Highway and Traffic Engineering Measures on Road Safety. Pergamon, Elsevier Science Ltd, Oxford, UK.
- Hauer, E., Bamfo, J., 1997. Two tools for finding what function links the dependent variable to the explanatory variables. In: Proceedings of the ICTCT 1997 Conference. Lund. Sweden.
- Heydecker, B.G., Wu, J., 2001. Identification of sites for road accident remedial work by bayesien statistical methods: an example of uncertain inference. Adv. Eng. Software 32, 859–869.
- Huang, Y., Cayford, R., May, A.D., 1992. Accident prediction models for freeway segments, Presented at the 71st Annual Meeting of the Transportation Research Board, Washington, DC.
- Hughes, R., Council, F., 1999. On establishing the relationship(s) between freeway safety and peak period operations: performance measurement and methodological considerations, Presented at the 78th Annual Meeting of Transportation Research Board, Washington, DC.
- Knuiman, M.W., Council, F.M., Reinfurt, D.W., 1993. Association of median width and highway accident rate (with discussion and closure). Transport. Res. Record 1401, 70–82.
- Lee, C., Hellinga, B., Saccomanno, F., 2003. Real-time crash prediction model for the application to crash prevention in freeway traffic (Paper No. 03-2749), Presented at the 82nd Annual Meeting of the Transportation Research Board, Washington, DC, Transport. Res. Record 1840, 67–77.
- Lee, C., Saccomanno, F., Hellinga, B., 2002. Analysis of crash precursors on instrumented freeways. Transport. Res. Record 1784, 1–8.
- Loader, C., 1999. Local Regression and Likelihood. Spinger-Verlag, New York.
- Lord, D., 2002. Issues related to the application of accident prediction models for the computation of accident risk on transportation networks. Transport. Res. Record 1784, 17–26.
- Lord, D., Persaud, B.N., 2000. Accident prediction models with and without trend: application of the generalized estimating equation (GEE) procedure. Transport. Res. Record 1717, 102–108.
- Mahalel, D., 1986. A note on accident risk. Transport. Res. Record 1068, 85, 80
- Maher, M.J., Hughes, P.C., Smith, M.J., Ghali, M.O., 1993. Accidentand travel time-minimising routing patterns in congested networks. Traffic Eng. Contr. 34 (9), 414–419.
- Maher, M.J., Summersgill, I., 1996. A comprehensive methodology for the fitting predictive accident models. Accid. Anal. Prev. 28 (3), 281–296.
- Martin, J.-L., 2002. Relationship between crash rate and hourly traffic flow on interurban motorways. Accid. Anal. Prev. 34, 619–629.
- Maycock, G., Hall, R.D., 1984. Accidents at 4-arm roundabouts, TRRL Laboratory Report 1120, Transportation and Road Research Laboratory, Crowthorne, Bershire.
- McCullagh, P., Nelder, J.A., 1989. Generalized Linear Models. Chapman and Hall, London, UK.
- Mensah, A., Hauer, E., 1998. Two problems of averaging arising in the estimation of the relationship between accidents and traffic flow. Transport. Res. Record 1635, 37–43.
- Miaou, S.-P., Lord, D., 2003. Modeling traffic crash-flow relationships for intersections: dispersion parameter, functional form, and bayes versus empirical bayes. Transport. Res. Record. 1840, 31–40.
- Myers, R.H., Montgomery, D.C., Vinning, G.G., 2002. Generalized Linear Models: with Application in Engineering and Sciences. Wiley, New York
- Payne, R.W. (Ed.), 2000. The Guide to Genstat. Lawes Agricultural Trust, Rothamsted Experimental Station. Oxford, UK.

- Persaud, B.N., 1992. Road Safety—A Review of the Ontario Experience and Relevant Work Elsewhere, PAV-92-02, Ministry of Transportation of Ontario, Toronto.
- Persaud, B.N., Dzbik, L., 1993. Accident prediction models for freeways. Transport. Res. Record 1401, 55–60.
- Persaud, B.N., Bahar, G., 2000. Applications of safety performance functions in the management of highway safety. In: Proceedings of the third Transportation Specialty Conference of the Canadian Society for Civil Engineering, London, Ontario, pp. 99–105.
- Persaud, B.N., Look, H., 2000. Relating safety to capacity to level of service for rural two-lane roads. In: Proceedings of the third Transportation Specialty Conference of the Canadian Society for Civil Engineers, London, Ontario, pp. 131–136.
- Persaud, B.N., Nguyen, T., 1998a. Safety consideration and capacity analysis. In: Processings of the third International Symposius on Highway Capacity, Copenhagen, Denmark, pp. 817–831.
- Persaud, B.N., Nguyen, T., 1998b. Disaggregate safety performance models for signalized intersections on ontario provincial roads. Transport. Res. Record 1635, 113–120.
- Roess, R.P., McShane, W.R., Prassas, E., 1998. Traffic Eng., second ed. Prentice Hall, Upper Saddle River, New Jersey.
- Schoen, J.A., May, A.D., Reilley, W., Urbanik, T., 1995. Speed-Flow Relationships for Basic Freeway Segments. NCH 3-45. JHK Associates, Transportation Research Board, Washington, DC.

- Taylor, L.R., Woiwod, I.P., Perry, J.N., 1978a. The density-dependence of spatial behavior and the rarity of randomness. J. Anim. Ecol. 47 (1), 383–406.
- Taylor, L.R., Woiwod, I.P., Perry, J.N., 1978b. The negative binomial as a dynamic ecological model for aggregation, and the density dependence of k. J. Anim. Ecol. 48 (1), 289–304.
- Tanner, J.C., 1953. Accidents at rural three-way junctions. J. Inst. Highway Eng. 2 (11), 56-67.
- TRB, 1992. Traffic Flow Theory, Update to the TRB Special Report 165, Washington, DC, (document accessed at http://www.tfhrc.gov/its/tft/tft.htm on 26 October, 2003), in press.
- TRB, 2001. Highway Capacity Manual, Transportation Research Board, Washington, DC.
- Turner, D.J., Thomas, R., 1986. Motorway accidents: an examination of accident totals, rates and severity and their relationship with traffic flow. Traffic Eng. Contr. 27 (78), 377–387.
- Vizioli A., Manar, A., 2002. Analyse macroscopique et microscopique de la relation entre le taux d'accidents et la densité, Presented at the 37th Annual Meeting of the AQTR, Quebec City, Quebec, (in French).
- Wood, G.R., 2002. Generalized linear accident models and goodness of fit testing. Accid. Anal. Prev. 34, 417–427.
- Zhou, M., Sisiopiku, V.P., 1997. Relationship between volume-to-capacity ratios and accident rates. Transport. Res. Record 1581, 47–52.