Examining Application of Aggregated and Disaggregated Poisson–Gamma Models Subjected to Low Sample Mean Bias

Dominique Lord and Maneesh Mahlawat

Two general classes of models have been proposed for modeling crash data: disaggregated (both with and without time trend) and aggregated models. Poisson–gamma models have traditionally been used under both of these model classes. As documented in previous studies, data sets characterized by small sample size and low mean values can significantly affect the performance of Poisson–gamma models, particularly those related to the estimation of the inverse dispersion parameter. Thus, guidance is needed on when to use aggregated models instead of disaggregated models as a function of the sample size and the sample mean value. The objective of this study was to estimate the conditions in which aggregated models (with a higher mean but a smaller sample size) could provide a more reliable estimate of the inverse dispersion parameter than disaggregated models (with a lower sample mean value but a larger sample size) or vice versa. To accomplish this objective, several simulation runs were performed for different values describing the mean, the sample size, and the inverse dispersion parameter. The simulation scenarios represented cases where 3, 5, and 10 years of data were available. To help illustrate the proposed guidance, aggregated and disaggregated models were estimated with crash data collected from four-lane rural highways in Texas. This paper provides guidelines about which model classes are more reliable as a function of the sample mean values, the sample size, and the amount of dispersion observed in the raw data.

Two of the most common probabilistic models used to model motor vehicle crashes remain the Poisson model and the related Poisson–gamma (or negative binomial) model. Since crash data have usually been shown to exhibit overdispersion (*1*)—meaning that the variance is greater than the mean—it is more suitable to use Poisson–gamma distributions than Poisson distributions when developing crash prediction models. The Poisson–gamma distribution offers a simple way to accommodate overdispersion, especially because the final equation has a closed form and because the mathematics used to manipulate the relationship between the mean and variance structures are relatively simple (*2*).

When safety analyses are conducted, data on crashes and related topics such as traffic flow and geometric-design features of highways are often collected at only a limited number of sites. This is caused by the potentially prohibitive expense of collecting these kinds of data (*3*). One way to increase the sample size is by collecting these data over many years, albeit at a limited number of sites (*2*). This method is usually more cost-effective because it requires fewer potential site visits and therefore limits the costs associated with those visits. One drawback to this method, however, is the possible difficulty in determining whether a sampling site's characteristics remain unchanged during the data collection period.

During the past few years, two general classes of models, aggregated and disaggregated, have been proposed for modeling crash data using such data sets. For aggregated models, crash data are usually summed over a specified time period and variables such as traffic flow are averaged over the same period. In this model class, the time period (e.g., the number of years or months) is used as an offset, which makes the model output the number of crashes for that given time period (*4*). For disaggregated models, each year (or time period *t*) is defined as a distinct observation. Disaggregated models can be developed using a different intercept (or as a covariate depending on the functional form of the model) to capture changes that vary over time (*3, 5–7*), or they can be developed with a single intercept. In the former, the models are often referred to as models with trend (or time trend), whereas the other category of models is known as a model without trend. Disaggregated models with trend have been shown to provide better goodness-of-fit statistical properties than do both disaggregated models without trend and aggregated models. (*6, 8*). In addition, time trend models can provide useful information about the trends in crashes or about year-to-year variations, including sudden drops or increases (e.g., in the number of reportable accidents) that may have been be unknown to or unidentified by the transportation safety analyst. An important characteristic associated with disaggregated models (with and without trend) is that the data are considered repeated measurements: each site is observed several times during the study period. Thus, repeated measurements are subjected to serial correlation. To account for the extra variation associated with this serial correlation, there is a need for estimating methods such as the generalized estimating equations (GEE) (*6, 9, 10*) and random effects models (*11, 12*). If these methods are not used, the standard error associated with the estimated coefficients may be underestimated (*6*).

As reported in previous research on highway safety (and elsewhere), Poisson–gamma models have been shown to become highly unstable, to provide a biased estimate of the inverse dispersion parameter, or to give unreliable goodness-of-fit statistics when they are estimated using data characterized by small sample size and low sample mean values (*13–17*). Given the high costs associated with collecting crash

D. Lord, Zachry Department of Civil Engineering, Texas A&M University, 3136 TAMU, College Station, TX 77843-3136. M. Mahlawat, Baez Consulting, LLC, 4925 Greenville Avenue, Suite 1300, Dallas, TX 75206. Corresponding author: D. Lord, d-lord@tamu.edu.

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data at multiple sites in conjunction with the possible biases from data characterized by small sample sizes and low sample mean values, it has not been investigated whether it is better to use aggregated models in which the sample mean is higher but the sample size is smaller, or to use disaggregated models in which the sample mean is lower but the sample size is larger.

The objective of this study was to determine the conditions under which aggregated models (with a higher mean but a smaller sample size) provide a more reliable estimate of the inverse dispersion parameter than disaggregated models (with a lower sample mean value but a larger sample size), and vice versa. To accomplish this objective, several simulation runs were estimated for different values describing the sample mean value, the sample size, and the inverse dispersion parameter. The simulation scenarios represented cases where 3, 5, and 10 years of data were available. To help illustrate the proposed guidance, this paper provides a description of an example application that compares aggregated and disaggregated models estimated from crash data collected from four-lane rural highways in Texas.

This paper is divided into five sections. The first section provides background on the characteristics of Poisson–gamma models and the issues posed by low sample mean values and small sample size. The second section describes the methodology used for simulating the data. The third section presents the results of the simulation analysis. The fourth section describes the example application. The last section summarizes key findings and offers ideas for further research.

BACKGROUND

Poisson–gamma models in highway safety applications have been shown to have the following probabilistic structure: the number of crashes at the *i*th entity (e.g., highway segment, intersections) and *t*th time period, Y_{it}, when conditional on its mean μ_{it}, is assumed to be Poisson distributed and independent over all entities and time periods as

$$Y_{it}|\mu_{it} \sim Po(\mu_{it}) \qquad i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \tag{1}$$

where *Po* is the Poisson variable with mean μ_{it}.

The mean of the Poisson is structured as

$$m_{it} = f(X;\beta)\exp(e_{it}) \tag{2}$$

where

$f(.)$ = function of covariates (X),
β = vector of unknown coefficients, and
e_{it} = model error independent of all the covariates.

It is usually assumed that $\exp(e_{it})$ is independent and gamma distributed with a mean equal to 1 and a variance $1/\phi$ for all i and t (with $\phi > 0$). With this characteristic, it can be shown that Y_{it}, conditional on $f(.)$ and ϕ, is distributed as a Poisson–gamma random variable with a mean $f(.)$ and a variance $f(.)(1 + f(.)/\phi)$ respectively. [Other variance functions exist for Poisson–gamma models, but they are not covered here since they are seldom used in highway safety studies. The reader is referred to Maher and Summersgill (*13*) and Cameron and Trivedi (*18*) for a description of alternative variance functions.] The probability density function of the Poisson–gamma structure described above is given by the following equation:

$$f(y_{it};\phi,\mu_{it}) = \binom{y_{it}+\phi-1}{\phi-1}\left(\frac{\phi}{\mu_{it}+\phi}\right)^{\phi}\left(\frac{\mu_{it}}{\mu_{it}+\phi}\right)^{y_{it}} \tag{3}$$

where

y_{it} = response variable for observation i and time period t,
μ_{it} = mean response for observation i and time period t, and
ϕ = inverse dispersion parameter of the Poisson–gamma distribution.

The term ϕ is usually defined as the inverse dispersion parameter of the Poisson–gamma distribution. [In the statistical and econometric literature, $\alpha = 1/\phi$ is usually defined as the dispersion parameter; in some published documents, the variable α has also been defined as the overdispersion parameter. However, according to Saha and Paul, "since α can take a positive as well as a negative value ($\alpha > -1/\mu$), it is called a dispersion parameter rather than an overdispersion parameter" (*19,* p. 179). As discussed in Lord and Miranda-Moreno (*16*), this formulation can no longer be referred to as a Poisson–gamma model because the parameter of the gamma distribution cannot be negative.] Traditionally, this term has been assumed to be fixed and a unique value applied to the entire data set in the study, but recently the term has been found to be dependent on the covariates (*20–22*). As has been described, the inverse dispersion parameter plays an important role in safety analyses, including in the computation of the weight factor for the empirical Bayes (EB) method (*2, 8*) and in the estimation of confidence intervals around the gamma mean and the predicted values of models applied to a new data set not used for the model development (*23, 24*).

Many researchers have examined the reliability (or bias) of the various estimators of the dispersion parameter under different scenarios. These estimators have included the method of moments (MM), the weighted regression (WR), and the maximum likelihood (ML) methods. In the past, all research studies have used simulations to examine the reliability of these methods. Table 1 summarizes the results of the key studies. The table shows that while earlier research studies mainly focused on the affect of small sample sizes on the performance of the ML estimator (MLE) and compared those results to those from other estimators, more recent studies have examined more extreme cases of sample mean values and overdispersion. For example, Lord (*15*) examined the effects of a very low sample mean ($\mu < 1$) combined with a small sample size on the estimation of the dispersion parameter. Crash data used for developing negative-binomial regression models often have a sample mean below 1. In another example, Lloyd-Smith (*25*) explored the bias, precision, and confidence interval coverage of the ML estimation of the dispersion parameter when these data are highly overdispersed ($\phi < 1$). (Highly dispersed data are commonly found in epidemiological studies.)

Although these researchers have come from a wide range of fields, their conclusions have much in common. First, when the data set is characterized by a small sample size, the MLE and the MM are less accurate than other estimators such as the quasi-likelihood method. Second, small sample sizes tend to overestimate the true dispersion parameter under all conditions. Third, the bias for the MLE gets larger as the sample mean decreases and the true ϕ increases (if known).

TABLE 1 Summary of Previous Studies of Estimators of Dispersion Parameters

Author (year)	Estimators	Results
Clark and Perry, 1989 (*26*)	MM, MQL[a]	Both become biased when $\mu \leq 3$, $n < 20$. Bias becomes worse when $\phi \rightarrow \infty$.
Piegorsch, 1990 (*27*)	MM, MQL, ML	ML is less accurate than MQL and MM when n is small. α was allowed to have negative values.
Dean, 1994 (*28*)	MM, ML	ML produces a biased estimate as n decreases and ϕ increases. The bias influences coefficients of NB model.
Toft et al., 2006 (*29*)	ML	Estimator is unstable (even for $\mu = 10$ and $n = 100$) as $\alpha \rightarrow 0$ (i.e., $\phi \rightarrow \infty$).
Lord, 2006 (*15*)	MM, WR, ML	All three estimators are biased and skewed when n and μ are small. ML method overestimates true dispersion parameter.
Lloyd-Smith, 2007 (*25*)	ML	MLE becomes more biased and less precise for higher ϕ. MLEs are not biased downward by any of the factors considered.

[a]Maximum quasi-likelihood estimator.

Unless the sample size is sufficiently large, bias with the MLE is inevitable. Recently, researchers have started to consider the potential for misestimating the inverse dispersion parameter in the development of regression models (*16, 19, 30*).

METHODOLOGY

This section describes the methodology used for simulating the data. The first part covers the three estimators used in this study. The second part provides details about the characteristics of the simulation analysis.

Dispersion Parameter Estimators

This study evaluated the three most common estimators (the MM estimator, the WR analysis estimator, and the MLE) used in highway safety studies for estimating inverse dispersion parameters. These estimators are the same ones used in the study by Lord (*15*) and are therefore briefly described here. To be consistent with previous work in highway safety, the results are presented for the inverse dispersion parameter $\phi = 1/\alpha$.

MM Estimator

With the MM estimator, the dispersion parameter is estimated using the method of moments. With this method, the value of dispersion parameter is used as an input to an iterative regression analysis, which is repeated until all the values (dispersion parameter, coefficients, etc.) converge. The estimator has usually been found to converge after a single iteration (*15*). The estimator is given by the following equation:

$$\hat{\alpha} = \frac{1}{(n-p)} \sum_{i=1,t=1}^{n,m} \frac{\left\{(y_{it} - \hat{\mu}_{it})^2 - \hat{\mu}_{it}\right\}}{\hat{\mu}_{it}^2} \tag{4}$$

where

n = sample size,
p = number of parameters,
m = number of time periods (for aggregated models, $m = 1$),
y_{it} = ith observation at time t, and
$\hat{\mu}_{it}$ = estimator of the population mean μ_{it}.

The MM estimator is being used less frequently, since most commercial statistical software packages use the maximum likelihood estimation method described below. The MM and WR (described next) can be used when models are recalibrated from one jurisdiction to another (*3*).

WR Analysis Estimator

With the WR estimator, the dispersion parameter is estimated using a weighted regression analysis, as proposed by Cameron and Trivedi (*31*). Although the functional form of the WR analysis estimator is similar to that with the MM estimator, the actual crash count is subtracted from the square of the difference between observed and predicted values. One advantage of this method is that confidence intervals or standard errors can be directly computed from the output of the regression analysis. The second estimator is given by the following equation:

$$\frac{(y_{it} - \hat{\mu}_{it})^2 - y_{it}}{\hat{\mu}_{it}} = \alpha\hat{\mu}_{it} + \epsilon \tag{5}$$

where ϵ is the error associated with the regression model or estimator.

Maximum Likelihood Estimator

With the MLE, the dispersion parameter is calculated using the ML method (*32*). The Newton–Raphson (NR) algorithm can be used to find the values of the log likelihood function. The NR algorithm can also be used to build confidence intervals associated with the estimator (*16*). The log likelihood function used is the following:

$$\ell(\alpha, \hat{\mu}_{it}) = \sum_{i=1,t=1}^{n,m} \left(\sum_{j=0}^{y_i - 1} \ln\{1 + \alpha j\} + y_{it} \ln\{\hat{\mu}_{it}\} - (y_{it} + \alpha^{-1}) \ln\{1 + \alpha\hat{\mu}_{it}\} \right) \tag{6}$$

The gradient elements of new log likelihood function are defined as follows:

$$\nabla_{\mu}\ell = \frac{y_{it}}{\hat{\mu}_{it}} - \frac{1+\alpha y_{it}}{1+\alpha\hat{\mu}_{it}} \tag{7a}$$

$$\nabla_{\alpha}\ell = \sum_{i=1,t=1}^{n,m}\left(\sum_{j=0}^{y_{it}-1}\frac{j}{1+\alpha j} + \alpha^{-2}\ln\left\{1+\alpha\hat{\mu}_{it}\right\} - \frac{\hat{\mu}_{it}\left(y_{it}+\alpha^{-1}\right)}{1+\alpha\hat{\mu}_{it}}\right) \tag{7b}$$

With the gradients, the NR scoring algorithm can be used to find the values of the log likelihood function through the ML method.

Simulation Framework

This section describes the simulation framework used to estimate the inverse dispersion parameter of the models using both disaggregated and aggregated data. The simulation was performed using a mixed distribution where sample mean values and count data were simulated successively. The simulation runs consisted of simulating Poisson–gamma distributions using a fixed sample population mean and covered different sample sizes and sample mean values that have been found to cause important biases in the estimation of the inverse dispersion parameter (*15*): $\mu = 0.5, 1.0$; $\phi = 0.5, 1, 2$; and $n = 50$.

Table 2 summarizes the scenarios and direct comparisons between the different simulation runs. For instance, Comparison 1 consisted of evaluating disaggregated and aggregated models as follows: if 3 years of data are available, the highway safety analyst can develop either an aggregated model where the sample mean is 1.5 crashes per 3 years (and use 3 years as an offset) with a sample size equal to 50, or a disaggregated model with a sample mean equal to 0.5 crashes per year with a (larger) sample size equal to 150 (3 years of data × 50 observations or sites). (Again, one could use a single or multiple intercepts.) Using an offset for aggregated models does not change the sample mean of the data. In fact, the offset only affects the value of the intercept term β_0. This has actually been validated in this study.

The same algorithm used by Lord (*15*) was employed for this study. The simulation was carried out using Genstat (*33*). The algorithm used for simulating the discrete counts is described below (defined as one simulation sample or run):

1. Generate a mean value ρ_k for observation k from a fixed sample population mean λ:

$$\rho_k = \lambda$$

2. Generate a value δ_k from a gamma distribution with mean equal to 1 and parameter ϕ:

$$\delta_k \sim \text{gamma}\left(\phi, \frac{1}{\phi}\right)$$

3. Calculate the mean μ_k for the observation k:

$$\mu_k = \rho_k \times \delta_k$$

4. Generate a discrete value Y_k for observation k from a Poisson distribution with mean μ_k:

$$Y_k \sim \text{Poisson}\left(\hat{\mu}_k\right)$$

5. Repeat these four steps 50 times, summarize the data, and compute the measures of effectiveness (MOEs).

The simulation samples were used to estimate a value for the inverse dispersion parameter ϕ by using randomly generated values derived from the Poisson–gamma distribution. The various scenarios were compared using the following MOEs: the coefficient of variation (CV), the square of the bias (Bias_Sqr), and the mean squared error (MSE). For comparison purposes, a smaller value of the MSE usually indicates a better or more reliable estimate. The MSE is equal to the Bias_Sqr and square of the standard deviation (SD) (or the variance). It is represented by Equation 8:

$$\text{MSE}\left(\hat{\phi}\right) = \left(\text{bias}\left(\hat{\phi}\right)\right)^2 + \left(\text{SD}\left(\hat{\phi}\right)\right)^2 \tag{8}$$

where $\text{bias}\left(\hat{\phi}\right) = E\left(\hat{\phi}\right) - \phi$ and $\text{SD}\left(\hat{\phi}\right) = \sqrt{\text{Var}\left(\hat{\phi}\right)}$.

The term $\text{Var}(\hat{\phi})$ is the variance of the simulated values for a particular estimator. This value was calculated for each simulation sample (50 simulation samples). The expected value $E(\hat{\phi})$ is the average (mean) value of ϕ of a given simulation sample.

TABLE 2 Scenarios Used for Simulation

		3 Years		5 Years		10 Years	
		$\phi = 0.5, 1.0, 2.0$		$\phi = 0.5, 1.0, 2.0$		$\phi = 0.5, 1.0, 2.0$	
Comparison	Model Class	Sample Mean	Sample Size	Sample Mean	Sample Size	Sample Mean	Sample Size
1	Disaggregated	0.5[a]	150	0.5[a]	250	0.5[a]	500
	Aggregated	1.5[b]	50	2.5[c]	50	5.0[d]	50
2	Disaggregated	1.0[a]	150	1.0[a]	250	1.0[a]	500
	Aggregated	3.0[b]	50	5.0[c]	50	10.0[d]	50

NOTE: The assumption used for the simulation runs is that the long-term mean is constant for the entire time period. However, even if the long-term mean varies for every year, the results will be the same as illustrated in the example application described in the next section.
[a]Crashes/year
[b]Crashes/3 years
[c]Crashes/5 years
[d]Crashes/10 years

TABLE 3 Simulation Results for 3-Year Period

	Comparison 1						Comparison 2					
	Disaggregated (*n* = 150)			Aggregated (*n* = 50)			Disaggregated (*n* = 150)			Aggregated (*n* = 50)		
	λ = 0.5 (crashes/year)			λ = 1.5 (crashes/3 years)			λ = 1.0 (crashes/year)			λ = 3.0 (crashes/3 years)		
	MM	WR	ML	MM	WR	ML	MM	WR	ML	MM	WR	ML
ϕ = 0.5												
Mean[a]	0.6412	0.6455	0.6087	0.6765	0.6904	0.6054	0.5739	0.5777	0.5316	0.5965	0.6087	0.5499
Min.	0.2234	0.2249	0.2935	0.2824	0.2881	0.2251	0.3369	0.3392	0.3648	0.2878	0.2936	0.3329
Max.	1.7070	1.7180	2.1250	1.4410	1.4700	1.0860	1.5580	1.5680	1.0790	1.1620	1.1860	1.0200
SD	0.3153	0.3173	0.3192	0.2671	0.2726	0.1884	0.2047	0.2060	0.1386	0.2175	0.2220	0.1588
Bias_Sqr	0.0199	0.0212	0.0118	0.0312	0.0362	0.0111	0.0055	0.0060	0.0010	0.0093	0.0118	0.0025
MSE	0.1193	0.1219	0.1137	0.1025	0.1105	0.0466	0.0473	0.0485	0.0202	0.0566	0.0611	0.0277
CV	0.4916	0.4916	0.5245	0.3948	0.3948	0.3112	0.3566	0.3565	0.2607	0.3645	0.3646	0.2888
ϕ = 1												
Mean[a]	1.5314	1.5415	1.4562	1.4198	1.4487	1.3013	1.1460	1.1536	1.0779	1.2226	1.2475	1.1551
Min.	0.4308	0.4337	0.4147	0.5412	0.5523	0.5863	0.4306	0.4335	0.4438	0.4605	0.4699	0.6300
Max.	10.9500	11.0200	8.9220	3.5220	3.5940	3.0900	2.6320	2.6500	2.7470	2.4250	2.4740	2.3120
SD	1.6044	1.6147	1.3595	0.7174	0.7320	0.6118	0.4254	0.4283	0.3824	0.4834	0.4932	0.3875
Bias_Sqr	0.2824	0.2933	0.2082	0.1762	0.2014	0.0908	0.0213	0.0236	0.0061	0.0495	0.0612	0.0241
MSE	2.8566	2.9007	2.0564	0.6908	0.7372	0.4651	0.2023	0.2070	0.1523	0.2832	0.3045	0.1742
CV	1.0477	1.0475	0.9336	0.5053	0.5053	0.4702	0.3712	0.3712	0.3548	0.3954	0.3954	0.3355
ϕ = 2												
Mean[a]	3.6357	3.3599	3.5608	3.1013	3.1646	3.1151	2.4670	2.4835	2.4997	2.6501	2.7042	2.4458
Min.	0.6265	0.6308	0.6598	0.7386	0.7536	0.6605	1.0970	1.1050	1.1160	0.9306	0.9495	0.9552
Max.	29.0600	29.2500	30.8800	10.7000	10.9200	10.7700	10.2000	10.2700	10.6300	9.7220	9.9210	9.4540
SD	4.8269	4.8585	5.1447	2.2441	2.2901	2.2648	1.4799	1.4900	1.6107	1.7487	1.7844	1.6616
Bias_Sqr	2.6755	2.7553	2.4362	1.2128	1.3564	1.2434	0.2181	0.2338	0.2497	0.4226	0.4959	0.1987
MSE	25.9744	26.3604	28.9043	6.2488	6.6008	6.3725	2.4082	2.4539	2.8442	3.4807	3.6800	2.9596
CV	1.3276	1.3275	1.4448	0.7236	0.7236	0.7270	0.5999	0.5999	0.6444	0.6599	0.6599	0.6794

NOTE: Shaded cells indicate the lowest MSE between aggregated and disaggregated models. Underlined values indicate a difference less than 50% compared with the lowest MSE used in the comparison. Min. = minimum, max. = maximum.
[a]50 iterations.

SIMULATION RESULTS

The simulation results for the 3-year, 5-year, and 10-year periods are summarized in Tables 3, 4, and 5, respectively. The shaded cells in these tables indicate the lowest MSE estimate between the aggregated and disaggregated models for a given sample mean value. Underlined values indicate a difference less than 50% compared with the lowest MSE used in the comparison.

The results shown in Tables 3, 4, and 5 indicate that the inverse dispersion parameter ($\phi = 0.5, 1, 2$) is estimated more efficiently when the sample mean is equal to 1.0 ($\lambda = 1.0$ crashes per year) than when it is lower for the same sample size, as expected. These results can be observed for all three estimators and are consistent with previous work on this topic (*14, 34*). Overall, the ML estimates perform slightly better than the other two estimates.

For the 3-year period, disaggregated models produced, in general, more reliable estimates, as seen with the MSE, when the sample mean value is equal to 1.0. For a sample mean equal to 0.50 crashes per year, aggregated models produced estimates that are more reliable. However, for $\phi = 2.0$, the aggregated models still provided less than desirable estimates.

For the 5-year period, the same characteristics can be seen as those observed for the 3-year period, although for $\phi = 1.0$ and $\lambda = 0.5$, the disaggregated model offers a faintly better MSE. However, for this time period, the inverse dispersion parameters are more accurately estimated than those inverse dispersion parameters estimated for the 3-year period for the same sample mean values. For $\phi = 2.0$ and $\lambda = 1.0$, disaggregated and aggregated models performed almost similarly.

For the 10-year period, the aggregated models estimated less biased estimates for $\phi = 1.0, 2.0$ and a sample mean equal to 0.5 crashes per year, while for $\phi = 0.5$ the disaggregated model performed better. For a sample mean equal to 1.0 crashes per year, disaggregated models were superior for $\phi \leq 1.0$, while the aggregated models provided a better estimate for $\phi = 2.0$, although the MSE values were almost equal to those of the aggregated models.

The simulation results show that selecting a disaggregated or aggregated model can significantly influence the estimation of the inverse dispersion parameter. Furthermore, the selection of the model class seems to be dependent on the amount of dispersion observed in the data. As a general rule, if the sample mean value is located around 0.5 crashes per year for the entire study period and the initial ϕ estimated from the raw data is above 1.0, the use of an aggregated model

TABLE 4 Simulation Results for 5-Year Period

	Comparison 1						Comparison 2					
	Disaggregated ($n = 250$)			Aggregated ($n = 50$)			Disaggregated ($n = 250$)			Aggregated ($n = 50$)		
	$\lambda = 0.5$ (crashes/year)			$\lambda = 2.5$ (crashes/5 years)			$\lambda = 1.0$ (crashes/year)			$\lambda = 5.0$ (crashes/5 years)		
	M	WR	ML	MM	WR	ML	MM	WR	ML	MM	WR	ML
$\phi = 0.5$												
Mean[a]	0.5679	0.5702	0.5548	0.6215	0.6342	0.5691	0.5381	0.5403	0.5098	0.5974	0.6096	0.5729
Min.	0.2519	0.2529	0.3283	0.3114	0.3178	0.3401	0.3100	0.3112	0.3328	0.2263	0.2309	0.2645
Max.	1.1250	1.1300	1.0860	0.9236	0.9425	0.9742	0.7933	0.7965	0.7329	1.4750	1.5050	1.1870
SD	0.2084	0.2093	0.1835	0.1646	0.1680	0.1483	0.1214	0.1219	0.0913	0.2175	0.2220	0.1641
Bias_Sqr	0.0046	0.0049	0.0030	0.0148	0.0180	0.0048	0.0015	0.0016	0.0001	0.0095	0.0120	0.0053
MSE	0.0481	0.0487	0.0367	0.0419	0.0462	0.0268	0.0162	0.0165	0.0084	0.0568	0.0613	0.0322
CV	0.3670	0.3670	0.3307	0.2649	0.2649	0.2605	0.2256	0.2256	0.1790	0.3642	0.3642	0.2864
$\phi = 1$												
Mean[a]	1.1287	1.1333	1.1109	1.2529	1.2785	1.2072	1.0889	1.0932	1.0724	1.2204	1.2453	1.1476
Min.	0.5330	0.5351	0.4202	0.3941	0.4022	0.5140	0.5920	0.5944	0.6744	0.3487	0.3558	0.5625
Max.	2.2410	2.2500	2.9730	2.8250	2.8830	3.9770	1.6150	1.6210	1.5980	2.9400	3.0000	2.1720
SD	0.4458	0.4476	0.4867	0.4858	0.4958	0.5733	0.2367	0.2375	0.2190	0.5117	0.5221	0.3781
Bias_Sqr	0.0166	0.0178	0.0123	0.0640	0.0776	0.0429	0.0079	0.0087	0.0052	0.0486	0.0602	0.0218
MSE	0.2153	0.2181	0.2491	0.3000	0.3233	0.3716	0.0639	0.0651	0.0532	0.3104	0.3328	0.1647
CV	0.3949	0.3950	0.4381	0.3878	0.3878	0.4749	0.2174	0.2173	0.2043	0.4193	0.4193	0.3294
$\phi = 2$												
Mean[a]	3.3786	3.3924	3.2376	2.5380	2.5898	2.6034	2.2800	2.2891	2.2317	2.2683	2.3145	2.1719
Min.	0.6676	0.6702	0.7376	0.9290	0.9479	1.0730	0.9546	0.9585	1.1390	0.8766	0.8945	1.1370
Max.	10.5300	10.5800	10.8100	9.1420	9.3280	9.1740	5.9630	5.9870	5.6430	4.4180	4.5080	4.4460
SD	2.3705	2.3807	2.1312	1.3195	1.3464	1.3655	0.8776	0.8811	0.8864	0.8777	0.8956	0.7645
Bias_Sqr	1.9007	1.9389	1.5316	0.2895	0.3479	0.3641	0.0784	0.0836	0.0537	0.0720	0.0989	0.0295
MSE	7.5198	7.6068	6.5658	2.0305	2.1607	2.2287	0.8486	0.8600	0.8394	0.8423	0.9011	0.6141
CV	0.7016	0.7018	0.6930	0.5199	0.5199	0.5245	0.3849	0.3849	0.3972	0.3869	0.3870	0.3520

NOTE: Shaded cells indicate the lowest MSE between aggregated and disaggregated models. Underlined values indicate a difference less than 50% compared with the lowest MSE used in the comparison.
[a]50 iterations.

is suggested in order to increase the sample mean of the data. Although the sample size will be reduced, the inverse dispersion parameter is less likely to be erroneously estimated (see the description in this paper's following section on example application). However, for a sample mean above 1.0, disaggregated models usually provide more reliable estimates, especially when the data are highly dispersed. As previously noted, disaggregated models with trend usually fit the data better than aggregated models (*6, 8*). For a description about the recommended minimum sample size to use when developing models as a function of sample size and sample mean values, the reader is referred to Lord (*15*). Finally, as documented in Lord (*15*), developing aggregated or disaggregated models using a sample size smaller than 50 is not recommended.

EXAMPLE APPLICATION

This section discusses an example to help illustrate the proposed guidance for selecting the appropriate model class on the basis of the sample size and the sample mean value. In this example, crash data were collected on four-lane undivided rural highways in Texas from 1997 to 2001. These data were initially collected for an NCHRP research project about a methodology to predict the performance of rural multilane highways (*35*). The original data set contained 1,499 segments, all with a minimum length of 0.1 mi. A sample of 50 segments was extracted from this data set.

The summary statistics for this sample are presented in Table 6. This table shows that the average number of crashes per year is 0.74. This table also shows that the raw data are overdispersed since the variance, $(\text{StdDev})^2$, is greater than the mean. According the guidelines previously noted, an aggregated model should be selected over a disaggregated model.

With the sample data described in Table 6, three models (as seen in Table 7) were estimated: an aggregated model using the generalized linear modeling (GLM) method and two disaggregated models without trend (i.e., one single intercept), one estimated using the GLM method and the other using the GEE method. As described above, the GLM estimating method does not account for the serial correlation, whereas the GEE does. Both dependent and autoregressive correlation structures were used for

TABLE 5 Simulation Results for 10-Year Period

	Comparison 1						Comparison 2					
	Disaggregated ($n = 500$)			Aggregated ($n = 50$)			Disaggregated ($n = 500$)			Aggregated ($n = 50$)		
	$\lambda = 0.5$ (crashes/year)			$\lambda = 5$ (crashes/10 years)			$\lambda = 1.0$ (crashes/year)			$\lambda = 10$ (crashes/10 years)		
	MM	WR	ML	MM	WR	ML	MM	WR	ML	MM	WR	ML
$\phi = 0.5$												
Mean[a]	0.5455	0.5466	0.5308	0.6196	0.6322	0.5672	0.5139	0.5149	0.5062	0.5671	0.5787	0.5330
Min.	0.2605	0.2611	0.3223	0.1954	0.1993	0.3235	0.3787	0.3795	0.4020	0.3008	0.3070	0.3329
Max.	0.8429	0.8446	0.8027	1.0940	1.1160	0.9110	0.6590	0.6603	0.6286	1.1070	1.1300	0.8570
SD	0.1203	0.1205	0.1082	0.1949	0.1989	0.1450	0.0691	0.0692	0.0575	0.1629	0.1662	0.1097
Bias_Sqr	0.0021	0.0022	0.0009	0.0143	0.0175	0.0045	0.0002	0.0002	0.0000	0.0045	0.0062	0.0011
MSE	0.0165	0.0167	0.0126	0.0523	0.0570	0.0255	0.0050	0.0050	0.0033	0.0310	0.0338	0.0131
CV	0.2205	0.2205	0.2038	0.3146	0.3145	0.2556	0.1344	0.1344	0.1135	0.2872	0.2872	0.2057
$\phi = 1$												
Mean[a]	1.1107	1.1129	1.0887	1.0932	1.1157	1.0704	1.0601	1.0622	1.0482	1.1126	1.1353	1.1205
Min.	0.6441	0.6454	0.6298	0.5306	0.5415	0.6588	0.6533	0.6546	0.7336	0.5274	0.5381	0.7115
Max.	2.8680	2.8730	2.8330	1.9230	1.9620	1.9520	1.8180	1.8220	1.5250	2.2960	2.3430	2.1170
SD	0.4016	0.4024	0.3928	0.3174	0.3239	0.3022	0.2264	0.2269	0.1671	0.3242	0.3309	0.2663
Bias_Sqr	0.0123	0.0128	0.0079	0.0087	0.0134	0.0050	0.0036	0.0039	0.0023	0.0127	0.0183	0.0145
MSE	<u>0.1735</u>	<u>0.1746</u>	<u>0.1621</u>	0.1094	0.1183	0.0963	0.0549	0.0553	0.0302	0.1178	0.1278	0.0854
CV	0.3616	0.3615	0.3608	0.2903	0.2903	0.2824	0.2136	0.2136	0.1594	0.2914	0.2914	0.2377
$\phi = 2$												
Mean[a]	2.4224	2.4272	2.3183	2.3033	2.3502	2.3099	2.0315	2.0356	2.0342	2.2111	2.2563	2.1838
Min.	1.0420	1.0440	1.0630	1.0420	1.0630	1.1610	1.1720	1.1740	1.2330	1.4160	1.4450	1.3670
Max.	8.8170	8.8350	7.5860	4.3820	4.4710	4.3810	4.6070	4.6160	4.5650	3.9630	4.0440	3.6770
SD	1.5083	1.5113	1.3627	0.8534	0.8708	0.7828	0.6917	0.6930	0.6748	0.6123	0.6249	0.5595
Bias_Sqr	0.1785	0.1825	0.1013	0.0920	0.1227	0.0961	0.0010	0.0013	0.0012	0.0446	0.0657	0.0338
MSE	2.4534	2.4666	1.9582	0.8203	0.8809	0.7089	<u>0.4794</u>	<u>0.4816</u>	<u>0.4565</u>	0.4195	0.4562	0.3468
CV	0.6226	0.6227	0.9528	0.3705	0.3705	0.3389	0.3405	0.3405	0.3317	0.2769	0.2769	0.2562

NOTE: Shaded cells indicate the lowest MSE between aggregated and disaggregated models. Underlined values indicate a difference less than 50% compared with the lowest MSE used in the comparison.
[a]50 iterations.

the GEE. As was expected by Lord et al. (*7*), using both correlation structures provided the same results.

To simplify the modeling process, a flow-only functional form was utilized. Although this functional form can be subject to omitted variables bias, this type of model is still considered to be the most popular functional form used by transportation safety analysts, mainly because it can easily be recalibrated (*3, 36*). The functional form was the following:

Aggregated:

$$\mu_i = \beta_0 L_i F_i^{\beta_1} \tag{9a}$$

Disaggregated (without trend):

$$\mu_{it} = \beta_0 L_i F_{it}^{\beta_1} \tag{9b}$$

where

- μ_i = estimated number of crashes per year for site i (to estimate the number of crashes for the time period under study, multiply μ_i by 5),
- μ_{it} = estimated number of crashes per year for site i and time t,
- L_i = length of the segment in miles for site i,
- F_i = average traffic flow [average daily traffic (ADT)] for site i (over 5 years), and
- F_{it} = ADT for site i and time t.

TABLE 6 Summary Statistics for Four-Lane Undivided Rural Segments in Texas

	Aggregated			Disaggregated	
Statistics	Length (miles)	Flow (veh/day)	Crashes (per 5 years)	Flow (veh/day)	Crashes (per year)
Mean	0.754	6,654.2	3.70	6,654.2	0.74
SD	0.758	3,929.4	5.35	3,952.3	1.31
Min.	0.122	1,530	0	1,350	0
Max.	4.324	16,580	28	18,800	9

TABLE 7 Modeling Results for Aggregated and Disaggregated Models

	Aggregated	Disaggregated (without trend)	
Variable	GLM	GLM	GEE[a]
Model Estimates			
Intercept (ln β_0)	−6.00 (2.19)[b]	−5.89 (1.38)	−5.56 (2.01)
Flow (β_1)	0.68 (0.25)	0.67 (0.16)	0.63 (0.22)
MLE (ϕ)	2.93 (1.25)	4.11 (2.45)	4.13 (2.43)
WR (ϕ)	3.77 (1.61)	4.72 (0.69)	4.75 (0.70)
MM (ϕ)	3.55	NA[c]	NA[c]
MLE (ϕ) full dataset[d]	2.25 (0.19)	2.27 (0.19)	2.27 (0.19)
Goodness-of-Fit			
−2 log likelihood	201.7	494.0	501.1
Deviance	52.58 (F = 9.1, d.f. = 48)	223.4 (F = 19.67, d.f. 249)	234.4 (F = 12.31, d.f. 249)
MAD[e]	2.12	2.13	2.11
MSPE[f]	10.48	10.36	10.42
Sample size	50	250[g]	250[g]
Sample mean (crashes)	3.70	0.74	0.74

[a]Estimated using a dependent correlation structure (note: the same results were estimated using an autoregressive correlation structure).
[b]Standard error.
[c]Provided a negative value (e.g., characterized by underdispersion) (also signs of an unstable estimate).
[d]Model using the full data set was estimated to determine the inverse dispersion parameter.
[e]Mean absolute deviance: $\frac{1}{n}\sum\left|y_{it}-\hat{\mu}_{it}\right|$.
[f]Mean squared prediction error: $\frac{1}{n}\sum\left(y_{it}-\hat{\mu}_{it}\right)^2$.
[g]50 sites × 5 years (each year is considered a different observation).

The modeling results are summarized in Table 7. First, this table shows that all three models provide similar values for the coefficients (within one standard error of each other). The similarity is confirmed by the goodness-of-fit analysis, as illustrated by the mean absolute deviance and the mean squared prediction error (*37*). In short, all three models offer the same statistical performance. Concerning the serial correlation, because a complete data set was used (i.e., no missing values), the same modeling results can be seen whether the coefficients of the models were estimated using the GLM or the GEE estimating methods (*7*). The only difference, as seen in this table, is associated with the standard error of the models' coefficients, which is larger when the serial correlation is accounted for in the modeling process.

Second, despite similar model performances, it can be observed that the inverse dispersion parameters appear to be misestimated for the disaggregated models, given the inverse dispersion parameters estimated from the full data set (and compared with the values estimated for the aggregated modes). Furthermore, the MM estimator provided an unreliable estimate, since the values were negative for the disaggregated models. For the aggregated model, the parameters also seemed to be misestimated, but the potential bias with the aggregated model is much smaller than those associated with the disaggregated models. If one uses the disaggregated model for estimating confidence intervals around predicted values and for identifying hazardous sites using the EB method, the analysis will be erroneous, at least more so than if the aggregated model was used. To minimize the bias, the next step would be to adjust the inverse dispersion parameter using the method proposed by Park and Lord (*17*) and Saha and Paul (*19*). In short, this example supports the simulation results described above.

SUMMARY AND CONCLUSIONS

The objective of this study was to develop guidance about which model class, aggregated or disaggregated, provides more reliable estimates of the inverse dispersion parameter in cases when the sample mean value and sample size are small. More specifically, the aim was to determine the conditions in which one model type should be selected over the other. To fulfill this objective, several simulation runs were produced for different values describing the sample mean value, the sample size, and the inverse dispersion parameter. The simulation scenarios represented cases where 3, 5, and 10 years of data were available. The study also presented an example of how this guidance could be applied.

In summary, the simulation results used for estimating the inverse dispersion parameter have shown the following properties:

- The estimation of the inverse dispersion parameter using an aggregated or disaggregated approach is influenced by the sample size and sample mean values as well as by the amount of dispersion observed in the original data set.

- For sample mean values around 0.5 crashes per year (for the entire study period), aggregated models are favored over disaggregated models, especially when the inverse dispersion parameter estimated from the raw data is above one.
- When the sample mean is larger than 1.0, disaggregated models should be selected over aggregated models; for these models, models with trend usually offer better statistical performances, as discussed in Lord and Persaud (*6*).
- Finally, developing aggregated or disaggregated models using a sample size smaller than 50 is not recommended.

Although this research provides useful information about when to use aggregated instead of disaggregated models, further research should be conducted to validate this work. Given the increasing use of full Bayes models in highway safety research, it is suggested to examine how the application of such models influences the posterior estimate of the inverse dispersion parameter. The simulation techniques proposed by Lord and Miranda-Moreno (*16*) could be used for such purpose.

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