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Analyzing the severity of accidents on the German Autobahn



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ABSTRACT

We study the severity of accidents on the German Autobahn in the state of North Rhine-Westphalia using data for the years 2009 until 2011. We use a multinomial logit model to identify statistically relevant factors explaining the severity of the most severe injury, which is classified into the four classes fatal, severe injury, light injury and property damage. Furthermore, to account for unobserved heterogeneity we use a random parameter model. We study the effect of a number of factors including traffic information, road conditions, type of accidents, speed limits, presence of intelligent traffic control systems, age and gender of the driver and location of the accident. Our findings are in line with studies in different settings and indicate that accidents during daylight and at interchanges or construction sites are less severe in general. Accidents caused by the collision with roadside objects, involving pedestrians and motorcycles, or caused by bad sight conditions tend to be more severe. We discuss the measures of the 2011 German traffic safety programm in the light of our results.

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1. Introduction

In November 2011 the German ministry for traffic, construction and city development published the traffic safety program for Germany under the motto "Each traffic death is one too many". The goal was to reduce fatalities on German roads by 40% by the year 2020. A positive trend can already be observed and despite an increasing traffic volume, in 2010 there were only 3648 fatalities, which is the lowest number since this statistic firstly was recorded in 1953. Continuing this trend is the central goal of the ministry (Bundesministerium für Verkehr and Bau und Stadtentwicklung, 2011).

In order to increase traffic safety it is of central importance to know the causing factors of accidents. Using data containing detailed accident information the determinants of accident frequency and severity can be analyzed using statistical methods. As mentioned by Savolainen et al. (2011), the factors influencing the accident frequency may sometimes be different from the ones influencing the severity and it may therefore be reasonable to analyze the two separately. For example, guardrails have been found to affect the severity but not the frequency of accidents. The frequency of accidents is studied in Chin and Quddus (2003), El-Basyouny and Sayed (2009), Shankar et al. (1998) or Shankar et al. (1995) among many others. Garnowski and Manner (2011) analyze accident

frequencies on German Autobahn connectors using data from the state of North Rhine-Westphalia for the years 2003 until 2005. To our knowledge, accident severities on German Autobahns have not yet been analyzed in the literature and in this paper we aim to fill that gap. Due to the fact that there is no obligatory speed limit on the German Autobahn this paper addresses the modeling of accident severities in an environment that differs from the ones that have been studied in the literature. Given the relevance of the German highway system in the center of Europe and the constantly increasing traffic this seems important. We use a similar, but not the same, data set as in Garnowski and Manner (2011) where now we have detailed information on accidents that occurred on the Autobahn in the state of North Rhine-Westphalia, the most densely populated state in Germany, for the years 2009 until 2011. The variable of interest is the severity of the most severe injury falling into the four categories fatal, severe injury, minor injury and property damage. Detailed information on traffic flow is obtained from a different data base and matched to our accident data.

The literature on the severity of traffic accidents is well developed. Lui et al. (1988) analyze the design of the vehicle, in particular the seat belt. The data they use is from the Fatality Accident Reporting System (FARS) introduced by the National Highway Traffic Safety Administration (NHTSA) in 1975. Many other studies rely on the FARS data, e.g. Kockelman and Kweon (2002). Other countries are also making an effort to collect detailed accident data that make detailed statistical analyses possible. Hongkong introduced the Computerized Traffic Accident Data System (TRADS) in 1990, which are analyzed in Yau (2004). Mao et al. (1997) use data from the Canadian Traffic Accident Information Databank (TRAID) in

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their study. While no studies have looked at data for the German Autobahn, the study by Christoforou et al. (2010) is related to our research as it analyses accident severities on a highway in the Paris region. It also provides an excellent overview of the literature on accident severity with a summary of the main findings.

A central goal of traffic accident analysis is to identify factors that can be influenced by policymakers in order to reduce the frequency and severity of accidents or to study the effectiveness of certain measures. Lee and Mannering (2002) analyze the effect that roadside conditions have on the frequency and severity of accidents. They note that "the marginal effect of these factors are computed to provide an indication of the effectiveness of potential countermeasures". Kim et al. (2007) empirically show that speed limits can have large effects on accidents involving cars and bicycles finding a threshold effect for the speed of 32.2 km/h. The effectiveness of ice warning signals on accidents caused by icy conditions is rejected by Carson and Mannering (2001). The current paper also aims at identifying factors that can be manipulated by policy makers and in the light of our empirical results we take a closer look at the measures suggested in the German traffic safety program. We find that intelligent traffic control systems, the prevention of leaving the road and better safety measures for pedestrians or motorcycles can reduce the severity of accidents.

Next to factors that can be influenced there are a number of other factors that have an effect on the severity of accidents. The characteristics of the person causing the accident, in particular the gender and age, are the ones that are most commonly looked at, see e.g. Abdel-Aty (2003), Milton et al. (2008), Yau (2004), or Chang and Mannering (1999). We study the effect of these two factors, but also look at the light conditions, seasonal effects or road conditions as done by many other authors. While all these factors may not be useful for immediate policy actions they may nevertheless turn out useful for indirect or long term measures. 1

Khorashadi et al. (2005) study the differences between accidents in rural and urban areas when trucks are involved and find significant differences for the two areas. Abdel-Aty (2003) also distinguishes between different locations looking at roadway segments, intersections and toll stations. We limit our analysis to highways, but we study the effect of accidents occurring on interchanges on the accident severity. Savolainen and Mannering (2007) and Shankar and Mannering (1996) study accident severities when motorcycles are involved. We also analyze the effects when pedestrians, motorcycles or trucks are involved in the accident.

From a methodological perspective two model classes are prominent in the literature. Multinomial models have been used by Kim et al. (2007) and Islam and Mannering (2006) and we use this model as our workhorse. An alternative approach is the ordered probit model used in Kockelman and Kweon (2002), Abdel-Aty (2003), or Christoforou et al. (2010). More recently, random coefficient (mixed) logit models have been applied to overcome inefficiencies of the models mentioned above and we make use of this model here. Studies applying these types of models are Milton et al. (2008), Eluru et al. (2008), Moore et al. (2011), Chen and Chen (2011), Kim et al. (2013), or Mehta and Lou (2013). Model selection is done by the commonly used general-to-specific approach as, e.g., in Moghaddam et al. (2009). A recent review of the methodology available in the literature can be found in Savolainen et al. (2011).

This paper makes the following contributions to the empirical literature on accident severity. First, we study the factors influencing accident severities in a setting that has not been looked at in the literature, namely on heavily used highways that do not have an obligatory speed limit. This allows identifying factors that are

Table 1Descriptive statistics of the accident severity.

Severity	2009	2010	2011	Total	Percentage
Fatal	67	67	66	200	0.34
Severe injury	915	873	912	2700	4.65
Minor injury	2747	2927	2774	8448	14.55
Property damage	17,028	18,108	11,594	46,730	80.46
Total	20,757	21,975	15,346	58,078	100

common to those found in similar studies on different road types as well as factors that differ. Second, we compare model specifications using both daily and hourly traffic information to draw conclusions about the importance of intraday seasonality, but also to check the robustness of our results. Finally, we discuss the measures of the 2011 traffic safety program considering our empirical findings.

The rest of the paper is structured as follows. The next section describes the data. In Section 3 we discuss the methodology and in Section 4 we present the empirical results. A conclusion and the discussion of our result in the context of the German traffic safety program can be found in Section 5.

2. Data

The data set used in this study contains almost 58,078 accidents that occurred on highways in the German state of North Rhine-Westphalia during the years 2009-2011. In fact, data was available since 2006. However, the crash reporting process changed in November 2008 due to an order by the ministry of the interior that obliged police officers to record light property damage into the accident file. Thus we face the problem of underreporting (Savolainen et al., 2011) for the period 2006–2008. A preliminary analysis included this data but we decided to remove it on the basis of likelihood ratio test for model stability. The data have been provided by the "Landesbetrieb Straßenbau Nordrhein-Westfalen", which is the institution responsible for the planning, construction and maintenance of the highways for the state. Whenever an accident occurs the highway police collects detailed information such as severity of the most severe injury, driver characteristics, cause of the accident, and weather conditions, which is then stored centrally. The severity of the accidents falls into the categories fatal, severe injury, minor injury and property damage. Table 1 shows the distribution of the accidents over time and category. Only 0.34% of all accidents were fatal, while the vast majority were either minor injuries or property damage.

The data file contains information on a number of things related to the accident and the ones that turned out to be relevant for explaining accident severities are listed in Table 2. Most variables are self-explaining. The variable Dry is equal to one when the road is dry and zero otherwise, i.e. when the road is slippery due to oil, leaves, rain or ice. The variables Age and Female refer to the person who caused the accident and may therefore differ from the most severely injured person. This makes the interpretation of the effects of these variables difficult, as in some cases the responsible person is the most severely injured person and in other cases he is not. This needs to be kept in mind when interpreting the corresponding effects. Some variables like the nationality of the driver had to be disregarded because data was not available for many accidents and including them would have reduced our sample size drastically. Note that there was a systematic error concerning the age variable in our data set, since in some cases the age of the driver was less than 16 years, which is impossible in Germany. We decided to remove these observations. After dealing with missing data we were left with a sample size of 37,735 to do the analysis.

For the traffic flow we have two potential sources of information. The accident data contains the average daily traffic at the site

¹ For example, daylight/night effects may be countered by illuminated highways as is done in Belgium.

Table 2 Descriptive statistics of the regressors.

Variable category	Variable	Description	Mean	sd
	Daylight	Dummy for daylight	0.670	0.470
	Winter	Dummy for December, January or February	0.271	0.445
	Dry	Dummy for dry road	0.639	0.480
	Persons involved	Number of involved persons	1.903	0.818
	Collision	Dummy for collision with roadside obstacle	0.385	0.487
Accident information	Sight	Cause due to sight obstruction	0.003	0.058
	Obstacle	Cause by obstacle on road	0.026	0.158
	Road condition	Cause due to bad road condition	0.161	0.367
	Motorcycle	Dummy for motorcycle involved	0.017	0.128
	Truck	Dummy for truck involved	0.414	0.492
	Pedestrian	Dummy for pedestrian involved	0.001	0.037
	Interchange	Dummy for accident on interchange	0.181	0.385
	Speed limit	Dummy for speed limit of 100 km/h or less	0.264	0.441
	Construction site	Dummy for construction site	0.096	0.295
Accident location	Traffic control	Dummy for intelligent traffic control system	0.006	0.079
	ADT	Average daily traffic in 1000	76.022	32.378
	AHT	Average hourly traffic in 1000	0.792	0.474
D	Female	Dummy for female	0.188	0.391
Responsible person	Age	Age of person in years	40.629	14.636

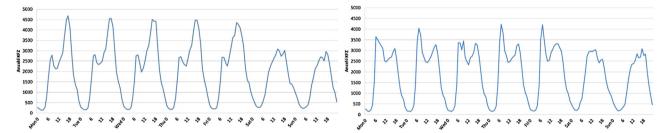


Fig. 1. Average hourly traffic Section 4.200 from Cologne Mehrheim to Refrath in the eastern (left) and western (right) direction.

of the accident, so daily traffic data is available for the complete data set. However, this disregards the day of the week or the time of the day. Since traffic flow data is recorded permanently on a very fine grid of the highways we use the data for hourly traffic flow from the corresponding database. Except for very few cases the distance between two measuring sections on the highway is no more than 10 km. The accidents were matched to the traffic flow data as follows. Each accident is matched to the closest previous measuring sections, unless the distance is more than 20 km or there is an exit or drive-up ramp between the accident location and the measuring section. On interchanges the matching is much more precise distinguishing individual lanes. However, we only had access to the traffic flow database for the area Rhineland, which covers 15,028 accidents. This reduction in the number of observation is unfortunate, as high frequency data is only available for a subset of our data and consequently the comparison of the model with hourly and daily traffic is based on different sample sizes. Nevertheless, we are still left with a large number of accidents and therefore also consider the analysis with hourly traffic data below. Note that due to technical reasons we were not able to obtain the data for the time and place of every accident.² Instead, we downloaded the data for two representative weeks³ for each accident location. We then averaged over these two weeks. Since we do not have any information on which lane an accident took place we additionally averaged over all driving lanes. In Fig. 1 you can see the traffic flow for

control section 4.200 between Cologne Mehrheim and Refrath in the eastern (from Cologne to Refrath) and western (from Refrath to Cologne) direction. The seasonality is apparent, as there is hardly any traffic during the night time and the peak hours are clearly observable. Furthermore, one can see that more people commute to work from Refrath to Cologne than the other way around, since in the eastern direction the peak is larger in the afternoon, whereas in the western direction it is higher in the morning. The traffic flow data also contains information about the average speed at the different sites. However, this information is incomplete and therefore cannot be used for the analysis. After accounting for missing data on the regressors we were left with 13,053 observations when using hourly traffic flow.

3. Methodology

Our dependent variable y_i , the severity given an accident has occurred, can be modeled using either ordered logit/probit models, or alternatively using the multinomial logit/probit model, see e.g. Cameron and Trivedi (2005) for a general treatment of these model classes. The variable y_i can take on one of the values $1, \ldots$ m. We do not consider the ordered logit model since there are two potential drawbacks with the approach. First, the influence of any variable can only be universally positive (negative) for the probability of the most severe category while being negative (positive) for the probability of the least severe category. The effect on intermediate categories is not clear. Usually this is not a problem, but there may be some cases where this is restrictive. For example the presence of an airbag may both reduce the probability of death and property damage, while increasing the probability of minor and major injuries. Second, the presence of underreporting causes biased parameter estimates and underreporting is a typical feature

² The limiting factor is not the existence of the data, but that the access is limited by aged equipment and software that does not allow downloading large quantities of data at once.

³ We used two weeks that did not contain any holidays and did not fall into the vacation time, namely April 19–25 and September 13–19 2010.

of accident data. The multinomial logit model does not suffer from these drawbacks. It is given by

$$p_{ij} = \frac{e^{\mathbf{x}_i'\beta_j}}{\sum_{l=1}^{m} e^{\mathbf{x}_i'\beta_l}}, \quad j = 1, \dots, m,$$
 (1)

where p_{ij} is the probability of observing category j for observation i. The vector \mathbf{x}_i contains the regressors for observation i and $\mathbf{\beta}_j$ is the coefficient vector for category j, which is allowed to differ for each category. In order to ensure that $\sum_{j=1}^m p_{ij} = 1$ a base level is chosen for which the coefficients are set to zero. In our case the base level is property damage.

An extension of this model is to allow (a subset of) the coefficient vector to be randomly distributed with distribution $f(\beta|\varphi)$, where φ refers to the parameters of the density (Bhat, 2001, 2003). Then the model is given by

$$p_{ij} = \int \frac{e^{\mathbf{x}_i' \boldsymbol{\beta}_j}}{\sum_{l=1}^m e^{\mathbf{x}_i' \boldsymbol{\beta}_l}} f(\boldsymbol{\beta} | \varphi) d\boldsymbol{\beta}. \tag{2}$$

We consider the random coefficients to be normally distributed. Estimation can be done by solving the integral by Monte Carlo simulation. Efficiency is increased by using Halton draws instead of pseudo-random numbers, see Train (1999, 2003) for details.

The elasticity of parameter estimates for continuous regressors can be computed by

Elasticity =
$$\frac{\partial p_{ij}}{\partial x_{ik}} \times \frac{x_{ik}}{p_{ij}} = [1 - p_{ij}]\beta_{ik}x_{ik}.$$
 (3)

For dummy variable we compute the pseudo-elasticities as in Washington et al. (2011) by

Pseudo-elasticity
$$= \frac{\Pr(y_i = j | \mathbf{x}, x_k = 1) - \Pr(y_i = j | \mathbf{x}, x_k = 0)}{\Pr(y_i = j | \mathbf{x}, x_k = 0)}$$

$$= \frac{exp[\triangle(\boldsymbol{\beta}_i \mathbf{X}_i)] \sum_{\forall I} exp(\beta_{kl} x_{kl})}{exp[\triangle(\boldsymbol{\beta}_i \mathbf{X}_i)] \sum_{\forall I_i} exp(\beta_{kl} x_{kl}) + \sum_{\forall I_j \neq I_n} exp(\beta_{kl} x_{kl})} - 1.$$

$$(4)$$

One can estimate the elasticities either at the average value of the regressors or average the elasticities over the sample. Since using the average value is not reasonable for dummy variables we decided to use average elasticities.

To test the adequacy of the choice of categories Anderson (1984) introduced the concept of indistinguishability of two categories when none of the independent variable has an effect on the odds of these categories. In order to test whether two categories can be joined a standard Wald test is performed.

4. Results

In this section we present our empirical results. The interpretation of the estimation results for the multinomial logit model is done in Section 4.1. We discuss two specifications determined by the use of the traffic variable, one using average daily traffic (ADT) and one using average hourly traffic (AHT) from two representative weeks as described in Section 2. The estimation results for the mixed logit model are presented in Section 4.2.

Before estimating the models, we look at the correlation between the independent variables to identify possible multicollinearity. The correlations are largest for the pairs *Dry-Road condition* (-0.60), *Speed limit-construction site* (0.50), *AHT-Daylight* (0.34), *Road condition-Collision* (0.38), *AHT-Collision* (-0.35) and *ADT-Interchange* (-0.27). Most of these correlations are obvious

Table 3Wald test for joining categories.

Daily traffic	Hourly traffic
6042.76(0)	1145.48(0)
7750.22(0)	2802.31(0)
6835.20(0)	1627.58(0)
614.21(0)	3439.92(0)
2281.02(0)	3052.66(0)
2416.38(0)	572.90(0)
	6042.76(0) 7750.22(0) 6835.20(0) 614.21(0) 2281.02(0)

Note: This table reports the results the Wald test for joining two categories for the multinomial logit model based on daily and hourly traffic flow data with *p*-value in parenthesis.

and none are exceedingly high. Therefore we do not expect problems with multicollinearity, but one should nevertheless keep these correlations in mind when interpreting the results.

4.1. Fixed parameter model and interpretation

Table 3 presents the results of the Wald test for the null hypothesis of indistinguishability of two categories for all pairs. The null hypothesis is clearly rejected for all pairs indicating that our classification is appropriate.

We start by reporting the estimation results using the average daily traffic, which allows for using the largest possible sample size. The estimated parameters can be found in Table 4, while the corresponding (pseudo-)elasticities are reported in Table 5. We see that during the day an accident is 73% less likely to be fatal and 7% more likely to result in a minor injury. The results for the variable Winter and Dry are rather counterintuitive as both Winter and slippery road conditions reduce the probability of death or injury and increase the probability of property damage. This is probably due to the fact that drivers acknowledge the riskiness of these conditions, but also because it is probably more likely to observe minor accidents that happen at low driven speed. For Persons involved the results are as to be expected from earlier findings in the literature and from intuition, i.e., more involved people increase the risk of fatalities or injuries. Collisions with roadside obstacles increase the risk of fatality and severe injury by 186% and 147%, respectively. For accidents whose main cause was sight obstruction fatality is 1446% less likely, but the risk of injuries is increased. This effect is rather large and a closer look at the data indicated that there are no accidents in the sample for which the variable Sight was equal to one and that was fatal. The same is true for the variables Obstacle and traffic control. Another look at the descriptive statistics in Table 2 shows that these variables have a very low mean. Therefore, for these variables the coefficients and elasticities for fatal accidents must be interpreted with care. When an obstacle on the road is the main cause property damage is 21% more likely, but severe injury and minor injury are, respectively, 162% and 120% less likely. Accidents due to bad road conditions also tend to result in property damage more often. When a motorcycle or a pedestrian is involved in the accident the likelihood of death or severe injury is much higher, while property damage becomes much less likely. The involvement of a truck makes a fatality more likely, but minor injuries less likely. Still the probability of observing property damage is increased by 11%, so the effect is not monotone. This can be explained by accidents involving two trucks, in which the drivers are rather well protected.

Concerning the accident location we see that accidents on interchanges are 269% less likely to be fatal and 132% less likely to result in severe injury. An explanation for this can be found in Fig. 2, which shows a boxplot of the average speed at interchanges compared to road segments. One can see that people drive much slower on interchanges making accidents less risky. A speed limit of 100 or less increases the risk of fatality by 41% and the risk of minor injury

⁴ In fact, the estimate for the constant would be affected by the presence of underreporting, but the estimates for the remaining parameters would still be consistent.

Table 4Estimated coefficients of the multinomial logit model with ADT.

Variable	Fatal	Severe injury	Minor injury
Accident information			
Daylight	-0.731 (0.171)***	$-0.23 (0.054)^{***}$	$0.065 (0.034)^*$
Winter	$-0.364(0.209)^*$	$-0.42 (0.066)^{***}$	$-0.234 (0.038)^{***}$
Dry	0.524 (0.227)**	$0.364 (0.072)^{***}$	0.175 (0.042)***
Persons involved	0.738 (0.047)***	0.619 (0.033)***	$0.604 (0.028)^{***}$
Collision	2.123 (0.185)***	1.701 (0.058)***	0.868 (0.037)***
Sight	-14.106 (0.251)***	1.382 (0.25)***	$1.029(0.199)^{***}$
Obstacle	-15.7 (0.352)***	$-1.834 (0.278)^{***}$	$-1.407(0.155)^{***}$
Road condition	-0.371 (0.321)	$-0.34 (0.093)^{***}$	$-0.137 (0.055)^{**}$
Motorcycle	4.362 (0.296)***	3.696 (0.137)***	1.999 (0.131)***
Truck	0.476 (0.168)***	$-0.128 (0.056)^{**}$	$-0.693(0.035)^{***}$
Pedestrian	6.452 (0.611)***	4.415 (0.522)***	3.198 (0.485)***
Accident location			
Interchange	-2.83 (1.011)***	$-1.463 \left(0.204\right)^{***}$	$-0.619(0.128)^{***}$
Speedlimit	0.562 (0.2)***	0.311 (0.065)***	0.684 (0.036)***
Construction site	-2.012 (0.535)***	$-0.645(0.111)^{***}$	$-0.538 (0.057)^{***}$
Traffic control	$-14.754 (0.445)^{***}$	0.281 (0.638)	0.941 (0.299)***
ADT	$-0.022 (0.004)^{***}$	-0.013 (0.001)***	$-0.004(0)^{***}$
Responsible person			
Female	-0.132 (0.253)	$0.523 (0.058)^{***}$	$0.428 (0.035)^{***}$
Age	0.012 (0.006)**	$-0.006(0.002)^{***}$	$-0.009(0.001)^{***}$
Constant	-6.885 (0.434)****	$-3.666(0.146)^{***}$	$-2.454(0.094)^{***}$

Note: This table reports the estimation results of the multinomial logit model with basis category property damage based on 37,735 observations. For the traffic flow (ADT) average daily values are used. The pseudo- R^2 is equal to 0.1148.

by 16%, while decreasing the risk of property damage by 15%. This is counterintuitive, as one would expect a speed limit to reduce the severity of accidents. A possible explanation is that speed limits are imposed at locations where many accidents tend to occur, so one would need data at the same location before and after the speed limits were imposed to see if they had a positive impact. At construction sites accidents tend to be much less severe, which is not surprising after controlling for *Speed limit* and *Pedestrian*. Many

accidents are caused by the narrow lanes and these tend to result in property damage only. The presence of intelligent traffic control drastically reduces the probability of fatalities by 1499%, which again must be interpreted with care due to the lack of occurrences. The probability of severe injuries is not influenced, while minor injuries are 71% more likely. The probability for property damage also decreases by a factor of 23%. The influence of the quantity of daily average traffic is negative for fatalities and both injury types.

Table 5 (Pseudo-)elasticities for the multinomial logit model with ADT.

Variable	Fatal	Severe injury	Minor injury	Property damage
Accident information				
Daylight	-0.725***	-0.224***	0.071**	0.006
Winter	-0.304	-0.360***	-0.174***	0.060***
Dry	0.475**	0.314***	0.125***	-0.050***
Persons involved	1.128***	0.899***	0.870***	-0.294***
Collision	1.858***	1.436***	0.603***	-0.265***
Sight	-14.457***	1.031***	0.678***	-0.351***
Obstacle	-15.488***	-1.623***	-1.195***	0.212***
Road condition	-0.330	-0.299***	-0.096**	0.04***
Motorcycle	3.069***	2.403***	0.706***	-1.293***
Truck	0.588***	-0.016	-0.581***	0.112***
Pedestrian	4.249***	2.211***	0.995***	-2.204***
Accident location				
Interchange	-2.691***	-1.324***	-0.480^{***}	0.139***
Speedlimit	0.414**	0.163***	0.537***	-0.148***
Construction site	-1.901***	-0.533***	-0.427***	0.111***
Traffic control	-14.986***	0.049	0.709***	-0.232**
ADT	-1.555***	-0.857***	-0.220^{***}	0.102***
Responsible person				
Female	-0.240	0.415***	0.320***	-0.108***
Age	0.544**	-0.164**	-0.293***	0.066***
Mean probability	0.005 (0.013)	0.054 (0.064)	0.165 (0.102)	0.776 (0.157)

Note: This table reports the elasticities corresponding to the estimation results in Table 4. Elasticities are averaged over all observations. For binary regressors we report the pseudo-elasticities using (4).

^{*} Statistical significance of the parameters is 10%.

^{**} Statistical significance of the parameters is 5%.

^{***} Statistical significance of the parameters is 1%.

^{*} Statistical significance of the parameters is 10%.

^{**} Statistical significance of the parameters is 5%.

^{***} Statistical significance of the parameters is 1%.

Table 6Estimated coefficients of the multinomial logit model with AHT.

Variable	Fatal	Severe injury	Minor injury
Accident information			
Dry	$1.376 (0.721)^*$	0.611 (0.144)***	0.396 (0.085)***
Persons involved	0.911 (0.11)***	0.683 (0.072)***	0.712 (0.067)***
Collision	2.101 (0.41)***	1.582 (0.112)***	0.74 (0.075)***
Sight	$-12.765(0.616)^{***}$	1.832 (0.638)***	1.841 (0.546)***
Obstacle	$-15.544(0.849)^{***}$	-2.941 (0.837)***	-1.565 (0.357)***
Road condition	0.199 (0.748)	-0.436 (0.176)**	$-0.057(0.104)^*$
Motorcycles	4.339 (0.553)***	3.423 (0.287)***	1.733 (0.29)***
Truck	-0.202 (0.351)	-0.403 (0.109)***	-0.952 (0.07)***
Pedestrian	5.546 (1.049)***	3.075 (0.872)***	2.303 (0.712)***
Accident location			
Interchange	$-2.032(0.987)^{**}$	$-0.52 (0.208)^{**}$	$-0.48 (0.132)^{***}$
Speed limit	$-0.515(0.442)^{***}$	$-0.192 (0.106)^*$	0.06 (0.063)
Traffic control	$-12.829(0.536)^{***}$	-13.968 (0.309)***	1.039 (0.38)***
AHT	$-1.544 (0.435)^{***}$	-0.339 (0.117)***	0.352 (0.066)***
Responsible person			
Female	0.021 (0.464)	0.587 (0.104)***	0.394 (0.067)***
Age	-0.01 (0.015)	$-0.006(0.003)^*$	-0.013 (0.002)***
Constant	-7.563 (1.156)***	-4.249 (0.261)***	-2.703 (0.182)***

Note: This table reports the estimation results of the multinomial logit model with basis category property damage based on 8612 observations. For the traffic flow (AHT) hourly values averaged over two representative weeks are used. The pseudo-R² is equal to 0.1183.

- * Statistical significance of the parameters is 10%.
- ** Statistical significance of the parameters is 5%.
- *** Statistical significance of the parameters is 1%.

This can most likely be explained by the lower driven speed when there is a lot of traffic.

Accidents caused by females tend to be less often fatal (-24%), but lead to severe and major injuries more often (42%) and (42%). Property damage is also slightly less likely (-11%). The effect of age is also not monotone with fatal accidents being (42%) more likely for every (42%) increase in age, while injuries are a bit less likely. The effect on property damage is negligible.

Tables 6 and 7 report the results when hourly traffic information is used. First note that the variables *Daylight*, *Winter* and *construction site* are not significant anymore. Besides that, most results are very similar to the ones using daily traffic information. The variables *Road condition*, *Truck* and *Age* are not significant for the category fatal anymore, but that is likely due to the small number of observations for this outcome. The findings for the variable *Speed limit* are now more in line with intuition after controlling for the daily seasonality in traffic flow. The probability of death is reduced by 51% and the probability of severe injury by 19%. However, this model does not contain the variable indicating a construction site, so *Speed limit* may pick up part of its effect given the high

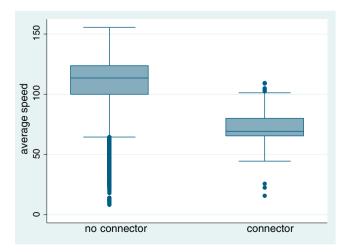


Fig. 2. Average speed at interchanges (connectors) and regular road segments.

correlation between these two variables. The effect of *AHT* is similar to the effect of *ADT* in the model above, but now the effect on minor injuries is positive with an elasticity of 0.24%.

A summary of our qualitative finding can be found in Table 10. As mentioned above, the models using daily and hourly traffic flow data basically lead to the same conclusions about the effects influencing accident severity with the exception of the effect of speedlimits. More importantly, our results are also mostly in line with previous finding in the literature on accident severity in a number of distinct settings. Some factors that are typically found to increase accident severities are night/darkness (Chimba and Sando, 2009; Quddus et al., 2009; Eluru et al., 2008; Gray et al., 2008), involvement of pedestrians (Chang and Wang, 2006; Eluru et al., 2008) and motorcycles (Chang and Wang, 2006; Yamamoto and Shankar, 2004; Wang and Abdel-Aty, 2008), and collisions (Savolainen and Mannering, 2007; Chang and Wang, 2006; Yamamoto and Shankar, 2004). Traffic volume is usually found to decrease accident severities (Milton et al., 2008; Christoforou et al., 2010). There are also several factors for which different studies find distinct effects. Some of those are the role of male drivers, weather or road conditions. Our study adds some findings for the effect these variables have on accident severities for the case of the German Autobahn.

4.2. Mixed logit model

As mentioned in Section 3, a common way of dealing with potential heterogeneity is to use a mixed logit model that allows certain parameters to be randomly distributed and we estimate this model using our data. The parameter estimates are given in Tables 8 and 9. The number of Halton draws to evaluate the log-likelihood function was 200 for the model with daily traffic data and 500 for the specification using hourly traffic information. This distinction had to be made due to the long computation time needed to estimate such a model for a large data set with many covariates. In fact, the specification search with respect to the random parameters had to

 $^{^{5}}$ The estimation was done using the STATA routine $\it{mixlogit}$, see Hole (2007) for details.

Table 7 (Pseudo-)elasticities for the multinomial logit model with AHT.

Variable	Fatal	Severe injury	Minor injury	Property damage
Accident information				
Dry	1.252*	0.486***	0.272***	-0.124***
Persons involved	1.298***	0.868***	0.922***	-0.421***
Collision	1.820***	1.301***	0.459***	-0.281***
Sight	-13.596***	1.001**	1.010***	-0.831**
Obstacle	-15.250***	-2.646***	-1.271***	0.295***
Road condition	0.237	-0.399**	-0.020	0.037
Motorcycles	3.057***	2.141***	0.451***	-1.282***
Truck	0.005	-0.196**	-0.746***	0.206***
Pedestrian	4.050***	1.580***	0.807***	-1.496***
Accident location				
Interchange	-1.910*	-0.398**	-0.357***	0.122***
Speed limit	-0.513	-0.190**	0.062	0.002
Traffic control	-13.079***	-14.218***	0.789***	-0.250
AHT	-1.271***	-0.312***	0.238***	-0.042***
Responsible person				
Female	-0.109	0.457***	0.264***	-0.131***
Age	-0.287	-0.116	-0.413***	0.120***
Mean probability	0.005 (0.016)	0.068 (0.071)	0.204 (0.124)	0.723 (0.174)

Note: This table reports the elasticities corresponding to the estimation results in Table 6. Elasticities are averaged over all observations. For binary regressors we report the pseudo-elasticities using (4).

- * Statistical significance of the parameters is 10%.
- ** Statistical significance of the parameters is 5%.

be done using only 50 Halton sequences for each evaluation of the log-likelihood, because a large number of models had to be estimated. In order to improve the numerical stability we decided to set the coefficients of the variables *Sight*, *Obstacle* and *traffic control*

Table 8Estimated coefficients of the mixed logit model with ADT.

Variable	Fatal	Severe injury	Minor injury
Accident information	1		
Daylight	$-0.986(0.242)^{***}$	$-0.288 (0.066)^{***}$	0.043 (0.052)
Winter	-0.582**(0.297)	$-0.538(0.081)^{***}$	$-0.299 (0.057)^{***}$
Dry	$0.854 (0.346)^{**}$	$0.436(0.088)^{***}$	$0.192 (0.062)^{***}$
Persons involved	-0.340(0.233)	$0.163(0.078)^{**}$	$0.937 (0.041)^{***}$
SD	0.999 (0.141)***	$0.782 (0.070)^{***}$	$0.605 (0.048)^{***}$
Collision	2.810 (0.320)***	2.161 (0.104)***	-0.182(0.217)
SD			3.124(0.313)***
Sight	-	1.438 (0.338)***	1.660 (0.311)***
Obstacle	-	$-1.806(0.296)^{***}$	$-1.677 (0.221)^{***}$
Road condition	-0.235(0.415)	$-0.428 (0.107)^{***}$	0.010 (0.090)
Motorcycle	5.436 (0.488)***	4.578 (0.214)***	2.495 (0.167)***
Truck	0.351 (0.497)	-0.072(0.070)	$-1.125 (0.060)^{***}$
SD	1.282 (0.539)**		
Pedestrian	8.831 (0.983)***	5.572 (0.637)***	3.829 (0.567)***
Accident location			
Interchange	$-3.637(1.196)^{***}$	$-1.852(0.275)^{***}$	$-2.634 (0.964)^{**}$
SD			3.606(1.105)***
Speedlimit	$0.660 (0.280)^{**}$	0.295 (0.080)***	1.002 (0.059)***
Construction site	$-2.595(0.732)^{***}$	$-0.854 (0.208)^{***}$	$-0.651 (0.083)^{***}$
SD		0.332 (0.651)	
Traffic control	-	-0.080(0.857)	1.408 (0.403)***
ADT	$-0.033(0.006)^{***}$	$-0.015(0.001)^{***}$	$-0.006(0.001)^{***}$
SD	$0.008 (0.005)^*$		
Responsible person			
Female	-0.146(0.327)	0.620 (0.074)***	0.567 (0.055)***
Age	-0.013 (0.014)	$-0.008(0.021)^{**}$	$-0.013 (0.002)^{***}$
SD	0.031 (0.007)***	0.006 (0.008)	
Constant	$-6.555 (0.640)^{***}$	-3.626 (0.175)***	-3.248 (0.143)***

Note: This table reports the estimation results of the mixed logit model with basis category property damage based on 37,735 observations. For the traffic flow (ADT) average daily values are used.

- * Statistical significance of the parameters is 10%.
- ** Statistical significance of the parameters is 5%.
- *** Statistical significance of the parameters is 1%.

equal to zero for the category death. The coefficient of *traffic control* was also restricted to equal zero for the category *severy injury* when using hourly traffic. The reason for this decision is the fact that there are no cases where these dummies are equal to one and an accident of the corresponding category occurs. This results in extremely variable and huge parameter estimates with unreasonably large standard errors. The model fit and the estimates of the remaining coefficients were not influenced by these restriction.

The estimation results for the random parameter models are qualitatively very close to the ones using fixed coefficient models. Noticeable differences can be seen for the variables *Persons involved*

Table 9Estimated coefficients of the mixed logit model with AHT.

Variable	Fatal	Severe injury	Minor injury
Accident information			
Dry	3.354 (2.172)	$0.742(0.172)^{***}$	0.490 (0.133)***
Persons involved	-2.628(1.850)	$0.339(0.120)^{***}$	1.187 (0.107)***
SD	2.439 (1.122)**	$0.733(0.115)^{***}$	$0.740(0.118)^{***}$
Collision	4.424 (1.823)**	2.043 (0.178)***	0.261 (0.280)
SD			2.534(0.471)***
Sight		$1.784(0.878)^*$	3.025 (0.835)***
Obstacle		$-3.146(0.909)^{***}$	$-1.483(0.423)^{***}$
Road condition	1.153 (1.995)	$-0.529 (0.200)^{***}$	0.049 (0.162)
Motorcycles	7.406 (2.110)***	$4.304(0.375)^{***}$	2.254 (0.365)***
Truck	-0.106(0.790)	$-0.376(0.135)^{***}$	$-1.518(0.151)^{***}$
Pedestrian	11.072 (4.348)***	3.852 (1.021)***	2.935 (0.988)***
Accident location			
Interchange	-3.340(2.064)	$-0.538 (0.228)^{**}$	$-0.727 (0.214)^{***}$
Speed limit	-0.825(0.808)	$-0.273(0.130)^{**}$	0.076 (0.097)
Traffic control	-	-	1.588 (0.575)***
AHT	$-3.119(1.243)^{***}$	$-0.440 (0.135)^{***}$	0.488 (0.104)***
Responsible person			
Female	-0.327(0.918)	0.691 (0.131)***	0.538 (0.112)***
Age	-0.023(0.024)	$-0.007(0.004)^*$	-0.020 (0.003)***
Constant	-9.996 (3.309)***	-4.535 (0.318)***	-3.962 (0.336)***

Note: This table reports the estimation results of the mixed logit model with basis category property damage based on 8612 observations. For the traffic flow (AHT) hourly values averaged over two representative weeks are used.

- * Statistical significance of the parameters is 10%.
- ** Statistical significance of the parameters is 5%.
- *** Statistical significance of the parameters is 1%.

^{***} Statistical significance of the parameters is 1%.

Table 10 Summary of qualitative findings.

	-		
Variable	Daily traffic data	Hourly traffic data	Overall effect
Accident information			
Daylight	↓	_	↓
Winter	↓	_	↓
Dry	↑	↑	1
Persons involved	<u></u>	†	<u>†</u>
Collision	↑	↑	↑
Sight	0↑↑	0↑↑	0↑↑
Obstacle	↓	↓	↓
Road condition	↓	0↓↓	↓
Motorcycle	↑	↑	↑
Truck	$\uparrow\downarrow\downarrow$	0↓↓	0↓↓
Pedestrian	↑	↑	↑
Accident location			
Interchange	↓	↓	↓
Speedlimit	↑	$\downarrow \downarrow 0$	0
Construction site	↓	-	↓
Traffic control	00↑	00↑	00↑
ADT/AHT	↓	$\downarrow\downarrow\uparrow\uparrow$	↓
Responsible person			
Female	0↑↑	0↑↑	0↑↑
Age	$\uparrow\downarrow\downarrow$	0↓↓	0↓↓

Note: This table summarizes the qualitative findings of our study, where \uparrow means that a factor generally increases the severity of accident, \downarrow means that it generally decreases the severity and a 0 means that no clear effect could be identified. When the effect is not the same for all categories one symbol is reported for fatal, severe injuries and minor injuries, respectively.

for all categories or *Collision* for the category *minor injury*. Furthermore, allowing for random coefficients significantly increase the log-likelihood, from -22576 to -22337 for the model with daily traffic information and from 5828 to 5765 for the model with hourly traffic information. Note that this increase in log-likelihood is not as large as in other applications of mixed logit models reported in the literature. The estimated standard deviations of the random coefficients are mostly rather large compared to the estimated coefficients, which indicates that both positive and negative effects are very likely for those variables. For the model with hourly traffic data only the variables *Persons involved* and *Collision* have random coefficients. For the model with daily traffic data there is more evidence for random coefficients. However, for most variables the standard deviation for the parameters was significant only for one of the categories with the exception of *Persons involved* and *Age*.

Overall the estimation results confirm the qualitative findings from the fixed parameter models in Section 4.1.

5. Discussion

From all the factors influencing the accident severity we found, summarized in Table 10, only some are of interest for policy makers. While the findings on given factors such as the gender of the driver or daylight conditions are interesting by themselves, the factors that can be manipulated directly or indirectly are the ones that one is eventually interested in. As mentioned in the introduction, the German government introduced a traffic safety program in 2011 with the aim to "guarantee safe mobility for all citizens" (Bundesministerium für Verkehr and Bau und Stadtentwicklung, 2011) and we want to discuss some of the recommendations given considering our empirical findings.

All model specifications indicate that accidents due to a collision with a roadside object tend to be more severe than others. Leaving the road often happens to tired and distracted drivers. Such accidents can potentially be prevented by the introduction of rumble strips, which are still very uncommon in Germany. This measure has been recommended by the safety programm

for particularly dangerous road segments. Furthermore, accidents involving motorcycles tends to be highly dangerous. This has been recognized in the safety programm, which recommends breaking systems with automatic anti-lock technology as a possible countermeasure. However, it requires some initiative from the producers. The increased severity of accidents involving large trucks is also acknowledged in the program. One problem is seen in a too high or unstable skid plate making accidents with trucks particularly dangerous. Furthermore, increased inspection requirements may help prevent accidents caused by trucks.

The effect of intelligent traffic control systems is not completely unambiguous. We likely face the problem of unobserved heterogeneity because according to the traffic safety plan they are usually placed at locations where many accidents or traffic jams occur. Furthermore, our sample contains only very few accidents that happened when a traffic control system was in effect. Therefore the results from the multinomial logit model should be interpreted with care. Still the fact that no accidents with fatalities or severe injuries have occurred when traffic control systems were present can be seen as an indication of their effectiveness and it seems useful to consider this costly measure in the future. The added advantage is that traffic jams can be reduced, which prevents significant economic costs. Still, to make reliable statements about the effects of traffic control systems more refined data would be required, e.g. information when temporary speed limit are in effect due to fog or heavy rain.

The effect of the driven speed could not be analyzed, but intuition and the finding of many previous studies indicate that this has a clear effect on the accident severity. Recalling that on many German highways there is no general speed limit it would nevertheless be interesting to study the effect of extremely high driven speeds (e.g. 180 km/h or more), as one can expect very large increases in accident severities. A further concern mentioned in the safety program is the safety at construction sites. After controlling for the involvement of pedestrians and speed limits the effects were not significant (multinomial logit model) or negative (ordered probit). Thus mainly injuries to workers seem to be the problem, which is suggested to be countered by a better organization of the construction sites.

Concerning the characteristics of the driver it was found that older drivers tend to be involved in more severe accidents. This is not surprising, as older drivers may not always be able to react appropriately in risky situations. Health checks for drivers exceeding a certain age may raise the awareness of these drivers, although legally it is not possible to prohibit anyone from driving due to age related health problems. It would be interesting to investigate the severity of the injury of the person causing the accident but this is not possible due to limitations in our data set.

Summing up, the measures suggested in the traffic safety program are supported by our empirical findings and should be followed. Nevertheless, more research would be required to make clear recommendations on how to spend limited resources in the most effective way. In particular, we believe that more and better data would allow us to draw more refined conclusions concerning the factors influencing the accident severity. A useful measure would be to collect accident data centrally for the whole country, but also to make detailed traffic flow and speed data more easily accessible. However, as the quality and quantity of the data has been improving over the past years we are optimistic that this trend will continue.

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