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A Bayesian Neural Network approach to estimating the Energy Equivalent Speed

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Abstract

To reduce the number and the gravity of accidents, it is necessary to analyse and reconstruct them. Accident modelling requires the modelling of the impact which in turn requires the estimation of the deformation energy. There are several tools available to evaluate the deformation energy absorbed by a vehicle during an impact. However, there is a growing demand for more precise and more powerful tools. In this work, we express the deformation energy absorbed by a vehicle during a crash as a function of the Energy Equivalent Speed (EES). The latter is a difficult parameter to estimate because the structural response of the vehicle during an impact depends on parameters concerning the vehicle, but also parameters concerning the impact. The objective of our work is to design a model to estimate the EES by using an original approach combining Bayesian and Neural Network approaches. Both of these tools are complementary and offer significant advantages, such as the guarantee of finding the optimal model and the implementation of error bars on the computed output. In this paper, we present the procedure for implementing this Bayesian Neural Network approach and the results obtained for the modelling of the EES: our model is able to estimate the EES of the car with a mean error of 1.34 m s^{-1}. Furthermore, we built a sensitivity analysis to study the relevance of model's inputs.

Keywords: Energy Equivalent Speed; Artificial Neural Network; Bayesian inference; Road traffic accident

1. Introduction

At the beginning of the XXth century, accidents were considered as unexpected and unpredictable events (Ferrandez, 1995). This fatalistic conception was replaced by a scientific one that tries to identify the causes of the accident, in order to reduce the number or the gravity of the accidents. To achieve this goal, it is necessary to analyse and to model the accident. The modelling of the accident requires the modelling of the impact, which itself necessitates the estimation of the deformation energy.

The deformation energies are difficult to estimate because the structural response of the vehicle during an impact depends on several parameters (Varat and Husher, 1999): those concerning the vehicle (material, make, type . . .), but also those concerning the impact (localization of the impact on the vehicle, direction of the impact, intensity of the impact . . .).

There are several tools available to determine the energy absorbed by a vehicle during an impact (McHenry, 2001; Coyle, 2004; Rövid, 2004). Most of them rely on an experimental database of crash tests.

Nevertheless, according to (Rövid, 2004), there is a growing demand for more precise, more user-friendly and more powerful methods.

For our study, we express the deformation energy according to the Energy Equivalent Speed (EES). The EES is a parameter which can be determined by human expertise from crash test database (Zeidler et al., 1986).

A previous neural model of the damaged car's EES in the case of a frontal crash was designed from a damaged car database, implemented by the LAB (Laboratory of Accidentology and Biomechanics) (Rivière et al., 2004). The use of Artificial Neural Network (ANN) was proving to be a suitable tool for modelling the EES. Indeed, the availability of data and the fact that ANN are data-driven approaches capable of performing a non-linear

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mapping between a set of input and output variables allowed us to have interesting results. However, the method employed presents some drawbacks.

In this paper, we improve this model by proposing a Bayesian Neural Network approach to modelling the EES.

Bayesian probability theory is currently experiencing an increase in popularity in the sciences as a means of probabilistic inference (Malakoff, 1999; D'Agostini, 2003). Bayesian methods can be used either for model selection problems (Bretthorst, 1990) or for parameter estimation problems (Bretthorst, 1990). In the field of Neural Network (NN), Bayesian methods offer significant advantages over the classical NN approach (MacKay, 1995).

In Section 2, we present several models which can be used to estimate the deformation energy absorbed by a vehicle during an impact. Section 3 deals with a classical NN approach to the modelling of the EES while Section 4 presents the principle of Bayesian learning for the implementation of a NN and its application to the estimation of the EES. The last part draws the conclusions of the study.

2. Modelling the EES

During a crash involving two vehicles, two phases can be distinguished: the compression phase and the restitution phase. The compression phase lasts from the contact of the vehicles to the point of maximum compression. During this phase, the energy is stocked until the point of maximum deformation. During the restitution phase, if there is a post-crash phase, this energy is released as deformation energy and kinetic energy.

So, the energetic balance during the collision between cars can be written as follows:

$$E_{kb1} + E_{rb1} + E_{kb2} + E_{rb2} = E_{ke1} + E_{re1} + E_{ke2} + E_{re2} + E_{d2} \quad (1)$$

where E_{kb1}, E_{kb2} are the kinetic energies at the beginning of impact for the vehicles 1 and 2; E_{ke1}, E_{ke2} the kinetic energies at the end of impact for the vehicles 1 and 2; E_{d1}, E_{d2} the deformation energies for the vehicles 1 and 2; E_{rb1}, E_{rb2} the rotational energies at the beginning of impact for the vehicles 1 and 2; and E_{re1}, E_{re2} the rotational energies at the end of impact for the vehicles 1 and 2, respectively.

The kinetic and rotational energies can be calculated easily:

$$E_k = \frac{1}{2}mv^2 \quad (2)$$

$$E_r = \frac{1}{2}J\omega^2 \quad (3)$$

where v is the speed of vehicle ($m\,s^{-1}$), m the mass of vehicle (kg), J the moment of inertia of vehicle ($kg\,m^2$), and ω the rotational speed of vehicle ($rad\,s^{-1}$).

The deformation energies are more difficult to estimate. They can be modelled in different ways.

For example, Rövid (2004) proposed a technique based on the combination of new methods of digital image processing and intelligent techniques (fuzzy- and Neural Network-based techniques) to estimate the deformation energy.

The accident reconstruction software CRASH[1] and the versions developed from the software[2] model the deformation energy as follows (McHenry, 2001):

$$E_d = \int \frac{B}{2}\left(\frac{A}{B} + C\right)^2 dW \quad (4)$$

where A ($kg\,m^{-1}$) and B ($kg\,m^{-2}$) are deformation coefficients obtained by empirical methods, in tables constituted by crash-tests, C is the depth of vehicle deformation, and W the total width of vehicle deformation.

For our study, we express the deformation energy as follows:

$$E_d = \frac{1}{2}m \times EES^2 \quad (5)$$

with m as the mass of the vehicle (kg), and EES the Energy Equivalent Speed ($m\,s^{-1}$).

The EES is a speed which can be estimated by purely empirical methods: the expert compares the deformations and the intrusions of the damaged car with a vehicle in the crash test database, and thereby estimates the EES. This method allows the estimation of the EES with a precision of more or less 1.39 $m\,s^{-1}$. Fig. 1 presents a vehicle damaged in a real crash and a vehicle damaged in a crash test: the comparison of deformations of both these vehicles allows the expert to estimate an EES of 16.1 $m\,s^{-1}$.

Thus, the EES is an important parameter to precisely determine. First, it contributes to determine the deformation energy, and so to reconstruct the pre-crash phase, from information about the post-crash.

Moreover, the estimation of the EES contributes to the characterization of the severity of the crash. Besides, Miltner and Salwender (1995) showed the relationship between the collision variables, including EES, and the injury severity.

3. A classical Neural Network approach to modelling the EES

ANNs are powerful mathematical tools capable of establishing a non-linear mapping between a set of inputs and one or several outputs. The Neural Network approach has proved successful in many fields. They offer a practical and rapid means for developing models provided there are enough data available.

In this section, we succinctly present the structure of a NN, the obtained results on the previous EES model and the limits of the study. A more detailed presentation of this study can be found in Rivière et al. (2004).

[1] CRASH (Calspan Reconstruction of Accident Speeds on the Highway) is accident reconstruction software, created in 70s by Raymond R. McHenry with the collaboration of the Calspan Corporation.

[2] In 1979, CRASH 3 was created for the National Accident Sampling System (NASS). In 1984, another version of CRASH was developed by the National Highway Traffic Safety Administration (NHTSA) and the Engineering Dynamics Corporation (EDC): Engineering Dynamics Corporation Reconstruction of Accidents Speeds on the Highway (EDCRASH).

Fig. 1. Estimation of the EES by empirical methods.

3.1. NN structure

To implement a NN, several types of structures can be used. In order to model the EES, we chose the *Multi-Layer Perceptron* (MLP) structure. The MLP structure consists of an input layer, one or several hidden layers and an output layer. The input layer gathers the model's inputs while the output layer yields the model's output (in our case, the EES). The hidden layer is characterized by several non-linear units (or neurons). The non-linear function (also called activation function) is usually the tangent hyperbolic function (tanh), given by Eq. (6).

$$\tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} \tag{6}$$

The parameters of the NN model are given by the so-called weights ω and biases b denoted here by the parameter vector $\boldsymbol{w}$ that link the layers between them.

Fig. 2 presents a one hidden layer MLP.

This MLP structure yields the following non-linear relationship between the output, y, and the inputs, x_i:

$$y = y(x; \boldsymbol{w}) = \sum_{j=1}^{k} \left[\omega_j \tanh \left(\sum_{i=1}^{n} \omega_{ji} x_i + b_{1j} \right) \right] + b_2 \tag{7}$$

where n is the total number of inputs, and k the total number of hidden neurons.

The methodology used to implement an ANN can be found in Bishop (1995).

3.2. Application to the modelling of the EES

The neural model of EES was implemented from the French LAB[3] database. This database contains about 4500 crashed cars from accidents that have occurred in France since 1970. The information about crash configuration, impact characteristics, car deformations and occupant injuries is collected by specially trained accident specialists using a retrospective methodology. The data is not collected on-scene but a few days or weeks after the accident. Careful car examination is performed at tow-away garages and injury assessments are collected from hospitals. The police report is also consulted. Important parameters are recorded or estimated such as direction of impact, depth of intrusion in different parts of the car, EES, impact speed, closing speed, seat belt use, type of restraint, occupant age and gender, injuries by type, severity, body region, etc.

There are basically two kinds of surveys. The first is regional, the second national. All cars crashed in an injury accident in an area of 2.5×10^6 to 3×10^6 m^2, in the west of Paris, are systematically examined regardless of crash configuration. All kinds of impact (frontal, side, rollover, etc.) are of interest. These crashes are assumed to be almost representative of car crashes in France. In addition, there are special investigations of serious accidents involving newer cars, regardless of their make and model, all over France. These investigations are assumed to give important insight into the evaluation of the effectiveness of newer car protection. Two hundred and fifty car crashes of the first type and 150 crashes of the second type are systematically collected, examined, coded and analysed each year.

Among the 4500 crashed cars, we were interested in the 1750 vehicles damaged in frontal crashes. In order to visualize and characterize these data, we built a factor analysis, and a clustering of the database. We determined three clusters, which are presented Fig. 3.

As we are interested in the modelling of EES, we studied the distribution of the EES in the database. We can note that 82% of vehicles of the database have an EES between 8.1 and 18.3 m s^{-1}. Eight percent of vehicles have an EES below 8.1 m s^{-1}. That is understandable, since a crash is studied and is added to the database only if there is at least one injury. And 10% of vehicles have an EES above 18.3 m s^{-1}; these cases are less widespread, and a crash can be studied only if the occupants of vehicles are alive.

From the thousand or so variables of the LAB database, we selected the parameters which influence the model's output: the EES. Sixteen inputs, described in Table 1, were selected.

Once the input variables had been selected, we built our learning and test sets. We split at random 1250 vehicles for the learning set and 500 vehicles for the test set. Then, we normalized the inputs: numerical and discrete data are normalized between 0.1 and 0.8. The categorical variables were normalized by a 1-of-n coding.

During the learning phase, we tried to compute the optimal number of units on the hidden layer from a cross-validation (CV) method. We found that a NN with 16 hidden neurons exhibits a minimal error between the output estimated by the model and the experimental output. In order to prevent overtraining, we employed the weight-decay regularization method. Again, the

[3] LAB: Laboratory of Accidentology and Biomechanics, France.

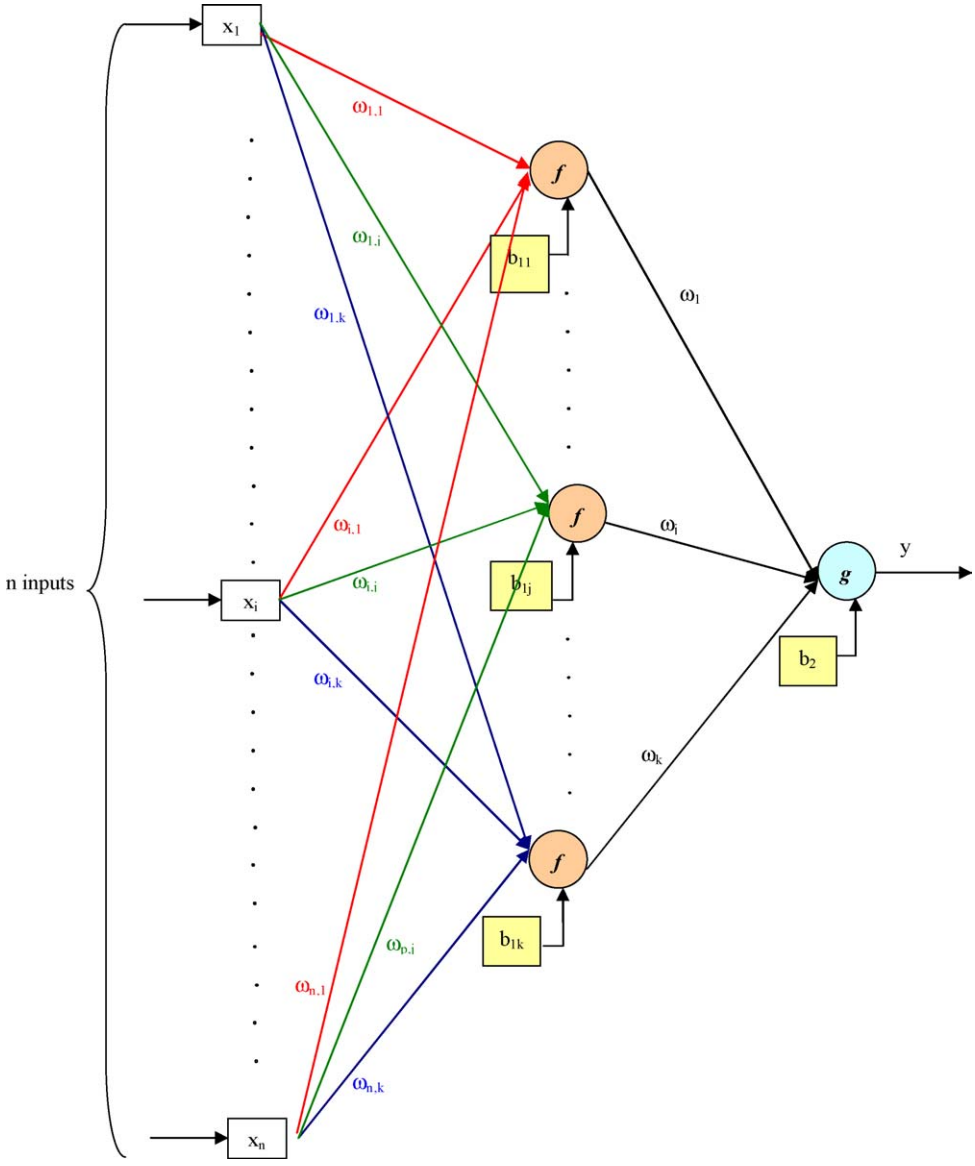


Fig. 2. MLP structure.

optimal value of the weight-decay term, X, must be searched through CV techniques. We found the value of 0.16 for the optimal weight-decay term.

We tested the NN built on the test set. The model estimates the EES of damaged cars with a mean error of 1.28 m s^{-1}. Fig. 4 presents the distribution of the error between the EES estimated by the model and the experimental EES of the test set.

We note that:

For 54% of damaged cars, the EES is estimated by the model with a precision of more or less 1.1 m s^{-1}.
For 73% of damaged cars, the EES is estimated by the model with a precision of more or less 1.7 m s^{-1}.

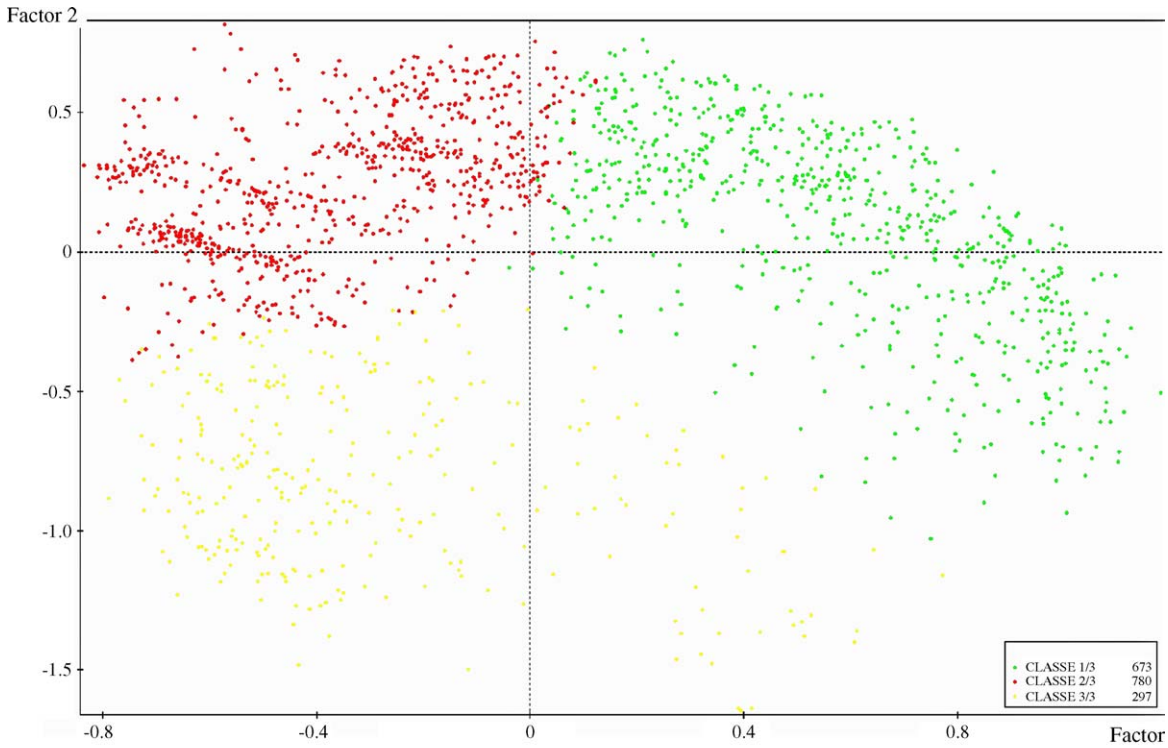
3.3. Limits of the classical approach

The modelling of EES with the classical NN approach gave us interesting results. Indeed, from the shock's and damaged car's characteristics, our model is able to estimate the EES of the car with a mean error of 1.28 m s^{-1}.

Nevertheless, the methodology employed presents several limits:

(a) The overtraining can be quickly reached when some precautions are not taken. First, we determined the optimal number of units on the hidden layer from a cross-validation technique. We found that the optimal structure is the NN with 16 hidden neurons. Moreover, in order to have a model which is not fitted to the noise, we applied the weight-decay regularization method. Again, the optimal weight-decay coefficient was computed by cross-validation.

Nevertheless, the CV technique presents several disadvantages. First, the CV technique needs a separate data set (so fewer data for the learning set), the validation set, in order to evaluate the optimal number of hidden neurons (i.e. optimal structure) and the optimal value of λ. Second, the



Cluster 1	Cluster 2	Cluster 3
The vehicles of this cluster have an EES between 16,67 and 19,44 $m.s^{-1}$. That corresponds to an intrusion between 50 and 150 cm, and an overlap between 40 and 60 %. In the passenger compartment, the driver side is damaged.	The vehicles of this cluster have an EES between 8,33 and 13,89 $m.s^{-1}$. That corresponds to an intrusion between 0 and 50 cm. There is no damage inside the vehicle.	The vehicles of this cluster have an EES between 13,89 and 19,44 $m.s^{-1}$. That corresponds to an intrusion between 50 and 100 cm, and an overlap between 80 and 100 %. Inside the vehicle, the passenger side is damaged.

Fig. 3. Clustering of the database.

determination of the optimal structure or the optimal regularization coefficient is based on a validation error which is known to be a noisy performance measure (MacKay, 2003). So, we are not warranted to have the optimal structure of the model and the optimal value of λ. Furthermore, the range of regularization coefficients must be pre-specified and can only be optimized in a discrete manner. Finally, last but not the least, CV is computationally demanding.

(b) In the previous model, the uncertainty on the model's parameters and the noise on data are not taken into account in the computation of the output's predictions.

(c) In other words, there is no guarantee that the classical approach produces the optimal model.

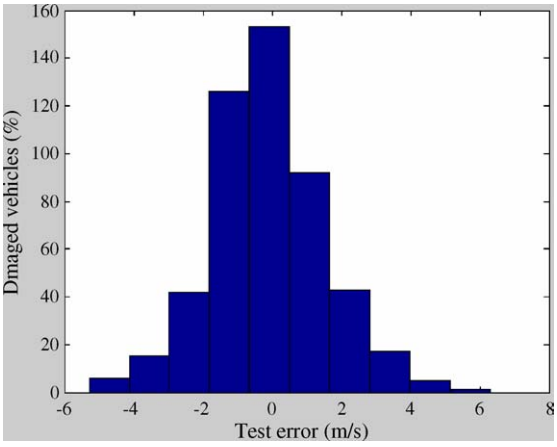


Fig. 4. Test error distribution.

In order to compensate for these limits, we employed a less conventional approach to implement the EES's neural model. This approach consists in using Bayesian techniques for the Neural Network learning.

4. Combining Bayesian and Neural approaches to modelling the EES

4.1. Bayesian probability theory

Bayesian probability theory is currently experiencing an increase in popularity in the sciences as a mean of probabilistic inference (Malakoff, 1999; D'Agostini, 2003). Cox (1946) showed that any method of scientific inference that satisfies

Table 1
Description of model's input variables

Description of the variable	Type of variable
Kerb weight (kg)	Numerical
Model year	Discrete
Vehicle make	Categorical
Spread of principal impact	Categorical
Direction of principal impact	Discrete
Deformation height of principal impact	Categorical
Principal impact intrusion (cm)	Numerical
Overlap of principal impact (%)	Discrete
Zone of internal deformation	Categorical
High intrusion: rearward movement of outboard dashboard, driver side (cm)	Numerical
High intrusion: rearward movement of outboard dashboard, passenger side (cm)	Numerical
Low intrusion: rearward movement of outboard footwell, driver side (cm)	Numerical
Low intrusion: rearward movement of footwell in the driver's axis (cm)	Numerical
Low intrusion: rearward movement of footwell in the passenger's axis (cm)	Numerical
Low intrusion: rearward movement of outboard footwell, passenger side (cm)	Numerical
Maximal reduction of the passenger compartment's width (cm)	Numerical

simple rules of logical and consistent reasoning must be equivalent to the use of ordinary probability theory as originally developed by Bayes (1763) and Laplace (1812), and treated by other authors like MacKay (1995), or Gelman et al. (2004). Two of these simple rules of probability theory are the sum rule and product rule (where prob stands either for a probability or a probability density function (p.d.f)):

$$\text{prob}(x|I) + \text{prob}(\bar{x}|I) = 1 \tag{8}$$

$$\text{prob}(x, y|I) = \text{prob}(x|y, I) \times \text{prob}(y|I) \tag{9}$$

where $\bar{x}$ represents the proposition that x is false, the vertical bar "|" means "given" and the comma is read as the conjunction "and". Two useful relationships are derived from these basic rules namely, the Bayes' theorem and the marginalisation relationship:

$$\text{prob}(x|y, I) = \frac{\text{prob}(y|x, I) \times \text{prob}(x|I)}{\text{prob}(y|I)} \tag{10}$$

$$\text{prob}(x|I) = \int \text{prob}(x, y|I)\text{d}y \tag{11}$$

where the symbol I denotes the relevant background and assumptions.

Notice that, for sake of clarity, the relevant background I will be omitted in the subsequent formulae related to the p.d.fs.

In the Bayesian context, a probability represents a degree-of-belief (or encodes a state of knowledge) that is how much something is true based on all the relevant information at hand.

However, this concept seems too vague and too subjective to the school of conventional statistics (i.e. frequentist approach) that defined probability as the long-run relative frequency with which an event occurred, given infinitely many repeated experimental trials. Indeed, the concept of degree-of-belief is criticized by the school of frequentists as it leads to subjectivity (because my belief could be different from yours). Although the frequency definition appears to be more objective, it fails to tackle most real-life scientific problems. Further, in a Bayesian view, all probabilities are always conditional (i.e. based on all the relevant background) and as stated by Jaynes (2003), objectivity requires only that two people having the same information should assign the same probability. A good review of the Bayesian approach is given by Jaynes (1986) and Loredo (1990).

Bayesian methods can be used either for model selection problems (Bretthorst, 1990) or for parameter estimation problems (Bretthorst, 1990). As Bayesian probability theory does not define a probability as a frequency of occurrence but rather as a reasonable degree-of-belief, it is possible to assign probabilities to propositions such as "The probability that parameter θ had value x when data was taken". In other words, in the Bayesian framework, questions of the form: "What is the best estimate of a parameter one can make from the data and prior information?" make perfect sense.

In a modelling application, Bayesian inference deals with the estimation of the values of p model parameters $\theta = (\theta_1, \theta_2, \ldots, \theta_p)$ about which there may be some prior beliefs. These prior beliefs can be expressed as a probability density function (p.d.f) called *prior*, $\pi(\theta)$ and may be interpreted as the probability placed on all possible parameter values before collecting any new data. The dependence of observations (or measurements) $D = (d_1, d_2, \ldots, d_N)$ on the p parameters θ can be also expressed as a p.d.f: $L(D/\theta)$ called the *likelihood function*. The latter is used to update the prior beliefs on θ, to account for the new data D. This updating is done through the Bayes' theorem:

$$\pi(\theta|D) = \frac{\pi(\theta)L(D|\theta)}{\int_\theta \pi(\theta)L(D|\theta)\text{d}\theta} \tag{12}$$

where $\pi(\theta/D)$ represents the *posterior* p.d.f and expresses the values of the parameter after observing the new data. In other words, the *prior* is modified by the *likelihood* function to yield the *posterior*.

In the Bayesian framework, uncertainties in parameter values and model output are naturally assessed. For instance, the position of the maximum of the *posterior* p.d.f represents a best estimate of the parameter; its width or spread about this optimal value gives an indication of the uncertainty in the estimate of the parameters. Error bars for the model output take into account in a natural way two contributions: one arising from the intrinsic noise in the data and one arising from uncertainties in the parameter values given by the width of the posterior p.d.f.

In this paper, the principles of Bayesian reasoning are outlined and applied to the estimation the NN parameters. Further, it will be shown that the overfitting problem can be solved by using a Bayesian approach to control model complexity (Bishop, 1995). Appendix A describes the principle of the Bayesian NN learning. The methodology is applied to modelling the EES.

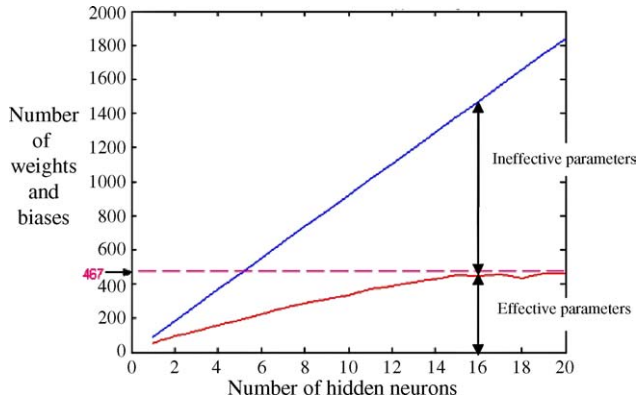


Fig. 5. Evolution of γ according to the number of hidden neurons.

4.2. Implementation of the EES's NN with a Bayesian learning and results

In this section, we apply the Bayesian techniques the implementation of the EES's model.[4]

4.2.1. Learning phase

The Bayesian NN learning consists in automatically determining the optimal structure of the NN. Indeed, the hyperparameters, α and β, which control the complexity of the model, the ordinary model parameters (weights and biases), and the parameter γ which corresponds to the effective number of weights and biases required to describe the model, are simultaneously optimized by using all the available data.

4.2.1.1. Determination of α and β. α and β were determined from EVIDENCE approach (Bishop, 1995). The network's learning was realized from both algorithms: Scaled Conjugate Gradients (SCG) and Quasi-Newton. As the SCG algorithm is more computationally efficient, we only present the results obtained with this algorithm.

In the EVIDENCE approach, the determination of hyperparameters, α and β, consists in studying their convergence, when the number of external cycles changes. We noted that the number of internal cycles has no (or very little) effect on the convergence of hyperparameters

To determine α and β, we chose an architecture with 16 hidden neurons (architecture of the previous NN) but we will see that this is not the optimal structure. We note that α converges on 12.5 and β on 504 after 16 external cycles.

4.2.1.2. Determination of the NN's optimal structure. Studying the convergence of the parameter γ allows us to determine the optimal structure of the model. γ corresponds to the effective number of weights and bias required to describe the model.

Fig. 5 presents the evolution of γ according to the number of units on hidden layer. We compared it with the curve of the total number of weights and biases for a given number of hidden nodes.

We can see that γ converges towards 467, which corresponds to approximately five units on the hidden layer. This means that among the 16 hidden neurons (1473 parameters) found in our previous work, only 5 of them are necessary.

During the learning phase, the ANN with five hidden units gives us a sum-of-squares error equal to 625. We note that $S(\boldsymbol{w})$ converges well to $N/2$.

4.2.2. Test phase

4.2.2.1. Expression of the EES. Having defined the network architecture and calculated the weights and the biases, we can express the EES according to inputs:

$$\text{EES} = \sum_{j=1}^{5}\left[\omega_j \tanh\left(\sum_{j=1}^{90}\omega_{ji}x_i + b_{1j}\right)\right] + b_2 \tag{13}$$

with x_i as the network inputs, ω_j the weight between the output neuron and the jth hidden neuron, ω_{ji} the weight between the jth hidden neuron and the ith input, b_{ij} the bias corresponding to the jth hidden neuron, and b_2 the bias corresponding to the output neuron.

4.2.2.2. Results of the test phase. We tested the EES's expression on the 500 damaged vehicles of the test set. We note that the model has captured the dynamic of the experimental data. Fig. 6 presents the distribution of the error between the EES estimated by the model and the experimental EES of the test base.

We note that:

For 52.4% of damaged cars, the EES is estimated by the model with a precision of more or less 1.1 m s^{-1}.
For 69.2% of damaged cars, the EES is estimated by the model with a precision of more or less 1.7 m s^{-1}.

Table 2 presents the characteristics of the test error distribution.

It seemed that the error peaks correspond to the EES values under-represented in the learning set. To confirm this idea, we observed the number of vehicles in the learning base and the

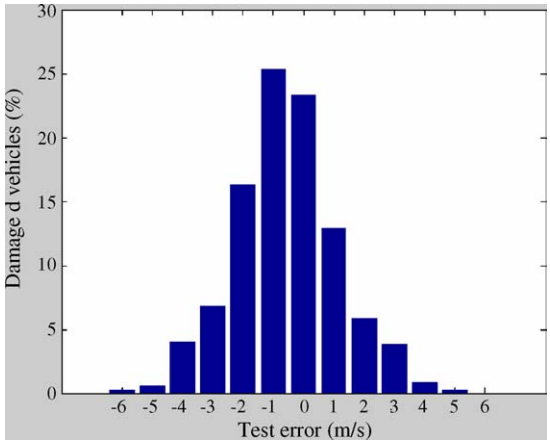


Fig. 6. Test error distribution.

[4] We used the Netlab's toolboxes (Nabney, 2002) for computations.

Table 2
Characteristics of the test error distribution

Absolute error ($m\,s^{-1}$)	
Minimum	0.003
Maximum	6.28
Mean	1.34
Standard deviation	1.10

mean absolute error during the test phase, according to the EES (Fig. 7).

The number of vehicles in the learning dataset is lower for the values of EES inferior to 8.1 $m\,s^{-1}$ or superior to 18.3 $m\,s^{-1}$ (see Section 3.2). We can note that for these range of values, the mean absolute error during the test phase is higher. It is understandable, since a good generalization requires a good learning, and a good learning requires a rich learning dataset with a high number of cases. Conversely, for EES between 8.1 and 18.3 $m\,s^{-1}$, the EES value is correctly approximated by the model.

4.2.2.3. Error bars on the network output. We have seen that the Bayesian approach enables the computation of error bars on the output, by taking the uncertainty of weights and data into account (see Appendix A).

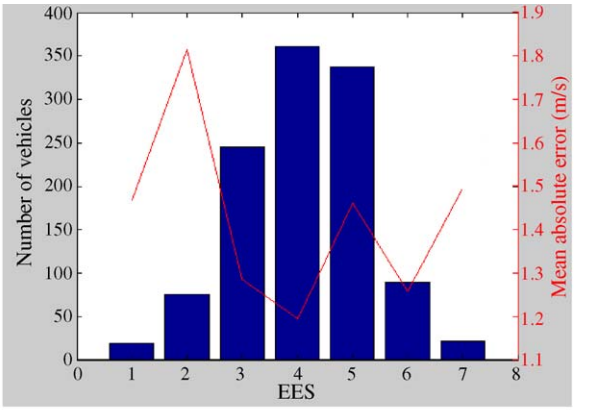
We present the EES estimation and the associated error bars for 100 examples of the test set (Fig. 8).

We can conclude that our model is concordant with the data.

4.3. Relevance of input variables

In this section, we propose to evaluate the relevance of the input variables in two steps: first, we treat the case where not enough inputs are collected to estimate the EES; second, we study the case where more inputs are available. The performance of our model will be assessed in these two cases.

In order to evaluate the relevance of the model's inputs on the output, and thus determine the mandatory input variables, we employed a Sensitivity Analysis (SA) method: the Extended



	1	2	3	4	5	6	7
EES ($m.s^{-1}$)	1.4-4.7	4.7-8.1	8.1-11.5	11.5-14.9	14.9-18.3	18.3-21.6	21.6-25

Fig. 7. Number of vehicles in the training set and mean absolute error during the generalization according to the EES.

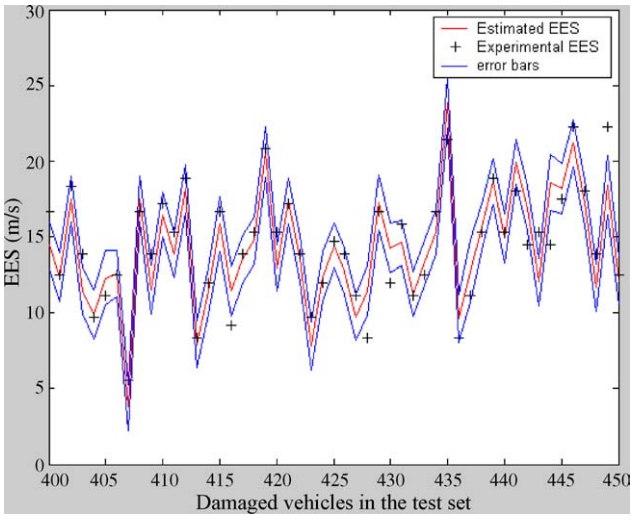


Fig. 8. Experimental EES and estimated EES with the associated error bars for 100 examples of the test set.

Fourier Amplitude Sensitivity Test method (Saltelli et al., 1999), called the EFAST method (see Fig. 9).

This method relies on a global Sensitivity Analysis of Model Output (SAMO) method. The EFAST principle is as follows: each input variable Z_i is assigned a frequency ω_i and each input oscillates periodically in the range [0.1, 0.9]. Notice that [0.1, 0.9] represents the minimal and maximal values taken by the input. As each input Z_i oscillates periodically between [0.1, 0.9] at the corresponding frequency ω_i, the model output Y exhibits different periodicities that result from the combination of the different frequencies $\omega_{i=1,\ldots,p}$, whatever the model F is. Notice that in our case, the model F is the Eq. (7). As stated by (Saltelli et al., 1999), if the ith input variable has strong influence on the output, the oscillations of Y at frequency ω_i shall be of high amplitude. This is a basis for computing a sensitivity measure for the input Z_i based on the evaluation of the Fourier amplitudes at the corresponding frequency ω_i and its harmonics. In other words, large Fourier amplitudes at the fundamental frequency ω_i and its harmonics indicate that the output is sensitive to the input variable Z_i. One major advantage in shifting the analysis into the frequency domain is that the spectral decomposition is equivalent to variance decomposition. Thus, each input is assigned

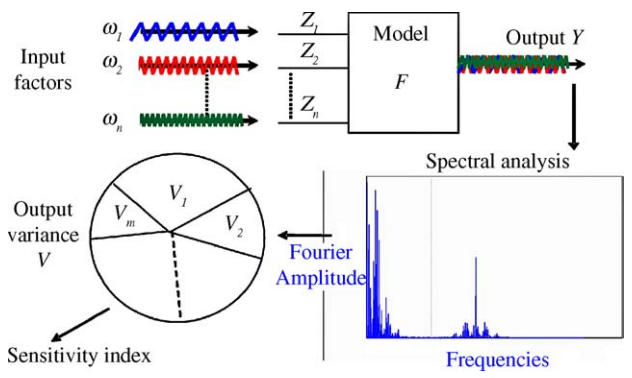
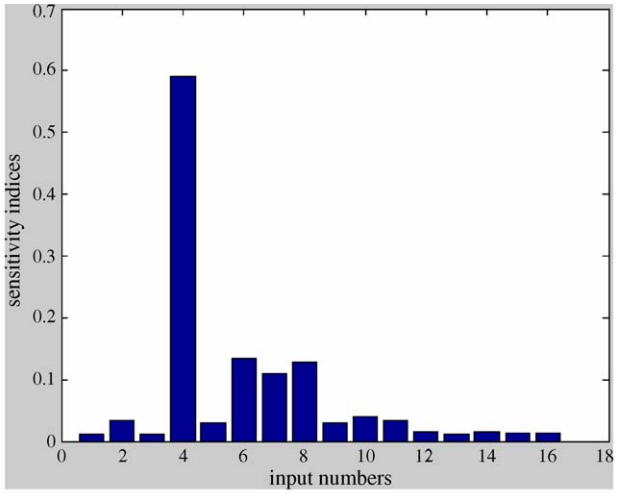


Fig. 9. General scheme of a quantitative Sensitivity Analysis method. The total variance is apportioned to the various input factors, as shown by the pie diagram.



N°	Description ofthe variable	Sensitivity indices
1	Kerb weight (kg)	0.01
2	Model year	0.03
3	Direction ofprincipal impact	0.01
4	Principal impact intrusion (cm)	0.59
5	Overlap ofprincipal impact (%)	0.03
6	High intrusion: rearward movement ofoutboard dashboard,driver side (cm)	0.14
7	High intrusion: rearward movement ofoutboard dashboard,passenger side (cm)	0.11
8	Low intrusion: rearward movement ofoutboard footwell, driverside (cm)	0.13
9	Low intrusion: rearward movement offootwell in the driver's axis(cm)	0.03
10	Low intrusion: rearward movement offootwell in the passenger'saxis (cm)	0.04
11	Low intrusion: rearward movement ofoutboard footwell, passengerside (cm)	0.04
12	Maximal reduction ofthe passenger compartment'swidth (cm)	0.02
13	Vehicle make	0.01
14	Spread ofprincipal impact	0.02
15	Deformationheight of principal impact	0.01
16	Zone of internal deformation	0.01

Fig. 10. Sensitivity analysis of inputs from the EFAST method.

a ratio (the fraction of variance which the input accounts for) that allows their ranking. The details of the computation of the sensitivity indices are not given here. The interested reader is referred to (Saltelli et al., 1999).

We applied the EFAST method to study the influence of model's inputs on the EES. Fig. 10 gives the obtained results.

We can note that the EFAST method exhibits three groups of input variables with different degrees of relevance. The first one includes the most relevant variable namely *Principal impact intrusion* (input number 4). The second one is formed by three variables (with a lesser relevance): *High intrusion: rearward movement of outboard dashboard, driver side* (input number 6), *High intrusion: rearward movement of outboard dashboard, passenger side* (input number 7), and *Low intrusion: rearward movement of outboard footwell, driver side* (input number 8). And the third one is made up of the less relevant variables.

Usually, in practice, one can consider that inputs do not influence the output if they account for less of 5% of the output variance. According to this rule, four input variables are absolutely necessary to estimate the EES: the variables included in the two first groups. To verify that and to determine the performance of the model with only these inputs, we trained our NN with five hidden neurons and four inputs. Then we tested it on the generalization dataset. Table 3 gives the characteristics of the test error distribution.

From four input variables, our model is able to estimate the EES with a mean error of 1.59 m s^{-1}, while from 16 variables it determines the EES with a mean error of 1.34 m s^{-1} (same order of magnitude). Thus, among the 16 input variables of the model, some of them can be missing, except the 4 variables determined by the SA (quoted above), which are absolutely necessary to the estimation of the EES.

Second, we were interested in the behaviour of the model if more input variables are available to estimate the EES. The Bayesian framework proposes a response through the Automatic Relevance Determination technique, called the ARD technique (Bishop, 1995).

So far, it has been assumed that a single hyperparameter α controls the whole of weights. However, it can be interesting to

Table 3
Characteristics of the test error distribution with four inputs

Absolute error (m s^{-1})	
Minimum	0.003
Maximum	6.74
Mean	1.59
Standard deviation	1.20

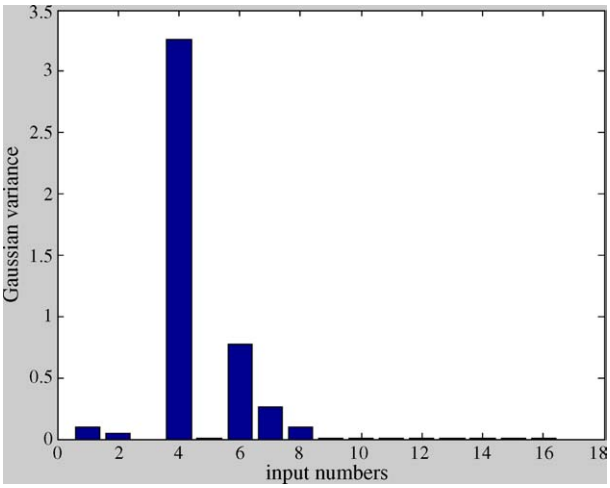


Fig. 11. Results obtained with the ARD method.

divide the weights into several groups g (for instance, one group for the input-layer weights, one for the second-layer weights, one for input layer biases and one for the second-layer biases). Again, these groups are controlled through a Gaussian prior by a hyperparameter α_g. The technique of ARD consists in assigning a separate regularization coefficient to each input. More precisely, all weights related to the same input are controlled by a same hyperparameter α_g. This hyperparameter is associated with a Gaussian prior with zero mean and variance $1/\alpha_g$. At the end of the training session, the weights with a large α_g (or conversely a small variance $1/\alpha_g$) are close to zero. Consequently, one can state that the corresponding input is irrelevant.

Fig. 11 shows the results obtained with the ARD technique. This figure represents the inverse of the values of the hyperparameters, namely the variance of the Gaussian associated with each input variable.

We can note that the ARD results confirm those obtained by EFAST. Indeed, both methods exhibits clearly input number 4 (principal impact intrusion). The other relevant input variables are input numbers 6 (the High intrusion: rearward movement of outboard dashboard, driver side), 7 (the High intrusion: rearward movement of outboard dashboard, passenger side), and 8 (low intrusion: rearward movement of outboard footwell, driver side).

Nonetheless, the most important characteristic of the Bayesian ARD technique is that it is possible to include a large number of input variables without fear of overfitting. If an input variable is found irrelevant then all the fan-out connections related to this input are automatically set to zero (or near-zero). Thus, if irrelevant variables are added to the estimation of the EES, they will not hurt the performance of the model.

5. Conclusions and discussions

The Energy Equivalent Speed contributes to determining the deformation energy, and so to reconstructing the pre-crash phase, from information about the post-crash. The reconstruction of the accident enables the identification of the accidental mechanisms which contributed to produce the crash, and thus to the implementation of the necessary means to decrease the number of accidents. Moreover, the estimation of the EES contributes to the characterization of the severity of the crash.

Today, the existing techniques for EES estimation are not very precise or difficult to implement. That is why we built this EES model by combining Bayesian and neural approaches.

This combination of techniques presents several advantages compared to the conventional approach.

First, the Bayesian learning enables the accurate determination of the optimal structure of the ANN, without a validation set. This aspect is important first, because it means that more data is available for the network learning, but also because the techniques, such as cross-validation, are sometimes criticized: they are computationally demanding and sometimes noisy. Thus, with the Bayesian learning, we found that an ANN with five hidden neurons is the most parsimonious.

Second, the hyperparameters and the ordinary model parameters (weights and biases) are optimized simultaneously: it is more computationally efficient and it guarantees obtaining the optimal model.

Furthermore, in the traditional neural learning, the weights are represented by scalars. In the Bayesian learning, they are represented by p.d.f. that are updated from the data of the learning set. As a result, the performance of the model is not only estimated by a scalar (test error), but by predictive error bars on the network output.

Lastly, the Bayesian learning, through the ARD technique, offers the possibility to automatically drive to zero the input variables that do not participate to the estimation of the EES. In other words, irrelevant added variables will not hurt the model. Moreover, the ARD technique allowed us to determine four relevant inputs. This result has been confirmed by a specific sensitivity analysis method, the EFAST method. We tested our model with these 4 inputs, and we obtained an error of the same order of magnitude than the 1 with 16 inputs. Indeed, from 16 input variables, our model is able to estimate the EES of a vehicle with a mean error of $1.34\,\mathrm{m\,s^{-1}}$, while from 4 inputs it determines the EES with a mean error of $1.59\,\mathrm{m\,s^{-1}}$.

As a result, this combination of Bayesian and neural techniques seems suitable for our issue. Indeed, from the crash's and damaged car's characteristics, our model is able to estimate the EES of the car with a mean error of $1.34\,\mathrm{m\,s^{-1}}$.

The use of the Bayesian technique results in a simpler model (5 hidden neurons versus 16 corresponding to 461 parameters versus 1473 parameters). Further, this reduction in complexity does not diminish the performance of the NN model.

For the present application and with the data at hand, regarding the performance of the model on the test set, the Bayesian approach does not lead to a clear improvement. However, this statement must be moderated as the performance of the Bayesian approach is obtained with a NN model whose complexity has been greatly reduced.

Our model could be improved. We note that during the test phase the larger errors correspond to cases rarely seen during the training phase. More specifically, there is a higher error level when the model estimates EES values below $8.1\,\mathrm{m\,s^{-1}}$ or above $18.3\,\mathrm{m\,s^{-1}}$. EES values between 8.1 and $18.3\,\mathrm{m\,s^{-1}}$ are relatively well estimated by the model. As a first means to improve

our model, we could add other accidents to the training set. In particular, we could add damaged cars with very low (below 8.1 m s^{-1}) or very high EES (above 18.3 m s^{-1}). This sensitivity of the model to the number of experimental data reinforces the need to use a Bayesian approach.

Appendix A. Principles of the Bayesian NN learning

A.1. A probabilistic approach to NN learning

The Bayesian approach considers a probability density function over weight space. So, in our case, the preceding parameter vector θ is given by the parameter vector $\boldsymbol{w}$. This p.d.f represents the degrees of belief taken by the different values of the weight vector. This p.d.f is set initially to some prior distribution and converted into a posterior distribution once the data have been observed through the use of Bayes' theorem.

So, instead of the single 'best' set of weights computed by the classical approach (through minimization of an error function), Bayesian methods yield a complete distribution for the NN parameters. This posterior distribution can then be used, for instance, to infer predictions of the network for new values of the input variables.

A.1.1. The prior

We choose to express the prior and the likelihood function as Gaussian distributions (Bishop, 1995):

$$p(\boldsymbol{w}|\alpha) = \frac{1}{Z_w(\alpha)} \mathrm{e}^{(-\alpha E_w)} \tag{A.1}$$

where

$$E_w = \frac{1}{2}\sum_{i=1}^{p} w_i^2$$

α is called a hyperparameter as it controls the distribution of other parameters. It represents the inverse of the variance on the set of weights and biases, $Z_w(\alpha)$ is a normalization factor.

A.1.2. The likelihood function

The likelihood function is given by:

$$l(D|\boldsymbol{w}, \beta) = \frac{1}{Z_D(\beta)} \exp(-\beta E_D) \tag{A.2}$$

where E_D is the error function, β is also called a hyperparameter and as stated above represents the inverse of the variance on the noise of the target output, $Z_D(\beta)$ is a normalization factor.

A.1.3. Posterior distribution

Once the prior p.d.f and the likelihood function have been defined, it is possible to infer the posterior p.d.f, through the use of Bayes' theorem:

$$p(\boldsymbol{w}|\alpha, \beta, D) = \frac{l(D|\boldsymbol{w}, \beta)p(\boldsymbol{w}|\alpha)}{p(D|\alpha, \beta)} \tag{A.3}$$

The optimal values of NN parameters (weights and biases) correspond to the maximum of the posterior p.d.f., this is equivalent to minimizing the quantity $S(\boldsymbol{w})$:

$$S(\boldsymbol{w}) = \frac{\beta}{2}\sum_{i=1}^{N}\{y(x^i; \boldsymbol{w}) - t^i\}^2 + \frac{\alpha}{2}\sum_{i=1}^{p} w_i^2 \tag{A.4}$$

A.2. Determination of error bars on the output

The Bayesian formalism is interesting in that it incorporates uncertainty about parameters into predictions. Under some approximations and using the Gaussian approximation of the posterior of weights, it can be shown (Bishop, 1995) that the distribution of outputs is also Gaussian and has the following form:

$$p(t|x, D) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{(t - y_{\mathrm{MP}})^2}{2\sigma_t^2}\right) \tag{A.5}$$

This output distribution has a mean given by $y_{\mathrm{MP}} = y(x, \boldsymbol{w}_{\mathrm{MP}})$ (the model response for the optimal estimate $\boldsymbol{w}_{\mathrm{MP}}$) and a variance given by:

$$\sigma_t^2 = \sigma^2 + g^T A^{-1} g \tag{A.6}$$

where $g = \nabla_{\boldsymbol{w}} y|_{\boldsymbol{w}_{\mathrm{MP}}}$ is the gradient of the model output evaluated at the best estimate.

One can interpret the standard deviation of the output distribution for the output as an error bar on the mean value y_{MP}. This error bar has two contributions: one arising from the intrinsic noise on the data and one arising from the width of the posterior distribution.

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