Examination of Crash Variances Estimated by Poisson–Gamma and Conway–Maxwell–Poisson Models

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The Poisson–gamma (negative binomial or NB) distribution is still the most common probabilistic distribution used by transportation safety analysts to model motor vehicle crashes. Recent studies have shown that the Conway–Maxwell–Poisson (COM–Poisson) distribution also is promising for developing crash prediction models. The objectives of this study were to investigate and compare the estimation of crash variance predicted by the COM–Poisson generalized linear model (GLM) and the traditional NB model. The comparison analysis was carried out with the most commonly employed functional forms, which linked crashes to the entering flows and other explanatory variables at intersections or on segments. To accomplish the objectives of the study, several NB and COM–Poisson GLMs (including flow-only models and models with several covariates) were developed and compared by using two data sets. The first data set contained crash data collected at signalized, four-legged intersections in Toronto, Ontario, Canada. The second data set included data collected on rural, four-lane, undivided highways in Texas. The results of this study show that the trend of crash variance prediction by COM–Poisson GLM is similar to that predicted by the NB model. The Spearman's rank correlation coefficients between the crash variance predicted by the COM–Poisson and the NB model confirmed that there was a perfect monotone increasing, and the values were highly correlated. This correlation means that a site characterized by a large variance would essentially be identified as such, whether the NB model or the COM–Poisson model was used.

In highway safety, the traditional Poisson and mixed-Poisson models are the most common probabilistic models used for analyzing crash data. Crash data have been found to often exhibit overdispersion (i.e., the variance is larger than the mean) and thus mixed-Poisson models [e.g., the Poisson–gamma or negative binomial (NB)] are generally preferred over the traditional Poisson model. The Conway–Maxwell–Poisson (COM–Poisson) distribution is also one of those generalizations of Poisson distribution, which can also be used for analyzing crash data. The COM–Poisson distribution was originally developed in 1962 as a method for modeling both underdispersed and overdispersed count data (*1*). The COM–Poisson distribution was then revisited by Shmueli et al. after a long period in which the distribution was not widely used (*2*). The COM–Poisson model can also handle underdispersed data [which the NB generalized linear model (GLM) cannot or has difficulties converging, see below] and data sets that contain intermingled over- and underdispersed counts (for dual-link models only, because the dispersion characteristic is captured by using the covariate-dependent shape parameter).

Recent research in highway safety has shown that the dispersion parameter of the Poisson–gamma model can potentially depend on the covariates of the model and could vary from one observation to another (*3–7*). This characteristic has been shown to be important, especially when the mean function is misspecified, such as in models that incorporate the entering traffic flows only (*8*). Furthermore, previous studies have reported that Poisson–gamma models with a varying dispersion parameter provide a better statistical fit (*9–11*). Similarly, the shape parameter of the COM–Poisson model provides a basis for using a link function to allow the amount of overdispersion or underdispersion to vary across measurements. It is also expected that COM–Poisson models with a varying shape parameter provide improved statistical fit.

The primary objective of this research was to examine whether the Poisson–gamma model and COM–Poisson model showed similar trends for estimating the crash variance. To accomplish the objectives of the study, NB and COM–Poisson GLMs were developed and compared by using two data sets. The first data set contained crash data collected at four-legged, signalized intersections in Toronto, Ontario, Canada. The second data set included data collected for rural, four-lane, undivided highways in Texas. Flow-only models and models with covariates were evaluated.

This paper is organized as follows. The first section provides a brief overview about the characteristics of Poisson–gamma models and COM–Poisson models. The second section describes the methodology for estimating and comparing the models. The third section presents the summary statistics of the two data sets. The fourth section presents the results of the analysis. The last section provides a summary of the research and outline avenues for further work.

BACKGROUND

This section provides a brief description about the characteristics of the Poisson–gamma and the COM–Poisson models, respectively.

Poisson–Gamma Model

The Poisson–gamma (or NB) distribution is the most common probabilistic distribution used by transportation safety analysts for modeling motor vehicle crashes (*3, 4, 12, 13*). The Poisson–gamma

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model has the following model structure (*14*): the number of crashes Y_{it} for a particular *i*th site and time period *t* when conditional on its mean μ_{it} is Poisson distributed and independent over all sites and time periods

$$Y_{it} \mid \mu_{it} \sim \text{Po}(\mu_{it}) \qquad i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \tag{1}$$

The mean of the Poisson is structured as

$$\mu_{it} = f(X;\ \beta)\exp(e_{it}) \tag{2}$$

where

$f(.)$ = function of the covariates (X),
β = vector of unknown coefficients, and
e_{it} = model error independent of all the covariates.

With this characteristic, it can be shown that Y_{it}, conditional on μ_{it} and ϕ, is distributed as a Poisson–gamma random variable with a mean μ_{it} and a variance $\mu_{it} + \mu_{it}^2/\phi$, respectively. [Other variance functions exist for the Poisson–gamma model, but they are not covered here because they are seldom used in highway safety studies. See Cameron and Trivedi (*15*) and Maher and Summersgill (*16*) for descriptions of alternative variance functions.] The probability density function of the Poisson–gamma structure described above is given by Equation 3.

$$f(y_{it};\ \phi,\ \mu_{it}) = \begin{pmatrix} y_{it} + \phi - 1 \\ \phi - 1 \end{pmatrix} \left(\frac{\phi}{\mu_{it} + \phi}\right)^{\phi} \left(\frac{\mu_{it}}{\mu_{it} + \phi}\right)^{y_{it}} \tag{3}$$

where

y_{it} = response variable for observation *i* and time period *t,*
μ_{it} = mean response for observation *i* and time period *t,* and
ϕ = inverse dispersion parameter of the Poisson–gamma distribution.

If $\phi \rightarrow \infty$, the crash variance equals the crash mean, and this model reverts to the standard Poisson regression model.

The term ϕ usually is defined as the inverse dispersion parameter of the Poisson–gamma distribution. (In the statistical and econometric literature, $\alpha = 1/\phi$ usually is defined as the dispersion parameter; in some published documents, the variable α has also been defined as the overdispersion parameter.) This term has traditionally been assumed to be fixed and a unique value applied to the entire data set in the study. As discussed above, recent research in highway safety has shown that the dispersion parameter could potentially depend on the covariates of the model and could vary from one observation to another (*3–7*).

COM–Poisson Model

The COM–Poisson distribution has recently been used for modeling motor vehicle crashes (*17, 18*). Shmueli et al. (*2*) elucidated the statistical properties of the COM–Poisson distribution by using the formulation given by Conway and Maxwell (*1*). Kadane et al. developed the conjugate distributions for the parameters of the COM–Poisson distribution (*19*). Its probability mass function is given by Equations 4 and 5.

$$P(Y = y) = \frac{1}{Z(\lambda, v)} \frac{\lambda^y}{(y!)^v} \tag{4}$$

$$Z(\lambda, v) = \sum_{n=0}^{\infty} \frac{\lambda^n}{(n!)^v} \tag{5}$$

where

Y = discrete count,
λ = centering parameter that is approximately the mean of the observations in many cases, and
v = the shape parameter of the COM–Poisson distribution.

The centering parameter λ is approximately the mean when v is close to 1; it differs substantially from the mean for small v. Given that v would be expected to be small for overdispersed data, this would make a COM–Poisson model on the basis of the original COM–Poisson formulation difficult to interpret and use for overdispersed data.

To circumvent this problem, Guikema and Coffelt proposed a reparameterization of the COM–Poisson distribution by substituting $\mu = \lambda^{1/v}$ to provide a clear centering parameter (*20*). This new formulation of the COM–Poisson is summarized in Equations 6 and 7.

$$P(Y = y) = \frac{1}{S(\mu, v)} \left(\frac{\mu^y}{y!}\right)^v \tag{6}$$

$$S(\mu, v) = \sum_{n=0}^{\infty} \left(\frac{\mu^n}{n!}\right)^v \tag{7}$$

The mean and variance of Y are given in terms of the new formulation as

$$E[Y] = \left[\frac{1}{v} \frac{\partial \log S}{\partial \log \mu}\right]$$

and

$$V[Y] = \left[\frac{1}{v^2} \frac{\partial^2 \log S}{\partial \log^2 \mu}\right]$$

with asymptotic approximations $E[Y] \approx \mu + 1/2v - 1/2$ and $\text{Var}[Y] \approx \mu/v$ especially accurate once $\mu > 10$. With this new parameterization, the integral part of μ is now the mode, which leaves μ as a reasonable approximation of the mean. The substitution $\mu = \lambda^{1/v}$ also allows v to keep its role as a shape parameter. That is, if $v < 1$, the variance is greater than the mean, while $v > 1$ leads to underdispersion.

Guikema and Coffelt developed a COM–Poisson GLM framework for modeling discrete count data (*20*). Their approach depended on Markov chain Monte Carlo for fitting a dual-link GLM on the basis of the COM–Poisson distribution. The approach also used a reformulation of the COM–Poisson to provide a more direct centering parameter than the original COM–Poisson formulation. Sellers and Shmueli developed a maximum likelihood estimate for a single-link GLM on the basis of the original COM–Poisson distribution (*21*). Equations 8 and 9 describe this modeling framework. The framework is in effect a dual-link GLM, in which both the mean and the variance depend on the covariates. In Equations 8 and 9, x_i and z_j are covariates, and there are assumed to be *p* covariates used in the centering link function and *q* covariates used in the shape link function [similar to the varying dispersion parameter of the Poisson–gamma model proposed by Miaou and Lord (*3*), Heydecker and Wu (*7*), Hauer (*9*), and Geedipally et al. (*22*)]. The sets of parameters used in the two-link functions do not necessarily have to be identical.

$$\ln(\mu) = \beta_0 + \sum_{i=1}^{p} \beta_i x_i \tag{8}$$

$$\ln(v) = \gamma_0 + \sum_{j=1}^{q} \gamma_j z_j \tag{9}$$

The GLM framework can model underdispersed data sets, overdispersed data sets, and data sets that contain intermingled underdispersed and overdispersed counts (for dual-link models only, because the dispersion characteristic is captured by using the covariate-dependent shape parameter). The variance is allowed to depend on the covariate values, which can be important if high (or low) values of some covariates tend to be variance-decreasing while high (or low) values of other covariates tend to be variance-increasing. The parameters have a direct link to either the mean or the variance, which provides insight into the behavior and driving factors in the problem, and the mean and variance of the predicted counts are readily approximated on the basis of the covariate values and regression parameter estimates.

METHODOLOGY

This section describes the methodology used for estimating different NB and COM–Poisson models. For each data set, COM–Poisson GLMs and NB models were initially estimated by using the fixed shape parameter and dispersion parameter, respectively. Then the models were developed by using different parameterizations for a varying shape and a varying dispersion parameter.

The functional form used for models was the following (the centering parameter is the mean with the NB model, whereas it is approximately the mode with the COM–Poisson model):

Toronto intersection data:

Centering parameter:

$$\mu_i = \beta_0 F_{\mathrm{Maj}\_i}^{\beta_1} F_{\mathrm{Min}\_i}^{\beta_2} \tag{10}$$

Shape parameter (of COM–Poisson model):

$$v_i = \gamma_0 F_{\mathrm{Maj}\_i}^{\gamma_1} F_{\mathrm{Min}\_i}^{\gamma_2} \tag{11}$$

Dispersion parameter (of NB model):

$$\phi_i = \gamma_0 F_{\mathrm{Maj}\_i}^{\gamma_1} F_{\mathrm{Min}\_i}^{\gamma_2} \tag{12}$$

Texas segment data:

Centering parameter:

$$\mu_j = \beta_0 L_j F_j^{\beta_1} e^{\beta_2 * \mathrm{LW}_j + \beta_3 * \mathrm{SW}_j + \beta_4 * \mathrm{CD}_j} \tag{13}$$

Shape parameter (of COM–Poisson model):

Model 1:

$$v_j = \gamma_0 L_j \tag{14}$$

Model 2:

$$v_j = \gamma_0 / L_j \tag{15}$$

Model 3:

$$v_j = \gamma_0 L_j^{\gamma_1} \tag{16}$$

Dispersion parameter (of NB model):

Model 1:

$$\phi_j = \gamma_0 L_j \tag{17}$$

Model 2:

$$\phi_j = \gamma_0 / L_j \tag{18}$$

Model 3:

$$\phi_j = \gamma_0 L_j^{\gamma_1} \tag{19}$$

where

μ_i = mean number of crashes for intersection i,
μ_j = mean number of crashes per year for segment j,
F_{Maj_i} = entering flow for the major approach [average annual daily traffic (AADT)] for intersection i,
F_{Min_i} = entering flow for the minor approach (AADT) for intersection i,
F_j = flow traveling on segment j (AADT) and time period t,
L_j = length in miles for segment j,
LW_j = lane width in ft for segment j,
SW_j = total shoulder width in ft for segment j,
CD_j = curve density (curves per mile) for segment j, and
β's, γ's = estimated coefficients.

The coefficients of the COM–Poisson GLMs and NB models were estimated by using the software WinBUGS (*23*). Vague or noninformative hyper-priors were used for the COM–Poisson and NB GLMs. Three Markov chains were used in the model estimation process. The Gelman–Rubin convergence statistic was used to verify that the simulation runs converged properly.

DATA DESCRIPTION

This section describes the characteristics of the two data sets. The first data set contained crash data collected in 1995 at four-legged, signalized intersections located in Toronto. The data had been used for several research projects and found to be of relatively good quality (*3, 17, 24–26*). In total, 868 signalized intersections were used in this data set. The second data set contained crash data collected from 1997 to 2001 at four-lane, rural, undivided segments in Texas. The data were provided by the Texas Department of Public Safety and the Texas Department of Transportation and were used for NCHRP Project 17-29 (*27*). The final database included 1,499 segments (≥0.1 mi). Table 1 presents the summary statistics for the two data sets used in this study.

RESULTS

This section presents the modeling results for the COM–Poisson GLMs as well as for the NB models and is divided into two parts. The first part explains the modeling results for the Toronto data. The second part provides details about the modeling results for the Texas data.

Toronto Data

Table 2 summarizes the results of the COM–Poisson and NB GLMs for the Toronto data. This table shows that the coefficients for the flow parameters were below 1, which indicates that the crash risk increased at a decreasing rate as traffic flow increased. The 95% marginal posterior credible intervals for each of the coefficients did not include the origin. The deviance information criteria (DIC) value showed that there was no significant difference in the fit among various models [this result supports the finding of Lord et al (*17*)]. The NB model with a varying dispersion parameter showed a slightly better fit, as expected, however.

TABLE 1 Summary Statistics for Toronto and Texas Data

Data Characteristic	Minimum	Maximum	Average	Total
Toronto				
Crashes	0	54	11.56 (10.02)	10,030
Major AADT	5,469	72,178	28,044.81 (10,660.4)	—
Minor AADT	53	42,644	11,010.18 (8,599.40)	—
Texas				
Crashes	0	97	2.84 (5.69)	4,253
Length (mi)	0.1	6.275	0.55 (0.67)	830.5
AADT	402	24,800	6,613.61 (4,010.01)	—
Lane width (ft)	9.75	16.5	12.57 (1.59)	—
Shoulder width (right + left) (ft)	0	40	9.96 (8.02)	—
Number of horizontal curves	0	16	0.70 (1.32)	1,052

NOTE: — = not applicable.

Figure 1 illustrates the frequency distribution of the varying shape parameter of COM–Poisson distribution across all observations. The frequency distribution can be approximated by a normal- or lognormal-shaped distribution. This figure shows that the highest frequency (i.e., the mode) occurred between 0.30 and 0.40, and the average shape parameter value was 0.36, which was slightly higher than the value found for the fixed shape parameter (i.e., 0.34).

Figure 2 illustrates the distribution of the varying (inverse) dispersion parameter across all observations. A wide variation in the dispersion parameter occurred across various observations. The figure shows that the highest frequency (i.e., the mode) occurred between 9 and 10, whereas the average dispersion parameter value was 7.0. The average value of the varying dispersion parameter was found, however, to be much closer to the fixed dispersion parameter (i.e., 7.1).

Figure 3 shows the comparison of crash variance predicted by the COM–Poisson and NB models. It can be seen that, for the sites with a crash mean of less than 20, both the models predicted almost the same variance. For the sites with a higher mean, however, the NB model predicted a slightly higher variance than the COM–Poisson model, although the shapes were similar. The discrepancy between the fit and the smaller variance can be explained by the fact that the variance, via the dispersion parameter, was estimated directly from the data and independently from the mean (*28*). The Spearman's rank correlation coefficient between the variances predicted by both models was 0.999, which means that there was a perfect monotone increasing, and the values were highly correlated. For this data set, the variance might be slightly better captured by the COM–Poisson than by the NB model, especially at larger mean values.

Figure 4 illustrates the frequency distribution of crash variance predicted by the COM–Poisson and NB models. As discussed above, the frequency of sites with low variance was higher with the COM–Poisson model than with the NB model.

Texas Data

Table 3 presents the results of the COM–Poisson GLM for the Texas data. This table shows that the coefficient for the flow parameter was above 1 for all models except Model 1, which indicates that the crash risk increased at an increasing rate as traffic flow increased. The 95% marginal posterior credible intervals for each of the coefficients did not include the origin. The DIC value showed that Model 3 fit the data better than all other models. Geedipally et al. also found Model 3 to be the best model (*22*).

Table 4 presents the results of the NB GLM for the Texas data. This table shows that the coefficient for the flow parameter was below 1 for all models, which indicates that the crash risk increased at a decreasing rate as traffic flow increased. The 95% marginal posterior credible intervals for each of the coefficients did not include the origin. The DIC value shows that Model 1 fit the data better than all other models, although Model 3 was a close second. The NB model fit the data slightly better than the COM–Poisson model.

TABLE 2 Modeling Results for COM–Poisson and NB GLMs with Toronto Data

	COM–Poisson Model		NB Model	
Estimate	Fixed Shape Parameter	Varying Shape Parameter	Fixed Dispersion Parameter	Varying Dispersion Parameter
$\mathrm{Ln}(\beta_0)$	−11.53 (0.4159)	−10.67 (0.5249)	−10.11 (0.4794)	−10.28 (0.4501)
β_1	0.6350 (0.04742)	0.5498 (0.04841)	0.6071 (0.046)	0.6161 (0.04695)
β_2	0.795 (0.03101)	0.7971 (0.03405)	0.6852 (0.021)	0.6943 (0.02295)
ν	0.3408 (0.02083)	—	—	—
ϕ	—	—	7.12 (0.619)	—
$\mathrm{Ln}(\gamma_0)$	—	4.882 (1.753)	—	2.764 (1.663)
γ_1	—	−0.5945 (0.1734)	—	−0.409 (0.1856)
γ_2	—	0.0133 (0.05463)	—	0.3671 (0.1049)
DIC	4,953.70	4,937.48	4,777.59	4,762.38

NOTE: — = not applicable.

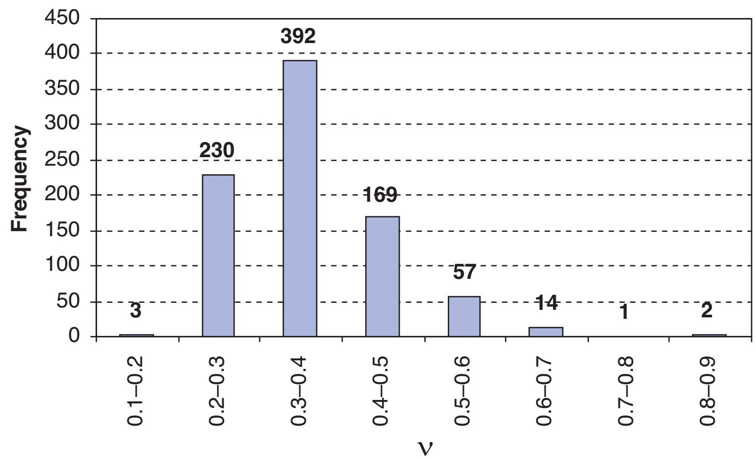

FIGURE 1 Frequency distribution of the varying shape parameter of COM–Poisson model (mean = 0.36).

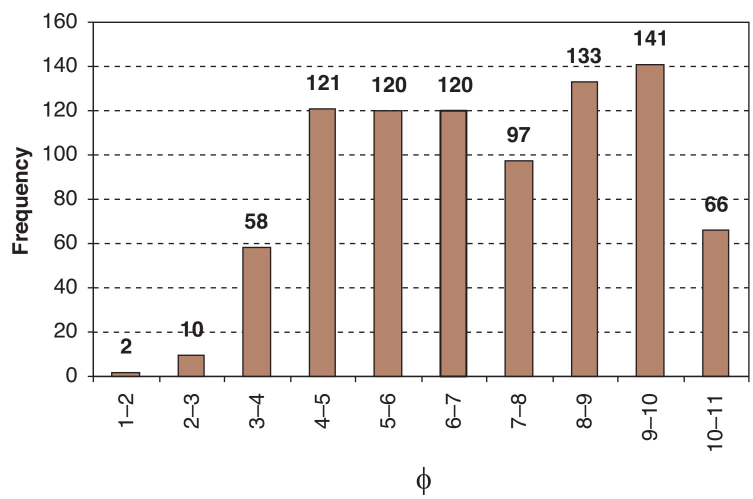

FIGURE 2 Frequency distribution of the inverse dispersion parameter of NB model (mean = 7.0).

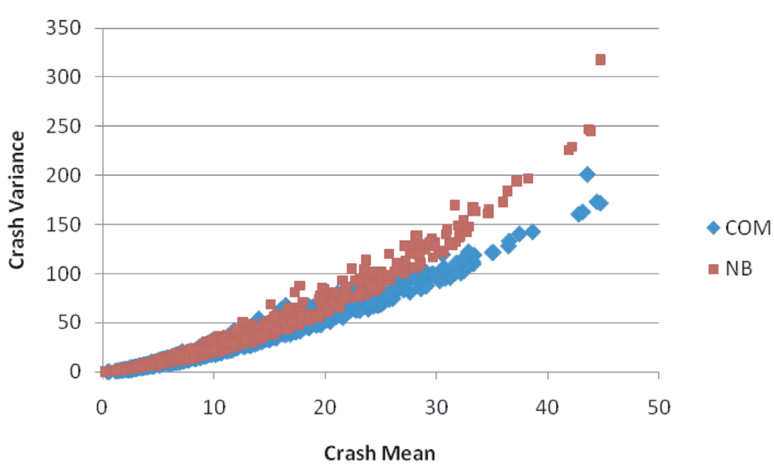

FIGURE 3 Crash variance versus crash mean for Toronto data.

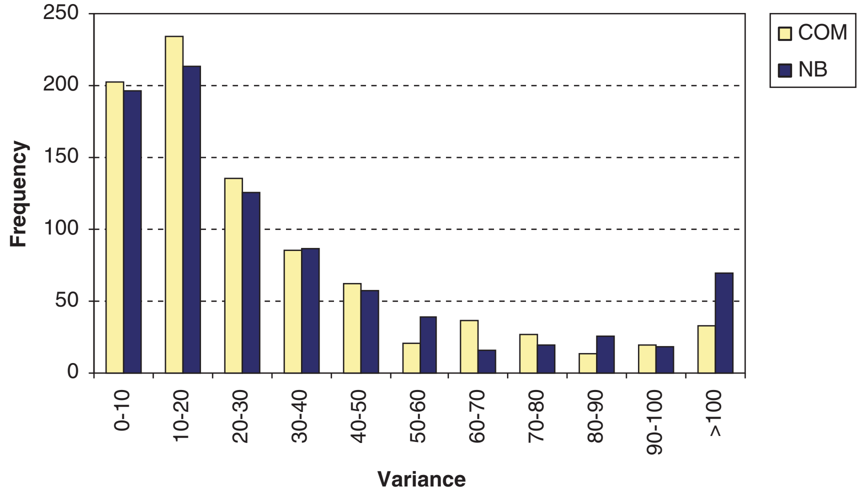

FIGURE 4 Frequency distribution of crash variance for Toronto data.

TABLE 3 Modeling Results for COM–Poisson GLM with Texas Data

Estimate	Fixed Shape	Model 1	Model 2	Model 3
$\ln(\beta_0)$	−8.845 (0.673)	−5.746 (0.239)	−27.18 (3.741)	−13.59 (0.947)
β_1	1.298 (0.081)	0.975 (0.029)	3.097 (0.366)	1.764 (0.108)
β_2	−0.104 (0.019)	−0.107 (0.017)	−0.101 (0.051)	−0.099 (0.026)
β_3	−0.018 (0.004)	−0.018 (0.003)	−0.019 (0.012)	−0.018 (0.006)
β_4	0.094 (0.013)	0.139 (0.009)	0.168 (0.039)	0.096 (0.018)
ν	0.419 (0.026)	—	—	—
$\ln(\gamma_0)$	—	−0.321 (0.026)	−2.984 (0.078)	−1.792 (0.019)
γ_1	—	—	—	−0.548 (0.043)
DIC	5,159.3	6,481.6	5,076.2	5,000.6

NOTE: — = not applicable.

Figure 5 shows the frequency distribution of the varying shape parameter of COM–Poisson distribution across various observations for the Texas data. Similar to the Toronto data, the frequency distribution of the Texas data could be approximated by a normal- or lognormal-shaped distribution. This figure shows that the highest frequency (i.e., the mode) occurred between 0.20 and 0.30, and the average shape parameter value was 0.313, which was slightly lower than the value found for the fixed shape parameter (i.e., 0.419).

Figure 6 illustrates the distribution of the varying (inverse) dispersion parameter of the NB model across various observations for Texas data. This frequency distribution could be approximated by a skewed normal or lognormal distribution. The figure shows that the highest frequency (i.e., the mode) occurred between 1 and 2, whereas the average dispersion parameter value was 2.45, which was much closer to the fixed dispersion parameter (i.e., 2.55).

Figure 7 shows the comparison of crash variance predicted by COM–Poisson and NB models. The figure illustrates that both the

TABLE 4 Modeling Results for NB GLM with Texas Data

Estimates	Fixed Dispersion	Model 1	Model 2	Model 3
$\ln(\beta_0)$	−6.384 (0.412)	−5.597 (0.285)	−6.752 (0.305)	−5.977 (0.388)
β_1	0.983 (0.043)	0.915 (0.033)	1.004 (0.031)	0.945 (0.038)
β_2	−0.055 (0.017)	−0.071 (0.014)	−0.043 (0.014)	−0.061 (0.014)
β_3	−0.010 (0.003)	−0.011 (0.003)	−0.009 (0.003)	−0.011 (0.003)
β_4	0.067 (0.012)	0.095 (0.014)	0.062 (0.010)	0.078 (0.013)
ϕ	2.55 (0.234)	—	—	—
$\ln(\gamma_0)$	—	1.485 (0.081)	1.052 (0.134)	1.144 (0.099)
γ_1	—	—	—	0.501 (0.091)
DIC	4,784.0	4,709.6	4,988.9	4,732.6

NOTE: — = not applicable.

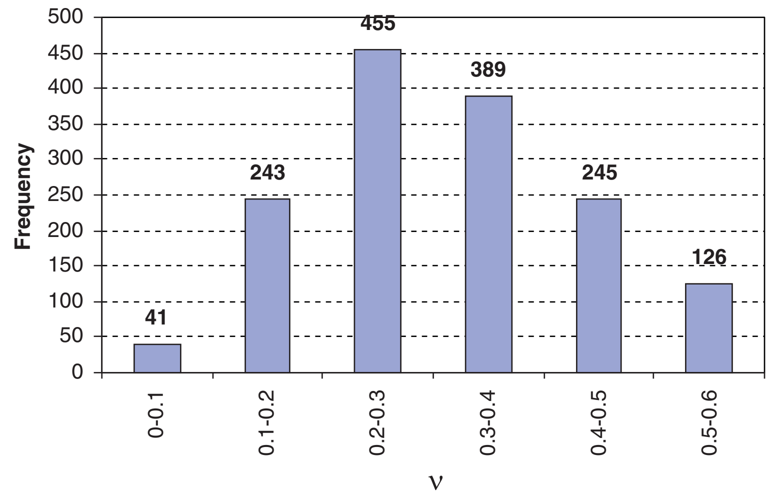

FIGURE 5 Frequency distribution of shape parameter of COM–Poisson model (mean = 0.313).

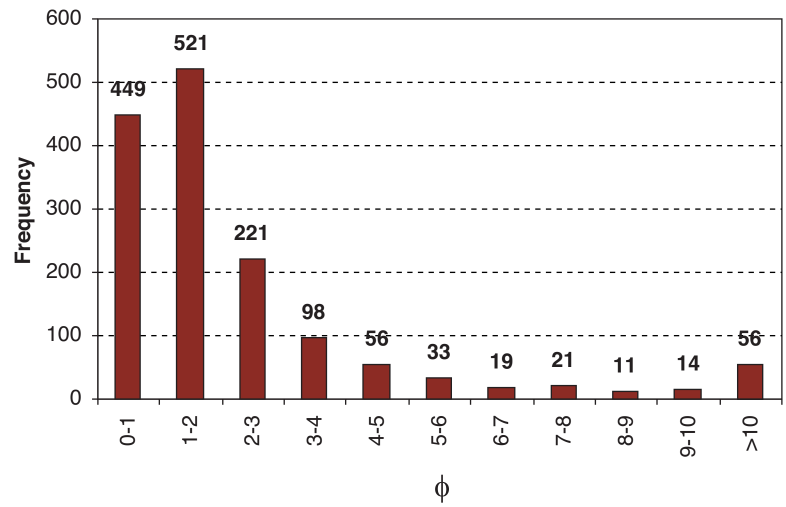

FIGURE 6 Frequency distribution of inverse dispersion parameter of NB model (mean = 2.45).

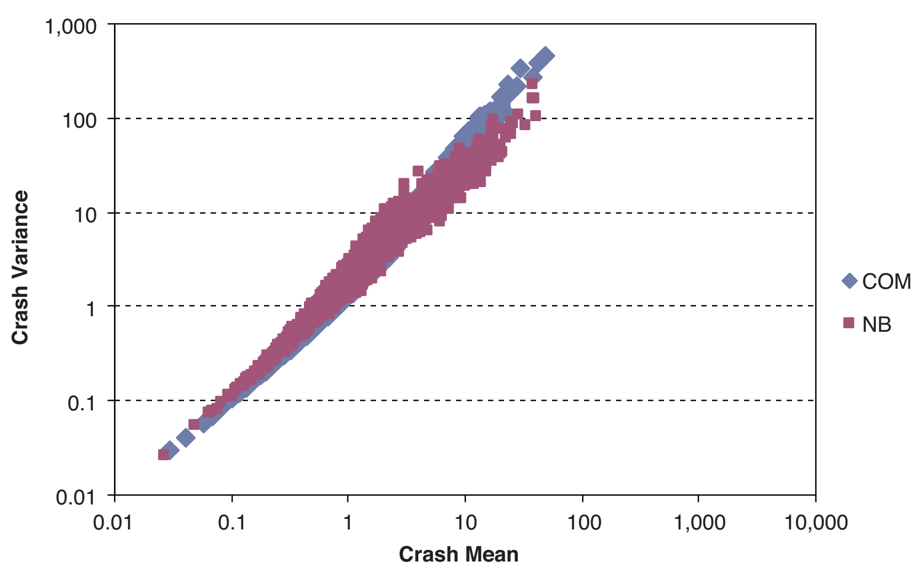

FIGURE 7 Crash variance versus crash mean for Texas data (*x*-axis and *y*-axis formatted under a logarithmic scale).

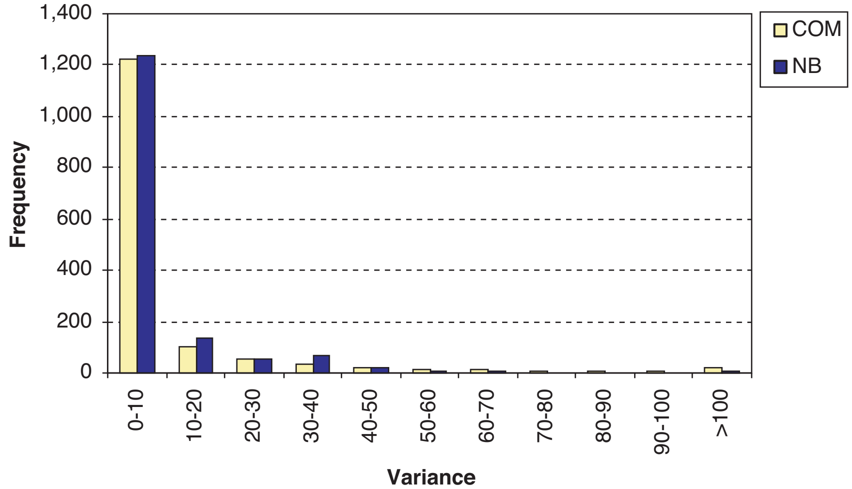

FIGURE 8 Frequency distribution of crash variance for Texas data.

models predict almost the same variance for a given mean. For higher crash means, however, the NB model predicted a slightly higher variance than the COM–Poisson model. The Spearman's correlation coefficient between the variances was 0.909, which confirms that the association was in the same direction and was positively correlated.

Figure 8 presents the frequency distribution of crash variance predicted by COM–Poisson and NB models. Unlike in the Toronto data, the frequency of sites between both models was almost the same for the Texas data.

SUMMARY AND CONCLUSIONS

This paper has documented the difference in the estimation of crash variance by using the COM–Poisson and NB models. The NB model is the most commonly used model for analyzing motor vehicle crashes. Recently, the COM–Poisson model was introduced for traffic crash data modeling. The COM–Poisson model introduces a covariate-dependent shape parameter, which captures the dispersion in the data. Thus the COM–Poisson model has a capability to handle data sets that contain intermingled over- and underdispersed counts.

The objectives of this study were to investigate and compare the estimation of crash variance predicted by the COM–Poisson and the NB models. To accomplish the study objectives, several NB and COM–Poisson GLMs were developed by using two data sets. The first data set contained crash data collected at four-legged, signalized intersections in Toronto. The second data set included data collected for rural, four-lane, undivided highways in Texas.

The results of this study show that the trend of crash variance prediction by COM–Poisson GLM was similar to that predicted by the NB model. The Spearman's rank correlation coefficients between the crash variance predicted by the COM–Poisson and NB models were 0.999 and 0.909 for Toronto data and Texas data, respectively. This means that a site that is characterized by a large variance will essentially be identified as such whether the NB or the COM–Poisson model is used. This characteristic was found for both flow-only and models with covariates. For the latter, the results may indicate that the variation observed for the variance was data-specific (*22*), rather than attributed to the model specification, as suggested by Mitra and Washington (*8*). Further work is needed on this topic.

Constraint of the parameters to remain constant across various observations may lead to inconsistent and biased estimates (*29*). To overcome or minimize this important problem in count data models, Anastasopoulos and Mannering suggested the use of random-parameter models (*30*). Further work is thus needed to find the difference in the estimation of crash variance between NB and COM–Poisson models when the random-parameter model is used in conjunction with a varying shape parameter. Such codes for the COM–Poisson model are not yet available.

The next step is to examine how these slight differences in observed variance for high means influence typical highway safety studies (e.g., evaluation of the effects of interventions and the identification of hazardous sites, when a varying dispersion or shape parameter is used). The approach taken by Geedipally and Lord could be used for such an evaluation (*31*). It is also recommended to conduct an analysis with the varying shape parameter for an underdispersed data set.

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