



# To illuminate or not to illuminate: Roadway lighting as it affects traffic safety at intersections

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## ARTICLE INFO

### Article history:

Received 14 May 2012

Received in revised form 3 October 2012

Accepted 20 December 2012

### Keywords:

Lighting

Safety

Regression analysis

Visual performance

Crash reduction

## ABSTRACT

A two-pronged effort to quantify the impact of lighting on traffic safety is presented. In the statistical approach, the effects of lighting on crash frequency for different intersection types in Minnesota were assessed using count regression models. The models included many geometric and traffic control variables to estimate the association between lighting and nighttime and daytime crashes and the resulting night-to-day crash ratios. Overall, the presence of roadway intersection lighting was found to be associated with an approximately 12% lower night-to-day crash ratio than unlighted intersections. In the parallel analytical approach, visual performance analyses based on roadway intersection lighting practices in Minnesota were made for the same intersection types investigated in the statistical approach. The results of both approaches were convergent, suggesting that visual performance improvements from roadway lighting could serve as input for predicting improvements in crash frequency. A provisional transfer function allows transportation engineers to evaluate alternative lighting systems in the design phase so selections based on expected benefits and costs can be made.

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## 1. Introduction

A primary purpose for installing roadway lighting is to increase the visual range afforded by vehicle headlamps while driving at night (IES, 2000). It is generally believed that roadway lighting improves safety by reducing the frequency of crashes occurring at night through improvements in driver visibility. Some studies have examined statistical associations between roadway lighting presence and traffic safety, making no explicit assumptions about the role of visibility in mitigating nighttime crashes. Generally, the results from these studies (IES, 1989; Elvik, 1995) have led to the general conclusion that roadway lighting is associated with a reduction in the night-to-day crash ratio (CIE, 1992). A night-to-day crash ratio reduction of approximately 30% has been suggested for the overall crash safety effect from roadway lighting (CIE, 1992).

Most studies of roadway lighting and safety consider lighting as a binary variable (i.e., present or not), but several attempts to relate specific characteristics of roadway lighting and safety have been made, using various photometric measures as surrogates for visibility. One early study was undertaken by Box (1971) where he compared the ratio of night-to-day crash rates along 22 lighted and unlighted highway sections. The author grouped the highway

sections into categories corresponding to mean horizontal illuminance levels between 3 and 6 lx, between 8 and 11 lx, and between 13 and 15 lx. Presumably, higher illuminances would result in greater visibility and might be expected to yield fewer nighttime crashes. The lighted sections had lower night-to-day crash rate ratios than the unlighted sections, but the lowest night-to-day crash rate ratio was found for the 3-to-6 lx category, with higher night-to-day crash rate ratios for the two higher illuminance categories. In a different study, Box (1976) evaluated the impact of reducing illuminance levels along a major highway from 14 lx to 9 lx and found that nighttime crash frequency increased by 10%, although daytime crash frequency also increased (by 4%). In comparison, Scott (1980) measured roadway luminance levels along 89 (each at least 1 km in length) two-lane roadway sections. Despite a great deal of variability in night-to-day crash ratios among all of the sites, a best-fitting exponential function to the data yielded a monotonically decreasing relationship between night-to-day crash ratios and luminance level, consistent with Box (1976) but inconsistent with Box (1971). The lack of agreement across these studies could be related to differences among the specific locations studied (e.g., roadway geometry or traffic control) or different evaluation paradigms (e.g., before/after or with/without comparisons). Even considering these differences, it is important to account for all the factors that could affect visibility. In particular, the impact of lighting on visual performance depends upon the contrast and the size of a hazard, not simply upon illuminance or luminance levels provided by the roadway lighting system.

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In an attempt to relate specific measures of visibility associated with roadway lighting to safety, Janoff et al. (1978) studied nighttime crash frequency and measured photometric conditions along several roadways, and found nighttime crash frequency to have a weak inverse relationship with a visibility metric they derived, which was based on the ratio between an object's luminance contrast and the contrast it would require to be just visible. Also, Keck (2001) summarized a different study of roadway lighting, visibility and safety and reported that a similar visibility metric was not correlated with the frequency of nighttime crashes unless headlamp illumination was also considered. Even then, the statistical association was modest in magnitude (coefficient of determination  $r^2 = 0.12$ ).

In addition to the above issues related to study locations and evaluation methods, a factor that makes it difficult to assess the relationship between fixed roadway illumination systems and traffic safety is that lighting is installed for a variety of reasons (e.g., security, esthetics). Moreover, lighting is usually not the only roadway safety feature that is installed when a roadway is designed or improved (IES, 1989). For example, lighting may be installed when converting a stop-controlled to a signalized intersection, or may be installed when threshold levels of pedestrian or vehicular volumes, or annual crash frequencies, are exceeded. Thus, the estimates of association between roadway lighting presence and nighttime crashes in past studies may be confounded by other modifications to the study site.

In the present paper, an exploratory strategy is presented to test the theoretical links among roadway lighting, visibility, and safety while accounting for as many potential safety-influencing variables as possible. To overcome previous limitations in the literature, our goal was to determine if we could establish convergence between statistical and analytical approaches to crash safety. Thus, both statistical and analytical approaches were undertaken to relate crash frequencies to visual performance levels for *the same set of roadway lighting conditions*. Donnell et al. (2010) merged roadway lighting presence and roadway geometric and traffic volume and control data with daytime and nighttime crash data to assess the statistical association between roadway intersection lighting presence and the night-to-day crash ratio in Minnesota. In their study, many potential safety-influencing variables that were not considered in previous lighting-safety research were included in the statistical model estimation. In a parallel but independent investigative domain, Rea et al. (2010) developed photometric simulations of a large variety of roadway intersection lighting configurations to make context-specific predictions of visual performance levels provided by roadway lighting systems varying in light level, spatial configuration, and ambient characteristics for drivers of different ages.

Here we present a methodology to link roadway lighting characteristics to visual performance levels and, therefore, to traffic crashes. Importantly, our objective was to test whether there is convergence between statistical and analytical approaches, which would bolster the expected relationships among lighting presence, improved visibility, and improved safety. Each approach reduces the inherent uncertainty associated with the other by providing an independent basis to explore the theoretical relationship between roadway intersection lighting and reductions in nighttime crashes through improvements in driver visual performance.

The present paper summarizes the parallel statistical and analytical approaches, developed by Donnell et al. (2010) and by Rea et al. (2010), respectively, used to probe the theoretical relationship described above. First, the background, methods and results of each independent approach are provided; next, the evidence for the convergence of these approaches is described. As stated above, the approaches used in this paper are described in previous publications (Donnell et al., 2010; Rea et al., 2010); the present

paper describes how these approaches were adapted for different roadway intersection types in Minnesota.

If the necessary links between roadway lighting, visual performance and traffic safety can in fact be forged through converging approaches, it could then be possible for traffic engineers to improve traffic safety by considering the visual performance levels and the costs of proposed roadway illumination systems through a provisional proposed transfer function relating visibility from lighting to crash safety.

## 2. Description of parallel approaches

In the present section, the statistical modeling and the visual performance modeling are described together with the findings from these two approaches.

### 2.1. Statistical modeling

#### 2.1.1. Background

A significant body of published literature consistently shows that fixed roadway lighting improves intersection safety. Observational before-after studies and with-without cross-sectional comparisons have been used to support these findings. The measures used to evaluate the effects of roadway lighting differ across studies. With regard to observational before-after studies, several authors used reported nighttime crash rates (Walker and Roberts, 1976; Lipinski and Wortman, 1978) as the safety performance measure. These studies found that roadway lighting was associated with reduced nighttime crash rates of 45–52%. Other authors (Walker and Roberts, 1976; Schwab et al., 1982; Green et al., 2003; Isebrands et al., 2004, 2006) reported 13–49% reductions in reported nighttime crash frequencies attributed to lighting. Reductions in reported night-to-day crash ratios of 22–40% have been documented in other observational before-after studies related to roadway lighting (Lipinski and Wortman, 1978; Preston and Schoenecker, 1999; Isebrands et al., 2004, 2006).

With regard to with-without cross-sectional comparisons, the published literature also indicates that roadway lighting appears to be an effective safety countermeasure. The results of previous studies have found that nighttime crash rates are 25% lower at lighted intersections when compared to unlighted intersections (Preston and Schoenecker, 1999); reported nighttime crash frequencies are 39% lower at lighted intersections (Schwab et al., 1982). Night-to-day crash ratios are 31% lower at intersections with lighting when compared to intersections without lighting (Isebrands et al., 2004, 2006).

More recently, an estimate of the safety effect of at-grade intersection lighting was published by Harwood et al. (2007), using the results of a meta-analysis of published literature from Elvik and Vaa (2004). Harwood et al. (2007) estimated that an appropriate accident modification factor for the presence of lighting is 0.96, or a 4% reduction in total crashes after the installation of roadway lighting (this corresponds to a larger percentage of nighttime crashes, since roadway lighting is only of benefit at night). This result is published in the first edition of the American Association of State Highway and Transportation Officials' *Highway Safety Manual* (AASHTO, 2010).

While many of the previous lighting-safety studies appear to offer similar results using a variety of measures (i.e., nighttime crash rates, nighttime crash frequencies, night-to-day crash ratios), there are limitations to these studies. First, much of the published literature evaluates the safety effects of roadway lighting based on crash rates or crash frequencies using reported crash data. Crash rates assume a linear relationship between crashes and traffic volume, which may not necessarily be the case, particularly at intersection locations (e.g., Poch and Mannering, 1996;

Bauer and Harwood, 1999; Washington et al., 2005). Reported crashes are rare events that vary randomly over time; thus, the use of expected crash frequencies in lighting-safety studies may be more meaningful. Secondly, many of the previous studies did not consider the relationship between intersection crashes and other safety-influencing features present at intersections (e.g., intersection form, intersection angle, posted speed limits, cross-section dimensions of approach legs, etc.). Statistical models of expected crash frequency should be conditioned on intersection features thought to influence crashes.

While Hauer (2005) offered the empirical Bayes observational before-after study (which considers expected crash frequency) as a superior method to evaluate the safety effects of a countermeasure, such a method is difficult to apply in the case of roadway lighting because installation of a lighting system is often accompanied by other intersection improvements (e.g., conversion of a stop-controlled to a signalized intersection or addition of a turn-lane). Thus, use of an observational before-after study is complicated by the inability to identify sites where *only* lighting was installed as a traffic safety countermeasure. A with-without comparison is thus a logical alternative evaluation method. Several studies have used this method (Schwab et al., 1982; Preston and Schoenecker, 1999; Isebrands et al., 2004, 2006) to assess the safety effects of intersection lighting. Past with-without lighting-safety studies used reported nighttime crash frequencies or rates, or night-to-day crash ratios, which are subject to the methodological limitations identified above. The present study estimates statistical models of nighttime and daytime crash frequencies as a function of intersection features and traffic control. This approach enables comparisons of expected nighttime crash frequencies at intersections with and without fixed lighting, and also enables comparisons of expected night-to-day crash ratios at locations with and without lighting. As noted previously, nighttime crash frequencies and night-to-day crash ratios are common performance metrics in past lighting research. With regard to night-to-day ratios, expected daytime crash frequencies are used as a form of control to assess the association between crashes and the presence of lighting, which is consistent with past research (e.g., CIE, 1992; Griffith, 1994). Because lighting is not turned on during the daytime, it is anticipated that roadway lighting will not be associated with daytime crashes. If this assumption holds, then the nighttime crash frequency estimates will be used to assess the safety effects of lighting. However, if there is an association between lighting and daytime crashes (e.g., Lacy et al., 2004), this same association would be present during the nighttime, which is the basis for using night-to-day ratios as the performance measure to evaluate lighting effectiveness. In this case, the night-to-day ratio will be used to assess the effects of lighting.

### 2.1.2. Statistical modeling method

In consideration of the aforementioned issues, the present analysis is based upon a with-without comparison of expected crash frequencies, conditioned on several intersection safety-influencing features, as the method to assess the safety effects of intersection lighting presence on motor vehicle crash frequency. Data files from Minnesota, which were acquired from the Federal Highway Administration's (FHWA) *Highway Safety Information System* (HSIS), contained the intersection-level traffic volume, geometric design, and roadway lighting presence data needed to estimate cross-sectional models of intersection safety. Four years (1999–2002, inclusive) of crash and roadway inventory data were available for the analysis. A total of 6464 intersections, all of which are maintained by the Minnesota Department of Transportation, were included in the analysis. Due to some missing geometric or traffic control data at some intersections, only 22,058 sample observations (out of a possible 25,832) were used for statistical modeling.

Descriptive statistics for the variables used in the statistical analysis are shown in Table 1.

A framework to estimate the safety effects of intersection lighting presence using Minnesota HSIS data is described by Donnell et al. (2010). The authors described the data structures, statistical modeling alternatives, possible endogeneity of lighting, and various analysis taxonomies. The entirety of the discussion is not repeated here; however, a brief review of various modeling alternatives is provided. Possible modeling methods include negative binomial regression to account for overdispersion in crash data (see Poch and Mannering, 1996; Bauer and Harwood, 1996; Washington et al., 2005 for intersection examples). Generalized estimating equations (e.g., Lord and Persaud, 2000; Wang et al., 2006; Donnell et al., 2009b) have been used to account for temporal correlation resulting from observing crash data annually. Random effects negative binomial (Shankar et al., 1997; Chin and Quddus, 2003) and negative multinomial (Ulfarsson and Shankar, 2003) models have been used to address issues of spatial and temporal correlation in crash data. Most recently, models of crash frequency have employed random parameters models (e.g., Anastasopoulos and Mannering, 2009) to account for the unobserved heterogeneity present across analysis locations. Lord and Mannering (2010) describe the advantages and disadvantages of using all of these methods to estimate statistical and econometric models of crash frequency.

As described by Lord and Mannering (2010), the negative binomial regression model accounts for overdispersion in crash data and is a straightforward model to estimate. Thus, the negative binomial model seems to be a logical choice to model expected intersection crash frequencies in the present study. The general functional form of the model is as follows:

$$\ln \lambda_i = \beta X_i + \varepsilon \quad (1)$$

where  $\lambda_i$  = expected number of crashes per year at intersection  $i$ ;  $\beta$  = vector of estimable regression parameters;  $X_i$  = vector of geometric design, lighting presence, and traffic volume data; and  $\varepsilon_i$  = gamma-distributed error term.

The mean-variance relationship for the negative binomial regression model is as follows:

$$\text{Var}(y_i) = E(y_i)[1 + \alpha E(y_i)] \quad (2)$$

where  $\text{Var}(y_i)$  = variance of observed crashes  $y$  at intersection  $i$ ;  $E(y_i)$  = expected annual crash frequency at intersection  $i$ ; and  $\alpha$  = overdispersion parameter.

The method of maximum likelihood is used to estimate the regression parameters in the negative binomial regression model. Ulfarsson and Shankar (2003) found that the negative binomial regression model outperformed the random-effects negative binomial regression model when indicator variables to account for serial correlation were included in the model. Thus, the present study applies a negative binomial regression model with indicator variables to account for temporal correlation resulting from the inclusion of yearly crash counts in the model.

A random parameters count regression model is also estimated in the present study to account for unobserved heterogeneity across observations. As described by Lord and Mannering (2010), the regression parameters estimated using a random parameters model can be written as follows:

$$\beta_i = \beta + \phi_i \quad (3)$$

where  $\phi_i$  = randomly distributed error term (Anastasopoulos and Mannering, 2009 found the normal distribution with mean 0 and variance  $\sigma^2$  to provide best fit in crash frequency models).

A simulation-based maximum likelihood procedure was used in the present study to estimate the parameters. In this process, 200 Halton draws were used because recent research by

**Table 1**

Variable definitions and descriptive statistics for Minnesota roadway intersection and crash data (Donnell et al., 2010).

Continuous variables	Min.	Max.	Mean	Standard deviation
Night crash frequency (per year)	0	28	0.366	0.969
Day crash frequency (per year)	0	55	1.121	2.457
Major road average daily traffic	40	77,430	8284	9381
Percent heavy vehicles on major road	0	61.11	8.888	5.109
Minor road average daily traffic	1	77,430	3164	5179
Categorical variables	Proportion in sample			
Area type indicator (1 = urban/suburban; 0 = rural)	1:0.446 0:0.554			
Traffic control indicator (1 = signal; 0 = stop-control)	1:0.137 0:0.863			
Lighting indicator (1 = present; 0 = not present)	1:0.421 0:0.579			
Intersection type indicator (1 = skew; 0 = cross or tee)	1:0.110 0:0.890			
Speed indicator <sup>a</sup> (1 = 50 mph or greater; 0 otherwise)	1:0.673 0:0.327			
No access control indicator <sup>a</sup> (1 = no access; 0 = partial access control)	1:0.943 0:0.057			
Depressed median indicator <sup>a</sup> (1 = depressed median; 0 = barrier or no median)	1:0.116 0:0.884			
Paved left-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	1:0.458 0:0.542			
Paved right-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	1:0.510 0:0.490			

<sup>a</sup> Data were used for the major intersecting roadway only.

Anastasopoulos and Mannering (2009) indicate that such a procedure produces accurate parameter estimates and converges more quickly than random draws. For a complete review of random parameters count models, readers are referred to Greene (2008).

### 2.1.3. Statistical modeling results

The results of the statistical analyses are shown in Table 2. Daytime and nighttime total crash frequency models are shown based on the negative binomial regression models without yearly indicator variables, negative binomial regression model with yearly indicators, and the random parameters negative binomial regression model. Negative binomial regression models for different intersection type subsets (e.g., rural signalized, rural stop-controlled, urban signalized, etc.) are included in Donnell et al. (2009a).

All of the geometric design and traffic control variables have the same sign and many have similar magnitudes in the daytime and nighttime crash frequency models across all formulations, suggesting that the effects of these variables do not change

lighting is higher than the baseline condition of no lighting present by about 5–8%, respectively, across the regression methods. The small magnitude of the lighting presence variable could suggest that more crashes are expected at intersections with lighting as a result of luminaires (i.e., fixed poles) present along the roadside. Assuming that this statistical association is consistent both day and night, the night-to-day crash ratio was used to adjust the nighttime crash frequency estimate relative to the daytime estimate.

The expected percent difference in the night-to-day crash ratio was computed using Eq. (4).

$$100 \left( \frac{(N/D)_w - (N/D)_{wo}}{(N/D)_{wo}} - 1 \right) \quad (4)$$

where  $N$ =expected number of nighttime crashes for all intersections included in the analysis;  $D$ =expected number of daytime crashes for all intersections included in the analysis;  $w$ =intersections with lighting; and  $wo$ =intersections without lighting.

The percent difference in the night-to-day crash ratio for total intersection crashes in Minnesota, based on the negative binomial model in Table 2, is computed as follows:

$$100 \left( \frac{(\exp(-0.079 \times 1)/\exp(0.048 \times 1))_w - (\exp(-0.079 \times 0)/\exp(0.048 \times 0))_{wo}}{(\exp(-0.079 \times 0)/\exp(0.048 \times 0))_{wo}} - 1 \right) = -11.8\% \quad (5)$$

considerably between the daytime and nighttime periods or across regression methods. A notable exception to this is the lighting presence indicator variable. As would be expected, the presence of roadway lighting is associated with fewer expected nighttime crashes than the baseline of no lighting present. In the daytime crash frequency models, the lighting presence indicator is 0.048, 0.050, and 0.078, respectively, across the three model formulations. This shows that the expected crash frequency at intersections with

For all Minnesota intersections in the analysis, the expected percent difference in the night-to-day crash ratio is approximately 12% lower at intersections with lighting when compared to intersections without lighting when applying the negative binomial regression model with no yearly indicator variables. When using the negative binomial regression model with yearly indicators and the random parameters negative binomial regression models, the expected percent difference in the night-to-day crash ratio is –11.9



**Table 2**  
Daytime and nighttime crash frequency models for all intersection crashes in Minnesota.

Variable	Negative binomial (no year indicators)		Negative binomial (with year indicators)		Random parameters			
	Daytime model		Daytime model		Daytime model		Nighttime model	
	Coeff. (SE <sup>b</sup> )	Coeff. (SE <sup>b</sup> )	Coeff. (SE <sup>b</sup> )	Coeff. (SE <sup>b</sup> )	Coeff. (SE <sup>b</sup> )	Scale <sup>c</sup> (SE <sup>b</sup> )	Coeff. (SE <sup>b</sup> )	Scale <sup>c</sup> (SE <sup>b</sup> )
Constant	−6.533 <sup>*</sup> (0.123)	−6.886 <sup>*</sup> (0.185)	−6.560 <sup>*</sup> (0.126)	−6.908 <sup>*</sup> (0.186)	−6.927 <sup>*</sup> (0.115)	0.593 <sup>*</sup> (0.008)	−7.033 <sup>*</sup> (0.176)	0.228 <sup>*</sup> (0.013)
Log major road average daily traffic (veh/day)	0.601 <sup>*</sup> (0.012)	0.572 <sup>*</sup> (0.018)	0.598 <sup>*</sup> (0.012)	0.569 <sup>*</sup> (0.019)	0.606 <sup>*</sup> (0.011)	0.037 <sup>*</sup> (0.001)	0.579 <sup>*</sup> (0.018)	0.006 <sup>*</sup> (0.001)
Log minor road average daily traffic (veh/day)	0.160 <sup>*</sup> (0.005)	0.127 <sup>*</sup> (0.008)	0.161 <sup>*</sup> (0.005)	0.126 <sup>*</sup> (0.008)	0.177 <sup>*</sup> (0.005)	0.007 <sup>*</sup> (0.001)	0.131 <sup>*</sup> (0.008)	0.023 <sup>*</sup> (0.002)
Percent heavy vehicles on major road (%)	−0.009 <sup>*</sup> (0.002)	−0.017 <sup>*</sup> (0.004)	−0.012 <sup>*</sup> (0.002)	−0.019 <sup>*</sup> (0.004)	−0.027 <sup>*</sup> (0.002)	0.055 <sup>*</sup> (0.001)	−0.034 <sup>*</sup> (0.004)	0.044 <sup>*</sup> (0.002)
Area type indicator (1 = urban/suburban; 0 = rural)	−0.100 <sup>*</sup> (0.020)	−0.421 <sup>*</sup> (0.032)	−0.101 <sup>*</sup> (0.021)	−0.421 <sup>*</sup> (0.032)	−0.155 <sup>*</sup> (0.021)	0.116 <sup>*</sup> (0.011)	−0.455 <sup>*</sup> (0.033)	0.031 (0.017)
Traffic control indicator (1 = signal control; 0 = stop-control)	0.646 <sup>*</sup> (0.032)	0.713 <sup>*</sup> (0.041)	0.647 <sup>*</sup> (0.032)	0.714 <sup>*</sup> (0.041)	0.722 <sup>*</sup> (0.022)	0.001 (0.014)	0.774 <sup>*</sup> (0.037)	0.039 (0.021)
Lighting indicator (1 = present; 0 = not present)	0.048 <sup>*</sup> (0.022)	−0.079 <sup>*</sup> (0.034)	0.050 <sup>*</sup> (0.023)	−0.077 <sup>*</sup> (0.035)	0.078 <sup>*</sup> (0.023)	0.041 <sup>*</sup> (0.010)	−0.110 <sup>*</sup> (0.036)	0.292 <sup>*</sup> (0.015)
Intersection type indicator (1 = skew; 0 = cross or tee)	0.488 <sup>*</sup> (0.020)	0.486 <sup>*</sup> (0.032)	0.480 <sup>*</sup> (0.021)	0.482 <sup>*</sup> (0.033)	0.476 <sup>*</sup> (0.023)	0.045 <sup>*</sup> (0.021)	0.381 <sup>*</sup> (0.037)	0.425 <sup>*</sup> (0.031)
Speed indicator <sup>a</sup> (1 = 50 mph or greater; 0 = otherwise)	−0.160 <sup>*</sup> (0.017)	−0.113 <sup>*</sup> (0.027)	−0.153 <sup>*</sup> (0.018)	−0.107 <sup>*</sup> (0.027)	−0.092 <sup>*</sup> (0.017)	0.082 <sup>*</sup> (0.011)	−0.097 <sup>*</sup> (0.027)	0.279 <sup>*</sup> (0.016)
Access control indicator <sup>a</sup> (1 = no control; 0 = partial control)	−0.042 (0.043)	−0.016 (0.051)	−0.048 <sup>*</sup> (0.042)	−0.018 (0.051)	−0.096 <sup>*</sup> (0.028)	0.379 <sup>*</sup> (0.009)	−0.160 <sup>*</sup> (0.040)	0.610 <sup>*</sup> (0.014)
Depressed median indicator <sup>a</sup> (1 = depressed median; 0 = barrier or no median)	0.085 <sup>*</sup> (0.028)	0.172 <sup>*</sup> (0.042)	0.089 <sup>*</sup> (0.028)	0.176 <sup>*</sup> (0.042)	0.164 <sup>*</sup> (0.025)	0.050 <sup>*</sup> (0.018)	0.208 <sup>*</sup> (0.038)	0.007 (0.026)
Paved left-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	−0.118 <sup>*</sup> (0.035)	−0.291 <sup>*</sup> (0.053)	−0.117 <sup>*</sup> (0.036)	−0.295 <sup>*</sup> (0.053)	−0.172 <sup>*</sup> (0.030)	0.143 <sup>*</sup> (0.011)	−0.288 <sup>*</sup> (0.046)	0.016 (0.018)
Paved right-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	0.081 <sup>*</sup> (0.035)	0.217 <sup>*</sup> (0.052)	0.080 <sup>*</sup> (0.036)	0.220 <sup>*</sup> (0.052)	0.127 <sup>*</sup> (0.029)	0.119 <sup>*</sup> (0.011)	0.239 <sup>*</sup> (0.045)	0.046 (0.016)
Year 2002 indicator (1 = year 2002; 0 = otherwise)			−0.011 (0.030)	−0.012 (0.042)				
Year 2003 indicator (1 = year 2003; 0 = otherwise)			0.249 <sup>*</sup> (0.026)	0.225 <sup>*</sup> (0.038)				
Year 2004 indicator (1 = year 2004; 0 = otherwise)			0.021 (0.030)	0.027 (0.041)				
Dispersion parameter ( $\alpha$ )	0.950 <sup>*</sup> (0.016)	0.896 <sup>*</sup> (0.028)	0.948 <sup>*</sup> (0.019)	0.895 <sup>*</sup> (0.032)	3.643 <sup>*</sup> (0.098)		3.080 <sup>*</sup> (0.173)	
Number of observations = 22,058	LL (constant only) = −30,979 LL (full model) = −27,087	LL (constant only) = −15,888 LL (full model) = −15,168	LL (constant only) = −30,932 LL (full model) = −27,033	LL (constant only) = −15,863 LL (full model) = −15,144	LL (constant only) = −54,438 LL (full model) = −25,560		LL (constant only) = −19,367 LL (full model) = −14,737	

The shaded row corresponds to the model values for lighting.

<sup>a</sup> Variable represents data available for major road approaches only.

<sup>b</sup> Standard error.

<sup>c</sup> Scale is the scale of the distribution for the random parameters as shown in Eq. (3).

<sup>\*</sup> Statistically significant at 95% confidence level.

and –17.1%, respectively. When considering the standard errors of the lighting presence indicator coefficients, the range of the expected percent difference in the night-to-day crash ratio is –6.9 to –16.7% for the negative binomial regression model without yearly indicators, –6.7 to –16.9% for the negative binomial regression model with yearly indicators, and –12.1 to –21.9% for the random parameters negative binomial regression model. It is also worth noting here, that Gross and Donnell (2011) applied an epidemiological case–control approach to the same data files used in the present study and reported that night-to-day crash ratios are 16.4% lower at lighted intersections than at unlighted intersections. Similarly, Sasidharan and Donnell (2013) applied a propensity scores and potential outcomes framework to the same Minnesota data, and found that the night-to-day crash ratio is 9.5% lower at lighted intersections when compared to unlighted intersections. The case–control and propensity scores-potential outcomes methods closely compare to the results obtained using the negative binomial regression models shown in Table 2. For this reason, and based on the review by Lord and Mannering (2010), which indicates that the random parameters negative binomial regression model is not easily transferable to other datasets, the remainder of the analysis focuses on interpretation of the negative binomial regression model.

Elasticities were computed for each of the independent variables included in the daytime and nighttime crash frequency models. The elasticity is a measure of responsiveness of one variable to a change in another variable. In the present study, the elasticity for a continuous variable is interpreted as a percent change in the expected daytime or nighttime crash frequency given a 1% change in the independent variable. Given variable  $k$  at intersection  $i$  during time period  $j$ , the elasticity for a continuous variable is (Washington et al., 2003):

$$E_{x_{ijk}}^{\lambda_{ij}} = \frac{\partial \lambda_{ij}}{\partial x_{ijk}} \times \frac{x_{ijk}}{\lambda_{ij}} \quad (6)$$

Eq. (6) reduces to  $\beta_k$  for a log-log functional form, and reduces to  $\beta_k x_{ijk}$  for a log-linear functional form. The elasticity for an indicator variable (i.e., pseudo-elasticity) is the percent change in the expected daytime or nighttime crash frequency given a change in the value of the indicator variable from zero to unity. The pseudo-elasticity for an indicator variable  $k$  at intersection  $i$  during time period  $j$  is defined as the following in the present study:

$$E_{x_{ijk}}^{\lambda_{ij}} = \exp(\beta_k) - 1 \quad (7)$$

Elasticities for the daytime and nighttime negative binomial crash frequency models are shown in Table 3. The elasticities in Table 3 for major and minor road traffic volumes are similar for the nighttime and daytime crash frequency models. The minor road elasticity is lower than the major road elasticity, indicating that the major road traffic volume has a greater effect on intersection crashes than the minor road traffic volume. A 1% increase in the heavy vehicle percentage on the major road is associated with a decrease in the expected daytime and nighttime crash frequency. An urban/suburban location is associated with fewer expected daytime and nighttime crashes than rural locations, likely due to lower speed and lower crash severities in urban/suburban areas. This may lead to a lower likelihood that a crash is reported. As anticipated, the expected daytime and nighttime crash frequency at signalized and skewed intersections is higher when compared to unsignalized and non-skewed intersections. The high-speed, access control, depressed median, paved left-shoulder, and paved right-shoulder indicators are consistent across the daytime and nighttime crash frequency models. As anticipated, the lighting presence indicator changes when comparing daytime and nighttime elasticities. This suggests that the presence of lighting is associated with a near

**Table 3**  
Elasticities for daytime and nighttime crash frequencies.

Variable	Elasticity or pseudo-elasticity (%)	
	Daytime crashes	Nighttime crashes
Log major road average daily traffic (veh/day)	0.601	0.572
Log minor road average daily traffic (veh/day)	0.160	0.127
Percent heavy vehicles on major road (%)	–0.082	–0.149
Area type indicator (1 = urban/suburban; 0 = rural)	–9.4	–34.4
Traffic control indicator (1 = signal control; 0 = stop-control)	90.5	103.8
Lighting indicator (1 = present; 0 = not present)	4.9	–7.6
Intersection type indicator (1 = skew; 0 = cross or tee)	62.6	62.3
Speed indicator <sup>a</sup> (1 = 50 mph or greater; 0 = otherwise)	–14.8	–10.7
Access control indicator <sup>a</sup> (1 = no control; 0 = partial control)	–4.1	–1.5
Depressed median indicator <sup>a</sup> (1 = depressed median; 0 = barrier or no median)	8.9	18.7
Paved left-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	–11.0	–25.2
Paved right-shoulder indicator <sup>a</sup> (1 = paved shoulder; 0 = unpaved or no shoulder)	8.3	24.0

<sup>a</sup> Data were available for major road only, so elasticity represents effect for major road approaches and not minor road approaches.

5% increase in daytime crashes, but is associated with a near 8% reduction in nighttime crashes.

Table 4 summarizes selected results from the statistical models for four specific intersection types in Minnesota (corresponding to signalized or unsignalized intersections, and intersections in either rural or in urban/suburban locations) based on negative binomial regression. The leftmost column in Table 4 includes the number of observations in the database over a period of four years. The numbers of observations in Table 4 are approximately the number of intersections of each type multiplied by four, but because a few locations changed status (e.g., by installation of lighting or construction work zone) during the four-year period, small deviations from this relationship exist. Data for urban and suburban intersections of a given type (signalized or unsignalized) were combined into a single category (urban/suburban), because early modeling efforts yielded nearly identical models for urban and suburban intersections, and because there is no universally accepted method to differentiate urban and suburban locations among transportation agencies. While all of the details for the Minnesota intersection models are not reported in Table 4, all of the daytime and nighttime models take into account the presence of lighting, average daily traffic volumes on the intersecting roads, posted speed limit, type of access control, percentage of heavy trucks, and other factors, which are also included in the model shown in Table 2. The statistical models account for many of the geometric and traffic control variables present at an intersection, and thus remove some of the effects that in the past might have been attributed to the presence of lighting reported in published research. However, the

**Table 4**

Summary results for statistical modeling of daytime and nighttime crash frequencies (with confidence ranges based on  $\pm 1$  standard error) for different intersection types (using the approach developed by Donnell et al., 2010). The ranges of values in the rightmost column are based on calculating night-to-day crash ratios to identify the range of values that could be calculated using the standard error bounding values for the difference in daytime and nighttime crashes.

Intersection type (count, %lighted)	Model		
	Difference in day crashes with lighting ( $\pm 1$ standard error)	Difference in night crashes with lighting ( $\pm 1$ standard error)	Difference in night-to-day crash ratio (based on $\pm 1$ standard error)
All intersections <sup>a</sup> (n = 22,058, 46%)	+5% (+2% to +8%)	–8% (–11% to –4%)	–12% (–18% to –5%)
Urban/suburban signalized (n = 2875, 97%)	+3% (–6% to +14%)	–3% (–17% to +13%)	–7% (–28% to +21%)
Urban/suburban unsignalized (n = 7730, 76%)	+5% (+0.3% to 11%)	–9% (–15% to –1%)	–13% (–24% to –2%)
Rural signalized (n = 352, 94%)	–2% (–18% to +17%)	–2% (–24% to +27%)	0% (–35% to +55%)
Rural unsignalized (n = 11,101, 10%)	+9% (+3% to +15%)	+7% (+0.2% to +15%)	–2% (–13% to +12%)

<sup>a</sup> From Table 2.

statistical models do not account for factors such as weather, or the effects of glare or other characteristics of the lighting systems on safety because these data were not available in databases that were linked to crash data in the present study.

Although the statistical modeling used in the present study represents a useful step forward in developing estimates of the statistical association between roadway lighting and night-to-day crash ratios, it is difficult to translate the results into engineering practice because lighting had to be treated as a binary variable (i.e., it was either present or not); that is, the presence or absence of lighting was all that was available in the HSIS data sets. Furthermore, the statistical modeling approaches described here do not test the theoretical cause-and-effect relationships among lighting, visibility and safety (Sasidharan and Donnell, 2013). As a result, alternative analyses must be performed to convert the statistical analyses into lighting design practice.

## 2.2. Visual performance modeling

### 2.2.1. Background

The approach taken by Rea et al. (2010) was, in effect, to develop support for the link between roadway lighting and visibility, within the theoretical framework described in the Introduction to this paper. In their analytical study, Rea and colleagues used photometrically accurate lighting calculation software together with a model of visual performance (relative visual performance, RVP; Rea and Ouellette, 1991) to assess the *visibility coverage area* afforded by different roadway lighting systems.

The majority of crashes at roadway intersections in Minnesota were multiple-vehicle crashes (Donnell et al., 2009a). This suggests, in agreement with Neuman et al. (2003), that one important cause of crashes at intersections is a driver's inability to accurately judge the relative speed and distance of traffic, either on the same or on the adjacent roadway. Making these judgments is important when deciding to proceed through, or make a turning maneuver, in an intersection.

Gap estimation, in which a driver estimates whether there is sufficient time between vehicles to enter or cross the traffic stream, is a critical visual task at intersections (Neuman et al., 2003). Gap estimation requires a driver to see the approaching vehicles. During daytime this visual task is straightforward provided there is adequate visibility along the line of sight. Since vehicles operate with their headlights on during nighttime, it might be assumed that gap estimation is also straightforward at night, but this is not necessarily the case. Certainly vehicle headlights are highly visible, but their high intensity detracts from one's ability to see the area surrounding the vehicle (Bullough et al., 2003), information which is necessary in judging the relative speed of the traveling

vehicle. In such cases, the presence of roadway lighting around the vehicle increases the luminance of the surrounding pavement and roadside surfaces, reducing disability glare from vehicle headlights (IES, 2000) and providing increased figure/ground information for gap estimation. The visual performance analyses conducted by Rea et al. (2010) assessed the visual information provided by different configurations of roadway lighting at intersections using a standard visual target (IES, 2000) to assess the visual figure/ground information required for accurate gap estimation at night.

They first created a virtual roadway intersection with drivers positioned at different critical points on the roadway. They next systematically quantified the many levels of visibility provided by roadway, vehicle and ambient lighting in and around the virtual roadway intersection, for a wide range of combinations of illuminance levels, spatial configurations and ambient light conditions. They then determined the visibility of a standard target when placed on the virtually illuminated roadway as it would be seen by drivers of different ages and in different viewing positions within the virtual environment to assess drivers' gap estimation ability throughout the intersection. Disability glare levels from virtual headlights were also determined as they might affect target visibility. The large amount of data generated for a given set of conditions was reduced to a single figure called the *visibility coverage area* afforded to drivers by the roadway intersection lighting system.

The modeling approach developed by Rea et al. (2010) is based on conventional photometric (i.e., photopic) units, which characterize a lighting system's ability to stimulate the cone photoreceptors of the visual system. An extensive body of research (Rea et al., 2004; IES, 2006; CIE, 2010) has demonstrated that at light levels typical for roadway lighting at night (IES, 2000), peripheral visual performance is supported by both the cone and rod photoreceptors. The rod photoreceptors are distributed throughout the peripheral retina in the eye, and they are relatively more sensitive to shorter (i.e., "blue") visible wavelengths of light than the cone photoreceptors in the central portion of the retina. Detection of objects in the visual periphery under roadway lighting conditions, therefore, is improved when a light source with greater relative short-wavelength output is used (e.g., light-emitting diodes or metal halide), even compared to one producing the same photopic light level but producing relatively little short-wavelength output (e.g., high pressure sodium). These findings may have significant implications for roadway lighting when unexpected hazards such as pedestrians or animals are first detected in the visual periphery, but are probably less important for visual tasks such as gap acceptance, where the primary visual judgment is an on-axis task, not a peripheral detection task. Since there are few if any rod photoreceptors in the central portion of the eye's retina, rods

would be expected to play little role in on-axis vision and therefore, conventional photometric units are used in the analyses described here.

### 2.2.2. Visual performance modeling method

The relative visual performance (RVP) model (Rea and Ouellette, 1991) provides a precise method for calculating the speed and accuracy with which visual information can be processed, based upon the following input parameters to the model:

- Size of the visual target
- Luminance of the background around the visual target
- Luminance contrast between the visual target and its background
- Age of the observer

Fig. 1 shows the plateau and escarpment characteristics of RVP, and Appendix A describes the RVP calculation method (Rea and Ouellette, 1991).

Fig. 1 is a three-dimensional surface plot of RVP for a target of a particular visual size, or solid angle, measured in microsteradians ( $\mu\text{sr}$ ). In the illustrated example, the size of the target is  $15 \mu\text{sr}$ , which corresponds to the size of a standard “small target visibility” (STV) target used by the Illuminating Engineering Society (IES, 2000) for the specification of roadway visibility when viewed from a distance of 46 m. Each point on the surface of Fig. 1 corresponds to the level of RVP associated with that target size at different contrasts and seen against different background luminances. The figure clearly illustrates that, for this target size, when both background luminance and luminance contrast are reduced, visual performance can drop precipitously. Conversely, as background luminance or luminance contrast increases, nearly asymptotic values of RVP can be reached such that further increases in either background luminance or luminance contrast do not result in substantial increases in speed and accuracy of processing visual information.

This plateau and escarpment characteristic is a fundamental nature of visual performance (Rea and Ouellette, 1991).

The RVP model was developed from two independent sets of data: measurements of speed and accuracy for error-checking of printed numbers varying in luminance contrast and light level (Rea, 1986) and reaction times to flashed increment and decrement targets varying in luminance contrast and size (Rea and Ouellette, 1988). Light levels for these data ranged from mesopic ( $0.17 \text{ cd/m}^2$ ) to photopic ( $255 \text{ cd/m}^2$ ), corresponding to outdoor and indoor lighting conditions (Rea, 2000). This model was subsequently validated for reading tasks (Bailey et al., 1993; Eklund et al., 2001), traffic sign legibility (Goodspeed and Rea, 1999; Schnell et al., 2009), and for roadway pedestrian identification under different lighting conditions in static conditions (Bullough et al., 2012) and under real-world nighttime driving conditions (Bullough and Skinner, 2009, 2012).

The RVP model differs from the visibility model used to specify STV in the IES (2000) recommendations. That model is based on the concept of “visibility level” (VL), defined as the ratio between a target’s contrast under specific lighting conditions, and the minimum contrast it would need to just achieve a threshold visibility. The model of threshold visibility used by the IES (2000) was developed by Adrian (1989) based on threshold visibility data collected by Blackwell (1946).

There are two key differences between the VL and RVP approaches to modeling visual performance. The first is that VL does not exhibit a plateau characteristic, so that asymptotic visual performance (e.g., in terms of response speed and accuracy) is associated with VLs of 5, 10 or even higher. Thus, VL increases while visual performance does not. The second difference is that visual performance above threshold but below saturation cannot be predicted accurately by VL. Curves of the luminance contrast required to achieve equal performance of a visual task, plotted as a function of the background luminance, are not parallel (Ross, 1978; Rea, 1986). Both of these flaws in the VL determination mean that VL

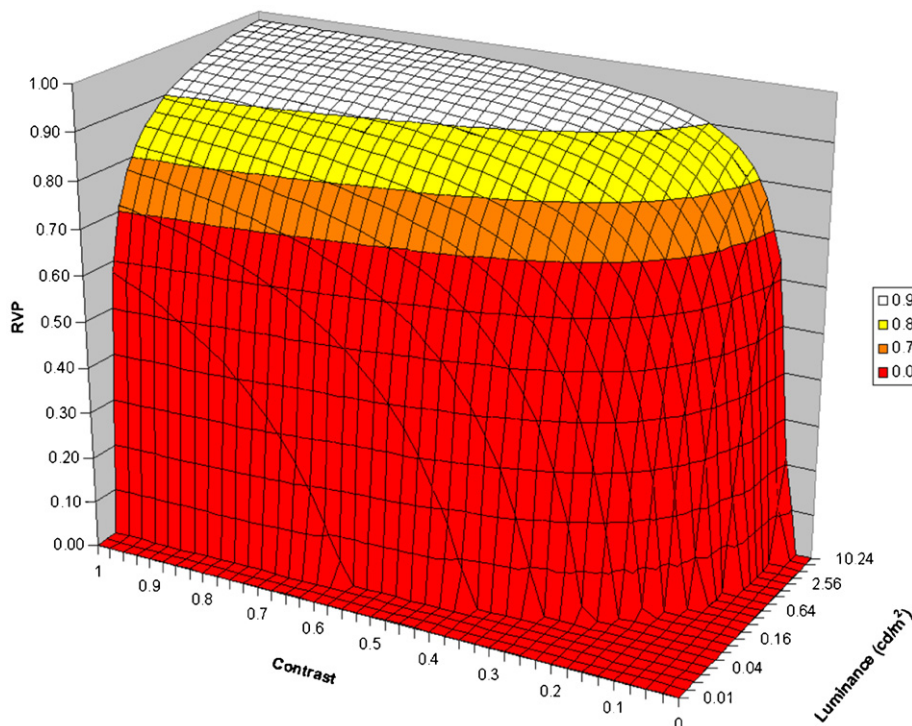


Fig. 1. Relationships among background luminance, target contrast and relative visual performance (RVP) for a target having a solid angular size of 15 microsteradians. This size corresponds to that of the small target ( $20 \text{ cm} \times 20 \text{ cm}$ ) used by the IES (2000) in the specification of visibility for roadway lighting, viewed from 46 m.



does not provide a rectification between the characteristics of a visual target and one's ability to quickly and accurately detect it.

Despite these fundamental differences between RVP and VL, the analysis presented here could be performed using any visibility model—RVP, VL or others (Clear and Berman, 1990; Kambich, 1991; Ising, 2008), and would all produce similar trends. Because the RVP model was developed using data corresponding to the mesopic and photopic luminance ranges found in roadway lighting practice (IES, 2000) and because it has been validated under a variety of conditions including nighttime driving (Bullough and Skinner, 2009, 2012), it was used in the present analytical modeling approach.

For simplicity, Rea et al. (2010) quantified visibility coverage area in terms of RVP scores, where RVP values  $\geq 0.9$  have an RVP score of 3, RVP values  $\geq 0.8$  and  $< 0.9$  have an RVP score of 2, RVP values  $\geq 0.7$  and  $< 0.8$  have an RVP score of 1, and RVP values  $< 0.7$  have an RVP score of 0. These RVP scores are illustrated by the shading of Fig. 1.

### 2.2.3. Visual performance modeling results

Roadway lighting can be applied in a variety of ways (e.g., continuous and regular pole spacing, staggered pole spacing, poles only at intersections, or poles only at property lines) in both rural or urban locations (Li et al., 2006) along roads with different driving speeds and traffic volumes. Potential visual hazards of different sizes and contrasts can be present in any location along the roadway, and since recommended light levels can differ from just a few lx to more than 20 lx depending upon roadway types and conflict areas (IES, 2000), a wide range of visual conditions are presented to drivers of different visual capabilities. Recognizing this diversity in visual conditions, Rea et al. (2010) developed the concept of *visibility coverage area* which is determined from the mean level of RVP scores provided to drivers of different ages and viewing positions by the roadway, vehicle and ambient lighting conditions surrounding an intersection. Modern, photometrically accurate computer software (AGI32, Lighting Analysts) was used to create realistic virtual lighting scenarios for a prototypical roadway intersection within the context of rural, suburban, urban and highly urbanized, metropolitan environments.

A standard target was selected and placed in the virtual scenario so that the levels of RVP could be computed for different driver ages and viewing positions within the virtual scenario. The visual target used by the IES to compute STV, a gray (50% reflectance) 20-cm square, was utilized to compute levels of RVP. This standard target was assumed (Rea et al., 2010) to meaningfully represent the reflectance and size of a potential hazard to drivers, being too large to be ignored by a driver yet small enough to be sensitive to differences in roadway lighting conditions. More specifically with regard to the crash analyses, this standard target was assumed to be useful as a stationary reference in the environment for drivers to assess the movement and direction of an object (e.g., another car) that might become a hazard to avoid. Fig. 2 illustrates the prototypical roadway intersection and one of the roadway lighting layouts used to assess *visibility coverage areas* using the photometrically accurate software. Shown too are the locations of the two virtual driver

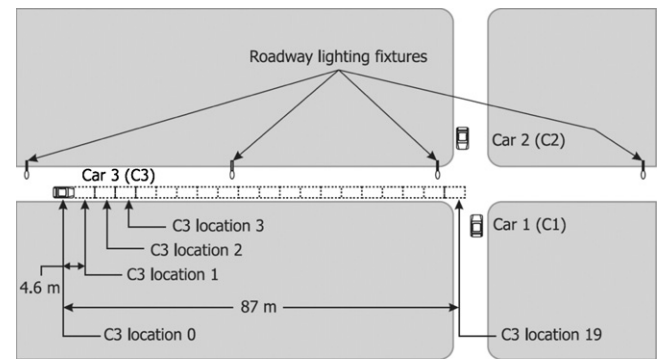


Fig. 2. Roadway scenario used in analytical approach. For localized lighting configurations, only the luminaire closest to the intersection was assumed to be on; for the extended lighting configurations, all of the luminaires illustrated were assumed to be on.

vehicles adjacent to the intersection (C1 and C2) and the 20 locations of the virtual approaching vehicle (C3) and the standard visual target (adjacent to the virtual car) used in the RVP calculations.

To provide a comparison with the results of the statistical approach, visual performance data for the four roadway intersection types listed in Table 4 were computed based on the analytical approach developed by Rea et al. (2010). It was possible to make this examination because roadway intersection lighting practices in Minnesota (MNDOT, 2006) follow a well-established method recommended by the IES (IES, 2000). The assumptions regarding representative lighting configurations for these roadway location types are listed in Table 5. In Table 5, *localized* lighting refers to a configuration of only one or two luminaires at the intersection's conflict area, and *extended* lighting refers to a configuration of continuous lighting (at least three luminaires) along the approach of the intersection's major highway. The assumptions for roadway lighting in the scenarios are not only based on the lighting standards for roadway lighting published by the Minnesota Department of Transportation (MNDOT, 2006), but also upon visual spot checks (i.e., pole locations and spacing) of lighting systems using Google Maps' Street View function (<http://maps.google.com>) for a small sample ( $n = 20$ ) of state and U.S. highway intersections in Minnesota that were found of different types:

- Rural unsignalized ( $n = 9$ ): four with no lighting, five with localized lighting
- Rural signalized ( $n = 1$ ): localized lighting
- Urban/suburban unsignalized ( $n = 3$ ): extended lighting
- Urban/suburban signalized ( $n = 7$ ): extended lighting

Again, the visibility coverage area is based upon the average RVP scores from three different driver age groups (30, 45 and 60 years) and for the different target locations in the prototypical roadway in Fig. 2. For the intersections with lower speed limits (in urban/suburban locations), the relevant target locations for assessing visibility coverage area are close to the intersection, that is, at positions 10–19. For rural intersections, with higher speed

Table 5

Characteristics of roadways and lighting systems for urban/suburban signalized, urban/suburban unsignalized, rural signalized and rural unsignalized intersections.

Roadway intersection type	Roadway illuminance (lx)	Intersection illuminance (lx)	Ambient illuminance (lx)	Speed limit (mph)	Extended/localized
Urban/suburban signalized	18	30	2	30	Extended
Urban/suburban unsignalized	9	15	0.2	30	Extended
Rural signalized	6	10	0.2	55	Localized
Rural unsignalized	6	10	0.02	55	Localized

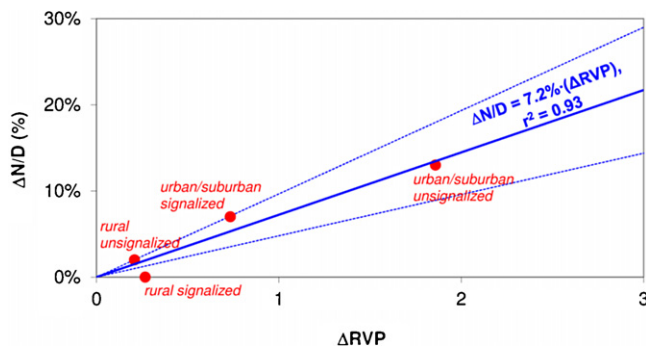
**Table 6**  
Calculated changes in RVP score for each intersection type and driver age, using the roadway locations illustrated in Fig. 2 (locations 10–19 for urban/suburban intersections and locations 0–9 for rural intersections).

C3	Age = 30				Age = 45				Age = 60			
Urban/suburban signalized												
10	0	1	1	1	0	1	0	0	0	2	1	1
11	1	1	1	1	2	1	1	1	1	1	1	1
12	2	1	1	1	2	1	1	1	2	1	0	1
13	1	1	1	1	2	1	1	1	3	2	0	0
14	1	1	0	1	1	1	1	1	2	1	0	0
15	1	0	0	0	1	1	1	1	2	1	1	1
16	1	0	0	0	1	1	0	1	1	1	1	1
17	0	0	0	0	1	0	0	1	1	1	1	1
18	0	0	0	0	0	0	0	0	1	0	1	1
19	0	0	0	0	0	0	0	0	0	0	1	1
Ave	0.53				0.73				0.95			
	0.73											
Urban/suburban unsignalized												
10	0	2	2	2	0	2	2	2	0	2	1	1
11	0	2	2	2	0	2	2	2	0	1	1	1
12	2	2	2	2	2	2	2	2	1	2	1	1
13	3	3	2	2	2	3	2	2	2	2	2	2
14	3	3	2	2	3	3	2	2	2	2	2	2
15	3	3	1	3	3	3	1	2	2	2	2	2
16	3	3	1	3	3	3	1	2	3	3	2	2
17	3	1	1	2	3	2	1	2	2	3	2	2
18	3	0	1	2	3	0	1	3	3	1	2	2
19	0	0	2	2	0	0	3	3	1	0	2	2
Ave	1.93				1.95				1.70			
	1.86											
Rural signalized												
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	0	0	0	0	0
3	0	0	1	1	0	0	0	0	0	0	0	0
4	0	0	1	1	0	0	0	0	0	0	0	0
5	0	0	2	2	0	0	1	1	0	0	0	0
6	0	0	1	1	0	0	1	1	0	0	0	0
7	0	0	1	2	0	0	1	1	0	0	0	0
8	0	0	2	2	0	0	1	1	0	0	0	0
9	0	0	2	2	0	0	1	1	0	0	0	0
Ave	0.55				0.27				0.00			
	0.27											
Rural unsignalized												
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0
4	0	0	1	1	0	0	0	0	0	0	0	0
5	0	0	2	2	0	0	1	1	0	0	0	0
6	0	0	1	1	0	0	0	0	0	0	0	0
7	0	0	1	1	0	0	1	1	0	0	0	0
8	0	0	1	1	0	0	1	1	0	0	0	0
9	0	0	2	2	0	0	1	1	0	0	0	0
Ave	0.43				0.20				0.00			
	0.21											

limits, the relevant target locations are further from the intersection, that is, positions 0–9. Visibility coverage areas for the four intersection types, with and without lighting, are then determined. The differences between RVP scores with roadway lighting and without are then calculated for every location and for the three age groups. These difference values are presented in the cells of Table 6 along with the overall average change in RVP scores for each of the four roadway intersection scenarios.

**3. Integration of results from the statistical and analytical approaches**

As described above, the visibility coverage area analysis developed by Rea et al. (2010) was used here together with the statistical associations between lighting and nighttime crashes based on Donnell et al. (2010) to examine the theoretical relationships among lighting, visibility and traffic safety. Since the statistical



**Fig. 3.** Comparison of the estimated improvement in visibility coverage area ( $\Delta RVP$ ) from roadway lighting at night and the reductions in the night-to-day crash ratios ( $\Delta N/D$ ) estimated from the statistical models. Also shown are the 95% confidence intervals for the best-fitting linear function (form:  $y = ax$ ) to the data.

associations between crash frequency and roadway lighting for different types of intersections had been modeled by Donnell et al. (2009a) and since the visibility coverage areas for those intersection types could be determined, it follows that if improved visibility is the mechanism by which vehicle crashes at night can be reduced with roadway intersection lighting, there should be a correlation between the predicted differences in night-to-day crash ratios associated with the presence of roadway lighting (from Table 4) and the impact in visual performance due to roadway lighting.

To provide a first test of this logical construct, each of the night-to-day crash ratio differences derived from the statistical models in Table 4 for each roadway lighting scenario were compared to the change in RVP scores for the corresponding visibility coverage areas in Minnesota in Table 6. As described in the Introduction, the statistical and visibility modeling approaches represent independent analyses of the likely impacts of lighting on safety and on visibility, but importantly both are based on the same engineering practices (i.e., those for roadway intersections in Minnesota (MNDOT, 2006)).

There was a strong positive correlation ( $r^2 = 0.93$ ) between the night-to-day crash ratio differences estimated from the statistical models in Table 4, and the changes in RVP score values from Table 6 for the corresponding visibility coverage areas. This correlation is illustrated in Fig. 3, and it supports the inference that lighting, as practiced in Minnesota, increases visibility, as measured using the visibility coverage area, which in turn improves safety, as defined by lower night-to-day crash ratios.

#### 4. Discussion

The present study formally examined the theoretical relationships among lighting, visibility and safety using two independent approaches, a statistical and an analytical approach, for the same lighting context. A key to the present study was that the roadway lighting conditions associated with crashes in Minnesota could be modeled in terms of the levels of visibility those same roadway lighting conditions provide. The close correspondence between the statistical and the analytical results (Fig. 3) provides preliminary empirical support for the theoretical links among roadway lighting, visual performance and traffic safety.

In the context of previous statistical studies of the association between roadway lighting and improvements in traffic safety, it is worth noting that the calculated changes in the night-to-day crash ratios for the models listed in Table 4 do not exceed 13% (as illustrated in Fig. 3). This value is substantially lower than the 30% value for the reduction in the night-to-day crash ratio often presented in the literature (e.g., CIE, 1992) as an estimate for the safety benefit of roadway lighting. The difference between the results reported here and the 30% value from the literature is likely a result of the

inclusion of several, but not necessarily all (e.g., weather) potential safety-influencing variables that are not always included in observational studies of roadway lighting. For example, Donnell et al. (2010) found a 28% difference in the night-to-day crash ratio (similar to the 30% value from CIE (1992)) associated with the presence of roadway intersection lighting from the observed data underlying the statistical models they developed, which yielded an estimated 12% night-to-day crash ratio difference associated with lighting when controlling for many safety-influencing features present at intersections included in the statistical approach.

It is assumed with the analytical approach described here that illumination, rather than delineation or visual guidance systems, are important to safety. For example, as pointed out by Rea et al. (2010), vehicle headlamps are high-brightness objects and easy to see at night, yet vehicle crashes are still common at night. In order to assess the relative hazard potential of another vehicle or of an object when driving at night it is often necessary to estimate its relative speed and direction of movement (Leibowitz and Owens, 1977). These judgments require not only high target conspicuity ("figure") but the context against which that target is seen ("ground"). By their nature, delineation and visual guidance systems, whether reflective, such as post-mounted delineators (Skinner and Bullough, 2009), or self-luminous, such as in-pavement crosswalk lights (Van Derlofske et al., 2003), identify roadway features that can be important for driving safety, such as sharp curves or pedestrian crosswalks, but do not provide figure-ground information deemed essential for driving safety (Simms, 1985; Beatson and Gianutsos, 2000) during situations when unexpected hazards occur (e.g., an animal moving across a roadway). For these reasons the night-to-day crash ratio reduction values illustrated in Fig. 3 should only be considered estimates of the beneficial effect of roadway illumination on nighttime crash safety.

An important contribution of the present study comes from the statistical approach developed by Donnell et al. (2010) in that it is based on expected rather than observed crashes, does not assume a linear relationship between crashes and traffic volume, and controls for many intersection-safety relationships that have not been considered in past lighting-safety research (e.g., geometry, traffic control). Still, the statistical approach was unable to consider all variables (e.g., weather conditions) and lighting design variables. In sharp contrast, the RVP model used in the analytical approach by Rea and Ouellette (1991) is a technique to evaluate the impact of lighting on visibility and this model of visual performance has been validated in a number of studies (Bailey et al., 1993; Goodspeed and Rea, 1999; Eklund et al., 2001; Schnell et al., 2009; Bullough et al., 2012). Despite its validity, however, it is completely silent with respect to crash frequency. What is needed, and has been attempted here, is a converging approach, combining the safety estimates with the visibility predictions where the lighting conditions are the same. To make this linkage, the RVP analysis was jointly considered with the statistical models through the use of a visibility coverage area metric developed by Rea et al. (2010). The empirical, provisional transfer function illustrated in Fig. 3 supports the theoretical relationships among lighting, visibility and traffic safety.

Once the relationships among lighting, visibility and traffic safety have been justified, different roadway lighting scenarios can be assessed by roadway engineers in terms of benefits and costs. These engineering analyses may then be used to inform public policy. The provisional transfer function in Fig. 3 can be used to estimate the expected crash frequency benefits of roadway lighting as it affects visual performance. Future research should be undertaken to develop relationships between lighting, visibility, and crash severity, which would permit the undertaking of benefit/cost analyses of different lighting configurations during the lighting design stages, because assessing the visual performance

associated with lighting is readily possible using existing lighting calculation software (Rea et al., 2010).

Despite the encouraging relationship illustrated in Fig. 3, the need to further validate these results should not be underestimated. Safety is an important reason for installing roadway lighting (Bullough, 2010), so it is important to provide additional empirical evidence for the linkages among lighting, visibility and traffic safety. Ideally, data from other states, provinces and countries should be examined. Unfortunately, safety and lighting presence data of the types examined by Donnell et al. (2010) are not routinely collected and therefore are not readily available for statistical modeling. Another approach is to continue to conduct case studies, either before-and-after or with-without comparisons, where proper control conditions are implemented. In this way it would be possible to formally examine whether the predicted lighting-dependent differences in visual performance translate into observed differences in crash frequency. With the clear linkages among lighting, visibility and safety offered here, this type of research can be undertaken.

### Acknowledgments

We gratefully acknowledge the input and technical contributions of our colleagues Peter Morante and Leora Radetsky from the Lighting Research Center, Rensselaer Polytechnic Institute.

### Appendix A.

This appendix provides the calculation methods for assessing the relative visual performance (RVP) of a target with a particular background luminance, luminance contrast and size, for an observer of a particular age (Rea and Ouellette, 1991).

Let  $L_b$  be the background luminance in  $\text{cd}/\text{m}^2$ . Let  $L_t$  be the luminance of the target. Calculate the target's luminance contrast ( $C$ , a dimensionless quantity):

$$C = \frac{|L_t - L_b|}{\max(L_t, L_b)} \quad (\text{A.1})$$

Let  $S$  be the size of the target in microsteradians ( $\mu\text{sr}$ ):

$$S = \frac{T}{1,000,000d^2} \quad (\text{A.2})$$

where  $T$  is the projected area of the target (in  $\text{m}^2$ ) and  $d$  is the viewing distance (in m).

Calculate the pupil radius  $P$  (in mm):

$$P = 2.39 - 1.22 \tanh(0.3 \log L_b) \quad (\text{A.3})$$

Let  $A$  be the observer's age in years. Calculate the age-corrected retinal illuminance  $E_r$  in trolands (Td):

$$E_r = \pi P^2 L_b [1 - 0.017(A - 20)] \quad (\text{A.4})$$

Calculate five intermediate values  $x_1, x_2, x_3, x_4$  and  $x_5$ :

$$x_1 = \log[\tanh(20,000S)] \quad (\text{A.5a})$$

$$x_2 = \log \left[ \log \left( 10 \frac{E_r}{\pi} \right) \right] \quad (\text{A.5b})$$

$$x_3 = 1 + [0.0025(A - 20)] \quad (\text{A.5c})$$

$$x_4 = \log[\tanh(5000S)] \quad (\text{A.5d})$$

$$x_5 = \log \left[ \tanh \left( 0.04 \frac{E_r}{\pi} \right) \right] \quad (\text{A.5e})$$

Calculate the threshold luminance contrast  $C_t$  (a dimensionless quantity):

$$C_t = x_3 10^{(-1.36 - 0.18x_1 - 0.81x_2 + 0.23x_1^2 - 0.077x_2^2 + 0.17x_1x_2)} \quad (\text{A.6})$$

Calculate the half-saturation constant  $K$ :

$$K = 10^{(-1.76 - 0.18x_4 - 0.031x_5 + 0.11x_4^2 + 0.17x_5^2 + 0.062x_4x_5)} \quad (\text{A.7})$$

Calculate the maximum response  $R_{\max}$ :

$$R_{\max} = 0.0002 \log(E_r) + 0.0027 \quad (\text{A.8})$$

Calculate the visual response time  $V$  (in ms):

$$V = \frac{(C - C_t)^{0.97} + K^{0.97}}{(C - C_t)^{0.97} R_{\max}} \quad (\text{A.9})$$

Calculate the relative visual performance (RVP):

$$\text{RVP} = 1.42 - \frac{V}{778.56} \quad (\text{A.10})$$

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