Modeling Crash Types: New Insights into the Effects of Covariates on Crashes at Rural Intersections

Do-Gyeong Kim¹; Simon Washington²; and Jutaek Oh³

Abstract: Many studies focused on the development of crash prediction models have resulted in aggregate crash prediction models to quantify the safety effects of geometric, traffic, and environmental factors on the expected number of total, fatal, injury, and/or property damage crashes at specific locations. Crash prediction models focused on predicting different crash types, however, have rarely been developed. Crash type models are useful for at least three reasons. The first is motivated by the need to identify sites that are high risk with respect to specific crash types but that may not be revealed through crash totals. Second, countermeasures are likely to affect only a subset of all crashes—usually called target crashes—and so examination of crash types will lead to improved ability to identify effective countermeasures. Finally, there is a priori reason to believe that different crash types (e.g., rear-end, angle, etc.) are associated with road geometry, the environment, and traffic variables in different ways and as a result justify the estimation of individual predictive models. The objectives of this paper are to (1) demonstrate that different crash types are associated to predictor variables in different ways (as theorized) and (2) show that estimation of crash type models may lead to greater insights regarding crash occurrence and countermeasure effectiveness. This paper first describes the estimation results of crash prediction models for angle, head-on, rear-end, sideswipe (same direction and opposite direction), and pedestrian-involved crash types. Serving as a basis for comparison, a crash prediction model is estimated for total crashes. Based on 837 motor vehicle crashes collected on two-lane rural intersections in the state of Georgia, six prediction models are estimated resulting in two Poisson (P) models and four NB (NB) models. The analysis reveals that factors such as the annual average daily traffic, the presence of turning lanes, and the number of driveways have a positive association with each type of crash, whereas median widths and the presence of lighting are negatively associated. For the best fitting models covariates are related to crash types in different ways, suggesting that crash types are associated with different precrash conditions and that modeling total crash frequency may not be helpful for identifying specific countermeasures.

DOI: 10.1061/(ASCE)0733-947X(2006)132:4(282)

CE Database subject headings: Traffic safety; Traffic accidents; Intersections; Pedestrians; Traffic models; Roads; Rural areas.

Introduction

Due to the complexity of traffic movements and the potential number of conflicts between motor vehicles, pedestrians, and bicyclists, there is a relatively large potential for crashes at intersections (compared to segments). From the desire to understand the effects of countermeasures and to identify factors that are associated with crashes, researchers routinely estimate crash prediction models of intersections using roadway geometric variables, environmental factors, and traffic conditions as predictors. Many researchers, furthermore, have focused on the development of aggregate crash prediction models, whereby

Note. Discussion open until September 1, 2006. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on April 6, 2004; approved on September 28, 2005. This paper is part of the *Journal of Transportation Engineering*, Vol. 132, No. 4, April 1, 2006. ©ASCE, ISSN 0733-947X/2006/4-282-292/\$25.00.

concentration is on the influences of geometric, environmental, and traffic variables on the total expected number of crashes at intersections (totals, fatals, injuries, etc.). Crash prediction models focused on predicting different crash types, however, have rarely been developed.

There are at least three important and defensible reasons for estimating models that predict and/or explain crash type as a function of geometric, environmental, and traffic factors. The first is the need to identify sites that are high risk with respect to specific crash types but that are not revealed through crash totals. For example, an urban intersection may produce fairly typical numbers of total crashes but possess outlying numbers of rear-end or angle crashes with respect to similar sites. Being able to determine the "expected" number of crashes by crash type enables a researcher to identify these outlying types of crashes, which in turn may indicate a specific deficiency at the intersection. A second use of these models is to gain an understanding of the differing effects of geometric, traffic, and environmental factors on crash type, so that countermeasure effects may be better understood. For example, it may be learned that pavement resurfacing increases roll over crashes but reduces rear-end crashes. Finally, there is a priori reason to believe that certain types of crashes are associated with road geometry, the environment, and traffic variables in different ways. For example, dark conditions at intersections may influence angle crashes more than run off road crashes—suggesting that these crash types may be affected by different coefficients and that comingling these crash types within

¹Graduate Research Assistant, Dept. of Civil Engineering, Univ. of Arizona, Tucson, AZ 85712-0072. E-mail: dokkang@u.arizona.edu

²Professor, Dept. of Civil and Environmental Engineering, Arizona State Univ., Temple, AZ 85787-5306. E-mail: Simon.Washington@asu.edu

³Research Associate, Dept. of Highway Research, The Korea Transport Institute, 2311 Daehwa-dong, Ilsan-gu, Goyang-si, Gyeonggi-do 411-701, Republic of Korea. E-mail: jutaek@koti.re.kr

crash totals may blur otherwise distinct and perhaps important differences in the crash process underlying these two crash types.

The objectives of this paper are to (1) demonstrate that different crash types are associated to predictor variables in different ways (as theorized), and (2) show that estimation of crash type models may lead to greater insights regarding crash occurrence and countermeasure effectiveness. The research represents a contribution to the profession through the insight that crash types are associated with different precrash conditions, and that certain advantages are gained by modeling crash types instead of crash totals (totals, fatals, injuries, etc.). Further research should explore the use of crash type models for identifying high risk locations and comparing these results to high risk sites identified using total crash models.

Careful attention is paid throughout to avoid the use of the phrases "crash causation" or "crash causes," and instead the word "association" is used throughout. Because the data used in the analysis are observed cross-sectional data, it is presumed that the strongest claim is one of association. Although causal relationships are possible and likely, this study cannot support such claims. It is believed, however, that all of the variables employed have potential to influence crash occurrence and belong squarely in predictive models.

Poisson (P) and negative binomial (NB) models are used to estimate the effects of variables on crash types, and "best fitting" models are presented and described. The paper begins with a review of previous research focused on modeling rural intersections, followed by a description of the data used in the analysis. A description of the modeling approach used to establish a relationship between explanatory variables and different types of crashes is followed by a discussion of the model estimation results. Finally, a summary of model findings and conclusions are presented.

Literature Review

Considerable past research has concentrated on assessing the safety effects of crash countermeasures. Much of this research (Vogt and Bared 1998; Vogt 1999; Harwood et al. 2000; Oh et al. 2003; Chin and Quddus 2003) has focused on modeling the relationship(s) between total, fatal, and injuries crashes with intersection geometric characteristics, environmental factors, and traffic related explanatory variables. Although this research is extremely useful for understanding and forecasting total crashes and for understanding the general impacts of countermeasures and/or road features, it does not reveal a disaggregate "picture" of crash events at intersections. In contrast, research conducted to investigate the safety effects of roadway geometric, traffic, and environmental factors on the different crash types is scant. An early study by Hauer et al. (1998) developed crash type prediction models for 15 different accident patterns at urban and suburban signalized intersections in Toronto, Canada. These models required turning movements at intersections as inputs-which in fact were shown to significantly improve predictive ability of the models. Another study conducted on Canadian roads (Persaud and Nguyen, 1998) developed two levels of models based on data inputs. The Level 2 models were similar to the model developed by Hauer et al. (1998), while the Level 1 models developed 'aggregate' prediction models for crash types rear-end, right-angle, and turning movement crashes. Another study by Shankar et al. (1995) resulted in the estimation of separate crash models to evaluate the effects of roadway geometric variables and environmental factors on different crash types, and mainly focused on identifying the safety effects of environmental variables on crash types rather than identifying the safety effects of road geometric variables. Shankar et al. (1995) concluded that separate regression models for specific types of crashes would have the potential for providing greater explanatory power relative to a single overall frequency model. Specifically, separate models allow coefficient estimates to vary by the type of crash, which agrees with intuition as described previously. Stutts et al. (1996) identified the effects of pedestrian characteristics, location, environment, and roadway on pedestrian—vehicle crashes.

For rural intersections, the crash models (Vogt and Bared 1998; Vogt 1999; Harwood et al. 2000; Lyon et al. 2003) developed by the Federal Highway Administration provide somewhat limited information on traffic flow and design countermeasure safety effects. The crash models have mainly been developed for incorporation in the Interactive Highway Safety Design Model, which is roadway design and redesign software that estimates safety effects of alternative designs, in addition to operational and other traditional aspects. Vogt and Bared (1998) formulated several crash models for both road segments and intersections of two-lane rural roads in the states of Minnesota and Washington. With respect to intersections, two intersection crash models were estimated for three- and four-legged stop controlled intersections with two lanes. In another study by Vogt (1999), which may be regarded as a continuation of the previous study, three negative binomial crash prediction models were estimated for three- and four-legged intersections with two lanes on minor and four lanes on major rural highways as well as models for signalized intersections of two-lane rural roads.

Based on the results of these studies, Oh et al. (2003) validated the logical defensibility of proposed models and assessed the transferability of models over future time periods and across different geographic locations. In their companion paper, Lyon et al. (2003) recalibrated the crash prediction models for five types of rural intersections, proposed by Vogt and Bared (1998) and Vogt (1999), based on the validation results. Tables 1 and 2 present explanatory variables found to be significant for each type of rural intersection, their safety effects on total crash frequencies, and model types used in these studies (Vogt and Bared 1998; Vogt 1999; Lyon et al. 2003), whereas Table 3 defines the variables used in these studies.

In addition to these studies, Harwood et al. (2000) presented a crash prediction algorithm for two-lane rural highway sections that includes road segments and three types of at-grade intersections. The crash prediction algorithm consists of a new approach that combines historical crash data, regression analysis, before-and-after studies, and expert judgment to develop accident modifications factors, which are then used to adjust base model (based on AADT only) crash predictions.

In contrast to the bulk of prior studies that have focused on total, fatal, and injury crash models, this study reports on the estimation of crash type models. It is a complement to past studies that have focused on crash type. First, the data are from the U.S. and not from Canada, as in two of the most comprehensive. Second, six crash types are used (e.g. instead of three used in Persaud and Nguyen, 1998). Third, possible simultaneity between crash types is tested and shown not to be significant although theoretically feasible. Finally, the issue of endogeneity is raised as a potential significant issue that should be addressed in future research.

Table 1. Summary of Previous Research (Three- and Four-Legged Stop Controlled Intersections)

	Three-legged stop con	Three-legged stop controlled intersections		trolled intersections
Variables Vogt and	Vogt and Bared (1998)	Lyon et al. (2003) ^a	Vogt and Bared (1998)	Lyon et al. (2003) ^a
Constant	-12.992	-8.825	-10.426	-9.248
Log of AADT1	0.8052	0.7001	0.6026	0.7079
Log of AADT2	0.5037	0.3785	0.6091	0.5153
HI	0.0339	0.0314	0.0449	
VCI	0.2901	0.1204	0.2885	0.0766
POSTSPD	0.0285		0.0187	
RHR	0.1726	0.1433		
RT MAJ	0.2671	-0.1887		
RT MIN		0.419		
LT MAJ		-0.155		
HAU	0.0045		-0.0049	
NODRWY			0.1235	0.1375
Estimated model	NB	NB	NB	NB

Note: Log of AADT1=logarithm of AADT on major road; Log of AADT2=logarithm of AADT on minor roads; HI=sum of degree of curve in degrees per hundred feet of each horizontal curve on major road any portion of which is within 76 m (250 ft.) of the intersection center divided by the number of such curves; VCI=sum of absolute change of grade in percent per hundred fee for each crest curve on major road any portion of which is within 76 m (250 ft.) of the intersection center divided by the number of such curves; POSTSPD=the average posted speed in miler per hour on major road in vicinity of the intersection; RHR=average roadside hazard rating within 76 m (250 ft.) of the intersection center along major road; RT MAJ=1 if a right-turn lane exists on major road, 0 otherwise; RT MIN=1 if a right-turn lane exists on mionor road, 0 otherwise; HAU=intersection angle variable in degree; NODRWY=the number of residential and commercial driveways on the major road within 76 m (250 ft.) of the intersection center.

^aModels which do not include a state indicator term are compared.

Data Description

Data from 38 counties in the state of Georgia consisting of crash files, road characteristic files, aerial photographs, and geographic information system (GIS) roadmaps were used to develop statistical models. Crash and road characteristic files were available for 1996 and 1997. Road characteristic files provided detailed information on roadway characteristics. Digital orthophotography quarter quadrangles aerial photos were used from 1994 and 2000 to extract information regarding intersection angle and degree of horizontal curvature of selected intersections by overlapping with GIS roadmaps. Descriptions of the variables used for the analysis are provided in Table 3.

For total crashes, any crash occurring at the intersection or within 76 m (250 ft) from the intersection center along the major and minor roads is included. To estimate crash type models, crashes were divided into seven different crash types: Total number of crashes, angle crashes, head-on crashes, rear-end crashes, same direction sideswipe crashes, opposite direction sideswipe crashes, and pedestrian-involved crashes. For this study, the total number of crashes does not include single vehicle crashes such as run-off road and fixed object crashes because single vehicle crashes were unavailable in the dataset.

A total of 165 rural intersections were included in the data: 114 nonsignalized intersections and 51 signalized intersections of two-lane four-legged roads, with a total of 837 crashes: 345 occurring at nonsignalized intersections and 492 occurring at signalized intersections. More than 40% of the crashes were angle crashes, and only 2.6 and 2.5% of the total crashes were head-on crashes and opposite direction sideswipe crashes, respectively. The basic statistics of the data are summarized in Table 4.

Analytical Approach

Two different approaches are generally used to estimate crash prediction models: Poisson regression, and negative binomial re-

gression. (Jovanis and Chang 1986; Washington et al. 2003), although various other modeling approaches are possible (see for example Lord et al. 2004). Crash counts are approximated well by a Poisson process (Joshua and Garber, 1990), since crash counts are discrete, positive integers. The Poisson regression model requires that the variance of the crash frequency is approximately equal to its mean. In much of the crash data, however, the variance of the crash frequency is greater than the mean and overdispersion occurs (Miaou et al. 1992). Miaou et al. introduced the negative binomial distribution for modeling traffic safety which accommodates greater variance in the data than allowed by the Poisson distribution. The overdispersion typically arises from variation in crash means across sites. As a result, the negative binomial regression model is the preferred modeling approach when overdispersion is present (Washington et al. 2003). In addition, a comprehensive and detailed review of alternative modeling approaches is provided in Lord et al. (2004).

The negative binomial regression model specifies a relationship between the expected number of crashes occurring at the *i*th element and the q parameters, X_{i1} , X_{i2} , ..., X_{iq} , as follows:

$$E(y_i) = \mu_i = \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_q X_{iq} + \varepsilon_i)$$

In addition, the negative binomial regression model includes a quadratic term in the variance to reflect overdispersion in the model variance. As a result, the negative binomial regression model takes the following form:

$$P(y_i) = \frac{(y_i + \alpha - 1)!}{y_i! (\alpha - 1)!} \frac{\mu^{y_i}}{(1 + \mu)^{y_i + \alpha}}$$

where α =overdispersion parameter and the variance is

$$Var(y_i) = \mu_i + \alpha(\mu_i)^2$$

The overdispersion or extra-Poisson variation is generally due to variables omitted from the model that explain variations in crashes between sites. If α is equal to 0, the negative binomial

Table 2. Summary of Previous Research (Stop Controlled and Signalized Intersections)

	N	Model 1		Model 2	Model 3	
Variables	Vogt (1999)	Lyon et al. (2003)	Vogt (1999)	Lyon et al. (2003)	Vogt (1999)	Lyon et al. (2003)
Constant	-15.46	-10.19	-11.11	-7.471	-6.084	-5.153
Log of AADT1	1.433	0.888	0.930	0.735	0.595	0.450
Log of AADT2	0.269	0.323	0.354	0.239	0.294	0.270
MEDWIDTH1	-0.061	-0.011				
NODRWY1	0.056					
COMDRWY1		0.068				0.054
VEI1		0.108				
HAU		0.010				
MEDTYPE1		-0.321				
PK%LEFT1			0.149	0.023		
PK%TRUCK				-0.048	0.029	
PK%THRU				0.025		
SDR2				-0.0003		
PROT_LT					-0.471	
PK%LEFT2					-0.017	
VEICOM					0.113	
SPD1						0.018
LIGHT						-0.294
HEICOM						-0.029
Estimated model	NB	NB	NB	NB	NB	NB

Note: Model 1=three-legged stop controlled intersections with two lanes on minor and four lanes on major road; Model 2=four-legged stop controlled intersections with two lanes on minor and four lanes on major road; and Models 3=signalized intersections of two-lane roads. Log of AADT1=logarithm of AADT on major road; Log of AADT2=logarithm of AADT on minor roads; MEDWIDTH1=median width on major road; NORDWY1=the number of driveways on the major road within 76 m (250 ft.) of the intersections center; COMDRWY1=the number of commercial driveways on the major road within 76 m (250 ft.) of the intersections center; VEI1=sum of absolute change of grade in percent per hundred fee for each curve on major road any portion of which is within 244 m (800 ft.) of the intersections center, divided by the number of such curves; HAU=intersection angle virable in degree; MEDTYPE1=median type on major road(0=no median, 1=painted, 2=curbed, 3-others); PK%LEFT1=peak left-turn percentage on major road (%); PK%TRUCK=peak truck percentage passing through the intersection (%); PK%TJRU=peak through percentage (%); SDR2=right-side sight distance on minor road (ft); PROT_LT=protected left land (0=no, 1=yes); PK%LEFT2=peak left-turn percentage on minor road (%); VEICOM=(1/2) (VEI1+VEI2); SPD1=the average posted speed on major road in vicinity of the intersection (mph); LIGHT=light at intersection (0=no, 1=yes); HEICOM=(1/2)(HEI1+HEI2); HEI1=sum of absolute change of grade in percent per hundred fee for each horizontal curve on major road any portion of which is within 244 m (800 ft.) of the intersections center, divided by the number of such curves.

reduces to a Poisson model. The value of α corresponds with the degree of overdispersion over and above that associated with the mean μ_i .

Similar to previous studies, the final model structure used is

$$E(\mu_i) = aAADT_{\text{major}}^{\alpha 1}AADT_{\text{minor}}^{\alpha 2} \exp \sum \beta_j x_{ij}$$

where $E(\mu_i)$ =expected number of crash type i; AADT_{major}=average annual daily traffic (AADT) for the major road; AADT_{minor}=average annual daily traffic (AADT) for the minor road; x_{ij} =variables describing road geometry and traffic information; and a, α_1 , α_2 , and β_i =estimated parameters.

Testing for simultaneity: Previous work has shown that estimation of crashes originating from the same entity (e.g., intersection, road segment, city, traffic analysis zone) may require simultaneous estimation techniques (Ladron de Guevara and Washington 2005) due to contemporaneous correlation of disturbances—representing a seemingly unrelated regression system (Zellner 1962). Simultaneity in this case may arise from common omitted variables across models of crash types as a result of common important omitted variables. In such a case, regression models applied equation-by-equation yield consistent but inefficient coefficient estimates [see Washington et al. (2003)]. By accounting for this correlation across errors using simultaneous equation models, more efficient parameter estimates can be obtained. However, parameter estimates in either singular or simul-

taneous estimation methods are unbiased (Washington et al. 2003).

If simultaneity is present across models (i.e., in various models of crash type) then the error terms (fitted minus observed values) will be highly correlated across models (Greene 2000; Washington et al. 2003). In this study simultaneity would suggest that an omitted explanatory variable (or variables) has been omitted from all models. Thus, an informal statistical test for simultaneity is to examine the correlation across estimated models and assess the degree of correlation. There are a number of conditions when simultaneity is not significant and singular estimation techniques can be used [adapted from Greene (2000)]:

- 1. When correlation across disturbances is sufficiently small;
- 2. If the equations are actually unrelated;
- 3. If the equations have identical explanatory variables; or
- If explanatory variables in one model are a subset of variables in another.

Simultaneity was examined in this study and determined not be significant. Error terms were not significantly correlated and many models share explanatory variables (e.g., AADT). Thus singular estimation techniques were applied.

Modeling Results

First a model of the total expected number of crashes was estimated. In addition, six individual models were also estimated for

Table 3. Variables Used in Study

Variables	Definition	Mean	Minimum	Maximum	S.D.
Dependent variables				<u> </u>	
TOTACC	Number of total crashes	5.07	0	53	6.64
ANGLE	Number of angle crashes	2.20	0	33	3.58
HDON	Number of head-on crashes	0.13	0	2	0.38
REND	Number of rear-end crashes	1.44	0	15	2.48
SDSAME	Number of sideswipe crashes for same direction	0.25	0	6	0.72
SDOPPO	Number of sideswipe crashes for opposite direction	0.13	0	3	0.40
PED	Number of pedestrian-involved crashes	0.92	0	4	0.98
Independent variables					
Log of AADTMAJ	Logarithm of AADT on major road	8.04	6.04	9.63	0.94
og of AADTMIN	Logarithm of AADT on minor road	6.57	4.38	9.25	0.99
MEDWDMAJ	Median width on major road in feet	0.75	0	14	2.95
MEDWDMIN	Median width on minor road in feet	0.21	0	12	1.56
SHLWDMAJ	Shoulder width on major road in feet	1.43	0	10	1.42
SHLWDMIN	Shoulder width on minor road in feet	0.64	0	10	1.12
SIGNAL	Intersection type (0 if nonsignalized intersection, 1 if signalized intersection)	0.31	0	1	0.46
RTLMAJ	Right-turn lane indicator (1 if at least one right-turn lane on the major road, 0 otherwise)	0.15	0	1	0.35
TLMAJ	Left-turn lane indicator (1 if at least one left-turn lane on the major road, 0 otherwise)	0.25	0	1	0.43
RTLMIN	Right-turn lane indicator (1 if at least one right-turn lane on the minor road, 0 otherwise)	0.12	0	1	0.33
LTLMIN	Left-turn lane indicator (1 if at least one left-turn lane on the minor road, 0 otherwise)	0.14	0	1	0.35
HZRATMAJ	Roadside hazard rating on major road (from 1, least hazardous case, to 7, most hazardous case)	3.47	1	6	1.16
HZRATMIN	Roadside hazard rating on minor road	3.64	1	7	1.12
DRWYMAJ	Number of driveways on major road within 250 ft of the intersection center	1.82	0	9	2.03
DRWYMIN	Number of driveways on minor road within 250 ft of the intersection center	1.90	0	11	1.77
LIGHTMAJ	Lighting indicator (1 if lighting exists on the major road, 0 otherwise)	0.20	0	1	0.40
LIGHTMIN	Lighting indicator (1 if lighting exists on the minor road, 0 otherwise)	0.19	0	1	0.39
ΓERNMAJ	Terrain on major road (0=flat, 1=rolling, 2=mountainous)	0.57	0	2	0.55
ΓERNMIN	Terrain on minor road (0=flat, 1=rolling, 2=mountainous)	0.69	0	2	0.55
SPDLIMAJ	Speed limit on major road in mph	46.35	25	55	8.10
SPDLIMIN	Speed limit on minor road in mph	38.70	20	55	6.77
SDMAJ	Sight distance on major road in feet	1068	235	2000	425.9
SDMIN	Sight distance on minor road in feet	825	224	2000	355.1
VIMAJ	Sum of absolute change of grade in percent per hundred feet for each curve on major road or minor road within	1.74	0	5.63	1.13
VIMIN	250 ft of the intersection center, divided by the number of such curves	2.51	0	8.93	1.69
HAU	Intersection angle variable in degree	0.44	-57.5	50.0	20.61

Note: S.D.=standard deviation.

Table 4. Summary of Crash Data

	Nonsignalized		Signalized		Total	
	Number	%	Number	%	Number	%
Number of intersections	114	69.1	51	30.9	165	100
Total crashes	345	41.2	492	58.8	837	100
Angle	152	18.2	211	25.2	363	43.4
Head-on	9	1.0	13	1.6	22	2.6
Rear-end	62	7.4	175	20.9	237	28.3
Sideswipe—same direction	14	1.7	28	3.3	42	5.0
Sideswipe—opposite direction	12	1.4	9	1.1	21	2.5
Pedestrian-related	96	11.5	56	6.7	152	18.2

crash types defined previously. Of course an array of models were estimated and compared, various collections of independent variables were fitted, and "best" models were selected using standard goodness of fit criteria and logical defensibility. All resultant models were of the negative binomial type except head-on and pedestrian-involved crash models, which were fitted using Poisson regression (overdispersion parameter for these crash type models was not significantly different from zero). It proved difficult to obtain statistically significant explanatory variables for the opposite direction sideswipe model. The primary reason is likely that crash frequencies of this type were too small to lead to meaningful results. Although these crash types were omitted from the analysis, an alternative approach might be to include these crash types with head-on crashes (they arise under similar precrash conditions) since these two crash types may respond similarly to specific countermeasures and intersection features. As a result nothing can be concluded about opposite direction sideswipe crashes in this analysis.

Total Crash Model

Estimation results for the total crash model are presented in Table 5. Six explanatory variables are included in this model: AADT on the major road, AADT on the minor road, median width on the major road, right-turn lane on the major road, the number of driveways on the major road, and lighting on the major road. All of these variables are statistically significant at the 95% significance level. Of the six variables, four variables have a positive relationship with total number of crashes, whereas two variables reveal a negative relationship.

Similar to prior studies, AADT is positively associated with total crashes. Increased traffic volumes (AADT) or exposure to risk on both the major and minor roads are associated with increased crashes at intersections. Median width for the major road is negatively associated with crashes also in agreement with prior research (Abdel-Aty and Radwan 2000; Chin and Quddus 2003; Oh et al. 2003). The explanation for this finding is that physical separation of travel directions on average improves safety. Surprising, the presence of exclusive left-turn lanes, which typically is associated with a reduction in crashes, was not found to be statistically significant. This is likely because angle crashes, which are affected by left-turn lanes, represent a subset of total crashes, and so the effect of left-turn lanes is muted by the remaining crash types. In contrast, the presence of a right-turn lane on a major road is associated with a higher number of crashes at intersections. There is a plausible explanation for this finding. Typically, right-turn lanes are installed when right turning movements are high and also when the opportunity for angle crashes is also high—so it may merely indicate the presence of a traffic movement this is not significant at similar sites without right-turn lanes. The number of commercial driveways close to the intersection was also associated with higher total crash frequencies at intersections. This is not surprising; in general, traffic movements are more complex at access points, thus traffic crashes are more likely to occur at access points, especially when access points are close to intersections (prohibition of access points near intersections has become a popular access management strategy). Finally, a positive relationship was revealed between lighting and safety, which suggests that the presence of lighting effectively reduces traffic crashes—providing greater visibility to drivers at night so they can perceive and react quicker to potential hazardous situations.

Angle Crash Model

For the negative binomial angle crash model five independent variables were found to be statistically significant, as shown in Table 6. The variables include AADT on the major road, AADT on the minor road, left-turn lane on the major road, the number of driveways on the major road, and lighting on the major road. In contrast with the total crash model, the presence of left-turn lanes on major roads was found to be significant (positively associated with angle crashes). In the angle crash model the presence of the

Table 5. Estimation Results for Total Crashes (Negative Binomial Regression Model)

	Estimated		
Variables	coefficient	t statistic	p value
Constant	-4.4552	-6.490	0.0000
Log of AADT on the major road	0.4356	5.974	0.0000
Log of AADT on the minor road	0.3196	3.692	0.0002
Major road median width in feet	-0.0757	-3.006	0.0026
Right-turn lane indicator	0.7408	3.330	0.0009
(1 if at least one right-turn lane on the major road, 0 otherwise)			
Number of driveways	0.1168	3.015	0.0026
on the major road within 250 ft of the intersection center			
Lighting indicator	-0.4785	-2.326	0.0200
(1 if light on the major road, 0 otherwise)			
α (dispersion parameter)	0.4139	5.064	0.0000
Number of observations	165		
Log-likelihood at zero	-464.54		
Log-likelihood at convergence	-394.52		
ρ^2	0.15		

Table 6. Estimation Results for Angle Crashes (Negative Binomial Regression Model)

	Estimated		
Variables	coefficient	t statistic	p value
Constant	-4.1811	-3.929	0.0001
Log of AADT on the major road	0.3799	3.170	0.0015
Log of AADT on the minor road	0.2164	2.017	0.0437
Left-turn lane indicator	0.8895	2.940	0.0033
(1 if at least one left-turn lane on the major road, 0 otherwise)			
Number of driveways on the major road within 250 ft of the intersection center	0.1126	2.392	0.0167
Lighting indicator (1 if light on the major road, 0 otherwise)	-0.7209	-2.803	0.0051
α (dispersion parameter)	0.5553	3.352	0.0008
Number of observations	165		
Log-likelihood at zero	-318.67		
Log-likelihood at convergence	-290.29		
$ \rho^2 $	0.09		

lane indicates installation of the left-turn lane for handling large turning movements or in reaction to higher than average crash counts at a location. In other words, the presence of turning lanes variable is potentially an endogenoue indicator variable. The endogeneity arises from the fact that crash warrants (left-turn or total crashes exceeding a threshold) may have been used to justify installations of the left turn lanes. Thus, the dependent variable influences the value of an independent variable, a problem the leads to biased parameter estimated in a regression (Greene, 2000). It is also possible that some left-turn lanes were not installed due to crash warrants but exposure warrants-in these cases a right hand side variable (left-turn lane indicator) would be influenced by the exposure variables (AADT). This condition is also generally undesirable in a regression. A promising solution to this problem is to adopt an instrumental variables approach, whereby a system of equations is such that

$$E(\mu_i) = aAADT_{\text{mai}}^{\alpha 1}AADT_{\text{min}}^{\alpha 2} \exp \sum \beta_i x_{ij}$$
 (1)

$$E(LTMAJ) = f(LT_{\text{maj}}, LT_{\text{min}}, AADT_{\text{maj}}, AADT_{\text{min}}, AngleCrashes)$$
(2)

and where $\exp \sum \beta_i x_{ij}$ includes E(LFMAJ)—the predicted value (instrument) for the presence of left turn lanes, and all other predictors as shown in Table 6. In Eq.(2), E(LFMAJ) is the function (most likely a logistic regression) of left turn (LT) and AADT on the major and minor roads and angle crashes. Unfortunately, without left-turning traffic volumes at the sampled sites (unlike Hauer et al. 1998), which are often a primary determinant in left turn lane installation warrants, such a model is not possible with the available data. This instrumental variables approach, however, would lead to unbiased parameter estimates associated with the model parameter for left turn lane indicator, and is an ideal topic for future research (when turning movement data are available). The presence of right-turn lanes and major road median width were not statistically significant. Because median width is mildly correlated with the presence of a left-turn lane (0.45), the positive role that median width typically plays-providing turning vehicles a refuge while turning—is thought to be subsumed by the presence of left-turn lane effect.

Table 7. Estimation Results for Head-On Crashes (Poisson Regression Model)

Variables	Estimated coefficient	t statistic	p value
Constant	-11.6926	-4.329	0.0000
Log of AADT on the major road	0.6585	1.899	0.0576
Log of AADT on the minor road	0.5864	2.786	0.0053
Number of observations	165		
Log-likelihood at zero	-67.71		
Log-likelihood at convergence	-56.46		
$ \underline{\rho^2} $	0.17		

Head-On Crash Model

For the Poisson head-on crash model shown in Table 7, only traffic volume variables for the major and minor roads are statistically significant. Generally, median width is expected to be negatively associated with head-on crashes, that is, a narrower median width prevents fewer head-on crashes by providing less recovery distance (and time), by providing less of a vehicle refuge, etc. Contrary to expectations, this variable was not statistically significant in this model. The reason might be due to the small number of observations—of the 837 total crashes, only 22 crashes (2.6%) were head-on crashes.

Rear-End Crash Model

The rear-end negative binomial crash model presented in Table 8 shares the same set of explanatory variables with the total crash model, albeit with different coefficient estimates (this is an argument, as discussed previously, for not using simultaneous model estimation). The directions of the associations with safety of the model variables are identical to the safety effects on total crashes: AADT, the presence of a right-turn lane, and density of driveways are positively associated with rear-end crashes. The sign associated with the presence of a right turn lane is again likely to be a cross-sectional data artifact and endogeneity problem: sites with right lanes had experienced high numbers of rear end crashes to justify their installation. As before, an instrumental variables approach making use fo right-turning traffic volumes could address the bias introduced by the endogeneity. In contract, median with and the presence of lighting are associated with fewer rear-end crashes. The relationship between median width and rear-end crashes is a bit puzzling and suggests that median width is serving as a proxy for an unobserved variable—most likely degree of urbanization, with greater widths positively associated with more urbanized and complex driving environments. Similar to the angle crash model, the effect of left-turn lane and median width are confounded—and only one variable enters the model, unlike previous results found by Mitra et al. (2002).

Sideswipe (Same Direction) Crash Model

A surprising finding for the sideswipe same direction crash model is that a statistically significant relationship between traffic volumes and sideswipe crashes could not be found. This is a bit troubling and is expected to represent an anomalous finding—it is anticipated that replicated studies would find a significant relationship. Only three variables were associated with sideswipe crashes as presented in Table 9. According to the modeling results, median width on major roads is negatively associated with sideswipe crashes, whereas the presence of a left-turn lane and the

Table 8. Estimation Results for Rear-End Crashes (Negative Binomial Regression Model)

	Estimated		
Variables	coefficient	t statistic	p value
Constant	-11.3326	-8.650	0.0000
Log of AADT on the major road	0.9571	6.747	0.0000
Log of AADT on the minor road	0.4610	4.083	0.0000
Major road median width in feet	-0.0838	-2.091	0.0366
Right-turn lane indicator (1 if at least one right-turn lane	0.7425	2.646	0.0081
on the major road, 0 otherwise)			
Number of driveways	0.1389	2.441	0.0147
on the major road within 250 ft of the intersection center			
Lighting indicator	-0.5073	-1.731	0.0834
(1 if light on the major road, 0 otherwise)			
α (dispersion parameter)	0.4119	2.177	0.0295
Number of observations	165		
Log-likelihood at zero	-214.29		
Log-likelihood at convergence	-206.12		
$ \underline{\rho^2} $	0.04		

density of nearby driveways are associated with higher crash frequencies; vehicles slowing to turn are struck by through moving vehicles. A similar result was observed in a previous study (Chin and Quddus 2003)—it appears that when left-turn lanes are present, motor vehicles in the approach spend proportionately more time side-by-side (e.g., a queue formed in the left-turn lane is passed by all adjacent lane through traffic) and proportionately more time weaving into and out of the left-turn lane—when combined lead to greater opportunities for same direction sideswipe crashes.

The number of driveways is also associated with an increase in sideswipe crashes. This may be explained by conflicts between through vehicles and merging vehicles from driveways—which in the presence of an already complicated driving task (intersection) presents increased risk. This finding supports access management policies near intersections.

Table 9. Estimation Results for Sideswipe (Same Direction) Crashes (Negative Binomial)

Variables	Estimated coefficient	t statistic	p value
Constant	-2.6925	-6.605	0.0000
Major road median width in feet	-0.2368	-2.335	0.0195
Left-turn lane indicator (1 if at least one left-turn lane on the major road, 0 otherwise)	1.8042	4.564	0.0000
Number of driveways on the major road within 250 ft of the intersection center	0.2698	2.975	0.0029
α (dispersion parameter)	0.7364	1.272	0.2034
Number of observations	165		
Log-likelihood at zero	-87.39		
Log-likelihood at convergence	-85.07		
ρ^2	0.03		

Table 10. Estimation Results for Pedestrian-Involved Crashes (Poisson Regression Model)

Variables	Estimated coefficient	t statistic	p value
Constant	-2.1935	-4.034	0.0001
Log of AADT on the minor road	0.3064	3.795	0.0001
Major road shoulder width in feet	0.1028	1.976	0.0482
Lighting indicator (1 if light on the major road, 0 otherwise)	-0.5794	-2.481	0.0131
Number of observations	165		
Log-likelihood at zero	-209.66		
Log-likelihood at convergence	-198.92		
$\underline{\rho^2}$	0.05		

Pedestrian-Involved Crash Model

A Poisson model was developed to estimate pedestrian-involved crashes, and is shown in Table 10. It should be noted at the outset that pedestrian exposure is entirely missing (for the obvious reason that pedestrian crossing counts are not routinely recorded), which complicates model estimation and subsequent interpretation. Thus, the variables that are available require a modeler to provide educated guesses as to what surrogate "pedestrianinvolved" variables are being captured by standard traffic related variables, a difficult task at best. Three variables were found to be statistically associated with pedestrian-involved crashes: AADT on minor roads, shoulder width on major roads, and lighting on major roads. Unlike the other models, AADT on major roads was not significant, but AADT on minor roads is positively associated with pedestrian-involved crashes. The reason for this finding is not entirely clear; however, it is suggested that AADT on the minor road is suggestive of a relatively busy intersection, with greater numbers of pedestrians.

Shoulder width is positively associated with pedestrian-involved crashes. This finding is somewhat intuitive and suggests that pedestrians may feel a greater sense of security (compared with narrow shoulders where they may refuse to walk) walking on a road with wide shoulders—increasing their exposure to potential conflict with motor vehicles. In addition, crossing a road-way with wider shoulders increases the exposure time to motor vehicles. The relationship of safety with AADT on minor roads may be indicative of higher road standards associated with a greater local built environment and increased pedestrian activity (again in the absence of pedestrian exposure). The presence of lighting is found to be negatively associated with pedestrian safety. Contrary to the other two variables, lighting appears not to be a surrogate for pedestrian activity, and suggests that lighting is beneficial for reducing pedestrian and motor vehicle conflicts.

Comparison of Coefficients and Variables across Crash Type Models

Table 11 provides a summary of model estimation results across all models estimated in this study. As stated previously, one of the justifications for modeling crash types is to identify which variables contribute to certain types of crashes and to compare how different significant variables affect safety for different crash types. The results show that a handful of the available roadway geometric and traffic volume variables affect the safety of two-lane rural intersections: AADT on major and minor roads, median width of major roads, shoulder width of major roads, the presence

Table 11. Comparison of Coefficients and Variables across Crash Type Models

			Estimated c	coefficients		
Variables	TC^2	AC^2	HD^2	RE^2	SS ²	PR ²
Constant	-4.4552	-4.1811	-11.6926	-11.3326	-2.6925	-2.1935
Log of AADT on the major road	0.4356	0.3799	0.6585	0.9571	n/s	n/s
Log of AADT on the minor road	0.3196	0.2164	0.5864	0.4610	n/s	0.3064
Major road median width in feet	-0.0757	n/s	n/s	-0.0838	-0.2368	n/s
Major road shoulder width in feet	n/s^1	n/s	n/s	n/s	n/s	0.1028
Right-turn lane indicator (1 if at least one right-turn lane on the major road, 0 otherwise)	0.7408	n/s	n/s	0.7425	n/s	n/s
Left-turn lane indicator (1 if at least one left-turn lane on the major road, 0 otherwise)	n/s	0.8895	n/s	n/s	1.8042	n/s
Number of driveways on the major road within 250 ft of the intersection center	0.1168	0.1126	n/s	0.1389	0.2698	n/s
Lighting indicator (1 if light on the major road, 0 otherwise)	-0.4785	-0.7209	n/s	-0.5073	n/s	-0.5794
Estimated models	NB	NB	P	NB	NB	P

Note: All of the variables are statistically significant at 5% significance level except a AADT on the major road for the head-on crash model and a lighting indicator variable for the rear-end crash model, which are significant at 10% significance level. n/s indicates that a zero-effect coefficient could not be rejected at 95% level of confidence; TC=total crash model; AC=angle crash model; HD=head-on crash model; RE=rear-end crash model; SS=sideswipe (same direction) crash model; PR=pedestrian-involved crash model; NB=negative binomial model; and P=Poisson model.

of turning lanes, the number of driveways on major roads, and the presence of lighting for major roads are found in various groupings in the models.

Among these variables, AADT on the major and minor roads in general are the most influential variables associated with crash risk. As AADT increases, exposure to risk (at the site) increases. Past research has shown that some safety performance functions with respect to AADT are nonlinear (e.g., exponential)—whereby increasing AADT eventually will lead to improvement in safety; nonlinear relationships between safety and AADT were also found in this study. AADT is generally not viewed as a controllable factor but instead an important predictor of crashes, since controlling AADT is generally not an option to engineers or planners.

With respect to influential roadway geometric variables, median widths of major roads are negatively associated with safety on total crashes, rear-end crashes, and sideswipe crashes.

Shoulder width is significant only for pedestrian-involved crashes. The number of driveways near an intersection (driveway density) is associated with higher crash frequencies for all crash types except head-on and pedestrian-involved crashes.

The turning-lane indicator variables revealed a variety of statistically significant effects. The presence of right-turn lanes on major roads was significant in the rear-end and total crash models, whereas the presence of left-turn lanes was significant in angle and same direction sideswipe crash models. The coefficient estimates for the right-turn indicator were effectively the same; whereas the coefficient estimates for the left-turn indicator variables differed by a factor of two.

Finally, the lighting indicator variable revealed a fairly consistent negative association with safety across models, although a zero effect could not be rejected for the head-on and same direction sideswipe crash type models, probably because the observed number of these two crash types during the observation period is small, representing 22 and 42 out of 837 crashes, respectively.

Examination of the coefficient estimates across crash type models clearly demonstrates, at least statistically, that crash types are associated with a different set of predictors. This result is not surprising, since precrash conditions will largely determine the crash type—and this result is in agreement with engineering expectations. For example, a relatively short yellow clearance interval at a signalized intersection may affect rear-end crashes differently than same direction sideswipe crashes. The insights afforded by estimating crash type models may lead to insights as to the relative effectiveness of various countermeasures and/or predictive variables.

The second justification claimed in this paper for estimating crash type models is that "hazardous" sites may reveal themselves through elevated crash type frequencies and not total crash frequencies. Although this paper is not focused on "hot spot identification" methodology, an inspection of a simple ranking of "highest risk" sites using the models estimated in this study can reveal this phenomenon. Table 12 lists the top 5 "high risk" sites identified using each of the models. To identify the sites listed in the table, the total and crash type models were used to predict crash frequencies for each of the sites. Predicted frequencies were compared to observed frequencies, and the top five "high-risk" sites (observed-predicted) were identified. The table shows that for the total crash model, Sites 129, 134, 124, 157, and 136 were ranked as "most hazardous." That is, the model of total crashes predicts the expected crash frequency for these sites; however these sites recorded significantly more than predicted (and based on similar sites). In comparison, the head-on crash model ranked only two of these sites as highest risk, Sites 129 and 124, and identified the highest risk site as Site 14. Similarly, the rear-end crash model identified Site 149 as most hazardous, because it recorded six more sideswipe crashes than expected compared to similar sites. Site 50, again not identified as top priority by the total crash model, recorded three more pedestrian crashes than expected compared to similar sites. Although it is acknowledged that sample sizes are small and that this analysis is preliminary (we do not assess "how outlying" observations are with respect to expected safety), the results do lend credence to the claim that crash type models may reveal "problems" at intersections (road

Table 12. Top Five Outlying Observations with Respect to Expected Crash Counts (in Order of Decreasing Rank)

				Rank		
Model		First	Second	Third	Fourth	Fifth
Total crash	Site	129	134	124	157	136
	Observed	53	23	27	23	25
	Predicted	19	10	18	16	20
	Residual	34	13	9	7	5
Angle crash	Site	129	134	124	136	159
	Observed	33	11	12	13	10
	Predicted	15	5	7	9	9
	Residual	18	6	5	4	1
Head-on crash	Site	14	129	124	158	
	Observed	2	2	1	1	
	Predicted	0	1	0	0	
	Residual	2	1	1	1	
Rear-end crash	Site	149	129	124	136	159
	Observed	8	15	10	10	8
	Predicted	2	10	6	8	6
	Residual	6	5	4	2	2
Sideswipe crash (same direction)	Site	157	131	149	130	158
	Observed	6	3	2	3	3
	Predicted	1	1	0	2	2
	Residual	5	2	2	1	1
Pedestrian-involved crash	Site	50	107	118	124	83
	Observed	4	3	3	3	3
	Predicted	1	1	1	2	2
	Residual	3	2	2	1	1

Note: Residual=observed carshes-predicted crashes.

segments, interchanges, etc.) that will not be revealed through total crash models. A more thorough hot spot identification analysis would consider site selection bias (regression to the mean) and might apply Bayesian methods to correct for this phenomenon.

Conclusions and Recommendations

The justification for this research rested upon the claim that modeling crash types provides certain advantages and is complementary to total crash modeling (totals, fatals, injuries, etc.). Much research has focused on estimating total crash models, whereas relatively little has focused on modeling crash types. Through analysis of rural intersections in Georgia, total crashes and crash type models were estimated for total, angle, head-on, rear-end, same direction sideswipe, and pedestrian involved crashes. The data were fit with negative binomial or Poisson regression models, resulting in satisfactory predictive models that are shown to be generally consistent with past research findings and with engineering knowledge of crash occurrence (with a few justifiable exceptions).

The results support the claim that crash types are associated with different conditions at rural intersections in Georgia—although the expectation is that analysis results would be similar elsewhere and for different roadway elements. Coefficient estimates varied considerably in some cases, and models were fit with different subsets of explanatory variables. One advantage of modeling crash type, therefore, is to gain insight as to the differences between conditions that lead to various crash types. Armed

with this knowledge it may be possible to better understand the impact of various countermeasures on crash type.

It was also argued, second, that crash type models might also provide insight for ranking sites (intersections in this study), with total crash models failing to identify specific problems related to specific crash types. Although unsophisticated, it was shown through comparison of expected to actual counts that crash type models may in fact identify different sites for further engineering scrutiny. This finding again bolsters the argument for the development of crash type models as complements to total crash models.

A larger and more comprehensive data set spanning several states would offer improved analysis potential. The inclusion of single-vehicle accidents is necessary to provide a complete picture of intersection crashes, but these crash types were unavailable. Turning movements at intersection approaches would likely lead to improvements in explanatory ability of models; however, turning movement counts are rarely known in practice and may limit practicality of the models. In addition, environmental variables and in particular inclement weather related data (i.e., wet pavement days, icy days, snow days, etc.) would be useful to include in predictive models but were unavailable for this study. As mentioned previously, using simultaneous equations and an instrumented variable for turning lane indicator variables, which may be endogenous—may improve properties of estimated parameters. Finally, a more sophisticated analysis of hot spot identification would reveal further insights into differences between total crash and crash type models and should be conducted. It is recommended that crash type models be estimated more routinely

in conjunction and as complements to total crash models. It should be emphasized that crash type models are intended as a complement to models of crash severity, which are used to answer different but equally important safety questions.

References

- Abdel-Aty, M. A., and Radwan, A. E. (2000). "Modeling traffic accident occurrence and involvement." *Accid. Anal Prev.*, 32(5), 633–642.
- Chin, H. C., and Quddus, M. A. (2003). "Applying the random effect negative binomial model to examine traffic accident occurrence at signalized intersections." *Accid. Anal Prev.*, 35(5), 253–259.
- Greene, W. (2000). Econometric analysis, Prentice-Hall, Upper Saddle River, N.J.
- Harwood, D. W., Council, F. M., Hauer, E., Hughes, W. E., and Vogt, A. (2000). "Prediction of the expected safety performance of rural twolane highways." FHWA-RD-99-207, Federal Highway Administration, Washington, D.C.
- Hauer, E., Ng, J. C.H., and Lovell, J. (1988). "Estimation of safety at signalized intersections." *Transportation Research Record 1185*, Transportation Research Board, Washington, D.C., 48–61..
- Joshua, S., and Garber, N. (1990). "Estimating truck accident rate and involvement using linear and Poisson regression models." *Transp. Plan. Technol.* 15, 41–58.
- Jovanis, P., and Chang, H. (1986). "Modeling the relationship of accidents to miles traveled." *Transportation Research Record 1068*, Transportation Research Board, Washington, D.C., 42–51.
- Ladron de Guevara, F., and Washington, S. (2005). "Forecasting crashes at the planning level. A simultaneous negative binomial crash model applied in Tucson, Arizona." J. Transp. Res. Board, in press.
- Lord, D., Washington, S., and Ivan, J. (2004). "Poisson, poisson-gamma, and zero-inflated regression models of motor vehicle crashes: Balancing statistical fit and theory." Accident analysis and prevention, Pergamon/Elsevier Science, New York.
- Lyon, C., Oh, J., Persaud, B., Washington, S., and Bared, J. (2003).

- "Empirical investigation of interactive highway safety design model accident prediction algorithm: rural intersections." *Transportation Research Record 1840*, Transportation Research Board, Washington, D.C., 78–86.
- Miaou, S., Hu, P., Wright, T., Rathi, A., and Davis, S. (1992). "Relation-ship between truck accidents and highway geometric design: a Poisson regression approach." *Transportation Research Record* 1376, Transportation Research Board, Washington, D.C., 10–18.
- Mitra, S., Chin, H. C., and Quddus, M. A. (2002). "Study of intersection accidents by maneuver type." *Transportation Research Record 1784*, Transportation Research Board, Washington, D.C., 43–50.
- Oh, J., Lyon, C., Washington, S., Persaud, B., and Bared, J. (2003). "Validation of the FHWA crash models for rural intersections: Lessons learned." *Transportation Research Record 1840*, Transportation Research Board, Washington, D.C., 41–49.
- Persaud, B. and Nguyen, T. (1998). "Disaggregate safety performance models for signalized intersection on Ontario Provincial roads." *Transportation Research Record* 1635, Transportation Research Board, Washington, D.C., 113–120.
- Shankar, V., Mannering, F., and Barfield, W. (1995). "Effect of roadway geometric and environmental factors on rural freeway accident frequencies." Accid. Anal Prev., 27(3), 371–389.
- Stutts, J., Hunter, W., and Pein, W. (1996). "Pedestrian-vehicle crash types: an update." *Transportation Research Record 1538*, Transportation Research Board, Washington, D.C., 68–74.
- Vogt, A. (1999). "Crash models for rural intersections: four-lane by two-lane stopcontrolled and two-lane by two-lane signalized." FHWA-RD-99-128, Federal Highway Administration, Washington, D.C.
- Vogt, A., and Bared, J. (1998). "Accident prediction models for two-lane rural roads: Segments and intersections." FHWA-RD-98-133, Federal Highway Administration, Washington, D.C.
- Washington, S., Karlaftis, M., and Mannering, F. (2003). Statistical and econometric methods for transportation data analysis, Chapman and Hall, Boca Raton, Fla.
- Zellner, A. (1962). "An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias." J. Am. Stat. Assoc., 57, 348–368.