Distribution :-

Exponential Distribution :

The p.d.f of the exponential distribution is given by

$$f(x) = \begin{cases} \alpha\, e^{-\alpha x}, & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Mean and Standard Deviation of the Exponential Distribution

Mean $(\mu) = \dfrac{1}{\alpha}$ S.D. $(\sigma) = \dfrac{1}{\alpha}$ $\Rightarrow$ $V(x) = \dfrac{1}{\alpha^2}$

Mean $E(x) = \int_0^\infty x f(x)\,dx$

$SD = \sqrt{V(x)}$

Mean of exponential dist^n :-

$\Rightarrow$ Mean $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

where $f(x) = \begin{cases} \alpha e^{-\alpha x} ; & 0 \le x \le \infty \\ 0 & \text{otherwise} \end{cases}$

$\int uv = u\int v - u'\iint v + u''\iiint v - \cdots$

Becomes zero at some point

OR

$\int uv = u\int v - \int\int v \cdot \text{diff } u$

$\therefore \mu = \int_0^{\infty} \underbrace{x}_{u}\, \alpha \underbrace{e^{-\alpha x}}_{v}\, dx$

$= \alpha \left[x \cdot \left(\frac{e^{-\alpha x}}{-\alpha} \right) - 1 \left(\frac{e^{-\alpha x}}{(-\alpha)^2} \right) \right]_0^{\infty}$

$= \alpha \left\{ -\frac{1}{\alpha} \left[\infty e^{-\alpha \infty} - 0 e^{0} \right] - \frac{1}{\alpha^2} \left[e^{-\infty} - e^{0} \right] \right\}$

$= \alpha \left\{ 0 - \frac{1}{\alpha^2} \left[0 - 1 \right] \right\}$

$\Rightarrow \mu = \alpha \left[\frac{1}{\alpha^2} \right]$

$\boxed{\mu = \frac{1}{\alpha}}$

Variance of Exponential dist

$$V(x) = E(x^2) - [E(x)]^2$$

where,

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$\therefore V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\frac{1}{\alpha}\right]^2$$

$$= \int_0^{\infty} x^2 \cdot \alpha e^{-\alpha x}\,dx - \frac{1}{\alpha^2}$$

$$= \alpha\left\{x^2\left(\frac{e^{-\alpha x}}{-\alpha}\right) - 2x\left(\frac{e^{-\alpha x}}{(-\alpha)^2}\right) + 2\left(\frac{e^{-\alpha x}}{(-\alpha)^3}\right)\right]_0^{\infty} - \frac{1}{\alpha^2}$$

$$= \alpha\left\{0 - 0 - \frac{2}{\alpha^3}\cdot\left[e^{-\infty} - e^0\right]\right\} - \frac{1}{\alpha^2}$$

$$V(x) = \alpha\left\{-\frac{2}{\alpha^3}[0-1]\right\} - \frac{1}{\alpha^2}$$

$$= \alpha\left\{\frac{2}{\alpha^3}\right\} - \frac{1}{\alpha^2}$$

$$= \frac{2}{\alpha^2} - \frac{1}{\alpha^2}$$

$$\boxed{V(x) = 1/\alpha^2}$$

$$\Rightarrow SD \text{ of } x = \sqrt{V(x)}$$

$$= \sqrt{\frac{1}{\alpha^2}}$$

$$\boxed{SD = \frac{1}{\alpha}}$$ //

If x is an exponential variate with mean 3 find **(i)** $P(x > 1)$ **(ii)** $P(x < 3)$

$\Rightarrow$ Exponential dist is given by $f(x) = \begin{cases} \alpha e^{-\alpha x} & ; \ 0 \le x \le \infty \\ 0 & \text{otherwise.} \end{cases}$

Given mean $\mu = 3$

WKT mean $\mu = \frac{1}{\alpha}$

$3 = \frac{1}{\alpha} \Rightarrow \boxed{\alpha = \frac{1}{3}}$

i) $P(x > 1) = \int_1^\infty f(x)\,dx$

$= \int_1^\infty \alpha e^{-\alpha x}\,dx$

$= \int_1^\infty \frac{1}{3} e^{-\frac{1}{3}x}\,dx$

$= \cancel{\frac{1}{3}} \left[\frac{e^{-\frac{1}{3}x}}{-\cancel{\frac{1}{3}}} \right]_1^\infty$

$= -\left[e^{-\infty} - e^{-\frac{1}{3}} \right]$

$= e^{-\frac{1}{3}} \Rightarrow \boxed{P(x>1) = 0.176}$

2) $P(X<3) = \int_0^3 f(x)\,dx$

$= \int_0^3 \frac{1}{3} e^{-\frac{1}{3}x}\,dx$

$= \frac{1}{3}\left[\frac{e^{-\frac{1}{3}x}}{-\frac{1}{3}}\right]_0^3$

$= -\left[e^{-\frac{1}{3}\times 3} - e^{0}\right]$

$= -\left[e^{-1} - 1\right]$

$= 0.6321$

$e^3 =$

$\boxed{= e(3)}$

If x is an exponential variate with mean 5, evaluate. → HW

(i) $P(0 < x < 1)$ **(ii)** $P(-\infty < x < 10)$ **(iii)** $P(x \leq 0 \text{ or } x \geq 1)$

1) $\int_0^1 f(x)\,dx$

2) $\int_0^{10} f(x)\,dx$

$\boxed{\alpha = 1/5}$

3) $0 + \int_1^{\infty} f(x)\,dx$

Ans: $P(0<x<1) = 0.1813$

2) $P(-\infty < x < 10) = 0.8647$

3) $P(x \leq 0 \text{ or } x \geq 1) = 0.8187$

The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes (ii) between 5 and 10 minutes.

$\Rightarrow$ Given mean $\mu = 5$

$\frac{1}{\alpha} = 5$

$\Rightarrow \boxed{\alpha = \frac{1}{5}}$

$\therefore \quad f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x \le \infty \\ 0 & \text{otherwise} \end{cases}$

$= \begin{cases} \frac{1}{5} e^{-\frac{1}{5}x} & 0 \le x \le \infty \\ 0 & \text{otherwise} \end{cases}$

1) $P(x < 5) = \int_0^5 f(x)\,dx$

$= \int_0^5 \frac{1}{5} e^{-\frac{1}{5}x}\,dx$

$= 0.6231$

2) $P(5 < x < 10) = \int_5^{10} f(x)\,dx$

$= \int_5^{10} \frac{1}{5} e^{-\frac{1}{5}x}\,dx$

$= 0.2325$

The sales per day in a shop is exponentially distributed with the average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on two consecutive days.

Given $\mu = 100$

$\frac{1}{\alpha} = 100$

$\boxed{\alpha = \frac{1}{100}}$

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & ; 0 \le x \le \infty \\ 0 & \text{otherwise.} \end{cases}$$

$$\boxed{f(x) = 0.01 e^{-0.01x} \quad ; 0 \le x \le \infty}$$

$\cancel{P(X > 30)}$? ? ? X

Let A be the amt for which the profit is 8%

$\Rightarrow A \cdot \frac{8}{100} = 30$

$A = \frac{30}{0.08} \Rightarrow A = 375$

$$P(A > 375) = \int_{375}^{\infty} f(x)\,dx$$

$$= \int_{375}^{\infty} 0.01 e^{-0.01x}\,dx$$

$$P(A>375) = \int_{375}^{\infty} 0.01\, e^{-0.01x}\, dx$$

$$= \cancel{0.01}\left[\frac{e^{-0.01x}}{-\cancel{0.01}}\right]_{375}^{\infty}$$

$$= -\left[\cancel{e^{-\infty}}^{\,0} - e^{-0.01\times 375}\right]$$

$$= -\left(0 - 0.0235\right)$$

$$P(A>375) = 0.0235$$

$\therefore$ The Prob that the net Profit exceeds Rs 30 for 2 consecutive days is

$0.0235 \times 2 = ??$ or $0.0235 \times 0.0235 = ??$

In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for :

(i) *10 minutes or more* ***(ii)*** *less than 10 minutes* ***(iii)*** *between 10 and 12 minutes*

$\Rightarrow \mu = 5$

$\frac{1}{\alpha} = 5$

$\boxed{\alpha = \frac{1}{5}}$

$$p(x) = \begin{cases} \alpha e^{-\alpha x} & 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

1) $P(X \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-\frac{1}{5}x}\, dx$

$= 0.1353$

2) $P(X < 10) = \int_{0}^{10} \frac{1}{5} e^{-\frac{1}{5}x}$

$= 0.8646$

3) $P(10 < X < 12) = \int_{10}^{12} \frac{1}{5} e^{-\frac{1}{5}x}\, dx$

$= 0.0446$