Binomial distribution

A random experiment having only two outcomes, say success (**S**) and failure (**F**) is called a **Bernoulli trial**. If p is the probability of success and $q = 1 - p$ is the probability of failure in a single Bernoulli trial, , then the probability of x successes out of n Bernoulli trials is given by

$$\boxed{p(x) = n_{c_x}\, p^x\, q^{n-x}}$$

We form the following probability of [x, P(x)] where x = 0, 1,2, …n

x	0	1	2	…	n
P(x)	q^n	$n_{c_1}\ q^{n-1}\, p$	$n_{c_2}\ q^{n-2}\, p^2$	…	p^n

It may be observed that the value of P(x) for different values x = 0, 1, 2, …n are the successive terms in the binomial expansion of $(q+p)^n$ and accordingly this distribution is called the **Binomial Distribution** or **Bernoulli Distribution**.

Mean and standard deviation of the Binomial Distribution :

$$\boxed{\text{Mean } (\mu) = np}$$

$$\boxed{\text{variance } (V) = n\,p\,q}$$

$$\boxed{\text{S.D. } (\sigma) = \sqrt{n\,p\,q}}$$

Poisson distribution

Poisson distribution is regarded as the limiting form of the binomial distribution when n is very large ($n \to \infty$) and p the probability of success is very small ($p \to 0$) so that n p tends to a fixed finite constant say m. (ie. $\boldsymbol{m = np}$)

$$\boxed{\boldsymbol{P(x) = \frac{m^x e^{-m}}{x!},}} \qquad \boldsymbol{x = 0, 1, 2, \ldots \infty}$$

This is known as the Poisson distribution of the random variable.

P (x) is also called Poisson Probability function and x is called a Poisson Variate. The distribution of probabilities for x = 0, 1, 2, 3,... is as follows.

X	0	1	2	3	...
P (x)	e^{-m}	$\frac{me^{-m}}{1!}$	$\frac{m^2e^{-m}}{2!}$	$\frac{m^3e^{-m}}{3!}$	

Mean and standard deviation of the Poisson distribution

$\boxed{\text{Mean } (\mu) = m}$ $\boxed{\text{Variance } (V) = m}$ Proof is imp $\boxed{\text{S.D. } (\sigma) = \sqrt{m}}$

Mean of Poisson Distribution :-

$$\mu = E(x) = \sum_{x} x P(x)$$

W.K.T. Poisson dist is $P(x) = \frac{m^x e^{-m}}{x!}$

$$\therefore \mu = E(x) = \sum_{x=0}^{\infty} x \cdot \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{m^1 \cdot m^{x-1} \cdot e^{-m}}{x(x-1)!}$$

$$= m e^{-m} \cdot \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= m e^{-m} \left[\frac{m^0}{0!} + \frac{m^1}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots \right]$$

$$\mu = E(x) = m \cancel{e^{-m}} \cdot \left[\cancel{e^{m}}\right]$$

$$\boxed{\mu = E(x) = m}$$

Variance of Poisson Distribution :-

$$V(X) = E(X^2) - \underbrace{[E(X)]^2}_{m^2}$$

$$= \sum_x x^2 P(x) - m^2$$

$$= \sum_{x=0}^{\infty} [x(x-1) + x]\, P(x) - m^2$$

$$= \sum_{x=0}^{\infty} x(x-1)P(x) + \underbrace{\sum x P(x)}_{m} - m^2$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-m} \cdot m^x}{x!} + m - m^2$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \cdot \frac{e^{-m} \cdot m^2 \, m^{x-2}}{\cancel{x(x-1)}(x-2)!} + m - m^2$$

$$V(x) = e^{-m} \cdot m^2 \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} + m - m^2$$

$$= e^{-m} \cdot m^2 \underbrace{\left[\frac{m^0}{0!} + \frac{m^1}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right]}_{e^m} + m - m^2$$

$$= \cancel{e^{-m}} \cdot m^2 \cdot \cancel{e^{m}} + m - m^2$$

$$= \cancel{m^2} + m \cancel{- m^2}$$

$$\boxed{V(x) = m} \; //$$

$$SD = \sqrt{V} = \sqrt{m} \; //$$

1) The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that (i) exactly 2 are defective (ii) at least 2 are defective (iii) none of them are defective

⇒ Probability of defective pen $\boxed{p = 1/10}$; $q = 1 - p \Rightarrow q = 1 - 1/10 \Rightarrow \boxed{q = 9/10}$

Total Pen manufactured $\boxed{n = 12}$

we have Binomial dist $P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$$P(x) = {}^{12}C_{x} \left(\frac{1}{10}\right)^{x} \left(\frac{9}{10}\right)^{12-x}$$

1) $P(x=2) = {}^{12}C_{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{12-2}$

$= 0.2301$ //

2) Atleast 2 are defective

Atleast - min or more

Atmost $\to$ max or less

$$P(x \geq 2) = \begin{cases} P(X=2) + P(X=3) + \cdots + P(X=12) \\ \text{OR} \\ 1 - P(X<2) \end{cases}$$

$$P(x \geq 2) = 1 - P(x<2)$$

$$= 1 - \left[P(x=0) + P(x=1)\right]$$

$$= 1 - \left\{ \underbrace{{}^{12}C_0 \left(\tfrac{1}{10}\right)^0 \cdot \left(\tfrac{9}{10}\right)^{12-0}} + {}^{12}C_1 \left(\tfrac{1}{10}\right)^1 \left(\tfrac{9}{10}\right)^{12-1} \right]$$

$$= 1 - \left[0.2824 + 0.3765\right]$$

$$= 0.3409 \,//$$

3) $P(x=0)$

$$= {}^{12}C_0 \left(\tfrac{1}{10}\right)^0 \left(\tfrac{9}{10}\right)^{12-0}$$

$$= 0.2824$$

When a coin is tossed 4 times, find the probability of getting
(i) *exactly one head* **(ii)** *atmost 3 heads* **(iii)** *atleast 2 heads*

$\Rightarrow p =$ Prob of getting head $= 1/2 \Rightarrow \boxed{p = 1/2}$ & $\boxed{q = 1/2}$ $n = 4$ //

we have B.D $P(x) = {}^nC_x \, p^x q^{n-x}$

$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} = 4C_x \left(\frac{1}{2}\right)^4$

1) $P(X=1) = 4C_1 \left(\frac{1}{2}\right)^4 = 0.25$

2) $P(X \leq 3) = 1 - P(X>3)$

$= 1 - [P(X=4)]$

$= 1 - \left[4C_4 \left(\frac{1}{2}\right)^4\right]$

$= 0.9375$ //

3) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[4C_0 \left(\frac{1}{2}\right)^4 + 4C_1 \left(\frac{1}{2}\right)^4\right]$

$= 0.6875$ //

In a consignment of electric lamps 5 % are defective. If a random sample of 8 lamps are inspected what is the probability that one or more lamps are defective ?

$\Rightarrow p = 0.05$; $q = 0.95$; $n = 8$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - {}^{8}C_{0} (0.05)^{0} (0.95)^{8-0}$

$= 0.3366$

The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70.

$p = 0.65$; $q = 0.35$ $n = 10$

$P(x) = {}^nC_x \, p^x q^{n-x}$

$= {}^{10}C_x (0.65)^x (0.35)^{10-x}$

$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= 0.5137$

The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70.

The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that **(i)** *no line is busy* **(ii)** *all lines are busy* **(iii)** *atleast one line is busy* **(iv)** *atmost 2 lines are busy.*

HW

$p = 0.1$; $q = 0.9$; $n = 10$

1) $P(X = 0) =$

2) $P(X = 10) =$

3) $P(X \geq 1) =$

4) $P(X \leq 2) =$

In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked ? Also find the probability of the same if there are 4 options for a correct answer.

Case 1 :- $p = \frac{1}{2}$; $q = 1/2$; $n = 10$ $\quad p(x) = {}^{n}C_x \, p^x \, q^{n-x}$ $\Rightarrow p(x) = {}^{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x}$

$$\boxed{= {}^{10}C_x \left(\tfrac{1}{2}\right)^{10}}$$

$$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$= \underline{\underline{0.3770}}$$

Case 2 :- $p = 1/4$; $q = 3/4$; $n = 10$

$$P(x \geq 6) = \underline{\underline{0.0197}}$$

In sampling a large number of parts manufactured by a company, the mean number of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

$\Rightarrow$ mean $\mu = np$

$2 = 20p \Rightarrow p = \frac{2}{20} = \frac{1}{10}$

$\boxed{p = \frac{1}{10}}$ $\boxed{q = \frac{9}{10}}$ $\boxed{n = 20}$

$P(x) = {}^nC_x p^x q^{n-x}$

$= {}^{20}C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{20-x}$

${}^{20}C_x \left(\frac{1}{10}\right)^x$

$\hookrightarrow$ calc

Alpha+)

$\Rightarrow P(x \geq 3) = 1 - P(x < 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - [\_\_ + \_\_ + \_\_]$

$= 1 - \{\_\_$

$= 0.323$

$\therefore$ For 1000 samples the no. of defectives is 1000×0.323

$= 323$

(VV2P)

If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students are 2.5 and $\sqrt{1.875}$. Find an estimate of the number of candidates answering correctly **(i)** *8 or more questions* **(ii)** *2 or less* **(iii)** *5 questions.*

$\Rightarrow$ mean $\mu = 2.5$ & $SD = \sqrt{1.875}$

WKT $\mu = np$ & $SD = \sqrt{npq}$

$\boxed{2.5 = np} \rightarrow *$

$\sqrt{1.875} = \sqrt{npq}$

$1.875 = \boxed{np}q$

$1.875 = 2.5q$

$\Rightarrow q = \dfrac{1.875}{2.5} = 0.75$ $\Rightarrow p = 1-q$

$= 1 - 0.75$

$= 0.25$ sub p in *

$\boxed{p = 0.25}$ // $\boxed{q = 0.75}$ // $\boxed{n = 10}$ //

1) $P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$= 4.88 \times 10^{-4}$

$P(X) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{10}C_{x} (0.25)^{x} (0.75)^{10-x}$

$\therefore$ For 4096 students we have

$4096 \times P(X \geq 8) = 4096 \times \underline{4.88 \times 10^{-4}}$

$\approx \underline{2}$ Students

$\therefore$ No. of students correctly answering 8 or more questions are $\underline{\underline{2}}$

16. *Find the binomial probability distribution which has mean 2 and variance 4/3.*

>> We know that for the binomial distribution mean $= np$ and variance $= npq$

Hence $np = 2$ and $npq = 4/3$ by using the data.

Further $2q = 4/3$ or $q = 2/3$ $\therefore$ $p = 1 - q = 1/3$

Since $np = 2$, we have $n(1/3) = 2$ $\therefore$ $n = 6$

The binomial probability function $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$ becomes

$$P(x) = {}^{6}C_{x}\,(1/3)^{x}(2/3)^{6-x}$$

The distribution of probabilities is as follows.

x	0	1	2	3
$P(x)$	$(2/3)^6$	${}^{6}C_{1}(1/3)(2/3)^5$	${}^{6}C_{2}(1/3)^2(2/3)^4$	${}^{6}C_{3}(1/3)^3(2/3)^3$
		4	5	6
		${}^{6}C_{4}(1/3)^4(2/3)^2$	${}^{6}C_{5}(1/3)^5(2/3)$	$(1/3)^6$

In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing **(i)** *no defective* **(ii)** *one defective* **(iii)** *two defective blades in a consignment of 10,000 packets.*

$\Rightarrow P(x) = \dfrac{m^x e^{-m}}{x!}$; m is mean

Given $p = 1/500$ & $n = 10$ $\therefore$ mean $\mu = m$
$= np$
$= 10 \times \frac{1}{500} = \boxed{\frac{1}{50} = m}$

$\therefore P(x) = \dfrac{\left(\frac{1}{50}\right)^x e^{-1/50}}{x!}$

i) $P(x=0) = \dfrac{\left(\frac{1}{50}\right)^0 e^{-1/50}}{0!} = 0.9801$

$\therefore$ No. of Packets with no defective Blades is $0.9801 \times 10{,}000$
$= 9801$ Packets

2) $P(X=1) = \dfrac{\left(\frac{1}{50}\right)^{1} \cdot e^{-1/50}}{1!} = 0.0196$

$\therefore$ No. of Packets with 1 defective Blade is

$0.0196 \times 10{,}000$

$= 196$ Packets

3) $P(X=2) = \dfrac{\left(\frac{1}{50}\right)^{2} \cdot e^{-1/50}}{2!} = 1.96 \times 10^{-4}$

$\therefore$ No. of Packets with 2 defective Blades are

$1.96 \times 10^{-4} \times 10{,}000$

$\simeq 2$ packets

The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with

(i) *no accident in a year*

(ii) *more than 3 accidents in a year.*

$\Rightarrow$ given $m=3$; $P(x) = \dfrac{m^x e^{-m}}{x!} \Rightarrow P(x) = \dfrac{3^x e^{-3}}{x!}$

1) $P(x=0) = \dfrac{3^0 e^{-3}}{0!} = 0.0497$; $\therefore$ No. of taxi drivers with no accidents are

0.0497×1000

$\simeq 50$ drivers

2) $P(X>3) = 1 - P(X \le 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[\dfrac{3^0 e^{-3}}{0!} + \dfrac{3^1 e^{-3}}{1!} + \dfrac{3^2 e^{-3}}{2!} + \dfrac{3^3 e^{-3}}{3!}\right] \Rightarrow P(x>3) = 0.3527$

$\therefore$ No. of taxi drivers with more than 3 accidents is

0.3527×1000

$\simeq 353$ drivers

2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains
(i) *no defective fuses* **(ii)** *3 or more defective fuses.*

* *

$\Rightarrow$ Given $p = 2\% = \frac{2}{100} = 0.02$ & $n = 200$

wkt mean of Poisson dist is m $\therefore m = np$
$= 200 \times 0.02$
$= 4$

$\therefore P(x) = \frac{m^x e^{-m}}{x!} = \frac{4^x e^{-4}}{x!}$

1) $P(x=0) = 0.0183$

2) $P(x \geq 3) = 1 - P(x<3)$
$= 1 - [P(x=0) + P(x=1) + P(x=2)]$
$= 0.7620$

If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction.

$\Rightarrow p = 0.001$ & $n = 2000$ $\Rightarrow m = np$

$= 2000 \times 0.001 \Rightarrow \boxed{m = 2}$

$$p(x) = \frac{m^x e^{-m}}{x!} = \frac{2^x e^{-2}}{2!}$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 0.3234$$

A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of **(i)** *no error during a micro second* **(ii)** *one error per micro second* **(iii)** *atleast one error per micro second* **(iv)** *two errors* **(v)** *atmost two errors.*

Hint: Given $p = 0.001$ & $n = 12$

HW

A shop has 4 disel generator sets which it hires every day. The demand for a gen set on an average is a poisson variate with value $5/2$. Obtain the probability that on a particular day **(i)** *there was no demand*

(ii) *a demand had to be refused.*

The probability that a news reader commits no mistake in reading the news is $1/e^3$.
Find the probability that on a particular news broadcast he commits
(i) *only 2 mistakes* **(ii)** *more than 3 mistakes* **(iii)** *atmost 3 mistakes.*

The probabilities of a Poisson variate taking the values 3 and 4 are equal. Calculate the probabilities of the variate taking the values 0 and 1.