



USN

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester B.E. (CSE/ISE) (Credit System) Degree Examinations

Make up / Supplementary Examinations – July 2018

16CS401 / 16IS401 – LINEAR ALGEBRA AND PROBABILITY THEORY

Duration: 3 Hours

Max. Marks: 100

- Note: 1) Answer Five full questions choosing One full question from each Unit.
2) Use of statistical tables is permitted.

	Unit – I	Marks	BT*
1. a)	i) Define subspace of a vector space V. ii) If W_1 and W_2 are subspaces of a vector space V then prove that $W_1 \cap W_2$ is also a subspace of V.	6	L^*5
b)	i) Find the coordinate vector of $v = (2,3,4)$ relative to the basis $\beta = \{(1,1,1), (1,0,1), (0,0,1)\}$. ii) Check whether the set $\{(1,2,3), (4,5,6), (2,1,0)\}$ is linearly independent or not.	7	$L1$
c)	Show that $V = \{(x, y) x, y \in R\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication defined by $c(x, y) = (cx, cy)$ is a vector space.	7	$L1$
2. a)	Express $u = (1, -2, 5)$ in R^3 as a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$, $u_3 = (2, -1, 1)$.	6	$L1$
b)	Define a basis for a vector space V. If $\{u_1, u_2, \dots, u_n\}$ is a basis for a vector space V, then prove that any vector in V can be expressed uniquely as a linear combination of vectors in the basis.	7	$L2$
c)	Obtain the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	7	$L1$
	Unit – II		
3. a)	Let V and W be vector space over the field F. Let T and U be linear transformation from V into W. Show that i) the function $(T+U)$ defined by $(T+U)(\alpha) = T\alpha + U\alpha$ is a linear transformation from V into W. ii) If c is any element of F, the function (cT) defined by $(cT)(\alpha) = c(T\alpha)$ is a linear transformation from V into W.	6	$L2$
b)	Find the matrix of the linear transformation $T: V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (x - y + 2z, 3x + y)$ with respect to the bases $\beta = \{(1,1,1), (1,2,3), (1,0,0)\}$, $\beta^1 = \{(1,1), (1,-1)\}$.	7	$L1$
c)	Let $F: R^4 \rightarrow R^3$ be defined by $F(x, y, z, w) = (x^2 y, xyz, x^2 + y^2 + zw^2)$ and $v = (0, 1, 2, -2)$. Find $F'(1, 2, -1, -2)$ and $D_v F(1, 2, -1, -2)$.	7	$L1$
4. a)	Define a linear transformation from vector space V into a vector space W. Let $T: V_3(R) \rightarrow V_2(R)$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$. Show that T is a linear transformation from $V_3(R)$ to $V_2(R)$.	6	$L1$

- b) Prove that every finite dimensional vector space V is isomorphic to F^n .
 c) i) Define range space and null space of a linear transformation T.
 ii) Find the range space and null space of a linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

7 L2

7 L1

Unit – III

5. a) State and prove Bayes' theorem.
 b) A random variable X has the density function

$$f(x) = \begin{cases} k(x+1) & , -1 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

- i) Find the value of k.
 ii) Find $E(X)$ and $V(X)$.
 c) The joint probability distribution of two dimensional discrete random variable (X, Y) is given by the following table.

X \ Y	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

- i) Find the marginal distribution of X and Y.
 ii) Check whether X and Y are independent or not.

7 L1

10

6. a) The probability distribution function of a random variable X is given by the following table

X	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	k

- i) find the value of k
 ii) find the mean and variance of X
 iii) find $P(-1 < X \leq 2)$ and $P(X < 1)$.

6 L1

- b) Suppose that two dimensional random variable (X, Y) has joint probability

$$\text{density function } f(x, y) = \begin{cases} \frac{1}{3}(4-x-y) & , 1 < x < 2, 0 < y < 2 \\ 0 & , \text{elsewhere} \end{cases}$$

- i) find the marginal distribution of X and Y
 ii) find $E(XY)$.

7 L1

- c) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{cases} x & , 0 \leq x < 1 \\ 2-x & , 1 \leq x < 2 \\ 0 & , \text{elsewhere} \end{cases}$$

find the c.d.f $F(x)$.

BT

7

Unit – IV

7. a) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes .What is the probability that a shower will last for i)less than 10 minutes ii)10 minutes or more.

6

- b) Obtain the mean and variance of binomial distribution with parameters n and p
 c) Calculate the coefficient of correlation and obtain the equation of regression line of x on y for the following data.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

8. a) Obtain the mean and variance of uniform distribution.
 b) If the probability that an individual suffers a bad reaction from an injection is 0.001, determine the probability that out of 2000 individuals i) exactly 3 ii) more than 2 individuals suffers a bad reaction.
 c) Suppose that the life lengths of two electric devices, say D₁ and D₂ have distributions N(40,36) and N(45,9) respectively.
 i) If the electronic device is to be used for a 45-hours period, which is to be preferred ?
 ii) If it is to be used for a 48 hours period, which device is to be preferred?

7 L5

7 L1

6 L5

7 L5

7 L1

Unit – V

9. a) Define moment generating function of a discrete and continuous random variable X. Show that if M_X(t) is the moment generating function of a random variable ,then M'_X(0) is the mean and M''_X(0) is E(X²).
 b) Let S² be the variance of a random sample of size 6 from the normal distribution N(μ,12), find P(2.30 < S² < 22.2).
 c) Define Student's t-distribution .Determine the value of b such that P(-b < X < b) = 0.9 , where X has t-distribution with 14 degrees of freedom.

6 L2

7 L1

7 L1

10. a) If $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is a stochastic matrix and $V = [v_1, v_2]$ is a probability vector.

6 L2

Show that VA is also a probability vector.

- b) In a cascade of binary communication channels, the symbols 1 and 0 are transmitted in successive stages. In any stage, the probability of transmitted 1 is received as 1 is 0.75 and the probability that a 0 is received as 0 is 0.5. Find the probability that
 i) 1 transmitted in the first stage is received correctly and 0 transmitted in the first stage is received as 1, after third stage.
 ii)If the probability of transmitting a 1 in the initial stage is $\frac{5}{8}$, find the probability that a 1 is received and a 0 is received after 3 stages.

7 L1

- c) Define probability vector. Find the unique fixed probability vector for the regular

$$\text{stochastic matrix } A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

7 L1

USN

--	--	--	--	--	--	--	--	--	--

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

IV Sem B.E. (CSE/ISE) Mid Semester Examinations - II, March 2017

15CS401 / 15IS401 – PROBABILITY THEORY AND NUMERICAL METHODS

Duration: 1 Hour

Max. Marks: 20

Note: Answer any One full question from each Unit.

Unit - I

Marks BT*

- a) Suppose three companies X, Y and Z produce T.V's. X produce twice as many as Y, while Y and Z produce the same number. It is known that 2% of X, 2% of Y and 4% of Z are defective. All the T.V's produced are put into one shop and then one T.V is chosen at random.
- What is the probability that the T.V is defective?
 - Suppose a T.V chosen is defective, what is the probability that this T.V is produced by company X?
- b) The probability density function of a random variable X is given by
- $$f(x) = \begin{cases} x; & 0 < x < 1 \\ 2-x; & 1 \leq x \leq 2 \\ 0; & \text{elsewhere} \end{cases}$$
- Find the value of $P(0.5 \leq X \leq 1.5)$.
 - Find the cumulative distribution function of X.
- a) From a lot containing 25 items, 5 of which are defective, 4 are chosen at random. Let X be the number of defective items in the sample. Obtain the probability distribution for X. Also find the mean and standard deviation of X.
- b) Suppose that the joint pdf of (X, Y) is given by $f(x, y) = \begin{cases} kxye^{-(x^2+y^2)}; & 0 < x < \infty, 0 < y < \infty \\ 0; & x, y \leq 0 \end{cases}$. Find the value of k. Prove that X and Y are independent.

5 L*4

5 L3

5 L5

5 L4

Unit - II

- a) Apply the fourth order Runge - Kutta method to find an approximate value of y at $x = 0.1$ where $y' = 3x + y$; $y(0) = 1$. Take $h = 0.1$.
- b) The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that:
- Exactly 2 will be defective.
 - At least 2 will be defective.
 - None will be defective.
- a) Using Modified Euler's method, find an approximate value of y at $x = 1.1$. Given that $\frac{dy}{dx} = x^2 + y^2$; $y(1) = 2$. Take $h = 0.1$.
- b) Define Poisson distribution. Obtain the mean and variance of a Poisson variate.

5 L3

5 L4

5 L3

5 L3

Bloom's Taxonomy, L* Level

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

IV Sem B.E. (CSE/ISE) Mid Semester Examinations - II, March 2017

15CS404/15IS404 – DATA COMMUNICATION

Max. Marks: 20

Note: Answer any One full question from each Unit.

Unit – I

Marks BT*

- 1 a) Define cyclic codes.
Let $g(x) = x^3 + x + 1$. Consider the information sequence 1001.

i) Find the codeword corresponding to the information sequence.

ii) Suppose the codeword has transmission error in the second bit. What does the receiver obtain when it does error checking?

b) What is constellation diagram? Explain the role of constellation diagram in analog transmission with an example.

c) List the characteristics of good polynomial generator.

4 L4

4 L4

3 L3

3 L2

3 L1

4 L4

3 L3

3 L2

3 L2

5 L2

5 L2

5 L2

5 L2

Unit – II

- 2 a) Calculate the Internet Checksum for the word 'DataWord' at sender & receiver site. Hint: ASCII value of A = 41 & a = 61.
- b) Draw the waveforms for BASK, BPSK and BFSK by taking an example 0011011010.
- c) Define linear block codes. With neat diagram explain the structure of encoder and decoder for simple parity-check code.

4 L4

3 L3

3 L2

3 L2

5 L2

5 L2

5 L2

3 a) Explain the data rate management in TDM.

b) What is multiplexing? Give the differences between FDM and WDM.

- 4 a) Explain analog hierarchy by considering an example of telephone company.
- b) Explain two different schemes of TDM.

5 a) Bloom's Taxonomy, L* Level

USN

--	--	--	--	--	--	--	--

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

IV Sem B.E. (CSE/ISE) Mid Semester Examinations - I, February 2017**15CS401/15IS401 – PROBABILITY THEORY AND NUMERICAL METHODS**

Duration: 1 Hour

Max. Marks: 20

*Note: Answer any One full question from each Unit.***Unit - I**

1. a) Prove that the first difference of a polynomial of degree n is a polynomial degree (n-1).
 b) The following table gives the velocity v of a particle at time t:

t(second)	0	2	4	6	8	10	12
v(m/sec)	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds (use Simpson's 1/3rd rule) and also the acceleration at t = 2 sec.

Marks

BT*

3 L*2

2. a) Derive Newton- Cotes quadrature formula.
 b) The population of certain town (as obtained from census data) is shown in the following table:

Year	1951	1961	1971	1981	1991
Population (in thousands)	19.96	39.65	58.81	77.21	94.61

Estimate the population in the year 1956. Also find the rate of growth of population in 1961.

7 L4

4 L3

6 L4

Unit - II

3. a) Using suitable interpolation formula, compute the value of x when f(x) = 10 from the data

x	1	2	4	8
f(x)	0	1	5	21

4 L3

3 L3

3 L2

- b) Using Regula-Falsi method find the root of the equation $x^3 - 4x + 1 = 0$ in (0, 1), carry out 3 iterations.

- c) Prove that Newton-Raphson method has second order convergence.

4. a) Prove that $\Delta^r y_k = \nabla^r y_{k+r}$. If $y(0) = 3$, $y(1) = 5$, $y(2) = 8$ and $y(3) = 12$ then find $(\nabla(\Delta y(1)))$.

4 L4

- b) Use the method of ordinary iteration to obtain the root of the equation $\cos x = 3x - 1$, near $x = 0.75$ (carry out 3 iterations)

3 L3

- c) Using Taylor's series method compute y at $x = 0.1$, given $y' = x + y$, $y(0) = 1$ and $h = 0.1$.

3 L3

BT* Bloom's Taxonomy, L* Level

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester B.E. (CSE/ISE) (Credit System) Degree Examinations
April – May 2017**15CS401 / 15IS401 – PROBABILITY THEORY AND NUMERICAL METHODS**

Duration: 3 Hours

Max. Marks: 100

Note: 1) Answer **Five full** questions choosing **One full** question from each Unit.
 2) Use of statistical table may be permitted.

Unit – IMarks 6 BT*
6 L*3

- a) State and prove Bayes' theorem.
 b) The percentage of alcohol ($100X$) in a certain compound is considered as random variable, where $X, 0 < X < 1$, has the probability density function:

$$f(x) = \begin{cases} 20x^3(1-x), & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Check that the above is a probability density function(pdf)
 (ii) Obtain the cumulative distribution function(cdf) for X
 (iii) Evaluate $P(X \leq \frac{2}{3})$

7 L4

- c) Suppose that the following table represents the joint distribution of the discrete random variable (X, Y)

	X 1	2	3
Y	1/12	1/6	0
1	0	1/9	1/5
2	1/18	1/4	2/15

- (i) Find marginal distributions of X and Y .
 (ii) Find variances of X and Y .

7 L3

- a) Consider a random variable with possible outcomes 1, 2, 3,... Suppose that

$$P(X = j) = \frac{1}{aj}, j = 1, 2, 3, \dots$$

- (i) For what values of 'a' is the above model meaningful?
 (ii) Compute $P(X \text{ is even})$
 (iii) Compute $P(X \text{ is divisible by } 3)$.

6 L3

- b) A random variable gives measurements X between 0 and 1 with probability

$$\text{density function } f(x) = \begin{cases} 12x^3 - 21x^2 + 10x, & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Find $E(X)$ and $V(X)$.
 (ii) Find the value of k such that $P\left(X \leq \frac{1}{2}\right) = k P\left(X > \frac{1}{2}\right)$.

7 L3

- c) Suppose that the joint pdf of two dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P\left(X > \frac{1}{2}\right)$, $P(Y < X)$ and $P(X + Y \geq 1)$.

7 L4

Unit – II

- a) The mean and standard deviation of the marks obtained by 1000 students in an examination are respectively 34.4 and 16.5. Assuming the normality of distribution, find the approximate number of students expected to obtain marks (i) between 30 and 60 (ii) less than 50.

6 L3

- b) If a random variable X has a Poisson distribution, then prove that $E(X) = V(X)$, where $E(X), V(X)$ are expectation and variance of X .

7 L2

- c) Find the coefficient of correlation and the line of regression of x on y for the following data :

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

7 L3

4. a) (i) Obtain the moment generating function of a binomial variate with parameters n and p .
(ii) Prove that if \bar{X} is the sample mean of a random sample $X_1, X_2, X_3, \dots, X_n$ of size n , then $E(\bar{X}) = \mu$ and $(\bar{X}) = \frac{\sigma^2}{n}$, where μ and σ^2 are mean and variance of population.
b) Define student's t-distribution. If X is a random variable having t-distribution with 10 degree of freedom, find $P(|X| > 2.228)$.
c) State and prove Central Limit theorem.

6 L3

6 L3

8 L3

Unit – III

5. a) Define Stochastic matrix. If A and B are 2×2 stochastic matrices, then prove that product AB is also stochastic matrix.
b) (i) Define Stochastic process and Markov chain.
(ii) Explain the classification of stochastic process.
c) A person playing chess game or tennis is as follows: If he plays chess game one week, then the probability that he switches to playing the tennis in next week is 0.2. On the other hand, if he plays tennis one week, then there is a probability of 0.7 that he will play tennis only in the next week as well. If he plays tennis on first week, find the probability that after four weeks
(i) he will play chess (ii) he will play tennis.

6 L3

7 L2

7 L3

6. a) (i) Define irreducible Markov chain and give one example.
(ii) Briefly explain the various classifications of states of Markov chain.
b) Define regular stochastic matrix. Determine which of the following are regular stochastic matrices? Justify your answer.

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

6 L2

7 L4

- Also find B^2 .
c) A salesman S sells in only 3 cities, A, B and C. Suppose that S never sells in the same city on successive days. If S sell in city A, then next day S sells in city B. However, if S sells in either B or C, then the next day S is twice likely to sell in city A as in the other city. Find out how often in the long run, S sells in each city?

7 L3

Unit – IV

7. a) The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds.

Time t (secs)	0	5	10	15	20
Velocity v (m/sec)	0	3	14	69	228

6 L3

7 L3

Find (i) Initial acceleration

(ii) Acceleration at $t = 10$.

b) Derive Newton's forward difference interpolation formula.

- c) A solid of revolution is formed by rotating about x -axis, the area between the x -axis, the lines $x = 0$, $x=1$ and the curve $3y=x^3$. Estimate the volume of the solid formed. Use simpson's $1/3$ rd rule, take 6 subintervals.

7 L4

- a) (i) Evaluate $\Delta^4(1-x)(1+x)(1-2x)(1+2x)$, take $h = 1$.
(ii) Form the backward differences of the function $f(x) = x^3 - 3x^2 - 5x - 7$ for $x = -1, 0, 1, 2, 3$. Continue the table to obtain $\Delta^3 f(0)$.
- b) Use suitable interpolation formula to obtain the root of the equation $f(x) = 0$, given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$ and $f(42) = 18$.
- c) (i) Derive Newton-Cote's quadrature formula.
(ii) Find the value of $\cos 1.70$ using the values given in the table below

x	1.70	1.74	1.78	1.82
sin x	0.9916	0.9857	0.9781	0.9691

6 L4

6 L3

8 L4

Unit - V

- a) Using Fourth order Runge-Kutta method find $y(0.1)$, given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0)=1$, take $h=0.1$.
- b) Solve the initial value problem $\frac{dy}{dx} = 1 + xy^2$, $y(0) = 1$, for $x = 0.4$ by using Milne's method, when it is given that
- | | | | |
|---|-------|-------|-------|
| x | 0.1 | 0.2 | 0.3 |
| Y | 1.105 | 1.223 | 1.355 |
- c) (i) Find the root of the equation $2x = \cos x + 3$, by iteration method take $x_0=1$ (carry out 3 iterations).
(ii) Use method of false position to obtain a real root of the equation $xe^x = 2$ on the interval $(0, 1)$ (carry out 3 iterations).
- a) Use Newton-Raphson method to find the root of the equation $e^x \sin x = 1$, which is near $x = 0$, carry out 5 iterations.
- b) Use Taylor's series method to approximate the value of y when $x = 0.2$, given that $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ by taking first five terms of Taylor's series expansion, take $h = 0.1$.
- c) Solve the following differential equation by Euler's modified method:
 $\frac{dy}{dx} = \log_{10}(x + y)$, $y(0) = 2$ at $x = 0.2$ and $x = 0.4$ with $h = 0.2$ (carry out 3 iterations in each step).

6 L3

8 L4

6 L3

7 L3

7 L3

* Bloom's Taxonomy, L* Level

USN []

NMAM INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester B.E. (CSE/ISE) Credit System) Degree Examinations
 Make up / Supplementary Examinations - July 2017
 15CS401 / 15IS401 – PROBABILITY THEORY AND NUMERICAL METHODS

Duration: 3 Hours

Max. Marks: 100

Note: 1) Answer **Five full** questions choosing **One full** question from each Unit.
 2) Use of statistical tables is permitted.

Unit – I

- a) A box contains 12 items of which 4 are defective. A sample of 3 items is selected at random from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X . Determine the mean and standard deviation of the distribution. Marks BT*

6 L*3

- b) Is the function $f(x) = \begin{cases} 0; & x < 2 \\ \frac{1}{18}(2x+3); & 2 \leq x \leq 4 \\ 0; & x > 4 \end{cases}$, a valid density function?

If so, find the probability $P(2 \leq X \leq 3)$ and $P(X > 3)$. Also find its cumulative density function. 7 L4

- c) Suppose that the two dimensional random (X, Y) has joint pdf $f(x, y) = \begin{cases} k(2x+3y); & 0 < x < 1, 0 < y < 1 \\ 0; & \text{elsewhere} \end{cases}$

- i) Evaluate the constant k .
 ii) Find the marginal distribution of X and Y .
 iii) Find $E(X)$ and $E(Y)$. 7 L4

- a) Suppose that the following table represents the joint pdf of (X, Y)

		X		
		1	2	3
Y	1	1/12	1/6	0
	2	0	1/9	1/5
	3	1/18	1/4	2/15

Evaluate i) the marginal distribution of X and Y . ii) $E(X)$ and $E(Y)$

6 L2

- b) i) State and prove the Bayes' theorem.
 ii) Three machines A , B and C respectively produce 25%, 30% and 45% of identical items. Of their outputs, respectively 5%, 4% and 3% are faulty. An item is selected at random and found to be faulty. What are the chances that it was produced by machine C . 7 L4

- c) Suppose that the joint pdf of (X, Y) is given by $f(x, y) = \begin{cases} kxye^{-(x^2+y^2)}; & 0 < x < \infty, 0 < y < \infty \\ 0; & x, y \leq 0 \end{cases}$. Find the value of k . Also prove that X and Y are independent. 7 L4

Unit – II

- a) Prove that the Poisson's distribution is the limiting form of the binomial distribution. 6 L2

- b) In a test on 2000 electric bulbs, it was found that the life span of a particular make was normally distributed with a mean life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for :
- more than 2150 hours
 - less than 1950 hours
 - more than 1920 hours but less than 2160 hours.

- c) Calculate the coefficient of correlation and obtain the lines of regression for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

7 L5

7 L4

6 L3

7 L3

7 L3

Unit – III

- a) Define probability vector. Find the unique fixed probability vector for the regular stochastic matrix

$$\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

6 L3

- b) Define the terms discrete Markov chain.

A habitual gambler is a member of two clubs A and B. He visits either of the clubs everyday for playing cards. He never visits club A on two consecutive days. But, if he visits club B on a particular day, then the next day he is likely to visit club B or club A. Find the transition matrix of this Markov chain. If the person had visited club B on Monday, find the probability that he visits club A on Thursday.

7 L3

- c) In a cascade of binary communication channels, the symbol 1 and 0 are transmitted in successive stage. In any stage, the probability of a transmitted 1 is received as 1 is 0.75 and the probability that a 0 is received as 0 is 0.5. Find the probability that:

- 1 transmitted in the first stage is received correctly and 0 transmitted in the first stage is received as 1 after third stage.

7 L4

- If the probability of transmitting a 1 in the initial stage is $5/8$, find the probability that 1 is received and a 0 received after three stages.

6 L3

- a) Define the term stochastic matrix. Prove, with reference to two second order stochastic matrices, that their product is also a stochastic matrix.

- b) In a particular city (modeled as either sunny or rainy), it was observed that, if the weather is sunny today then 90% chances that it will be sunny the next day. But if it rains today, then only 50% chance that it will rain the next day. Obtain the transition matrix of this chain. The weather on a day is known to be sunny. Find the probability that it will rain after 3 days.

7 L4

- c) Also, find the steady state probabilities of the transition matrix. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to A as to B. If C was the first person to throw the ball, find the probabilities that after three throws (i) A has the ball, (ii) B has the ball and (iii) C has the ball.

7 L4

7 L4

- a) Derive the Newton's forward difference interpolation formula for equally spaced tabular points.

- b) Find the root of the equation $f(x) = 0$ where $f(x)$ is tabulated as follows:

x	-2	-1	2	4
$f(x)$	-9	-1	11	69

using a suitable interpolation formula.

- c) A function $y = f(x)$ is given by the following table:

x	1	1.2	1.4	1.6	1.8	2.0
y	0.0	0.128	0.544	1.296	2.432	4.0

Find the approximate values of $f'(1.1)$ and $f''(1.1)$ by suitable interpolation formula

- a) For the following table of values, estimate $f(7.5)$.

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

- b) i) If $f(x) = x(x+1)(x+2)(x+3)$ and $h = 1$, prove that

$$\Delta f(x) = 4(x+1)(x+2)(x+3). \text{ Hence find } \Delta^2 f(x).$$

- ii) Show that: $\Delta - \nabla = \Delta \nabla$ where Δ and ∇ are respectively, the forward and backward difference operators.

- c) Derive the Newton cotes quadrature formula. Evaluate $\int_0^2 \sqrt{8x^3 + 1} dx$ by taking seven ordinates using Simpson's 1/3rd rule.

Unit – V

- a) Using the method of false position, find a root of the equation

$$x^2 - \log_e x = 12 \text{ in the interval } (3, 4). \text{ Carry out 3 iterations.}$$

- b) Apply the fourth order Runge - Kutta method to find an approximate value of y at $x = 0.2$ where $y' = x + y + xy$; $y(0) = 1$. Take $h = 0.1$.

- c) The differential equation $y' = x^2 + y^2 - 2$ satisfies the following data:

x	-0.1	0	0.1	0.2
y	1.09	1.0	0.89	0.7605

- Use Milne's predictor – corrector method to obtain the value of $y(0.3)$.

- a) Prove that the Newton – Raphson method has quadratic convergence.

- b) Using the Taylor series for $f(x)$, find the value of $y(0.2)$ from the differential equation $y' = x^2 + y^2$; $y(0) = 1$ taking $h = 0.1$. Expand up to 4th degree terms.

- c) Using Modified Euler's method, find an approximate value of y at $x = 1.2$.

$$\text{Given that } \frac{dy}{dx} = x + y; y(1) = 2. \text{ Take } h = 0.1.$$

Consider two approximations in each stage.

USN

--	--	--	--	--	--	--	--

NMAM INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belagavi)

IV Sem B.E. (CS/IS) Mid Semester Examinations - I, February 2018

16CS401/16IS401 – LINEAR ALGEBRA AND PROBABILITY THEORY

Time: 1 Hour

Max. Marks: 20

Note: Answer any One full question from each Unit.

	Unit – I	Marks	BT*
a) i) Define a vector space V over the field F. ii) Check whether $V = \{(x, y) x, y \in R\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 y_2, x_2 y_1)$ and scalar multiplication defined by $c(x, y) = (cx, cy)$ is a vector space or not.			
b) If W_1 and W_2 are subspaces of a vector space V then prove that $W_1 \cap W_2$ is also a subspace of V.	5	L*3	
c) Check whether the set $\{(1,1,1,1), (0,1,1,1), (0,1,1,1), (0,0,1,1)\}$ is linearly independent.	3	L4	
a) Let W be the subspace of R^4 spanned by the vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. Extend this basis to become a basis of R^4 .	2	L3	
b) If $\{u_1, u_2, \dots, u_n\}$ is a basis for a vector space V then prove that any vector in V can be expressed uniquely as a linear combination of vectors in the basis.	4	L3	
c) Express $u = (1, -2, 5)$ in R^3 as a linear combination of the vectors $u_1 = (1, 1, 1), u_2 = (1, 2, 3), u_3 = (2, -1, 1)$.	3	L4	
	3	L3	

Unit - II

a) Find the matrix of the linear transformation $T : V_2 \rightarrow V_3$ defined by $T(x, y) = (x + y, x, 3x - y)$ with respect to the bases $\beta = \{(1, 1), (3, 1)\}$ and $\beta' = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.	5	L3
b) i) Define range space and null space of a linear transformation T. ii) Find the range space and null space of T of a linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (y - x, y - z)$.	5	L3
a) Let $T : V_3 \rightarrow V_2$ be defined by the rule $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$. Show that T is a linear transformation from $V_3(R)$ to $V_2(R)$.	4	L4
b) Let $V_3(R)$ be a finite dimensional vector space. Find the coordinate vector of $u = (3, 1, -4)$ relative to the basis $u_1 = (1, 1, 1), u_2 = (0, 1, 1), u_3 = (0, 0, 1)$	3	L3
c) Find eigen values of $\begin{bmatrix} 13 & -4 & 2 \\ -4 & 13 & -2 \\ 2 & -2 & 10 \end{bmatrix}$	3	L3

USN []

NMAM INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belagavi)**IV Sem B.E. (CSE/ISE) Mid Semester Examinations – II, March 2018****16CS401/16IS401 – LINEAR ALGEBRA AND PROBABILITY THEORY**

Duration: 1 Hour

Max. Marks: 20

*Note: Answer any One full question from each Unit.***Unit – I**

1. a) The probability distribution of a random variable X is given by following table:

x	-2	-1	0	1	2	3
P(x)	0.1	0.1	0.2	0.2	0.3	0.1

Find (i) $P(x>-1)$ (ii) mean and variance of X .

- b) Define Stochastic Independence. Two dimensional random variables X and Y

$$\text{have joint p.d.f } f(x, y) = \begin{cases} kx(x-y), & 0 < x < 2, -x < y < x \\ 0 & \text{elsewhere} \end{cases}$$

find (i) k (ii) marginal p.d.f of X and Y

Marks BT*

4 L*1

6 L1

4 L2

2. a) State and prove Bayes' theorem.

b) (i) Show that $E(aX + b) = aE(X) + b$ & $V(aX + b) = a^2V(X)$

- (ii) Let X and Y be the random variable with the joint p.d.f given by the following table. Obtain Co-variance and check whether they are independent.

Y \ X		-2	-1	1	2
-1	1/16	1/8	1/8	1/16	
0	1/16	1/16	1/16	1/16	
1	1/16	1/8	1/8	1/16	

6 L1

Unit – II

3. a) If $F(x, y, z, w) = (x^2 - y^2, 2xy, zx, z^2w^2x^2)$ and $v = (2, 1, -2, -1)$ find $F'(1, 2, -1, -2)$ and $D_v F(1, 2, -1, -2)$.

4 L1

- b) If $f(x, y, z) = x^2y^2 - z^2 + 2x - 4y + z$. Find Hessian matrix of $f(x, y, z)$.

2 L1

- c) The probability that a man aged 60 will live up to 70 is 0.65. Out of 10 men now at the age of 60, find the probability that (i) at least 7 will live up to 70 (ii) exactly 9 will live up to 70.

4 L1

- a) The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective (ii) at least 2 will be defective (iii) at most 2 defective.

4 L1

- b) Obtain mean and variance of Poisson distribution.

6 L3

* Bloom's Taxonomy, L* Level

USN []

NMAM INSTITUTE OF TECHNOLOGY, NITTE
 (An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester B.E. (CS/IS) (Credit System) Degree Examinations
 April - May 2018

16CS401/16IS401 - LINEAR ALGEBRA AND PROBABILITY THEORY

Duration: 3 Hours

Max. Marks: 100

- Note: 1) Answer **Five full** questions choosing **One full** question from **each Unit**.
 2) Statistical tables permitted.

Unit - I

- | | Marks | BT* |
|---|-------|-----|
| a) Define a basis of a vector space V. Check whether the set $\{(1,2,-1), (1,-2,1), (-3,2,-1)\}$ is a basis for R^3 or not. | 6 | L*3 |
| b) If W_1 and W_2 are subspaces of a vector space V then prove that $W_1 \cap W_2$ is also a subspace of V. | 7 | L5 |
| c) i) Define a vector space V over the field F.
ii) Check whether $V = \{(x, y) x, y \in R\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 y_2, x_2 y_1)$ and scalar multiplication defined by $c(x, y) = (cx, cy)$ is a vector space or not. | 7 | L3 |
| a) Let $V_3(R)$ be a finite dimensional vector space. Find the coordinate vector of $u = (5,3,4)$ relative to the basis $u_1 = (1,-1,0), u_2 = (1,1,0), u_3 = (0,1,1)$. | 6 | L3 |
| b) Prove that a set of vectors $\{u_1, u_2, \dots, u_n\}$ is linearly dependent iff at least one of them is a linear combination of other vectors. | 7 | L3 |
| c) Let W be the subspace of R^4 spanned by the vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. Extend this basis to become a basis of R^4 . | 7 | L3 |

Unit - II

- | | | |
|--|---|----|
| a) If $F(x, y, z, w) = (x^2 - y^2, 2xy, zx, z^2 w^2 x^2)$ and $v = (2, 1, -2, -1)$ find $F^1(1, 2, -1, -2)$ and $D_v F(1, 2, -1, -2)$. | 6 | L3 |
| b) Find the matrix of the linear transformation $T : V_2 \rightarrow V_3$ defined by $T(x, y) = (x + y, x, 3x - y)$ with respect to the bases $\beta = \{(1, 1), (3, 1)\}$ and $\beta^1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. | 7 | L4 |
| c) i) Define range space and null space of a linear transformation T.
ii) Find the range space and null space of T of a linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. | 7 | L3 |
| a) Let $T : V_3(R) \rightarrow V_2(R)$ be defined by the rule $T(x, y, z) = (x + y, y + z)$. Show that T is a linear transformation from $V_3(R)$ to $V_2(R)$. | 6 | L3 |
| b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 13 & -4 & 2 \\ -4 & 13 & -2 \\ 2 & -2 & 10 \end{bmatrix}$ | 7 | L4 |

- c) Prove that every finite dimensional vector space V is isomorphic to F^n (where F is R or C)

7 L3

Unit - III

- i. a) A random variable has the following probability distribution

x	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	k

Find (i) value of K (ii) $E(X)$ (iii) $P(-1 < X \leq 2)$.

- b) In a certain college 4% of boy students and 1% of girl students are taller than 1.8 m. Furthermore, 60% of the students are girls. If a student is selected at random and is found taller than 1.8m, what is the probability that the student is a girl?

6 L5

- c) The joint p.d.f of two continuous random variable X and Y is
 $f(x, y) = 6e^{-2x-3y}$, $x, y \geq 0$

7 L3

= 0, elsewhere. Show that X and Y are independent.

7 L3

- i. a) State and prove Bayes' theorem on probability.

6 L3

- b) Suppose that the joint p.d.f of of two dimensional random variable (X, Y) is given

$$\text{by } f(x, y) = x^2 + \frac{xy}{3}, 0 \leq x \leq 1, 0 \leq y \leq 2 \\ = 0, \text{elsewhere}$$

Compute i) $P\left(X < \frac{1}{2}, Y < \frac{1}{2}\right)$ ii) $E(X)$

7 L3

- c) The joint p.d.f of two discrete random variables X and Y is given below.

X/Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

i) Find the marginal p.d.f's of X and Y

7 L4

ii) Evaluate $P[X + Y \geq 7]$

iii) Evaluate $P[XY > 2]$.

Unit - IV

7. a) The length of a telephone conversation follows an exponential distribution with mean 3 minutes. Find the probability that a call

6 L3

i) Ends less than 3 minutes and ii) takes between 3 and 5 minutes.

7 L3

- b) Obtain mean and variance of a Poisson distribution.

- c) Find the coefficient of correlation between x and y and regression line of y on x from the following data

x	3	6	5	4	4	6	7	5
y	3	2	3	5	3	6	6	4

7 L3

- d) a) In Poisson distribution , prove that i) $P(X \text{ is even}) = \frac{1}{2}(1 + e^{-2\lambda})$

6 L3

$$\text{ii) } P(X \text{ is odd}) = \frac{1}{2}(1 - e^{-2\lambda}).$$

- b) The number of telephone lines busy at an instant of time is a binomial variate with probability 0.2. If at an instant 10 lines are chosen at random, what is the probability that i) 5 lines are busy ii) at most 2 lines are busy iii) all lines are busy?
- c) In a normal distribution 31% of the items are under 45 and 48% are over 64. Find the mean and variance of the distribution.

7 L2
7 L3

Unit - V

- a) Prove that the product of two second order stochastic matrices is also a stochastic matrix.
- b) In a cascade of binary communication channels, the symbol 1 and 0 are transmitted in successive stage. In any stage, the probability of a transmitted 1 is received as 1 is 0.75 and the probability that a 0 is received as 0 is 0.5. Find the probability that 1 transmitted in the first stage is received correctly and 0 transmitted in the first stage is received as 1 after 3 stages.
- c) Find the unique fixed probability vector for the regular stochastic matrix

6 L4
7 L4

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

7 L3
6 L3
7 L4

- a) i) Define t-distribution
ii) Let X have a t-distribution with 14 d.o.f, determine 'b' such that $P(-b < t < b) = 0.90$.
- b) Obtain the moment generating function of a binomial variate with parameters 'n' and 'p'. Using this find mean and variance of a binomial variate.
- c) i) Define sample mean \bar{X} and sample variance S^2
ii) Prove that if \bar{X} is the sample mean of a random sample of size n,

$$E(\bar{X}) = \mu, \quad V(\bar{X}) = \frac{\sigma^2}{n}, \text{ where } \mu \text{ and } \sigma^2 \text{ are the mean and variance of the population.}$$

7 L4

Bloom's Taxonomy, L* Level

USN _____

NMAM INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belagavi)Fourth Semester B.E. (CSE/ISE) (Credit System) Degree Examinations
Make up / Supplementary Examinations – July 2018

16CS401 / 16IS401 – LINEAR ALGEBRA AND PROBABILITY THEORY

Duration: 3 Hours

Note: 1) Answer **Five full** questions choosing **One full** question from each Unit.
 2) Use of statistical tables is permitted.

Max. Marks: 100

Unit – I

- | | |
|---|-----------|
| 1. a) i) Define subspace of a vector space V.
ii) If W_1 and W_2 are subspaces of a vector space V then prove that $W_1 \cap W_2$ is also a subspace of V.
b) i) Find the coordinate vector of $v = (2,3,4)$ relative to the basis $\beta = \{(1,1,1), (1,0,1), (0,0,1)\}$.
ii) Check whether the set $\{(1,2,3), (4,5,6), (2,1,0)\}$ is linearly independent or not.
c) Show that $V = \{(x, y) x, y \in R\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication defined by $c(x, y) = (cx, cy)$ is a vector space. | Marks BT* |
| | 6 L*5 |
| | 7 L1 |
| | 7 L1 |
| 2. a) Express $u = (1, -2, 5)$ in R^3 as a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$, $u_3 = (2, -1, 1)$. | 6 L1 |
| b) Define a basis for a vector space V. If $\{u_1, u_2, \dots, u_n\}$ is a basis for a vector space V, then prove that any vector in V can be expressed uniquely as a linear combination of vectors in the basis. | 7 L2 |
| c) Obtain the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ | 7 L1 |

Unit – II

- | | |
|---|------|
| 3. a) Let V and W be vector space over the field F. Let T and U be linear transformation from V into W.
Show that i) the function $(T+U)$ defined by $(T+U)(\alpha) = T\alpha + U\alpha$ is a linear transformation from V into W.
ii) If c is any element of F, the function (cT) defined by $(cT)(\alpha) = c(T\alpha)$ is a linear transformation from V into W.
b) Find the matrix of the linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (x - y + 2z, 3x + y)$ with respect to the bases $\beta = \{(1,1,1), (1,2,3), (1,0,0)\}$, $\beta' = \{(1,1), (1,-1)\}$.
c) Let $F : R^4 \rightarrow R^3$ be defined by $F(x, y, z, w) = (x^2 y, xyz, x^2 + y^2 + zw^2)$ and $v = (0, 1, 2, -2)$. Find $F'(1, 2, -1, -2)$ and $D_v F(1, 2, -1, -2)$. | 6 L2 |
| | 7 L1 |
| | 7 L1 |
| a) Define a linear transformation from vector space V into a vector space W. Let $T : V_3(R) \rightarrow V_2(R)$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$. Show that T is a linear transformation from $V_3(R)$ to $V_2(R)$. | 6 L1 |

- Take up / Supplementary – July 2018
- a) Prove that every finite dimensional vector space V is isomorphic to F^n .
 b) Define range space and null space of a linear transformation T .
 c) Find the range space and null space of a linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

7 L2

- a) State and prove Bayes' theorem.

7 L1

- b) A random variable X has the density function

6 L5

$$f(x) = \begin{cases} k(x+1) & , -1 < x < 1 \\ 0 & , \text{ elsewhere} \end{cases}$$

- i) Find the value of k .

- ii) Find $E(X)$ and $V(X)$.

- c) The joint probability distribution of two dimensional discrete random variable (X, Y) is given by the following table.

7 L1

X \ Y	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

- i) Find the marginal distribution of X and Y .

- ii) Check whether X and Y are independent or not.

7 L1

- a) The probability distribution function of a random variable X is given by the following table

X	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	k

- i) find the value of k

- ii) find the mean and variance of X

- iii) find $P(-1 < X \leq 2)$ and $P(X < 1)$.

6 L1

- b) Suppose that two dimensional random variable (X, Y) has joint probability

$$\text{density function } f(x, y) = \begin{cases} \frac{1}{3}(4-x-y) & , 1 < x < 2, 0 < y < 2 \\ 0 & , \text{ elsewhere} \end{cases}$$

- i) find the marginal distribution of X and Y

- ii) find $E(XY)$.

7 L1

- c) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{cases} x & , 0 \leq x < 1 \\ 2-x & , 1 \leq x < 2 \\ 0 & , \text{ elsewhere} \end{cases}$$

7 L1

find the c.d.f $F(x)$

- d) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for i) less than 10

6 L1

Unit – IV

- i) Prove that every finite dimensional vector space V is isomorphic to P^* .
 ii) Define range space and null space of a linear transformation T .
 iii) Find the range space and null space of a linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

7 L2

5. a) State and prove Bayes' theorem.
 b) A random variable X has the density function

$$f(x) = \begin{cases} k(x+1) & , -1 < x < 1 \\ 0 & , \text{ elsewhere} \end{cases}$$

7 L1

- i) Find the value of k .
 ii) Find $E(X)$ and $V(X)$.
 c) The joint probability distribution of two dimensional discrete random variable (X, Y) is given by the following table.

	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

7 L1

- i) Find the marginal distribution of X and Y .
 ii) Check whether X and Y are independent or not.

7 L1

- a) The probability distribution function of a random variable X is given by the following table

X	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	k

- i) find the value of k .
 ii) find the mean and variance of X .
 iii) find $P(-1 < X \leq 2)$ and $P(X < 1)$.

6 L1

- b) Suppose that two dimensional random variable (X, Y) has joint probability

density function $f(x, y) = \begin{cases} \frac{1}{3}(4-x-y) & , 1 < x < 2, 0 < y < 2 \\ 0 & , \text{ elsewhere} \end{cases}$

- i) Find the marginal distribution of X and Y .

7 L1

- ii) Find $E(XY)$.

- c) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{cases} x & , 0 \leq x < 1 \\ 2-x & , 1 \leq x < 2 \\ 0 & , \text{ elsewhere} \end{cases}$$

- Find the c.d.f $F(x)$.

7 L1

4. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for i) less than 10 minutes ii) 10 minutes or more.

6 L1

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

- a) Obtain the mean and variance of uniform distribution.
 b) If the probability that an individual suffers a bad reaction from an injection is 0.001, determine the probability that out of 2000 individuals i) exactly 3 ii) more than 2 individuals suffer a bad reaction.
 c) Suppose that the life lengths of two electric devices, say D_1 and D_2 have distributions $N(40,36)$ and $N(45,9)$ respectively.
 i) If the electronic device is to be used for a 45-hours period, which is to be preferred?
 ii) If it is to be used for a 48 hours period, which device is to be preferred?

7 L5

7 L1

6 L5

7 L5

7 L1

Unit - V

- a) Define moment generating function of a discrete and continuous random variable X. Show that if $M_X(t)$ is the moment generating function of a random variable, then $M'_X(0)$ is the mean and $M''_X(0)$ is $E(X^2)$.
 b) Let S^2 be the variance of a random sample of size 6 from the normal distribution $N(\mu, 12)$, find $P(2.30 < S^2 < 22.2)$.
 c) Define Student's t-distribution. Determine the value of b such that $P(-b < X < b) = 0.9$, where X has t-distribution with 14 degrees of freedom.

6 L2

7 L1

7 L1

- a) If $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is a stochastic matrix and $V = [v_1, v_2]$ is a probability vector.

6 L2

- Show that VA is also a probability vector.
 b) In a cascade of binary communication channels, the symbols 1 and 0 are transmitted in successive stages. In any stage, the probability of transmitted 1 is received as 1 is 0.75 and the probability that a 0 is received as 0 is 0.5. Find the probability that

- i) 1 transmitted in the first stage is received correctly and 0 transmitted in the first stage is received as 1, after third stage.

7 L1

- ii) If the probability of transmitting a 1 in the initial stage is $\frac{5}{8}$, find the

- probability that a 1 is received and a 0 is received after 3 stages.

- c) Define probability vector. Find the unique fixed probability vector for the regular

Stochastic matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

7 L1

Bloom's Taxonomy, L* Level
