

# Score-based Denoising Diffusion Models

## - a tutorial

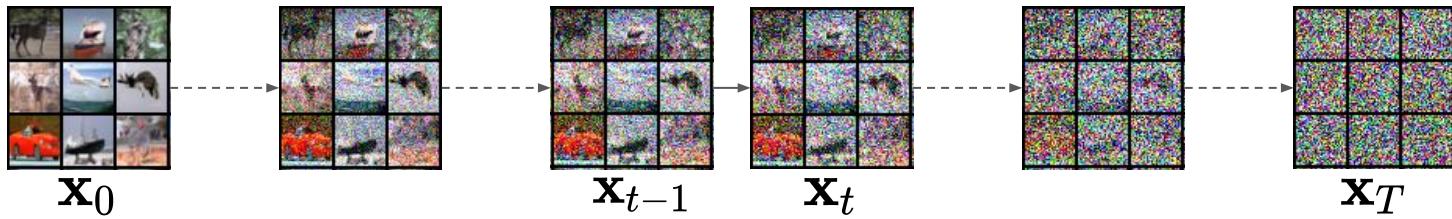
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Supervisor: Prof. Christopher Pal

1. Denoising Diffusion Probabilistic Model (DDPM)
2. Score-based Generative Model (SGM)
3. MCVD: Masked Conditional Video Diffusion

# DDPM : Forward process



$$q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) = \mathcal{N}(\sqrt{1 - \beta_t} \mathbf{x}_{t-1}, \beta_t \mathbf{I})$$

;  $0 < \beta_1 < \dots < \beta_t < \dots < \beta_T = 1$



$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

$$; \bar{\alpha}_t = \prod_{s=1}^t (1 - \beta_s)$$

$$\implies \mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$$

$$; \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$



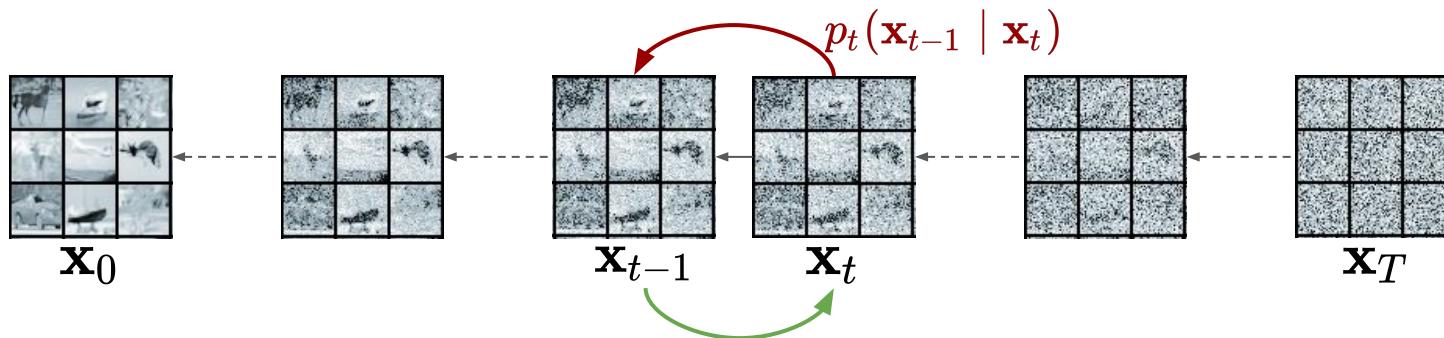
$$\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} \epsilon$$

$$(1) \quad \mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$$

$$(2) \quad \mathbf{x}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon) / \sqrt{\bar{\alpha}_t}$$

$$(3) \quad \epsilon = \frac{1}{\sqrt{1 - \bar{\alpha}_t}} (\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)$$

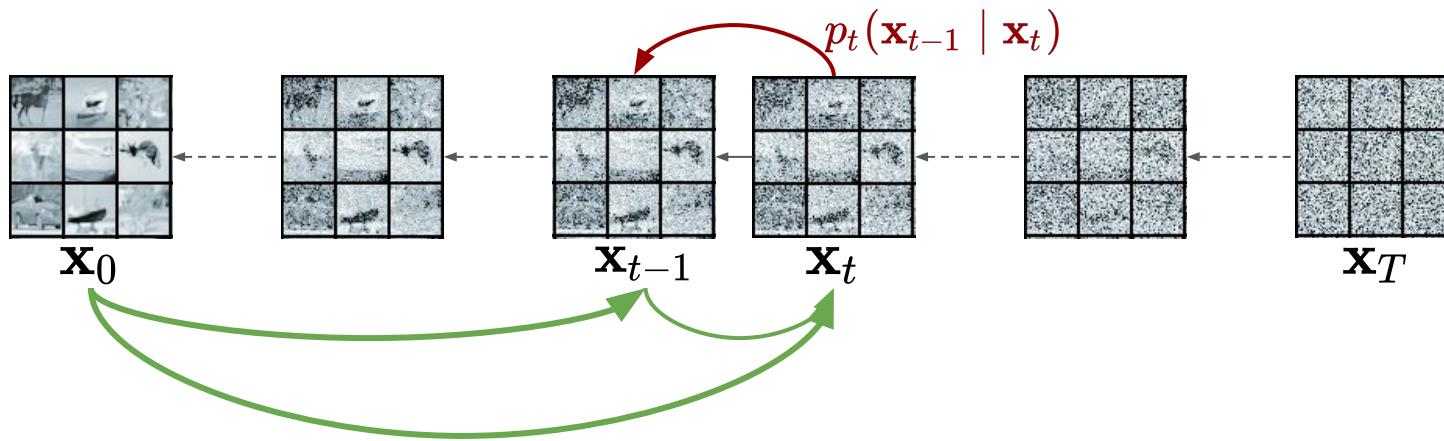
# DDPM : Reverse process



$$p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t) = \frac{q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) \checkmark p_t(\mathbf{x}_{t-1})?}{p_t(\mathbf{x}_t)?}$$

(Bayes' theorem)

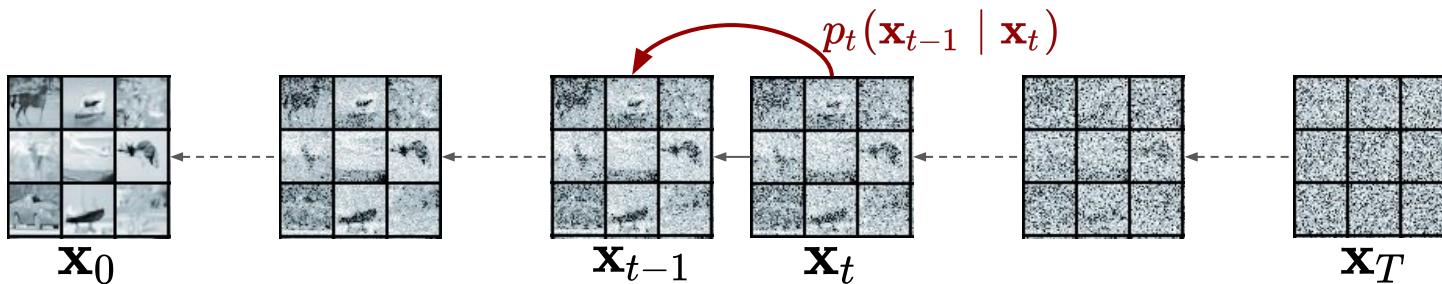
# DDPM : Reverse process



$$p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \boxed{\mathbf{x}_0}) = \frac{q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) \checkmark q_t(\mathbf{x}_{t-1} \mid \boxed{\mathbf{x}_0}) \checkmark}{q_t(\mathbf{x}_t \mid \boxed{\mathbf{x}_0}) \checkmark}$$

(Condition on  $\mathbf{x}_0$ )

# DDPM : Reverse process



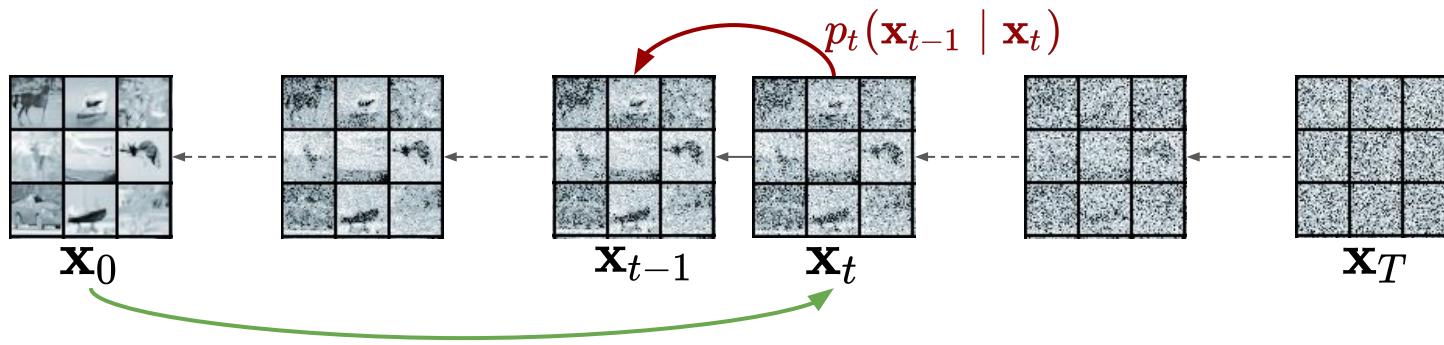
$$p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0), \tilde{\beta}_t \mathbf{I})$$

$$\tilde{\mu}(\mathbf{x}_t, \mathbf{x}_0) = \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1-\bar{\alpha}_t}\mathbf{x}_0 + \frac{\sqrt{1-\beta_t}(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t}\mathbf{x}_t ; \quad \tilde{\beta}_t = \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\beta_t$$

$p(\mathbf{u}) = \mathcal{N}(\mathbf{u} \mid \mathbf{m}\mathbf{u}, \mathbf{\Lambda}^{-1}),$   
 $p(\mathbf{v} \mid \mathbf{u}) = \mathcal{N}(\mathbf{v} \mid \mathbf{A}\mathbf{u} + \mathbf{b}, \mathbf{L}^{-1})$   
 $\Rightarrow p(\mathbf{v}) = \mathcal{N}(\mathbf{v} \mid \mathbf{A}\mu + \mathbf{b}, \mathbf{L}^{-1} + \mathbf{A}\mathbf{\Lambda}^{-1}\mathbf{A}^T),$   
 $\Rightarrow p(\mathbf{u} \mid \mathbf{v}) = \mathcal{N}(\mathbf{u} \mid \mathbf{C}(\mathbf{A}^T\mathbf{L}(\mathbf{v} - \mathbf{b}) + \mathbf{\Lambda}\mathbf{m}\mathbf{u}), \mathbf{C})$   
 $[\mathbf{C} = (\mathbf{\Lambda} + \mathbf{A}^T\mathbf{L}\mathbf{A})^{-1}]$   
 where  $\mathbf{u} = \mathbf{x}_{t-1} \mid \mathbf{x}_0$  ,  $\mathbf{v} = \mathbf{x}_t$

[arxiv.org/abs/2006.11239](https://arxiv.org/abs/2006.11239)

# DDPM : Reverse process

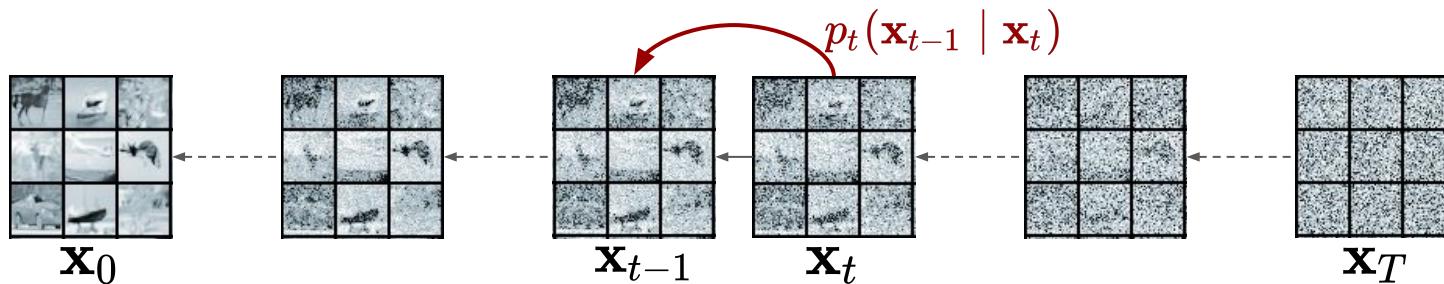


1  $\mathbf{x}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \quad \epsilon \quad ) / \sqrt{\bar{\alpha}_t}$

2  $p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\tilde{\mu}_t(\mathbf{x}_t, \mathbf{x}_0), \tilde{\beta}_t \mathbf{I})$

From (2):  
 $q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$   
 $\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$   
 $(2) \mathbf{x}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon) / \sqrt{\bar{\alpha}_t}$

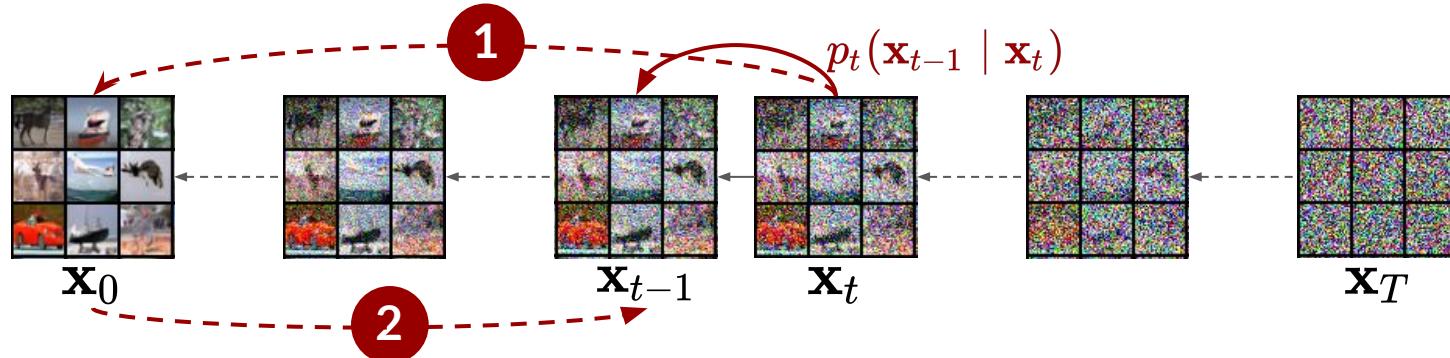
# DDPM : Reverse process



Deep neural network!

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \hat{\mathbf{x}}_0) = \mathcal{N}(\tilde{\mu}_t(\mathbf{x}_t, \hat{\mathbf{x}}_0), \tilde{\beta}_t \mathbf{I})$



$$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

**DDPM**

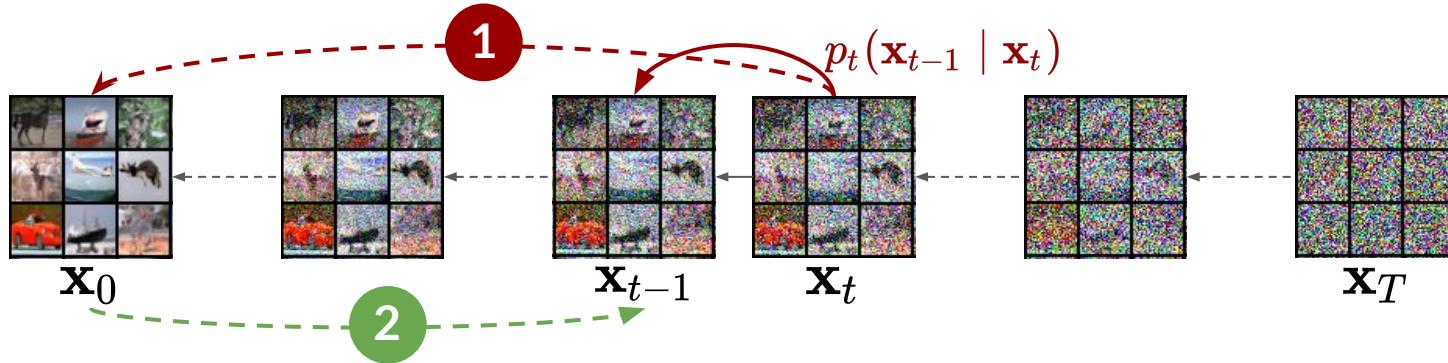
for  $t = T \rightarrow 0 :$

$$1 \quad \hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$$

$$2 \quad \mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t}(1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t} \mathbf{z}_t$$

$$(\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}))$$

# DDIM : Generation



$$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

DDIM

for  $t = T \rightarrow 0 :$

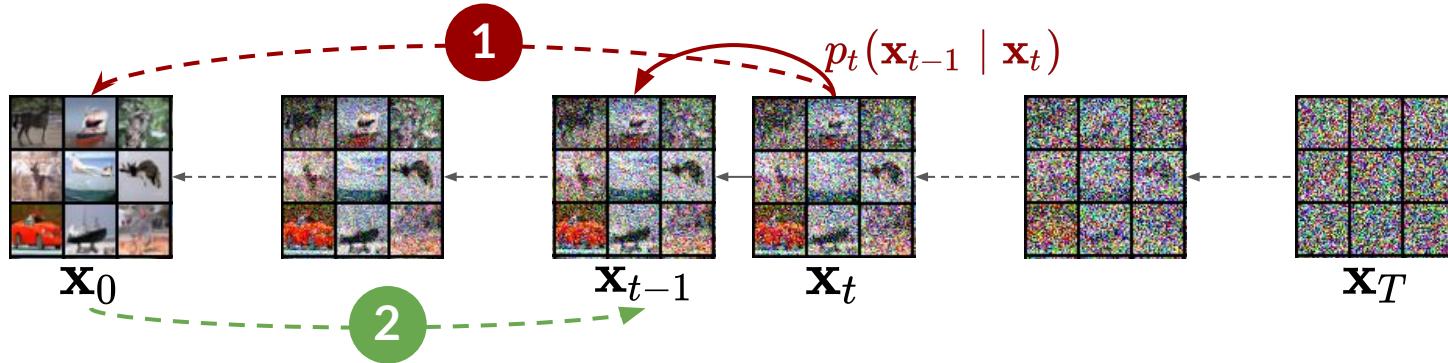
1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \hat{\mathbf{x}}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} \epsilon_\theta(\mathbf{x}_t, t)$

Deterministic!

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

$$(1) \quad \mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$$
$$\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} \epsilon$$



$$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

**DDIM**

for  $t = T \rightarrow 0 :$

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \hat{\mathbf{x}}_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \sigma_t^2} \epsilon_\theta(\mathbf{x}_t, t) + \sigma_t \mathbf{z}_t$

Deterministic  $\Rightarrow \sigma_t = 0$

$$(\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}))$$

## DDPM

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

$$p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}\left(\frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1-\bar{\alpha}_t}\mathbf{x}_0 + \frac{\sqrt{1-\beta_t}(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t}\mathbf{x}_t ; \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\beta_t \mathbf{I}\right)$$

$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

for  $t = T \rightarrow 0$  :

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1-\bar{\alpha}_t}\hat{\mathbf{x}}_0 + \frac{\sqrt{1-\beta_t}(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t}\mathbf{x}_t + \sqrt{\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t}\beta_t} \mathbf{z}_t$

## DDIM

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

$$p_t(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}\left(\sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \sigma_t^2} \epsilon ; \sigma_t^2 \mathbf{I}\right)$$

$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

for  $t = T \rightarrow 0$  :

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \hat{\mathbf{x}}_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \sigma_t^2} \epsilon_\theta(\mathbf{x}_t, t) + \sigma_t \mathbf{z}_t$

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\epsilon - \epsilon_\theta(\mathbf{x}_t, t)\|_2^2 \right]$$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon)$

## 1. Noise matching

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \|\epsilon - \epsilon_\theta(\mathbf{x}_t, t)\|_2^2$$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon)$

## 2. Generation

$$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

**DDPM**

for  $t = T \rightarrow 0$  :

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t}(1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t \mathbf{z}_t$

$(\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}))$

## 1. Training

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\epsilon - \epsilon_\theta(\mathbf{x}_t, t)\|_2^2 \right]$$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon)$

## 2. Generation

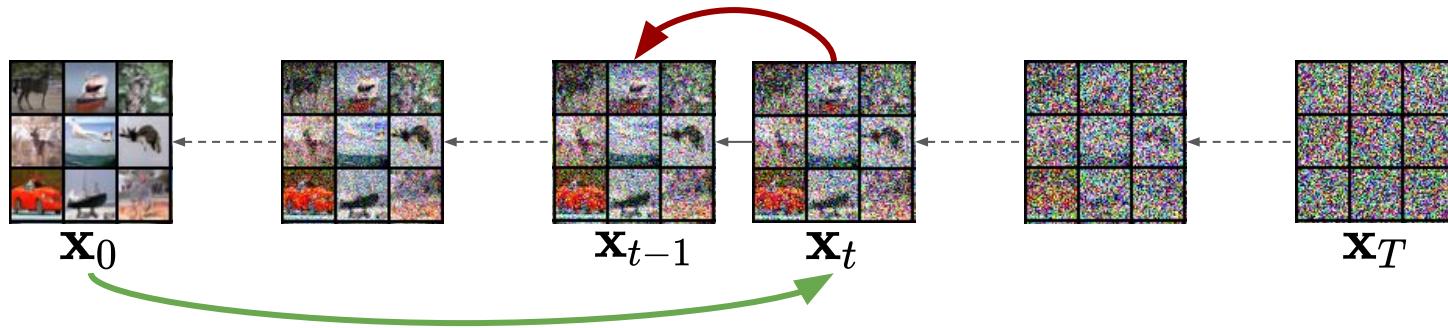
1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t} (1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t \mathbf{z}_t$

$(\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}))$

[arxiv.org/abs/2006.11239](https://arxiv.org/abs/2006.11239)

1. Denoising Diffusion Probabilistic Model (DDPM)
- 2. Score-based Generative Model (SGM)**
3. MCVD: Masked Conditional Video Diffusion



$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \sigma_t^2 \mathbf{I}) \quad ; \sigma_1 < \dots < \sigma_t < \dots < \sigma_T$$

$$\implies \mathbf{x}_t = \mathbf{x}_0 + \sigma_t \boldsymbol{\epsilon} \quad ; \boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

## Langevin Dynamics:



$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \nabla_{\mathbf{x}} \log p(\mathbf{x}_t) + \sqrt{2\lambda_t} \mathbf{z}_t \quad ; \lambda_t = \lambda \sigma_t^2 / \sigma_1^2; \mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

(Gradient ascent)

(Stochastic)

## 1. Score Matching:

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t)\|_2^2 \right]$$

$(\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon)$

## Denoising Score Matching:

$$\approx \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t | \mathbf{x}_0)\|_2^2 \right]$$

$$J_{ESMq}(\theta) = \mathbb{E}_{q(\mathbf{x})} \left[ \frac{1}{2} \left\| \psi(\mathbf{x}; \theta) - \frac{\partial \log q(\mathbf{x})}{\partial \mathbf{x}} \right\|^2 \right]$$

$$J_{DSMq_\sigma}(\theta) = \mathbb{E}_{q_\sigma(\mathbf{x}, \tilde{\mathbf{x}})} \left[ \frac{1}{2} \left\| \psi(\tilde{\mathbf{x}}; \theta) - \frac{\partial \log q_\sigma(\tilde{\mathbf{x}} | \mathbf{x})}{\partial \tilde{\mathbf{x}}} \right\|^2 \right]$$

$$J_{ESMq_\sigma} \curvearrowleft J_{DSMq_\sigma}$$

# Score (gradient of log density)

## 1. Score Matching:

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \sigma_t^2 \mathbf{I})$$

$$= \frac{1}{\sqrt{(2\pi)^{|\mathbf{x}|} \sigma_t^2}} \exp(-\frac{1}{2\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0)^T (\mathbf{x}_t - \mathbf{x}_0))$$

$$\Rightarrow \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \text{const} - \frac{1}{2\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0)^T (\mathbf{x}_t - \mathbf{x}_0)$$

Denoising Score Matching:

$$\Rightarrow (\text{Score}) \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = -\frac{1}{\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0) = \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2}$$

$$\approx \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0)\|_2^2$$

$$\|\mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2}\|_2^2$$

## 1. Score Matching:

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t)\|_2^2 \right]$$

( $\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon$ )

## Denoising Score Matching:

$$\approx \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t | \mathbf{x}_0)\|_2^2 \right]$$

$$= \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2}\|_2^2 \right]$$

( $\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon$ )

## 1. Denoising Score Matching:

$$\ell(\theta) := \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \left\| \mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2} \right\|_2^2 \right]$$

( $\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon$ )

Variance of score:

$$\mathbb{E} \left[ \left\| \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) \right\|_2^2 \right] = \mathbb{E} \left[ \left\| -\frac{\mathbf{x}_t - \mathbf{x}_0}{\sigma_t^2} \right\|_2^2 \right] = \mathbb{E} \left[ \left\| \frac{\sigma_t \epsilon}{\sigma_t^2} \right\|_2^2 \right] = \frac{1}{\sigma_t^2} \mathbb{E} \left[ \|\epsilon\|_2^2 \right] = \frac{1}{\sigma_t^2} \dim(\epsilon)$$

(Un)weighted objective function:

$$\mathcal{L}(\theta) = \sigma_t^2 \ell(\theta) := \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \left\| \sigma_t \mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t} \right\|_2^2 \right]$$

( $\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon$ )

Objective  
of SGM

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left\| \sigma_t \mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t} \right\|_2^2$$

$(\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon)$

- $\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon \implies \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t} = \epsilon$
- Make:  $\sigma_t \mathbf{s}_\theta(\mathbf{x}_t, t) = -\epsilon_\theta(\mathbf{x}_t, t)$  ----- How?

$$\implies \mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left\| \epsilon - \epsilon_\theta(\mathbf{x}_t, t) \right\|_2^2$$

$(\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon)$

Objective  
of DDPM!

# Score (gradient of log density)

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \sigma_t^2 \mathbf{I})$$

$$= \frac{1}{\sqrt{(2\pi)^{|\mathbf{x}|} \sigma_t^2}} \exp(-\frac{1}{2\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0)^T (\mathbf{x}_t - \mathbf{x}_0))$$

$$\Rightarrow \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \text{const} - \frac{1}{2\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0)^T (\mathbf{x}_t - \mathbf{x}_0)$$

$$\Rightarrow (\text{Score}) \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = -\frac{1}{\sigma_t^2} (\mathbf{x}_t - \mathbf{x}_0) = \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2}$$

$$\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon \quad (3) \quad \epsilon = \frac{1}{\sigma_t} (\mathbf{x}_t - \mathbf{x}_0)$$

$$\Rightarrow (\text{Score}) \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = -\frac{1}{\sigma_t} \epsilon$$

∴ Estimating  $\epsilon$   
is equivalent to  
estimating a  
scaled version  
of the **Score**!

## 1. Training

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \left| \left| \mathbf{s}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_0 - \mathbf{x}_t}{\sigma_t^2} \right| \right|_2^2 \right]$$

$(\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon)$

## 2. Generation

$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \mathbf{s}_\theta(\mathbf{x}_t, t) + \sqrt{2\lambda_t} \mathbf{z}_t \quad ; \lambda_t = \lambda \sigma_t^2 / \sigma_1^2; \mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

## 1. Training

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[ \|\epsilon - \epsilon_\theta(\mathbf{x}_t, t)\|_2^2 \right] \\ (\mathbf{x}_t = \mathbf{x}_0 + \sigma_t \epsilon)$$

## 2. Generation

$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \frac{-1}{\sigma_t} \epsilon_\theta(\mathbf{x}_t, t) + \sqrt{2\lambda_t} \mathbf{z}_t \quad ; \lambda_t = \lambda \sigma_t^2 / \sigma_1^2; \mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

## 1. Training

$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \|\epsilon - \epsilon_\theta(\mathbf{x}_t, t)\|_2^2$$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon)$

## 2. Generation

**1**  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$   $\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

**2**  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t} (1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t} \mathbf{z}_t$

## DDPM

$$q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) = \mathcal{N}(\sqrt{1 - \beta_t} \mathbf{x}_0, \beta_t \mathbf{I})$$

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t} (1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t} \mathbf{z}_t$

## SGM

$$q_t(\mathbf{x}_t \mid \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_0, (\sigma_t^2 - \sigma_{t-1}^2) \mathbf{I})$$

$$q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \sigma_t^2 \mathbf{I})$$

$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \mathbf{s}_\theta(\mathbf{x}_t, t) + \sqrt{2\lambda_t} \mathbf{z}_t$$

**Forward:**  $d\mathbf{x} = \mathbf{f}(\mathbf{x}, t) dt + g(t) d\mathbf{w}$

**Reverse:**  $d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - g(t)^2 \nabla_{\mathbf{x}} \log p_t(\mathbf{x})] dt + g(t) d\bar{\mathbf{w}}$

## DDPM

$$q_t(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\sqrt{1-\beta_t} \mathbf{x}_0, \beta_t \mathbf{I})$$

$$q_t(\mathbf{x}_t | \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

$$1 \quad \hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$$

$$2 \quad \mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t} (1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t} \mathbf{z}_t$$

## SGM

$$q_t(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_0, (\sigma_t^2 - \sigma_{t-1}^2) \mathbf{I})$$

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$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \mathbf{s}_\theta(\mathbf{x}_t, t) + \sqrt{2\lambda_t} \mathbf{z}_t$$

**Forward:**  $d\mathbf{x} = \mathbf{f}(\mathbf{x}, t) dt + g(t) d\mathbf{w}$

**Reverse:**  $d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - g(t)^2 \nabla_{\mathbf{x}} \log p_t(\mathbf{x})] dt + g(t) d\bar{\mathbf{w}}$

## DDPM

$$q_t(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\sqrt{1 - \beta_t} \mathbf{x}_0, \beta_t \mathbf{I})$$

$$q_t(\mathbf{x}_t | \mathbf{x}_0) = \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I})$$

1  $\hat{\mathbf{x}}_0 = (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \epsilon_\theta(\mathbf{x}_t, t)) / \sqrt{\bar{\alpha}_t}$

2  $\mathbf{x}_{t-1} = \frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1 - \bar{\alpha}_t} \hat{\mathbf{x}}_0 + \frac{\sqrt{1 - \beta_t} (1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t + \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t} \mathbf{z}_t$

$$d\mathbf{x} = -\frac{1}{2} \beta(t) \mathbf{x} dt + \sqrt{\beta(t)} d\mathbf{w}$$

Variance Preserving

## SGM

$$q_t(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_0, (\sigma_t^2 - \sigma_{t-1}^2) \mathbf{I})$$

$$q_t(\mathbf{x}_t | \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \sigma_t^2 \mathbf{I})$$

$$\mathbf{x}_{t-1} = \mathbf{x}_t + \lambda_t \mathbf{s}_\theta(\mathbf{x}_t, t) + \sqrt{2\lambda_t} \mathbf{z}_t$$

$$d\mathbf{x} = \sqrt{\frac{d[\sigma^2(t)]}{dt}} d\mathbf{w}$$

Variance Exploding

# Score (gradient of log density) for DDPM



$$\begin{aligned} q_t(\mathbf{x}_t \mid \mathbf{x}_0) &= \mathcal{N}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I}) \\ &= \frac{1}{\sqrt{(2\pi)^{|\mathbf{x}|}(1-\bar{\alpha}_t)}} \exp\left(-\frac{1}{2(1-\bar{\alpha}_t)}(\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)^T (\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)\right) \end{aligned}$$

$$\Rightarrow \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = \text{const} - \frac{1}{2(1-\bar{\alpha}_t)}(\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)^T (\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)$$

$$\Rightarrow (\text{Score}) \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = -\frac{1}{1-\bar{\alpha}_t}(\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)$$

$$\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t} \epsilon \quad (3) \quad \epsilon = \frac{1}{\sqrt{1-\bar{\alpha}_t}} (\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0)$$

$$\Rightarrow (\text{Score}) \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t \mid \mathbf{x}_0) = -\frac{1}{\sqrt{1-\bar{\alpha}_t}} \epsilon$$

∴ Estimating  $\epsilon$   
is equivalent to  
estimating a  
scaled version  
of the Score!

1. Denoising Diffusion Probabilistic Model (DDPM)
2. Score-based Generative Model (SGM)
3. **MCVD: Masked Conditional Video Diffusion**

# MCVD: Masked Conditional Video Diffusion



$p$  past frames:  $\mathbf{p} = \{\mathbf{p}^i\}_{i=1}^p$

$k$  current frames:  $\mathbf{x}_0 = \{\mathbf{x}_0^i\}_{i=1}^k$

$f$  future frames:  $\mathbf{f} = \{\mathbf{f}^i\}_{i=1}^f$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon)$

- **Video Prediction:**

$$\mathbb{E}_{t, [\mathbf{p}, \mathbf{x}_0] \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} ||\epsilon - \epsilon_\theta(\mathbf{x}_t | \boxed{\mathbf{p}}, t)||_2^2$$

- **Video Generation:**

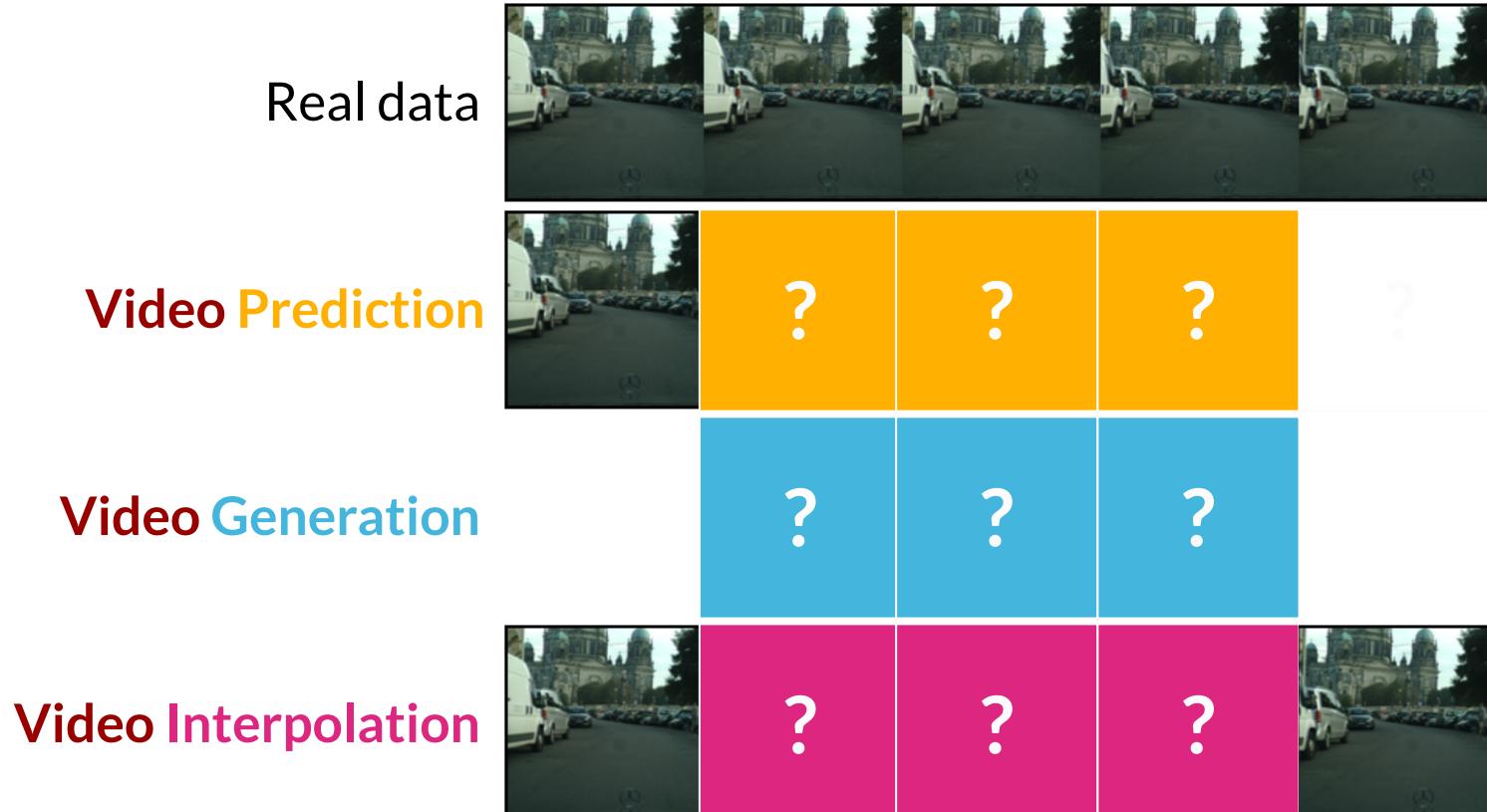
$$\mathbb{E}_{t, \mathbf{x}_0 \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} ||\epsilon - \epsilon_\theta(\mathbf{x}_t | t)||_2^2$$

- **Video Interpolation:**

$$\mathbb{E}_{t, [\mathbf{p}, \mathbf{x}_0, \mathbf{f}] \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} ||\epsilon - \epsilon_\theta(\mathbf{x}_t | \boxed{\mathbf{p}}, \boxed{\mathbf{f}}, t)||_2^2$$

[arxiv.org/abs/2205.09853](https://arxiv.org/abs/2205.09853)

# MCVD: Masked Conditional Video Diffusion



# MCVD: Masked Conditional Video Diffusion



$p$  past frames:  $\mathbf{p} = \{\mathbf{p}^i\}_{i=1}^p$

$k$  current frames:  $\mathbf{x}_0 = \{\mathbf{x}_0^i\}_{i=1}^k$

$f$  future frames:  $\mathbf{f} = \{\mathbf{f}^i\}_{i=1}^f$

$(\mathbf{x}_t = \sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon)$

- **Video Prediction:**

$$\mathbb{E}_{t, [\mathbf{p}, \mathbf{x}_0] \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \|\epsilon - \epsilon_\theta(\mathbf{x}_t \mid \mathbf{p}, t)\|_2^2$$

Random masking!

- **Video Prediction + Generation:**

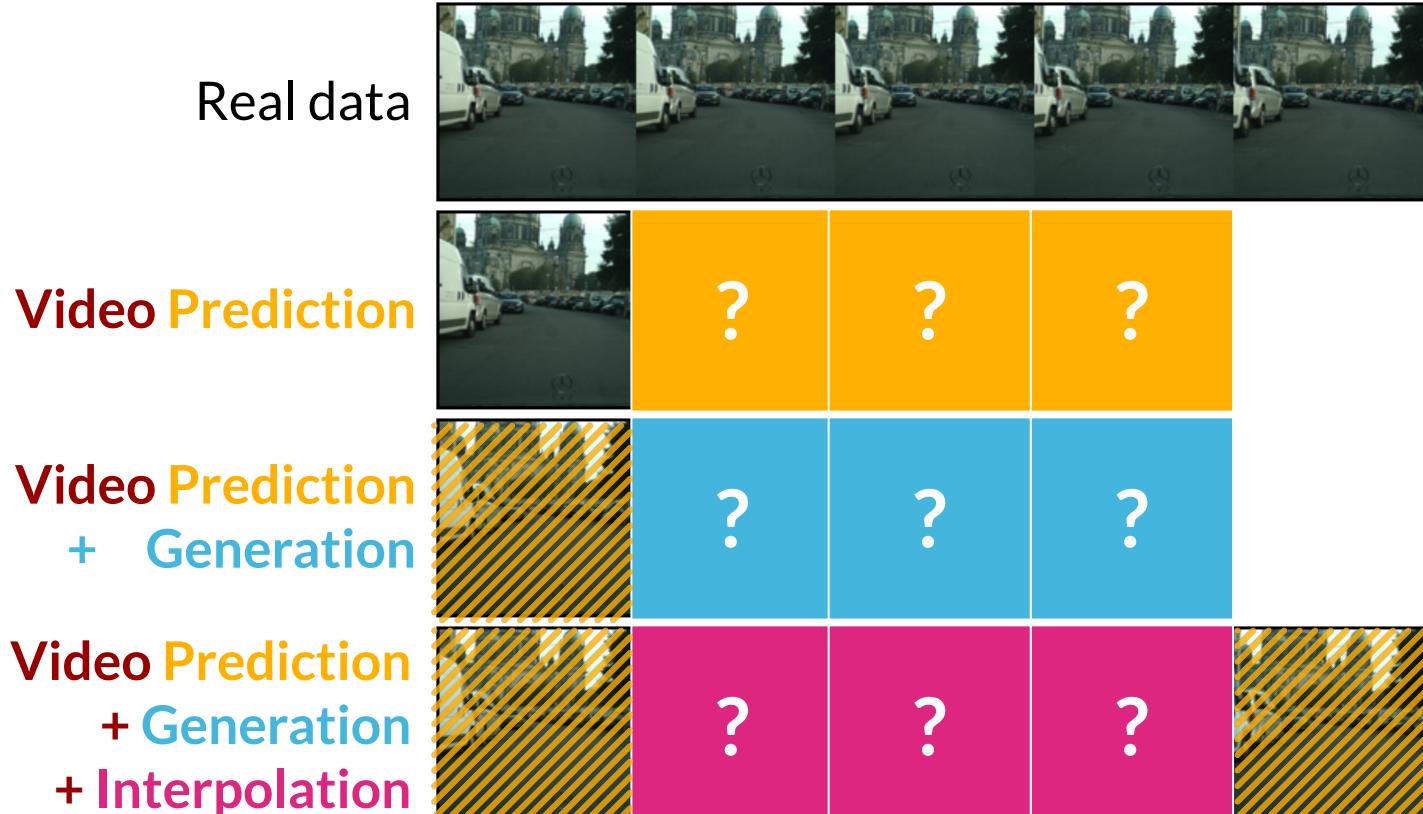
$$\mathbb{E}_{t, [\mathbf{p}, \mathbf{x}_0, \mathbf{f}] \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I}), m_p \sim \mathcal{B}(p_{\text{mask}})} \|\epsilon - \epsilon_\theta(\mathbf{x}_t \mid m_p \mathbf{p}, t)\|_2^2$$

- **Video Prediction + Generation + Interpolation:**

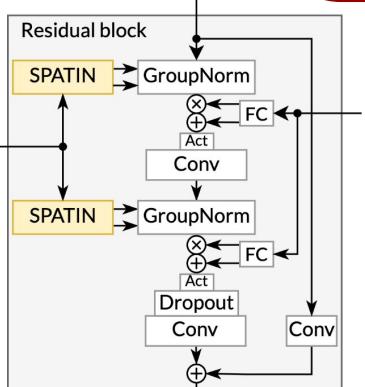
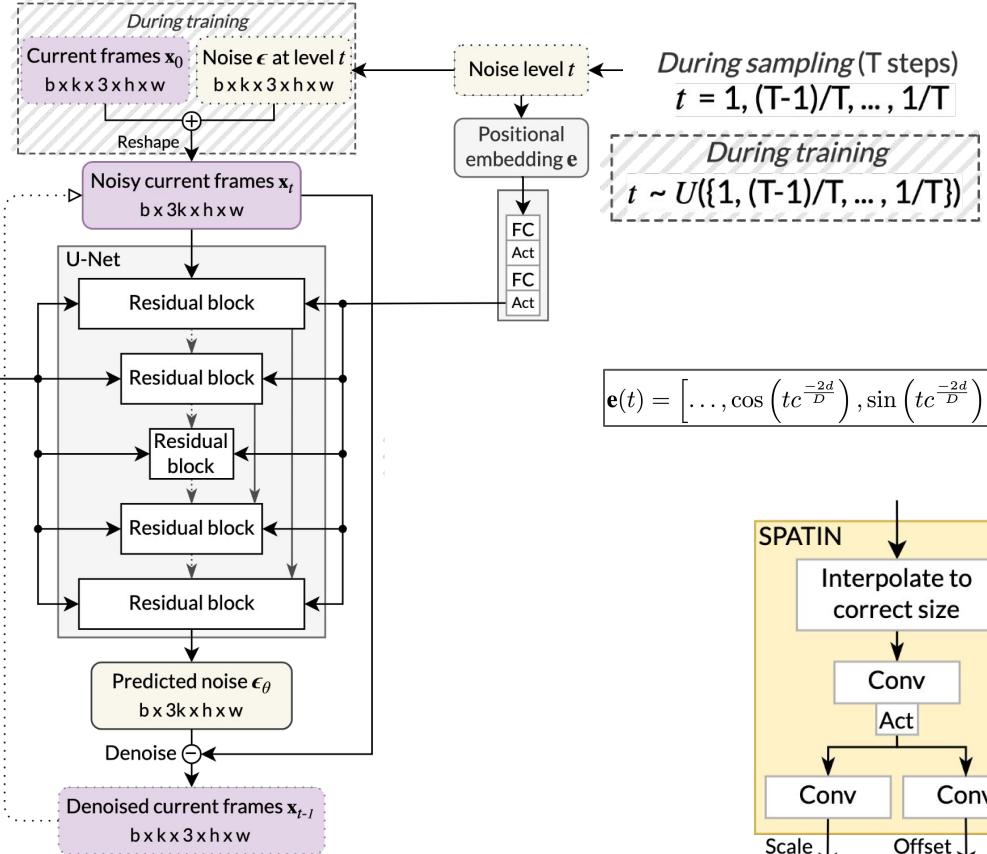
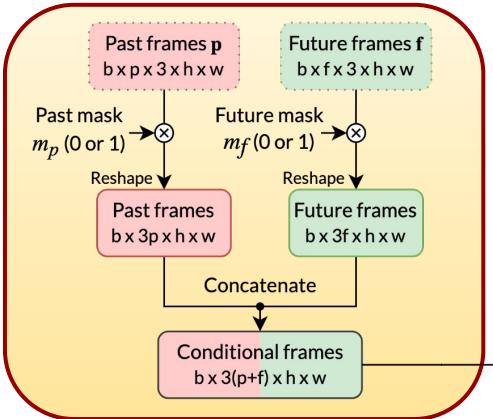
$$\mathbb{E}_{t, [\mathbf{p}, \mathbf{x}_0, \mathbf{f}] \sim p_{\text{data}}, \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I}), (m_p, m_f) \sim \mathcal{B}(p_{\text{mask}})} \|\epsilon - \epsilon_\theta(\mathbf{x}_t \mid m_p \mathbf{p}, m_f \mathbf{f}, t)\|_2^2$$

[arxiv.org/abs/2205.09853](https://arxiv.org/abs/2205.09853)

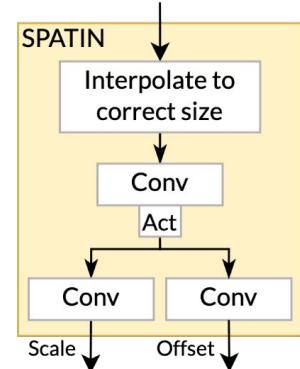
# MCVD: Masked Conditional Video Diffusion



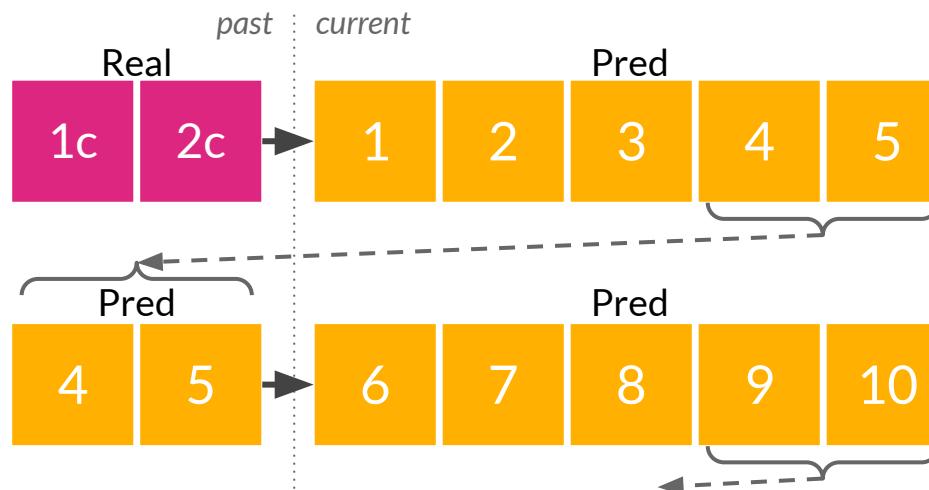
# MCVD: Masked Conditional Video Diffusion



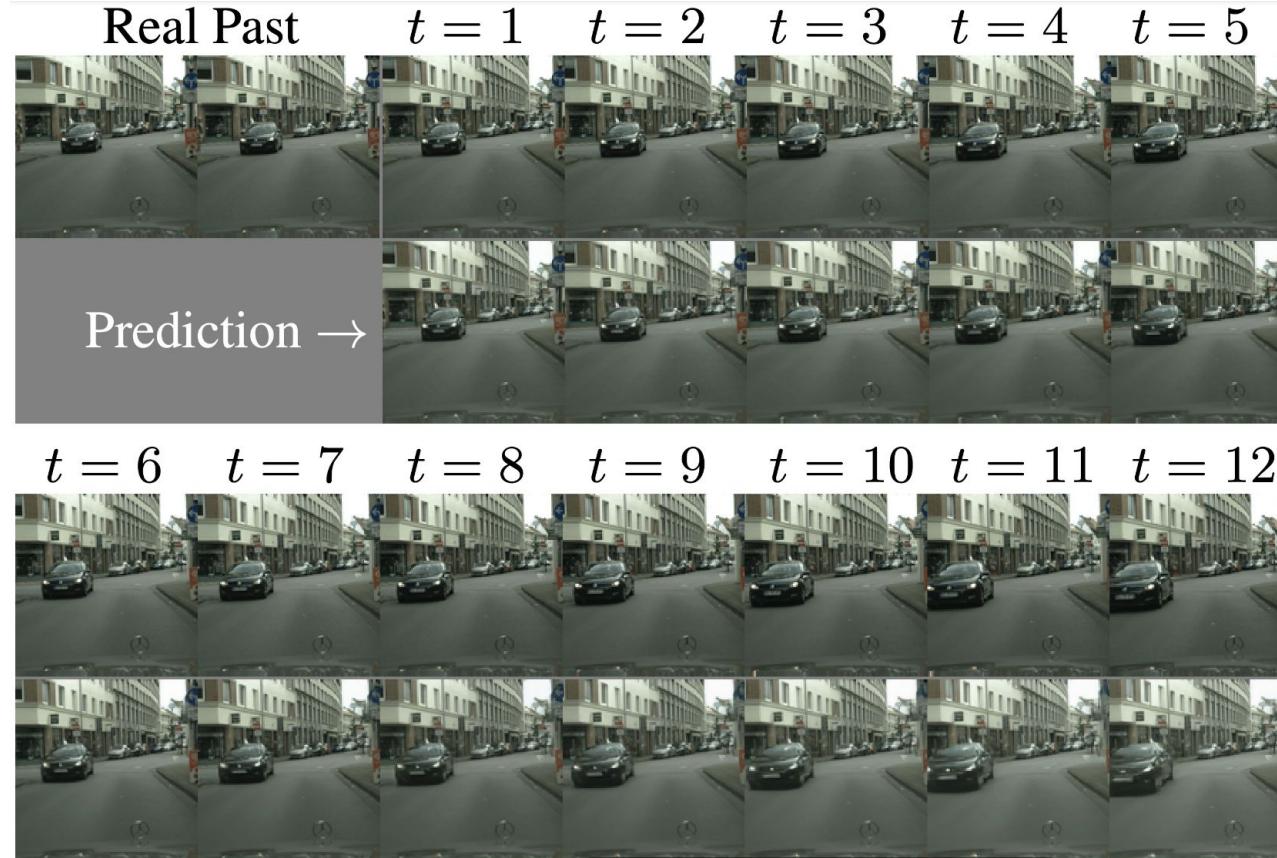
[arxiv.org/abs/2205.09853](https://arxiv.org/abs/2205.09853)



## Block-autoregressive generation:



# MCVD: Masked Conditional Video Diffusion



# MCVD: Masked Conditional Video Diffusion



(128x128)

Cityscapes [2 → 28; trained on $k$ ]	$k$	FVD↓	LPIPS↓
SVG-LP Denton and Fergus [2018]	10	1300.26	$0.549 \pm 0.06$
vRNN 1L Castrejón et al. [2019]	10	682.08	$0.304 \pm 0.10$
Hier-vRNN Castrejón et al. [2019]	10	567.51	$0.264 \pm 0.07$
GHVAE Wu et al. [2021]	10	418.00	$0.193 \pm 0.014$
MCVD spatin (Ours)	<b>5</b>	184.81	$0.121 \pm 0.05$
MCVD concat (Ours)	<b>5</b>	<b>141.31</b>	<b>0.112 ± 0.05</b>

(64x64)

BAIR [past $p \rightarrow pred$ ; trained on $k$ ]	$p$	$k$	$pred$	FVD↓	PSNR↑	SSIM↑
LVT [Rakhimov et al., 2020]	1	15	15	125.8	–	–
DVD-GAN-FP [Clark et al., 2019]	1	15	15	109.8	–	–
MCVD spatin (Ours)	1	<b>5</b>	15	103.8	18.8	0.826
TrIVD-GAN-FP [Luc et al., 2020]	1	15	15	103.3	–	–
VideoGPT [Yan et al., 2021]	1	15	15	103.3	–	–
CCVS [Le Moing et al., 2021]	1	15	15	99.0	–	–
MCVD concat (Ours)	1	<b>5</b>	15	98.8	18.8	0.829
MCVD spatin past-mask (Ours)	1	<b>5</b>	15	96.5	18.8	0.828
MCVD concat past-mask (Ours)	1	<b>5</b>	15	<b>95.6</b>	<b>18.8</b>	<b>0.832</b>
Video Transformer [Weissenborn et al., 2019]	1	15	15	94-96 <sup>a</sup>	–	–
FitVid [Babaeizadeh et al., 2021]	1	15	15	93.6	–	–
MCVD concat past-future-mask (Ours)	1	<b>5</b>	15	<b>89.5</b>	16.9	0.780
SAVP [Lee et al., 2018]	2	14	14	116.4	–	–
MCVD spatin (Ours)	2	<b>5</b>	14	94.1	19.1	0.836
MCVD spatin past-mask (Ours)	2	<b>5</b>	14	90.5	<b>19.2</b>	0.837
MCVD concat (Ours)	2	<b>5</b>	14	90.5	19.1	0.834
MCVD concat past-future-mask (Ours)	2	<b>5</b>	14	89.6	17.1	0.787
MCVD concat past-mask (Ours)	2	<b>5</b>	14	<b>87.9</b>	19.1	<b>0.838</b>
SVG-LP [Akan et al., 2021]	2	10	28	256.6	–	0.816
SLAMP [Akan et al., 2021]	2	10	28	245.0	19.7	0.818
SAVP [Lee et al., 2018]	2	10	28	143.4	–	0.795
Hier-vRNN Castrejón et al. [2019]	2	10	28	143.4	–	<b>0.822</b>
MCVD spatin (Ours)	2	<b>5</b>	28	132.1	17.5	0.779
MCVD spatin past-mask (Ours)	2	<b>5</b>	28	127.9	17.7	0.789
MCVD concat (Ours)	2	<b>5</b>	28	120.6	17.6	0.785
MCVD concat past-mask (Ours)	2	<b>5</b>	28	119.0	<b>17.7</b>	0.797
MCVD concat past-future-mask (Ours)	2	<b>5</b>	28	<b>118.4</b>	16.2	0.745

[arxiv.org/abs/2205.09853](https://arxiv.org/abs/2205.09853)

[mask-cond-video-diffusion.github.io](https://mask-cond-video-diffusion.github.io)



# Thank you!