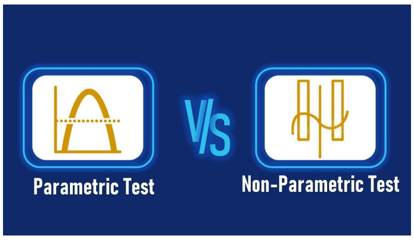


Advanced Hypothesis Testing

Please note that any topics that are not covered in today's lecture will be covered in the next lecture.

Content

1. Recap of One-Way ANOVA
2. Two-Way ANOVA
3. KS - Test
4. A/B Testing
5. Parametric vs Non-parametric.

Recap of One-Way ANOVA:

- In One-Way ANOVA, we explored how to compare means across **more than two** groups when we had just **one categorical independent variable**.
- We used it to determine whether there were statistically significant differences between those groups.
- When we perform One-Way ANOVA, we obtain an F-statistic and a p-value.
  - The p-value tells us whether the differences between the groups are statistically significant.
  - If the p-value is small (typically less than alpha), we conclude that there are significant differences between at least two groups.

Two Way ANOVA

Motivation: Now, why do we need Two-Way ANOVA?

Imagine you're working as a data scientist for a beverage company, and your company produces and sells soft drinks.

- You want to understand the factors that influence the sales of your drinks
- So, you decide to examine **two different factors**: the flavour of the drink and the location where it's sold.

Now, you've probably already grasped the importance of considering these two factors.

- The flavour of the drink (e.g., lemon, cola, and orange) and
- The location (e.g., North, South, East, and West) can both have an impact on sales.

This is where **Two-Way ANOVA** comes into play.

Two-way ANOVA allows us to investigate how **two independent categorical variables** interact and impact a continuous dependent variable, in this case, the sales of soft drinks.

In this example, "**Flavour**" and "**Location**" are our **independent variables**, and **"Sales"** is our **dependent variable**.

- With One-Way ANOVA, we could only investigate one of these factors at a time.
- But Two-Way ANOVA enables us to look at both factors simultaneously and assess how they interact.

Here's how it works:

1. **Main Effects:** Two-way ANOVA assesses the main effects of each factor. In our case, it would evaluate the effect of "Flavour" and the effect of "Location" on sales independently.
2. **Interaction Effect:** It also examines the interaction between the two factors. In other words, it helps us understand whether the impact of "Flavor" on sales depends on the "Location," and vice versa.

This allows us to answer questions like:

- Is there a significant difference in sales between the three flavours?
- Is there a significant difference in sales between the four locations?
- Is there an interaction effect between flavour and location on sales?

By conducting Two-Way ANOVA, we can determine if these factors have a statistically significant impact on sales.

This information can be vital for marketing strategies, product development, and resource allocation.

Let's solve the above business case with actual data.

```
import pandas as pd
import numpy as np
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

```
!wget --no-check-certificate https://drive.google.com/uc?id=1Vy0OPYInhYxuZzSn415D
```

```
--2024-01-18 09:45:16--  https://drive.google.com/uc?id=1Vy0OPYInhYxuZzSn415D
Resolving drive.google.com (drive.google.com)... 142.250.152.101, 142.250.152
Connecting to drive.google.com (drive.google.com)|142.250.152.101|:443... con
HTTP request sent, awaiting response... 303 See Other
Location: https://drive.usercontent.google.com/download?id=1Vy0OPYInhYxuZzSn4
--2024-01-18 09:45:16--  https://drive.usercontent.google.com/download?id=1Vy
Resolving drive.usercontent.google.com (drive.usercontent.google.com)... 209.
Connecting to drive.usercontent.google.com (drive.usercontent.google.com)|209
HTTP request sent, awaiting response... 200 OK
Length: 1541 (1.5K) [application/octet-stream]
Saving to: ‘two_way_anova.csv’

two_way_anova.csv   100%[===================>]   1.50K  --.-KB/s    in 0s

2024-01-18 09:45:17 (32.1 MB/s) - ‘two_way_anova.csv’ saved [1541/1541]
```

```
df=pd.read_csv('/content/two_way_anova.csv')
df.head()
```

	Flavour	Location	Sales
0	Orange	West	141
1	Lemon	West	178
2	Orange	West	170
3	Orange	East	76
4	Lemon	East	170

```
df.shape
```

```
(100, 3)
```

```
df['Flavour'].unique()
```

```
array(['Orange', 'Lemon', 'Cola'], dtype=object)
```

```
df['Location'].unique()
```

```
array(['West', 'East', 'North', 'South'], dtype=object)
```

We have the data of three different flavours & four different locations and their sales.

STEP 1:

> **What should be the null and alternate hypothesis?**

1. Null Hypotheses for Main Effects:
   - **Null Hypothesis for Flavor:** There is no significant difference in sales between the three flavors (Lemon, Cola, Orange).
   - **Null Hypothesis for Location:** There is no significant difference in sales between the four locations (North, South, East, West).
2. Alternative Hypotheses for Main Effects:
   - **Alternative Hypothesis for Flavor:** There is a significant difference in sales between at least two flavours.
   - **Alternative Hypothesis for Location:** There is a significant difference in sales between at least two locations.

1. Null Hypothesis for Interaction Effect:
   - **Null Hypothesis for Interaction Effect:** There is no interaction effect between the choice of flavour and the location of sale on sales. (i.e.,In other words, the impact of flavor on sales does not depend on the location, and vice versa.)
2. Alternative Hypothesis for Interaction Effect:
   - **Alternative Hypothesis for Interaction Effect:** There is a significant interaction effect between the choice of flavour and the location of sale on sales. (i.e., In other words, the impact of flavour on sales depends on the location, or the impact of location on sales depends on the flavour.)

STEP 2:

> **What is the distribution it follows?**

- Gaussian distribution.

STEP 3:

We perform Two-way ANOVA test and calculate the P-Value

Reference: [Documentation]

```
# perform two-way ANOVA

# fit an ols model on the dataframe
# use 'fit()' to fit the linear model
# ols('dependent variable ~ C(independent variable1) * (independent variable2)',
# ~ : This symbol separates the dependent variable from the independent variables
# C(): This indicates that the variable following it is treated as a categorical
# the + sign is used to include independent variables without an interaction
# the * sign is used to include independent variables with an interaction

test = ols('Sales ~ C(Flavour) * C(Location)', data=df).fit()

# create a table for a 2-way ANOVA test
# Pass the linear model 'test'
# 'typ = 2' performs two-way ANOVA
anova_table = sm.stats.anova_lm(test, typ = 2)

# Display the results
print(anova_table)
```

```
                              sum_sq    df         F    PR(>F)
C(Flavour)               6919.558981   2.0  1.968465  0.145773
C(Location)              2059.273884   3.0  0.390546  0.760092
C(Flavour):C(Location)  11802.257765   6.0  1.119163  0.357804
Residual               154669.016331  88.0       NaN       NaN
```

Brief explanation of each term represents:

1. **sum_sq (Sum of Squares):** It measures the variation in the dependent variable that can be attributed to the effect of the independent variable or factor being considered.
    - In Two-Way ANOVA, you have a sum of squares for each factor (e.g., Flavour, Location) and the interaction between them.
2. **df (Degrees of Freedom):** It represents the number of values in the final calculation of a statistic that are free to vary.
    - In ANOVA, it's associated with the factor being tested. For example, "df" for Flavour is the number of levels of Flavour minus 1.
3. **F (F-Statistic):** It's a measure of the ratio of the variance between groups (explained variance) to the variance within groups (unexplained variance).
    - A high F-value indicates a significant difference between groups, while a low F-value suggests that the groups are similar.
4. **PR(>F) (p-value):** This is the probability associated with the F-statistic. It indicates the likelihood that the observed differences in group means occurred by chance.
    - A low p-value (typically less than 0.05) suggests that the factor has a significant effect, while a high p-value suggests a lack of significance.

5. Residual: This represents the variation in the data that is not accounted for by the factors (Flavour, Location, and their interaction).
    - It is the unexplained or leftover variability after considering the effects of the specified factors.
    - We will learn more about this in future modules.

STEP 4:

We defined $\alpha = 0.05$ for confidence level 95%

The above table shows the summary of results from our Two-Way ANOVA analysis.

1. **Flavour:**
    - `PR(>F)` : The p-value is 0.145773, which is greater than the typical significance level of 0.05. As a result, you do not have enough evidence to **reject the null hypothesis**, suggesting that there is no significant effect of flavor on sales.
2. **Location:**
    - `PR(>F)` : The p-value is 0.760092, which is much greater than 0.05. Therefore, you do not have enough evidence to **reject the null hypothesis**, suggesting that there is no significant effect of location on sales.
3. **Flavour:Location (Interaction Effect):**
    - `PR(>F)` : The p-value is 0.357804, which is greater than 0.05. This suggests that there is no significant interaction in the effect between flavour and location on sales.

Overall, the data suggests that neither the choice of flavour, the location of sale, nor their interaction significantly affect the sales of the soft drinks.

KS - Test

Motivation:

Imagine you're working as a data analyst for a pharmaceutical company, and your company is researching the effectiveness of two different medicines, Medicine M1 and Medicine M2.

- You want to determine whether these two medicines have similar recovery time distributions when administered to patients.
- So, you decide to examine the distribution of recovery times for both medicines to see if there are any significant differences.

Now, you might be thinking, "Why not just use a Z-Test, like we did in other cases?"

- Well, there's a fundamental difference between the KS-Test and a Z-Test.

In a Z-Test:

- You make certain assumptions about the population distribution, such as normality, and you often need to know the population standard deviation.
- It's designed to compare sample means and is suited for situations when you have a clear understanding of the population and its parameters.

But in the case of the KS-Test:

- You don't need to make any specific assumptions about the population distribution.
  - It's **distribution-free**, which means you can use it even when you're not sure about the underlying distribution of the data.
- The KS-Test assesses the similarity of the entire distribution, not just the means.
  - It's suitable when you want to compare the overall distribution shapes and see if they're the same or significantly different.

So, Two-Sample Z-Tests and the KS-Test serve different purposes:

- A Two-Sample Z-Test is handy when you want to compare means and make specific assumptions about the population.
- But the KS-Test is your go-to when you want to check if two datasets have the same distribution, even if you're uncertain about that distribution.

In our example with Medicine M1 and Medicine M2, the **KS-Test** will help us determine whether the recovery time distributions for these two medicines are the same or significantly different without assuming a specific population distribution. This can be crucial in the pharmaceutical industry, where patient responses can vary widely.

Let's explore another statistical test called the **Kolmogorov-Smirnov Test**, often referred to as the **KS Test**.

- This test is used to determine whether two sets of data follow the same distribution or not.
- To understand this better, let's break it down into simpler terms.

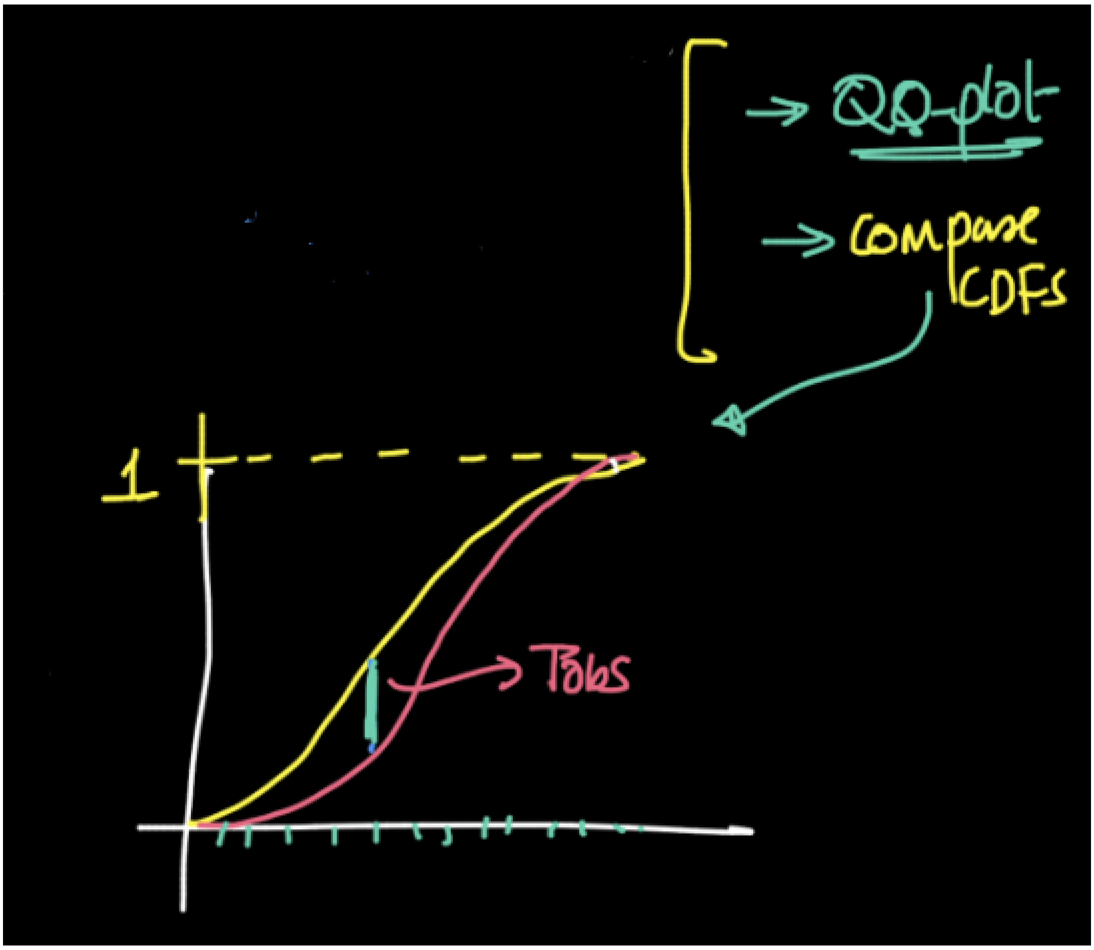
Imagine you have two sets of data, X and Y. You want to know whether they follow the same distribution.

- But first, let's start with a basic question:
  - Suppose there is x variable = $x_1, x_2 \ldots, x_n$
  - Does this follow a Gaussian (normal) distribution?

To check this, we're going to create another set of data,

- $y = y_1, y_2, \ldots, y_m$ in such a way that it has the same average (mu) as X and the same standard deviation (sigma) as X. This means that Y will have the same centre and spread as X.

Note: n and m in x_n and y_m are not to small means size of the sample x and y is not too small



Now, we want to test a hypothesis:

- Null Hypothesis (H_0): The distribution of X is the same as the distribution of Y.
- Alternative Hypothesis (H_1): The distribution of X is different from the distribution of Y.

We can use a few methods we've discussed in previous classes to compare these two distributions.

- One method is the **QQ-plot**, which helps us visualize the distributions,
- and another is to compare the **Cumulative Distribution Functions**(CDFs).

But today, we're going to focus on the Kolmogorov-Smirnov Test.

Now, here's the essential part:

- The Kolmogorov-Smirnov Test calculates a test statistic, which we'll call KS-Test.
- This statistic tells us how much the CDFs of X and Y differ from each other. In simple terms, it measures the gap between the two CDFs.

- In this case (2 sample KS test), the Kolmogorov–Smirnov statistic (T_{ks}) is given by.
- T_{ks} = $sup\ |CDF(x) - CDF(y)|$
- where $|CDF(x) - CDF(y)|$ is ab absolute value of gap between 2 CDFs.

  sup is supremum function.

Under the null hypothesis, when X and Y follow the same distribution, the test statistic (T_{ks}) tends towards zero. This is because their CDFs overlap with each other.

In contrast, **under the alternative hypothesis**, when X and Y have different distributions, the test statistic (T_{ks}) is a very large positive value. This means there's a significant difference between the two CDFs, indicating that the distributions are not the same.

Kolmogorov distribution [edit]

The Kolmogorov distribution is the distribution of the random variable

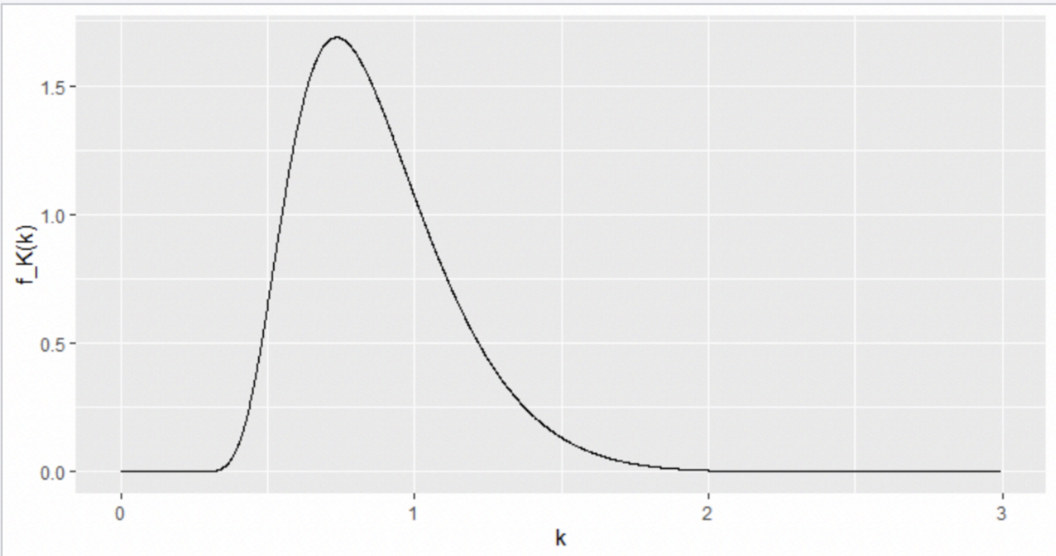
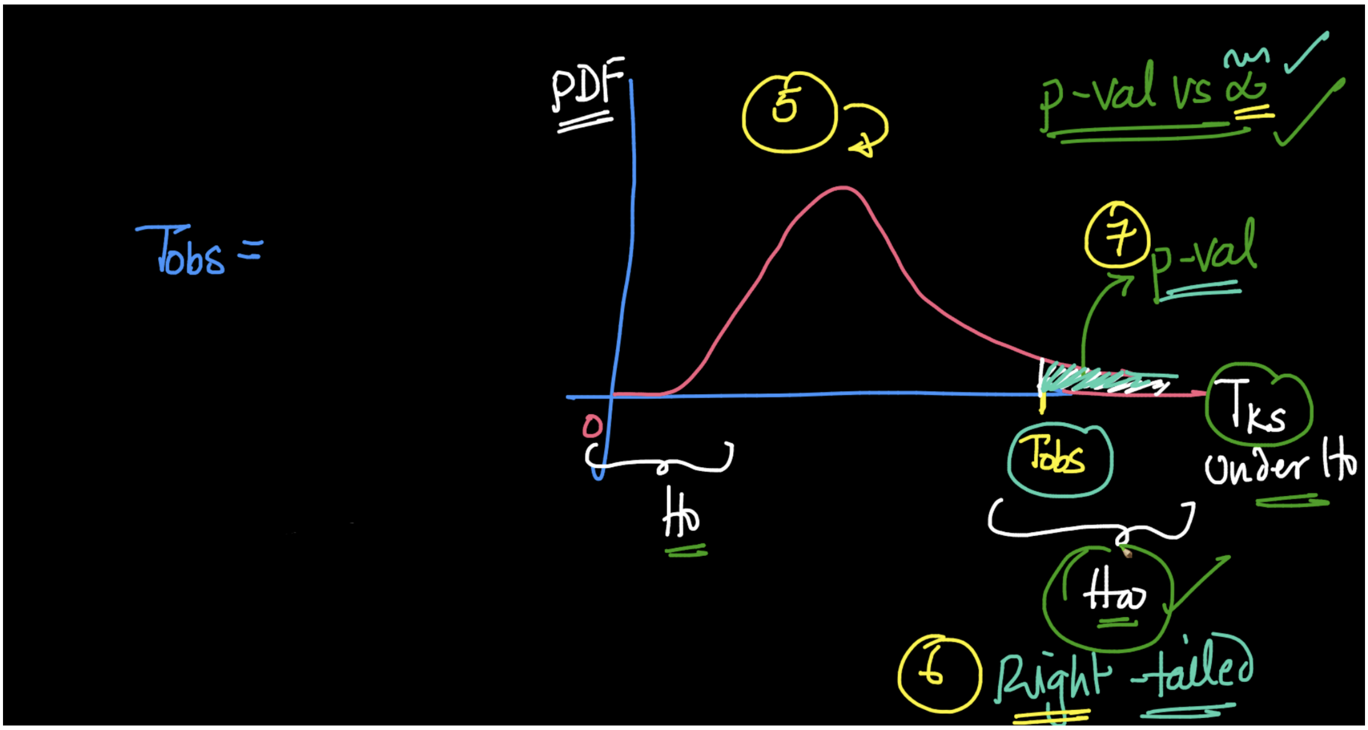


Illustration of the Kolmogorov distribution's PDF.

So, what distribution does this test statistic follow under the null hypothesis?

- It follows the **Kolmogorov distribution**. And if you look at the Kolmogorov distribution, you'll notice it has no negative values.



- The Kolmogorov-Smirnov Test is a **Right-tailed test**, which means it looks for significant differences on the right side of the distribution.

Now, let's relate this to a real-world example.

Let's take the same example of the recovery times of two different medicines, just like we discussed in our Two-Sample Z-Test lecture.

- We want to know if both medicines have similar recovery time distributions.
- This is where the Kolmogorov-Smirnov Test can help us determine if the recovery times for both medicines are drawn from the same distribution or not.

```
from scipy import stats
import numpy as np
import matplotlib.pyplot as plt
```

STEP 1:

> **What should be the null and alternate hypothesis?**

- Null Hypothesis (H_0): The distribution of sample 1 is the same as the distribution of sample 2.
- Alternative Hypothesis (H_1): The distribution of sample 1 is different from the distribution of sample 2.

STEP 2:

> **What is the distribution it follows?**

- Kolmogorov distribution .

STEP 3:

Let's take data on recovery times.

```
# recovery times of patients who took medicine-1
r1 = [8.82420842, 7.47774471, 7.55712098, 7.98131439, 6.82771606,
       7.48566433, 9.15385732, 5.84040502, 8.26124313, 8.4728876 ,
       6.82582186, 7.00490974, 8.43423058, 6.72099932, 6.97495982,
       5.93748053, 5.40707847, 6.16385557, 6.71421056, 4.42396183,
       6.87285228, 8.00313581, 6.69035041, 7.83622942, 8.70984957,
       5.56284584, 9.08093437, 4.98165193, 7.67769408, 6.04738478,
       7.64921582, 7.31051639, 6.74463303, 7.27356973, 8.16787232,
       6.90990965, 7.06439167, 6.62921957, 6.08283539, 6.2458137 ,
       8.65173634, 5.76080646, 6.20573219, 8.91561004, 6.22560201,
       5.67542104, 6.97412435, 8.31354697, 8.14172701, 8.26099345,
       7.87612791, 6.24835109, 9.95324783, 6.59504627, 6.17365145,
       6.05676895, 7.23030223, 7.71311809, 7.37163804, 5.69798738,
       5.71056902, 7.94556876, 7.47234105, 6.85346234, 4.77892053,
       6.92631063, 6.10681151, 7.06277198, 7.18023164, 7.78285327,
       7.85500885, 6.54349161, 8.25949958, 6.44289198, 7.16705977,
       6.03517015, 7.61274786, 7.032845  , 6.78161745, 7.07917968,
       6.21549342, 5.34267439, 6.73039933, 7.70562561, 8.15117049,
       6.72564324, 6.68220904, 8.50359274, 7.52912703, 7.34572493,
       5.95734283, 6.58259396, 6.49394335, 8.68069592, 8.60547125,
       6.8905056 , 7.72575925, 6.84801609, 7.96999724, 7.10420915]
```

```
# recovery times of patietnts who took medicine-2
r2 = [ 9.56597358,  7.49291458,  8.73841824,  7.63523452,  4.12559277,
        7.3679259 ,  9.87873565,  6.14516559,  8.19923821,  7.30169992,
       10.24606417,  6.83814477,  7.01611267,  6.15716049,  8.29590714,
       12.3333305 ,  8.22144016,  6.06830071,  3.75820649,  6.69220157,
       10.08721618,  9.70580422,  7.31050006, 11.40145721,  5.64818498,
        7.38914449,  8.43740074,  6.3451435 ,  7.05694361,  8.1997151 ,
        9.03059061,  7.76904679,  6.92375578,  5.78318543,  8.99027781,
        7.56186529,  5.27095372,  8.32896688, 11.52935757,  7.08119961,
        9.48825066,  9.14072759,  7.30357663,  8.62183754, 10.40999814,
        8.70096763,  7.04645384,  6.378799  , 10.5098363 ,  7.36078888,
        7.33403615,  8.07396248,  6.18309499,  7.24668404,  9.03430611,
        8.99016584,  6.78606416,  8.436418  ,  6.85877947, 10.10405772,
        6.74943076,  7.57812376,  7.12920671,  9.38065269,  9.57139966,
        6.4484012 ,  6.93877043,  9.22141667,  8.34815638,  7.73980671,
        7.17840767,  9.27913457,  6.49963224,  9.92287292,  7.63978639,
        9.53931977,  9.02602273,  6.79374185,  8.59715131,  8.37747338,
        8.78161815,  6.78716383,  8.28473394,  8.20283798, 12.50518811,
       10.19772574,  8.93758457,  8.9540311 ,  8.28927558,  6.28935098,
        7.69447559,  9.66777701, 10.33898342,  8.71199578,  5.12781581,
        9.70954569,  9.13685031,  7.28989718,  8.0868909 ,  7.42937556,
        7.31356749,  9.92345816,  8.60211814,  9.33228465,  8.14132658,
        6.17871495, 10.28358242,  7.31898597,  7.95085527,  6.20331719,
        9.19119762,  6.98600628,  7.05314883, 10.57921482,  6.83637574,
        7.86199283,  8.23350975,  5.87625665,  7.78945364,  8.83612492]
```

```
d1 = np.array(r1)
d2 = np.array(r2)
n1 = len(d1)
n2 = len(d2)
n1, n2
```

```
(100, 120)
```

STEP 4:

We perform KS-test and calculate the P-Value

```
statistic, p_value =  stats.kstest(d1, d2)
print("KS Statistic:", statistic)
print("P-value:", p_value)
```

```
KS Statistic: 0.3233333333333333
P-value: 1.516338798228849e-05
```

STEP 5:

We defined $\alpha = 0.01$ for confidence level 99%

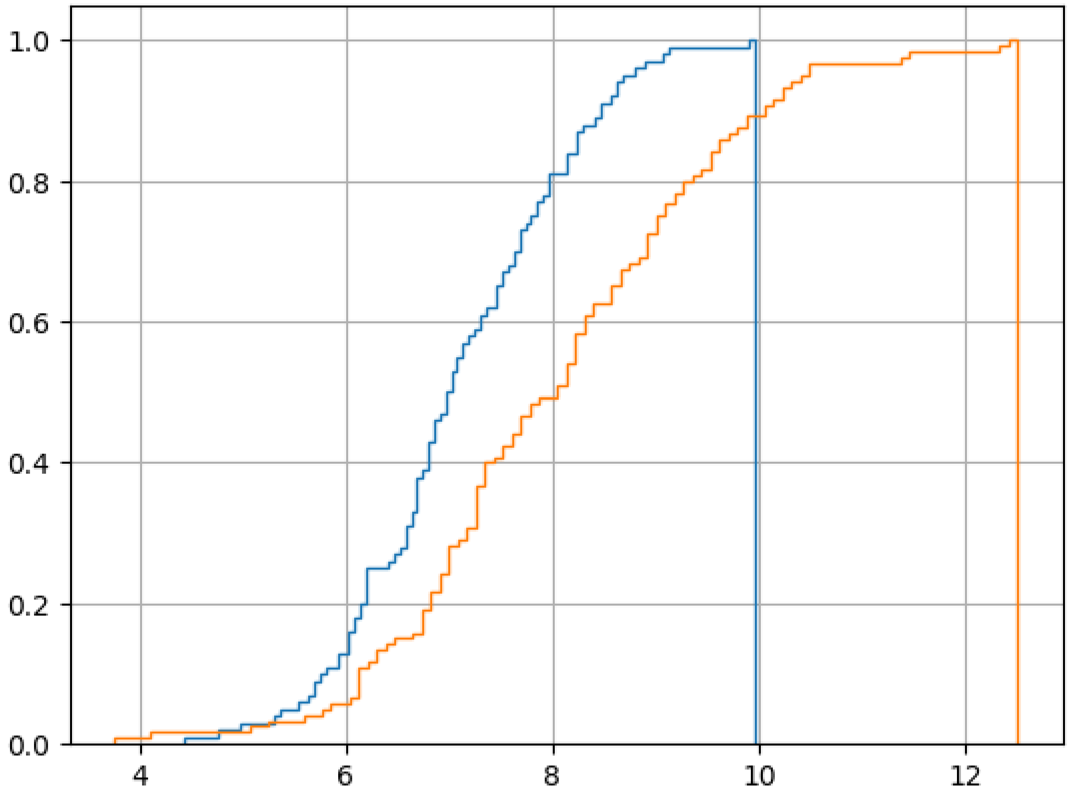
```
# Check if the p-value is greater than 0.01
if p_value > 0.01:
    print("The p-value is greater than 0.01,two samples have the same distributic
else:
    print("The p-value is less than or equal to 0.01, two samples have different
```

```
The p-value is less than or equal to 0.01, two samples have different distrib
```

STEP 6:

Plotting the data two visualise the distributions

```
plt.grid()
a = plt.hist(d1, bins=100, cumulative=True, label='CDF', density=True, histtype='
b = plt.hist(d2, bins=100, cumulative=True, label='CDF', density=True, histtype='
plt.show()
```



From the above graphs, we can see that there is a huge gap between the two distributions.

Let's take another set of data for the same use case.

STEP 1:

> **What should be the null and alternate hypothesis?**

- Null Hypothesis (H_0): The distribution of sample 1 is the same as the distribution of sample 2.
- Alternative Hypothesis (H_1): The distribution of sample 1 is different from the distribution of sample 2.

STEP 2:

> **What is the distribution it follows?**

- Kolmogorov distribution .

STEP 3:

Let's take data on recovery times.

```
data1 = [ 4.96714153,  1.38264301,  6.47688538, 15.23029856,  2.34153375,
         2.34136957, 15.79212816,  7.67434729,  4.69474386,  5.42560044,
         4.63417693,  4.65729754,  2.41962272, 19.13280245, 17.24917833,
         5.62287529, 10.1283112 ,  3.14247333,  9.08024076, 14.12303701,
        14.65648769,  2.257763  ,  0.67528205, 14.24748186,  5.44382725,
         1.1092259 , 11.50993577,  3.75698018,  6.0063869 ,  2.9169375 ,
         6.01706612, 18.52278185,  0.13497225, 10.57710929,  8.22544912,
        12.2084365 ,  2.08863595, 19.59670124, 13.28186049,  1.96861236,
         7.3846658 ,  1.71368281,  1.15648282,  3.01103696, 14.7852199 ,
         7.19844208,  4.60638771, 10.57122226,  3.4361829 , 17.63040155,
         3.24083969,  3.8508228 ,  6.76922   ,  6.11676289, 10.30999522,
         9.31280119,  8.39217523,  3.09212376,  3.31263431,  9.75545127,
         4.79174238,  1.85658977, 11.06334974, 11.96206624,  8.12525822,
        13.56240029,  0.72010122, 10.03532898,  3.61636025,  6.45119755,
         3.61395606, 15.38036566,  0.35826039, 15.64643656, 26.19745104,
         8.21902504,  0.87047068,  2.9900735 ,  0.91760777, 19.87568915,
         2.19671888,  3.57112572, 14.77894045,  5.18270218,  8.08493603,
         5.01757044,  9.15402118,  3.2875111 ,  5.29760204,  5.13267433,
         0.97077549,  9.68644991,  7.02053094,  3.27662147,  3.92108153,
        14.63514948,  2.96120277,  2.61055272,  0.05113457,  2.34587133]
```

```
data2 = [14.15370742,  4.20645323,  3.42714517,  8.02277269,  1.61285712,
         4.04050857, 18.86185901,  1.74577813,  2.57550391,  0.74445916,
        19.18771215,  0.26513875,  0.6023021 , 24.63242112,  1.92360965,
         3.01547342,  0.3471177 , 11.68678038, 11.42822815,  7.51933033,
         7.91031947,  9.09387455, 14.02794311, 14.01851063,  5.86857094,
        21.90455626,  9.90536325,  5.6629773 ,  0.99651365,  5.03475654,
        15.50663431,  0.68562975, 10.62303714,  4.73592431,  9.19424234,
        15.49934405,  7.83253292,  3.22061516,  8.13517217, 12.30864316,
         2.27459935, 13.07142754, 16.07483235,  1.84633859,  2.59882794,
         7.81822872, 12.36950711, 13.20456613,  5.21941566,  2.96984673,
         2.5049285 ,  3.46448209,  6.80024722,  2.32253697,  2.93072473,
         7.14351418, 18.65774511,  4.73832921, 11.91303497,  6.56553609,
         9.7468167 ,  7.87084604, 11.58595579,  8.20682318,  9.63376129,
         4.12780927,  8.2206016 , 18.96792983,  2.45388116,  7.53736164,
         8.8951443 ,  8.15810285,  0.77101709,  3.41151975,  2.76690799,
         8.27183249,  0.13001892, 14.53534077,  2.64656833, 27.20169167,
         6.25667348,  8.57157556, 10.70892498,  4.82472415,  2.23462785,
         7.14000494,  4.73237625,  0.72828913,  8.46793718, 15.14847225,
         4.46514952,  8.56398794,  2.14093744, 12.45738779,  1.73180926,
         3.8531738 ,  8.83857436,  1.53725106,  0.58208718, 11.42970298,
         3.5778736 ,  5.60784526, 10.83051243, 10.53802052, 13.77669368,
         9.3782504 ,  5.15035267,  5.13785951,  5.15047686, 38.52731491,
         5.70890511, 11.3556564 ,  9.54001763,  6.51391251,  3.15269245,
         7.5896922 ,  7.72825215,  2.36818607,  4.85363548,  0.81874139]


data1 = np.array(data1)
data2 = np.array(data2)
n1 = len(data1)
n2 = len(data2)
n1, n2
```

```
(100, 120)
```

STEP 4:

We perform KS-test and calculate the P-Value

```
statistic, p_value =  stats.kstest(data1, data2)
print("KS Statistic:", statistic)
print("P-value:", p_value)
```

```
KS Statistic: 0.095
P-value: 0.671374753605883
```

STEP 5:

We defined $\alpha = 0.01$ for confidence level 99%

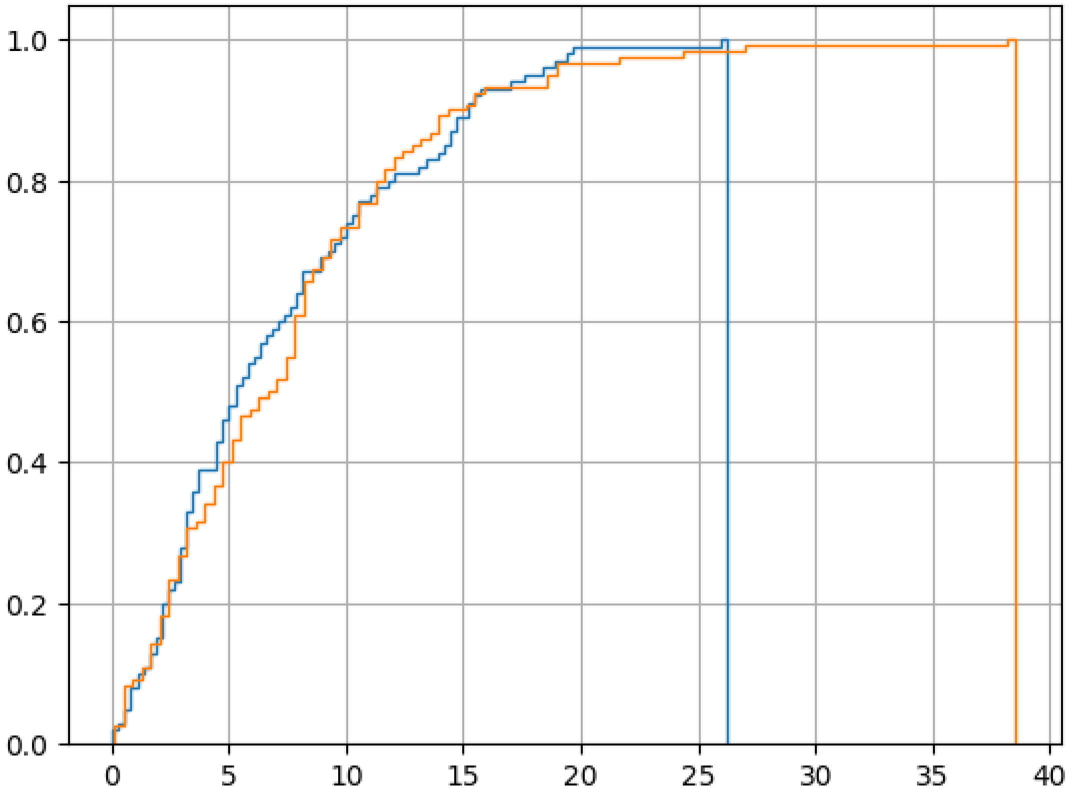
```
# Check if the p-value is greater than 0.01
if p_value > 0.01:
    print("The p-value is greater than 0.01,two samples have the same distributic
else:
    print("The p-value is less than or equal to 0.01, two samples have different
```

```
The p-value is greater than 0.01,two samples have the same distribution.
```

STEP 6:

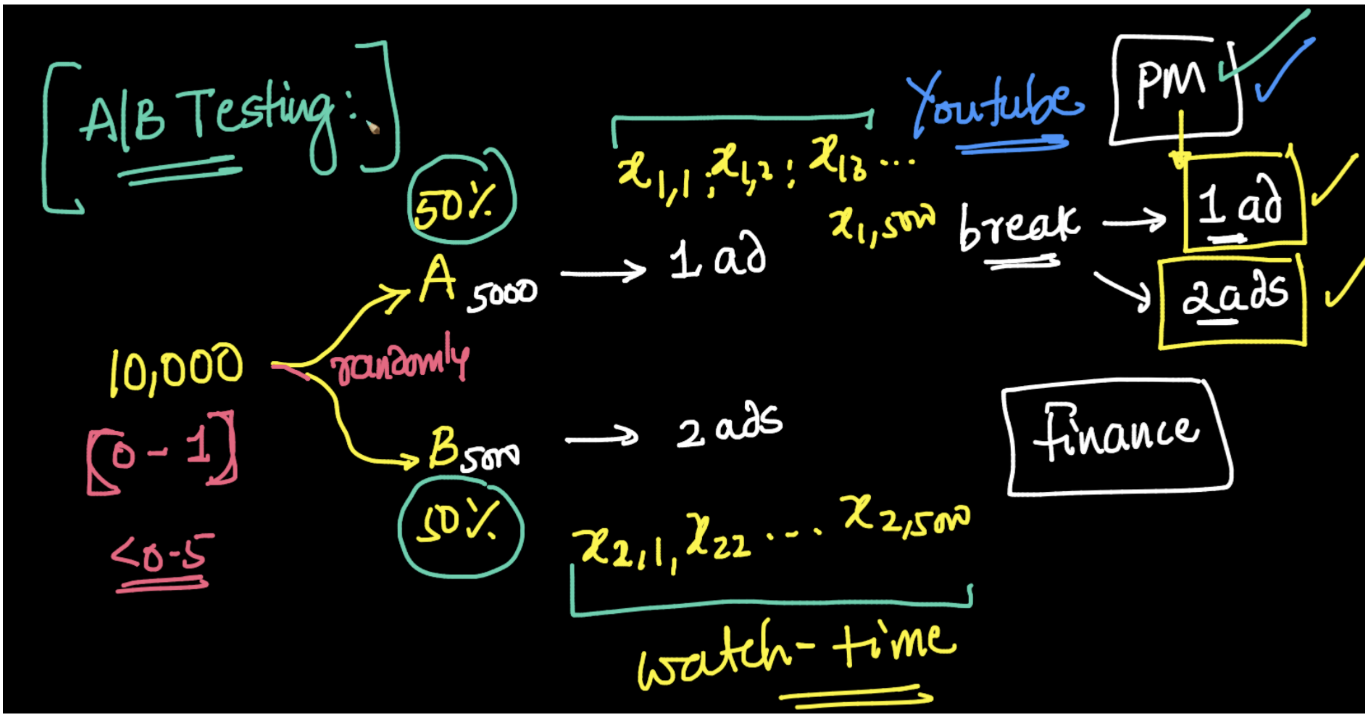
Plotting the data two visualise the distributions

```
plt.grid()
a = plt.hist(data1, bins=100, cumulative=True, label='CDF', density=True, histtyp
b = plt.hist(data2, bins=100, cumulative=True, label='CDF', density=True, histtyp
plt.show()
```



From the above graphs, we can see that there is no huge gap between the two distributions. They almost overlap.

A/B Testing



Let's say you are a data scientist at YouTube and you may want to analyze user watch time based on the number of ads in the ad break.

- Should YouTube just roll out this 2 ads per ad break feature to all its users, thinking it will generate more revenue?
- Or should it just stick to 1 ad per ad break thinking the watch time of users will drop?
  - Well, we obviously can't rely just on gut feeling.

We have to test the feature on some people before we roll it out to the masses

- Whenever we want to test the consequences of a new feature
- We usually perform tests on two groups

1. Treatment Group

- To which we introduce the new feature
- Treat them with a new feature

2. Control Group

- To which we do NOT introduce the new feature
- No new treatment

Everything else between the Treatment and Control groups remains SAME like the Day and Time of the experiment ... etc etc

- Just the new feature is introduced to the Treatment Group and NOT to the Control Group

In general, it's called Experimentation

In the "software world", we call it **"A/B Testing"**

- Group A is our Treatment Group - to which we introduce the new feature
- Group B is our Control Group - to which we DON'T introduce the new feature

Now, Let's see this with an actual example

Let's say we pick some YouTube users

- We divide the users into 2 groups **randomly** - A and B
- We show **2 ads** per ad break to **Group A** --> **Treatment Group**
- We show just **1 ad** per ad break to **Group B** --> **Control Group**

We collect data about the Mean Watch Time per day of every user in the two groups

The question that we will answer here is:

Whether YouTube should roll out the new feature of 2 ads per ad break? Or should it just stick to showing 1 ad per ad break?

What are our Null and Alternate Hypotheses?

- Null Hypothesis (H_0): Mean Watch Time of Users given 2 ads = Mean Watch Time of Users given 1 ad per ad break
- Alternative Hypothesis (H_1): Mean Watch Time of Users given 2 ads <> Mean Watch Time of Users given 1 ad per ad break

```
import seaborn as sns
from scipy import stats
```

```
!wget --no-check-certificate https://drive.google.com/uc?id=1CS513bBqabMfrUhVcqm_
```

```
--2024-01-18 09:46:42--  https://drive.google.com/uc?id=1CS513bBqabMfrUhVcqm_
Resolving drive.google.com (drive.google.com)... 142.250.128.138, 142.250.128
Connecting to drive.google.com (drive.google.com)|142.250.128.138|:443... con
HTTP request sent, awaiting response... 303 See Other
Location: https://drive.usercontent.google.com/download?id=1CS513bBqabMfrUhVc
--2024-01-18 09:46:42--  https://drive.usercontent.google.com/download?id=1CS
Resolving drive.usercontent.google.com (drive.usercontent.google.com)... 209.
Connecting to drive.usercontent.google.com (drive.usercontent.google.com)|209
HTTP request sent, awaiting response... 200 OK
Length: 883665 (863K) [application/octet-stream]
Saving to: ‘ab_test_data.csv’

ab_test_data.csv    100%[===================>] 862.95K  --.-KB/s    in 0.01s

2024-01-18 09:46:43 (78.0 MB/s) - ‘ab_test_data.csv’ saved [883665/883665]
```

```
ab_test_data = pd.read_csv("/content/ab_test_data.csv")
```

Let's check the data

```
ab_test_data.head(10)
```

	date	customer_id	premium	watch_time_hrs	customer_segmnt
0	2018-09-11	402	0	7.173618	control
1	2018-02-28	227	0	0.836170	control
2	2018-10-18	812	1	4.402078	treatment
3	2018-05-22	43	0	3.982454	control
4	2018-07-18	307	0	7.513302	control
5	2018-09-10	238	0	1.456961	control
6	2018-02-21	691	1	3.800375	treatment
7	2018-04-27	199	0	4.574446	control
8	2018-05-28	105	0	3.425942	control
9	2018-09-24	604	0	3.959896	treatment

```
sum((ab_test_data["premium"] == 0) & (ab_test_data["customer_segmnt"] == "control
```

    8459

```
sum((ab_test_data["premium"] == 1) & (ab_test_data["customer_segmnt"] == "treatme
```

    2012

```
sum((ab_test_data["premium"] == 1) & (ab_test_data["customer_segmnt"] == "control
```

    1514

```
sum((ab_test_data["premium"] == 0) & (ab_test_data["customer_segmnt"] == "treatme
```

    7975

```
ab_test_data.shape
```

    (19960, 5)

As we can see, our A/B Test Data has:

- Date on which observation was taken
- Customer ID for each user

- premium - It tells whether a customer has a premium subscription (ad-free) to YouTube or NOT.
    - 0 means the customer has NOT purchased a premium subscription
    - 1 means the customer has purchased a premium subscription
    - We will not use this attribute for our Hypothesis Testing today
- **Watch time** of each user in **hours**
- Whether the user was in **Treatment Group or Control Group**

Keep in mind that:

- Users belonging to the **Treatment Group** are shown **2 ads** per ad break
- Users in the **Control Group** are shown **1 ad** per ad break
- We have data of total 19,960 users

Let's check how many users we have in each group

> **Question:** Can anyone tell us how can we check no. of users in each of the Treatment and Control Groups?

```
ab_test_data['customer_segmnt'].value_counts()
```

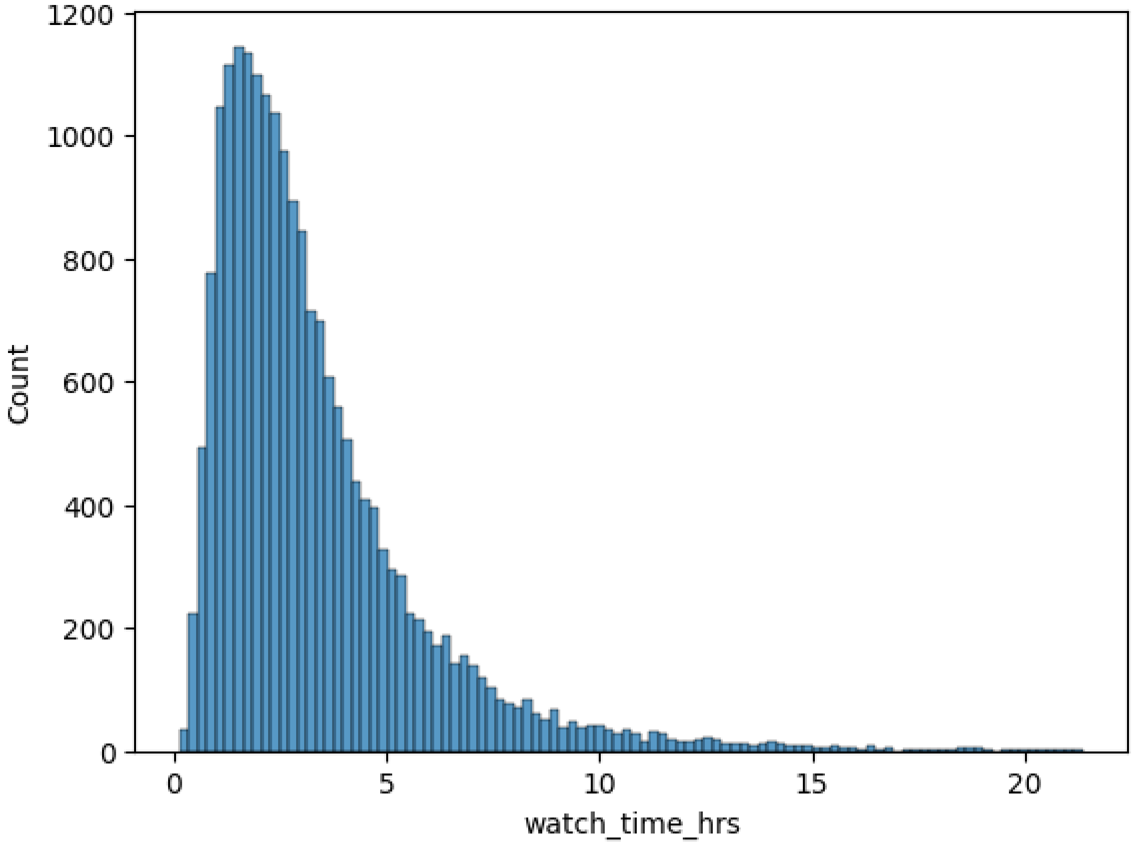
```
treatment    9987
control      9973
Name: customer_segmnt, dtype: int64
```

As you can see:

- We have the 9987 users in the Treatment Group
- We have the 9973 users in Control Group

Let's Explore the data further. Let's visualize the distribution of `watch_time_hrs` now

```
sns.histplot(ab_test_data['watch_time_hrs'], bins=100)
plt.show()
```



Does it look like a Normal Distribution?

- No, the data is right-skewed.
- We can perform Log-normal or Box-cox transformations to make the data normal. But, it's not required now.

Doing a Comparison of Means

- We are doing a Comparison of Means Hypothesis Test here Now, Let's find the mean watch time for each group (Treatment and Control)

```
ab_test_data.groupby("customer_segmnt")["watch_time_hrs"].mean()
```

```
customer_segmnt
control      3.609960
treatment    3.054294
Name: watch_time_hrs, dtype: float64
```

As you can notice:

- The mean watch time for the Control Group is 3.6 hrs
- The mean watch time for the Treatment Group is 3.05 hrs

Mean_Watch_Time (Treatment) < Mean_Watch_Time (Group)

- People in Group A who got pushed 2 ads per ad break are spending less time on average than People in Group B who watched 1 ad per ad break

> Can we say that pushing 2 ads per ad break is going to reduce the time spent by users on YouTube?

- No
- We can't just decide by looking at means

Where are the Confidence and the p-value we discussed earlier?

- We need Confidence - Need to compute it
- We can't stop YouTube from pushing 2 ads per ad break just because means are different
- There will be a loss in revenue from ads
- We need to calculate the significance of this difference as well

Is the difference in the two means (3.6 and 3.05) even Statistically Significant?

- Does pushing 2 ads make any significant difference?
- This is where the t-test comes in

Let's perform a t-test on the two means and compute confidence or p-value.

- First, we'll separate out the two groups

```
ab_test_control_data = ab_test_data[ab_test_data["customer_segmnt"] == "control"]
ab_test_treatment_data = ab_test_data[ab_test_data["customer_segmnt"] == "treatme
```

```
ab_test_control_data.shape[0]
```

```
9973
```

```
ab_test_treatment_data.shape[0]
```

```
9987
```

Now, Let's check the difference in the means of two groups

```
ab_test_control_data["watch_time_hrs"].mean() - ab_test_treatment_data["watch_tim
```

```
0.555666548844524
```

There's a difference of 0.55 hours between the mean watch times of the Control and Treatment Groups

Is this difference Statistically Significant enough for us to discard the idea of 2 ads per ad break?

- We'll conduct the independent t-test to get confidence in our result.

```
statistic, p_value = stats.ttest_ind(ab_test_control_data["watch_time_hrs"], ab_t

# The two groups are independent
# So, we'll use the method for independent t-test

print("Test Statistic:", statistic)
print("P-value:", p_value)
```

```
    Test Statistic: 15.96034913022092
    P-value: 5.438408586231319e-57
```

Question: Check the p-value and tell if can we Reject the Null Hypothesis and go with the Alternate Hypothesis.

In other words:

> **Question:** Are the Mean Watch Times Statistically Significantly Different??

Our p-value is way less than

- p-value < 0.05
- This means we have enough evidence to reject our Null Hypothesis. We can say with a confidence level of 95%
- There is a significant drop in watch time when users are shown 2 ads per ad break
- There is a Statistically Significant Difference b/w Mean Watch Times of Treatment Group and Control Group
- So, our recommendation is that YouTube should NOT roll out the feature of 2 ads per ad break :)

Also note:

- alternative argument allows us to specify whether to conduct a one-sided or two-sided t-test
- alternative argument defines the alternative hypothesis
- ttest_ind() by default does two-sided t-test
- Default value of alternative = 'two-sided'

We can change it to conduct a one-sided t-test by changing the value of the alternative agrument

- 'less': one-sided
- 'greater': one-sided

Let's see that in the code now

```
statistic, p_value = stats.ttest_ind(ab_test_control_data["watch_time_hrs"], ab_t

# The two groups are independent
# So, we'll use the method for independent t-test

print("Test Statistic:", statistic)
print("P-value:", p_value)
```

```
Test Statistic: 15.96034913022092
P-value: 2.719204293115659e-57
```

- p-value has decreased, but the t-statistic remains somewhat the same
- p-value is still < 0.05

Our rejection of the Null Hypothesis still stands.

> **For which test A/B testing is applicable?**

It is applicable to various test but in this scenario we are applying it for independent t test.

Parametric vs Non-parametric

Parametric Hypothesis Testing:

Assumptions:

- Parametric tests make specific assumptions about the population distribution from which the data is drawn.
- Common assumptions include normality (data follows a normal distribution) and variance is constant across groups or conditions.
- Parametric tests are typically used when the data reasonably follows the assumed distribution and other assumptions are met.
- Parametric tests tend to be more powerful (i.e., better at detecting true effects) than non-parametric tests when the assumptions are met.
- This is especially true when the sample size is large.

Non-Parametric Hypothesis Testing:

Assumptions:

- Non-parametric tests make fewer or no assumptions about the population distribution.
- They are distribution-free or rely on fewer assumptions, such as independence of observations.
- Non-parametric tests are useful when the assumptions for parametric tests are violated.

- They are also suitable for data types that don't fit well with parametric assumptions, such as ordinal or skewed data.
- Non-parametric tests are generally less powerful than parametric tests when data conforms to parametric assumptions.
- However, they can be more robust and appropriate when dealing with non-normally distributed data.

Let's see for each test we have learned till now, indicating whether it is parametric or non-parametric and the reasons why:

One Sample Z-Test:

- Type: Parametric
- Reason: Assumes a known population standard deviation and a normally distributed population.

Two Sample Z-Test:

- Type: Parametric
- Reason: Similar to the one-sample Z-test, it assumes known population standard deviations and normally distributed populations.

One Sample T-Test:

- Type: Parametric
- Reason: Assumes a normally distributed population but does not require knowledge of the population standard deviation.

Two Sample Independent T-Test:

- Type: Parametric
- Reason: Assumes normally distributed populations and equal variances between the two groups.

Paired T-Test:

- Type: Parametric
- Reason: Assumes normally distributed population differences.

One Sample Z-Test Proportion: