DAV-3

HYPOTHESIS TESTING

(Class starts @ 9:05 PM)

Lecture 4: T-test

#Agenda

1. Recap Z proportion Test
2. T-test
3. One Sample & Two sample T test
4. Paired T Test

Z-test Recap

Name of Test	Test Statistics.
1. One sample Z test	$Z_{st} = \dfrac{x - \mu}{\sigma / \sqrt{n}}$
2. Two sample Z test	$Z_{st} = \dfrac{m_1 - m_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

(Three sample Z test)

(Doesn't Exist)

We'll Study about this later

(Ref: ANOVA)

1 Sample Test Vs 2 Sample Test

population parameters
Vs
Sample parameter

(One Sample Test)

Ex: Shampoo Example
firm A

Sample parameter
Vs
Sample parameter

(Two Sample Test)

Ex: $M_1 = M_2$ (drug)
firm A Vs firm B
Engineer example

Z-proportion Test (Any proportion, percentage or ratio)

lets say you're a DS @ Amazon · Designing web application.

① Your PM asked you to add a new feature "to make more customer buy their product" "To increase their proportion of sales"

10

$$\text{Proportion of Sales} = \frac{\text{No of customer buy}^n \text{ the product}}{\text{No of customer visiting the web page}}$$

100

ratio

(10/100)

ratio = (10/100) ≠

One Sample Z-proportion Test

$$Z_{st} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

"ratio"

$$\frac{25}{100}$$

where

$\hat{p}$ = observed value of proportion

p = specified proportion under H_0

One-Sample Z-proportion Test

Question:

- You are a product manager for a company that has recently launched a new product.
- Customer satisfaction is a critical metric, and you want to determine if the proportion of satisfied customers with the new product meets your target satisfaction level of 70%.
- You collected a random sample of 150 customer reviews, and 115 of them expressed satisfaction with the product.

$\hat{p}$: 115/150

p : 70%

Step 1: H_0 : $\boxed{p = 70\%}$ → specified

H_a : $p \;!= 70\%$

Pon = 115/150

Step 2: Distribution/Test Statistics

Normal

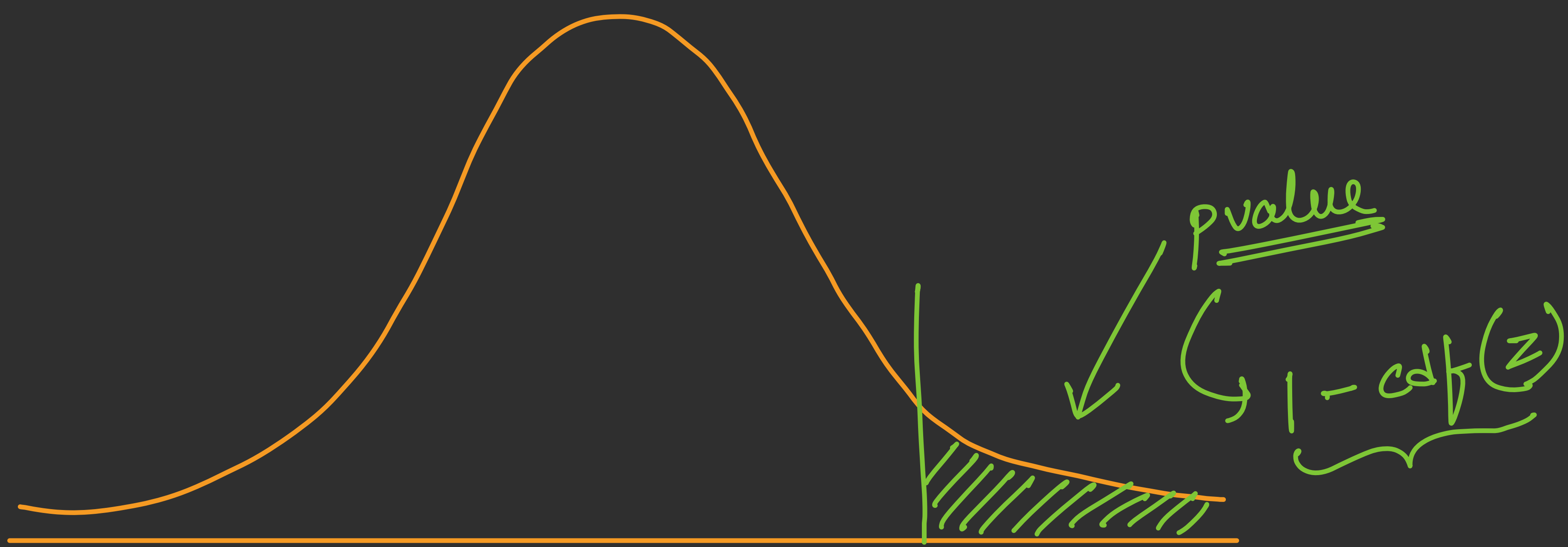
Step 3. Type of Tailed Test

Two Tailed Test

Step 4: p-value

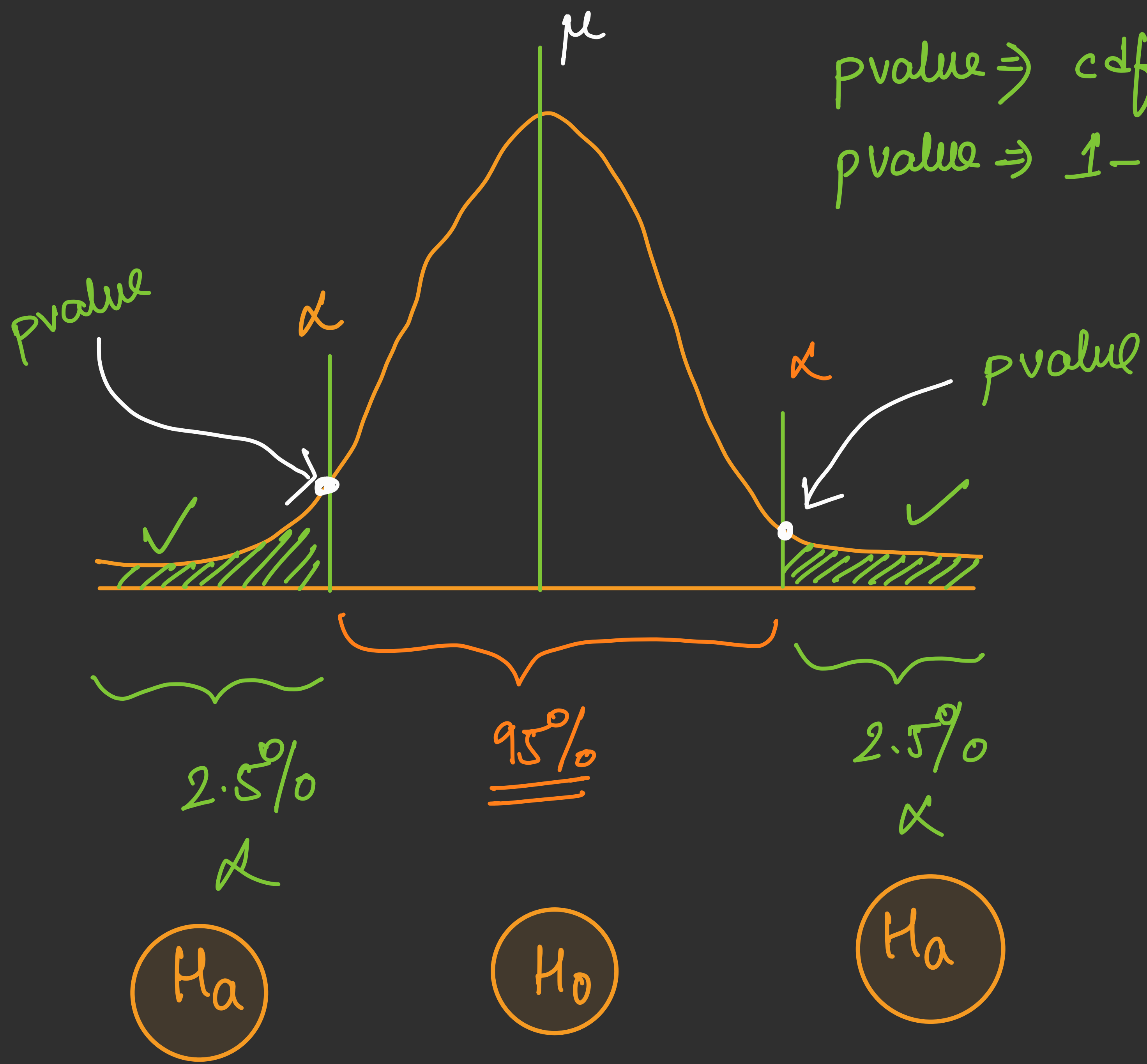
Step 5. p-value < α

$$p\text{ value} \Rightarrow \underbrace{P(\text{Data} \mid H_0 = \text{True})}$$



pvalue $\Rightarrow$ cdf(z)

pvalue $\Rightarrow$ 1 − cdf(z)



$$pvalue \Rightarrow (1 - cdf(abs(z))) \times 2$$

Formula

Two Tailed Test

$\underbrace{-ve}$ or $\underbrace{+ve}$

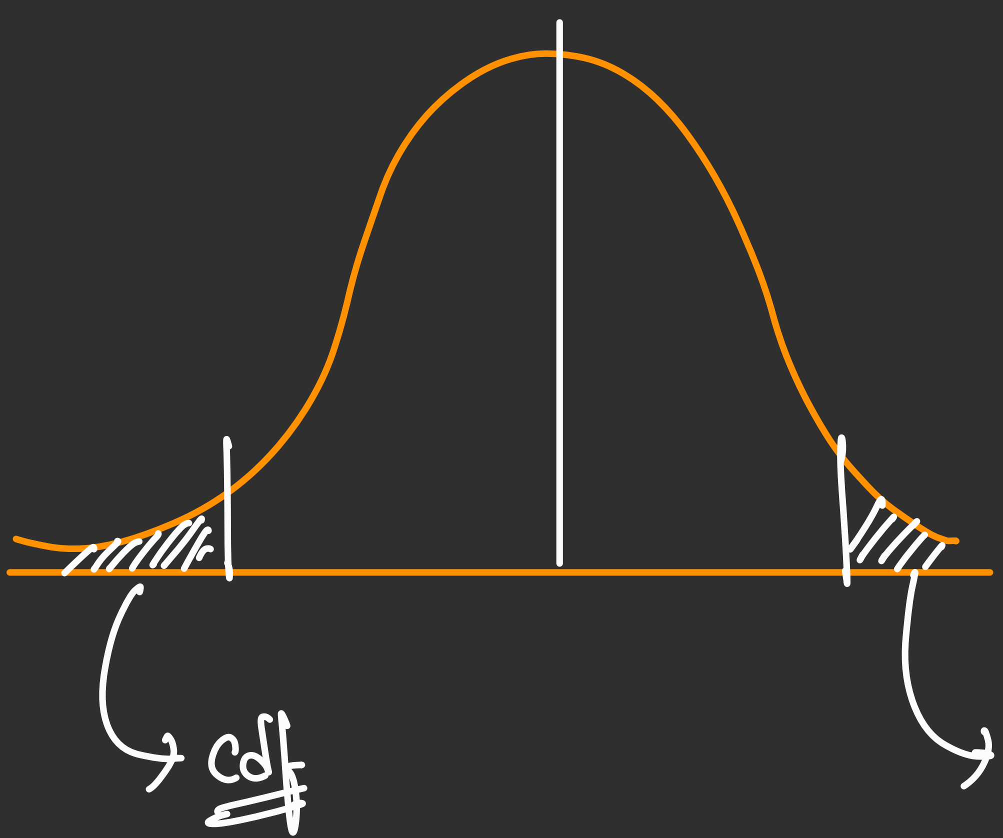
(avoid any bias that can get created by this)

$$\text{pvalue} \Rightarrow 2 \times (\text{cdf}(\text{abs}(z)))$$

$$(1 - \text{cdf}) \times 2$$

$$(1 - \text{cdf} + 1 - \text{cdf})$$

$$2 - 2\text{cdf}$$



Two Sample Z-proportion Test

Online Retail Store → keen to optimize web pages → Boost Sales

→ user engagement

↳ "In e-commerce,

Metric : "Conversion Rate"

↳ It measures the proportion of visitors who make a purchase

(Aka sales conversion rate) ✓

100 → 25 purchase

$$\text{Conv Rate} = \frac{25}{100} = 25\%$$

Retail website

① A: old webpage $\longrightarrow$ conv rate$_1$ = $\frac{\text{conversions}_1}{\text{visits}_1} = \hat{p}_1$

25

100

1000

② B: new webpage $\longrightarrow$ conv rate$_2$ = $\frac{\text{conv}_2}{\text{visit}_2} = \hat{p}_2$

500

$\frac{10}{50}$

1000 & 500 are your sample size

A B

Conditions for Z-proportion Test

① Sample size ≥ 30 ✓
② Data should be normally distributed ✓ } CLT

$\left(\hat{p} = x/n\right)$

Test Statistics (2 sample Z-proportion Test)

$$\boxed{Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}}$$

$$Z_{st} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Under $H_0 \longrightarrow Z_{st} \rightarrow (\hat{p}_1 = \hat{p}_2)$ $\underline{\sim}$ $\underline{\underline{0}}$

Under $H_a \longrightarrow Z_{st} \rightarrow (\hat{p}_1 \neq \hat{p}_2) \longrightarrow$ $\underbrace{\text{small/large}}$ difference

$$\hat{p} = \frac{conv_1 + conv_2}{visit_1 + visit_2}$$

$\rightarrow$ Total Conv Rate

Question:

You are the manager of an e-commerce website, and you have recently implemented a new web page in hopes of increasing sales.

To evaluate the effectiveness of the new page, you collected data on the conversion rates for both the old and new web pages. The conversion rate is defined as the proportion of visitors who make a purchase.

- For the old web page (Web Page A), you had **1000** visitors, resulting in **50** conversions.
- For the new web page (Web Page B), you had **500** visitors, resulting in **30** conversions.

Now, you want to determine if there is a statistically significant difference in the conversion rates between the old and new web pages.

Step 1: $H_0 \Rightarrow \hat{p}_1 = \hat{p}_2$

$H_a \Rightarrow \hat{p}_1 \neq \hat{p}_2$

Step 2: Distribution? (Normal)

Step 3: Type of Tailed Test (Two Tailed)

Step 4: pvalue

Step 5: pvalue $< \alpha$

T- test

Let's say you are a Research Scientist working on a new cognitive enhancement pill

- The goal is to develop a pill that can significantly improve IQ scores in individuals.
- The researchers believe that the new pill will lead to a significant increase in average IQ scores for the population.

Ho

Case I : No Impact (IQ remains same)

popln size is < 30
can't use z test
t-test comes into picture
↳ size < 30

Ha

Case II : IQ Increases }

Q) Why NOT Z-Test? [Interview Questions]

① Senario Complexity

→ In real life, you often won't have the pop^n std.

$$\boxed{Z_{st} = \frac{x - \mu}{(\sigma/\sqrt{n})}}$$

$\sigma \rightarrow pop^n$ std

② Sample size < 30

z test when population SD given & sample size was > 30

in t-test when population SD is not given & sample size is less than 30

Use Case : Improve IQ with pill

① Given Data:

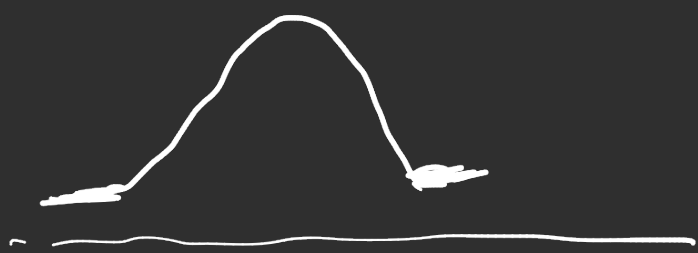
Avg pop^n IQ : 100

pop^n std $\longrightarrow$ unknown

Collected data: [102, 98, 115, 104, 96, 82, 76, 102, 180]

Step 1 : $H_0 : \boxed{\mu = 100}$

$H_a : \boxed{\mu > 100}$



Step 2 : Distribution

Not Normal

(T-distribution)

$$Z = \frac{x - \mu}{\sigma / \sqrt{n}}$$

$\sigma \longrightarrow X$

popln SD

sample SD

Types of T-test (Student T Test)

1. With 1 sample,
   → With 1 sample, you have bunch of samples you are comparing with a single number (pop mean)
2. With 2 samples where samples are "independent"
3. With 2 samples where samples are "dependent".

→ Paired T-Test

2 Sample T-Test

We are comparing the means in t-test

Test Statistic for one-sample T test

$$\boxed{T_{st} = \frac{x - \mu}{s/\sqrt{n}}}$$

$\begin{cases} n = \text{sample size} \\ s = \text{sample std deviation} \end{cases}$

sample std

sample std error

$$SE = \frac{SD}{\sqrt{n}}$$

$\left(\frac{\sigma}{\sqrt{n}}\right)$ Standard Error

$\left(\frac{s}{\sqrt{n}}\right)$ Sample Standard Error

Two-sample T-Test
(Independent)

$M_1 == M_2$

$$\boxed{T_{st} = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

n_1 :
n_2 : } Sample of each grou

x_1 :
x_2 : } Sample mean of the sample data of each group

μ_1 :
μ_2 : } Pop^n mean of each grop

s_1
s_2 } Sample std of each group

Two sample T-test

Two Samples

M_1 Vs M_2

Students Vs Students

IQ_1 IQ_2

$$T_{st} = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1^2} + \frac{s_2^2}{n_2^2}}}$$

Option 1

$H_0: \mu_1 = \mu_2$

$H_a: \boxed{\mu_1 \neq \mu_2}$

(Two tailed)

Option 2

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

(Right tailed)

Option 3

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 < \mu_2$

(left tailed)

(Reject H_0) (Accept H_0) (Reject H_0)

min test $\Rightarrow$ "2"

1) Start with a two tailed test (If Nothing is mentioned in question)

2) Then perform any <u>one tailed test</u>

Paired T-test

1) Two sample T-test

The samples are independent

$\boxed{IQ_1} \Rightarrow$ [100, 92, 82, 68, 110, ...]

$\boxed{IQ_2} =$ [110, 36, 89, 116, 112 ...]

(independent)

Paired T-test (Dependent Test)

		(Before) Module Test	Revision Module	(After) Module Test
x_1	Noreen	82		84
x_2	Suman	74		72
x_3	Jiyansh	68		66 = 68
x_4	Shashank	92		95

We are taking before and after of the same sample

One sample Z test

$$Z_{st} = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$$

One sample T-test

$$T_{st} = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$$

σ Vs s

[If you have a large sample, your T-test will behave like z-test]

$s \simeq \sigma$ (Under what situation this can happen)

(Law of large No)

$n \uparrow$ SE $\downarrow$ (σ, μ)

SS > 30

↳ T-test

SS < 30

↳ T-test