DAV-3

HYPOTHESIS TESTING

(Class starts @ 9:05 PM)

Lecture 3: Z-Test Continued…..

Class starts at 9:05 PM

#Agenda

→ Quick Info

1. Confidence Interval ✓
2. Power of Test ✓
3. Two sample Z-Test ✓
4. Z-propoportion Test ✓
   - One sample Z-prop Test
   - Two sample Z-prop Test
5. T-test ✓ → for the next class

Recap

1) One sample Z-test

Distribution / Test Statistic

$$Z = \frac{x - \mu}{(\sigma/\sqrt{n})}$$

"Test Statistic"

$\checkmark\checkmark$ Firm A

pop^n details

2000 Stores

$\mu = 1800$

$$\left\{ \begin{array}{l} \mu_s = 1850 \\ n = 50 \text{ stores} \end{array} \right.$$

One sample Z-test

$H_0: \mu = 1800$

$H_a: \mu > 1800$

pop^n para Vs (Sample para One)

Ways to Accept / Reject H_0 }

1) pvalue

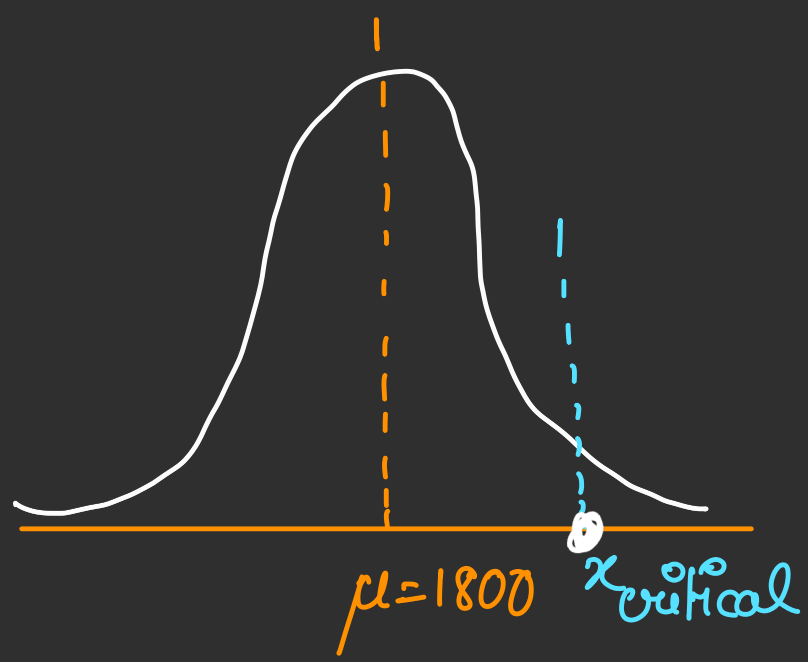
2) $x_{critical}$ value method

$x_{critical} \rightarrow$ $Z_{critical}$

$\downarrow$

norm.ppf(CL)

$$Z_{critical} = \frac{x - \mu}{(\sigma/\sqrt{n})}$$



ztest → p-value

if p-value < α

"Reject null"

else "fail to Reject Null"

Recap

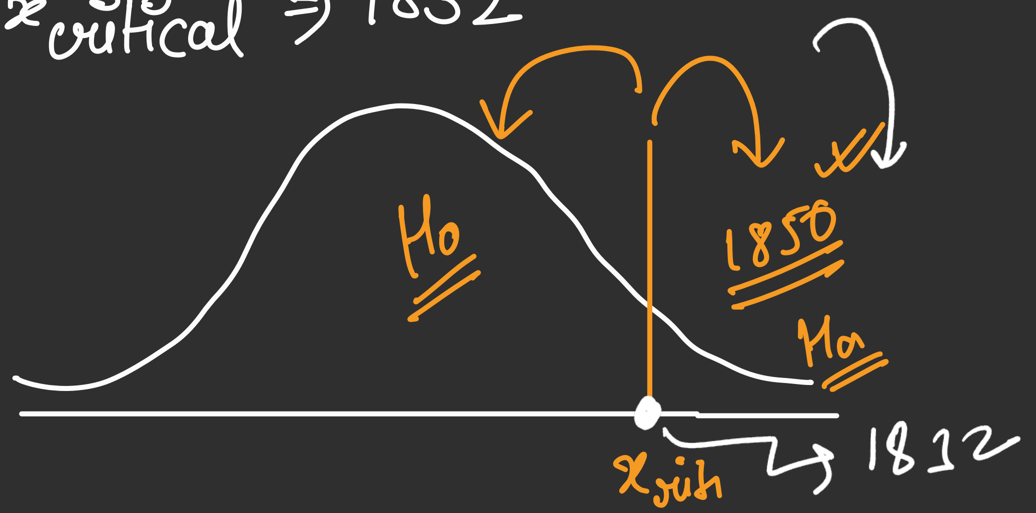
Firm A

$\mu = 1800$

sample mean = 1850

sample = 50

$x_{critical} \Rightarrow 1832$



Firm B

$\mu = 1800$

sample mean = 1900

sample = 5

$x_{critical} \Rightarrow 1903$

$1850 > 1832$

Our firm A is doing well

Confidence Interval

CLT → gives you point estimate

Firm A capable of driving Result? (What do you think?)

I will get $\boxed{80}$ marks $\boxed{80 \ \& \ 90}$

$\boxed{\text{Confidence Interval} \Rightarrow \text{point estimate} \pm \text{Margin of Error}}$

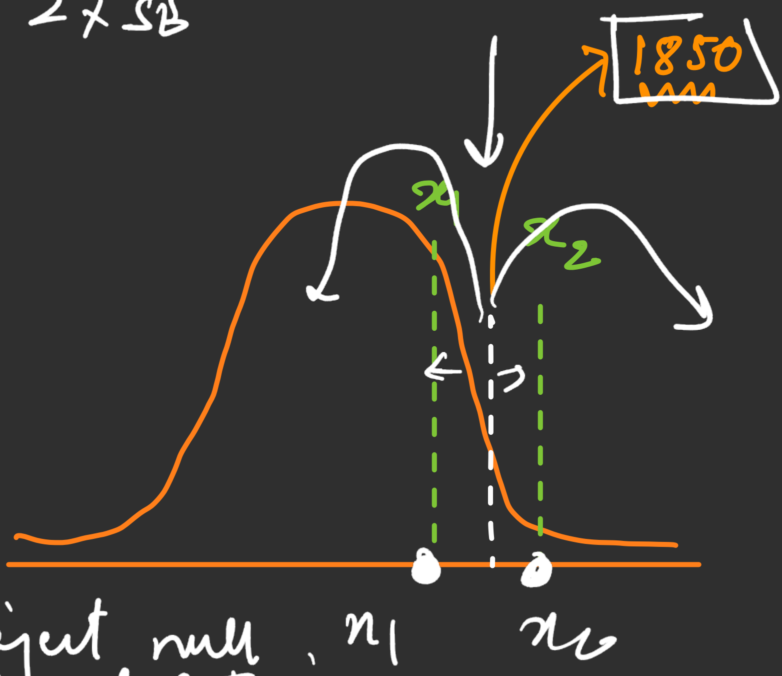
$\downarrow$

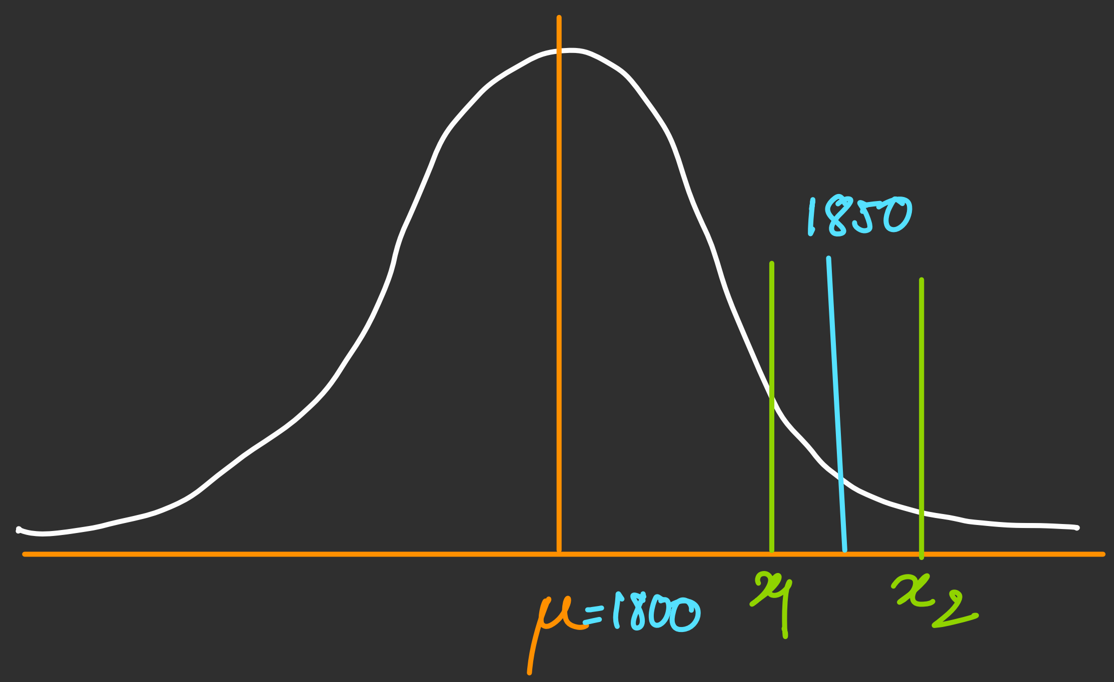
$2 \times SE$

$\left[\text{sample mean} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)\right]$

Goal ⇒ To find x_1 & x_2

If popln mean lies between x_1 & x_2 then you fail to reject null hypothesis




1850
μ=1800
x1
x2

For Example

given information

pop_mean = 1800

sample mean = 1850

pop_std = 100

sample size = 50

$\alpha = 0.01$

① Assumption

$H_0: \mu = 1800$

$H_a: \mu > 1800$

② Distribution (Test Statistic) = Normal

③ Type of Tailed Test $\longrightarrow$ Right Tailed Test

④ Calculate Confidence Interval

$\hookrightarrow$ sample mean $\pm$ z $\left(\frac{\sigma}{\sqrt{n}}\right)$ $\rightarrow$ norm.ppf $(1-\alpha)$

$1-\alpha$

$1-5\% \rightarrow 95\%$

$1\% -$

Power of Test

(Judge in the court Example)

Actual

Predicted

	Actually the Person is....	
	Innocent 0	Guilty
Innocent 0	TN (00)	FN (10)
Guilty	FP (01)	TP (11)

Terms

TN → True Negative

TP → True Positive

FN (β) → False Negative

FP (α) → False Positive

Rejecting H_0 (calling them guilty) ⟶ "Positive" 1

Failing to Reject H_0 (calling them Innocent) ⟶ "Negative" 0

H_0 is false

H_1 is true

	Actually the Person is....	
	H_0 is True	H_0 is False
Not Rejecting	$1-\alpha$	β
Rejecting	α 01	$1-\beta$

H_0 (sig level)

$\beta \rightarrow$ FN (Power)

$\alpha \rightarrow$ FP

$1-\beta \rightarrow$ TP

H_0: Person is Innocent

H_a: Person is guilty

Statistical (Power)

Innocent, Guilty

$\boxed{H_a \text{ or } H_1 \text{ is true}}$

$\alpha + TN = 1 \quad TN = 1-\alpha$

$TP + FN \Rightarrow \beta + 1-\beta = \underline{\underline{1}}$

$\beta = P(\text{fail to reject } H_0 | H_1 \text{ is true})$

Goal: To reduce β. In other words, we can say to increase $\underline{\underline{TP}}$ $(1-\beta)$

$$\boxed{P(FN \mid H_1 \text{ is true}) + P(TP \mid H_1 \text{ is true}) = 1}$$

$\beta \quad + \quad TP = 1$

$TP = 1-\beta$

Actual

		H_0 is True	H_0 is False (H_1 is true)
Predicted	Not Reject H_0	TN $1-\alpha$	$\boxed{FN}$ β → Type 2 error
	Rejecting H_0	FP (α)	TP ($1-\beta$)

$(1-\beta)$ is power of test

when H_1 is true and you do not reject H_0

$$\beta + TP = 1$$

$$\underbrace{P(FN \mid H_1 \text{ is true})} + P(TP \mid H_1 \text{ is true}) = 1$$

$$\beta + TP = 1$$

$$TP = 1-\beta$$

If $\boxed{H_1}$ is true, Two decisions

1) fail to reject H_0 → FN

2) reject $\cancel{H_0}$ → TP

Failing to reject H_0 when H_a is true → FN (Type 2 error)

What is the relationship b/w α & β?

$FP(\alpha) \uparrow \downarrow$

$FN(\beta) \downarrow \uparrow$

$\beta \rightarrow$ Type 2 Error

$\alpha \rightarrow$ Type I Error

$\alpha \rightarrow FP$

$\beta \rightarrow FN$

$$\boxed{\left\{ \alpha \propto \frac{1}{\beta} \right\}}$$

Hence we always need to find a trade off b/w them.

Factors influencing Power

$(1-\beta) \longrightarrow$ Power of Test

1. Sample size (n) ↑ (Better the power)
2. Significance level (α) ↑ → ~~Reducing~~ Power (β) ↓
3. Variability (σ) ↑ Power (β) ↓ FN
4. $\boxed{\text{Effect size } (d)}$

$\sqrt{\text{variance}}$

SD

Variance

SE ↑

$$\boxed{\alpha\uparrow \quad \beta\downarrow \quad (1-\beta)\uparrow}$$

$$\alpha\uparrow \quad \beta\downarrow \quad (1-\beta)\uparrow$$

Effect size (d)

Diet plan → A

1 ↓ 2 ↓ 3 ↓ 4 ↑ 5 ↓ 6 ↓

↓

It captures the $\underbrace{\text{magnitude}}_{\text{avg}}$ of the change

Effect size captures sense of direction

1 - 6 → reduce by 1 kg

4 → increase by 10 kg

1 ↓ 2 ↓ 3 ↓ 4 5 ↓ 6 ↓

B, C, D

-5, -4, -3, -2, -1, +100

Passing marks → $\underline{\underline{40}}$

1	2	3	4
(42)	(83)	(72)	(85)
P	P	P	P

Cohen's d formula

$$\text{Effect size } (d) = \frac{\boxed{\text{sample mean}} - \boxed{\text{pop}^n\text{ mean}}}{\boxed{\text{sample std}}}$$

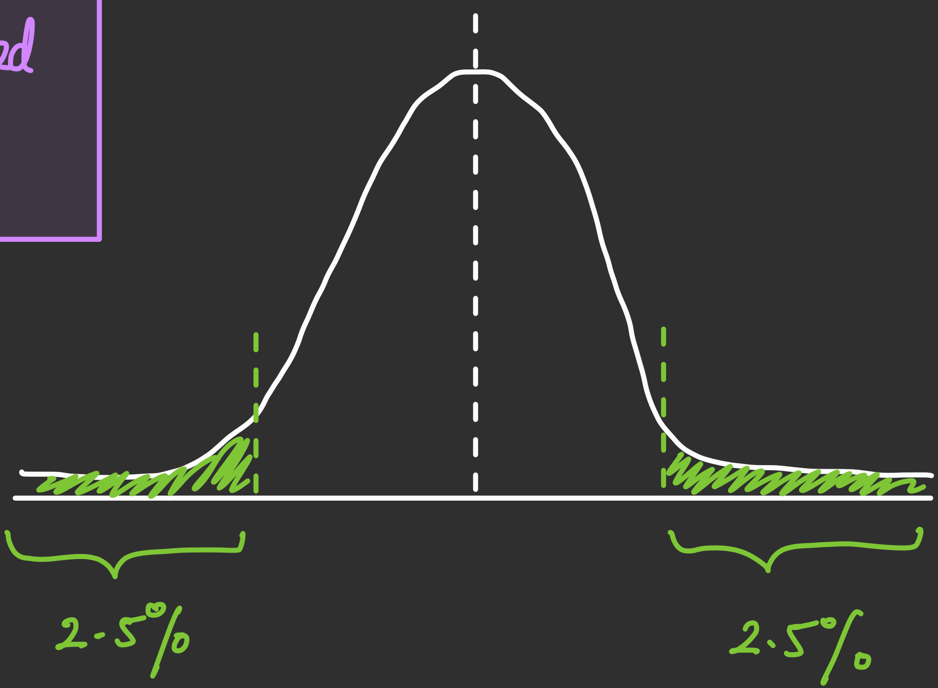
sample std → Coming from sample

→ (s)

$(\sigma/\sqrt{n})$ ✗

$$\text{Z score} \Rightarrow \frac{\text{sample mean} - \text{pop}^n\text{ mean}}{\boxed{\dfrac{\text{pop}^n\text{ std}}{\sqrt{ss}}}}$$

Q) What will be α for a two tailed test?



$\alpha = 5\%$

$$CL = 1 - (\alpha/2)$$

2 samples

Two sample Z-Test

there is no population

In one sample z test we were comparing sample mean with popn mean

1 Sample Test Vs 2 Sample Test

During the cov'd

M_1 Pfizer → [S_1, S_2 -- S_{10}]

New Covidchild → [X_1, X_2 -- X_{10}]

population parameters
Vs
Sample parameter

(One Sample Test)

Ex: Shampoo Example

firm A

Sample parameter
Vs
Sample parameter

(Two Sample Test)

Ex: $M_1 = M_2$ (drug)

firm A Vs firm B

Two sample Z-Test

ICMR, WHO, FDA

$\boxed{M_1 \& M_2}$ (Covid Medicine)

$\left\{ \begin{array}{l} \text{Medicine } (M_1) = m_1 \\ \text{Medicine } (M_2) = m_2 \end{array} \right.$ (Avg recovery time)

We need to find which medicine give quick recovery?

Goal: To check whether RT for M_1 & M_2 are same or not...

① Assumption

$H_0: \boxed{m_1 = m_2}$

$H_a: \boxed{m_1 \neq m_2}$

② Distribution (Test Statistic)

Normal

③ Type of Tailed Test

$\boxed{\text{Two Tailed Test}}$

4)

Test Statistics

1 sample

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Z score

2 sample

$$z = \frac{\boxed{m_1 - m_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

1 2

m_1 & m_2 → sample means

σ_1 & σ_2 → pop^n std / sample std

n_1 & n_2 → sample size

$M_1 - M_2 = 0$ z score $= 0$ null hypothesis is true

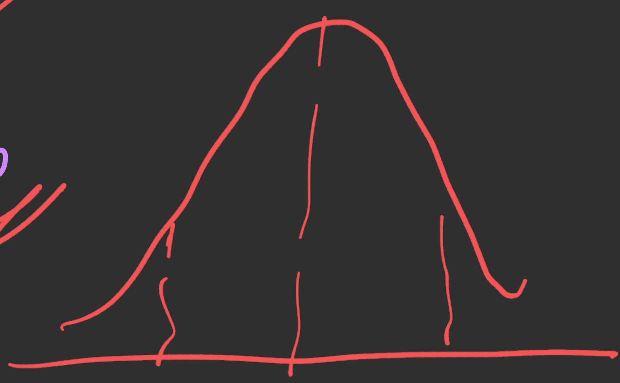
For M_1 (100 Patients)

$$\left\{ \begin{array}{l} x_1 \longrightarrow RT_1 \\ x_2 \longrightarrow RT_2 \\ \vdots \\ x_{100} \longrightarrow RT_{100} \end{array} \right.$$

For M_2 (90 Patients)

$$\left\{ \begin{array}{l} x_1 \longrightarrow R_{t_1} \\ x_2 \longrightarrow RT_2 \\ x_3 \longrightarrow RT_3 \\ \vdots \\ x_{90} \longrightarrow RT_{90} \end{array} \right.$$

$x_1 \neq x_1$



Test Statistic

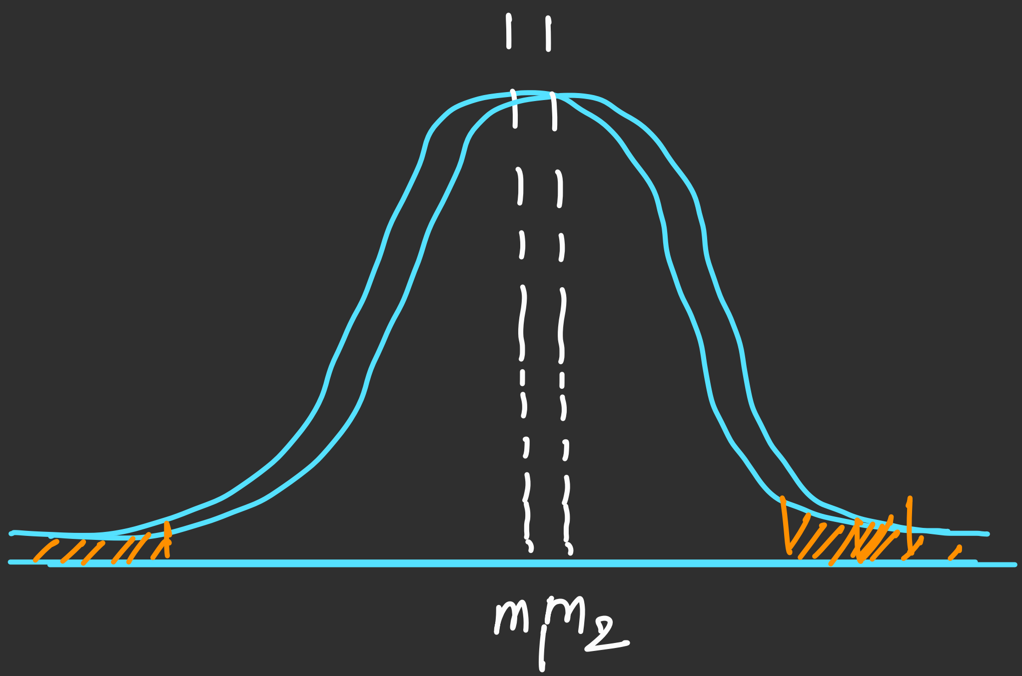
$$\boxed{z = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

Under H_0 → value of TS

↳ $\simeq 0$

$p_{value} \Rightarrow 2 \times \left(1 - \text{norm.cdf}(z)\right)$

compare pvalue with α



$$Z_{value} = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \Rightarrow \text{one value}$$

Till Now

1) One Sample Z test ⇒ (Pop para Vs Sample para)

$$z = \frac{x - \mu}{\alpha / \sqrt{n}}$$

2) Two Sample Z test ⇒ (Sample para Vs Sample para)

$$Z = \frac{m_1 - m_2}{\sqrt{\frac{a_1^2}{n_1} + \frac{a_2^2}{n_2}}}$$

Z-proportion Test

lets say you're a DS @ Amazon · Designing web application.

① Your PM asked you to add a new feature "to make more customer buy their product" "To increase their proportion of Sales"

z proportion test

$$\text{Proportion of Sales} = \frac{\boxed{\text{No of customer buy}^n \text{ the product}}}{\text{No of customer visiting the webpage}}$$

ratio

100 − 10

(10/100)

Proportion of Sales → 10/100

One Sample Z-proportion Test

$$Z_{st} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

"ratio"

$$\frac{25}{100}$$

where

$\hat{p}$ = observed value of proportion

p = specified proportion under H_0

One-Sample Z-proportion Test

Question:

- You are a product manager for a company that has recently launched a new product.
- Customer satisfaction is a critical metric, and you want to determine if the proportion of satisfied customers with the new product meets your target satisfaction level of $\boxed{70}$%.
- You collected a random sample of 150 customer reviews, and 115 of them expressed satisfaction with the product.

$\hat{p}$: $\boxed{115/150}$

p : 70%

Step 1: H_0 : $\boxed{p = 70\%}$

H_a : $p \; \boxed{!=} \; 70\%$

Step 2: Distribution / Test Statistics

Normal

Step 3. Type of Tailed Test

Two Tailed Test

$\boxed{\textbf{Step 4:} \quad \text{p-value}}$ → 2 [illegible]

Step 5. $\boxed{\text{p-value} < \alpha}$

$$\text{pvalue} \Rightarrow \left(1 - \text{cdf}(\text{abs}(z))\right) \times 2$$

Two Tailed Test

Formula

Burger

process A

Before / Existing

New Eng

Improved process

process B

After / To be

H_0: No change in process

H_a: Change in process ($pA \neq pB$ / $pA > pB$ / $pA < pB$)

[Old Burger ⇒ X]

New Burger

88%

New

Claim

Evidence

(85/100)

$ML_1 \longrightarrow$ Existing

$ML_2 \longrightarrow$ New

$$\begin{cases} H_0 : & ML_1 == ML_2 \\ H_a : & ML_2 > ML_1 \end{cases}$$

$CL = \underbrace{0.96}$

mean = 250

std = 30

$$1 - \overbrace{\underbrace{(1 - CL)}_{\alpha}}/2$$

$CL = 1 - \alpha$

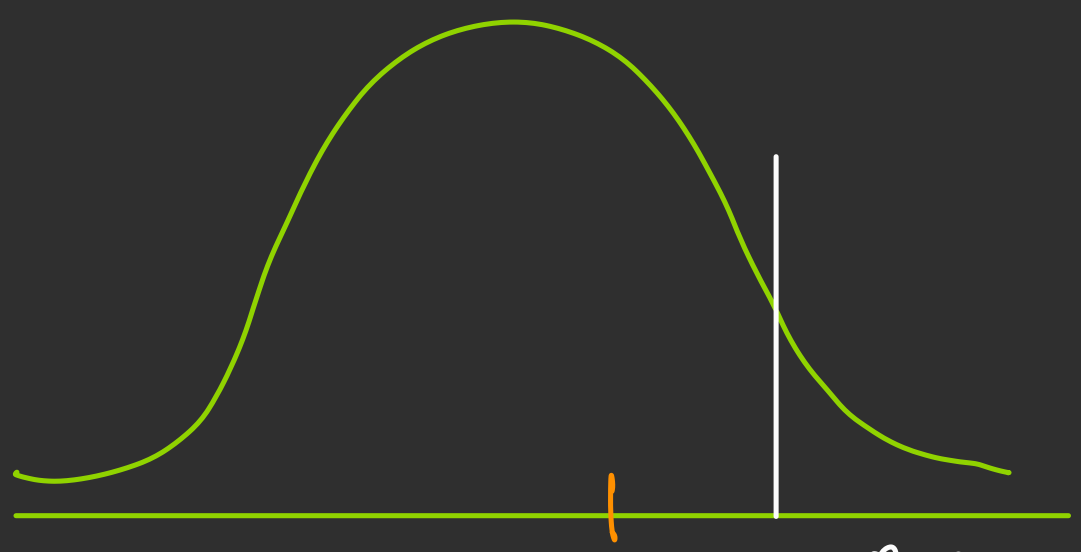
$\alpha = \underline{\underline{1 - CL}}$

$Z_{critical} \Rightarrow$ norm.ppf$(1 - \alpha/2)$

$\underline{\underline{x}}$ ~~α~~

$$Z_{crit} = \frac{x - \mu}{(\sigma/\sqrt{n})}$$

$n = 1$



Zcore < Zcritical ✓✓

xvalue

xcritical

Two Sample Z-proportion Test

Online Retail Store → keen to optimize web pages → Boost Sales

→ user engagement

"In e-commerce, Metric: "Conversion Rate"

It measures the proportion of visitors who make a purchase (Aka sales conversion rate) ✓

100 → 25 purchase

$$\text{Conv Rate} = \frac{25}{100} = 25\%$$

Retail website

1 A: old webpage $\longrightarrow$ conv rate$_1$ = $\frac{\text{conversions}_1}{\text{visits}_1}$ = $\hat{p}_1$

25

1000

100

2 B: new webpage $\longrightarrow$ conv rate$_2$ = $\frac{\text{conv}_2}{\text{visit}_2}$ = $\hat{p}_2$

500

$\frac{10}{50}$

1000 & 500 are your sample size

A B

Conditions for Z-proportion Test

1. Sample size ≥ 30 ✓
2. Data should be normally distributed ✓

CLT

$(\hat{p} = x/n)$

Test Statistics (2 sample Z-proportion Test)

$$\boxed{Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

Ho :- Project not impactful

Ha :- Project impactful

goal → Reject Ho when there is strong evidence that project is impactful

Reality	Decision	Error
Project not impact	Approve project	FP
Project impactful	Stopped project	Type II FN
Project impact	Approve Project →	TP

$\alpha = 0.02$ (easy to approve)

fewer impactful project min (more hard projects can be approved)

$$\boxed{Z_{st} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}}$$

Under $H_0 \longrightarrow Z_{st} \rightarrow (\hat{p}_1 = \hat{p}_2) \;\; \underline{\sim} \;\; \underline{\underline{0}}$

Under $H_a \longrightarrow Z_{st} \rightarrow (\hat{p}_1 \neq \hat{p}_2) \longrightarrow$ $\underbrace{\text{small/large}}$ difference

$$\hat{p} = \frac{conv_1 + conv_2}{visit_1 + visit_2}$$

$\rightarrow$ Total Conv Rate

Z value

Z distribution

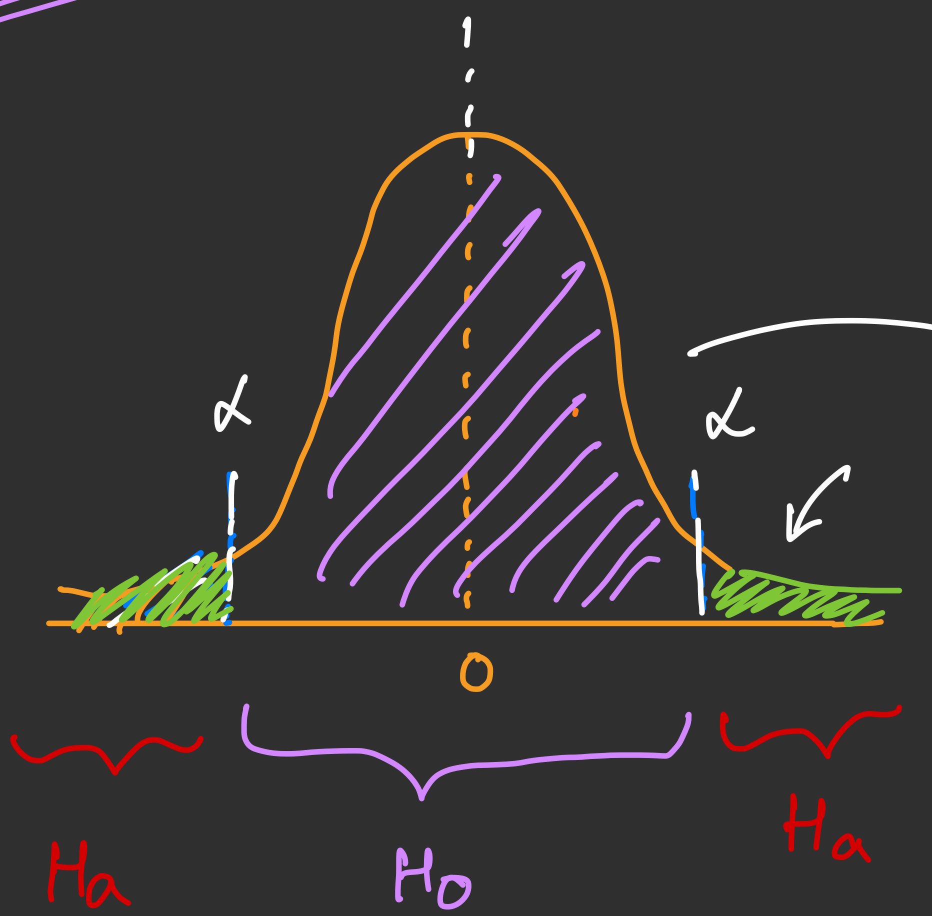
$(\mu = 0, \sigma = 1)$

$$Z = \frac{x - \mu}{\sigma}$$

Z_1

Z_2

2 sided Tailed Test



Standard Normal Distribution

$(\mu = 0$

$\sigma = 1)$

p-value $< \alpha$

Reject Ho

Question:

You are the manager of an e-commerce website, and you have recently implemented a new web page in hopes of increasing sales.

To evaluate the effectiveness of the new page, you collected data on the conversion rates for both the old and new web pages. The conversion rate is defined as the proportion of visitors who make a purchase.

- For the old web page (Web Page A), you had **1000** visitors, resulting in **50** conversions.
- For the new web page (Web Page B), you had **500** visitors, resulting in **30** conversions.

Now, you want to determine if there is a statistically significant difference in the conversion rates between the old and new web pages.

Step 1: $H_0 \Rightarrow \hat{p}_1 = \hat{p}_2$

$H_a \Rightarrow \hat{p}_1 \neq \hat{p}_2$

Step 2: Distribution ?
(Normal)

Step 3: Type of Tailed Test
(Two Tailed)

Step 4: pvalue

Step 5: pvalue $< \alpha$

T-test

Let's say you are a Research Scientist working on a new cognitive enhancement pill

- The goal is to develop a pill that can significantly improve IQ scores in individuals.
- The researchers believe that the new pill will lead to a significant increase in average IQ scores for the population.

Case I : No Impact (IQ remains same)

Case II : IQ Increases

Q) Why NOT Z-Test? [Interview Questions]

1. Senario Complexity

→ In real life, you often won't have the pop^n std.

$$\boxed{Z_{st} = \frac{x - \mu}{(\sigma/\sqrt{n})}}$$

$\sigma \rightarrow$ pop^n std

2. Sample size < 30

Use Case : Improve IQ with pill

① Given Data:

Avg pop^n IQ : 100 pop^n std $\longrightarrow$ unknown

Collected data: [102, 98, 115, 104, 96, 82, 76, 102, 180]

Step 1 : $H_0 : \mu = 100$

$H_a : \mu > 100$

Step 2 : Distribution

$\hookrightarrow$ Not Normal

(T-distribution)

$$\boxed{Z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}} \quad \sigma \longrightarrow \times$$

Test Statistic for one-sample T test

$$\left(T_{st} = \frac{x - \mu}{\underbrace{\overset{\text{sample std}}{s}/\sqrt{n}}_{\text{sample std error}}} \right)$$

$\begin{cases} n = \text{sample size} \\ s = \text{sample std deviation} \end{cases}$

Two-sample T-Test
(Independent)

$M_1 == M_2$

$$T_{st} = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

n_1 :
n_2 : } Sample of each group

x_1 :
x_2 : } Sample mean of the sample data of each group

μ_1 :
μ_2 : } Pop^n mean of each group

s_1
s_2 } Sample std of each group

Two schools

School 1	IQ_1	School 2	IQ_2
x_1	✓	x_1	✓
x_2	✓	x_2	✓
x_3	✓	x_3	✓
x_4	✓	x_4	✓
x_5	✓	x_5	✓
	x_1		x_2