22/02/17

UNIT - 4

chapter 1 :- Back tracking

1) General method

2) Applications.

(i) N - Queen's problem.

(ii) Sum of subsets

(iii) Graph coloring

(iv) Hamiltonian cycles.

chapter 2 :- Branch and Bound

1) General method

2) Applications.

(i) Travelling sales person problem.

(ii) 0/1 knapsack problem using

(i) Least count Branch & Bound.

(ii) FIFO Branch & Bound.

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⇒ **N-Queens problem** :- Here we have to discuss 4 Queen & 8 Queens problem.

(i) **4 Queen problem** :- Consider a 4x4 chess board and it contains 4 Queens Q_1, Q_2, Q_3 & Q_4. The objective is to place the 4 Queens on chess board in such a way that no two queens should be placed in the same row, same column and same diagonal position.

⇒ The explicity constraints are 4 queens are to be placed on 4x4 chess board in 4x4 ways. The implicity constraints are no two queens are in the same row, same column and same diagonal position. Let (x_1, x_2, x_3, x_4) are the solution vectors

STEP 1:- Consider empty 4x4 chess board as shown in below

STEP 2 :- place the queen Q_1 in 1st row & 1st column i.e

Q_1			

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STEP 3 :- Now place the queen 'Q_2' in 2nd row and 3rd column, 4th column. Now choose 2nd row 3rd column i.e.

Q_1			
		Q_2	

STEP 4 :- We are unable to place 'Q_3' in 3rd row. Apply the backtracking method we go back to 'Q_2' and place it somewhere else i.e 2nd row & 4th column.

Q_1			
			Q_2

STEP 5 :- Now place Queen Q_3 in 3rd row, 2nd column

Q_1			
			Q_2
	Q_3		

STEP 6 :- The fourth Queen, should be placed in 4th row but Q_1, Q_2, Q_3 attack to Q_4, so apply the back-tracking method, we go back to Q_3 and remove it. Again it is not possible, go back to Q_2 and remove it. Again it is not possible, go back to Q_1 move it to next column as shown below.

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	Q_1		

STEP :- 7 :- Now place Queen Q_2 in 2nd row, 4th column

	Q_1		
			Q_2

Step 8 :- Now place Queen Q_3 in 3rd, 1 column, because no one attack to Q_3

	Q_1		
			Q_2
Q_3			

STEP 9 :- place Queen Q_4 in 4th row, 3rd column because no one attack to Q_4

	Q_1		
			Q_2
Q_3			
		Q_4	

$\therefore (x_1, x_2, x_3, x_4) = (2, 4, 1, 3)$

⇒ **Draw the state space tree for 4-Queen's problem :-**

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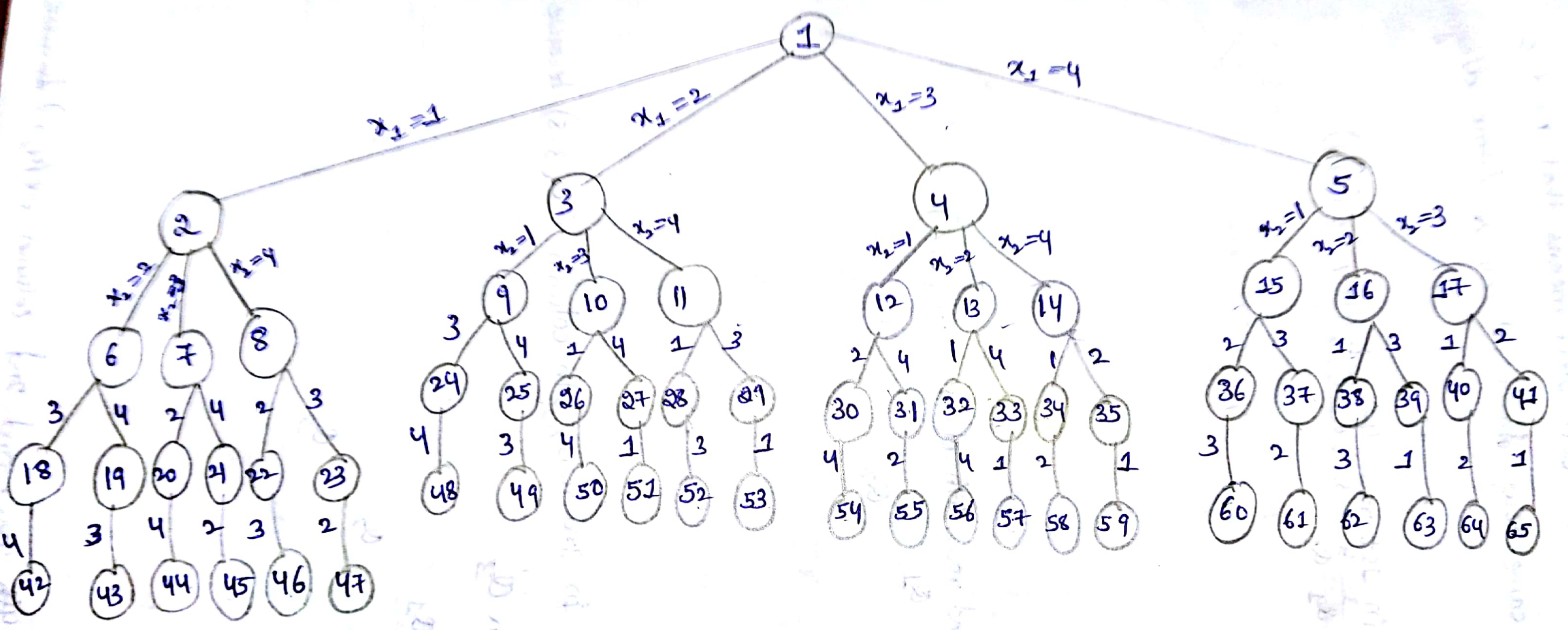

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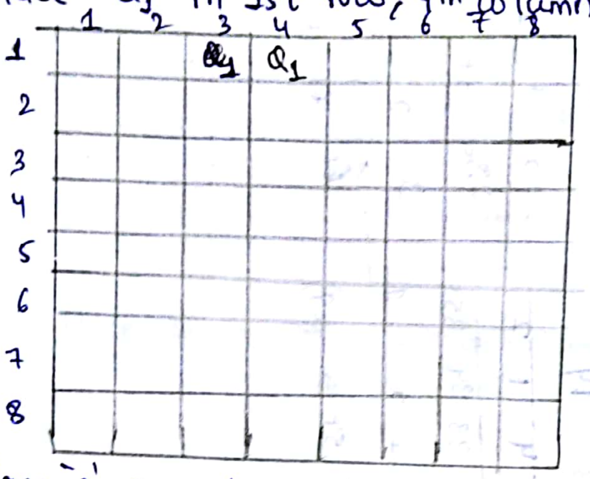
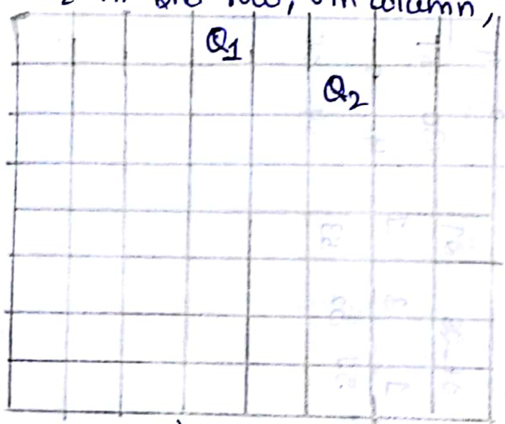
⇒ **8 Queen's problem** :- It stated that on 8×8 chess board playing 8 Queen's i.e $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$ & Q_8

No two Queen's can attack each other means that no two Queens are placed in same row, same column & same diagonal position.

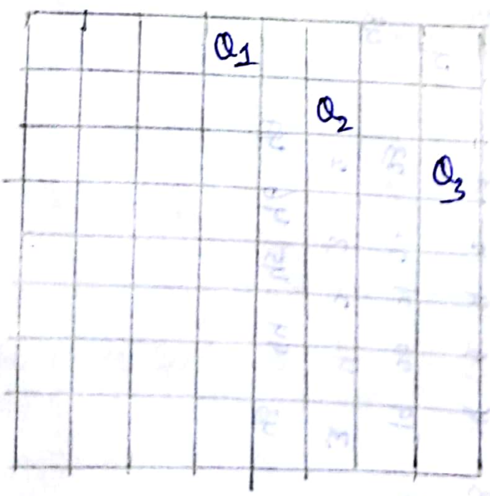
STEP 1 :- place 'Q_1' in 1st row, 4th column, i.e (1,4)



STEP 2 :- place 'Q_2' in 2nd row, 6th column, i.e (2,6)



STEP 3 :- place Queen 'Q_3' in 3rd row, 8th column i.e (3,8), because no one attack to Queen 'Q_3'



STEP 4 :- Place Queen 'Q_4' in 4th row, and 2nd column i.e (4,2) because

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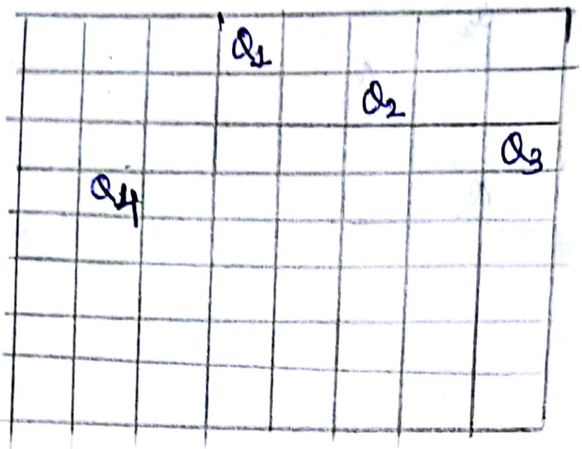
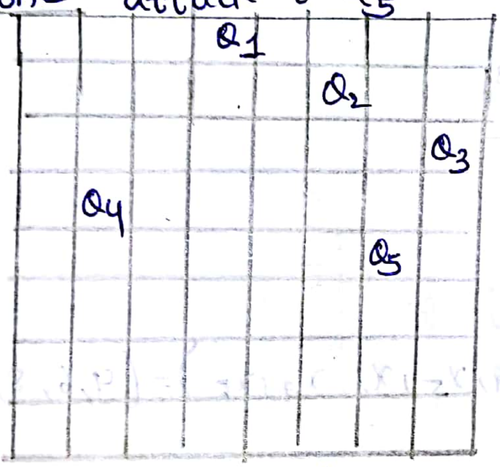
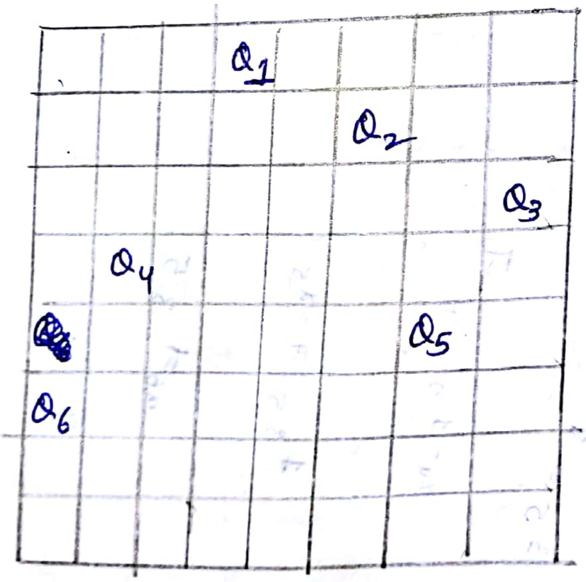
no one attack to Q_4



STEP 5 :- place Queen Q_5 in ~~at~~ 5th row, 7th col i.e (5,7) because no one attack to Q_5



STEP 6:- place Queen Q_6 in 6th row, 1st col i.e (6,1) because no one attack Q_6



STEP 7:- place Queen Q_7 in 7th row, 3rd col i.e (7,3) because no one ~~no~~ attack Q_7

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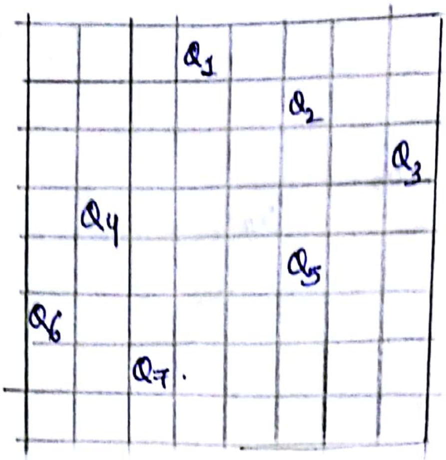
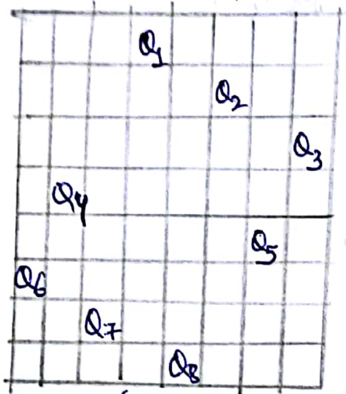
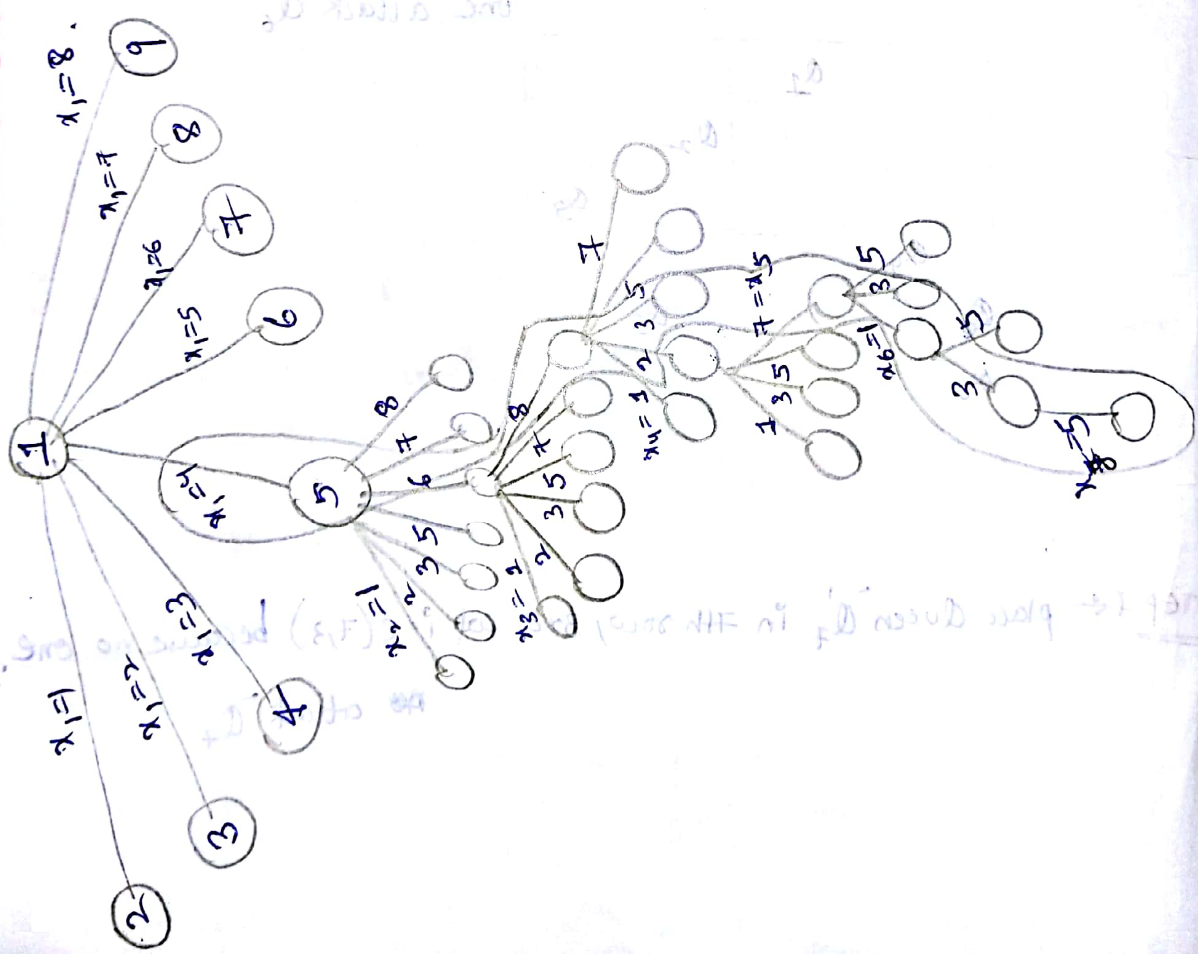


STEP 8 :- place Queen 'Q_7' in 8 th row, 5th col i.e (8,5) because no one attack 'Q_8'.



The solution is $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (4, 6, 8, 2, 7, 1, 3, 5)$



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⇒ Sum of Subsets problem :- Let $S = \{S_1, S_2, S_3 \dots S_n\}$ be a set of 'n' positive integers then we have to find a subset where sum is equals to given positive integers D (or) M (or) C.

procedure :-

STEP 1 :- Start with an empty set

STEP 2 :- Add next element from least to subset.

STEP 3 :- If the subset having sum "M" then stop with that subset as a solution.

STEP 4 :- If the subset is not feasible or if we have reached the end of the set, apply the backtraking to the subset until we find most suitable value.

STEP 5 :- If the subset is feasible then repeat step 2.

STEP 6 :- If we have visited all elements without finding a suitable subset and if no backtracking is possible then stop with no solution.

⇒ $M = 31$, $W = \{7, 11, 13, 24\}$ Draw the state space tree by using sum of subset problem.

Sol :- Given that $M = 31$

$W = \{7, 11, 13, 24\}$

1) Initially sum of subset is empty. The sum of all the weights from set $W = 7 + 11 + 13 + 24 = 55 \Rightarrow \therefore \boxed{W = 55}$

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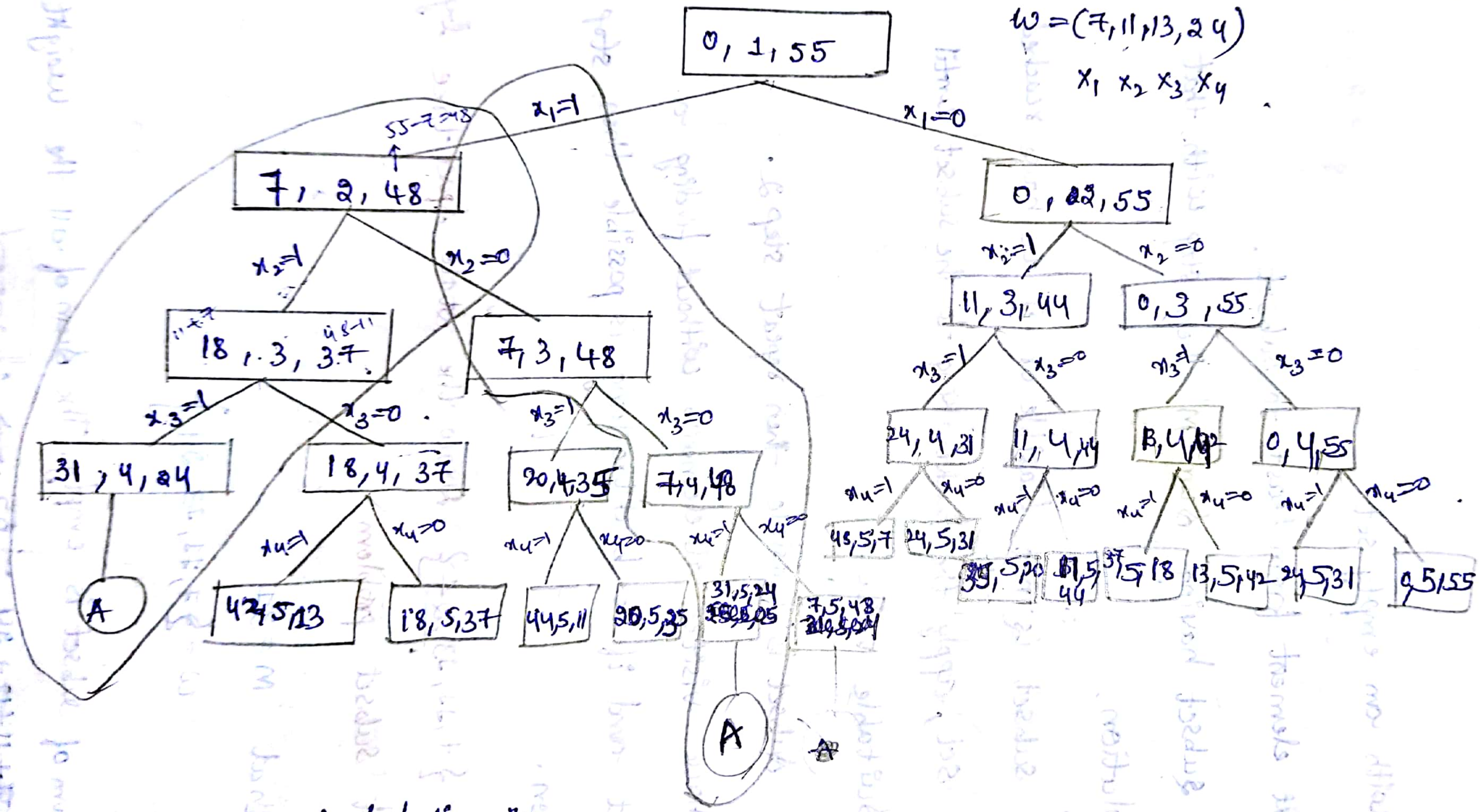
∴ state space tree can be represented as.

$w = (7, 11, 13, 24)$

$x_1 \ x_2 \ x_3 \ x_4$



In every step we should get 55

∴ Solution is :

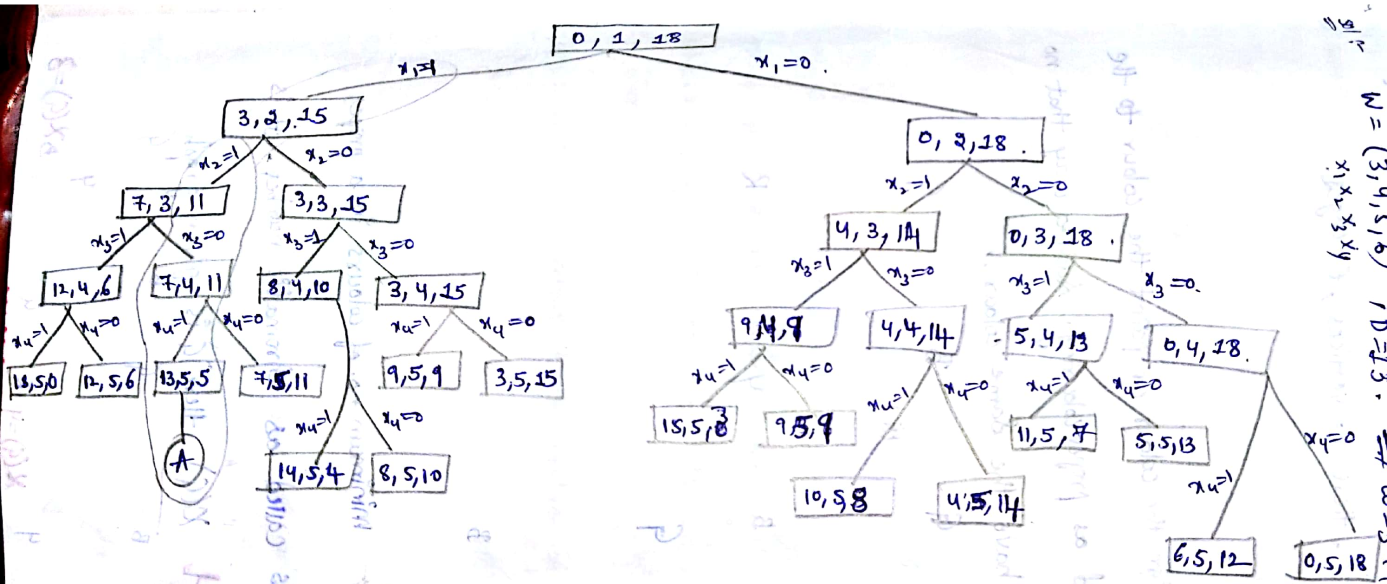
∴ $(x_1, x_2, x_3, x_4) = (1, 1, 1)$

$(x_1, x_2, x_3, x_4) = (1, 0, 0, 1)$

$S = \{3,4,5,6\}$, $D = 13$. Draw the state space tree by using sum of subsets problem.

Sol: $W = (3,4,5,6)$, $D = 13$. $\Rightarrow W = 3+4+5+6 = 18$

$x_1\ x_2\ x_3\ x_4$



∴ Solution is

$\therefore (x_1, x_2, x_3, x_4) = (1, 1, 0, 1)$

5m

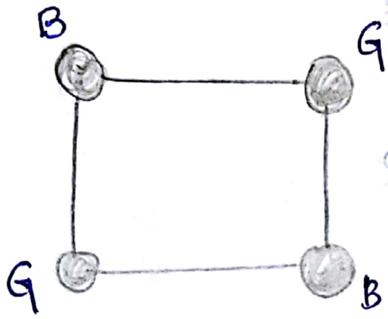
⇒ Graph colouring :-

Graph is a collection of vertices + edges and it is represented by $G=(V,E)$. Here V = vertices, E = edges.

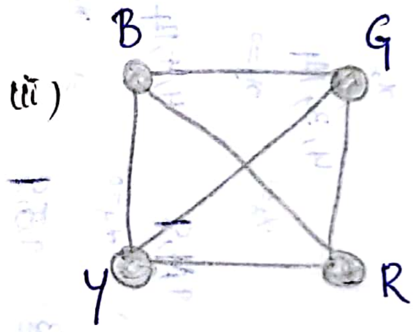
⇒ **Graph colouring :-**

→ **proper colouring :-** Assign the colour or paint the colour to the specified vertices is called as proper colouring. Such a way that no two adjacent vertices have the same colour.

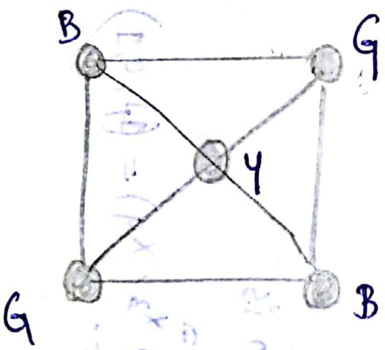
Ex :- (i)



(ii)



(iii)



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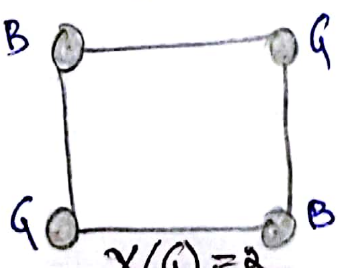
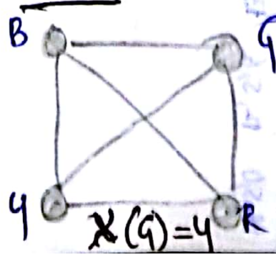
⇒ **Chromatic number :-** Minimum no. of colours in a proper coloring of graph "G" is called as chromatic number. It is represented by the symbol "$\chi(G)$" Here G is given graph.

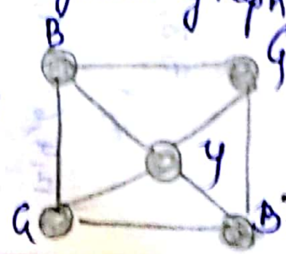
Ex :-



$\chi(G)=2$



$\chi(G)=4$



$\chi(G)=3$

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consider a given graph, we require 3 colours to the colour the graph. hence the chromatic no. is $\chi(G)=3$ and parally we apply the back tracking technique to solve the graph colouring problem as follows.

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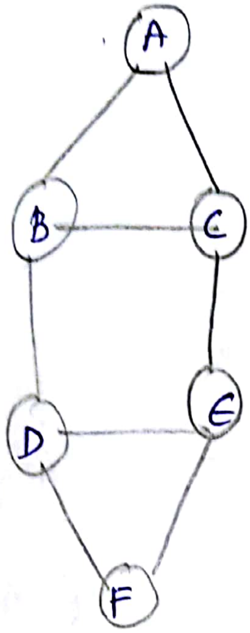
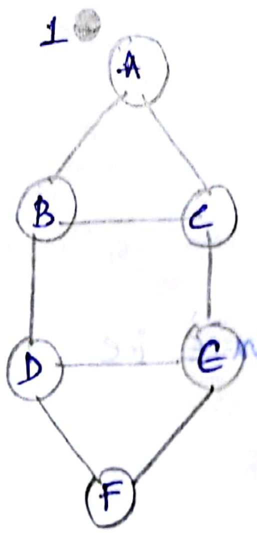


In the above graph 'G' consists of vertices from 'A' to 'F'. There are 3 colours used i.e red, green, blue. For our understanding we indicate one is red, two is green & three is blue.

STEP 1:- Assign red colour to the vertex 'A' i.e



STEP 2:- Assign green colour to the vertex 'B' i.e.

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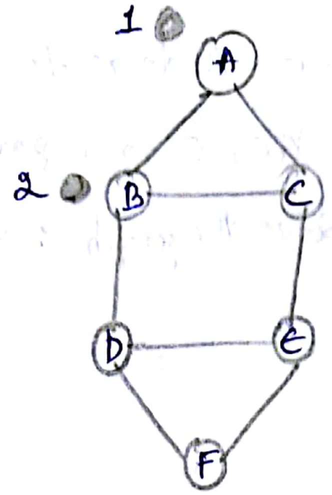
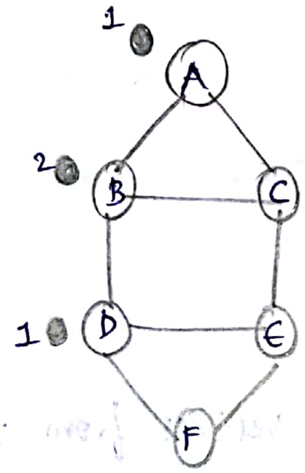
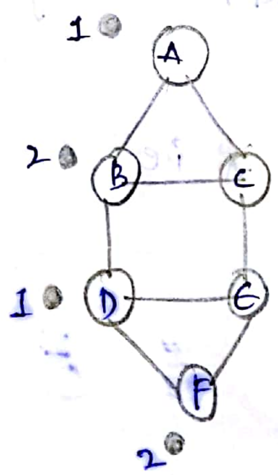


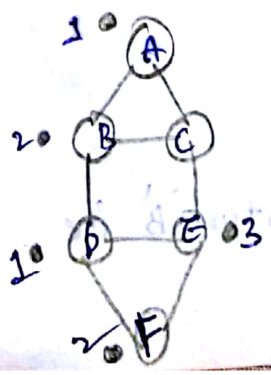
STEP 3 :- Assign red colour to vertex 'D' i.e



STEP 4 :- Assign green colour to vertex 'F' i.e



STEP 5 :- Assign blue colour to vertex 'E' i.e .



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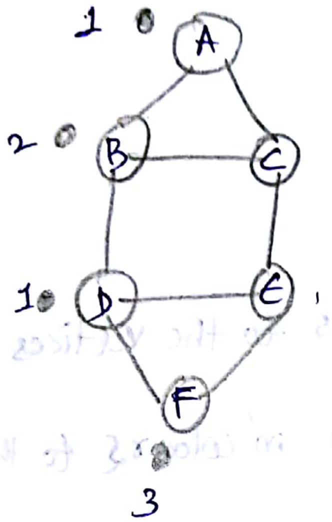
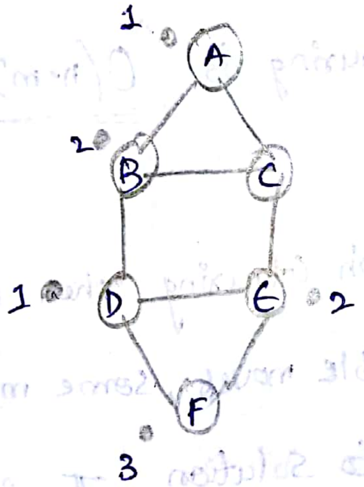
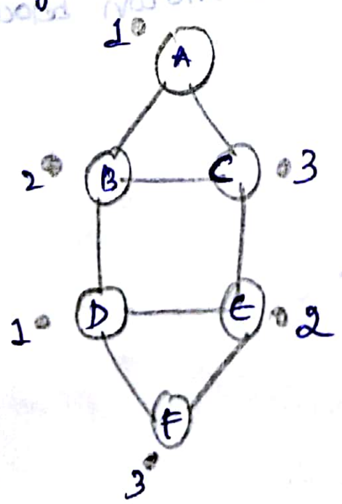
STEP 6:- Their is no possible to assign red, green, blue colours to the vertex 'C' & apply the back tracking method & come back to vertex 'E' & 'F' and remove their colours. and assign blue colour to the vertex 'F' i.e ,



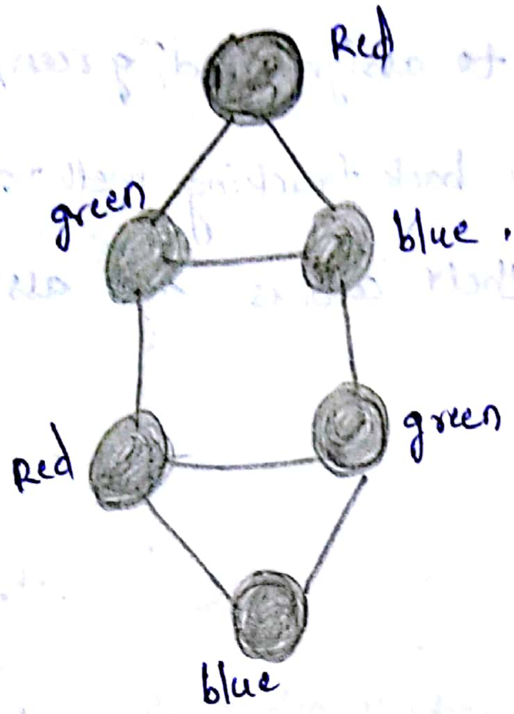
STEP 7:- Assign green colour to vertex 'E' i.e



STEP 8:- Now assign colour blue to vertex 'C' i.e .



Hence the graph colouring of given graph is



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⇒ **NOTE** :- If we assign the colours to the vertices is called as graph colouring. similarly if we assign 'm' colours to the given graph 'G' is called as "m' colouring graph."

⇒ The time complexity of graph colouring is "$O(n \cdot m^n)$"
where n – vertices, m – colouring graph.

⇒ Draw the state space tree for graph colouring when n=3, & m=3

State space tree contains all possible moves, some moves leads to solution or may not leads to solution. The following trees contains only possible solutions as shown below.

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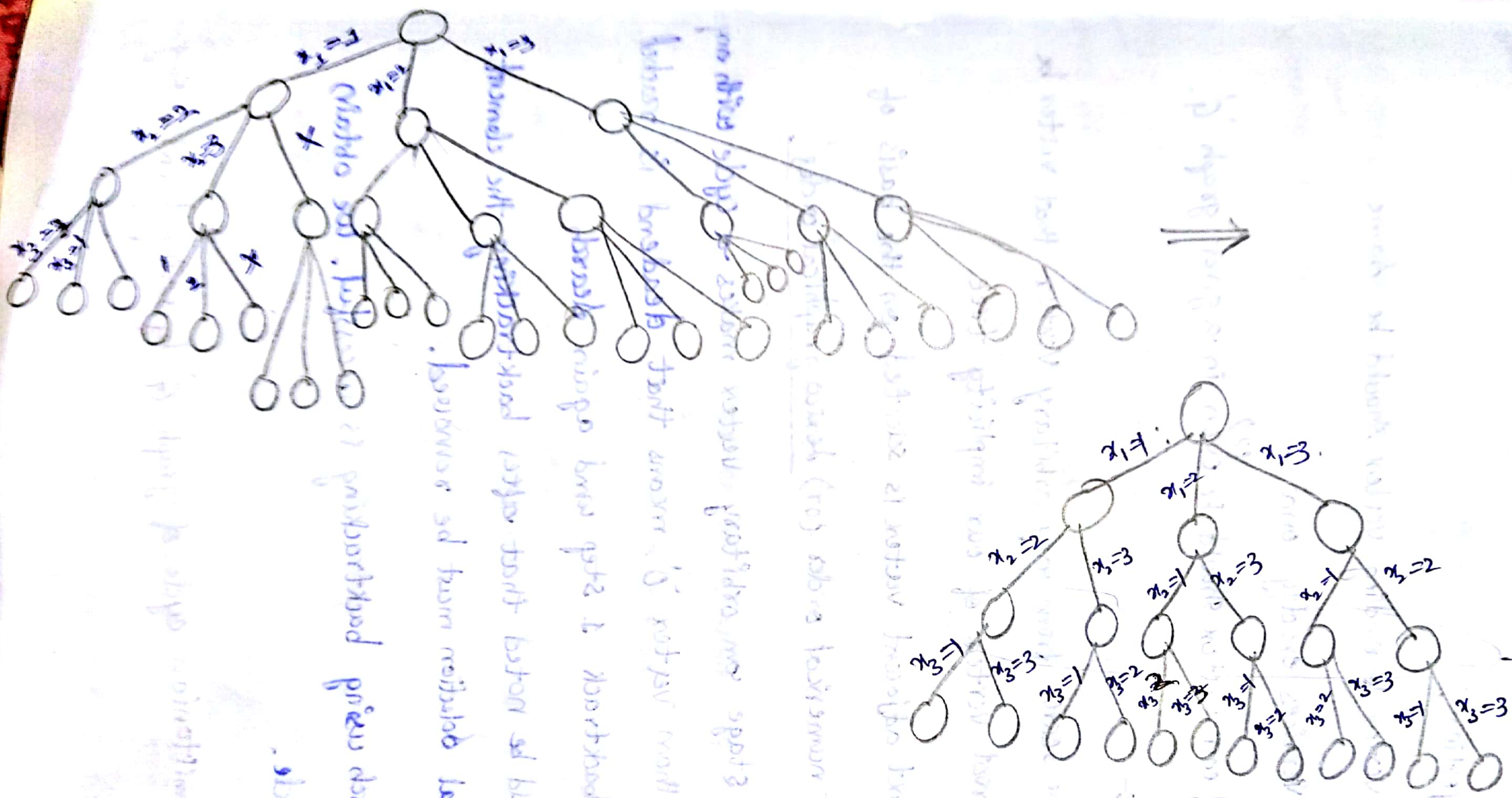

→ should be equa

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⇒ Hamiltonian cycle :-

A graph 'G' is said to be a hamiltonian cycle. If it contains following properties.

(i) Starting vertex & ending vertex should be same.

(ii) Visit all vertices exactly once.

(iii) Degree of each vertex should be even (2) in a given graph 'G'.

⇒ procedure :-

STEP1) Start the search from any arbitary vertex that vertex be treated as root vertex of our implicity tree

STEP2) The next adjacent vertex is selected on the basis of alphabetical or numerical order (or) "lexico graphical order".

STEP3) At any stage an orbitary vertex makes a cycle with any vertex other than vertex '0'. means that deadend is reached. In this case backtrack 1 step and again search

STEP4) It should be noted that after backtracking the element from the partial solution must be removed.

STEP5) The search using backtracking is successful. We obtain hamiltonian cycle.

⇒ Find the hamiltonian cycle of graph 'G' by using backtracking technique.

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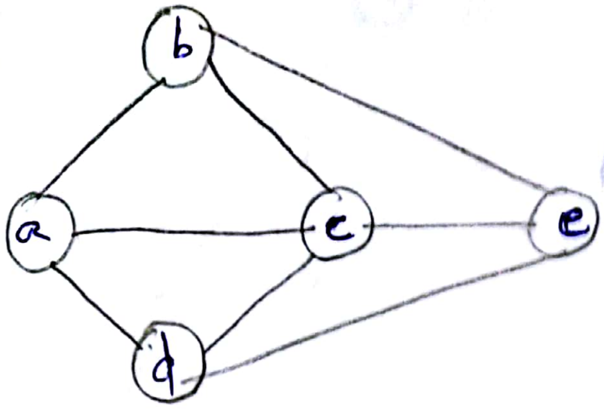
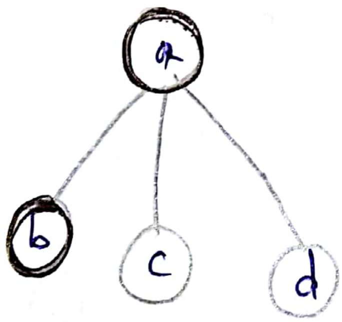
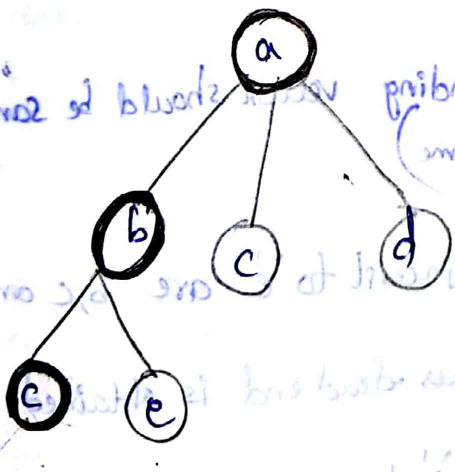


Sol :- STEP1 :- Initially start search with vertex 'A' that vertex be treated as root vertex of our implicity tree i.e

STEP2 :- Add all adjacent vertices to the vertex 'a' and it comes first in lexicographical order. i.e b,c,d.



STEP3 :- Add all adjacent vertices to the vertex 'b' i.e a,c,e there 'a' is already visited then we remove it & add c,e i.e



STEP4 :- Add all adjacent vertices to the vertex 'c' i.e a,b,d,e a,b already visited then we remove them & add d,e i.e .

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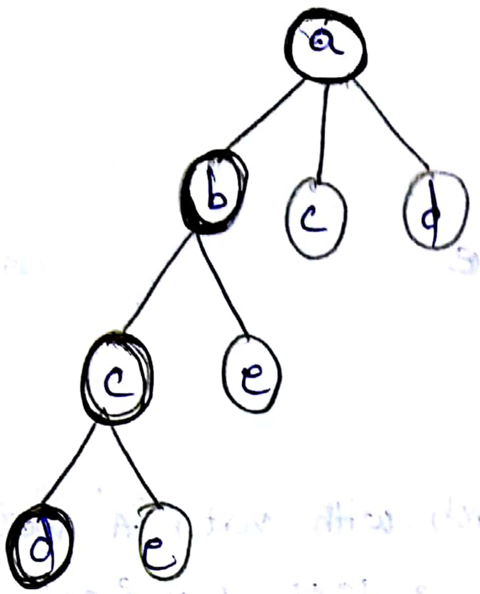
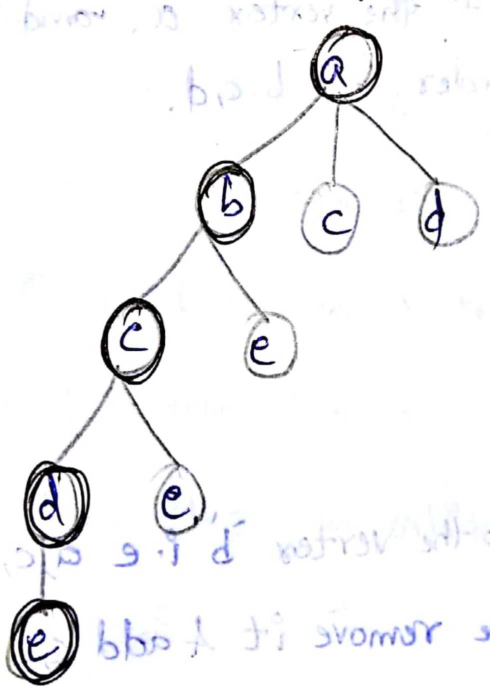


Step 5 :- Add all adjacent vertices to the vertex 'd' i.e a, c, e but a & c are already visited so add 'e' i.e



Dead End (because starting & ending vertex should be same but here not same)

⇒ In the above tree the vertex adjacent to 'e' are b, c and d but they are already visited thus dead end is obtained. so, backtrack 1 step & remove vertex 'e' from the partial solution.

i.e

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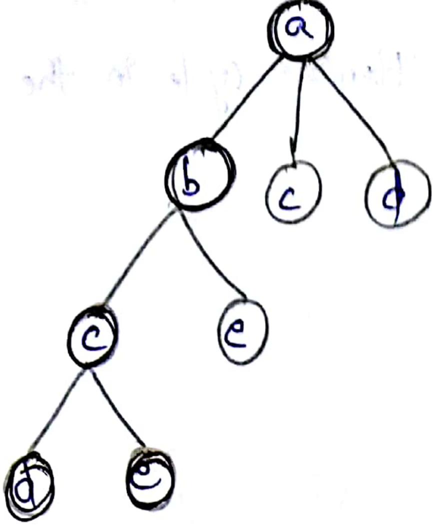
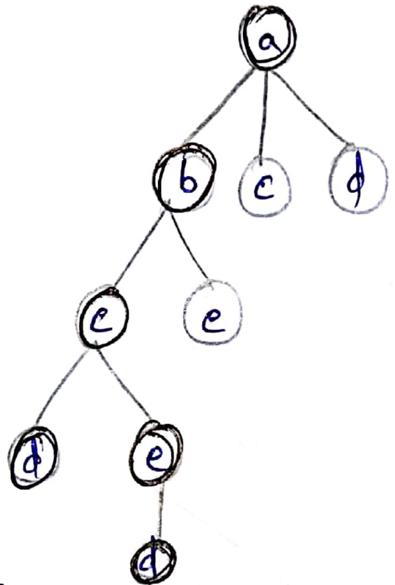
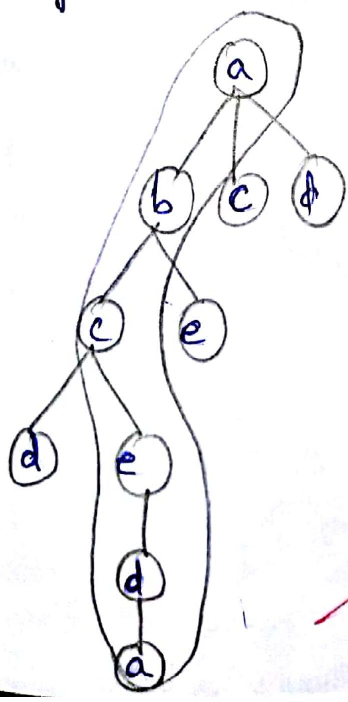


⇒ Now add all adjacent vertices of 'e' i.e b, c, d but b, c already visited in that path so add d to vertex e.



⇒ Add adjacent vertices to vertex 'd' i.e a, c, b but c & b are already visited now add a to vertex 'd' i.e



a - b - c - e - d - a

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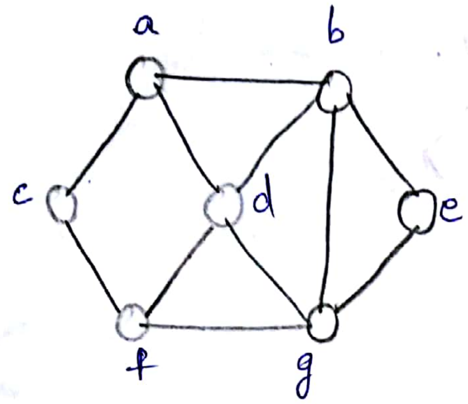
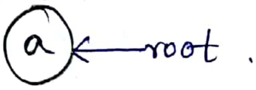
⟹ Apply backtracking to find hamiltonian cycle in the following graph

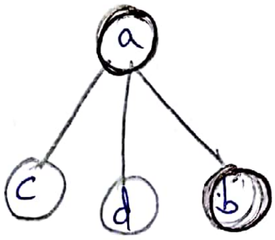


Sol :-

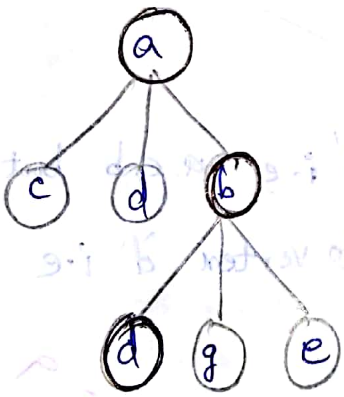
STEP 1 :-



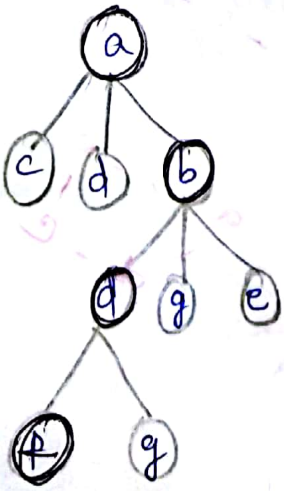
STEP 2 :-



STEP 3 :-



STEP 4 :-



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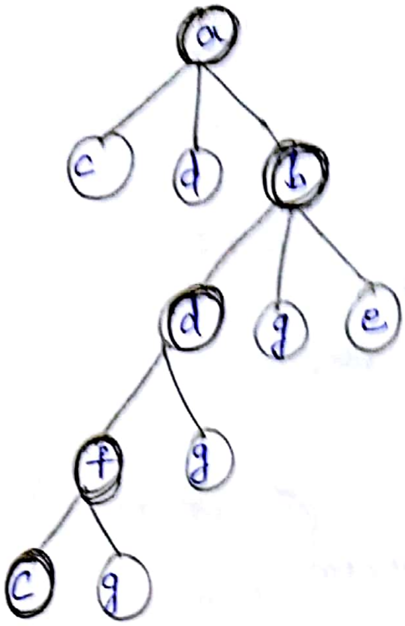
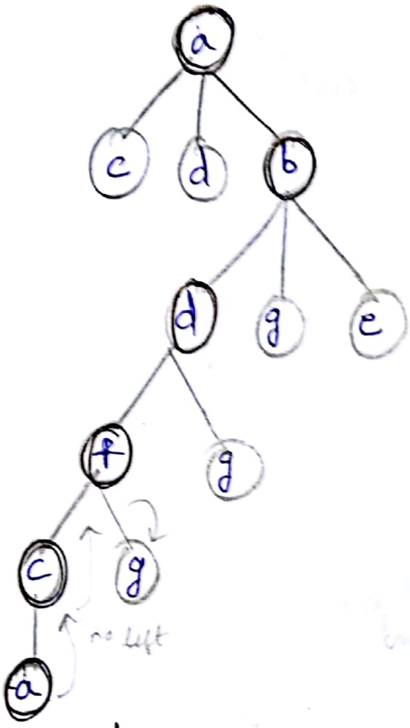
STEP 5 :-



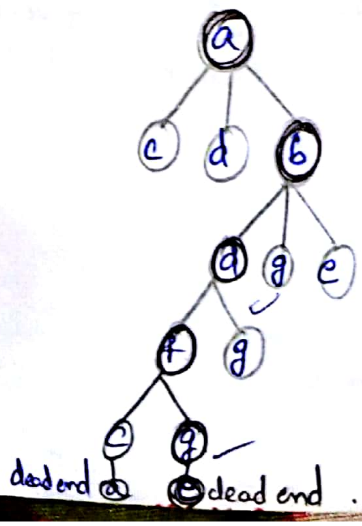
STEP 6 :-



Dead end.

⇒ Here we got only 5 vertices but there are only 7 vertices so Dead end . & back track .

⇒ STEP 7 :-



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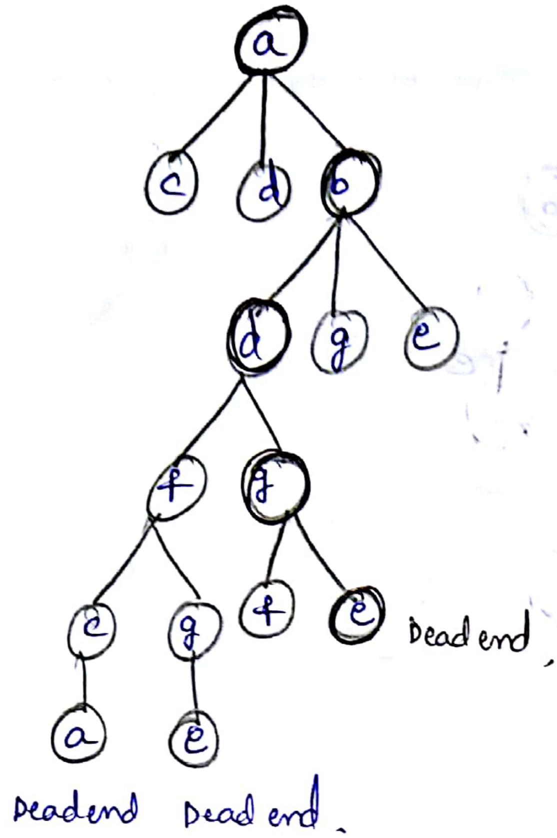
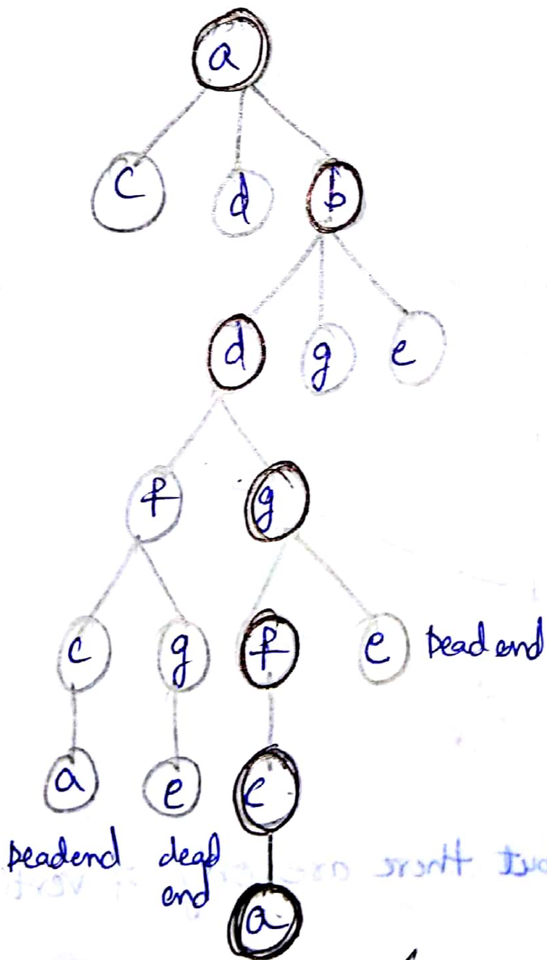
STEP 8 :-



STEP 9 :-



Deadend. (6 vertices but we have 7 vertices)
back track.

STEP 10 :-

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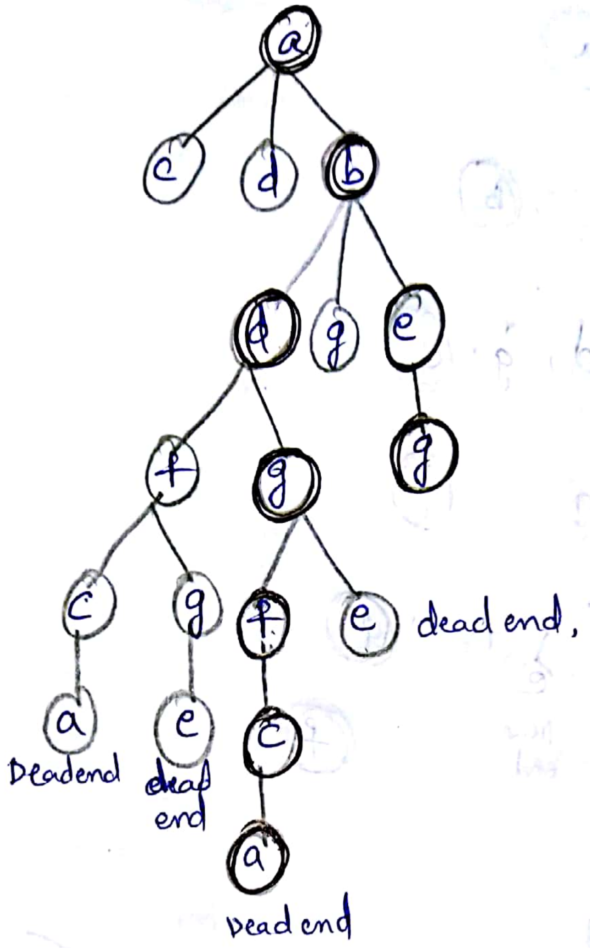
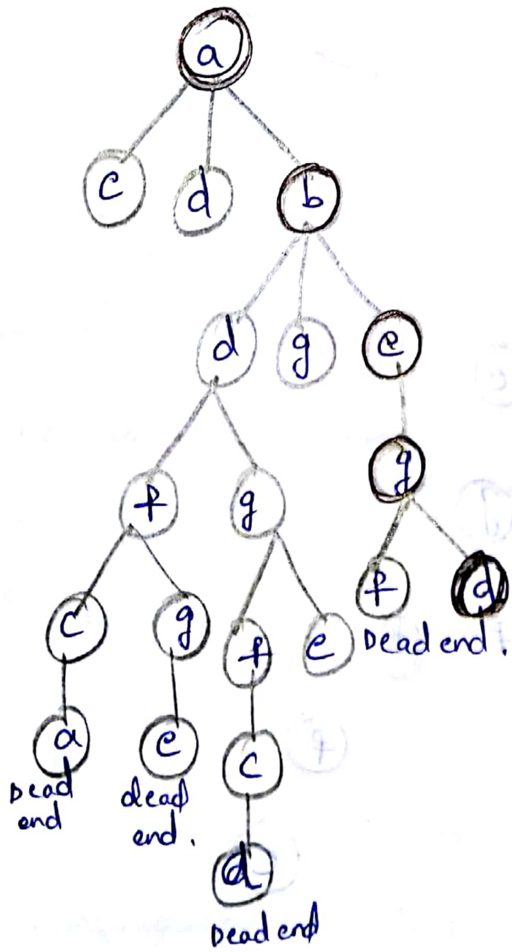


STEP 11 :-



STEP 12 :-

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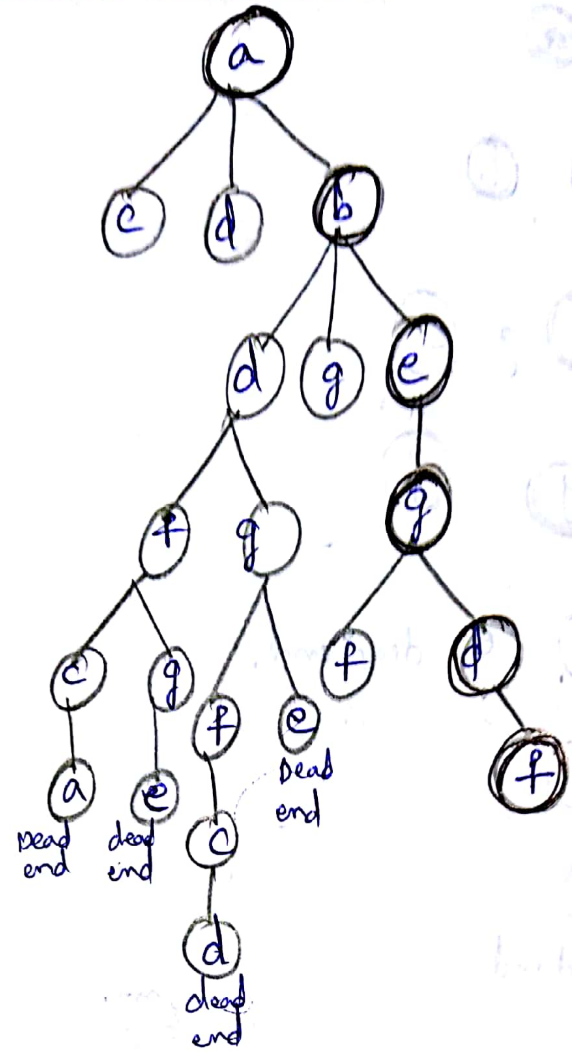
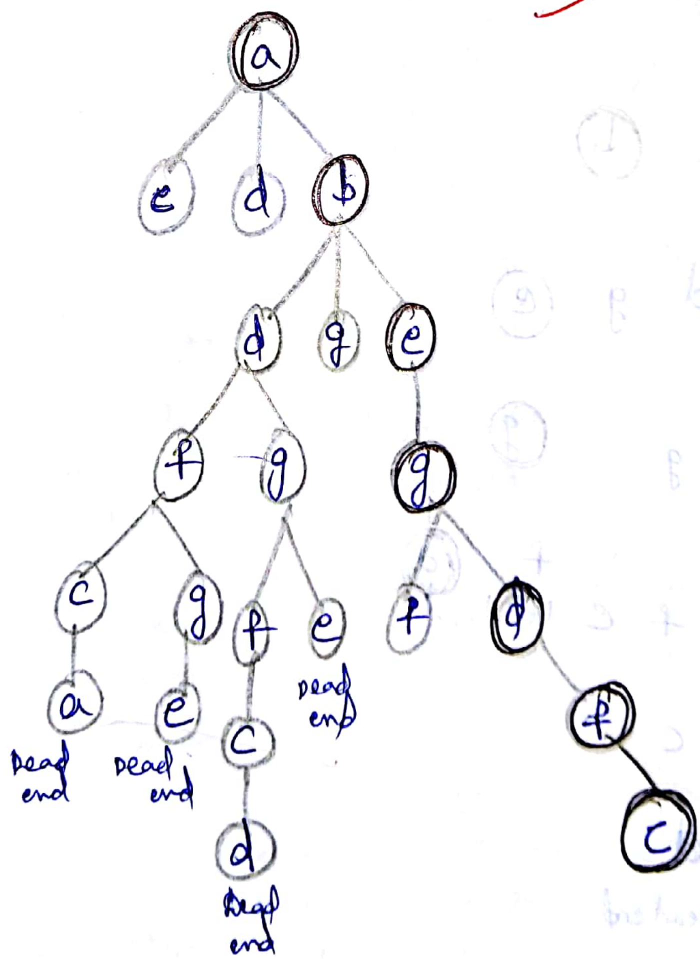


STEP 13 :-



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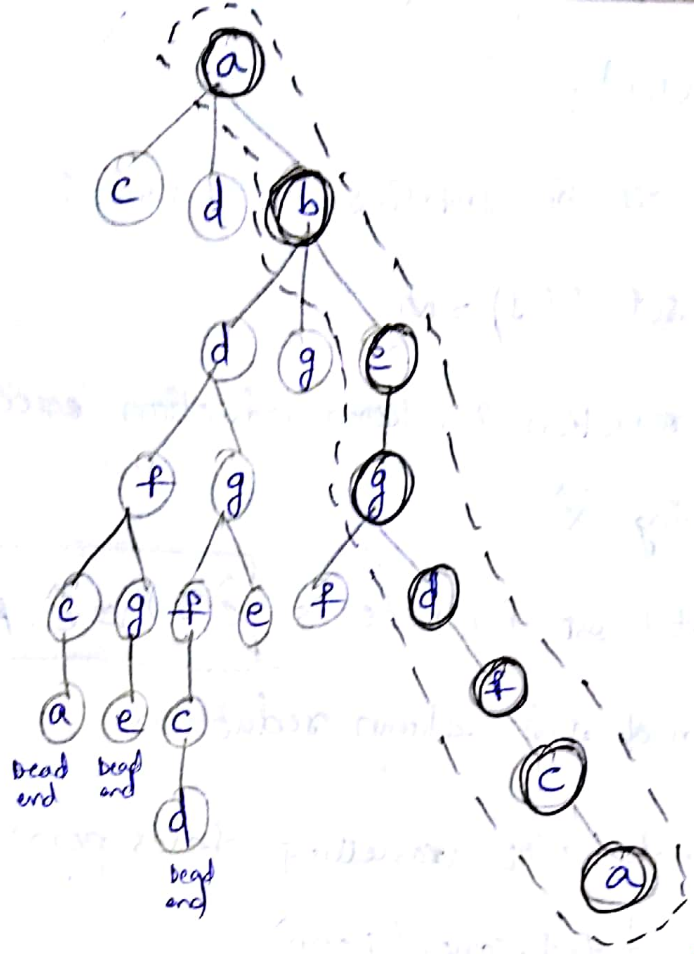
STEP 14:-



03/03/17

10m

⇒ Travelling sales person problem using branch & bound :- (LCBB) least cost

If there are 'n' cities and cost of travelling from one city to another city, a salesman start from any one of the state & has to visit all the cities exactly once and has to return to the starting place with shortest distance with minimum cost.

Let $G = (V, E)$ be a directed graph defining an instance of the travelling sales person problem, C_{ij} be the cost of the edg (i, j). $C_{ij} = \infty$ (infinity) if (i, j) does not belongs to given graph 'G'.

Procedure :-

STEP 1 :- Let 'A' be the reduced cost matrix for |R| and 'S' be the child of 'R' such that edge of (R, S) corresponds to including edge (i,j).

STEP 2 :- Change all the entities in a row 'i' and column 'j' of A to ∞. and set $(j,1) = \infty$

STEP 3 :- Apply row reduction & column reduction except for rows & columns containing '∞'.

STEP 4 :- The total cost for node is $\boxed{\hat{c}(S) = \hat{c}(R) + A(i,j) + r}$

Here r = sum of row & column reduction.

⇒ Solve the instance of travelling sales person problem by using branch & bound technique. (LCBB)

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

Sol :- Row reduction :-

$$\begin{array}{c} \\ \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{array} \begin{array}{c} \text{least value in row} \\ 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

$r = 21$

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column reduction :-

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$1 \quad 0 \quad 3 \quad 0 \quad 0 \quad (r=4)$

$\therefore$ 1st reduced matrix

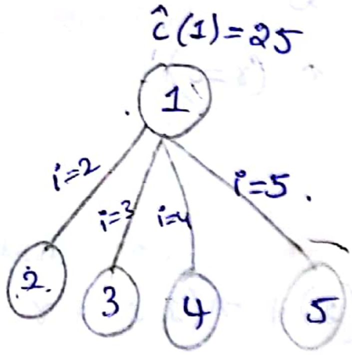
$$\boxed{A = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}}$$

$\therefore \; r = 21 + 4 = 25$

Initially

$$\boxed{\hat{c}(1) = 25 = r}$$

$\therefore$ The state space tree is



$\Rightarrow$ consider path (1,2) :-

It means that set '∞' to 1st row & 2nd column and set (2,1) = ∞ of reduced matrix 'A'. i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ 0 \\ - \end{matrix} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}.$$

$r = 0$

Column reduction:-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

$0 \quad - \quad 0 \quad 0 \quad 0 \quad (r=0)$

$\therefore r = 0.$

from reduced matrix.

$\hat{c}(2) = \hat{c}(1) + A(1,2) + r. \qquad (\hat{c}(S) = \hat{c}(R) + (A(i,j)) + r)$

child   parent.

$= 25 + 10 + 0$

$$\boxed{\hat{c}(2) = 35}$$

→ consider path (1,3):-

It means set '∞' to 1st row, 3rd column and set (3,1) = ∞ of reduced matrix 'A' i.e.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$r = 0$

Column reduction:-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$11 \quad 0 \quad - \quad 0 \quad 0 \quad (r=11)$

$\therefore r = 11 + 0 = 11.$

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$$\therefore \hat{c}(3) = \hat{c}(1) + A(1,3) + r.$$
$$= 25 + 17 + 11$$

$$\boxed{\hat{c}(3) = 53}$$

$$\begin{array}{r} 25 \\ 11 \\ 17 \\ \hline 53 \end{array}$$

⟹ <u>consider path (1,4) :-</u>

It means set ∞ to 1st row, 4th column & set $(4,1)=\infty$ of reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

<u>Row reduction :-</u>

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{array}{c} \leftarrow \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\ (r=0) \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

<u>Column reduction :-</u>

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} (r=0) \\ \begin{array}{ccccc} 0 & 0 & 0 & - & 0 \end{array} \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

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$\therefore r = 0 + 0 = 0$.

$\therefore \hat{c}(4) = \hat{c}(1) + A(1,4) + r$.

$= 25 + 0 + 0$

$$\boxed{\hat{c}(4) = 25}$$

⇒ consider path (1,5) :-

It means set '∞' to 1st row, 5th column & set (5,1) = ∞ of reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 2 \\ 0 \\ 3 \\ 0 \end{matrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$r = 5$

Column reduction :-

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & - \end{array} \; (r=0)$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

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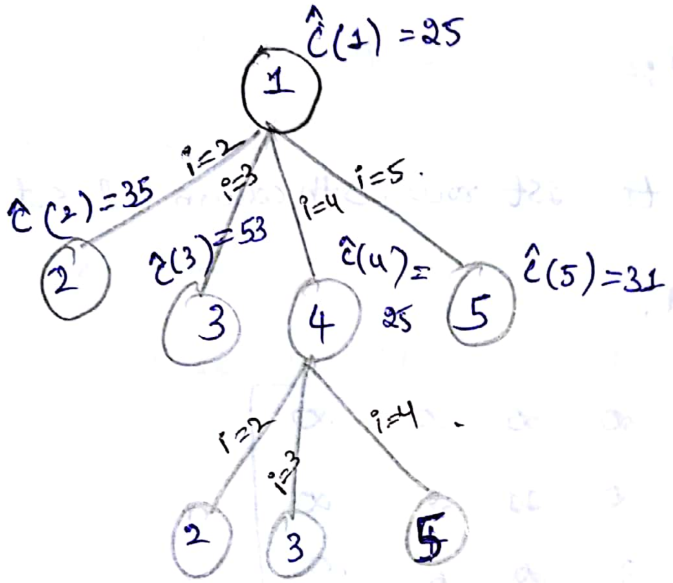
$(r = 5+0 = 5)$

$\therefore \hat{c}(5) = \hat{c}(1) + A(1,5) + r$

$= 25 + 1 + 5$

$\boxed{\hat{c}(5) = 31}$

$\therefore$ State space tree is:



Compare (1,2), (1,3), (1,4), (1,5). Here (1,4) has minimum cost then consider 2nd reduced cost matrix for path (1,4) i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

→ consider path (4,2):-

Consider path (4,2) means that set '∞' to 4th row & 2nd column & set (2,1) = ∞ of 2nd reduced matrix of A'.

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35

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

Row reduction:-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \begin{array}{c} - \\ 0 \\ 0 \\ - \\ 0 \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \\ 0 - 0 - 0 \ (r=0) \end{array}$$

Column reduction:-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \xrightarrow{\text{Column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \\ 0 - 0 - 0 \ (r=0) \end{array}$$

$$\hat{c}(2) = \hat{c}(4) + A(4,2) + r.$$

$$= 25 + 3 + 0$$

$$\boxed{\hat{c}(2) = 28}$$

→ consider path (4,3) :-

Consider path (4,3) means that set '∞' to 4th ~~column~~ row, 3rd column and set $(3,1) = \infty$ to the reduced 2nd matrix A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}.$$

<u>Row reduction:-</u>

$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} & \begin{matrix} - \\ 0 \\ 2 \\ - \\ 0 \end{matrix} \\ & (r=2) \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

<u>column reduction:-</u>

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \\ \begin{matrix} 11 & 0 & - & - & 0 \end{matrix}\ (r=11) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$\therefore \boxed{r = 13.}$

$$\hat{c}(3) = \hat{c}(4) + A(4,3) + r = 25 + 12 + 13 = 50.$$

$$\boxed{\hat{c}(3) = 50}$$

$\Rightarrow$ <u>consider path $(4,5)$:-</u>

It means set 4th row, 5th column as ∞ & set $(5,1) = \infty$ of 2nd reduced matrix A'.

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$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

Row reduction:

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ 11 \\ 0 \\ - \\ 0 \end{matrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

$(r = 11)$

Column reduction:

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

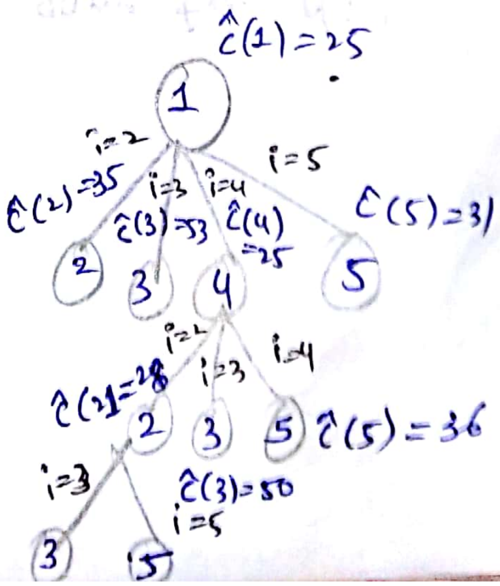
$0 \quad 0 \quad 0 \quad - \quad - \quad (r=0)$

$$\hat{c}(5) = \hat{c}(4) + A(4,5) + r$$

$$= 25 + 0 + 11 = 36$$

$$\boxed{\hat{c}(5) = 36}$$

The state space tree is :



Here compare (4,2) (4,3) (4,5) & (4,2) has minimum cost then consider 3rd reduced matrix of A i.e (4,2) i.e.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

⇒ Consider path (2,3) :-

It means set '∞' to 2nd row, 3rd column and set (3,1) = ∞ of 3rd reduced matrix A.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix}$$

Row Reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ 2 \\ - \\ 11 \end{matrix} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

$(r = 13)$

Column Reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \xrightarrow{\text{Column Reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

0 — — — 0 $(r=0)$

$\therefore r = 13 + 0 = 13$.

$\therefore \hat{c}(3) = \hat{c}(2) + A(2,3) + r$

$= 28 + 11 + 13$

$\boxed{\hat{c}(3) = 52}$

Consider path (2,5) :-

It means set ∞ to 2nd row, 5th column & also $A(5,1) = \infty$ of 3rd reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

Reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ 0 \\ - \\ 0 \end{matrix} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$(r=0)$

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$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$0 \quad - \quad 0 \quad - \quad 0 \quad (r=0)$

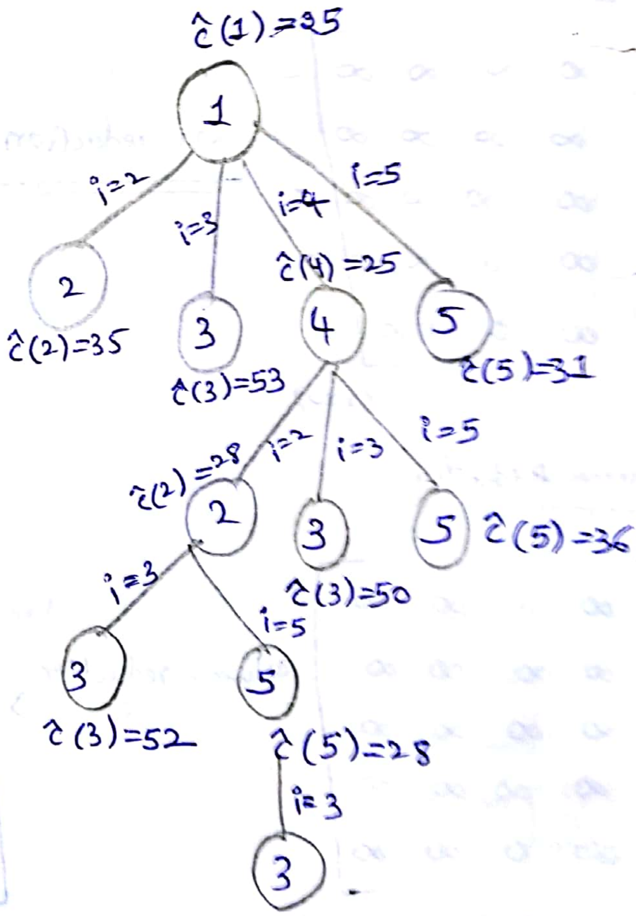
$\therefore r = 0 + 0 = 0$

$\therefore \hat{c}(5) = \hat{c}(2) + A(2,5) + r$

$= 28 + 0 + 0$

$\boxed{\hat{c}(5) = 28}$

$\therefore$ State space tree is :



Here compare (2,3), (2,5) and (2,5) has minimum cost then consider 4th reduced matrix (2,5) of 'A' i.e

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Consider path (5,3):-

It means set ∞ to 5th row, 3rd column & set $(3,1) = \infty$ of 4th reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Row Reduction:-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \end{matrix} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$(r=0)$

Column Reduction:-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \\ - \quad - \quad - \quad - \quad - (r=0) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\therefore r=0.$

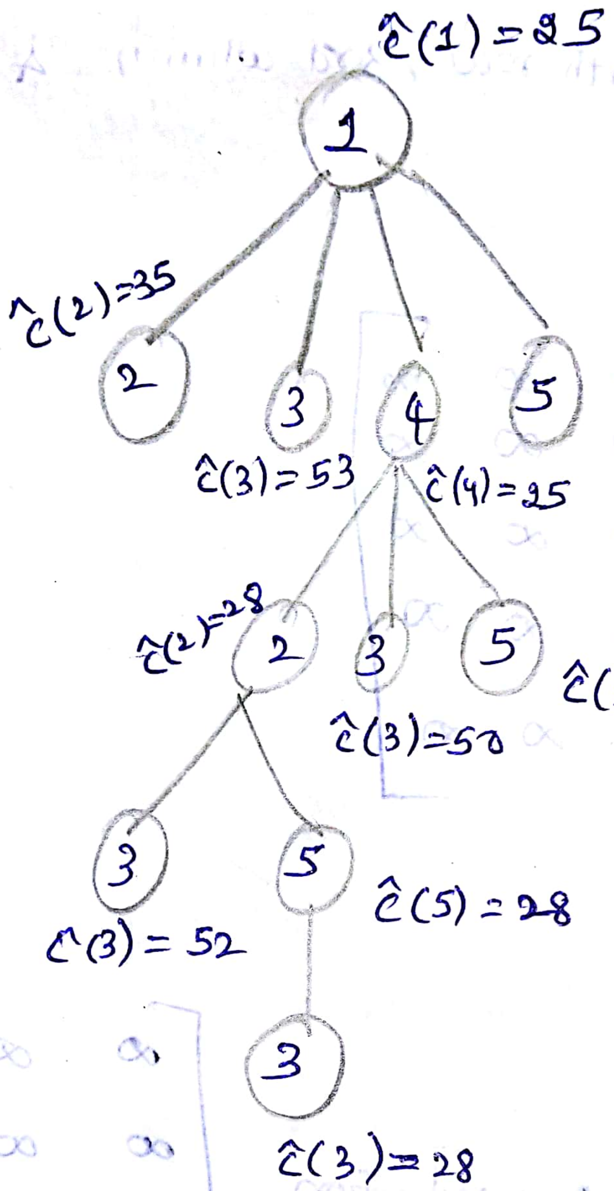
$$\hat{c}(3) = \hat{c}(5) + A(5,3) + r$$

$$= 28 + 0 + 0$$

$$\boxed{\hat{c}(3) = 28}$$

Scanned by CamScanner

∴ state space tree is :



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