Quantum Winter Hackathon

QUESTION 1

Team Quantico | 08-12-2020

Q1) Using the Finite Difference Method, discretize the 1-D Wave Equation with 2nd order spatial accuracy and 1st order time accuracy with implicit time integration scheme.

Data:

Sol:

The given PDE is

$$\frac{\partial u}{\partial t} + C\frac{\partial u}{\partial x} = 0 \;; \qquad .....(1)$$

$$\frac{\partial u}{\partial t} = U_t \quad \& \quad \frac{\partial u}{\partial x} = U_x$$

This is a First Order PDE which can be written as;

$U_t + CU_x = 0$

The given is the domain: [a, b] = [0, 2] and N = 41

X (Domain)

Therefore $\Delta x = \frac{(b-a)}{N-1}$ i.e., $\Delta x = 0.05$

Where $x_{i+1} = x_i + \Delta x$

X (Discretized)

Spatial Discretization:

$\frac{\partial u}{\partial x} = U_x \approx \frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x}$ (central difference and second order in x)

Substituting in (1)

$$\frac{\partial u}{\partial t} + C\,\frac{U_{i+1}^{n} - U_{i-1}^{n}}{2\Delta x} = 0$$ (2^{nd} Order Spatial Accuracy) … (2)

Time Discretization:

$$\frac{\partial u}{\partial t} = U_t \approx \frac{U_i^n - U_i^{n-1}}{\Delta t}$$

(backward difference and first order in t) .... (3)

Substituting in (3) in (2) we get

$$\frac{U_i^n - U_i^{n-1}}{\Delta t} + C\,\frac{U_{i+1}^{n} - U_{i-1}^{n}}{2\Delta x} = 0$$ .... (A)

Solving "A" Further

$$U_i^n = U_i^{n-1} - C\,\frac{\Delta t}{2\Delta x}\left(U_{i+1}^{n} - U_{i-1}^{n}\right)$$

This is the approximate solution of the given PDE.