



Quantum Winter Hackathon

QUESTION 1

Team Quantico | 08-12-2020

1 Solution of 1D Wave Equation using Finite Difference Method

The wave equation given is

$$\frac{\partial u}{\partial t} + C \frac{\partial u}{\partial x} = 0$$

Our goal is to discretize the above equation in 2^{nd} order spatial accuracy and 1^{st} order time accuracy with the implicit time integration scheme.

$$\frac{\partial u}{\partial t} = -C \frac{\partial u}{\partial x}$$

Let $\frac{\partial u}{\partial x} = U_x$ and $\frac{\partial u}{\partial t} = U_t$ and the PDE becomes $U_x + CU_t = 0$. — — — — —eq 1

2 Spatial Discretization

Let the space step be Δx where

$$\Delta x = \frac{b - a}{N - 1}$$

N is the number of points and $x \in [a, b]$.

We find the points on x axis such that $x_{i+1} = x_i + \Delta x$ where $i = 0, 1, 2, \dots, 40$.

The central difference formula can be applied to approximate U_x as

$$U_x = \frac{\partial u}{\partial x} = \frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} \text{ — — — — —eq 2}$$

at a time instant t_n .

Substituting eq 2 in eq 1 we get

$$U_t + C \frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} = 0 \text{ — — — — —eq 3}$$

3 Time Discretization

Let the time step be Δt where $t_n = t_0 + \Delta t$

The forward difference formula can be applied to approximate U_t as

$$U_t = \frac{\partial u}{\partial t} = \frac{U_i^{n+1} - U_i^n}{\Delta t} \text{ — — — — —eq 4}$$

using implicit time integration scheme

Substituting eq 4 in eq 3 we get

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + C \frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} = 0 \text{ — — — — —eq 5}$$

which is the approximation of given PDE.

By rearranging eq 5 we get

$$U_i^{n+1} = U_i^n - \frac{C\Delta t}{\Delta x}(U_{i+1}^n - U_i^n) \text{ --- eq 6}$$

eq 6 can be used to find U at any given point in space and time.