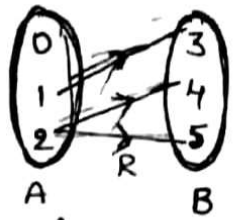
N Relation :- Any set of ordered pairs defines a
1) "binary relation". So it is called simply a relation.
An ordered pair means that each set is specified by two objects in a given fixed order. We denote an ordered pairs by (a,b). We also define that

$(a,b) = (c,d) \Rightarrow a=c, b=d$

If R is a relation from A to B if R is subset of $A \times B$. We say that R is a binary relation on A.

Eg:- $A = \{0,1,2\}$ $B = \{3,4,5\}$ Let $R = \{(1,3),(2,4),(2,5)\}$

$f : A \to B$



R is subset of $A \times B$. As such R is a relation from A to B and 1R3, 2R4, 2R5 this relation shown in diagram, called the "arrow diagram.

Matrix of a relation :-

Consider the sets $A = \{a_1, a_2, ---- a_m\}$ and $B = \{b_1, b_2, ---- b_n\}$ of order m and n respectively. Then $A \times B$ consists of all ordered pairs of the form (a_i, b_j)

Let $m_{ij} = (a_i, b_j)$ and assign the values 1 & 0 to m_{ij} according to the following rule

$m_{ij} = \{ 1$ if $(a_i, b_j) \in R$

The $m \times n$ matrix formed by m_{ij} is called the adjacency matrix (or) the matrix of the relation R is denoted by M_R (or) $M(R)$ 3

Eg:- Let $A = \{0, 1, 2\}$ and $B = \{p, q\}$ and the relation R from A to B defined by $R = \{(0,p), (1,q), (2,p)\}$ then find matrix representation.

Here $A = \{a_1, a_2, a_3\}$ & $B = \{b_1, b_2\}$

where $a_1 = 0$, $a_2 = 1$, $a_3 = 2$, $b_1 = p$, $b_2 = q$

we note that

$m_{11} = (a_1, b_1) = (0, p) = 1$ because $(0, p) \in R$

$m_{12} = (a_1, b_2) = (0, q) = 0$ because $(0, q) \notin R$

$m_{21} = (a_2, b_1) = (1, p) = 0$ because $(1, p) \notin R$

$m_{22} = (a_2, b_2) = (1, q) = 1$ because $(1, q) \in R$

$m_{31} = (a_3, b_1) = (2, p) = 1$ because $(2, p) \in R$

$m_{32} = (a_3, b_2) = (2, q) = 0$ because $(2, q) \notin R$

The matrix of relation R is

$$M_R = \begin{array}{cc} & \begin{array}{cc} b_1 & b_2 \\ p & q \end{array} \\ \begin{array}{cc} a_1 & 0 \\ a_2 & 1 \\ a_3 & 2 \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}_{3 \times 2} \end{array}$$

Eg:- Set $A = \{1, 2, 3, 4\}$ and a relation R defined on A by $R = \{(1,2), (1,3), (2,4), (3,2)\}$. Then here $A = \{a_1, a_2, a_3, a_4\} = B$, where $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$

$m_{ij} = a_i, b_j = (a_i, a_j) = (i, j) : A = B$

where $i = 1, 2, 3, 4$ (4)

$j = 1, 2, 3, 4$

$M_{11} = (a_1, a_1) = (1,1) = 0$ because $(1,1) \notin R$

$m_{12} = (1, 2) = 1$

$m_{13} = (1,3) = 1$

$m_{21} = (a_2, a_1) = (2,1) = 0$

$m_{31} = (3,1) = 0$

$m_{41} = (4,1) = 0$

$m_{22} = (2,2) = 0$

$m_{23} = (2,3) = 0$

$m_{24} = (2,4) = 1$

$m_{32} = (3,2) = 1$

$m_{33} = (3,3) = 0$

$m_{34} = (3,4) = 0$

$m_{42} = (4,2) = 0$

$m_{43} = (4,3) = 0$

$m_{44} = (4,4) = 0$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Di graph of a relation :-

The pictorial representation of R is called a directed graph or Di graph of relation R.

A small circle is drawn for each element of A. This circles are called "vertices or nodes".

An arrow is drawn from the vertex a_i to the vertex a_j if and iff $a_i R a_j$. This is called an edge

The no. of edges (arrows) terminating at a vertex called in degree of that vertex the no. of edges a vertex is called the out degree of that vertex

Eg:- Let $A = \{a, b, c, d\}$ and R be a relation on A, that has the matrix $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ find

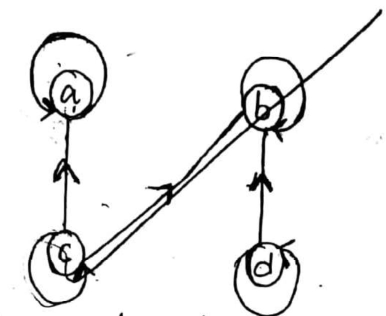
i) digraph ii) in degree iii) out degree

Sol:- Given relation R

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 1 & 1 & 1 & 0 \\ d & 0 & 1 & 0 & 1 \end{array}$$

$R = \{(a,a), (b,b), (c,a), (c,b), (c,c), (d,b), (d,d)\}$

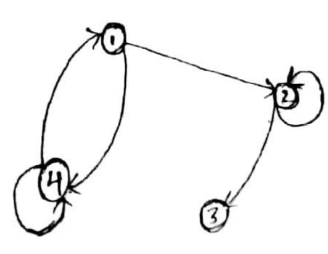
The digraph of this relation is



The in degree and out degree of the vertices as shown in below

Vertex	a	b	c	d
Indegree	2	3	1	1
Out degree	1	1	3	2

Eg:- Find the relation by the digraph given below. Also write down the matrix



Sol

$A = \{1, 2, 3, 4\}$

$R = \{(1,2)\ (2,2)\ (1,4)\ (2,3)\ (4,1)\ (4,4)\}$

$$M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

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Eg: Let $A = \{1, 2\}$ and $B = \{p, q, r, s\}$ and the relation R from A to B defined by $R = \{(1,q)\ (1,r)\ (2,p)\ (2,q)\ (2,s)\}$ write down the matrix of R.

Sol

Given $A = \{1, 2\}$

$B = \{p, q, r, s\}$

$$M_R = \begin{array}{c|c} & \begin{matrix} p & q & r & s \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Operations on relations :-

1) Union :- $R_1 \cup R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ (or) } (a,b) \in R_2\}$

2) Intersection :- $R_1 \cap R_2 = \{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ (and) } (a,b) \in R_2\}$

3) $R_1 - R_2$:- $\{(a,b) \in A \times B \mid (a,b) \in R_1 \text{ and } (a,b) \notin R_2\}$

4) $\overline{R}$:- $\{(a,b) \in A \times B \mid (a,b) \in \overline{R} \text{ and } (a,b) \notin R\}$

5) $(R^c)^c$:- R

A relation R from a set A to B that Converse of R denoted by R^c. is defined as a relation from B to A with the property that a, b belongs to R^c $((a,b) \in R^c)$ If and only iff $(a,b) \in R$

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** Consider the sets $A = \{a, b, c\}$ $B = \{1, 2, 3\}$ the relation (36)
$R = \{(a,1)(b,1)(c,2)(c,3)\}$ and $S = \{(a,1)(a,2)(b,1)(b,2)\}$
from $A \to B$ determine i) $\overline{R}$ ii) $\overline{S}$ iii) $R \cup S$ iv) $R \cap S$
v) R^c vi) S^c.

Sol

Note:- $\overline{R} = (A \times B) - R$, $\overline{S} = (A \times B) - S$ (7)

$A = \{a, b, c\}$ $B = \{1, 2, 3\}$

i) $\overline{R} = (A \times B) - R$

$A \times B = (a,1)(a,2)(a,3)(b,1)(b,2)(b,3)(c,1)(c,2)(c,3)$

$\overline{R} = \{(a,1)(a,2)(a,3)(b,1)(b,2)(b,3)(c,1)(c,2)(c,3)\}$
$- \{(a,1)(b,1)(c,2)(c,3)\}$

$\overline{R} = \{(a,2)(a,3)(b,2)(b,3)(c,1)\}$

ii) $\overline{S} = (A \times B) - S$

$\overline{S} = \{(a,1)(a,2)(a,3)(b,1)(b,2)(b,3)(c,1)(c,2)(c,3)\} - \{(a,1)(a,2)$
$(b,1)(b,2)\}$

$\overline{S} = \{(a,3)(b,3)(c,1)(c,2)(c,3)\}$

iii) $R \cup S$

$= \{(a,1)(b,1)(c,2)(c,3)\} \cup \{(a,1)(b,1)(a,2)(b,2)\}$

$= \{(a,1)(b,1)(a,2)(b,2)(c,2)(c,3)\}$

iv) $R \cap S$

$= \{(a,1)(b,1)(c,2)(c,3)\} \cap \{(a,1)(b,1)(a,2)(b,2)\}$

$= \{(a,1)(b,1)\}$

v) R^c

$R = \{(a,1)(b,1)(c,2)(c,3)\}$

$R^c = \{(1,a)(1,b)(2,c)(3,c)\}$

vi) S^c

$S = \{(a,1)(a,2)(b,1)(b,2)\}$

$S^c = \{(1,a)(2,a)(1,b)(2,b)\}$

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Eg: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ The relations R, S from $A \rightarrow B$ are represented by the following matrices determine i) $\overline{R}$ ii) $R \cup S$ iii) $R \cap S$ iv) S^c v) and their matrix representation.

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_S = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Sol

$R = \{(1,1)\ (1,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\}$

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,4)\ (3,2)\ (3,4)\}$

i) $\overline{R} = (A \times B) - R$

$A \times B = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)(3,3)(3,4)\}$

$\overline{R} = \{(1,2)(1,4)(2,1)(2,2)(2,3)(3,4)\}$

ii) $R \cup S$

$\{(1,1)(1,2)(1,3)(1,4)(2,4)(3,1)(3,2)(3,3)(3,4)\}$

iii) $R \cap S$

$\{(1,1)\ (1,3)\ (2,4)\ (3,2)\}$

iv) S^c.

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,4)\ (3,2)\ (3,4)\}$

$S^c = \{(1,1)\ (2,1)\ (3,1)\ (4,1)\ (4,2)\ (2,3)\ (4,3)\}$

v) $M\overline{R}$:

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

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$M_{R \cup S}$:-

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$M_{R \cap S}$:-

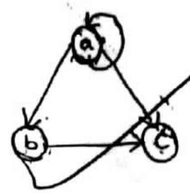
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

M_{S^c} :-

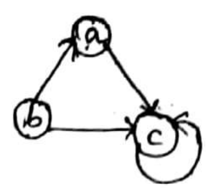
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Eg:- The digraph of two relations R and S and the set $A = \{a, b, c\}$ are given below draw the di-graph of i) $\bar{R}$ ii) $R \cup S$ iii) $R \cap S$ iv) $\bar{S}$ v) R^c vi) S^c

R =



S =



Sol:

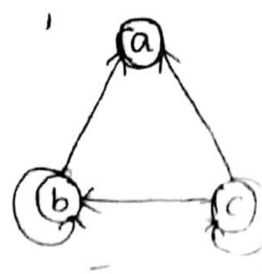
$R = \{(a,a)\ (a,b)\ (a,c)\ (b,c)\}$

$S = \{(b,a)\ (a,c)\ (c,c)\ (b,c)\}$

i) $\bar{R} = A \times A - R$

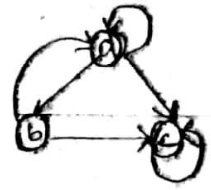
$= \{(a,a)\ (a,b)\ (a,c)\ (b,a)\ (b,b)\ (b,c)\ (c,a)\ (c,b)\ (c,c) - (a,a)\ (a,b)\ (a,c)\ (b,c)\}$

$= \{(b,a)\ (b,b)\ (c,a)\ (c,b)\ (c,c)\}$



ii) $R \cup S$

$= \{(a,a)(a,b)(a,c)(b,c)(b,a)(c,c)\}$



iii) $R \cap S$

$= \{(a,a)(a,b)(a,c)(b,c)\} \cap \{(b,a)(a,c)(c,c)(b,c)\}$

$= \{(a,c)(b,c)\}$

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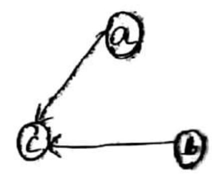
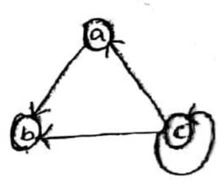


iv) S^c

$S = \{(b,a)(a,c)(c,c)(b,c)\}$

$S^c = \{(a,b)(c,a)(c,c)(c,b)\}$



v) R^c

$R = \{(a,a)(a,b)(a,c)(b,c)\}$

$R^c = \{(a,a)(b,a)(c,a)(c,b)\}$

iv) $\bar{S} = A \times B - R$

$= A \times A - R$

$A \times A = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

$\bar{S} = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$
$- \{(b,a)(a,c)(c,c)(b,c)\}$

$\{(a,a)(a,b)(b,b)(c,a)(c,b)\}$

Transitive relation:-

A relation R on set A is said to be a transitiv relation if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $(a,b,c) \in A$. It follows that R is not transitive if their exists $(a,b,c) \in A$ such th $(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$.

Eg: If we consider a set $A = \{1,2,3\}$ the relation

$R_1 = \{(1,1)\underbrace{(1,2)(2,3)}(1,3)\underbrace{(3,1)(3,2)}\}$, $R_2 = \{(1,2)(2,3)(1,3)(3,1)\}$

(a,c) ∈ R (a,b) ∈ R (b,c) ∈ R

on A then R_1 is transitive R_2 is not transitive

① Let $A = \{1,2,3\}$ determine the nature of the following relations on A

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i) $R_1 = \{(1,2)(2,1)(1,3)(3,1)\}$.
ii) $R_2 = \{(1,1)(2,2)(3,3)(2,3)\}$
iii) $R_3 = \{(1,1)(2,2)(3,3)\}$
iv) $R_4 = \{(1,1)(2,2)(3,3)(2,3)(3,2)\}$
v) $R_5 = \{(1,1)(2,3)(3,3)\}$
vi) $R_6 = \{(2,3)(3,4)(2,4)\}$
vii) $R_7 = \{(1,3)(3,2)\}$.

Sol:-

i) $R_1 = \{(1,2)(2,1)(1,3)(3,1)\}$

R_1 is symmetric and it is not reflecxive.

ii) $R_2 = \{(1,1)(2,2)(3,3)(2,3)\}$

R_2 is reflecxive and it is not symmetric

iii) $R_3 = \{(1,1)(2,2)(3,3)\}$

R_3 is reflecxive

iv) $R_4 = \{(1,1)(2,2)(3,3)(2,3)(3,2)\}$

R_4 is reflecxive and also symmetric so it is compatabilit relation

v) $R_5 = \{(1,1)(2,3)(3,3)\}$

It is not reflecxive and not symmetric

vi) $R_6 = \{(2,3)(3,4)(2,4)\}$ when $A = \{1,2,3,4\}$ it is transitive.

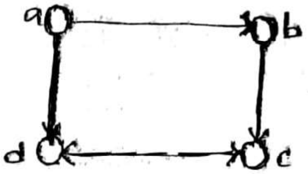
vii) R [illegible] not symmetric

2) Q5 Let $A = \{1,2,3,4\}$ determine the nature of the following relations on A

i) $R_1 = \{(1,1)(1,2)(2,1)(2,2)(3,3)(3,4)(4,3)(4,4)\}$

ii) $R_2 = \{(1,2)(1,3)(3,1)(1,1)(3,3)(3,2)(1,4)(4,2)(3,4)\}$

iii) $R_3 = \{$ Represented by the following graph



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Sol: i) $R_1 = \{(1,1)(1,2)(2,1)(2,2)(3,3)(3,4)(4,3)(4,4)\}$

It is reflexive because $\{(1,1)(2,2)(3,3)(4,4)\}$

It is symmetric because $\{(1,2)(2,1)\}$

It is transitive because $\{(1,2)(2,1)(1,1)\}$

Then it is also compatability relation because it is both reflexive and symmetric

ii) $R_2 = \{(1,2)(1,3)(3,1)(1,1)(3,3)(3,2)(1,4)(4,2)(3,4)\}$

It is not reflexive

It is symmetric because $\{(1,3)(3,1)\}$

It is transitive because $\{(1,3)(3,1)(1,1)\}$

iii) $R_3 = \{(a,b)(a,c)(a,d)(b,c)(c,d)\}$.

It is not reflexive, not transitive not symmetric

3) find the nature of the relations represented by the following matrices

i) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ iii) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Properties of binary relations :-

i) Reflective relation :

→ A Relation R on a set A is said to be reflexiv if $(a,a) \in R$, for all $a \in A$

→ It follows that R is not reflexive if there is some $a \in A$ such that $(a,a) \notin R$

Eg:- If $A = \{1, 2, 3, 4\}$ then the relation $R = \{(1,1)(2,2)(3,3)\}$ is not reflexive because $4 \in A$ but $(4,4) \notin R$.

Note:- A reflexive relation must have one's (1's) on it's diagonal

ii) Irreflective relation :-

→ A relation R on a set A is said to be irreflexive relation if $(a,a) \notin R$, for any $a \in A$. That is a relation R is irreflexive if no elements of A is related to it self by R.

Eg:- Consider the relation $R = \{(1,1)(1,2)\}$ define on the set $A = \{1, 2, 3\}$. The relation is not reflexive becaus $(2,2) \notin R$, $(3,3) \notin R$ the relation not irreflexive because $(1,1) \in R$.

Note:- A irreflexive relation must have 0's on it's diagonal

iii) Symmetric relation :-

→ A relation R on a set A is said to be symmetric if $(b,a) \in R$ whenever $(a,b) \in R$ for all $(a,b) \in A$.

→ It follows that R is not symmetric if their exists $(a,b) \in A$ such that $(a,b) \in R$ but $(b,a) \notin R$

→ A relation which is not symmetric is called an "Asymmetric relation

Eg :- If A $\{1,2,3\}$ & $R_1 = \{(1,1)(1,2)(2$ ... R_2 $\{(1,2)(2,1)(3,1)$

... asymmetric becau

iv) Compatability relation :-

→ A relation R on set A is said to be compatability relation which contain both reflecxive and symmetric relation.

Reflecxive :- If $(a,a) \in R$ for all $a \in A$

Symmetric :- $(b,a) \in R$ whenever $(a,b) \in R$ for all $(a,b) \in A$

Eg:- $R_1 = \{(1,1)\ (2,2)\ (3,3)\ (1,3)\ (3,1)\}$ (13)

$R_2 = \{(1,1)\ (2,2)\ (1,2)\ (2,1)\}$.

$R_3 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\}$ are relations on set $A = \{1, 2, 3\}$ then R_1 is compatability relation because it is reflecxive i.e., $(1,1) \in R$ $(2,2) \in R$ $(3,3) \in R$ on set A and symmetric relation i.e., $(1,3) \in R$, $(3,1) \in R$. R_2 is not a compatability relation because R_2 is symmetric but not reflecxive. R_3 is reflecxive but not symmetric so it is not compatability relation

v) Anti symmetric relation :-

→ A relation R on set A is said to be anti symmetric if whenever $(a,b) \in R$ and $(b,a) \in R$ then $a = b$

⇒ It follows that R is not anti symmetric if their exists $(a,b) \in A$ such that $(a,b) \in R$ and $(b,a) \in R$ but $a \neq b$.

Eg:- Let $A = \{1, 2, 3\}$, $R_1 = \{(1,1)\ (2,2)\}$, $R_2 = \{(1,2)\ (2,1)\ (2,3)\}$

we check that R_1 is both symmetric and anti symmetric and R_2 is neither symmetric nor anti symmetric.

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R = \{(1,2)(1,3)(2,1)(2,2)(3,1)(3,3)(3,4)(4,3)(4,4)\}$

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It is not reflexive

It is symmetric because $\{(1,2)(2,1)\}$

It is also transitive.

ii)
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R = \{(1,1)(1,3)(2,2)(2,4)(3,1)(3,3)(4,2)(4,4)\}$

It is reflexive because $\{(1,1)(2,2)(3,3)(4,4)\}$

It is symmetric because $\{(1,3)(3,1)\}$

Then it is compatability

It is transitive because $\{(3,4)(4,3)(4,4)\}$

iii)
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$R = \{(1,3)(1,4)(2,3)(3,4)(4,1)\}$

It is irreflexive (diagonal 0's)

It is symmetric because $\{(1,4)(4,1)\}$

Equivalence :-

A relation R on set A is said to be an equivalence relation on A if

i) R is reflexive
ii) R is symmetric
iii) R is transitive

} on A

Eg: (1) Let $A = \{1,2,3,4\}$ and $R = \{(1,1)(1,2)(2,1)(2,2)(3,4)(3,3)(4,4)\}$ be a relation on A and verify that R is an equivalence relation or not.

Sol:

$R = \{(1,1)(1,2)(2,1)(2,2)(3,4)(4,3)(3,3)(4,4)\}$

R is reflexive because $\{(1,1)(2,2)(3,3)(4,4)\}$

R is symmetric because $\{(1,2)(2,1)\} \in R$

R is transitive because $\{(1,2)(2,1)(1,1)\}$

$\therefore$ R is equivalence relation.

(2) Let $A = \{1,2,3,4\}$ $R = \{(1,1)(2,1)(2,2)(3,1)(3,3)(1,3)(4,1)(4,4)\}$ be a relation on A.

Sol:

$R = \{(1,1)(2,1)(2,2)(3,1)(3,3)(1,3)(4,1)(4,4)\}$

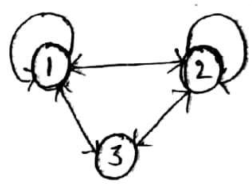
R is reflexive because $\{(1,1)(2,2)(3,3)(4,4)\} \in R$

R is not symmetric because $(1,2) \notin R$ $(2,1) \in R$

$(3,1) \in R$ $(1,3) \in R$, $(4,1) \in R$, $(1,4) \notin R$

Since R is not symmetric then it is not equivalence relation.

) The digraph of relation R on the set $A = \{1,2,3,4\}$ in given below. Determine whether R is an equivalence relation or not



Sol: Given Relation $R = \{(1,1)(1,2)(1,3)(2,2)(2,1)(2,3)(3,1)(3,2)\}$

The relation R is symmetric and transitive but not reflexive because $(3,3) \notin R$.

$\therefore$ R is not an equivalence relation

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Partial order relations:

A binary relation R on set P is called a partial order relations if R is

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i) Reflexive
ii) Anti symmetric
iii) Transitive.

bit
We denote a partial ordering by the symbol "≤".

If ≤ is a partial ordering on P, then the ordered pair is (P, ≤) is called a partially ordered set or "poset

Hasse diagram (or) poset diagram:-

A partial ordering '≤' on a set P can be represent by means of a diagram known as Hasse-diagram. of (P, ≤)

Some rules of Hasse diagram:-

1) The partial order is reflexive at every vertex in the digraph of a partial order. There would be a loop while drawing the digraph of a partial order we need not exhibit such loops'

2) In the digraph of a partial order, there is an edge from a vertex 'a' to vertex 'b' and there is an edge from vertex 'b' to 'a', vertex 'c' then there should be an edge from 'a' to 'c' because it is transitive. we need not exhibit an edge from a to c

3) To simplify the format of the digraph of a partial order we represent the vertices by dots (or) small circles

4) Draw the digraph in a such way that "all edges point upward"

5) We need not ... the edges

1) Let $P = \{1, 2, 3, 4\}$ and $\leq$ be the relation "less than (or) equal to" then drawn the Hasse diagram.

Sol

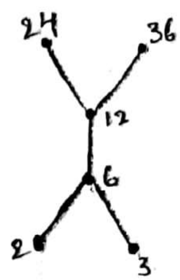
$1 \leq 2$

$2 \leq 3$

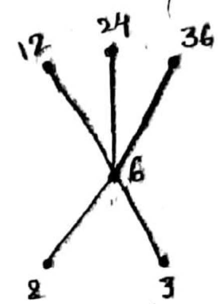
$3 \leq 4$

(or)

2) Let $X = \{2, 3, 6, 12, 24, 36\}$ relation $\leq$ be such that $x \leq y$ if x divides y.



(or)



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3) Draw the hasse-diagram represented with positive divisors of 36:

Sol: The set of all the divisors of 36 is

$P = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

The relation R of divisobility is a partial order and this set we note that 1 is related to all elements $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

2 is related to $\{2, 4, 6, 12, 18, 36\}$

3 is related to $\{3, 6, 9, 12, 18, 36\}$

4 is related to $\{4, 12, 36\}$

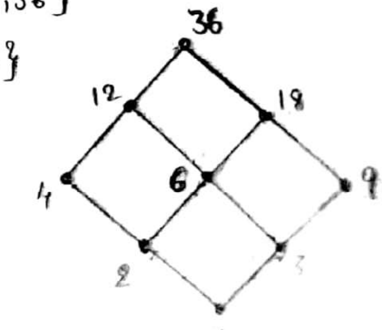
6 is related to $\{6, 12, 18, 36\}$

9 is related to $\{9, 18, 36\}$

12 is related to $\{12, 36\}$

18 is related to $\{18, 36\}$

36 is related to $\{36\}$



4) Let A be a given finite set and P(A) it's power set. Let $\subseteq$ be the inclusion relation on the elements P(A) draw the Hasse diagram of $(P(A), \subseteq)$ for

i) $A=\{a\}$ ii) $A=\{a,b\}$ iii) $A=\{a,b,c\}$ iv) $A=\{a,b,c,d\}$

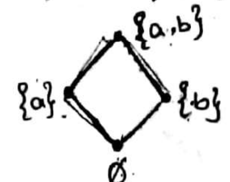
Sol:

i) $A=\{a\} = (\emptyset, a)$

$2^1 = 2$

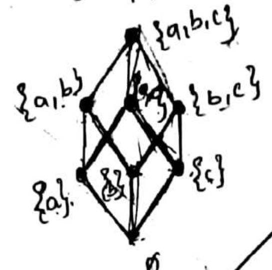
$2^2 = 4$.

ii) $A=\{a,b\} = (\emptyset, a, b, (a,b))$



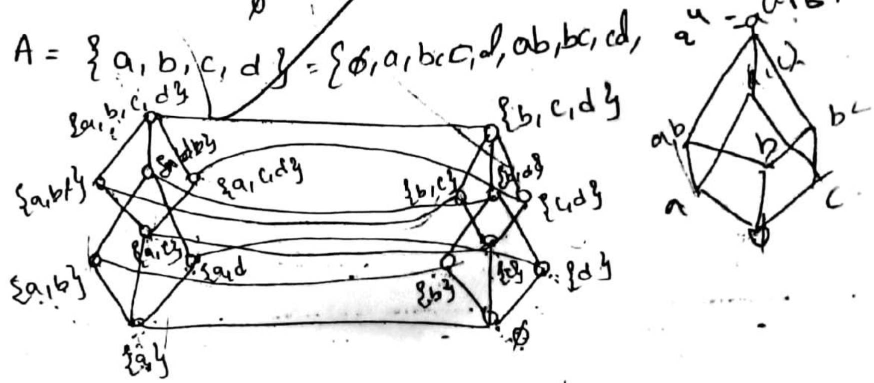
iii) $A=\{a,b,c\} = \emptyset, a, b, c, (a,b), (b,c), (c,a), (a,b,c)$

$2^3 = 8$.

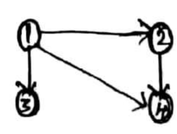


iv) $A=\{a,b,c,d\} = \{\emptyset, a, b, c, d, ab, bc, cd,$

2^4



5. A partial order R on set $A=\{1,2,3,4\}$ is represented by the following digraph from the Hasse diagram for R.

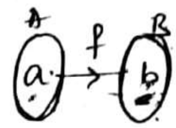


Sol

The graph $R=\{(1,2)\,(1,3)\,(1,4)\,(2,4)\}$ the hasse diagram for R is

Functions :- Let A and B are two non empty sets then a function (or) maping $f: A \to B$ is a relation from A to B such that for each 'a' in 'A' there is a unique b in 'B' such that $(a,b) \in f$. Then we write $b = f(a)$. Here B is called the Image of a and a is called the pre - image of b.

A function f from $A \to B$ is denoted by $f: A \to B$ the pictorial representation of f as shown in below.



$f : A \to B$.

Note:- Every function is a relation but every relation is not a function. for the function, $f: A \to B$, A is called the domain of f, B is called the co domain of f.

Sum of observations of a function $f: A \to B$:-

* Every element of A has an image in B, and ~~in~~ an element of 'a' of 'A' under 'f' as ~~shown~~ two images in B then the two images cannot be different.
* An element $b \in B$ need not have a pre image in 'A' under 'f'.
* If an element $b \in B$ has a pre - image $a \in A$ under f the pre- image need not be unique. In other words two different elements of A can have the same image in B under f.
* The Statement $(a,b) \in f$, afb and $b = f(a)$ are equivalent
* If g is function from $A \to B$ $(g: A \to B)$ then $f = g$ if and only iff $f(a) = g(a)$ for every $a \in A$

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1) Let $A = \{1,2,3\}$ & $B = \{-1,0\}$ and R be a relation from A to B defined by $R = \{(1,-1)(1,0)(2,-1)(3,0)\}$ if R is a function from $A \to B$.

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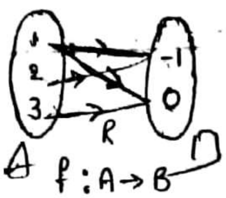
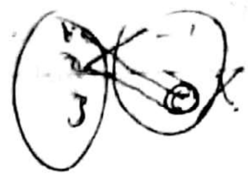
Sol:



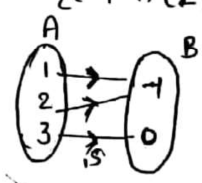


$f: A \to B$

The element 1(A) is related to different elements -1 & 0 Therefore R is not a function.

2) Let $A = \{1,2,3\}$ & $B = \{-1,0\}$ and S be a relation from A to B defined by $S = \{(1,-1)(2,-1)(3,0)\}$ is a function.

Sol



$S: A \to B$

The elements of A is related to unique element of B that is 1 is related to -1, 2 is related to -1, 3 is related to 0.

$\therefore$ S is a function

3) Let $A = \{0, \pm 1, \pm 2, 3\}$ consider the function $f: A \to R$ define by $f(x) = x^3 - 2x^2 + 3x + 1$ for $x \in A$ find the range of f

Sol Given that

$$A = \{0, \pm 1, \pm 2, 3\}$$

$$f(x) = x^3 - 2x^2 + 3x + 1$$

$x = 0$

$$f(0) = 0^3 - 2(0)^2 + 3(0) + 1$$

$$f(0) = 1$$

When $x = -1$

$$f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 1$$

$$= -1 - 2 - 3 + 1$$

$$f(-1) = -5$$

when $x = 1$

$f(1) = (1)^3 - 2(1)^2 + 3(1) + 1$

$= 1 - 2 + 3 + 1$

$f(1) = 3$

when $x = -2$

$f(-2) = (-2)^3 - 2(-2)^2 + 3(-2) + 1$

$= -8 - 8 - 6 + 1$

$f(-2) = -21$

when $x = 2$

$f(2) = (2)^3 - 2(2)^2 + 3(2) + 1$

$= 8 - 8 + 6 + 1$

$f(2) = 7$

when $x = 3$

$f(3) = (3)^3 - 2(3)^2 + 3(3) + 1$

$= 27 - 18 + 9 + 1$

$f(3) = 19$

$f(A) = \{1, 3, -5, -21, 7, 19\}$

4) Let $A = \{0, \pm 2, \pm 4, \pm 6\}$ consider the function $f: A \to B$ define by $f(x) = x^4 - 2x^2$ for $x \in A$ find the range of f

sol Given that

$A = \{0, \pm 2, \pm 4, \pm 6\}$

$f(x) = x^4 - 2x^2$

when $x = 0$

$f(0) = (0)^4 - 2(0)^2$

$= 0$

(23) (44)

when $x = 2$

$$f(2) = (2)^4 - 2(2)^2$$
$$= 16 - 8$$
$$f(2) = 8$$

when $x = -2$

$$f(-2) = (-2)^4 - 2(-2)^2$$
$$= 16 - 8$$
$$= 8$$

when $x = 4$

$$f(4) = (4)^4 - 2(4)^2$$
$$= 256 - 32$$
$$f(4) = 224$$

when $x = -4$

$$f(-4) = (-4)^4 - 2(-4)^2$$
$$= 256 - 32$$
$$f(-4) = 224$$

when $x = 6$

$$f(6) = (6)^4 - 2(6)^2$$
$$f(6) = 1224$$

when $x = -6$

$$f(-6) = (-6)^4 - 2(-6)^2$$
$$f(-6) = 1224$$

$$f(A) = \{0, 8, 8, 224, 224, 1224, 1224\}$$

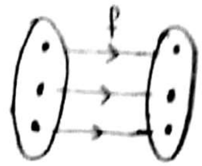
N* Types of functions :-

* **Onto function** :- A function $f: A \to B$ is said to be a onto function if every element of B has a pre-image in A under f.

When f is an onto function from $A \to B$

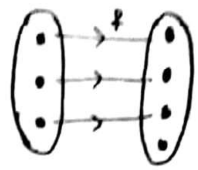
An onto function is also called as a surjective function.

Eg:- 1.



Surjective

2.



not Surjective.

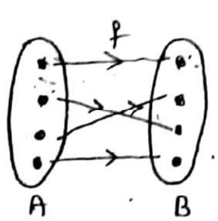
* <u>One to one function</u> :-

* A function $f: A \to B$ is said to be a one to one function if different elements of A have different images in B under f. i.e., if whenever $a_1, a_2 \in A$ with $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$

(or)

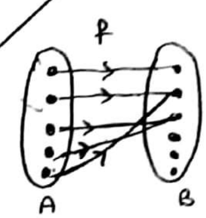
* Equivalently that if whenever $f(a_1) = f(a_2)$ for $a_1, a_2 \in A$ then $a_1 = a_2$.

Eg:- 1)



One to one function

2)



not one to one function

* Thus if $A: A \to B$ is a one to one function then every element of A has a unique image 'B' and every element of $f(A)$ has a unique preimage in A.

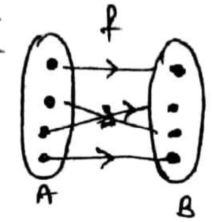
* A one to one function is also called an <u>injective function</u>.

* Bijective function :

* A function $f: A \to B$ is said to be bijective functi which is both one to one and onto functions.

* A bijective function is also called a one to one correspondence.

Eg:-

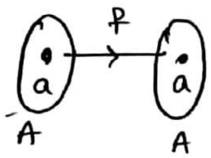


one to one correspondence

* Identity function :-

* A function $f: A \to A$ such that $f(a) = a$ for every $a \in A$ is called the identity function (or) identity maping on A. It is represented by the symbol 'I'.

Eg:-



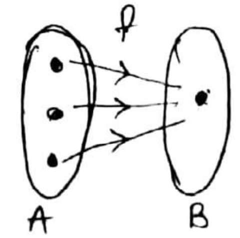
* Constant function :-

A function $f: A \to B$ such that $f(a) = c$ for every $a \in A$ where c is a fixed element of B is called a constant function.

In otherwise a functi f from $A \to B$ is constant function if all elements of A have the same image in B so that.

$f(a) = \{c\}$

Eg:-



Constant function.

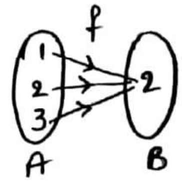
1) Find the nature of the following functions define on set $A = \{1, 2, 3\}$ i) $f = \{(1,1)(2,2)(3,3)\}$ ii) $g = \{(1,2)(2,2)(3,2)\}$ iii) $h = \{(1,2)(2,2)(3,1)\}$ iv) $P = \{(1,2)(2,3)(3,1)\}$

Sol:- i) $f = \{(1,1)(2,2)(3,3)\}$

we note that for every $a \in A$, $(a,a) \in f$ i.e., $a = f(a)$ there fore $(1,1)(2,2)(3,3)$ are identity function.

ii) $g = \{(1,2)(2,2)(3,2)\}$

we note that every $a \in A$ has two as its image i.e., $g(1) = 2$, $g(2) = 2$, $g(3) = 3$ there fore g is constant function.



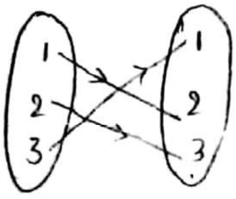
iii) $h = \{(1,2)(2,2)(3,1)\}$

we note 'h' is neither identity function nor a constant function. The range of h is $(2,1) \in A$. the element 3 has no preimage under 'h'.

There fore 'h' is not onto & not one-to-one

iv) $P = \{(1,2)(2,3)(3,1)\}$

We note that every element of A has a unique image and every element of A has unique pre image under p. there fore p is both one to one and onto functions. There fore p is bijective.



2) A function $f: R \to R$ and $g: R \to R$ are defined by
$f(x) = 3x+7$, $g(x) = x(x^3-1)$ $\forall x \in R$ verify that f is one to one function but g is not. (Q 7)

Sol: For any $x_1, x_2 \in R$, we have $f(x_1) = 3x_1 + 7$
$f(x_2) = 3x_2 + 7$ if $f(x_1) = f(x_2)$ we have.

$$3x_1 + 7 = 3x_2 + 7$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$

$\therefore f$ is one to one function

$g(x_1) = g(x_2)$
$x_1(x_1^3 - 1) = x_2(x_2^3 - 1)$
$x_1^3 x - x_1 = x_2 \cdot x_2^3 - x_2$
$g(x_1) \neq g(x_2)$

We note that $g(\underset{x_1}{0}) = 0(0^3 - 1)$

$$= 0$$

$$g(\underset{x_2}{1}) = 1(1^3 - 1)$$

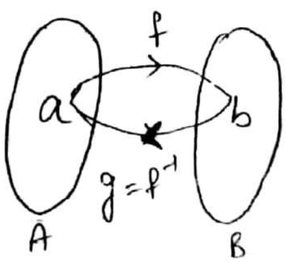
$$= 0$$

$\therefore x_1 = 0$, $x_2 = 1$, we have $g(x_1) = g(x_2)$ but $x_1 \neq x_2$

$\therefore g$ is not one-to-one function.

Invertable function :- (inverse function)

A function $f: A \to B$ is said to be invertable if there exists a function $g: B \to A$ such that $g\circ f = I_A$ and $f \circ g = I_B$, where I_A is the identity function on A & I_B is the identity function on B then g is called inverse of f and we write $g = f^{-1}$.



1) Let $A = \{1,2,3,4\}$ and f & g be functions from $A \to A$ given by $f = \{(1,4)(2,1)(3,2)(4,3)\}$ & $g = \{(1,2)(2,3)(3,4)(4,1)\}$ Prove that f and g are inverse of each other. (SE)

Sol Given that $A = \{1, 2, 3, 4\}$

$f = \{(1,4)(2,1)(3,2)(4,3)\}$

$g = \{(1,2)(2,3)(3,4)(4,1)\}$

$gof(1) = g[f(1)]$
$= g(4)$
$= 1 = I_A(1).$

$gof(2) = g[f(2)]$
$= g[1]$
$= 2 = I_A(2)$

$gof(3) = g[f(3)]$
$= g[2]$
$= 3 = I_A(3)$

$gof(4) = g[f(4)]$
$= g[3]$
$= 4 = I_A(4)$

$fog(1) = f[g(1)]$
$= f(2)$
$= 1 = I_A(1).$

$fog(2) = f[g(2)]$
$= f(3)$
$= 2 = I_A(2)$

$fog(3) = f[g(3)]$
$= f(4)$
$= 3 = I_A(3)$

$fog(4) = f[g(4)]$
$= f(1)$
$= 4 = I_A(4)$

For all $x \in A$, we have $gof(x) = I_A(x)$ and $fog(x) = I_A(x)$

$\therefore$ g is inverse of f and f is inverse of g

Let $f(x) = x^2 - 3x + 2$ find i) $f(x^2)$ ii) $f(y-x)$ iii) $f(x+3)$ (47)

(29)

Sol $f(x) = x^2 - 3x + 2$

i) $f(x^2) = (x^2)^2 - 3(x^2) + 2$

$f(x^2) = x^4 - 3x^2 + 2$

ii) $f(y-x) = (y-x)^2 - 3(y-x) + 2$

$= y^2 + x^2 - 2xy - 3y + 3x + 2$

$f(y-x) = x^2 + y^2 - 2xy + 3x - 3y + 2$

iii) $f(x+3) = (x+3)^2 - 3(x+3) + 2$

$= x^2 + \cancel{9} + 6x - 3x - \cancel{9} + 2$

$f(x+3) = x^2 + 3x + 2$

3) Let $f(x) = 2x - 3$ and $g(x) = x^2 + 3x + 5$ find gof & fog.

Sol

$f(x) = 2x - 3$

$g(x) = x^2 + 3x + 5$

$gof(x) = g[f(x)]$ $\quad \because f(x) = 2x - 3$

$= g[2x-3]$

$g(2x-3) = (2x-3)^2 + 3(2x-3) + 5$

$= 4x^2 + \cancel{9} - 12x + 6x - \cancel{9} + 5$

$= 4x^2 - 6x + 5$

$fog(x) = f[g(x)]$

$= f[x^2 + 3x + 5]$ $\quad \because g(x) = x^2 + 3x + 5$

$f[x^2 + 3x + 5] = 2(x^2 + 3x + 5) - 3$

$= 2x^2 + 6x + 10 - 3$

$= 2x^2 + 6x + 7$

4) Let f and g be a functions from the +ve real numbers to +ve real numbers defined by $f(x) = 2x$ & $g(x) = x^2$ calculate fog & gof.

(10)

Sol

Given, $f(x) = 2x$

$g(x) = x^2$

$fog(x) = f[g(x)]$

$= f(x^2)$

$f(x^2) = 2x^2$

$= 2x^2$

$gof(x) = g[f(x)]$

$= g[2x]$

$g(2x) = (2x)^2$

$= 4x^2$

Imp

find the Inverse of following functions

i) $f(x) = \dfrac{10}{\sqrt[5]{7-3x}}$ ii) $f(x) = 4 \cdot e^{(5x+2)}$

Sol

i) $f(x) = \dfrac{10}{\sqrt[5]{7-3x}}$

Let $f(x) = y \Rightarrow x = f^{-1}(y)$

$$\frac{10}{\sqrt[5]{7-3x}} = y$$

$$10 = y\left(\sqrt[5]{7-3x}\right)$$

$$10 = y(7-3x)^{1/5}$$

On multiplying with powers 5 on both sides

$$10^5 = y^5 (7-3x)^{1/5 \times 5}$$

$$10^5 = y^5 (7-3x)$$

$10 = 7y^5 - 3xy^5$

$10^5 - 7y^5 = -3xy^5$

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$7y^5 - 10^5 = 3xy^5$

$\frac{7y^5 - 10^5}{3y^5} = x$

$\therefore x = \frac{7y^5 - 10^5}{3y^5} = f^{-1}(y)$

$\therefore y = f(x)$

$f^{-1}(x) = \frac{7x^5 - 10^5}{3x^5} \quad \forall\, x \in R$

ii). $f(x) = 4.e^{(6x+2)}$

Let $f(x) = y$

$4.e^{(6x+2)} = y$

Applying log on both sides

$\log(4.\,e^{(6x+2)}) = \log y$

$\log_x x = 1$

$\log_{10} 10 = 1$

$\log 4 + \log e^{6x+2} = \log y$

$0.602 + \log(e^{6x}.e^2) = \log y$

$0.602 + \log e^{6x} + \log e^2 = \log y$

$0.602 + 6x \log e + 2 \log e = \log y$ $\quad \log a^b = b \log a$

$0.602 + 6x.1 + 2.1 = \log y$

$0.602 + 6x + 2 = \log y$

$6x = \log y - 2.602$

$x = \frac{\log y - 2.602}{6}$

$y = f(x) \quad , \quad x = f^{-1}(y)$

$$f^{-1}(x) = \frac{\log x - 2.602}{6} \quad \forall x \in R$$ (32)

(or)

$$4 \cdot e^{(6x+2)} = y$$

$$e^{6x+2} = \frac{y}{4}$$

$$\log e^{6x+2} = \log\left(\frac{y}{4}\right)$$ $\log \frac{y}{x} = \log y - \log x$

$$6x+2 = \log y - \log 4$$ $\log_e e = 1$

$\log m^n = n \log m$

$$6x = \log y - \log 4 - 2$$

$$x = \frac{\log y - \log 4 - 2}{6}$$

$$x = \frac{\log y - 0.602 - 2}{6}$$

$$x = \frac{\log y - 2.602}{6}$$

$$y = f(x)$$

$$x = f^{-1}(y)$$

$$f^{-1}(x) = \frac{\log x - 2.602}{6} \quad \forall x \in R$$

6) Let $f(x) = x+5$ and $g(x) = 2x+3$ find

i) $fog(2)$ ii) $gof(7)$

Sol:

$$f(x) = x+5$$

$$g(x) = 2x+3$$

i) $fog(2) = f[g(2)]$

$= f[2x+3]$

$= f(2(2)+3)$

$= f(7)$

ii) $gof(7) = g[f(7)]$

$= g[x+5]$

$= g[7+5]$

$= g(12)$

$= 2x+3$

$= 2(12)+3$

$= 24+3$

$= 27$

Imp

7) If $f: R \to R$ & $g: R \to R$ define by $f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2+1}$ and $h(x) = x^4$ then find

a) $(fogoh)(x)$ b) $(hogof)(x)$ c) $(gog)(x)$ d) $(goh)(x)$

a) $fogoh(x) = f[goh](x)$

$= f[g(h(x))]$

$= f[g(x^4)]$

$= f\left[\frac{1}{(x^2)^4+1}\right]$

$= f\left[\frac{1}{x^8+1}\right]$

$= \left(\frac{1}{x^8+1}\right)^3 - 4\left(\frac{1}{x^8+1}\right)$

b) $hogof(x) = h[gof](x)$

$= h[g(f(x))]$

$= h[g(x^3-4x)]$

$= h\left[\left(\frac{1}{x^2+1}\right)^3 - 4\left(\frac{1}{x^2+1}\right)\right]$

$= \left(\frac{1}{(x^4)^2+1}\right)^3 - 4\left(\frac{1}{(x^4)^2+1}\right)$

$= \left(\frac{1}{x^8 \ldots}\right)^3 - 4\left(\frac{1}{x^8 \ldots}\right)$

c) $gog(x) = g[g(x)]$

$= g\left[\dfrac{1}{x^2+1}\right]$

$= \dfrac{1}{\left(\frac{1}{x^2+1}\right)^2+1}$

$= \dfrac{1}{\frac{1}{(x^2+1)^2}+1}$

$\dfrac{(x^2+1)^2}{1+(x^2+1)^2}$

d) $goh(x) = g[h(x)]$

$= g[x^4]$

$= \cancel{\left(\dfrac{1}{x^2+1}\right)^4}$

$= \dfrac{1}{x^8+1}$

b) $hogof(x) = hog(x^3-4x)$

$= h\left[\dfrac{1}{(x^3-4x)^2+1}\right]$

$= h\left(\dfrac{1}{x^6+16x^2-8x^4+1}\right)$

$= h\left(\dfrac{1}{x^6-8x^4+16x^2+1}\right)$

$= \left(\dfrac{1}{x^6-8x^4+16x^2+1}\right)^4$

$= \dfrac{1}{(x^6-8x^4+16x^2+1)^4}$

$= \left(\dfrac{1}{x^3 \text{[illegible]}}\right)^4$

2) $[\sim q \wedge (p \to q)] \to \sim p$

p	q	$\sim p$	① $\sim q$	② $p \to q$	③ ① $\wedge$ ②	③ $\to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

3) $[[(p \vee q) \wedge (p \to r)] \wedge (q \to r)] \to r$

p	q	r	① $p \vee q$	② $p \to r$	③ ① $\wedge$ ②	④ $q \to r$	⑤ ③ $\wedge$ ④	⑤ $\to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

4) $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

6) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	① $p \to q$	② $q \to r$	③ ① $\wedge$ ②	④ $p \to r$	③ $\to$ ④
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$[(p \lor q) \land (\sim p \lor r)] \rightarrow (q \lor r)$

p	q	r	(1) $p \lor q$	$\sim p$	(2) $\sim p \lor r$	(3) (1) $\land$ (2)	(4) $q \lor r$	(3) $\rightarrow$ (4)
T	T	T	T	F	T	T	T	T
T	T	F	T	F	F	F	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	F	T	T

7) $[(p \rightarrow q) \rightarrow r] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$

p	q	r	(1) $p \rightarrow q$	(2) (1) $\rightarrow r$	(3) $p \rightarrow q$	(4) $p \rightarrow r$	(5) (3) $\rightarrow$ (4)	(2) $\rightarrow$ (5)
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F	T
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	F	T	T	T	T

Let P, Q and R are the prepositions

p : you have the flee

q : you miss the final examination

r : you pass the course.

Write the prepositions into statement form

1. $p \rightarrow q$
2. $\sim p \rightarrow r$
3. $q \rightarrow \sim r$
4. $p \lor q \lor r$
5. $(p \rightarrow \sim r) \lor (q \rightarrow \sim r)$
6. $(p \land q) \lor (\sim q \land r)$

1. $p \rightarrow q$

If you have the flee then you miss the final examination.

2. $\sim p \rightarrow r$

If you not have the flee then you pass the course.

3. $q \rightarrow \sim r$

If you miss the final examination then you not pass the course.

4. $p \vee q \vee r$

You have the flee (or) you miss the final examination (or) you pass the course.

5. $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$

If you have the flee then you not pass the cou (or) if you miss the final examination then you n pass the course.

6. $(p \wedge q) \vee (\sim q \wedge r)$

If you have the flee and you miss the fina examination (or) you not miss the final examinat and you pass the course.

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Tautological Implications :-

P is any propostional statement (or) compound statement and q is propostional statement (or) compound statement, p is tautologically implied by q if $P \to q$ is a tautology. It is represented by the symbol "$P \Rightarrow q$"

Eg:- $P \wedge q \Rightarrow P$

P	q	$P \wedge q$	$P \wedge q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

Since all the entries in the last coloumn are true. $(P \wedge q) \to P$ is a talitology. Hence $\boxed{(P \wedge q) \Rightarrow P}$

(11) 7M ***

Eg ② $P \to (Q \to R) \Rightarrow [(P \to Q) \to (P \to R)]$

Sol

P	Q	R	① $Q \to R$	④ $P \to ①$	② $P \to Q$	③ $P \to R$	⑤ $② \to ③$	$④ \to ⑤$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Tautology

2nd 8) The function $f: R \to R$ be defined by i) $f(x) = x^2+1$ find $f^{-1}(-8)$, $f^{-1}(17)$ ii) $f(x) = x^2+3$ find $f^{-1}(7)$, $f^{-1}(19)$ 52

Sol

i) $f(x) = x^2+1$

$f(x) = y$

$f(x) = x^2+1$ 35

$f(x) = y$

$x^2+1 = y \Rightarrow x^2 = y-1$

$x = f^{-1}(y)$

$x = \sqrt{y-1}$

$y = f(x) \Rightarrow f^{-1}(y) = \sqrt{y-1} \quad \forall y \in R$

$f^{-1}(x) = \sqrt{x-1}$

$f^{-1}(-8)$ means that $x = -8$

$\sqrt{-8-1} = \sqrt{-9} = \pm 3i$

$= [3i, -3i] = \emptyset$

$f^{-1}(17)$ means that $x = 17$

$\sqrt{17-1} = \sqrt{16} = 4$

$= [4, -4]$

ii) $f(x) = x^2+3$

$f(x) = y$

$x^2+3 = y$

$x^2 = y-3 \Rightarrow x = \sqrt{y-3}$

$y = f(x) = f^{-1}(y) = \sqrt{y-1} \quad \forall y \in R$

$f^{-1}(x) = \sqrt{x-3}$

$f^{-1}(7)$ means that $x = 7$

$\sqrt{7-3} = \sqrt{4} = \pm 2$

$= [2, -2]$

$f^{-1}(19)$ means that $x = 19$

$\sqrt{19-3} = \sqrt{16} = +4 \Rightarrow [4, -4]$

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Recursive function :-

A function called it self is called a recursive function any function $f: N^n \to N$ is called total because it is defined for every n-tuple in N^n

→ If $f: D \to N$ where $D \subseteq N^n$ then f is called partial

Eg:- $f(x,y) = x+y$ which is defined $\forall\, x, y \in N$ and hence is a total function $g(x,y) = x-y$ is defined for only those $x, y \in N$ which satisfy $x \geq y$ hence $g(x,y)$ is partial. A set of three functions called the initial function, which are used in defining other functions the initial functions are

→ zero functions , $Z: Z(x) = 0$
→ successor function , $S: S(x) = x+1$
→ projection function , $U_i^n: U_i^n(x_1, x_2 \cdots x_n) = x_i$

Note:-

The projection function is also called the generalised identity function.

$$U_1^2(x, y) = x$$

$$U_2^3(2, 4, 6) = 4 \ldots \text{etc}$$

① show that $\{\langle x, x\rangle / x \in N\}$ which defines the relation of equality is premitive recursive.

Sol

Given function $f(x,y) = \underline{x-y}$ Now

$\underline{x-y+1}$

$(x-y)+1$

$f(x-y)+1$

$\underline{f(x,y)+1}$

$S(x,y)$

as $f(x,0) = x = P_1(x)$

$f(x,y+1) = S\left(P_3^3(x,y,f(x,y))\right)$

Projective function

$\therefore$ $S P_1^1, P_2^2, P_3^3$ are initial function

$\therefore$ $f(x,y)$ is primitive recursive function

② Show that $f(x,y) = x^y$ is primitive recursive $\{(x,x) / x \in N$

Sol:

$f(x,0) = x^0$

$= 1 \;\; \forall (x \neq 0)$ and we get $x^0 = 0 \; \forall (x=0)$

$f(x,y+1) = x^{y+1}$

$= x^y \cdot x$

$= x * f(x,y) = U_1^1(x,y,f(x,y)) * U_3^3(x,y,f(x,y))$

Now $f(x,0) = S(Z(x))$ $\left(\because Z(x) = 0, \; S(x) = 0+1\right)$

$f(x,y+1) = S(f(x,y+1))$

$= S(x,y)$

$\boxed{\begin{aligned} & f(x,y+1) \\ & x+y+1 \\ & (x+y)+1 \\ & f(x,y)+1 \\ & S(x,y) \end{aligned}}$

③ show that $f(x,y) = x+y$, $(x,y) \in N$ is primitive recursive.

$f(x,y) = x+y$

$f(x,0) = x+0$

$= x = U_1^1(x)$

$f(x,y+1) = x+y+1$

$= S(x+y)$

$= S(f(x,y))$

$S(U_3^3(x,y,f(x,y)))$

$\therefore$ it follows that 'f' comes from primitive recursive of U_1^1 and U_3^3 (28)

$\therefore$ f is primitive recursive

4) Show that $f(x,y) = x * y$ $(x, y \in N)$ is primitive recursive.

Sol

$$f(x,0) = x * 0$$
$$= 0 \rightarrow (1)$$
$$f(x,y+1) = x*(y+1)$$
$$= x*y+x$$
$$= S(x*y+x)$$
$$= S(f(x*y,x))$$
$$= S(U_3^3(x,y,f(x*y,x)))$$

imp

5) Find the value of $f(2,5)$ by using $f(x,y) = x+y$ and the initial value $(2,0) = 2$

Sol

$$f(x,y) = x+y$$
$$f(2,1) = f(2,0)+1 = 2+1 = 3$$
$$f(2,2) = f(2,1)+1 = 3+1 = 4$$
$$f(2,3) = f(2,2)+1 = 4+1 = 5$$
$$f(2,4) = f(2,3)+1 = 5+1 = 6$$
$$f(2,5) = f(2,4)+1 = 6+1 = 7$$

HA

6) Let $f: R \rightarrow R$ is defined as $f(x) = 3x^3+3$ then find inverse function

Sol

Given $f(x) = 3x^3+3$

$f(x) = y \Rightarrow x = f^{-1}(y)$

$$3x^3 + 3 = y$$

39 54

$$3x^3 = y - 3$$

$$x^3 = \frac{y-3}{3}$$

$$x = \sqrt[3]{\frac{y-3}{3}}$$

$$f^{-1}(y) = \sqrt[3]{\frac{y-3}{3}} \quad \forall\, y \in R$$

$$f^{-1}(x) = \sqrt[3]{\frac{x-3}{3}} \quad \forall\, x \in R$$

7M

7) Let $f : R \to R$ and $g : R \to R$ and $h : R \to R$ is defined as $f(x) = 2x+1 \ \forall\, x \in R$ and $g(x) = 3x+2 \ \forall\, x \in R$ $h(x) = 2x - 2$ $\forall\, x \in R$ then find.

i) fog ii) gof iii) $fo(goh)$ iv) $fo(hog)$ v) $go(foh)$ vi) $go(fof)$ vii) $ho(gof)$

Sol

Given $f(x) = 2x+1$

$g(x) = 3x+2$

$h(x) = 2x-2$

i) $fog(x) = f[g(x)]$

$= f[3x+2]$

$= 2(3x+2)+1$

$= 6x+4+1$

$= 6x+5$

ii) $gof(x) = g[f(x)]$

$= g[2x+1]$

$= 3(2x+1)+2$

$= 6x+3+2$

$= 6x+5$

iii) $fo(goh) = f[g(h(x))]$

$= f[g(2x-2)]$

$= f[3(2x-2)+2]$

$= f[6x-4]$

$= 2(6x-4)+1$

$= 12x-8+1$

iv) $f o(h o g) = f[h(g(x))]$

$= f[h(3x+2)]$

$= f[2(3x+2)-2]$

$= f[6x+4-2]$

$= f[6x+2]$

$= 2(6x+2)+1$

$= 12x+4+1$

$= 12x+5$

v) $g o(f o h) = g[f(h(x))]$ 10

$= g[f(2x-2)]$

$= g[2(2x-2)+1]$

$= g[4x-4+1]$

$= g[4x-3]$

$= 3(4x-3)+2$

$= 12x-9+2$

$= 12x-7$

vi) $g o(f o f) = g[f(f(x))]$

$= g[f(2x+1)]$

$= g[2(2x+1)+1]$

$= g[4x+2+1]$

$= g[4x+3]$

$= 3(4x+3)+2$

$= 12x+9+2$

$= 12x+11$

vii) $h o(g o f) = h[g(f(x))]$

$= h[g(2x+1)]$

$= h[3(2x+1)+2]$

$= h[6x+3+2]$

$= h[6x+5]$

$= 2(6x+5)-2$

$= 12x+10-2$

$= 12x+8$

Q. Let $A = \{1,2,3,4,6,12\}$. On A define the relation 'R' by 'a' relation to 'b' if and only if 'a' divides 'b' prove that 'R' is a partial order(s) on A draw the Hasse diagram from this relation.

RAT - Post order
RST - equivalent

Sol $A = \{1,2,3,4,6,12\}$

$R = \{(1,1)(1,2)(1,3)(1,4)(1,6)(1,12)$
$(2,2)(2,4)(2,6)(2,12)$
$(3,3)(3,6)(3,12)\ (4,4)(4,12)(6,6)(6,12)(12,12)\}$

(4)

$R = \{(a,b) \in A / a$ divideds $b\}$

R is reflexive because $(a,a) \in R \quad \forall a \in A$

$\{(1,1)(2,2)(3,3)(4,4)(6,6)(12,12)\}$

R is Transitive because $(a,b) \in R$, $(b,c) \in R$

$(a,c) \in R \quad \forall (a,b,c) \in A$

$\{(1,2)(2,4)(1,4)\}$

Further for all $(a,b) \in A$, if a divides b and b divided a Then $a=b$ hence R is anti symmetric

$\therefore$ R is partial order on A

Hasse diagram:-

$\leq$

The given relation

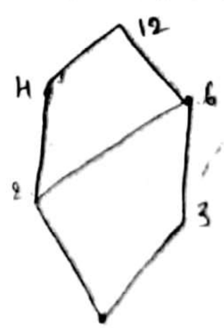
$R = \{\cancel{(1,1)} (1,2) (1,3) (1,4) (1,6) (1,12)$
$\cancel{(2,2)} (2,4) (2,6) (2,12)$
$\cancel{(3,3)} (3,6) (3,12)$
$\cancel{(4,4)} (4,12)$
$\cancel{(6,6)} (6,12)$
$\cancel{(12,12)}\}$

($\because$ Remove the reflexive terms)

$R = \{(1,2)(1,3) \cancel{(1,4)} \cancel{(1,6)} \cancel{(1,12)}$
$(2,4)$ $(2,6)$ $\cancel{(2,12)}$
$(3,6)$ $\cancel{(3,12)}$
$(4,12)$
$(6,12)\}$

($\because$ Remove the transitive terms)

$R = \{(1,2)(1,3)(2,4)(2,6)(3,6)(4,12)(6,12)\}$



9) Let $A = \{1,2,3,6,8,12\}$ on A. define the partial ordering relation R by aRb if and only iff a/b.

1) draw the Hasse diagram 4

ii). Write down relation matrix for R 4

Sol

$A = \{1,2,3,6,8,12\}$

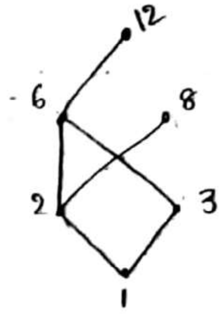
$R = \{ \cancel{(1,1)}\ (1,2)\ (1,3)\ (1,6)\ (1,8)\ (1,12)$
$\cancel{(2,2)}\ (2,6)\ (2,8)\ (2,12)$
$\cancel{(3,3)}\ (3,6)\ (3,12)$
$\cancel{(6,6)}\ (6,12)$
$\cancel{(8,8)}$
$\cancel{(12,12)} \}$

(∵ Remove the reflexive terms)

$R = \{ (1,2)\ (1,3)\ \cancel{(1,6)}\ \cancel{(1,8)}\ \cancel{(1,12)}$
$(2,6)\ (2,8)\ \cancel{(2,12)}$
$(3,6)\ \cancel{(3,12)}$
$(6,12) \}$

(∵ Remove the transitive terms)

$R = \{ (1,2)\ (1,3)\ (2,6)\ (2,8)\ (3,6)\ (6,12) \}$



10) Let $A = \{1,2,3,4,6,8,12\}$ on A. define the partial ordering relation R by aRb if and only iff a/b

i) draw the hasse diagram

ii) Write down relation matrix for R.

Sol:

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$A = \{1, 2, 3, 4, 6, 8, 12\}$

$R = \{\cancel{(1,1)}\,(1,2)\,(1,3)\,(1,4)\,(1,6)\,(1,8)\,(1,12)$
$\cancel{(2,2)}\,(2,4)\,(2,6)\,(2,8)\,(2,12)$
$\cancel{(3,3)}\,(3,6)\,(3,12)$
$\cancel{(4,4)}\,(4,8)\,(4,12)$
$\cancel{(6,6)}\,(6,12)$
$\cancel{(8,8)}$
$\cancel{(12,12)}\}$

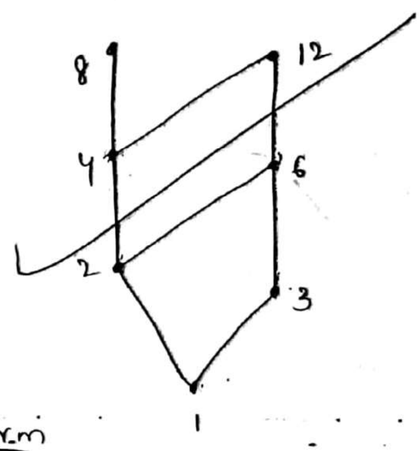
43

(∵ Remove the reflexive terms

$R = \{(1,2)\,(1,3)\,\cancel{(1,4)}\,\cancel{(1,6)}\,\cancel{(1,8)}\,\cancel{(1,12)}$
$(2,4)\,(2,6)\,\cancel{(2,8)}\,\cancel{(2,12)}$
$(3,6)\,\cancel{(3,12)}$
$(4,8)\,(4,12)$
$(6,12)\}$

(∵ Remove the transitive terms

$R = \{(1,2)(1,3)\,(2,4)\,(2,6)\,(3,6)\,(4,8),(4,12)\,(6,12)\}$



ii) Matrix form

$$\begin{array}{c|c} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 6 & 8 & 12 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ 8 \\ 12 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

11) Draw the Hasse diagram of relation R and $A = \{1, 2, 3, 4, 5\}$ whose the matrix is given below.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Sol:-

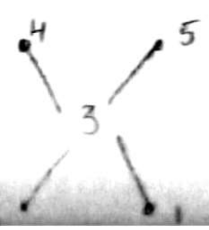
Given that $A = \{1, 2, 3, 4, 5\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{ \cancel{(1,1)}\ (1,3)\ (1,4)\ (1,5)$
$\cancel{(2,2)}\ (2,3)\ (2,4)\ (2,5)$
$\cancel{(3,3)}\ (3,4)\ (3,5)$
$\cancel{(4,4)}$
$\cancel{(5,5)} \}$

$R = \{ (1,3)\ \cancel{(1,4)}\ \cancel{(1,5)}$
$(2,3)\ \cancel{(2,4)}\ \cancel{(2,5)}$
$(3,4)\ (3,5) \}$

$R = \{ (1,3)\ (2,3)\ (3,4)\ (3,5) \}$



Draw the Hasse diagram for the divisibility rela
on the set A in each of the following cases.

45

i) $A = \{3, 6, 12, 36, 72\}$

ii) $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

iii) $A = \{2, 3, 6, 12, 24, 36\}$

iv) $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$

v) $A = \{2, 3, 5, 6, 7, 11, 12, 35, 385\}$

Sol

i) $A = \{3, 6, 12, 36, 72\}$

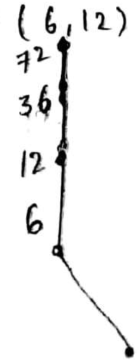
$R = \{\cancel{(3,3)}\ (3,6)\ (3,12)\ (3,36)\ (3,72)$
$\cancel{(6,6)}\ (6,12)\ (6,36)\ (6,72)$
$\cancel{(12,12)}\ (12,36)\ (12,72)$
$\cancel{(36,36)}\ (36,72)$
$\cancel{(72,72)}\}$

∴ Remove reflexive terms.

$R = \{(3,6)\ \cancel{(3,12)}\ \cancel{(3,36)}\ \cancel{(3,72)}$
$(6,12)\ \cancel{(6,36)}\ \cancel{(6,72)}$
$(12,36)\ \cancel{(12,72)}$
$(36,72)\}$

∴ Remove transitive terms

$R = \{(3,6)\ (6,12)\ (12,36)\ (36,72)\}$



ii) $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$R = \{$ ~~(1,1)~~ (1,2) (1,3) (1,5) (1,6) (1,10) (1,15) (1,30)

~~(2,2)~~ (2,6) (2,10) (2,30)

~~(3,3)~~ (3,6) (3,15) (3,30)

~~(5,5)~~ (5,10) (5,15) (5,30)

~~(6,6)~~ (6,30)

~~(10,10)~~ (10,30)

~~(15,15)~~ (15,30)

~~(30,30)~~ $\}$

Remove reflexive terms

$R = \{$ (1,2) (1,3) (1,5) ~~(1,6)~~ ~~(1,10)~~ ~~(1,15)~~ ~~(1,30)~~

(2,6) (2,10) ~~(2,30)~~

(3,6) (3,15) ~~(3,30)~~

(5,10) (5,15) ~~(5,30)~~

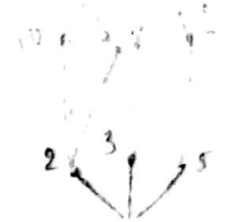
(6,30)

(10,30)

(15,30) $\}$

Remove Transitive terms

$R = \{$ (1,2) (1,3) (1,5) (2,6) (2,10) (3,6) (3,15) (5,10) (5,15) (6,30) (10,30) (15,30) $\}$

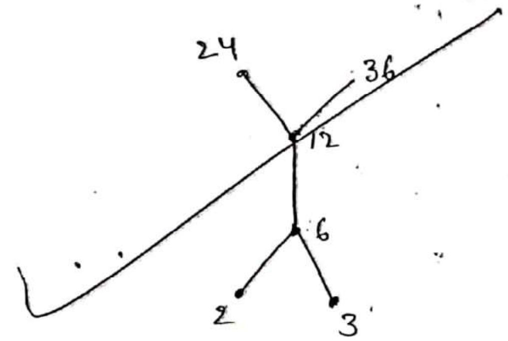


iii) $A = \{2,3,6,12,24,36\}$ (47)

$R = \{$ ~~(2,2)~~ (2,6) (2,12) (2,24) (2,36)
~~(3,3)~~ (3,6) (3,12) (3,24) (3,36)
~~(6,6)~~ (6,12) (6,24) (6,36)
~~(12,12)~~ (12,24) (12,36)
~~(24,24)~~
~~(36,36)~~ $\}$

$R = \{$ (2,6) ~~(2,12)~~ ~~(2,24)~~ ~~(2,36)~~
(3,6) ~~(3,12)~~ ~~(3,24)~~ ~~(3,36)~~
(6,12) ~~(6,24)~~ ~~(6,36)~~
(12,24) (12,36) $\}$

$R = \{(2,6)\ (3,6)\ (6,12)\ (12,24)\ (12,36)\}$



iv) $A = \{1,2,3,4,6,8,9,12,18,24\}$

$R = \{$ ~~(1,1)~~ (1,2) (1,3) (1,4) (1,6) (1,8) (1,9) (1,12) (1,18) (1,24)
~~(2,2)~~ (2,4) (2,6) (2,8) (2,12) (2,18) (2,24)
~~(3,3)~~ (3,6) (3,9) (3,12) (3,18) (3,24)
~~(4,4)~~ (4,8) (4,12) (4,24)
~~(6,6)~~ (6,12) (6,18) (6,24)
~~(8,8)~~ (8,24)
~~(9,9)~~ (9,18)
~~(12,12)~~ (12,24)
~~(18,18)~~

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R = {(1,2) (1,3) ~~(1,4)~~ ~~(1,6)~~ ~~(1,8)~~ ~~(1,9)~~ ~~(1,12)~~ ~~(1,18)~~ ~~(1,24)~~

(2,4) (2,6) ~~(2,8)~~ ~~(2,12)~~ ~~(2,18)~~ ~~(2,24)~~

(3,6) (3,9) ~~(3,12)~~ ~~(3,18)~~ ~~(3,24)~~

(4,8) (4,12) ~~(4,24)~~

(6,12) (6,18) ~~(6,24)~~

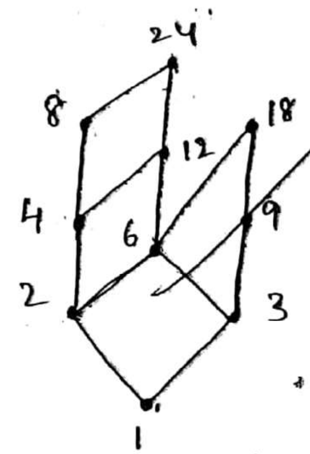
(8,24)

(9,18) ~~(9,24)~~

(12,24)}

48

R = {(1,2) (1,3) (2,4) (2,6) (3,6) (3,9) (4,8) (4,12) (6,12) (6,18) (8,24) (9,18) (12,24)}



v) A = {2, 3, 5, 6, 7, 11, 12, 35, 385}

R = {~~(2,2)~~ (2,6) (2,12) ~~(2,385)~~

~~(3,3)~~ (3,6) (3,12) ~~(3,385)~~

~~(5,5)~~ (5,35) (5, 385)

~~(6,6)~~ (6,12)

~~(7,7)~~ (7,35) (7, 385)

~~(11,11)~~ (11,385)

~~(12,12)~~ ~~(12,385)~~

~~(35,35)~~ (35,385)

~~(385,385)~~ }

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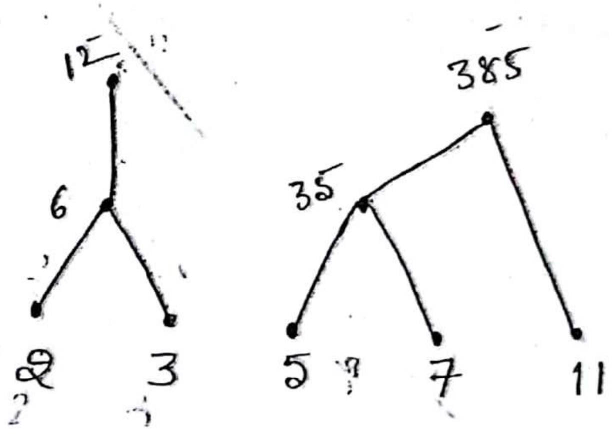
$R = \{ (2,6)\ \cancel{(2,12)}$
$(3,6)\ \cancel{(3,12)}$
$(5,35)\ \cancel{(5,385)}$
$(6,12)\ \cancel{(8,}$
$(7,35)\ \cancel{(7,385)}$
$(11,385)$
$(35,385) \}$

$R = \{ (2,6)\ (3,6)\ (5,35)\ (6,12)\ (7,35)\ (11,385)\ (35,3$



1) Write down the Hasse diagram of positive diviser of 45.

(50)

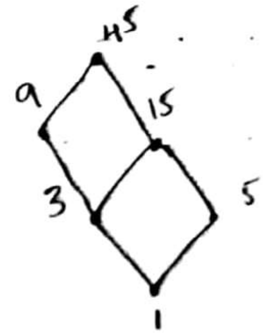
Set of all the divisors of 45 is

$A = \{1, 3, 5, 9, 15, 45\}$

$R = \{ \cancel{(1,1)}, (1,3)\ (1,5)\ (1,9)\ (1,15)\ (1,45)$
$\cancel{(3,3)}\ (3,9)\ (3,15)\ (3,45)$
$\cancel{(5,5)}\ (5,15)\ (5,45)$
$\cancel{(9,9)}\ (9,45)$
$\cancel{(15,15)}\ (15,45)$
$\cancel{(45,45)} \}$

$R = \{ (1,3)\ (1,5)\ \cancel{(1,9)}\ \cancel{(1,15)}\ \cancel{(1,45)}$
$(3,9)\ (3,15)\ \cancel{(3,45)}$
$(5,15)\ \cancel{(5,45)}$
$(9,45)$
$(15,45) \}$

$R = \{ (1,3)\ (1,5)\ (3,9)\ (3,15)\ (5,15)\ (9,45)\ (15,45) \}$



2,6
2×1 3×2
6×3

Lattice :-

57 60

Lattice introduced as post $(P, \leq)$ in which every pair has greatest lower bound (GLB) and least upper bound (LUB) is called lattice.

GLB :- (greatest lower bound)

Greatest lower bound of $(a, b) = a * b$ (or) $a \cdot b$ (or) gcd of a and b (or) $a \wedge b$

Eg :- GLB of $(2, 3) = 6 = 6 =$ gcd $(2, 3) = 1$.

LUB :- (least upper bound)

LUB of $(a, b) = a + b = a \oplus b =$ LCM of a and b $= a$

4M Eg :- Let $P = \{2, 3, 6, 12\}$ then prove that $(P, \leq)$ this notation is a lattice (or) not

Sol

Given that $P = \{2, 3, 6, 12\}$

Consider one pair $(2, 3)$ from the set P

GLB of $(2, 3) = 1 \notin P$ means that it does not GLB from the set P

LUB of $(2, 3) = 6 \in P$ then $(P, \leq)$ is not a lattice

2. If A is finite set and $P(A)$ is power set then prove that $(P(A), \leq)$ is a lattice for

i) $A = \{a\}$

ii) $A = \{a, b\}$

5

Sol:- i) $A = \{a\}$

$$P(A) = \{\{\emptyset\}, \{a\}\}$$

$$\text{GLB of } (\emptyset, \{a\}) = \emptyset \cap \{a\}$$
$$= \emptyset \in P(A)$$

$\therefore (\emptyset, \{a\})$ has GLB

$$P(A) = \{\{\emptyset\}, \{a\}\}$$

$$\text{LUB of } (\emptyset, \{a\}) = \emptyset \cup \{a\}$$
$$= \{a\} \in P(A)$$

$(\emptyset, \{a\})$ has LUB.

$\therefore (P(A), \leq)$ is a Lattice.

ii) $A = \{a, b\}$

$$P(A) = \{\{\emptyset\}, \{a\}, \{b\}, \{a, b\}\}$$

$(\varphi \wedge P = \varphi)$

$$\text{GLB of } (\emptyset, \{a\}) = \emptyset \cap \{a\}$$
$$= \emptyset \in P(A)$$

$\therefore (\emptyset, \{a\})$ has GLB

LUB of $(\emptyset, \{a\}) = \emptyset \cup \{a\}$

$= \{a\} \in P(A)$

$\therefore (\emptyset, \{a\})$ has LUB

$\therefore (\emptyset, \{a\})$ has GLB & LUB $\longrightarrow$ (1)

GLB of $(\emptyset, \{b\}) = \emptyset \cap \{b\}$

$= \emptyset \in P(A)$

$(\emptyset, \{b\})$ has GLB

LUB of $(\emptyset, \{b\}) = \emptyset \cup \{b\}$

$= \{b\} \in P(A)$

$\therefore (\emptyset, \{b\})$ has LUB

$(\emptyset, \{b\})$ has GLB & LUB $\longrightarrow$ (2)

GLB of $(\{\emptyset\}, \{a,b\}) = \emptyset \cap \{a,b\}$

$= \emptyset \in P(A)$

$\therefore (\{\emptyset\}, \{a,b\})$ has GLB

LUB of $(\emptyset, \{a,b\}) = \emptyset \cup \{a,b\}$

$= \{a,b\} \in P(A)$

$\therefore (\emptyset, \{a,b\})$ has LUB

$(\emptyset, \{a,b\})$ has GLB & LUB $\longrightarrow$ (3)

GLB of $(\{a\}, \{b\}) = \{a\} \cap \{b\}$

$= \emptyset \in P(A)$

$(\{a\}, \{b\})$ has GLB

LUB of $(\{a\}, \{b\}) = \{a\} \cup \{b\}$

$(\{a\}, \{b\})$ has GUB & LUB $\rightarrow$ (4)

GUB of $(\{b\}, \{a,b\}) = \{b\} \cap \{a,b\}$

$\overset{\{a,b\}}{\emptyset} \in P(A)$

GUB $(\{b\}, \{a,b\})$ has GUB

LUB of $(\{b\}, \{a,b\}) = \{b\} \cup \{a,b\}$

$= \overset{(a,b)}{\emptyset} \in P(A)$

$(\{b\}, \{a,b\})$ has GUB & LUB $\rightarrow$ (5)

GUB of $(\{a\}, \{a,b\}) = \{a\} \cap \{a,b\}$

$= \overset{\{a\}}{\emptyset} \in P(A)$

$(\{a\}, \{a,b\})$ has GUB

LUB of $(\{a\}, \{a,b\}) = \{a\} \cup \{a,b\}$

$= \overset{(a,b)}{\emptyset} \in P(A)$

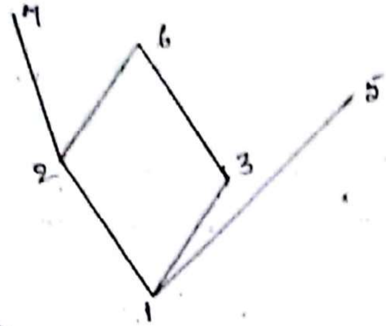
$(\{a\}, \{a,b\})$ has LUB

$\therefore$ $(\{a\}, \{a,b\})$ has GUB & LUB $\rightarrow$ (6)

from (1), (2), (3), (4), (5) & (6) equations are

$(P(A), \leq)$ has Lattice

3. GLB LUB



(62)

(55)

Sol

(1, 2)

A = {(1, 2), 3, 4, 5, 6}

In above hasse diagram 4, 6, 5 are maximal elements ~~ade~~ and '1' is minimal element 2, 3 are least upper bound and 1 is greatest lower bound

MAximal – 4, 6, 5

MInimal – 1

LUB – 2, 3

GLB – 1

Bounded Lattice :-

A lattice (L, R) is said to be bounded if it has greatest element and least element.

In a bounded lattice a greatest elemer is denoted by 'I' and least element is denoted by '0'.

Note:-

$$a \vee 0 = a$$

$$a \wedge 0 = 0$$

$$a \vee I = I$$

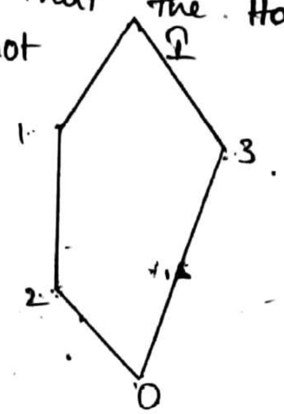
$$a \wedge I = a$$

Distributive lattice:-

A Lattice (L, R) is said to be distributive if for any $a, b, c \in L$, the following distributive laws hold.

* $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
* $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Eg:-1. prove that the Hasse diagram is distributive lattice (or) not



Sol

Assume that

$a = 1$

$b = 2$

$c = 3$

By distributive law we get

$1 \wedge (2 \vee 3) = (1 \wedge 2) \vee (1 \wedge 3)$

$1 \wedge I = 2 \vee 0 \qquad (\because 2 \vee 3 = I)$

$1 = 2$

$\therefore$ It is not a distributive lattice

2. prove that in the above Hasse diagram is distributive lattice (or) not.

57

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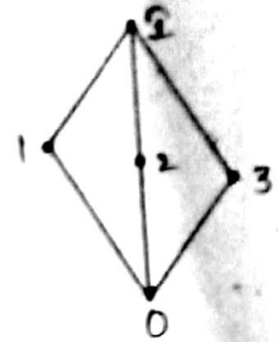


Sol

Assume that

$a = 1$

$b = 2$

$c = 3$

By distributive law we get

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

$$1 \wedge (2 \vee 3) = (1 \wedge 2) \vee (1 \wedge 3)$$

$$1 \wedge I = 0 \vee 0$$

$$1 = 0$$

$\therefore$ It is not a distributive lattice

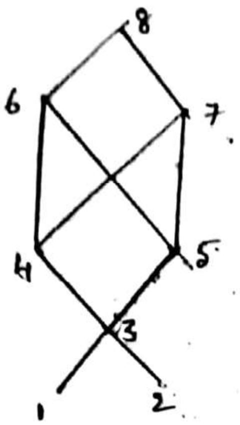
3. Consider the poset $A = \{1,2,3,4,5,6,7,8\}$ under the partial order whose hasse diagrame is as shown bellow. Consider the subsets $B = \{1,2\}$ & $C = \{3,4,5\}$ i) find all the lower and upper bounds of B and C

ii) a) GLB (B)

b) LUB (B)

c) GLB (C)

d) LUB (C)



Sol :- i) Upper bounds of $B = \{1, 2\}$ is

$= \{3, 4, 5, 6, 7, 8\}$

Upper bounds of $C = \{3, 4, 5\}$ is

$= \{6, 7, 8\}$

Lower bounds of $B = \{1, 2\}$ is

= None

Lower bounds of $C = \{3, 4, 5\}$ is

= ~~None~~ $\{1, 2\}$

Because every gretest lower bound occur exactly once

ii) a) GLB (B) = None

b) LUB (B) = 3

c) GLB (c) = 3 2

d) LUB (c) = ~~None~~ $\{1, 2\}$ 6

properties of Lattice:-

* Idempotent properties:-

i) $a \vee a = a$
ii) $a \wedge a = a$

* Commutative properties:-

i) $a \vee b = b \vee a$
ii) $a \wedge b = b \wedge a$

* Assosiative properties:-

i) $a \vee (b \vee c) = (a \vee b) \vee c$
ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

* Absorption properties:-

i) $a \vee (a \wedge b) = a$
ii) $a \wedge (a \vee b) = a$