TOPICS

UNIT - 1 Mathematical foundation of computer science

1. Statements and notations
2. Connectives
3. Well formed formulaes
4. Truth tables
5. Tautology
6. Equivalence Implication
7. Normal form
8. Quantifiers
9. Universal quantifier.

Statements and Notations

Statement :- Any declarative sentence which is either true (or) false but not both is called a statement.

Eg :-

1. Japan is a country
2. Rose is red
3. $5+5=10$
4. What is the meaning of tautology
5. $5+x=7$
6. Ramesh will arrive hyderabad by tomorrow
7. Hyderabad is a twin city.

Explanation :-

⇒ Eg 1, 3 and 7 are statements therefore examples 1 and 3 always true and example 7 is always false.

⇒ Eg 1, 3 and 7 are either true (or) false but not both hence they are statements.

⇒ Eg 4 is not a declaration sentence hence not a statement

⇒ Eg 2, 5 and 6 do not take any particular value.

⇒ They may be true (or) false depending on the situation hence they are not considered as a statements.

Notations :-

statements are usually represented by using notations like p, q / P, Q / a, b / A, B / x, a / x, y /

The concept of Notation is usually used when we connect two (or) more statements using connectives.

Connectives :-

Any word (or) expression used to connect two (or) more statements is called as connective.

The three basic connectives like

1. Negation (Not) ~ ¬
2. Conjunction (AND) ∧
3. disjunction. (OR) ∨.

Negation (Not) :-

The negation of given statement is represented using the connective negation (~).

If p is any statement then the negation of given statement is 'not' p and negation of p is denoted by ~p.

Eg:- p : Onida is a good company

~p : Onida is not a good company

Truth table :-

p	~p
T	F
F	T

Conjunction (AND) :-

The compound statement formed by connecting the two statements using the connective 'and' is called conjunction of two statements.

Note :- If p, q are two statements then any of the two statements are true the resultant statement is true otherwise false it is denoted by p∧q

Eg:- p : Japan is a country
q : Onida is a good company.
$p \wedge q$: Japan is a country and onida is a good company.

Truth table :-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR) :-

Let p and q are any primary statements then the disjunction of p and q is also a statement which is return as "$p \vee q$" and read as p or q it is called disjunction statement of $p \vee q$

Eg:- p : I will buy a car
q : I will buy a computer
$p \vee q$: I will buy a car (or) computer

Truth table :-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The above truth table of $p \vee q$ has the truth value false if both p & q truth values 'F' otherwise it has the truth value true.

Conditional statement ($\rightarrow$):-

Let p and q are any two primary statements then the conditional of p and q is also a statement which is return as "$p \rightarrow q$" and read as p implies q is called as a condition statement of $p \rightarrow q$

Eg:- p : Ranjani works hard

q : She will pass the exam

$p \rightarrow q$: If ranjani works hard then she will pass the exam.

Truth table:-

$p \rightarrow q$ has the truth value false if p truth value is true and q truth value is false otherwise true.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Note :- Converse and inverse and contrapositive of a conditional statement.

If $p \rightarrow q$ is a conditional statement then

Bits

1. The converse of $p \rightarrow q$ is "$q \rightarrow p$"
2. The inverse of $p \rightarrow q$ "$\sim p \rightarrow \sim q$"
3. The contrapositive of $p \rightarrow q$ is "$\sim q \rightarrow \sim p$"

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Eg:- Write converse, Inverse and contrapositive of the following statements i) $P \to (Q \to R)$

ii) $[P \wedge (P \to Q)] \to Q$

i) Sol Given statements is $P \to (Q \to R)$

Converse :

$$(Q \to R) \to P$$

Inverse :

$$\sim P \to \sim (Q \to R)$$

Contrapositive :

$$\sim (Q \to R) \to \sim P$$

ii) Sol $[P \wedge (P \to Q)] \to Q$

Converse : Here $P = P \wedge (P \to Q)$, $q = Q$

$$Q \to [P \wedge (P \to Q)]$$

Inverse :

$$\sim [P \wedge (P \to Q)] \to \sim Q$$

Contrapositive :

$$\sim Q \to \sim [P \wedge (P \to Q)]$$

Biconditional statements :-

Let p and q any two primary statements then the biconditional of p and q is also a statements which is return as "$P \leftrightarrow Q$" and read as p double implise Q is called as a biconditional of P and Q

Truth table :-

If any two statements are true and two statements are false then the resultant statement is true

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1. Note :- NAND statement ($\uparrow$)

Truth table :

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

$\sim(p \wedge q)$

2. Note :- NOR statement ($\downarrow$)

Truth table :

$\sim(p \vee q)$

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

3. $p \uparrow q = \sim(p \wedge q)$

$= \sim p \vee \sim q$

4. $p \downarrow q = \sim(p \vee q)$

$= \sim p \wedge \sim q$

Eg:- 1 Using the statements

R : Mark is rich

H : Mark is happy. write the following statements in symbolic form.

a : Mark is poor but happy

b : Mark is rich (or) un happy

c : Mark is neither rich nor happy

d : Mark is poor (or) is both rich and unhappy.

Sol

Given statements are R : Mark is rich

H: Mark is happy

a) Mark is poor but happy : $\sim R \vee H$

b) Mark is rich or un happy : $R \vee \sim H$

c) Mark is neither rich nor happy : $\sim R \wedge \sim H$

d) Mark is poor (or) is both rich and un happy : $\sim R \vee (R \wedge \sim H)$

2- Write the following statement in symbolic form. If either mr. srinu takes calculars (or) mr. swami takes graph theory then mr. mahesh will takes computer programming.

Sol

Given statements are

S : Mr. srinu takes calculars

T : Mr. swami takes graph theory

Q : Mr. mahesh take computer programing

$$\boxed{(S \vee T) \rightarrow Q}$$

Construct truth tables for the following statements 3)

1. $(P \to Q) \to P \wedge Q$
2. $(P \wedge Q) \vee (P \vee Q)$
3. $(P \wedge Q) \leftrightarrow (P \vee Q)$
4. $(\sim P \wedge \sim Q) \to (P \vee \sim Q)$
5. $(P \to Q) \vee \sim (P \to Q)$
6. $(P \wedge Q \vee R) \to [(P \wedge Q) \vee (P \vee R)]$
7. $[(P \to R) \wedge (P \to Q)] \vee (P \to \sim Q)$
8. $[(P \wedge R) \vee (P \wedge Q)] \leftrightarrow (P \to Q)$
9. $(P \to \sim R) \vee (\sim P \wedge \sim Q)$
10. $[(P \to Q) \vee (\sim P \wedge \sim Q)] \wedge [(P \to \sim R) \vee (\sim P \leftrightarrow Q)]$

Solutions :-

1) $(P \to Q) \to P \wedge Q$

P	Q	$P \to Q$	$P \wedge Q$	$(P \to Q) \to P \wedge Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

2) $(P \wedge Q) \vee (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \vee (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

3) $(P \wedge Q) \leftrightarrow (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \leftrightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

4) $(\sim P \wedge \sim Q) \rightarrow (P \vee \sim Q)$

P	Q	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$P \vee \sim Q$	$(\sim P \wedge \sim Q) \rightarrow (P \vee \sim Q)$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

5) $(P \rightarrow Q) \vee \sim(P \rightarrow Q)$

P	Q	$P \rightarrow Q$	$\sim(P \rightarrow Q)$	$(P \rightarrow Q) \vee \sim(P \rightarrow Q)$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

6) $(P \wedge Q \vee R) \rightarrow [(P \wedge Q) \vee (P \vee R)]$

P	Q	R	① $P \wedge Q$	② $P \vee R$	③ $(P \wedge Q) \vee (P \vee R)$	④ $P \wedge Q \vee R$	④ → ③
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	F	F	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

7) $[(P\to R)\wedge(P\to Q)]\vee(P\to \sim Q)$

P	Q	R	① $P\to R$	② $P\to Q$	③ ①∧②	$\sim Q$	④ $P\to\sim Q$	③∨④
T	T	T	T	T	T	F	F	T
T	T	F	F	T	F	F	F	F
T	F	T	T	F	F	T	T	T
T	F	F	F	F	F	T	T	T
F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

8) $[(P\wedge R)\vee(P\wedge Q)]\leftrightarrow(P\to Q)$

P	Q	R	① $P\wedge R$	② $P\wedge Q$	③ ①∨②	④ $P\to Q$	③↔④
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	F	F
T	F	F	F	F	F	F	T
F	T	T	F	F	F	T	F
F	T	F	F	F	F	T	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	T	F

9) $(P\to\sim R)\vee(\sim P\wedge\sim Q)$

P	Q	R	$\sim P$	$\sim Q$	$\sim R$	① $P\to\sim R$	② $\sim P\wedge\sim Q$	①∨②
T	T	T	F	F	F	F	F	F
T	T	F	F	F	T	T	F	T
T	F	T	F	T	F	F	F	F
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	T	F	T
F	T	F	T	F	T	T	F	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

10) $[(P \to Q) \vee (\sim P \wedge \sim Q)] \wedge [(P \to \sim R) \vee (\sim P \leftrightarrow Q)]$

P	Q	R	$\sim P$	$\sim Q$	$\sim R$	① $P \to Q$	② $\sim P \wedge \sim Q$	⑤ ①$\vee$②	③ $P \to \sim R$	④ $\sim P \leftrightarrow Q$	⑥ ③$\vee$④	⑤$\wedge$⑥
T	T	T	F	F	F	T	F	T	F	F	F	F
T	T	F	F	F	T	T	F	T	T	F	T	T
T	F	T	F	T	F	F	F	F	F	T	T	F
T	F	F	F	T	T	F	F	F	T	T	T	F
F	T	T	T	F	F	T	F	T	T	T	T	T
F	T	F	T	F	T	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T	F	T	T
F	F	F	T	T	T	T	T	T	T	F	T	T

Well formed formulae :-

* A statement formula is an expression which is a string consisting of variables, paranthesis and connective symbols ($\sim, \wedge, \vee, \rightarrow, \leftrightarrow$) which produce a statement when the variables are replaced by statements now we can give a recursive defination of a statement formula.

A well formed formula can be generated by the following rules.

1> A statement variables standing above is a well formed formula (wff)

2> If A is a wff then $\sim A$ is a well formed formula.

3> If A and B are well formed formula then $A \wedge B$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B$ is a well formed formula.

4> A string of symbols containing the statement variables connectives and paranthesis, if it can be obtained by finately many application of rules (1 & 2 & 3 rules)

5> A set of all well formed formula is the smallest set of string such that

a) Every statement variable is in the set

b) If a and b are in the set then $\sim A (A \wedge B)$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B$.

Eg:- $\sim(P \wedge Q)$, $[P \rightarrow (P \vee Q)]$, $[P \rightarrow (Q \rightarrow R)]$

Tautology :-

A statement formula is set to be tautology if it is true for all truth values of its component statement.

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Eg:1 :-

P	~P	P ∨ ~P
T	F	T
F	T	T

Eg:2 :- $(P \wedge Q) \vee \sim(P \wedge Q)$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \wedge Q) \vee \sim(P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

[Last Column Contain all Trues it Called Tautology]

Contradiction :-

A statement formula is said to be contradiction if it is false for all truth Value of its compound statement

Eg: 1 :-

P	~P	$P \wedge \sim P$
T	F	F
F	T	F

Eg: 2 :- $(P \vee Q) \wedge \sim(P \vee Q)$

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$(P \vee Q) \wedge \sim(P \vee Q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

[Last Column Contain all False it is Called Contradiction]

Contigency :- A statement formula is said to be contigency if it is neither tautology (or) contradiction is called contigency

Eg:- $(P \to Q) \wedge (P \wedge Q)$

P	Q	$P \to Q$	$P \wedge Q$	$(P \to Q) \wedge (P \wedge Q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

[If True & False : It is Called Contigency]

Determine weather the following statement is tautology are not.

1) $[\sim p \wedge (p \to q)] \to \sim q$
2) $[\sim q \wedge (p \to q)] \to \sim p$
3) $[[(p \vee q) \wedge (p \to r)] \wedge (q \to r)] \to r$
4) $(p \wedge q) \to (p \to q)$
5) $[(p \to q) \wedge (q \to r)] \to (p \to r)$
6) $[(p \vee q) \wedge (\sim p \vee r)] \to (q \vee r)$
7) $[(p \to q) \to r] \to [(p \to q) \to (p \to r)]$

Solutions :-

1) $[\sim p \wedge (p \to q)] \to \sim q$

p	q	$\sim p$	$\sim q$ ②	$p \to q$	$\sim p \wedge (p \to q)$ ①	① → ②
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

2) $[\sim q \wedge (p \to q)] \to \sim p$

p	q	$\sim p$	① $\sim q$	② $p \to q$	③ ① $\wedge$ ②	③ $\to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

3) $[[(p \vee q) \wedge (p \to r)] \wedge (q \to r)] \to r$

p	q	r	① $p \vee q$	② $p \to r$	③ ① $\wedge$ ②	④ $q \to r$	⑤ ③ $\wedge$ ④	⑤ $\to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

4) $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

6) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	① $p \to q$	② $q \to r$	③ ① $\wedge$ ②	④ $p \to r$	③ $\to$ ④
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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1. $p \rightarrow q$

If you have the flee then you miss the final examination.

2. $\sim p \rightarrow r$

If you not have the flee then you pass the course.

3. $q \rightarrow \sim r$

If you miss the final examination then you not pass the course.

4. $p \vee q \vee r$

you have the flee (or) you miss the final examination (or) you pass the course.

5. $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$

If you have the flee, then you not pass the course If you miss the final examination then you not (or) if you miss the final examination then you not pass the course.

6. $(p \wedge q) \vee (\sim q \wedge r)$

If you have the flee and you miss the final examination (or) you not miss the final examination and you pass the course.

Tautological Implications :-

P is any propositional statement (or) compound statement and q is propositional statement (or) compound statement, p is tautologically implied by q if $p \to q$ is a tautology. It is represented by the symbol "$P \Rightarrow q$"

Eg:- $P \wedge q \Rightarrow P$

P	q	$P \wedge q$	$P \wedge q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

Since all the entries in the last coloumn are true. $(P \wedge q) \to P$ is a talitology. Hence $\boxed{(P \wedge q) \Rightarrow P}$

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Eg ② $P \to (Q \to R) \Rightarrow [(P \to Q) \to (P \to R)]$

Sol

P	Q	R	① $Q \to R$	④ $P \to ①$	② $P \to Q$	③ $P \to R$	⑤ $② \to ③$	$④ \to ⑤$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Tautology

One compound statement $\Rightarrow$ another compound statement is ~~tabo~~tautology

(4)	(5)	(4) $\rightarrow$ (5)
T	T	T
F	F	T
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T

Tautology.

$P\rightarrow(Q\rightarrow R) \Rightarrow [(P\rightarrow Q)\rightarrow(P\rightarrow R)]$ is a tautology

Hence it is a tautologically implication.

(2) $Q \Rightarrow (P\rightarrow R)$

P	Q	R	$P\rightarrow R$	$Q\Rightarrow(P\rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Hence it is not a tautology

Equivalence Implication :-

Two compound statements are said to be logically equivalent is their truth values are same and it is denoted by the symbol " $\Leftrightarrow$ "

Problems :-

1. Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.

Sol Given Statements are

$\sim(p \vee (\sim p \wedge q)) \rightarrow$ I

$\sim p \wedge \sim q \longrightarrow$ II

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$	$\sim(p \vee (\sim p \wedge q))$
T	T	F	F	F	F	F
T	F	F	T	F	T	F
F	T	T	F	T	T	F
F	F	T	T	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

From the above table given compound statement values are same. They are logically equivalence $\therefore \sim(p \vee (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q$

2. Show that $\sim p \leftrightarrow q$ and $p \leftrightarrow \sim q$ are logically equivalent.

Sol Given compound statement are

$$\sim p \leftrightarrow q \rightarrow (1)$$
$$p \leftrightarrow \sim q \rightarrow (2)$$

p	q	$\sim p$	$\sim p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

p	q	$\sim q$	$p \leftrightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

From the above truth table are same and they are logically equivalence

$$\therefore \sim p \leftrightarrow q \Leftrightarrow p \leftrightarrow \sim q$$

3. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

4. Show that $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are logically equivalent.

3\. Sol Given compound statements are

$p \leftrightarrow q \rightarrow (1)$
$(p \wedge q) \vee (\sim p \wedge \sim q) \rightarrow (2)$ } Bi Conditional.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

From the above truth table are same and they are logically equivalence.

4\. Sol Given compound statements are

$p \rightarrow q \rightarrow (1)$
$\sim q \rightarrow \sim p \rightarrow (2)$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

From the above truth table are ~~not~~ same and they are ~~not~~ logically equivalence.

Properties of logical equivalence :-

1. Identity law :-

$$P \wedge T \equiv P$$
$$P \vee F \equiv P$$

2. Domination law :-

$$P \vee T \equiv T$$
$$P \wedge F \equiv F$$

3. Idempotent law :-

$$P \vee P \equiv P$$
$$P \wedge P \equiv P$$

4. Commutative ~~Communication~~ law :-

$$P \vee q \equiv q \vee P$$
$$P \wedge q \equiv q \wedge P$$

5. Assosiative law :-

$$(P \vee q) \vee r \equiv P \vee (q \vee r)$$
$$(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$$

6. Distributive law :-

$$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$$
$$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$$

7. Demorgan's law :-

$$\sim(P \wedge q) \equiv \sim P \vee \sim q$$
$$\sim(P \vee q) \equiv \sim P \wedge \sim q$$

8. Absorption law :-

$$P \vee (P \wedge q) \equiv P$$
$$P \wedge (P \vee q) \equiv P$$

9. Inverse law :-

$$P \vee \sim P \equiv T$$
$$P \wedge \sim P \equiv F$$

Note :- $\sim(\sim p) \equiv p$

Eg :-

p	$\sim p$	$\sim(\sim p) = p$
T	F	T
F	T	F

1. Show that $p \vee [p \wedge (p \vee q)]$ and p is a logically equivalent with out using truth table.

Sol Given statement is

$= p \vee [p \wedge (p \vee q)]$

$= p \vee p$ (By absorption law $p \wedge (p \vee q) \equiv p$)

$= p$ (By idempotent law $p \vee p \equiv p$)

$\boxed{\therefore\ p \vee [p \wedge (p \vee q)] \Leftrightarrow p}$

2. Show that $\sim[p \vee (\sim p \wedge q)]$ and $\sim p \wedge \sim q$ are logically equivalent with out ~~yo~~ using truth table.

Sol Given statements are

$= \sim[p \vee (\sim p \wedge q)]$

$= \sim p \wedge \sim(\sim p \wedge q)$ (by demorgan law $\sim(p \vee q) = \sim p \wedge \sim q$)

$= \sim p \wedge [\sim(\sim p) \vee \sim q]$ (Again demorgan law $\sim(p \wedge q) = \sim p \vee \sim q$)

$= \sim p \wedge [p \vee \sim q]$

(By distributive law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$)

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

(By Inverse law

$\sim p \wedge p \equiv F$)

$\equiv F \vee (\sim p \wedge \sim q)$

(By identity law

$p \vee F \equiv p$).

$\equiv \sim p \wedge \sim q$

$$\boxed{\therefore \sim (p \vee (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q.}$$

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3. Show that $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

Sol Given

L.H.S

$[p \vee q \vee (\sim p \wedge \sim q \wedge r)]$

$\equiv p \vee q \vee ((\sim p \wedge \sim q) \wedge r)]$ by Assosiative law

$\equiv p \vee q \vee (\sim (p \vee q) \wedge r)$ by Demorgans law

$\equiv (p \vee q) \vee [\sim (p \vee q) \wedge r]$

$\equiv (p \vee q) \vee (\sim p \vee q)) \wedge (p \vee q) \vee r$ by distributive law

$\equiv T \wedge (p \vee q) \vee r$ by inverse law

$\equiv (p \vee q) \vee r$ by identity law

$\equiv p \vee q \vee r$ by Assosiative law

L.H.S = R.H.S

Note :-

$$p \to q \Longleftrightarrow \sim p \vee q$$

4. Show that $[(\sim p \vee \sim q) \to (p \wedge q \wedge r)] \Longleftrightarrow p \wedge q$

Sol Given Statement

L.H.S

$$\therefore [\underbrace{(\sim p \vee \sim q)}_{p} \underbrace{\to}_{\to} \underbrace{(p \wedge q \wedge r)}_{q}] \qquad (\because p \to q \Longleftrightarrow \sim p \vee q)$$

$$= \sim(\sim p \vee \sim q) \vee (p \wedge q \wedge r)$$

$$= (p \wedge q) \vee (p \wedge q \wedge r) \qquad \text{By demorgan's law}$$

$$= \boxed{\underbrace{(p \wedge q)}_{p} \underbrace{\vee}_{\vee} (\underbrace{(p \wedge q)}_{(p \wedge q)} \wedge r) \qquad \text{By Assosiative}} \text{ law}$$

$$= p \wedge q \qquad \text{By Absorption law}$$

L.H.S = R.H.S

$$[(\sim p \vee \sim q) \to (p \wedge q \wedge r)] \Longleftrightarrow p \wedge q$$

5. Show that $p \to (q \to r) \Longleftrightarrow (p \wedge q) \to r$

Sol Given Statement

L.H.S $\quad \underbrace{p}_{p} \underbrace{\to}_{\to} \underbrace{(q \to r)}_{q} \qquad (\because p \to q \Longleftrightarrow \sim p \vee q)$

$$= \sim p \vee (q \to r) \qquad (\because p \to q \Longleftrightarrow \sim p \vee q)$$

$$= \sim p \vee (\sim q \vee r)$$

$$= (\sim p \vee \sim q) \vee r \qquad \text{By Associative law}$$

$$= \sim (p \wedge q) \vee r \qquad \text{By Demorgans law}$$

$$= (p \wedge q) \to r \qquad (\because p \to q \Longleftrightarrow \sim p \vee q)$$

L.H.S = R.H.S

6. Let x be a spacified real number. Write down the negation of the following condition
"If x is an Integer, then x is a rational number"

Sol

Let

p: x is an integer

q: x is rational number

$p \rightarrow q$

Apply negation

$\sim(p \rightarrow q)$

$\sim(\sim p \vee q)$ $\quad \because p \rightarrow q \equiv \sim p \vee q$

$\boxed{p \wedge \sim q}$

"if x is an integer and x is not a rational number"

7. Let x be a spacified number. Write down the negation of the following condition.
"if x is an Integer or a fraction then x is a rational number".

Sol

Let

p: x is an integer

q: x is an fraction

r: x is an rational number

$(p \vee q) \rightarrow r$

Apply negation

$\sim[(p \vee q) \rightarrow r]$ $\quad \because p \rightarrow q = \sim p \vee q$

$\sim[\sim(p \vee q) \vee r]$

$(p \vee q) \wedge \sim r$

x is an integer (or) fraction and x is not a rational number.

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8) Let x be a specified number. write down the negation of following condition.

" if x is not a real number then it is not a rational number and not an irrational number"

Sol Let

p : x is not a real number

q : x is not a rational number

r : x is not an irrational numbe

$\sim p \rightarrow (\sim q \wedge \sim r)$

Apply negation

$\sim[(\sim p) \rightarrow (\sim q \wedge \sim r)]$ $(\because p \rightarrow q \equiv \sim p \vee q)$

$\sim[\sim(\sim p) \vee (\sim q \wedge \sim r)]$

$\sim[p \vee \sim(q \vee r)]$

$\sim p \wedge \sim(\sim q \vee \sim r)$

$\sim p \wedge q \vee r$.

x is not a real number and x is rational number (or) irrational number

Quantifiers :- The symbol which is used to write th statements by using any one of the following 6 words They are.

1. for All.
2. for Every
3. for any
4. for some
5. Their exist at least one
6. Their exist some.

The above six word are called as quatifiers.

Eg:- 1. Some women are clevers.
2. Some real numbers are rational
3. Their exist a men.
4. Something is good.
5. All women are good
6. Some monkeys have tails .... etc.

* They are two types of quantifiers.

1. Universal quatifier..
2. Existential quantifier

Universal Quantifier :-

The quantifier "All" is called as universal quantifier ($\forall$, for every, for any). It is denoted by the symbol: '$\forall x$' (or) '(x)' (or) 'V_{all}'.

For our understanding we can use first symbol $\forall x$:

→ If we want to perform universal quantifier we can use ("$\longrightarrow$") -

1. Eg:- Every apple is blue.

Sol The above statement can be written as

$A(x)$: x is an apple.

$B(x)$: x is Blue:

for all x, if x is an apple then x is blue.

$$\boxed{\forall x [A(x) \rightarrow B(x)]}$$

2. Eg:- All men are mortal.

$A(x)$: x is men

$B(x)$: x is mortal

for all x, if x is men then x is mortal

$$\boxed{\forall x [A(x) \rightarrow B(x)]}$$

3. Eg:- Any integer is either positive (or) negative

Sol The above statement can be written as

$A(x)$: x is an integer

$B(x)$: x is +ve or −ve

for all x, if x is an integer then x is either +ve (or) −ve

$$\boxed{\forall x [A(x) \rightarrow B(x)]}$$

Write each of the following in symbolic form

1. All men are good
2. No men are good
3. Some men are good
4. Some men are not good.

Sol 1. All men are good

$A(x)$: x is men

$B(x)$: x is good

for all x, if x is men then x is good

$$\forall x\,[A(x) \rightarrow B(x)]$$

2. No men are good.

$M(x)$: x is men

$G(x)$: x is good

$\sim G(x)$: x is not good

for all x, if x is men then x is not good

$$\forall x\,[M(x) \rightarrow \sim G(x)]$$

3. Some men are good

$M(x)$: x is a men

$G(x)$: x is good

There exist an x, such that x is men and x is good

$$\exists x\,[M(x) \wedge G(x)]$$

4. Some men are not good

$M(x)$: x is a men

$G(x)$: x is good

$\sim G(x)$: x is not good

There exist an x, such that x is men and x is not good

$$\exists x \left[M(x) \wedge \sim G(x) \right]$$

Existential Quantifier :-

The quantifier "some" is called as a existential quantifier (Some, for some, Their exist atleast one, Their exist some) It is denoted by the symbol is '$\exists$'. If we want to perform existential quantifier we can use "$\wedge$".

1. Eg:- Some monkeys have tails.

Sol

Let

$M(x)$: x is a monkey

$T(x)$: x has tails

Their exist x such that, x is a monkey and x has a tail

$$\boxed{\exists x \{ M(x) \wedge T(x) \}}$$

1. Write each of the following in symbolic form

1. All monkeys have tails
2. No monkey have tail
3. Some monkey have tails
4. Some monkey have no tails.

1. All monkeys have tails.

Let $M(x)$: x is a monkey

$T(x)$: x has tail

for all x, if x is a monkey then x has tail.

$$\forall x\,[M(x) \rightarrow T(x)]$$

2. No monkey have tail

Let $M(x)$: x is a monkey

$T(x)$: x has tail

$\sim T(x)$: x has no tail

for all x, if x is a monkey then x has no tail

$$\forall x\,[M(x) \rightarrow \sim T(x)]$$

3. Some monkeys have tails

Let $M(x)$: x is a monkey

$T(x)$: x has tail

Their exist x such that, x is a monkey and x has has a tails

$$\exists x\,[M(x) \wedge T(x)]$$

4. Some monkeys have no tails.

Let

$M(x)$: x is a monkey

$T(x)$: x has tail

$\sim T(x)$: x has no tail

Their exist an x such that, x is a monkey and x has no tail

$$\exists x\,[M(x) \wedge \sim T(x)]$$

2. If any one is good then John is good.

Let

$P(x)$: x is a person

$G(x)$: x is good

$G_1(J)$: John is good

There exist an x such that, if x is person and x is good then John is good

$$\exists x \; (P(x) \wedge G(x)) \rightarrow G_1(J)$$

3. i) Some dogs are animal ~~and some cats~~.

ii) Some cats are animal.

Sol

i) $\exists x \; D(x) \wedge A(x)$

ii) $\exists x \; C(x) \wedge A(x)$

Normal Forms :-

In normal forms we use the word "product" in place of conjunction. We use word "sume" in place of disjunction.

i) Elementary product

ii) Elementary sum

i) Elementary product :-

A product of the variables and their negations in a formula is called elementary product.

Eg:- $P, q, \sim P, \sim q, P \wedge q, \sim P \wedge q, \sim P \wedge \sim q, P \wedge \sim q$ .... etc

ii) Elementary sums :-

A sume of variables and their negations in a formula is called elementary sums.

Eg:- $P, q, \sim P, \sim q, P \vee q, \sim P \vee q, \sim P \vee \sim q, P \vee \sim q$ .... et

DNF :- (Disjunction Normal Form)

A formula which is equivalent to a given formula which consist of sum of elementary products is called as DNF.

It can represented in the form of ( ) ∨ ( ) ∨ ( )

Eg:- $P \wedge (P \rightarrow q)$

Sol

$= P \wedge (\sim P \vee q)$

$= (P \wedge \sim P) \vee (P \wedge q)$

$= P \vee (P \wedge q)$

Procedure for DNF :- $\rightarrow$ $\leftrightarrow$

Step 1 I. If the conectives $\longrightarrow$, $\longleftrightarrow$ are appears in a given formula then an equivalent formula obtained in which $\rightarrow$, $\leftrightarrow$ are replaced by $\wedge, \vee, \sim$ symbols.

$$p \rightarrow q \equiv \sim p \vee q$$

$$P \leftrightarrow q \equiv (P \rightarrow q) \wedge (q \rightarrow P)$$

$$\equiv (\sim p \vee q) \wedge (\sim q \vee P)$$

$$\equiv (P \wedge q) \vee (\sim p \wedge \sim q)$$

Step 2 :- If the negation is obtained in a formula (on) and part of the formala and not the variable.

Eg:- $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(\sim p) \equiv p$

Step 3 :- Now applying distributive law untill sum of elementary product is obtained.

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r).$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$$

Eg:- Find the DNF of $\sim[p \to (q \wedge r)]$

Sol Given Statement is

$$\equiv \sim[p \to (q \wedge r)]$$

$$\equiv \sim[\sim p \vee (q \wedge r)] \qquad (\because p \to q \equiv \sim p \vee q)$$

$$\equiv [\sim(\sim p) \wedge \sim(q \wedge r)] \qquad \sim(\sim p) \equiv p, \quad \sim(q \wedge r) = \sim q \vee \sim r$$

$$\equiv p \wedge \sim(q \wedge r)$$

$$\equiv p \wedge (\sim q \vee \sim r) \qquad \text{(By Distributive law)}$$

$$\equiv \boxed{(p \wedge \sim q) \vee (p \wedge \sim r)}$$

$\therefore$ The DNF is $(p \wedge \sim q) \vee (p \wedge \sim r)$

2. **CNF :-** (conjuntion Normal form)

A formula which is equivalent to a given formula which consist of product of Elementary sums is called a CNF. $(\ ) \wedge (\ ) \wedge (\ )$

Eg:- $p \wedge (p \to q)$

$$\equiv p \wedge (\sim p \vee q) \qquad (p \to q \equiv \sim p \vee q)$$

procedure for CNF :-

Step :-1 If the concetive $\to, \leftrightarrow$ are appears in a given formula then an equivelent formula obtained in which $\to, \leftrightarrow$ are replaceated by $\wedge, \vee, \sim$

$$p \to q \equiv \sim p \vee q$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$$

$$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

— If the negation is obtained the formula (or) and part of the formula not the variable.

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$
$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(\sim p) \equiv p$$

Step:-3 Now applying distributive law untill we get the product of elementary sums.

1. Eg:- Find CNF of $\sim[p \to (q \wedge r)]$

Given statement is

$$\equiv \sim[p \to (q \wedge r)]$$
$$\equiv \sim[\sim p \vee (q \wedge r)]$$
$$\equiv [\sim(\sim p) \wedge \sim(q \wedge r)]$$
$$\equiv p \wedge \sim(q \wedge r)$$
$$\equiv \boxed{p \wedge (\sim q \vee \sim r)}$$

∴ The CNF is $p \wedge (\sim q \vee \sim r)$

1M 2009 IMP
1.*** Find CNF of $\sim(p \vee q) \leftrightarrow (p \wedge q)$

Sol Given statement is

$$\underbrace{\sim(p \vee q)}_{p} \leftrightarrow \underbrace{(p \wedge q)}_{q}$$

$- [\sim(p \vee q) \wedge (p \wedge q)] \vee [(\sim(\sim(p \vee q)) \wedge \sim(p \wedge q))]$

$(\ ) \vee (\ ) \vee (\ ) -$

$(\ ) \vee (\ ) \vee (\ )$

DNF →

$(\ ) \wedge (\ ) \wedge (\ )$

CNF →

note: $p \leftrightarrow q = (p \to q) \wedge (q \to p)$ ← CNF

$= (\sim p \vee q) \wedge (\sim q \vee p)$

$(p \wedge q) \vee (\sim p \wedge \sim q)$ ← DNF

2. Obtain DNF of $[P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))]$.

Sol Given Statement is

$\equiv P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$\equiv \sim P \vee ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$ $\quad (\because P \to Q \equiv \sim P \vee Q)$

$\equiv \sim P \vee ((\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P))$

$\equiv \sim P \vee ((\sim P \vee Q) \wedge (Q \wedge P))$ $\quad \sim(P \vee Q) \equiv \sim P \wedge \sim Q$, $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

$\equiv \sim P \vee ((\sim P \wedge (Q \wedge P)) \vee Q \wedge (Q \wedge P))$ $\quad$ Distributive law

$\equiv \sim P \vee ((\sim P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P))$ $\quad$ Associative law

$\equiv \sim P \vee ((F \wedge Q) \vee (Q \wedge P))$ $\quad (\sim P \wedge P = F)$, $F \vee P = P$, $P \wedge P \equiv P$

$\equiv \sim P \vee (F \vee (Q \wedge P))$

$\equiv \sim P \vee (Q \wedge P)$

$\equiv \sim P \vee (P \wedge Q)$

$\therefore$ The required DNF is $\boxed{\sim P \vee (P \wedge Q)}$

3. Obtain DNF $(P \wedge \sim(Q \vee R)) \vee (((P \wedge Q) \vee \sim R) \wedge P)$.

Sol Given Statement is

$\equiv (P \wedge \sim(Q \vee R)) \vee (((P \wedge Q) \vee \sim R) \wedge P)$ $\quad \because$ demorgan's law

$\equiv (P \wedge (\sim Q \wedge \sim R)) \vee (((P \wedge Q) \vee \sim R) \wedge P)$ $\quad \because$ distributive law

$\equiv (P \wedge \sim Q \wedge \sim R) \vee [(P \wedge Q) \wedge P \vee (\sim R \wedge P)]$

$\equiv (P \wedge \sim Q \wedge \sim R) \vee [(P \wedge Q \wedge P) \vee (\sim R \wedge P)]$ $\quad P \wedge P \equiv P$

$\equiv (P \wedge \sim Q \wedge \sim R) \vee (P \wedge Q) \vee (\sim R \wedge P)$

4. Obtain DNF $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

Given Statement is

$$\underbrace{\sim(P \vee Q)}_{p} \leftrightarrow \underbrace{(P \wedge Q)}_{q}$$

$$= [\sim(P \vee q) \wedge (P \wedge Q)] \vee [\sim(\sim(P \vee Q) \wedge \sim(P \wedge Q)]$$

$$= [(\sim P \wedge \sim q) \wedge (P \wedge Q)] \vee [(P \vee Q) \wedge (\sim P \vee \sim Q)]$$

$$= [\sim P \wedge \sim q \wedge P \wedge Q] \vee [(P \vee Q) \wedge (\sim P \vee \sim Q)]$$

$$= [F \wedge F] \vee [(P \vee Q) \wedge (\sim P \vee \sim Q)]$$

$$= F \vee [(P \vee Q) \wedge (\sim P \vee \sim Q)]$$

$$\equiv (P \vee Q) \wedge (\sim P \vee \sim Q)$$

$$\equiv (P \wedge \sim P) \vee (P \wedge \sim Q) \vee (Q \wedge \sim P) \vee (Q \wedge \sim Q)$$

$$\equiv F \vee (P \wedge \sim Q) \vee (Q \wedge \sim P) \vee F$$

$$= (P \wedge \sim Q) \vee (Q \wedge \sim P)$$

5. Obtain DNF of $\sim(P \rightarrow (Q \wedge R))$

Given Statement is

$$= \sim(P \rightarrow (Q \wedge R))$$

$$\equiv \sim(\sim P \vee (Q \wedge R))$$

$$\equiv \sim(\sim P) \wedge \sim(Q \wedge R)$$

$$\equiv P \wedge (\sim Q \vee \sim R)$$

$$\text{PLA } (C) = (P \wedge \sim Q) \vee (P \wedge \sim R)$$

6. Obtain CNF of $[Q \vee (P \wedge R)] \wedge \sim[(P \vee R) \wedge Q]$

Sol Given statement is

$$\equiv [Q \vee (P \wedge R)] \wedge \sim[(P \vee R) \wedge Q]$$

$$\equiv [Q \vee (P \wedge R)] \wedge (\sim(P \vee R) \vee \sim Q)$$

$$\equiv [(Q \vee P) \wedge (Q \vee R)] \wedge [(\sim P \wedge \sim R) \vee \sim Q]$$

$$\equiv [(Q \vee P) \wedge (Q \vee R)] \wedge [(\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)]$$

$$\equiv (Q \vee P) \wedge (Q \vee R) \wedge (\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)$$

H.W

7. Obtain DNF for the following.

i) $(P \wedge \sim(Q \wedge R)) \vee (P \rightarrow Q)$

ii) $P \vee (\sim P \rightarrow (Q \vee (Q \rightarrow R)))$

iii) $(\sim P \vee \sim Q) \rightarrow (\sim P \wedge R)$

iv) $P \vee (\sim P \wedge \sim Q \vee R)$

v) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

8. Obtain CNF for the following

i) $P \rightarrow [(P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P)]$

ii) $[P \wedge \sim(Q \vee R)] \vee [((P \wedge Q) \vee \sim R) \wedge P]$

Principle Disjunction Normal Form :- (PDNF)

Minterms :-

→ The conjunction of variables (or) its negations, but not both.

→ Let P and Q are any two statements variable then the minterm are $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$.

→ Every minterm should ~~be~~ have exactly one truth value has true.

→ We consider every minterm exactly once

→ If given formula having n variables then that formula having 2^n minterms

1. Eg:- If P and Q are any two statement variables then we have 4 minterms they are $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$.

2. Eg:- If P, Q and R any three variables then we have 8 minterms They are $P \wedge Q \wedge R$, $P \wedge \sim Q \wedge R$, $P \wedge Q \wedge \sim R$, $\sim P \wedge Q \wedge R$, $P \wedge \sim Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge R$, $\sim P \wedge Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge \sim R$

Note:- If we want to perform PDNF it can represented in the form of $(\ ) \vee (\ ) \vee (\ )$

Minterms (products)

Procedure to obtain PDNF :-

A formula which is equivalent to given formula and it containing disjunction of minterms is called as PDNF of given formula.

Step 1:- Replace →, ↔ by using $\wedge, \vee, \sim$ symbols. to get the equivalence formula.

Step 2:- Apply $\sim$ to the variables by using demorgan's laws. To be followed by distributive law.

Obtain disjunction & minterms by introducing missing factors.

Step 4:- Eleminate if we have any contradiction

Step 5:- Consider single minterms implies of identical minterms.

1. Eg:- Obtain PDNF of $P \to Q$

Sol: Given Statement is

$\equiv P \to Q$

$\equiv \sim P \vee Q$

$\equiv \sim P \wedge (Q \vee \sim Q) \vee Q \wedge (P \vee \sim P)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q) \vee (\sim P \wedge Q)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

$\equiv (P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

$\therefore$ The required PDNF is $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

2. Obtain PDNF of $(\sim P \vee \sim Q)$

Sol: Given Statement is

$\equiv (\sim P \vee \sim Q)$

$\equiv \sim P \wedge (Q \vee \sim Q) \vee \sim Q \wedge (P \vee \sim P)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (\sim Q \wedge P) \vee (\sim Q \wedge \sim P)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge \sim Q)$

$\equiv (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\therefore$ The required PDNF is $(\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

3. Obtain PDNF of $(P \vee \sim Q)$

Sol Given statement is

$$= P \vee \sim Q$$

$$= P \wedge (Q \vee \sim Q) \vee \sim Q \wedge (P \vee \sim P)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim Q \wedge P) \vee (\sim Q \wedge \sim P)$$

$$\equiv (P \wedge Q) \vee (P \wedge \sim Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$$

4. Obtain PDNF of $(P \leftrightarrow q)$

Given statement is

$$\equiv (P \leftrightarrow q)$$

$$\equiv (P \wedge q) \vee (\sim P \wedge \sim Q)$$

Principle conjuctive Normal Form :- (PCNF)

Maxterms:- The disjunction of variables. (or) its negation but not both is called maxterm.

1. Eg:- If P and Q are any two variables then we have maxterms are $P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, $\sim P \vee \sim Q$.

2. ⇒ Every maxterm should have exactly one truth value as false.

Procedure to obtain PCNF :-

Step :- A formula which is equivalent to given formula and its containing conjunction of maxterms is called PCNF of given formula.

If we want to represent PCNF in the form of

$(\;) \wedge (\;) \wedge (\;).$

maxterms.

Step :- 1 Replace $\rightarrow$, $\leftrightarrow$, by using $\wedge, \vee, \sim$ symbols to get the equivalence formula.

Step :- 2 Apply negation to the variables by using demorgan's law to be followed by the distributive law.

Step :- 3

Eleminate if we have any contradiction

Step :- 4

Consider single maxterm implies of identical maxterms.

Planar graph
M.S.T
DFS Algorithm
Hami Cycle
Euler

1. Obtain PCNF for $(P \wedge \sim Q)$

Sol

Given statement is

$$\equiv P \wedge \sim Q$$

$$\equiv [P \vee (Q \wedge \sim Q)] \wedge [\sim Q \vee (P \wedge \sim P)]$$

$$\equiv (P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim Q \vee P) \wedge (\sim Q \vee \sim P)$$

$$\equiv (P \vee Q) \wedge (P \vee \sim Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$$

$$\equiv (P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$$

$\therefore$ The required PCNF is $(P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

2. Obtain PCNF for $\sim (P \vee Q)$

Sol

Given statement is

$$\equiv \sim (P \vee Q)$$

$$\equiv \sim P \wedge \sim Q$$

$$\equiv \sim P \vee (Q \wedge \sim Q) \wedge \sim Q \vee (P \wedge \sim P)$$

$$\equiv (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (\sim Q \vee P) \wedge (\sim Q \vee \sim P)$$

$$\equiv (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$$

$$\equiv (\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$$

$\therefore$ The required PCNF is $(\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

EXTRA PROBLEMS:-

1. Show that $(\sim P) \rightarrow (P \rightarrow Q)$ is a tautology, without using truth table.

Sol

Given statement is

$$\equiv (\sim P) \rightarrow (P \rightarrow Q)$$

$$\equiv \sim(\sim P) \vee (P \rightarrow Q)$$

$$\equiv P \vee (\sim P \vee Q)$$

$$\equiv (P \vee \sim P) \vee Q$$

$$\equiv T \vee Q$$

$$\equiv T \quad \text{(Tautology)}$$

2. Express $P \leftrightarrow Q$ in terms of $\{\sim, \vee\}$ only

Given statement is

$$\equiv P \leftrightarrow Q$$

$$\equiv (P \to Q) \wedge (Q \to P)$$

$$\equiv (\sim P \vee Q) \wedge (\sim Q \vee P) \qquad \sim(\sim P) \equiv P$$

$$\equiv \sim\Big[\sim[(\sim P \vee Q) \wedge (\sim Q \vee P)]\Big]$$

$$\equiv \sim\Big[\sim(\sim P \vee Q) \vee \sim(\sim Q \vee P)\Big]$$

3. Show that i) $\sim(P \uparrow Q) \Leftrightarrow \sim P \downarrow \sim Q$

ii) $\sim(P \downarrow Q) \Leftrightarrow \sim P \uparrow \sim Q$ with out using truth table.

Sol i) Given statement is

$$\sim(P \uparrow Q) \Leftrightarrow \sim P \downarrow \sim Q$$

R.H.S

$$\equiv \sim P \downarrow \sim Q \qquad P \downarrow Q \equiv \sim(P \vee Q)$$

$$\equiv \sim[\sim P \vee \sim Q]$$

$$\equiv \sim[\sim(P \wedge Q)]$$

$$\equiv \sim[P \uparrow Q]$$

ii) Given statement is

$$\sim(P \downarrow Q) \Leftrightarrow \sim P \uparrow \sim Q$$

R.H.S $\quad \sim P \uparrow \sim Q \qquad P \uparrow Q \equiv \sim(P \wedge Q)$

$$\sim[\sim P \wedge \sim Q]$$

$$\sim[\sim(P \vee Q)]$$

$$\sim[P \downarrow Q]$$

Note :-

1. $p \uparrow q \equiv q \uparrow p$

2. $p \downarrow q \equiv q \downarrow p$

3. $p \uparrow q \equiv \sim(p \wedge q)$

4. $p \downarrow q \equiv \sim(p \vee q)$

5. $p \uparrow p \equiv \sim p$, 7. $p \downarrow p \equiv \sim p$

6. $q \uparrow q \equiv \sim q$ 8. $q \downarrow q \equiv \sim q$

4. Express $p \uparrow q$ interms of $(\downarrow)$

Sol Given statement is

$p \uparrow q$ interms $\uparrow$

$$\equiv p \uparrow q$$

$$\equiv \sim(p \wedge q)$$

$$\equiv \sim p \vee \sim q$$

$$\equiv \sim[\sim(\sim p \vee \sim q)]$$

$$\equiv \sim[\sim p \downarrow \sim q]$$

$$\equiv \sim[(p \downarrow p) \downarrow (q \downarrow q)]$$

$$\equiv [(p \downarrow p) \downarrow (q \downarrow q)] \downarrow [(p \downarrow p) \downarrow (q \downarrow q)]$$

$p \uparrow p \equiv \sim p$

$p \downarrow p \equiv \sim p$.

5. Express $p \rightarrow (\sim p \rightarrow q)$ i) in terms of $\uparrow$ only

ii) in terms of $\downarrow$ only

Sol i) Given statement is

$$\equiv p \rightarrow (\sim p \rightarrow q)$$

$$\equiv \sim p \vee (\sim p \rightarrow q)$$

$$\equiv \sim p \vee (\sim(\sim p) \vee q)$$

$\sim(\sim p) \equiv p$

$$\equiv \sim p \vee (p \vee q)$$

$$\equiv \sim[\sim[\sim p \vee (p \vee q)]]$$

$\sim(\sim p) \wedge \sim(p \vee q)$

$$\equiv \sim[p \wedge \sim(p \vee q)]$$

~~show that~~ $\equiv P \uparrow \sim(p \vee q)$

$\equiv P \uparrow \sim p \wedge \sim q$

$\sim p$

$\sim(p \wedge q)$

$\equiv P \uparrow ((p \uparrow p) \wedge (q \uparrow q))$

$\equiv P \uparrow ((p \wedge q) \uparrow (p \wedge q) \uparrow (p \wedge q) \uparrow (p \wedge q))$

$\equiv P \uparrow (\sim(p \wedge q) \uparrow \sim(p \wedge q))$

$\equiv P \uparrow (p \uparrow q) \uparrow (p \uparrow q)$

ii) Given statement is

$\equiv P \to (\sim p \to q)$

$\equiv \sim P \to (\sim p \to q)$

$\equiv \sim p \to [\sim(\sim p) \vee q]$

$\equiv \sim p \to [p \vee q]$

$\equiv \sim p \vee (p \vee q)$

$\equiv \sim [\sim(\sim p \vee (p \vee q))]$

$\equiv \sim[\sim p \downarrow (p \vee q)]$

$\equiv \sim[\sim p \downarrow \sim(\sim(p \vee q))]$

$\equiv \sim[\sim p \downarrow \sim(p \downarrow q)]$

$\equiv \sim[(p \downarrow p) \downarrow [(p \downarrow q) \downarrow (p \downarrow q)]]$

$\equiv \sim[(p \downarrow p) \downarrow (p \downarrow q) \downarrow (p \downarrow q)]$

$\equiv [(p \downarrow p) \downarrow (p \downarrow q) \downarrow (p \downarrow q)] \downarrow [(p \downarrow p) \downarrow (p \downarrow q) \downarrow (p \downarrow q)]$

1 - UNIT (BITS)

1. $P \wedge Q$ has the truth value T whenever both P and Q have the truth value : True
2. $P \vee Q$ has the truth value F only when both P and Q have the truth value : False
3. $P \vee Q$ is true if either P is True or Q is True or both P and Q are : True
4. $P \rightarrow Q$ has a truth value F when Q has the truth value False and P has the truth value True ; otherwise it has the truth value : True
5. The converse of $P \rightarrow Q$ is : $Q \rightarrow P$
6. The contrapositive of $P \rightarrow Q$ is : $\sim Q \rightarrow \sim P$
7. The inverse of $P \rightarrow Q$ is : $\sim P \rightarrow \sim Q$
8. $P \rightarrow Q$ and $\sim Q \rightarrow \sim P$ have the same truth value.
9. $P \rightarrow Q$ has the truth value T whenever both P and Q have identical truth value.
10. $P \rightleftarrows Q$ and $(P \rightarrow Q) \wedge (Q \rightarrow P)$ are : equivalent
11. $P \wedge \sim P$ is always : False
12. $P \vee \sim P$ is always : True
13. $P \vee \sim P$ is a : Tautology
14. $P \wedge \sim P$ is a : Contradiction
15. $(P \vee Q) \rightarrow P$ is not a : Tautology
16. $(P \wedge (P \rightleftarrows Q)) \rightarrow Q$ is a : Tautology
17. Any substitution instance of a tautology is a : Tautology
18. If $A \rightleftarrows B$ is a tautology, then : $A \Leftrightarrow B$
19. If $A \rightarrow B$ is a tautology, then we write : $A \Leftrightarrow B$ (or) A is tautologically implies to B
20. $A \Rightarrow B$ guarantees that B has the truth value T whenever A has the : True.

21. Both implication and equivalence are : Transitive

22. $\downarrow, \uparrow$ are : Functionally complete

23. A product (conjunction) of variables and their negations is called an : Elementary Product

24. A sum (disjunction) of variables and their negations is called an : Elementary sum

25. If A has the truth value T for atleast one combination of truth value assigned to $P_1, P_2, P_3, \ldots P_n$ then A is said to be : satisfiable

26. DNF is the : sum of elementary products

27. CNF is the : Product of elementary sums

28. PDNF is the : Disjunction of minterms only

29. PCNF is the : Conjunction of maxterms only

30. PCNF, PDNF are unique except for the rearrangment of the terms.

31. Rules of interference of statement calculus : Rule P, Rule T, Rule CP

32. Rules of predicate calculus are : Rule US, Rule UG, Rule ES, Rule EG

33. If $H_1, H_2, \ldots H_m$ and P imply Q then $H_1, H_2, \ldots H_m$ implies : $P \to Q$

34. The connective $\uparrow$ is not : Associative

35.
$$\begin{array}{r} P \to Q \\ P \\ \hline \therefore\ ? \end{array} \quad Q$$

12/07/23

PREDICATES

1. Predicative logic
2. Free and bound variables
3. Rules of inference
4. Consistency
5. Proof of contradiction
6. Automatic Theorem proving

Predicate :-

A common part (or) factor in a statement is called as predicate.

Eg :- 1. Ramu is a bachelor

2. Ranjani is a bachelor

In above two statement we have one common part i.e., "is a bachelor." This common part (or) factor is called predicate.

Predicative logic :-

The logic based on predicates is called as predicate logic means obtaining symbolic forms for the predicate statement. We can use capital Letters to denoted the predicates.

* We can use lower case to denote the individual variable names

Eg :- Ranjani is a bachelor.

is a bachelor is denoted by "B"

Ranjani is denoted by "r"

Symbolic form :- $B(r)$

Eg :- Ramu is a bachelor

is a bachelor is denoted by "A"

Ramu is denoted by "p"

Symbolic form :- $A(p)$

Types of predicates:-

If any predicate is associated with m individual variable names then that predicate is called "M place predicate."

One place predicate :-

If any predicate associated with only one predicate individual name then this predicate is called one place predicate.

Eg:- Ramu is a bachelor

A(R)

Two place predicate :-

If any predicate associated with two individual name then this predicate is called Two place predicate.

Eg:- Anushka is tallor than kajal.

T (a,k)

Three place predicate :-

If any predicate associated with three individual name then this predicate is called Three place predicate

Eg:- Sita rank is between ramu and ranjani

R (s, r, r)

Free and bounded variables:- Theory 4M }
problem 4M }

x bound part of the formula :- If a given formula containing the form of $\forall x\ p(x)$ then this formula is called as x bound part of the formula

y bound part of the formula :- If a given formula containing the form of $\forall x\ p(y)$ then this formula is called as y bound part of the formula.

x bound occarance of x :- Any occarance of x in x bound part of the formula is called as bound occarance of x at this time x is called "bounded variable."

Free occarance of x :- Any occarance of x in x bound part of the formula is not a bound occarance then this occarance is called as free occarance.

If any quantifier having atomic statement formula then the use of paranthesis is optional.

Scope of the quantifier :- A statement formula which is immediatly appears after the quantifier is called scope of the quantifier.

Eg:- 1. $\forall x\ p(x, y)$

Sol Given statement is $\forall x\ p(x, y)$

The scope of quantifier is $p(x, y)$

bounded occarance is x

Free occarance is y.

Eg : 2 :- $\forall x \, [(P(x) \rightarrow R(x))]$

Sol Given statement is $\forall x \, [P(x) \rightarrow R(x)]$

The scope of quantifier is $P(x) \rightarrow R(x)$

bounded occarance is x

Free

Eg : 3 :- $\forall x . [R(x) \rightarrow \exists y \, R(x,y)]$

Sol

Given Statement is $\forall x \, [R(x) \rightarrow \exists y \, R(x,y)]$

Scope of $\forall x$ quantifier is $R(x) \rightarrow \exists y \, R(x,y)$

bounded occarance is x

bounded occarance is y

scope of $\exists y$ quantifier is $R(x,y)$

Eg : 4 $\exists x \quad P(x) \wedge Q(x)$

Sol scope of $\exists x$ quantifier $P(x) \wedge Q(x)$

In $P(x)$ bounded occarance is x

In $Q(x)$ Free occarance is x

Eg : 5 $\exists x \, [P(x) \wedge Q(x)]$

Sol Given statement is $\exists x \, [P(x) \wedge Q(x)]$

Scope of $\exists x$ quantifier $P(x) \wedge Q(x)$

bounded occarance is x

Rules of inference :-

* Logic is the study of inference. A procedure which is used to obtain perticular formula is valide consequences of given set of premisis (series of statements). Here we are using two rules of inference which are called

1. Rule P
2. Rule T

Rule P :- A premesis may be introduced at any point in the derivation

Rule T :- A formula 'S' may be introduced in a derivation, if is tautologically implied by any one (or) more of the preciding formulas in the derivation

Rules of implication :-

I_1 : $P \wedge Q \Rightarrow P$

I_2 : $P \wedge Q \Rightarrow Q$

I_3 : $P \Rightarrow P \vee Q$

I_4 : $Q \Rightarrow P \vee Q$

I_5 : $\sim P \Rightarrow P \rightarrow Q$

I_6 : $Q \Rightarrow P \rightarrow Q$

I_7 : $\sim(P \rightarrow Q) \Rightarrow \sim Q$

I_8 : $\sim(P \rightarrow Q) \Rightarrow P$

I_9 : $P, Q \Rightarrow P \wedge Q$

* I_{10} : $\sim P, P \vee Q \Rightarrow Q$

* I_{11} : $P, P \rightarrow Q \Rightarrow Q$

* I_{12} : $\sim Q, P \rightarrow Q \Rightarrow \sim P$

* I_{13} : $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$

I_{14} : $P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$

I_{15} : $P \rightarrow R, Q \rightarrow R \Rightarrow (P \vee Q) \rightarrow R$

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Rules of equivalence :-

E_1 : $\sim(\sim P) \Leftrightarrow P$

E_2 : $P \wedge Q \Leftrightarrow Q \wedge P$

E_3 : $P \vee Q \Leftrightarrow Q \vee P$

E_4 : $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

E_5 : $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

E_6 : $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

E_7 : $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

E_8 : $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

E_9 : $\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

E_{10} : $P \vee P \Leftrightarrow P$

E_{11} : $P \wedge P \Leftrightarrow P$

E_{12} : $R \vee (P \wedge \sim P) \Leftrightarrow R$

E_{13} : $R \wedge (P \vee \sim P) \Leftrightarrow R$

E_{14} : $R \vee (P \vee \sim P) \Leftrightarrow T$

E_{15} : $R \wedge (P \wedge \sim P) \Leftrightarrow F$

E_{16} : $P \rightarrow Q \Leftrightarrow \sim P \vee Q$

E_{17} : $\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$

E_{18} : $P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

E_{19} : $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

E_{20} : $\sim(P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \sim Q$

E_{21} : $P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

E_{22} : $P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$

1. Demonstrate that R is a valide inference from the premisis $P \to Q$, $Q \to R$ and P.

Sol

(1)	$P \to Q$	rule P
(2)	P.	rule P
(3)	Q.	rule T from (1),(2) I_{11}
(4)	$Q \to R$	rule P
(5)	R	rule T from (3),(4) I_{11}

2. Show that $R \vee S$ follows logically from the premisis $C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$, $(A \wedge \sim B) \to (R \vee S)$

Sol

(1)	$C \vee D \to \sim H$	rule P
(2)	$\sim H \to (A \wedge \sim B)$	rule P
(3)	$(C \vee D) \to (A \wedge \sim B)$	rule T from (1),(2) I_{13}
(4)	$(A \wedge \sim B) \to (R \vee S)$	rule P
(5)	$(C \vee D) \to (R \vee S)$	rule T. from (3),(4) I_{13}
(6)	$C \vee D$	rule P
(7)	$R \vee S$	rule T from (5),(6) I_{11}

3. Show that R is valide inference of given premisis $P \to Q$, $Q \to R$, P

Sol

(1)	$P \to Q$	rule P
(2)	P	rule P
(3)	Q	rule T from (1),(2) I_{11}
(4)	$Q \to R$	rule P
(5)	R	rule T from (3)&(4) I_{11}

4. Show that $R \wedge (P \vee Q)$ is a valid conclu... the premises $P \vee Q$, $Q \to R$, $P \to M$, $\sim M$

Sol

①	$P \to M$	rule P	
②	$\sim M$	rule P	
③	$\sim P$	rule T from ①,②	I_{12}
q ④	$P \vee Q$	rule P	
⑤	Q	rule T from ③ ④	I_{10}
⑥	$Q \to R$	rule P	
P ⑦	R	rule T from ⑤,⑥	I_{11}
⑧	$R \wedge (P \vee Q)$	rule T from ④,⑦	I_9

$(M \wedge \sim) \to M$ Imp

5. Show that $S \vee R$ tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

$\because \sim p \to q \Leftrightarrow \sim(\sim p) \vee q \Leftrightarrow p \vee q$

①	$P \vee Q$	rule P	
②	$\sim P \to Q$	rule T from ①	E_1 & E_{16}
③	$Q \to S$	rule P	
④	$\sim P \to S$	rule T from ② & ③	I_{13}
⑤	$\sim S \to P$	rule T from ④	E_{18} & E_1
⑥	$P \to R$	rule P	
⑦	$\sim S \to R$	rule T from ⑤ & ⑥	I_{13}
⑧	$S \vee R$	rule T from ⑦	E_1 & E_{16}

$p \to q \Leftrightarrow \sim q \to \sim p$ E_{18} & E_1

$p \to q, q \to r$

$P \to Q : \sim P \vee Q$

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4. Show that $R \wedge (P \vee Q)$ is a valid conclusion [from] the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, $\sim M$

Sol

	①	$P \rightarrow M$	rule P
	②	$\sim M$	rule P
	③	$\sim P$	rule T from ①,② I_{12}
q	④	$P \vee Q$	rule P
	⑤	Q	rule T from ③ ④ I_{10}
	⑥	$Q \rightarrow R$	rule P
P	⑦	R	rule T from ⑤,⑥ I_{11}
	⑧	$R \wedge (P \vee Q)$	rule T from ④,⑦ I_9

7M (or) 7M
*** Imp

5. Show that $S \vee R$ tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

①	$P \vee Q$	rule P
②	$\sim P \rightarrow Q$	rule T from ① E_1 & E_{16}
③	$Q \rightarrow S$	rule P
④	$\sim P \rightarrow S$	rule T from ② & ③ ~~E_3~~ I_{13}
⑤	$\sim S \rightarrow P$	rule T from ④ E_{18} & E_1
⑥	$P \rightarrow R$	rule P
⑦	$\sim S \rightarrow R$	rule T from ⑤ & ⑥ I_{13}
⑧	$S \vee R$	rule T from ⑦ E_1 & E_{16}

$\because \sim P \rightarrow Q \Leftrightarrow \sim(\sim P) \vee Q \Leftrightarrow P \vee Q$

$P \rightarrow Q$, $Q \rightarrow P$

$P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$ E_{18} & E_1

$P \rightarrow Q : \sim P \vee Q$

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Consistency :-

Let H_1 and H_2 any two statement formulas then the conjunction of H_1 and H_2 has the truth value true then H_1 and H_2 as consistency formula.

Eg:- Let H_1 is $p \vee q$ and H_2 is $\sim p \vee \sim q$

P	q	H_1 $p \vee q$	$\sim p$	$\sim q$	H_2 $\sim p \vee \sim q$	$H_1 \wedge H_2$
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F

In above truth table the conjunction of $H_1 \wedge H_2$ has truth value true in ② & ③ at the time two formulas H_1 and H_2 are called as consistency formula.

proof of tautology :-

If we are using the notation of consistency in a procedure then this procedure is called as "proof of tautology".

$H_1 \wedge H_2 \Leftrightarrow T$

$\Leftrightarrow R \vee \sim R$

Altenatively a set of formulas $H_1, H_2, H_3 \cdots\cdots H_m$ is consist of their conjunction implies a tautology.

$\therefore H_1 \wedge H_2 \wedge H_3 \wedge H_4 \cdots\cdots\cdots \wedge H_m \Leftrightarrow R \vee \sim R$

In-consistency :-

Let H_1 and H_2 are any two statement formulas then the conjunction of H_1 and H_2 has truth values flase then $H_2 \wedge H_2$ as inconsistency formula.

In above truth table the conjunction of $H_1 \wedge H_2$ has truth value false in ① & ④ at this time two formulas H_1 & H_2 are called inconsistence.

$\therefore H_1 \wedge H_2 \iff F$

$\iff R \wedge \sim R$

$\therefore H_1 \wedge H_2 \wedge H_3 \wedge H_4 \cdots\cdots\cdots \wedge H_m \iff R \wedge \sim R$

Imp

4 Show that the following premesis are inconsistence

1) If Jack misses many classes through illness, then he fails high school.

2) If Jack fails high school, then he is uneducated

3) If Jack reads a lot of books, Then he is not uneducated.

4) Jack misses many class through illness, and reads a lot of books

Sol

$P \wedge \sim P = F$

$P \vee \sim P = T$

E : Jack misses many classes

S : Jack fails high school

A : Jack reads a lot of books

H : Jack is uneducated

The premesis are $E \to S$, $S \to H$, $A \to \sim H$, $E \wedge A$

(1)	$E \to S$	Rule P
(2)	$S \to H$	Rule P.
(3)	$E \to H$	Rule T from (1), (2) I_{13}
(4)	$A \to \sim H$	Rule P
(5)	$\sim A \to H$	Rule T from (4) E_{18}, E_1
(6)	$E \to \sim A$	Rule T from (3) & (5) I_{13}
(7)	$\sim E \vee \sim A$	Rule T from (6) E_{16}
(8)	$\sim [E \wedge A]$	Rule T from (7) E_8

(9) $E \wedge A$ rule P

(10) $(E \wedge A) \wedge \sim(E \wedge A)$ rule T from (8) & (9) I_9

$\therefore$ The given statements are inconsistence

Proof Contradiction :- (or) Direct proof

The Indirect method of proof is equivalent to what is known as the proof by contradiction. The lines of argument in this method of proof of the statement $P \rightarrow Q$ are as the follows.

1. Hypothesis :- Assume that $p \rightarrow q$ is false i.e., p is true and q is false.

2. Analysis :- Starting with the hypothesis that q is false and rules of logic and other known facts infer that p is false.

3. Conclusion :- Because of the contradiction arrived in the analysis that $p \rightarrow q$ is true.

Eg:- provide a proof by contradiction of the following statements for every integer 'n', if n^2 is odd then 'n' is odd

Sol

Let n be any integer. then the given statement reads $P \rightarrow Q$ where

P: n^2 is odd

Q: n is odd

Assume that $P \rightarrow Q$ is false i.e., P is true and Q is false.

Now Q is false means that n is even, so that "$n = 2K$" for some integer 'K'

$\therefore \quad n^2 = (2k)^2$

$n^2 = 4k^2$

from which evident n^2 is an even i.e., p is false. This contradicts the assumption that p is true. The given conditional $p \to q$ is true.

Eg:- prove that if 'm' is an even integer, then 'm+7' is an odd integer.

Sol

Given Statement is. $p \to q$

p: m is an even integer

q: m+7 is an odd integer

Assume that $p \to q$ is false i.e., p is true and q is false.

Now q is false means that m+7 is an even for some integer 2k. Hence m+7 = 2k.

~~m+7=2k~~ $m+7 = 2k$

$m = 2k - 7$

$m = (2k - 8) + 1$

$m = 2(k-4) + 1$

Which show that 'm' is an odd. This means p is false and assumption that p is true the given conditional $p \to q$ is true.

Show that $\sim P$ from the premisis $\sim Q$, $P \to Q$

1. $P \to Q$ Rule P.
2. $\sim Q$ rule P
3. $\sim P$ rule T from 1 & 2 I_{12}

(or)

1. $P \to Q$ rule P
2. $\sim q \to \sim p$ rule T from 1 E_{18}
3. $\sim q$ rule P
4. $\sim p$ rule T from 2 & 3 I_{11}

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