UNIT -III

ELEMENTARY COMBINATORICS

Topics

Basis of counting, Combinations & Permutations, with repetitions, Constrained repetitions, Binomial Coefficients, Binomial Multinomial theorems, the principles of Inclusion – Exclusion

1 Basic Counting Principles

1 Sum Rule Principle: Assume some event E can occur in m ways and a second event F can occur in n ways, and suppose both events cannot occur simultaneously. Then E or F can occur in m + n ways.

In general, if there are n events and no two events occurs in same time then the event can occur in n_1+n_2..........n ways.

Example: If 8 male processor and 5 female processor teaching DMS then the student can choose professor in 8+5=13 ways.

2 Product Rule Principle: Suppose there is an event E which can occur in m ways and, independent of this event, there is a second event F which can occur in n ways. Then combinations of E and F can occur in mn ways.

In general, if there are n events occurring independently then all events can occur in the order indicated as n_1 x n_2 x n_3.........n ways.

Example: In class, there are 4 boys and 10 girls if a boy and a girl have to be chosen for the class monitor, the students can choose class monitor in 4 x 10 = 40 ways.

2 Permutation and Combinations:

1 Permutation:

Any arrangement of a set of n objects in a given order is called Permutation of Object. Any arrangement of any $r \leq n$ of these objects in a given order is called an r-permutation or a permutation of n object taken r at a time.

It is denoted by P (n, r)

$$P(n, r) = \frac{n!}{(n-r)!}$$

Theorem: Prove that the number of permutations of n things taken all at a time is n!.

Proof: We know that

$$n_{P_n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Example: $4 \times n_{p3} = n+1_{P3}$

Solution: $4 \times \frac{n!}{(n-3)!} = \frac{(n+1)!}{(n+1-3)!}$

$$\frac{4 \times n!}{(n-3)!} = \frac{(n+1) \times n!}{(n-2)(n-3)!}$$

$4(n-2) = (n+1)$

$4n - 8 = n+1$

$3n = 9$

$n = 3.$

1.1 Permutation with Restrictions:

The number of permutations of n different objects taken r at a time in which p particular objects do not occur is

$$n - p_{P_r}$$

The number of permutations of n different objects taken r at a time in which p particular objects are present is

$$n - p\, P_{r-p} \times r P_p$$

Example: How many 6-digit numbers can be formed by using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 if every number is to start with '30' with no digit repeated?

Solution: All the numbers begin with '30.'So, we have to choose 4-digits from the remaining 7-digits.

∴ Total number of numbers that begins with '30' is

$7_{P4} = \frac{7!}{(7-4)!} = \frac{7 X 6 X 5 X 4 X 3!}{3!}$ =840.

1.2 Permutations with Repeated Objects:

Theorem: Prove that the number of different permutations of n distinct objects taken at a time when every object is allowed to repeat any number of times is given by n^r.

Proof: Assume that with n objects we have to fill r place when repetition of the object is allowed.

Therefore, the number of ways of filling the first place is = n
The number of ways of filling the second place = n
....
....
The number of ways of filling the rth place = n
Thus, the total number of ways of filling r places with n elements is
= n. n. n..............r times $=n^r$.

1.3 Circular Permutations:

There are two cases of circular-permutations:-

(a) If clockwise and anti clock-wise orders are different, then total number of circular-permutations is given by (n-1)!

(b) If clock-wise and anti-clock-wise orders are taken as not different, then total number of circular-permutations is given by (n-1)!/2!

Note: Number of circular-permutations of 'n' different things taken 'r' at a time:-

(a) If clock-wise and anti-clockwise orders are taken as different, then total number of circular-permutations = ${}^nP_r/r$

(b) If clock-wise and anti-clockwise orders are taken as not different, then total number of circular – permutation = ${}^nP_r/2r$

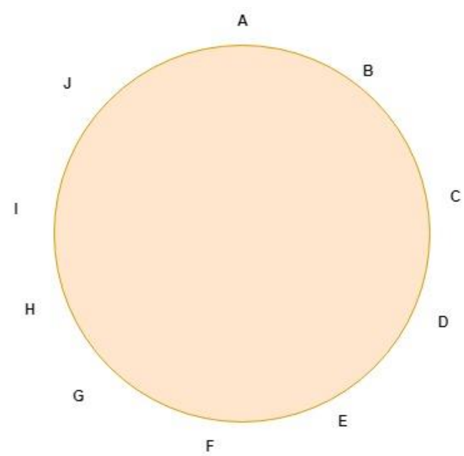
Example: How many necklace of 12 beads each can be made from 18 beads of different colours?

Ans. Here clock-wise and anti-clockwise arrangement s are same.

Hence total number of circular–permutations: ${}^{18}P_{12}/2x12$

= 18!/(6 x 24)

A permutation which is done around a circle is called Circular Permutation.



Example: In how many ways can get these letters a, b, c, d, e, f, g, h, i, j arranged in a circle?

Solution: (10 - 1) = 9! = 362880

Theorem: Prove that the number of circular permutations of n different objects is (n-1)!

Proof: Let us consider that K be the number of permutations required.

For each such circular permutations of K, there are n corresponding linear permutations. As shown earlier, we start from every object of n object in the circular permutations. Thus, for K circular permutations, we have K...n linear permutations.

Therefore, $K.\, n = n!$ or $K = \frac{n!}{n}$

$$K = \frac{n \times (n-1)!}{n}$$

$$K = (n-1)!$$

Hence Proved.

2. Combination:

A Combination is a selection of some or all, objects from a set of given objects, where the order of the objects does not matter. The number of combinations of n objects, taken r at a time represented by n_{Cr} or C (n, r).

$$n_{C_r} = \frac{n!}{r!\,(n-r)!}$$

Proof: The number of permutations of n different things, taken r at a time is given by

$$n_{P_r} = \frac{n!}{(n-r)!}$$

As there is no matter about the order of arrangement of the objects, therefore, to every combination of r things, there are r! arrangements i.e.,

$$n_{P_r} = r!\ n_{C_r} \quad \text{or} \quad n_{C_r} = \frac{n_{P_r}}{r!} = \frac{n!}{(n-r)!\,r!},\ n \geq r$$

Thus,

$$n_{C_r} = \frac{n!}{r!(n-r)!}$$

Example: A farmer purchased 3 cows, 2 pigs, and 4 hens from a man who has 6 cows, 5 pigs, and 8 hens. Find the number m of choices that the farmer has.

The farmer can choose the cows in C (6, 3) ways, the pigs in C (5, 2) ways, and the hens in C (8, 4) ways. Thus the number m of choices follows:

$$m = \binom{6}{3}\binom{5}{4}\binom{8}{4} = \frac{6.5.4}{3.2.1} \times \frac{5.4}{4} \times \frac{8.7.6.5}{4.3.2.1} = 20 \times 10 \times 70 = 14000$$

Restricted – Permutations:

(a) Number of permutations of ‘n’ things, taken ‘r’ at a time, when a particular thing is to be always included in each arrangement $= r\ {}^{n-1}P_{r-1}$

(b) Number of permutations of ‘n’ things, taken ‘r’ at a time, when a particular thing is fixed: $= {}^{n-1}P_{r-1}$

(c) Number of permutations of ‘n’ things, taken ‘r’ at a time, when a particular thing is never taken: $= {}^{n-1}P_{r}$.

(d) Number of permutations of ‘n’ things, taken ‘r’ at a time, when ‘m’ specified things always come together $= m! \times (n-m+1)!$

(e) Number of permutations of ‘n’ things, taken all at a time, when ‘m’ specified things always come together $= n! - [m! \times (n-m+1)!]$

Example: How many words can be formed with the letters of the word ‘OMEGA’ when:

(i) ‘O’ and ‘A’ occupying end places.
(ii) ‘E’ being always in the middle
(iii) Vowels occupying odd-places
(iv) Vowels being never together.

Ans.

(i) When ‘O’ and ‘A’ occupying end-places

=> M.E.G. (OA)

Here (OA) are fixed, hence M, E, G can be arranged in 3! ways

But (O,A) can be arranged themselves is 2! ways.

=> Total number of words = $3! \times 2! = 12$ ways.

(ii) When ‘E’ is fixed in the middle

=> O.M.(E), G.A.

Hence four-letter O.M.G.A. can be arranged in 4! i.e 24 ways.

(iii) Three vowels (O,E,A,) can be arranged in the odd-places (1^{st}, 3^{rd} and 5^{th}) = 3! ways.

And two consonants (M,G,) can be arranged in the even-place (2^{nd}, 4^{th}) = 2 ! ways

=> Total number of ways= $3! \times 2! = 12$ ways.

(iv) Total number of words = 5! = 120!

If all the vowels come together, then we have: (O.E.A.), M,G

These can be arranged in 3! ways.

But (O,E.A.) can be arranged themselves in 3! ways.

=> Number of ways, when vowels come-together = 3! x 3!

= 36 ways

=> Number of ways, when vowels being never-together

= 120-36 = 84 ways.

NOTE:1

Number of Combination of 'n' different things, taken 'r' at a time is given by:-

${}^{n}C_{r}$= n! / r ! x (n-r)!

Note: ${}^{n}C_{r} = {}^{n}C_{n-r}$

or ${}^{n}C_{r}$ = n!/r!x(n-r)! and ${}^{n}C_{n-r}$ = n!/(n-r)!x(n-(n-r))!

= n!/(n-r)!xr!

Restricted – Combinations

(a) Number of combinations of 'n' different things taken 'r' at a time, when 'p' particular things are always included = ${}^{n-p}C_{r-p}$.

(b) Number of combination of 'n' different things, taken 'r' at a time, when 'p' particular things are always to be excluded = ${}^{n-p}C_{r}$

Example: In how many ways can a cricket-eleven be chosen out of 15 players? if

(i) A particular player is always chosen,

(ii) A particular is never chosen.

Ans:

(i) A particular player is always chosen, it means that 10 players are selected out of the remaining 14 players.

=. Required number of ways = ${}^{14}C_{10} = {}^{14}C_{4}$

= 14!/4!x19! = 1365

(ii) A particular players is never chosen, it means that 11 players are selected out of 14 players.

=> Required number of ways = ${}^{14}C_{11}$

= $14!/11! \times 3!$ = 364

(iii) Number of ways of selecting zero or more things from ‘n’ different things is given by:- $2^n - 1$

Example: John has 8 friends. In how many ways can he invite one or more of them to dinner?

Ans. John can select one or more than one of his 8 friends.

=> Required number of ways = $2^8 - 1 = 255$.

NOTE: Number of ways of selecting zero or more things from ‘n’ identical things is given by :- n+1

Example: In how many ways, can zero or more letters be selected form the letters AAAAA?

Ans. Number of ways of :

Selecting zero 'A's = 1

Selecting one 'A's = 1

Selecting two 'A's = 1

Selecting three 'A's = 1

Selecting four 'A's = 1

Selecting five 'A's = 1

=> Required number of ways = 6 [5+1]

NOTE: Number of ways of selecting one or more things from ‘p’ identical things of one type ‘q’ identical things of another type, ‘r’ identical things of the third type and ‘n’ different things is given by :-

$(p+1)(q+1)(r+1)2^n - 1$

Example: Find the number of different choices that can be made from 3 apples, 4 bananas and 5 mangoes, if at least one fruit is to be chosen.

Ans:

Number of ways of selecting apples = (3+1) = 4 ways.

Number of ways of selecting bananas = (4+1) = 5 ways.

Number of ways of selecting mangoes = (5+1) = 6 ways.

Total number of ways of selecting fruits = 4 x 5 x 6

But this includes, when no fruits i.e. zero fruits is selected

=> Number of ways of selecting at least one fruit = (4x5x6) -1 = 119

Note :- There was no fruit of a different type, hence here n=o

=> $2^n = 2^0 = 1$

NOTE: Number of ways of selecting ‘r’ things from ‘n’ identical things is ‘1’.

Example: In how many ways 5 balls can be selected from ‘12’ identical red balls?
Ans. The balls are identical, total number of ways of selecting 5 balls = 1.
Example: How many numbers of four digits can be formed with digits 1, 2, 3, 4 and 5?
Ans. Here n = 5 [Number of digits]

And r = 4 [Number of places to be filled-up]

Required number is ${}^5P_4 = 5!/1! = 5 \times 4 \times 3 \times 2 \times 1$

Difference Between Permutation And Combination

We have provided the permutation and combination differences in the table below:

Permutation	Combination
A selection of r objects from a set of n objects in which the order of the selection matters.	The number of possible combination of r objects from a set on n objects where the order of selection doesn’t matter.
Permutation is used for lists (order matters).	Combination is used for groups (order doesn’t matter).
It denotes the arrangement of objects.	It does not denote the arrangement of objects.
We can derive multiple permutations from a single combination.	Only a single combination can be derived from a single permutation.
They are defined as ordered elements.	They are defined as unordered sets.

Binomial Coefficients:

Mathematical Functions:

2.1 Factorial Function: The product of the first n natural number is called factorial n. It is denoted by n!, read "n Factorial."

The Factorial n can also be written as

1. $n! = n\,(n-1)\,(n-2)\,(n-3)......1.$
2. $= 1$ and $0! = 1.$

Example1: Find the value of 5!

Solution:

```
5! = 5 x (5-1) (5-2) (5-3) (5-4)

 = 5 x 4 x 3 x 2 x 1 = 120
```

Example2: Find the value of $\frac{10!}{8!.}$

Solution: $\frac{10!}{8!.} = \frac{10 \times 9 \times 8!}{8!} = 10 \times 9 = 90$

2.2 Binomial Coefficients: Binomial Coefficient is represented by n_{Cr} where r and n are positive integer with $r \leq n$ is defined as follows:

$$\left(\begin{matrix} n \\ r \end{matrix} = \frac{n\,(n-1)(n-2)\dots\dots\dots\dots(n-r+1)}{1 * 2 * 3 \dots\dots\dots\dots\dots(r-1)\,r}\right)$$

Example: $8_{C2} = \frac{8!}{2!\,6!} = \frac{8 \times 7 \times 6}{2 \times 6} = 28.$

The Pigeonhole Principle

If n pigeonholes are occupied by n+1 or more pigeons, then at least one pigeonhole is occupied by greater than one pigeon. Generalized pigeonhole principle is: - If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

Example1: Find the minimum number of students in a class to be sure that three of them are born in the same month.

Solution: Here n = 12 months are the Pigeonholes
And k + 1 = 3
K = 2

Example2: Show that at least two people must have their birthday in the same month if 13 people are assembled in a room.

Solution: We assigned each person the month of the year on which he was born. Since there are 12 months in a year.

So, according to the pigeonhole principle, there must be at least two people assigned to the same month.

Inclusion-Exclusion Principle:

Let $A_1, A_2 \ldots\ldots A_r$ be the subset of Universal set U. Then the number m of the element which do not appear in any subset $A_1, A_2 \ldots\ldots A_r$ of U.

$$m = n(A_1^c \cap A_2^c \cap \ldots \ldots \cap A_r^c) = |U| - S_1 + S_2 - S_3 + \ldots\ldots\ldots (-1)^r S_r.$$

Example: Let U be the set of positive integer not exceeding 1000. Then |U|= 1000 Find |S| where S is the set of such integer which is not divisible by 3, 5 or 7?

Solution: Let A be the subset of integer which is divisible by 3
Let B be the subset of integer which is divisible by 5
Let C be the subset of integer which is divisible by 7

Then $S = A^c \cap B^c \cap C^c$ since each element of S is not divisible by 3, 5, or 7.

By Integer division,

$|A|= 1000/3 = 333$
$|B|= 1000/5 = 200$
$|C| = 1000/7 = 142$
$|A \cap B|=1000/15=66$
$|B \cap C|=1000/21=47$

$|C \cap A| = 1000/35 = 28$
$|A \cap B \cap C| = 1000/105 = 9$

Thus by Inclusion-Exclusion Principle

$|S| = 1000 - (333+200+142) + (66+47+28) - 9$
$|S| = 1000 - 675 + 141 - 9 = 457$

Important Problems(previous question papers)

1. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

Answer :
Number of ways of selecting 3 consonants out of 7 = $^7C_3$
Number of ways of selecting 2 vowels out of 4 = $^4C_2$
Number of ways of selecting 3 consonants out of 7 and 2 vowels out of 4 = $^7C_3 \times {}^4C_2$
$= (7 \times 6 \times 5)/(3 \times 2 \times 1) \times (4 \times 3)/(2 \times 1) = 210$
It means that we can have 210 groups where each group contains total 5 letters(3 consonants and 2 vowels).
Number of ways of arranging 5 letters among themselves = 5!
= 5 x 4 x 3 x 2 x 1 = 120
Hence, required number of ways = 210 x 120 = 25200

2. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Answer :
In a group of 6 boys and 4 girls, four children are to be selected such that
at least one boy should be there.
Hence we have 4 choices as given below
We can select 4 boys ------(Option 1).
Number of ways to this = $^6C_4$
We can select 3 boys and 1 girl ------(Option 2)
Number of ways to this = $^6C_3 \times {}^4C_1$
We can select 2 boys and 2 girls ------(Option 3)
Number of ways to this = $^6C_2 \times {}^4C_2$
We can select 1 boy and 3 girls ------(Option 4)
Number of ways to this = $^6C_1 \times {}^4C_3$
Total number of ways
$= ({}^6C_4) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_1 \times {}^4C_3)$
$= ({}^6C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_1 \times {}^4C_1)$ [*Applied the formula* $^nC_r = {}^nC_{(n-r)}$]
$= [6 \times 5)/(2 \times 1] + [(6 \times 5 \times 4)/(3 \times 2 \times 1) \times 4] + [(6 \times 5)/(2 \times 1)(4 \times 3)/(2 \times 1)] + [6 \times 4]$
= 15 + 80 + 90 + 24
= 209

3. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

Answer :
From a group of 7 men and 6 women, five persons are to be selected with at least 3 men.
Hence we have the following 3 choices
We can select 5 men ------(Option 1)
Number of ways to do this = ${}^7C_5$
We can select 4 men and 1 woman ------(Option 2)
Number of ways to do this = ${}^7C_4 \times {}^6C_1$
We can select 3 men and 2 women ------(Option 3)
Number of ways to do this = ${}^7C_3 \times {}^6C_2$
Total number of ways
= ${}^7C_5 + [{}^7C_4 \times {}^6C_1] + [{}^7C_3 \times {}^6C_2]$
= ${}^7C_2 + [{}^7C_3 \times {}^6C_1] + [{}^7C_3 \times {}^6C_2]$ [*Applied the formula* ${}^nC_r = {}^nC_{(n-r)}$]
$=[7\times6)/(2\times1]+[(7\times6\times5)/(3\times2\times1)\times6]+[(7\times6\times5)/(3\times2\times1)\times(6\times5)/(2\times1)]$
= 21 + 210 + 525 = 756

4. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

Answer :
The word 'OPTICAL' has 7 letters. It has the vowels 'O','I','A' in it and these 3 vowels should always come together. Hence these three vowels can be grouped and considered as a single letter. That is, PTCL(OIA).
Hence we can assume total letters as 5. and all these letters are different.
Number of ways to arrange these letters = 5! = [5 x 4 x 3 x 2 x 1] = 120
All The 3 vowels (OIA) are different
Number of ways to arrange these vowels among themselves = 3! = [3 x 2 x 1] = 6
Hence, required number of ways = 120 x 6 = 720

5. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

Answer :
The word 'CORPORATION' has 11 letters. It has the vowels 'O','O','A','I','O' in it and these 5 vowels should always come together. Hence these 5 vowels can be grouped and considered as a single letter. that is, CRPRTN(OOAIO).
Hence we can assume total letters as 7. But in these 7 letters, 'R' occurs 2 times and rest of the letters are different.
Number of ways to arrange these letters = $7!/2!=(7\times6\times5\times4\times3\times2\times1)/(2\times1)=2520$
In the 5 vowels (OOAIO), 'O' occurs 3 and rest of the vowels are different.
Number of ways to arrange these vowels among themselves
$= 5!/3!=(5\times4\times3\times2\times1)/(3\times2\times1)=20$

Hence, required number of ways = 2520 x 20 = 50400

6. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

Answer :
We need to select 5 men from 7 men and 2 women from 3 women
Number of ways to do this
= ${}^{7}C_{5}$ x ${}^{3}C_{2}$
= ${}^{7}C_{2}$ x ${}^{3}C_{1}$ [*Applied the formula* ${}^{n}C_{r} = {}^{n}C_{(n-r)}$]
$=(7\times6)/(2\times1)\times3$
= 21 x 3 = 63

7. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?

Answer :
The word 'MATHEMATICS' has 11 letters. It has the vowels 'A','E','A','I' in it and these 4 vowels must always come together. Hence these 4 vowels can be grouped and considered as a single letter. That is, MTHMTCS(AEAI).

Hence we can assume total letters as 8. But in these 8 letters, 'M' occurs 2 times, 'T' occurs 2 times but rest of the letters are different.
Hence,number of ways to arrange these letters =
$8!/(2!)(2!)=(8\times7\times6\times5\times4\times3\times2\times1)/(2\times1)(2\times1)=10080$
In the 4 vowels (AEAI), 'A' occurs 2 times and rest of the vowels are different.
Number of ways to arrange these vowels among themselves
$= 4!/2!=(4\times3\times2\times1)/(2\times1)=12$
Hence, required number of ways = 10080 x 12 = 120960

8. There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?

Answer :
We need to select 5 men from 8 men and 6 women from 10 women
Number of ways to do this
= ${}^{8}C_{5}$ x ${}^{10}C_{6}$
= ${}^{8}C_{3}$ x ${}^{10}C_{4}$ [*Applied the formula* ${}^{n}C_{r} = {}^{n}C_{(n-r)}$]
$=(8\times7\times6)/(3\times2\times1)(10\times9\times8\times7)/(4\times3\times2\times1)$
= 56 x 210
= 11760

9**. How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?**

Answer : Option A
The word 'LOGARITHMS' has 10 different letters.
Hence, the number of 3-letter words(with or without meaning) formed by using these letters
= $^{10}P_3$
= 10 x 9 x 8
= 720

10. In how many different ways can the letters of the word 'LEADING' be arranged such that the vowels should always come together?

Answer :
The word 'LEADING' has 7 letters. It has the vowels 'E','A','I' in it and
these 3 vowels should always come together. Hence these 3 vowels can be grouped
and considered as a single letter. that is, LDNG(EAI).
Hence we can assume total letters as 5 and all these letters are different.
Number of ways to arrange these letters = 5! = 5 x 4 x 3 x 2 x 1 = 120
In the 3 vowels (EAI), all the vowels are different.
Number of ways to arrange these vowels among themselves = 3! = 3 x 2 x 1= 6
Hence, required number of ways = 120 x 6= 720

11. A coin is tossed 3 times. Find out the number of possible outcomes.

Answer :
When a coin is tossed once, there are two possible outcomes - Head(H) and Tale(T)
Hence, when a coin is tossed 3 times, the number of possible outcomes
= 2 x 2 x 2 = 8
(The possible outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

12. In how many different ways can the letters of the word 'DETAIL' be arranged such that the vowels must occupy only the odd positions?

Answer :
The word 'DETAIL' has 6 letters which has 3 vowels (EAI) and 3 consonants(DTL)
The 3 vowels(EAI) must occupy only the odd positions.
Let's mark the positions as (1) (2) (3) (4) (5) (6).
Now, the 3 vowels should only occupy the 3 positions marked as (1),(3) and (5) in any order.
Hence, number of ways to arrange these vowels = $^{3}P_3$
= 3! = 3 x 2 x 1 = 6
Now we have 3 consonants(DTL) which can be arranged in the remaining 3 positions in any order
Hence, number of ways to arrange these consonants = $^{3}P_3$
= 3! = 3 x 2 x 1 = 6
Total number of ways
= number of ways to arrange the vowels x number of ways to arrange the consonants
= 6 x 6 = 36

13. A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag, if at least one black ball is to be included in the draw?

Answer :
From 2 white balls, 3 black balls and 4 red balls, 3 balls are to be selected such that at least one black ball should be there.
Hence we have 3 choices as given below
We can select 3 black balls --------------------------(Option 1)
We can select 2 black balls and 1 non-black ball------(Option 2)
We can select 1 black ball and 2 non-black balls------(Option 3)
Number of ways to select 3 black balls = 3C3
Number of ways to select 2 black balls and 1 non-black ball = 3C2 x 6C1
Number of ways to select 1 black ball and 2 non-black balls = 3C1 x 6C2
Total number of ways
= 3C3 + (3C2 x 6C1) + (3C1 x 6C2)
= 1 + (3C1 x 6C1) + (3C1 x 6C2) [*Applied the formula* $^nC_r = {^nC_{(n-r)}}$]
$=1+[3\times6]+[3\times(6\times5)/(2\times1)]$
= 1 + 18 + 45
= 64

14. In how many different ways can the letters of the word 'JUDGE' be arranged such that the vowels always come together?

Answer :
The word 'JUDGE' has 5 letters. It has 2 vowels (UE) in it and these 2 vowels should always come together. Hence these 2 vowels can be grouped and considered as a single letter. That is, JDG(UE).
Hence we can assume total letters as 4 and all these letters are different.
Number of ways to arrange these letters = 4!= 4 x 3 x 2 x 1 = 24
In the 2 vowels (UE), all the vowels are different.
Number of ways to arrange these vowels among themselves = 2! = 2 x 1 = 2
Total number of ways = 24 x 2 = 48

15. In how many ways can the letters of the word 'LEADER' be arranged?

Answer :
The word 'LEADER' has 6 letters.
But in these 6 letters, 'E' occurs 2 times and rest of the letters are different.
Hence,number of ways to arrange these letters
$= 6!/2!=(6\times5\times4\times3\times2\times1)/2\times1=360$

16. How many words can be formed by using all letters of the word 'BIHAR'?

Answer :
The word 'BIHAR' has 5 letters and all these 5 letters are different.
Total words formed by using all these 5 letters = $^5P_5$ = 5!
= 5 x 4 x 3 x 2 x 1 = 120

17. How many arrangements can be made out of the letters of the word 'ENGINEERING' ?

Answer :
The word 'ENGINEERING' has 11 letters.
But in these 11 letters, 'E' occurs 3 times,'N' occurs 3 times, 'G' occurs 2 times,
'I' occurs 2 times and rest of the letters are different.
Hence,number of ways to arrange these letters
= 11!/[(3!)(3!)(2!)(2!)]=277200

18. How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?

Answer :
A number is divisible by 5 if the its last digit is a 0 or 5
We need to find out how many 3 digit numbers can be formed from the 6 digits (2,3,5,6,7,9) which are divisible by 5.
Since the 3 digit number should be divisible by 5, we should take the digit 5 from the 6 digits(2,3,5,6,7,9) and fix it at the unit place.
There is only 1 way of doing this

		1

Since the number 5 is placed at unit place, we have now five digits(2,3,6,7,9) remaining.
Any of these 5 digits can be placed at tens place

	5	1

Since the digits 5 is placed at unit place and another one digits is placed at tens place,
we have now four digits remaining. Any of these 4 digits can be placed at hundreds place.

4	5	1

Required Number of three digit numbers = 4 x 5 x 1 = 20

19. How many words with or without meaning, can be formed by using all the letters of the word, 'DELHI' using each letter exactly once?

Answer :
The word 'DELHI' has 5 letters and all these letters are different.
Total words (with or without meaning) formed by using all these 5 letters using each letter exactly once
= Number of arrangements of 5 letters taken all at a time
= ${}^{5}P_{5}$ = 5! = 5 x 4 x 3 x 2 x 1 = 120

20. What is the value of ${}^{100}P_{2}$?

Answer : ${}^{100}P_{2}$ = 100 x 99 = 9900

21. In how many different ways can the letters of the word 'RUMOUR' be arranged?

Answer :

The word 'RUMOUR' has 6 letters.
But in these 6 letters, 'R' occurs 2 times, 'U' occurs 2 times and rest of the letters are different.
Hence, number of ways to arrange these letters
$= 6!/(2!)(2!)=(6×5×4×3×2×1)/(2×1)(2×1)=180$

22. There are 6 periods in each working day of a school. In how many ways can one organize 5 subjects such that each subject is allowed at least one period?

Answer :
Explanation :
5 subjects can be arranged in 6 periods in $^6P_5$ ways.
Remaining 1 period can be arranged in $^5P_1$ ways.
Two subjects are alike in each of the arrangement. So we need to divide by 2! to avoid overcounting.
Total number of arrangements = $(^6P_5 \text{ x } ^5P_1)/2! = 1800$
Alternatively this can be derived using the following approach.
5 subjects can be selected in $^5C_5$ ways.
Remaining 1 subject can be selected in $^5C_1$ ways.
These 6 subjects can be arranged themselves in 6! ways.
Since two subjects are same, we need to divide by 2!
Total number of arrangements = $(^5C_5 \times {}^5C_1 \times 6!)/2! = 1800$

23. How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?

Answer : :
The first two places can only be filled by 3 and 5 respectively and there is only 1 way of doing this
Given that no digit appears more than once. Hence we have 8 digits remaining(0,1,2,4,6,7,8,9)
So, the next 4 places can be filled with the remaining 8 digits in $^8P_4$ ways
Total number of ways = $^8P_4$ = 8 x 7 x 6 x 5 = 1680

24. An event manager has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?

Answer :
He has has 10 patterns of chairs and 8 patterns of tables
Hence, A chair can be arranged in 10 ways and
A table can be arranged in 8 ways
Hence one chair and one table can be arranged in 10 x 8 ways = 80 ways

25. 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?

Answer :
He can go in any bus out of the 25 buses.
Hence He can go in 25 ways.
Since he can not come back in the same bus that he used for travelling,

He can return in 24 ways.
Total number of ways = 25 x 24 = 600

26. A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out the number of ways of selecting the balls of different colours?

Answer :
1 red ball can be selected in $^4C_1$ ways
1 white ball can be selected in $^3C_1$ ways
1 blue ball can be selected in $^2C_1$ ways
Total number of ways
= $^4C_1$ x $^3C_1$ x $^2C_1$
=4 x 3 x 2
= 24

27. A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways can he do that?

Answer :
Number of ways to choose 8 questions from part P = $^{10}C_8$
Number of ways to choose 4 questions from part Q = $^{10}C_4$
Total number of ways
= $^{10}C_8$ x $^{10}C_4$
= $^{10}C_2$ x $^{10}C_4$ [*Applied the formula* $^nC_r = {}^nC_{(n-r)}$]
=(10×9)/(2×1)(10×9×8×7)/(4×3×2×1)
=45 x 210
= 9450

28. In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?

Answer :
In a circle, 5 boys can be arranged in 4! ways
Given that the boys and the girls alternate.
Hence there are 5 places for girls which can be arranged in 5! ways
Total number of ways = 4! x 5! = 24 x 120 = 2880

29. Find out the number of ways in which 6 rings of different types can be worn in 3 fingers?

Answer :
The first ring can be worn in any of the 3 fingers
=> There are 3 ways of wearing the first ring
Similarly each of the remaining 5 rings also can be worn in 3 ways
Hence total number of ways
=3×3×3×3×3×6=3$_6$=729

30. In how many ways can 5 man draw water from 5 taps if no tap can be used more than once?

Answer :
1st man can draw water from any of the 5 taps
2nd man can draw water from any of the remaining 4 taps
3rd man can draw water from any of the remaining 3 taps
4th man can draw water from any of the remaining 2 taps
5th man can draw water from remaining 1 tap

5	4	3	2	1

Hence total number of ways = 5 x 4 x 3 x 2 x 1 = 120

31. In how many ways can 11 persons be arranged in a row such that 3 particular persons should always be together?

Answer :
Given that three particular persons should always be together. Hence, just group these three persons together and consider as a single person.
Hence we can take total number of persons as 9. These 9 persons can be arranged in
$^{9}P_{9} = 9!$ Ways
We had grouped three persons together. These three persons can be arranged among themselves in $^{3}P_{3} = 3!$ Ways
Hence, the required number of ways = $9! \times 3!$

32. In how many ways can 9 different colour balls be arranged in a row so that black, white, red and green balls are never together?

Answer :
Total number of ways in which 9 different colour balls can be arranged in a row
= $^{9}P_{9} = 9!$ **------- (A)**
Now we will find out the total number of ways in which 9 different colour balls can be arranged in a row so that black, white, red and green balls are always together.
We have total 9 balls. Since black, white, red and green balls are always together, group these 4 balls together and consider as a single ball.
Hence we can take total number of balls as 6. These 6 balls can be arranged in $^{6}P_{6} = 6!$ Ways
We had grouped 4 balls together. These 4 balls can be arranged among themselves in $^{4}P_{4} = 4!$ ways
Hence, total number of ways in which 9 different colour balls be arranged in a row so that black, white, red and green balls are always together = $6! \times 4!$ **------(B)**
From (A) and (B),
Total number of ways in which 9 different colour balls can be arranged in a row so that black, white, red and green balls are never together
$= 9! - (6! \times 4!) = (6! \times 7 \times 8 \times 9) - (6! \times 4!) = 6! (7 \times 8 \times 9 - 4!)$
$= 6! (504 - 24) = 6! \times 480 = 720 \times 480 = 345600$

33. A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that no two of the civil engineers will sit together?

Answer

The 11 software engineers can be arranged in

${}^{11}P_{11}$ = 11! Ways **---(A)**

Now we need to arrange civil engineers such that no two civil engineers can be seated together. i.e., we can arrange 7 civil engineers in any of the 12 (=11+1) positions marked as * below

* 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 *10 * 11 *

(Where 1, 2… 11 represents software engineers)

This can be done in ${}^{12}P_7$ ways **---(B)**

From (A) and (B),

the required number of ways = 11! × ${}^{12}P_7$

=11!×12!/5!

34. A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that all the civil engineers do not sit together?

Answer :

Total number of engineers = 11 + 7 = 18

Total number of ways in which the 18 engineers can be arranged in a row

= ${}^{18}P_{18}$ = 18! **---(A)**

Now we will find out the total number of ways in which the 18 engineers can be arranged so that all the 7 civil engineers will always sit together.

For this, group all the 7 civil engineers and consider as a single civil engineer.

Hence, we can take total number of engineers as 12. (∵ 11 + 1)

These 12 engineers can be arranged in ${}^{12}P_{12}$ = 12! ways

We had grouped 7 civil engineers. But these 7 civil engineers can be arranged among themselves in ${}^{7}P_{7}$ = 7! ways

Hence, total number of ways in which the 18 engineers can be arranged so that the 7 civil engineers will always sit together = 12! × 7! **---(B)**

From (A) and (B),

Total number of ways in which 11 software engineers and 7 civil engineers can be seated in a row so that all the civil engineers will not sit together

= 18! – (12! × 7!)

35. In how many ways can 11 software engineers and 10 civil engineers be seated in a row so that they are positioned alternatively?

Answer :

The 10 civil engineers can be arranged in a row in

${}^{10}P_{10}$ = 10! Ways **---(A)**

Now we need to arrange software engineers such that software engineers and civil engineers are seated alternatively. i.e., we can arrange 11 software engineers in the 11 positions marked as * below

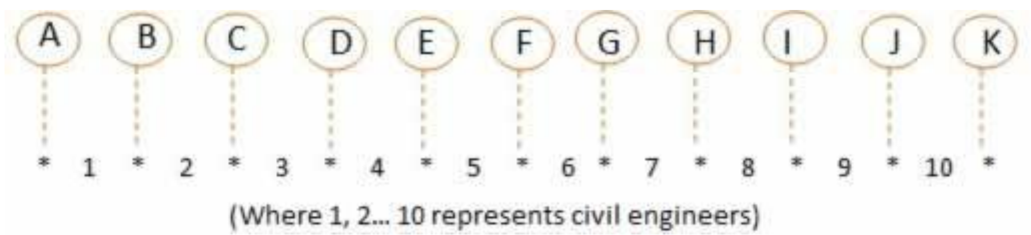
* 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 *10 *

(Where 1, 2… 10 represents civil engineers)

This can be done in $^{11}P_{11} = 11!$ ways **---(B)**
From (A) and (B),
The required number of ways = $10! \times 11!$

36. In how many ways can 10 software engineers and 10 civil engineers be seated in a row so that they are positioned alternatively?

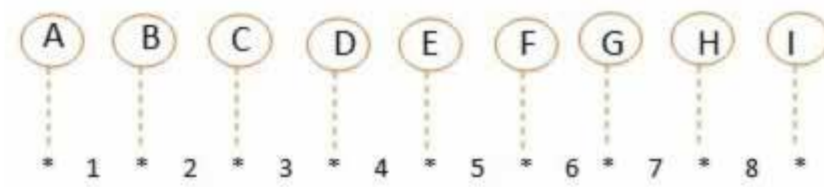
Answer :
The 10 civil engineers can be arranged in a row in
$^{10}P_{10} = 10!$ Ways **---(A)**
Now we need to arrange software engineers such that software engineers and civil engineers are seated alternatively. i.e., we can arrange 10 software engineers either in the 10 positions marked as A,B,C,D,E,F,G,H,I,J or in the 10 positions marked as B,C,D,E,F,G,H,I,J,K, as shown below



10 software engineers can be arranged in the 10 positions marked as A,B,C,D,E,F,G,H,I,J in $^{10}P_{10} = 10!$ Ways
10 software engineers can be arranged in the 10 positions marked as B,C,D,E,F,G,H,I,J,K in $^{10}P_{10} = 10!$ Ways
10 software engineers can be arranged in the 10 positions marked as A,B,C,D,E,F,G,H,I,J or in the 10 positions marked as B,C,D,E,F,G,H,I,J,K in $10! + 10! = 2 \times 10!$ Ways **---(B)**
From (A) and (B),
The required number of ways = $10! \times (2 \times 10!) = 2 \times (10!)^2$

37. Kiran has 8 black balls and 8 white balls. In how many ways can he arrange these balls in a row so that balls of different colours are alternate?

Answer
The 8 black balls can be arranged in $^{8}P_{8} = 8!$ Ways **---(A)**
Now we need to arrange white balls such that white balls and black balls are positioned alternatively. i.e., we can arrange 8 white balls either in the 8 positions marked as A,B,C,D,E,F,G,H or in the 8 positions marked as B,C,D,E,F,G,H,I as shown below



(where 1, 2... 8 represent black balls)

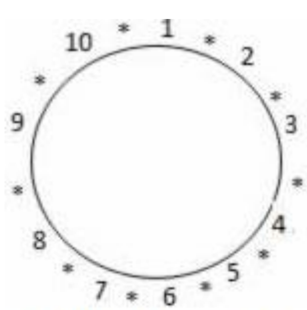
8 white balls can be arranged in the 8 positions
marked as A,B,C,D,E,F,G,H in ${}^8P_8 = 8!$ Ways
8 white balls can be arranged in the 8 positions
marked as B,C,D,E,F,G,H,I in ${}^8P_8 = 8!$ Ways
8 white balls can be arranged in the 8 positions marked as A,B,C,D,E,F,G,H
or in the 8 positions marked as B,C,D,E,F,G,H,I
in $8! + 8! = 2 \times 8!$ Ways **---(B)**
From (A) and (B), the required number of ways = $8! \times 2 \times 8! = 2 \times (8!)^2$

38. A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that all the civil engineers are always together?

Answer :
All the 7 civil engineers are always together. Hence, group all the 7 civil engineers
and consider as a single civil engineer.
Hence, we can take total number of engineers as 12. ($\because$ 11 + 1)
These 12 engineers can be arranged in ${}^{12}P_{12} = 12!$ Ways **---(B)**
We had grouped 7 civil engineers. These 7 civil engineers can be arranged among
themselves in ${}^7P_7 = 7!$ Ways **---(B)**
From (A) and (B),
The required number of ways = $12! \times 7!$

39. A company has 10 software engineers and 6 civil engineers. In how many ways can they be seated in a round table so that no two of the civil engineers will sit together?

Answer :
10 software engineers can be arranged in a round table in $(10-1)! = 9!$ ways **---(A)**
[Reference : Circular Permutations: Case 1]
Now we need to arrange civil engineers such that no two civil engineers can be seated together.
i.e., we can arrange 6 civil engineers in any of the 10 positions marked as * below



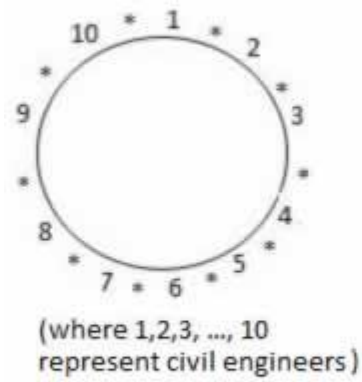
(where 1,2,3, ..., 10
represent software engineers)

This can be done in ${}^{10}P_6$ ways **---(B)**
From (A) and (B),
The required number of ways = $9! \times {}^{10}P_6$
$=9!\times10!/4!$

40. A company has 10 software engineers and 6 civil engineers. In how many ways can they be seated in a round table so that all the civil engineers do not sit together?

Answer :
Total number of engineers = 10 + 6 = 16
Total number of ways in which the 16 engineers can be arranged in a round table
= (16 – 1)! = 15!**---(A)**
[Reference : Circular Permutations: Case 1]
Now we will find out the total number of ways in which the 16 engineers can be arranged in a round table so that all the 6 civil engineers will always sit together.
For this, group all the 6 civil engineers and consider as a single civil engineer.
Hence, we can take total number of engineers as 11. (∵ 10 + 1)
These 11 engineers can be arranged in a round table in (11-1)! = 10! ways
We had grouped 6 civil engineers. These 6 civil engineers can be arranged among themselves in ${}^{6}P_6$ = 6! ways
Hence, total number of ways in which the 16 engineers can be arranged in a round table so that the 6 civil engineers will always sit together = 10! × 6! **---(B)**
From (A) and (B),
Total number of ways in which 10 software engineers and 6 civil engineers can be seated in a round table so that all the civil engineers do not sit together
= 15! – (10! × 6!)

41. In how many ways can 10 software engineers and 10 civil engineers be seated in a round table so that they are positioned alternatively?
Answer :
The 10 civil engineers can be arranged in a round table in
(10-1)! = 9! Ways **---(A)**
[Reference : Circular Permutations: Case 1]
Now we need to arrange software engineers the round table such that software engineers and civil engineers are seated alternatively. i.e., we can arrange 10 software engineers in the 10 positions marked as * as shown below



This can be done in $^{10}P_{10}$ = 10! Ways **---(B)**
From (A) and (B), The required number of ways = 9! × 10!

42. A company has 10 software engineers and 6 civil engineers. In how many ways can they be seated in a round table so that all the civil engineers are together?

Answer :
We need to find out the total number of ways in which 10 software engineers and 6 civil engineers can be arranged in a round table so that all the 6 civil engineers will always sit together.
For this, group all the 6 civil engineers and consider as a single civil engineer.
Hence, we can take total number of engineers as 11. (∵ 10 + 1)
These 11 engineers can be arranged in a round table in (11-1)! = 10! ways **---(A)**
[Reference : Circular Permutations: Case 1]
We had grouped 6 civil engineers. These 6 civil engineers can be arranged among themselves in $^{6}P_{6}$ = 6! ways **---(B)**
From (A) and (B), The required number of ways = 10! × 6!

43. How many 8 digit mobile numbers can be formed if any digit can be repeated and 0 can also start the mobile number?

Answer :
Here the digits(0,1,2,3,4,5,6,7,8,9) can be repeated and 0 can also be used to start the mobile number.
Hence, any of the 10 digits can be placed at any place of the 8 digit number

10	10	10	10	10	10	10	10

Hence, the total number of 8 digit mobile numbers that can be formed using all the digits (0,1,2,3,4,5,6,7,8,9) (with repetition of the digits and 0 can also be used to start the number) = 10^8

44. How many 8 digits mobile numbers can be formed if at least one of their digits is repeated and 0 can also start the mobile number?

Answer :
Initially we will find out the number of 8 digits mobile numbers that can be formed if any digit can be repeated (with 0 can also start the mobile number)

The digits can be repeated and 0 can also be used to start the mobile number.
Hence, any of the 10 digits(0,1,2,3,4,5,6,7,8,9) can be placed at any place of the 8 digit number

10	10	10	10	10	10	10	10

Hence, the total number of 8 digit mobile numbers that can be formed using all the digits (0,1,2,3,4,5,6,7,8,9) if any digit can be repeated (with 0 can also start the mobile number) = 10^8 **---(A)**
Now we will find out the number of 8 digits mobile numbers that can be formed if no digit can be repeated (with 0 can also start the mobile number)
In this case, any of the 10 digits can be placed at the 1st position.
Since one digit is placed at the 1st position, any of the remaining 9 digits can be placed at 2nd position.
Since 1 digit is placed at the 1st position and another digit is placed at the 2nd position, any of the remaining 8 digits can be placed at the 3rd position. So on …

10	9	8	7	6	5	4	3

i.e., the number of 8 digits mobile numbers that can be formed if no digit can be repeated (with 0 can also start the mobile number)
= ${}^{10}P_8$ **---(B)**
(In fact you should directly get (A) and (B) without any calculations from the definition of permutations itself)
From(A) and (B),
the number of 8 digits mobile numbers that can be formed if at least one of their digits is repeated and 0 can also start the mobile number
= $10^8 - {}^{10}P_8$

45. How many 8 digits mobile numbers can be formed if at least one of their digits is repeated and 0 cannot be used to start the mobile number?

Answer :
Initially we will find out the number of 8 digits mobile numbers that can be formed if any digit can be repeated and 0 cannot be used to start the mobile number
The digits can be repeated. 0 cannot be used to start the mobile number.
Hence, any of the 9 digits (∵ any digit except 0) can be placed at the 1st position.
Then, any of the 10 digits can be placed at any of the the remaining 7 positions of the 8 digit number

9	10	10	10	10	10	10	10

Hence, the total number of 8 digit mobile numbers that can be formed using all the digits (0,1,2,3,4,5,6,7,8,9) if any digit can be repeated and 0 cannot be used to start the mobile number = 9×10^7**---(A)**
Now we will find out the number of 8 digits mobile numbers that can be formed if no digit can be repeated and 0 cannot be used to start the mobile number
Here, any of the 9 digits (∵ any digit except 0) can be placed at the

1^{st} position.
Since one digit is placed at the 1^{st} position, any of the remaining
9 digits can be placed at 2^{nd} position.
Since 1 digit is placed at the 1^{st} position and another digit is placed
at the 2^{nd} position, any of the remaining 8 digits can be placed at the
3rd position.
So on …

9	9	8	7	6	5	4	3

i.e., the number of 8 digits mobile numbers that can be formed if no digit can be
repeated and 0 cannot be used to start the mobile number
$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3$
$= 9 \times {}^{9}P_{7}$ **---(B)**
From(A) and (B),
the number of 8 digits mobile numbers that can be formed if at least one of their
digits is repeated and 0 cannot be used to start the mobile number
$= 9 \times 10^7 - 9 \times {}^{9}P_{7}$

46. How many possible outcomes are there when five dice are rolled in which at least one dice shows 6?

Answer :
Initially we will find out the total number of possible outcomes when 5 dice are rolled.
Outcome of first die can be any number from (1,2,3,4,5,6).
i.e, outcome of first die can happen in 6 ways
Similarly outcome of each of the other 4 dice can also happen in 6 ways

6	6	6	6	6

Hence, total number of possible outcomes when 5 dice are rolled = 6^5 **---(A)**
Now we will find out the total number of possible outcomes when 5 dice are rolled in which 6 does not appear in any dice.
In this case, outcome of first die can be any number from (1,2,3,4,5).
i.e, outcome of first die can happen in 5 ways.
Similarly outcome of each of the other 4 dice can also happen in 5 ways

5	5	5	5	5

Hence, total number of possible outcomes when 5 dice are rolled
in which 6 does not appear in any dice = 5^5 **---(B)**
From (A) and (B), the total number of possible outcomes when five dice are rolled
in which at least one dice shows 6 = $6^5 - 5^5$

47. A board meeting of a company is organized in a room for 24 persons along the two sides of a table with 12 chairs in each side. 6 persons wants to sit on a particular side and 3 persons wants to sit on the other side. In how many ways can they be seated?

Answer : Option C
Explanation :

First, arrange the 6 persons in the 12 chairs on the particular side.
The 6 persons can sit in the 12 chairs on the particular side
in ${}^{12}P_6$ ways. **---(A)**
Now, arrange the 3 persons in the 12 chairs on the other side.
The 3 persons can sit in the 12 chairs on the other side
in ${}^{12}P_3$ ways. **-----(B)**
Remaining persons = 24 – 6 – 3 = 15
Remaining chairs = 24 – 6 – 3 = 15
i.e., now we need to arrange the remaining 15 persons in the remaining 15 chairs.
This can be done in ${}^{15}P_{15} = 15!$ ways. **-----(C)**
From (A), (B) and (C),
Required number of ways = ${}^{12}P_6 \times {}^{12}P_3 \times 15!$

48. How many numbers not exceeding 10000 can be made using the digits 2,4,5,6,8 if repetition of digits is allowed?

Answer :
Given that the numbers should not exceed 10000
Hence numbers can be 1 digit numbers or 2 digit numbers or 3 digit numbers
or 4 digit numbers
Given that repetition of the digits is allowed.
A. Count of 1 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed)
The unit digit can be filled by any of the 5 digits (2,4,5,6,8)

$\boxed{5}$

Hence the total count of 1 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed) = 5 **---(A)**
B. Count of 2 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed)
Since repetition is allowed, any of the 5 digits(2,4,5,6,8) can be placed
in unit place and tens place.

$\boxed{5}\boxed{5}$

Hence the total count of 2 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed) = 5^2 **---(B)**
C. Count of 3 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed)
Since repetition is allowed, any of the 5 digits (2,4,5,6,8) can be placed
in unit place , tens place and hundreds place.

$\boxed{5}\boxed{5}\boxed{5}$

Hence the total count of 3 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed) = 5^3 **---(C)**
D. Count of 4 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed)
Since repetition is allowed, any of the 5 digits (2,4,5,6,8) can be placed
in unit place, tens place, hundreds place and thousands place

$\boxed{5}\boxed{5}\boxed{5}\boxed{5}$

Hence the total count of 4 digit numbers that can be formed
using the 5 digits (2,4,5,6,8) (repetition allowed) = 5^4 **---(D)**
From (A), (B), (C), and (D),
total count of numbers not exceeding 10000 that can be made
using the digits 2,4,5,6,8 (with repetition of digits)
$= 5 + 5^2 + 5^3 + 5^4$
$=780$

49. How many 5 digit numbers can be formed using the digits 1,2,3,4,… 9 such that no two consecutive digits are the same?

Answer :
Here, no two consecutive digits can be the same
The ten thousands place can be filled by any of the 9 digits (1,2,3,4,… 9)

$\boxed{9}\boxed{}\boxed{}\boxed{}\boxed{}$

Repletion is allowed here. Only restriction is that no two consecutive digits can be the same. Hence the digit we placed in the ten thousands place cannot be used at the thousands place. Hence thousands place can be filled by any of the 8 digits.

$\boxed{9}\boxed{8}\boxed{}\boxed{}\boxed{}$

Similarly, hundreds place, tens place and unit place can be filled by any of the 8 digits

$\boxed{9}\boxed{8}\boxed{8}\boxed{8}\boxed{8}$

Hence, the required count of 5 digit numbers that can be formed
using the digits 1,2,3,4,… 9 such that no two consecutive digits are same
$= 9 \times 8^4$

50 A company has 10 software engineers and 6 civil engineers. In how many ways can a committee of 4 engineers be formed from them such that the committee must contain exactly 1 civil engineer?

Answer :
The committee should have 4 engineers. But the committee must contain exactly 1 civil engineer.
Hence, select 3 software engineers from 10 software engineers
and select 1 civil engineer from 6 civil engineers
Total number of ways this can be done = ${}^{10}C_3 \times {}^{6}C_1$
$=[(10\times9\times8)/(3\times2\times1)]\times6=10\times9\times8=720$

51 A box contains 20 balls. In how many ways can 8 balls be selected if each ball can be repeated any number of times?

Answer :
It is a question of combination with repetition
[Reference : Combinations with Repetition]

Number of combinations of n distinct things taking r at a time when each thing may be repeated any number of times is $^{(n+r-1)}\mathbf{C_r}$
Here, n=20, r=8
Hence, require number of ways = $^{(n+r-1)}C_r$ = $^{(20+8-1)}C_8$ = $^{27}C_8$

52. A box contains 12 black balls, 7 red balls and 6 blue balls. In how many ways can one or more balls be selected?

Answer :
Number of ways of selecting one or more than one objects out of S_1 alike objects of one kind, S_2 alike objects of the second kind and S_3 alike objects of the third kind is
$(S_1 + 1) (S_2 + 1)(S_3 + 1) - 1$
Hence, require number of ways = (12 + 1)(7 + 1)(6 + 1) – 1 = (13 × 8 × 7) – 1 = 728- 1 = 727

53. In how many ways can seven '#' symbol and five '*' symbol be arranged in a line so that no two '*' symbols occur together?

Answer :
There are 7 identical '#' symbols and 5 identical '*' symbols.
We need to arrange these 12 symbols in a line so that no two '*' symbols occur together.
The seven '#' symbols can be arranged in 1 way **---(A)**
(Because all these symbols are identical and order is not important)
Now there are 8 positions to arrange the five '*' symbols
so that no two '*' symbols occur together as indicated in the diagram below

	#		#		#		#		#		#		#	
1		2		3		4		5		6		7		8

The five '*' symbols can be placed in these 8 positions in $^8C_5$ ways**---(B)**
(Because all these symbols are identical and order is not important)
From(A) and (B),
the required number of ways = 1 × $^8C_5$ = $^8C_5$
= $^8C_3$ [∵ $^nC_r = {}^nC_{(n - r)}$]
$=8\times7\times63\times2\times1=8\times7=56$

54. Naresh has 10 friends and he wants to invite 6 of them to a party. How many times will 3 particular fiends always attend the party?

Answer :
Initially invite the 3 particular friends. This can only be done in 1 way **---(A)**
Now he needs to invite 3 (=6-3) more friends from the remaining 7 (=10-3) friends.
This can be done in $^7C_3$ ways **---(B)**
From(A) and (B), required number of ways = 1 × $^7C_3$ = $^7C_3$
$=(7\times6\times5)/(3\times2\times1)=7\times5=35$

55. Naresh has 10 friends and he wants to invite 6 of them to a party. How many times will 3 particular fiends never attend the party?

Answer :

Remove the 3 particular friends. This can only be done in 1 way **---(A)**
Now he needs to invite 6 friends from the remaining 7 (=10-3) friends.
This can be done in $^7C_6$ ways **---(B)**
From(A) and (B), required number of ways = $1 \times {^7C_6} = {^7C_6}$
$= {^7C_1}$[$\because {^nC_r} = {^nC_{(n-r)}}$]
= 7

56. In how many ways can 10 engineers and 4 doctors be seated at a round table without any restriction?

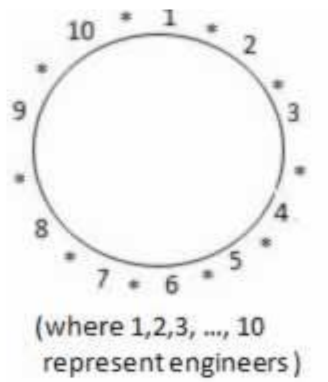
Answer :
Number of circular permutations (arrangements) of n different things is
(n-1)!
[Reference : Circular Permutations: Case 1]
Here, n = 10 + 4 = 14
Hence, the number of arrangements possible = (14-1)! = 13!

57. In how many ways can 10 engineers and 4 doctors be seated at a round table if all the 4 doctors sit together?

Answer :
Number of circular permutations (arrangements) of n different things is
(n-1)!
[Circular Permutations: Case 1]
Since all the 4 doctors sit together, group them together and consider as a single doctor.
Hence, n = total number of persons = 10 + 1 = 11
These 11 persons can be seated at a round table in (11-1)! = 10! ways **---(A)**
However these 4 doctors can be arranged among themselves in 4! Ways **---(B)**
From (A) and (B), required number of ways = 10! × 4!

58. In how many ways can 10 engineers and 4 doctors be seated at a round table if no two doctors sit together?

Answer :
Number of circular permutations (arrangements) of n different things is
(n-1)!
[Reference : Circular Permutations: Case 1]
No two doctors sit together. Hence, let's initially arrange the 10 engineers at a round table.
Total number of ways in which this can be done = (10-1)! = 9! **---(A)**
Now there are 10 positions left (marked as *) to place the four doctors as shown below
so that no two doctors can sit together.



Total number of ways in which this can be done = $^{10}P_4$**---(A)**
From (A) and (B), required number of ways = $9! \times {}^{10}P_4$

59. In how many ways can 10 engineers and 4 doctors be seated at a round table if all the 4 doctors do not sit together?

Answer :
Number of circular permutations (arrangements) of n different things is
(n-1)!
[Reference : Circular Permutations: Case 1]
Initially let's find out the total number of ways in which 10 engineers and 4 doctors can be seated at a round table
In this case, n = total number of persons = 10 + 4 = 14
Hence the total number of ways in which 10 engineers and 4 doctors can be seated at a round table = (14 – 1)! = 13! **---(A)**
Now let's find out the total number of ways in which 10 engineers and 4 doctors can be seated at a round table where all the 4 doctors sit together.
Since all the 4 doctors sit together, group them together and consider as a single doctor.
Hence, n = total number of persons = 10 + 1 = 11
These 11 persons can be seated at a round table in (11-1)! = 10! ways **---(B)**
However these 4 doctors can be arranged among themselves in 4! Ways **---(C)**
From (B) and (C), the total number of ways in which 10 engineers and 4 doctors can be seated at a round table where all the 4 doctors sit together
= $10! \times 4!$ **---(D)**
From (A) and (D),
The total number of ways in which 10 engineers and 4 doctors can be seated at a round table if all the 4 doctors do not sit together = **$13! - (10! \times 4!)$**

60 In a chess competition involving some men and women, every player needs to play exactly one game with every other player. It was found that in 45 games, both the players were women and in 190 games, both players were men. What is the number of games in which one person was a man and other person was a woman?

Answer :
Total number of women = w
Let total number of men = m

Total number of games in which both players were women = 45
=> ${}^{w}C_2 = 45$
$$=> w(w-1)2=45$$
=> w(w-1) = 90
=> **w = 10**
Total number of games in which both players were men = 190
=> ${}^{m}C_2 = 190$
$$=> m(m-1)2=190$$
=> m(m-1) = 380
=> **m = 20**
We have got that
Total number of women = 10
Total number of men = 20
Total number of games in which one person was a man and other person was a woman
= ${}^{20}C_1 \times {}^{10}C_1 = 20 \times 10 = 200$

61. There are 6 boxes numbered 1,2,...,6. Each box needs to be filled up either with a red or a blue ball in such a way that at least 1 box contains a blue ball and the boxes containing blue balls are consecutively numbered. The total number of ways in which this can be done is

Answer :
Case 1: One Box Contains a Blue Ball

The blue ball can be placed into any of the 6 boxes. i.e, 6 ways of doing this.
Red balls can be filled in the remaining boxes. Since red balls are identical,
there is only 1 way of doing this.

Total number of ways = 6 × 1 = 6
Case 2: Two Boxes Contain Blue Balls
Two blue balls can be placed into (Box 1 and Box 2) or (Box 2 and Box 3)
or (Box 3 and Box 4) or (Box 4 and Box 5) or (Box 5 and Box 6).
i.e, 5 ways of doing this.
Red balls can be filled in the remaining boxes.
Since red balls are identical, there is only 1 way of doing this.
Total number of ways = 5 × 1 = 5
Case 3: Three Boxes Contain Blue Balls
Three blue balls can be placed into (Box 1 , Box 2 and Box 3)
or (Box 2, Box 3 and Box 4) or (Box 3 , Box 4 and Box 5) or (Box 4, Box 5 and Box 6).
i.e, 4 ways of doing this.
Red balls can be filled in the remaining boxes.
Since red balls are identical, there is only 1 way of doing this.
Total number of ways = 4 × 1 = 4
Case 4: Four Boxes Contain Blue Balls
Four blue balls can be placed into (Box 1 , Box 2, Box 3 and Box 4)
or (Box 2, Box 3, Box 4 and Box 5) or (Box 3 , Box 4, Box 5 and Box 6)

i.e, 3 ways of doing this.
Red balls can be filled in the remaining boxes. Since red balls are identical, there is only 1 way of doing this.
Total number of ways = 3 × 1 = 3

Case 5: Five Boxes Contain Blue Balls

Five blue balls can be placed into (Box 1, Box 2, Box 3, Box 4 and Box 5) or (Box 2, Box 3, Box 4, Box 5 and Box 6)
i.e, 2 ways of doing this.
Red balls can be filled in the remaining boxes. Since red balls are identical, there is only 1 way of doing this.
Total number of ways = 2 × 1 = 2

Case 6: All the Six Boxes Contain Blue Balls

Six blue balls can be placed into (Box 1 , Box 2, Box 3, Box 4,Box 5 and Box 6)
Total number of ways to do this = 1
Hence, the required number of ways = 6 + 5 + 4 + 3 + 2 +1 = 21