Multi-view learning for time series:

~~In this section we~~

Definition (Univariate time series):

"A univariate time series $X = [x_1, x_2, \cdots, x_m]$, $X \in \mathbb{R}^m$ is an ordered ~~set of~~ sequence of single dimentional vector, where x_i denotes the ith time stamp observation and length of series is m."

Definition (~~a~~ Multi-view ~~of~~ univariate time Series)

"A multi-view univariate time series is a set of subset ~~of~~ $V = \{X_{(v)}\}_{v=1}^{K}$ of univariate time series $X \in \mathbb{R}^m$, where $\underset{\{K\text{-total no. of views}\}}{\cancel{\wedge}}$ vth view from X is observed as $X_v \subseteq X$ & $X_v \in \mathbb{R}^{m_v}$, ~~$p \leq m$~~. $m_v < m$, ~~and~~ $|X| = \sum_{v=1}^{K} |X_{(v)}|$ ~~and~~ view ~~of~~ univariate series denoted as $X_v = [x_t, x_{t+1}, x_{t+2}, \cdots x_{t+m_v}]$"

definition (~~Single~~ v-th View ~~of~~ of univariate time Series):

"~~A vth view of~~ ~~a~~ ~~univariate~~ time series $X_v = [x_t, x_{t+1}, x_{t+2}, \cdots x_{t+m_v}]$ ~~is the subset of univariate time Series X~~

" A vth view $X_v = [x_t, x_{t+1}, x_{t+2}, \cdots x_{t+m_v}]$ of univariate of time series X is ~~the subset of~~ $X_v \subset X \in \mathbb{R}^m$ & $X_v \in \mathbb{R}^{m_v}$, where $m_v < m$, and m (& m_v) is the length of time series ~~of~~ X and X_v. "

Definition (Ergodic Property of univariat time Series): "A time series has the long-term ~~p~~ time ~~process~~ mean of a process is equivalent to the (mean of) all possible realization of the ~~the~~ series."

Let's $x = [x_1, x_2, x_3, \ldots x_m]$ is a univariate time series, where ith ~~point~~ data point $x_i \in X$, $X \in \mathbb{R}^m$ and m is the length of the time series which follow the ~~Ergodi~~ Ergodic properties [see definition]. It means, ~~data~~ time series X should not have the following:

(a) <u>High-variability</u>: The Cauchy-distributed iid X has $\bar{x} \overset{d}{=} x_1$ which means high variability of marginal distribution function.

(b) <u>Lack of stability</u>: The X distribution changes too much for marginal distribution function such as variance approaches to infinity.

(c) <u>Absorbing state</u>: The range of probability measure varies from 0 to 1.

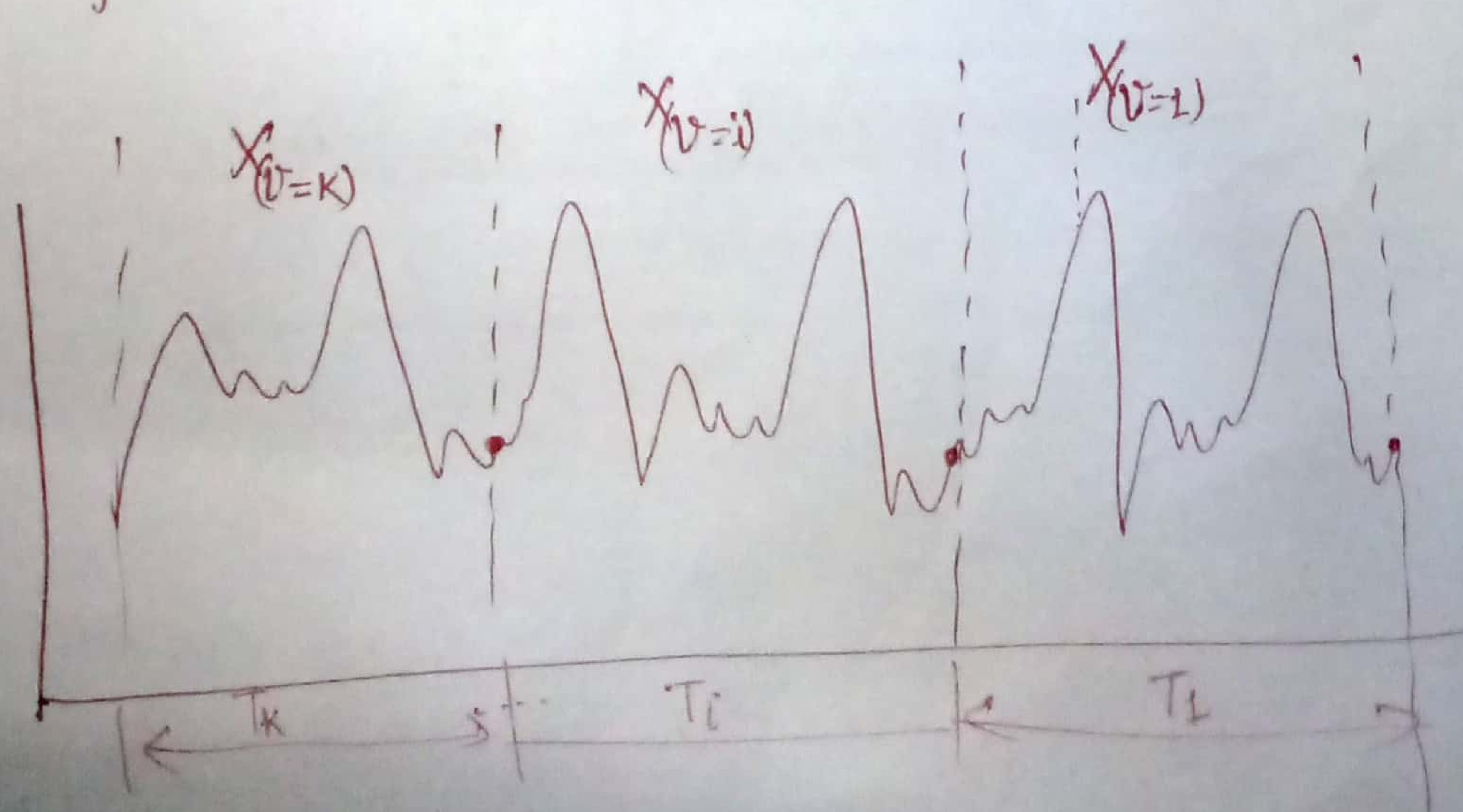
Additionally, it is also assumed that the univariate time series X' is (atleast) weakly stationary (~~see definition~~), or strict stationary (~~see definition~~)

[Content-A]

(for K-no. of partition)

Generation of views $\{X_v \in X\}$ ~~from univariate time series X~~:

Content-B: Lets a view $X_v^K \in X$ denoted as $X_v^K = [x_t, x_{t+1}, x_{t+2}, \ldots x_{t+m_v}]$ with m_v length of series, ~~where~~ is the vth view of the X, where $\{X\} = \{\bigcup_{v=1}^{K} \{X_v^K\}\}$ and $m = \sum_{v=1}^{K} m_v$, ($m_v = T_K$ (from eqn(?))) K-no. of view of X (see the definition).



Contd (A) :

~~Time Series~~ Measuring strength of Seasonality by decomposition of series X,

the decomposition of time series X can be represented as

$$X_t = X^T + X^S + X^R \quad \cdots (*)$$

where ~~X^T is smoth~~ Trend component : X^T, Seasonal Component : X^S and remainder component : X^R are denoted respectively.

The strength of seasonality (F_S) can be defined as eqn (?) :

$$F_S = \max\left(0,\ 1 - \frac{Var(X^R)}{Var(X^S + X^R)}\right) \cdots (*)$$

where $Var(X^R)$ and $Var(X^S + X^R)$ are variance of remainder and detrended data over seasional and remainder respectively. And F_S close to 0 and 1 ~~denotes the exibt~~ exhibits the no seasionallity and strong ~~seasesip~~ seasionality.

Identifying Seasional for ~~min~~ lowest lag ℓ :

for time series X

The autocorrelation function (ACF) coefficient of for lag ℓ can be obtained as: in eqn (?):

$$ACF(\ell) = \left(\frac{1}{m}\right) \frac{\sum_{t=\ell+1}^{m} \{X_{(t)} X_{(t-\ell)}\}}{\phi},$$

where $\ell = 1, 2, 3, \ldots m/2$, and ϕ consistant estimator can be obtained as in eqn (?) :

$$\phi = \frac{1}{(m-p)} \sum_{t=1}^{m}$$

$$\rho_\ell = \cancel{Cov(X_t, X_{t+\ell})} = \frac{E[(X_t - \mu)(X_{t+\ell} - \mu)]}{\sigma_x^2} \cdots (*)$$

where μ is constant mean ($\mu = E(X_t)$) and $\sigma_x^2 = \frac{1}{m}\sum_{t=1}^{m}(X_t - \bar{X})^2$,

$\bar{X} = \left(\frac{1}{m}\right)\sum_{t=1}^{m} X_t$ is the sample mean. The coefficient ρ_L of auto correlation function has range $[-1,+1]$, where close to $+1$ ~~exib~~ exhibites the strong positive correlation between X and its lagged value L at a specific lag. ~~So $\rho_L \approx 1$ shall be~~ is ~~considered~~. Let consider ~~the~~ ρ_L^+ ~~coefficient ρ_L^+ has all~~ that is a set of autocorrelation function coefficient of lag has positive correlation, which can denoted as

$$\rho_L^+ = \{\rho_L > 0\}_{L=1}^{p} \quad \text{------} (*)$$

Then the lowest lag value will indicate the smallest ~~period~~ behavior in the series X, time stamp that has seasion ~~bel~~ behavior can be identified as

$$(L^+)_{lowest} = \arg\min_{L \in \mathbb{R}} \{\rho_L^+\} \quad \text{-----} (*)$$

end of Content (A)

Content (B)

Lets consider ~~the time stamp T_s~~ $T_{(L^+)lowest}$ ~~T_s~~ is the time-stamp of ~~lowest~~ length of seasional with lowest $(L^+)_{lowest}$ lag. Theng the series X can be ~~observed~~ partitioned with K-seasional repeatition for L^+ lags, where ~~maximum~~ no. of partition can be ~~done~~ obtained corresponding to $\{L^+\}_{lowest}$.

~~can be written as (eqn(?)):~~
~~Now, the partitions of~~ X ~~is considered as view of the~~
~~series~~

T_K^*

The ~~time stamp~~ length of time-stamp of k-th partition (i.e. ~~view of~~ a view of X) can be obtained as eqn(?):—

$$T_K = (T^*) / (T_{(L^+)lowest} \times K) \quad \text{---} (*)$$

where T_x is the length of time stamp of X, and $T_{\{L^+\}_{lowest}}$ is the ~~time~~ length of ~~lowest~~ time-stamp of lowest log

End of Content - B

~~The possible no. of maximum no. of partition would be $\frac{T_x}{T_{\{L^+\}_{lowest}}}$.~~

The total no. of possible partition ranges as $\left[1, \frac{T_x}{T_{\{L^+\}_{lowest}}}\right]$:

~~# Learning through Stacked CNN-BiLSTM~~

Stacked CNN-BiLSTM :