NC STATE UNIVERSITY

OR 501: Introduction to Operations Research

PROJECT REPORT

AGGREGATE PLANNING PROBLEM – SKYPAS AIRLINES INC.

Submitted By

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Abstract

This research investigates the operational challenges faced by SKYPAS Airline Inc. and proposes an optimal strategy for fleet management, crew allocation, and route optimization. The study analyzes three key strategies: demand-driven, stable workforce, and a hybrid approach. A comprehensive mathematical model, incorporating factors like passenger demand, aircraft availability, crew constraints, and operational costs, is developed to evaluate these strategies. By implementing a demand-driven approach, SKYPAS can minimize costs while meeting passenger demand. However, this strategy requires flexible workforce management. The stable workforce strategy offers stability but may lead to higher costs due to potential underutilization. The hybrid approach balances these two, providing a compromise between cost efficiency and workforce stability. The results of this study provide valuable insights for airlines to optimize their operations and enhance their bottom line.

Introduction

The project focuses on tackling operational challenges for SKYPAS Airline Inc., a growing commercial aviation company. The challenge was to solve the aggregate planning problem and find an optimal strategy for six months while balancing fleet management, crew allocation and route optimization while minimizing operational costs.

Leveraging the forecasted monthly passenger demands, crew requirements, and aircraft specifications, our team formulated a comprehensive mathematical model to address key decisions such as flight scheduling, aircraft leasing, and workforce management. We incorporated constraints to ensure passenger demands were met, aircraft availability was not exceeded, proper crew assignments were implemented, and work-hour limitations were respected.

This project explores three operational strategies: a demand-driven workforce strategy, a stable workforce strategy, and a hybrid optimization strategy. Each strategy aims to balance different aspects of operational efficiency and paints a more comprehensive picture for the decision-makers choosing a strategy, as the comparative analysis of the three strategies offers valuable insights into the trade-offs between different aspects of operational efficiency.

Problem Description and Mathematical Formulation

SKYPAS Airlines Inc. is facing a critical juncture in its operational planning. The company's management team is tasked with developing an optimal aggregate plan for the next six months, from January to June. This plan must address the complex interplay of fleet management, crew allocation, and route optimization while minimizing overall costs and meeting fluctuating passenger demands.

SKYPAS currently operates a network of five distinct routes, each with unique demand patterns and operational challenges. The company has observed significant variations in passenger numbers across these routes over the past few months, and projections indicate this trend will continue. For the upcoming six-month period, the forecasted monthly demands for each route are outlined in Table 1.

Routes	January	February	March	April	May	June
1	2500	2750	2400	2900	3100	3200
2	2500	2600	2200	2800	3100	3300
3	2300	2100	1900	2300	2500	2700
4	2000	3000	2700	3200	3400	3600
5	2600	2900	2500	3100	3400	3600

Table 1. Forecasted monthly demand for each of SKYPAS's routes from January to June.

SKYPAS owns ten Airbus A320 (Type-1) aircraft to service its routes. However, given the projected increase in demand, the company can lease additional aircraft. The types of aircraft available for rent, their passenger capacities, their respective leasing costs, and the crew required to operate them are outlined in Table 2.

Fleet Type	Aircraft	Capacity	Available Fleet	Leasing Cost	Crew Requirements
1	Airbus A320	150	10	-	1 pilot, 1 co-pilot, 2 attendants
2	Boeing 777	200	-	$50,000	1 pilot, 1 co-pilot, 3 attendants
3	Airbus A380	250	-	$75,000	1 pilot, 2 co-pilots, 4 attendants

Table 2. The aircraft that are available for rent and their specifications.

SKYPAS currently employs eight pilots, twelve co-pilots, and twenty flight attendants. According to the Air Travel Employee Union's guidelines, employees are limited to 160 monthly working hours. Table 3 shows the salaries for each crew position and the costs associated with hiring, firing, and training each position in each month.

Employee Type	Salary ($)	Hiring Cost ($)	Firing Cost ($)	Training Cost ($)
Pilot	6,000	4,000	2,000	1,000
Co-Pilot	5,000	3,500	1,800	900
Flight Attendants	3,000	2,000	1,000	700

Table 3. Crew positions and the associated costs per month.

The flight durations and fuel costs for each of SKYPAS's routes are shown in Table 4.

Routes	Flight Duration (Hours)	Fuel Costs ($)
1	5	5,000
2	4	4,500
3	6	4,000
4	3	5,500
5	4.5	5,200

Table 4. The flight duration in hours and the fuel costs per flight for each route.

A comprehensive aggregate plan must be developed that addresses the following key decisions: How many flights should be scheduled on each route, using which aircraft types, each month? How many additional aircraft of each type (if any) should be leased each month? How should the workforce be managed in terms of hiring, firing, and retaining pilots, co-pilots, and flight attendants each month?

Some important constraints to be considered in this problem are that all passenger demand must be met or exceeded on all routes for each month and the number of scheduled flights does not exceed the number of available aircraft. Each flight must have the minimum required crew for the corresponding aircraft type and no crew member can be scheduled more than 160 hours in a month.

Three strategies will be explored to discover the optimal workforce levels so SKYPAS's cost of operation is minimized. The first is a demand-driven workforce strategy that adjusts the workforce each month to maintain just enough staff to meet flight demand. The second is a stable workforce strategy which maintains a relatively stable workforce across all 6 months. The final strategy is a hybrid optimization strategy that optimizes between hiring/firing and idle time costs.

The mathematical formulation for this is as follows:

Objective Function:

$$\text{Minimize} \quad \sum_{i,k,j} \left((X_{i,k,j} \times \text{Fuel Cost}_{i,j}) + \sum_{\text{type}} (\text{Wage per Crew Type} \times C_j^{\text{type}}) \right)$$
$$+ \sum_{\text{type}} (\text{Hiring Cost per Crew Type} \times H_j^{\text{type}}) + \sum_{\text{type}} (\text{Firing Cost per Crew Type} \times F_j^{\text{type}})$$
$$+ \sum_{\text{type}} (\text{Training Cost per Crew Type} \times T_j^{\text{type}}) + \sum_{k,j} (\text{Leased Fleet}_k \times \text{Leasing Cost}_k)$$

Subject to:

Passenger Demand Constraint

$$\sum_k P_k \times X_{i,k,j} \geq D_{i,j} \quad \forall i, j$$

Crew Assignment Constraints

$$C_j^{\text{pilot}} \geq \sum_{i,k} \left(M_k^{\text{pilot}} \times X_{i,k,j} \right)$$

$$C_j^{\text{copilot}} \geq \sum_{i,k} \left(M_k^{\text{copilot}} \times X_{i,k,j} \right)$$

$$C_j^{\text{attendant}} \geq \sum_{i,k} \left(M_k^{\text{attendant}} \times X_{i,k,j} \right)$$

Workforce Balance Constraints

$$C_j^{\text{type}} = C_{j-1}^{\text{type}} + H_j^{\text{type}} - F_j^{\text{type}} - T_j^{\text{type}} \quad \forall j$$

$$T_j^{\text{type}} = H_{j-1}^{\text{type}} \quad \forall j$$

Capacity Constraint

$$\sum_{i,k} X_{i,k,j} \leq \frac{160 \times C_j^{\text{type}}}{H_k} \quad \forall j, \text{type}$$

Non-negativity restrictions:

$$X_{i,j,k}, Fuel\ Cost_i, Leasing\ Cost_k, C_j^{type}, H_j^{type}, F_j^{type}, T_j^{type}, Leased\ Fleet_k \geq 0$$

Numerical Study

Demand Driven Workforce Strategy

This strategy focuses on closely aligning the workforce levels with monthly demand fluctuations. It helps with flexible staffing for the company as it adjusts the number of pilots, co-pilots, and flight attendants each month to match flight requirements.

Decision Variables

X(i,j,k): Number of flights on route i in month j using aircraft type k
leased_fleet(k,j): Number of leased aircraft of type k in month j
staff(e,j): Number of staff of type e in month j
hire(e,j): Number of staff of type e hired in month j
fire(e,j): Number of staff of type e fired in month j

Objective Function Minimize Z (Total cost):

$$\mathbf{Z} = \sum_{i,j,k} X_{i,j,k} \cdot \text{fuel\_cost}_i + \sum_{k>1,j} \text{leased\_fleet}_{k,j} \cdot \text{leasing\_cost}_k + \sum_{e,j} \text{staff}_{e,j} \cdot \text{salary}_e$$
$$+ \textstyle\sum_{e,j} \text{hire}_{e,j} \cdot (hiring\_cost_e + training\_cost_e) + \sum_{e,j} \text{fire}_{e,j} \cdot (firing\_cost_e)$$

Subject to

1. **Passenger Demand Constraints:**

$$\sum_k X_{i,j,k} \cdot \text{capacity}_k \geq \text{demand}_{i,j} \quad \forall i, j$$

2. **Type-1 aircraft Utilization Constraint:**

$$\sum_i X_{i,j,1} = \text{owned\_fleet}_1 \quad \forall j$$

3. **Demand Satisfaction Constraint:**

$$\sum_{k>1} X_{i,j,k} \cdot \text{capacity}_k \geq \max(0, \text{demand}_{i,j} - \text{owned\_fleet}_1 \cdot \text{capacity}_1) \quad \forall i, j$$

4. **Aircraft Balance Constraint:**

$$\sum_i X_{i,j,k} \leq \text{leased\_fleet}_{k,j} \quad \forall k > 1, j$$

5. **Crew Satisfaction Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{crew\_req}_{k,e} \leq \text{staff}_{e,j} \quad \forall e, j$$

6. **Crew Work Capacity Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{flight\_duration}_i \cdot \text{crew\_req}_{k,e} \leq \text{staff}_{e,j} \cdot \text{max\_hours} \quad \forall e, j$$

7. **Workforce Balance Constraint:**

$$\text{staff}_{e,j} = \text{staff}_{e,j-1} + \text{hire}_{e,j} - \text{fire}_{e,j} \quad \forall e, j > 1$$

8. **Initial Staff:**

$$\text{staff}_{e,1} = \text{initial\_staff}_e + \text{hire}_{e,1} - \text{fire}_{e,1} \quad \forall e$$

Non-negativity restrictions:

Integers: X(i,j,k), leased_fleet(k,j), staff(e,j), hire(e,j), fire(e,j)

X(i,j,k), leased_fleet(k,j), staff(e,j), hire(e,j), fire(e,j) ≥ 0

For all i (routes) = 1, 2, 3, 4, 5; j (months) = 1, 2, 3, 4, 5, 6; k (fleet type) = 1, 2, 3; e (crew type) = pilot, copilot, attendant

Stable Workforce Strategy

This approach aims to maintain a consistent workforce level throughout the six-month planning period. It has constant staff across all the months, i.e., it keeps the number of employees relatively unchanged across all months and prioritizes workforce stability over short-term demand fluctuations.

Decision Variables

X(i,j,k): Number of flights on route i in month j using aircraft type k
leased_fleet(k,j): Number of leased aircraft of type k in month j
staff(e): Number of staff of type e
hire(e): Number of staff of type e hired at the beginning
fire(e): Number of staff of type e fired at the beginning

Objective Function Minimize Z (Total Cost):

$$\mathbf{Z} = \sum_{i,j,k} X_{i,j,k} \cdot \text{fuel\_cost}_i + \sum_{k>1,j} \text{leased\_fleet}_{k,j} \cdot \text{leasing\_cost}_k$$
$$+ \sum_e \text{staff}_e \cdot \text{salary}_e \cdot 6 + \sum_e \text{hire}_e \cdot (\text{hiring\_cost}_e + \text{training\_cost}_e)$$
$$+ \sum_e \text{fire}_e \cdot \text{firing\_cost}_e$$

Subject to

1. **Passenger Demand Constraint:**

$$\sum_k X_{i,j,k} \cdot \text{capacity}_k \geq \text{demand}_{i,j} \quad \forall i,j$$

2. **Type-1 Aircraft Utilization Constraint:**

$$\sum_i X_{i,j,1} = \text{owned\_fleet}_1 \quad \forall j$$

3. **Demand Satisfaction Constraint:**

$$\sum_{k>1} X_{i,j,k} \cdot \text{capacity}_k \geq \max(0, \text{demand}_{i,j} - \text{owned\_fleet}_1 \cdot \text{capacity}_1) \quad \forall i,j$$

4. **Aircraft Balance Constraint:**

$$\sum_i X_{i,j,k} \leq \text{leased\_fleet}_{k,j} \quad \forall k>1, j$$

5. **Crew Satisfaction Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{crew\_req}_{k,e} \leq \text{staff}_e \quad \forall e,j$$

6. **Crew Work Capacity Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{flight\_duration}_i \cdot \text{crew\_req}_{k,e} \leq \text{staff}_e \cdot \text{max\_hours} \quad \forall e,j$$

7. **Workforce Balance Constraint:**

$$\text{staff}_e = \text{initial\_staff}_e + \text{hire}_e - \text{fire}_e \quad \forall e$$

8. **Initial Staff:**

$$\text{staff}_e \geq \text{initial\_staff}_e \quad \forall e$$

Non-negativity restrictions:

Integers: X(i,j,k), leased_fleet(k,j), staff(e), hire(e), fire(e)

X(i,j,k), leased_fleet(k,j), staff(e), hire(e), fire(e) ≥ 0

For all i (routes) = 1, 2, 3, 4, 5; j (months) = 1, 2, 3, 4, 5, 6; k (fleet type) = 1, 2, 3; e (crew type) = pilot, copilot, attendant

Hybrid Optimization Strategy

This strategy seeks to balance workforce stability with operational efficiency. It allows for some workforce adjustments while avoiding extreme fluctuations and considering idle time costs. The strategy involves some hiring and firing, but less than the demand-driven strategy. It also combines elements of both demand-driven and stable workforce strategies to optimize overall costs and efficiency.

Decision Variables

X(i,j,k): The number of flights on route i in month j using aircraft type k
leased_fleet(k,j): The number of leased aircraft of type k in month j
staff(e,j): The number of staff of type e in month j
hire(e,j): The number of staff of type e hired in month j
fire(e,j): The number of staff of type e fired in month j
idle hours(e,j): The idle hours for each employee type e in each month j

Objective Function Minimize Z (Total Cost):

$$Z = \sum_{i,j,k} X_{i,j,k} \cdot \text{fuel\_cost}_i + \sum_{k>1,j} \text{leased\_fleet}_{k,j} \cdot \text{leasing\_cost}_k + \sum_{e,j} \text{staff}_{e,j} \cdot \text{salary}_e$$
$$+ \sum_{e,j} \text{hire}_{e,j} \cdot (\text{hiring\_cost}_e + \text{training\_cost}_e) + \sum_{e,j} \text{fire}_{e,j} \cdot \text{firing\_cost}_e$$
$$+ \sum_{e,j} \text{idle\_hours}_{e,j} \cdot \frac{\text{salary}_e}{\text{max\_hours}} \cdot \text{idle\_cost\_factor}$$

Subject to

1. **Passenger Demand Constraint:**

$$\sum_{k} X_{i,j,k} \cdot \text{capacity}_k \geq \text{demand}_{i,j} \quad \forall i, j$$

2. **Type-1 Aircraft Utilization Constraint:**

$$\sum_{i} X_{i,j,1} = \text{owned\_fleet}_1 \quad \forall j$$

3. **Demand Satisfaction Constraint:**

$$\sum_{k>1} X_{i,j,k} \cdot \text{capacity}_k \geq \max(0, \text{demand}_{i,j} - \text{owned\_fleet}_1 \cdot \text{capacity}_1) \quad \forall i, j$$

4. **Aircraft Balance Constraint:**

$$\sum_{i} X_{i,j,k} \leq \text{leased\_fleet}_{k,j} \quad \forall k > 1, j$$

5. **Crew Satisfaction Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{crew\_req}_{k,e} \leq \text{staff}_{e,j} \quad \forall e, j$$

6. **Crew Work Capacity Constraint:**

$$\sum_{i,k} X_{i,j,k} \cdot \text{flight\_duration}_i \cdot \text{crew\_req}_{k,e} + \text{idle\_hours}_{e,j} = \text{staff}_{e,j} \cdot \text{max\_hours} \quad \forall e, j$$

7. **Workforce Balance Constraint:**

$$\text{staff}_{e,j} = \text{staff}_{e,j-1} + \text{hire}_{e,j} - \text{fire}_{e,j} \quad \forall e, j > 1$$

8. **Initial Staff:**

$$\text{staff}_{e,1} = \text{initial\_staff}_e + \text{hire}_{e,1} - \text{fire}_{e,1} \quad \forall e$$

9. **Idle Time Calculation:**

$$\text{idle\_hours}_{e,j} = \text{staff}_{e,j} \cdot \text{max\_hours} - \sum_{i,k} X_{i,j,k} \cdot \text{flight\_duration}_i \cdot \text{crew\_req}_{k,e} \quad \forall e, j$$

Non-negativity restrictions:

Integers: X(i,j,k), leased_fleet(k,j), staff(e,j), hire(e,j), fire(e,j), idle_hours(e,j)

X(i,j,k), leased_fleet(k,j), staff(e,j), hire(e,j), fire(e,j), idle_hours(e,j) ≥ 0

For all i (routes) = 1, 2, 3, 4, 5; j (months) = 1, 2, 3, 4, 5, 6; k (fleet type) = 1, 2, 3; e (crew type) = pilot, copilot, attendant

Results

Strategy	Objective Function Minimized Cost ($)
Demand-Driven	**30,590,800**
Stable	**31,974,300**
Hybrid – Optimization	**34,731,367**

Demand-Driven Strategy

In a demand-driven strategy, we found that the optimal objective function is $30,590,800. The following is the number of flights of Type 2 and 3 leased for each period:

Flight Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Type-2	52	58	51	64	70	72
Type-3	0	1	0	0	0	2

Number of Staff in each month:

Staff Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Pilot	62	69	68	74	80	84
Co-pilot	62	70	70	74	80	86
Attendant	176	198	198	212	230	244

Number of staff hired in each month:

Staff Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Pilot	54	7	0	5	6	4
Co-pilot	50	8	0	4	6	6
Attendant	156	22	0	14	18	14

Stable Workforce Strategy

In a stable workforce strategy, we found that the optimal objective function is $31,974,300. Here 84 pilots, 86 copilots, and 244.000 attendants will work throughout the 6 months with the initial hiring of 76 pilots, 74 copilots, and 224 attendants. The following is the number of flights of Type 2 and 3 leased for each period:

Flight Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Type 2	52	58	51	64	70	72
Type 3	0	1	0	0	0	2

Hybrid – Optimization Strategy

In the hybrid optimization strategy, the optimal objective function value is $34,731,367. The following is the number of flights of Type 2 and 3 leased for each period:

Flight Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Type 2	52	58	51	64	70	72
Type 3	0	1	0	0	0	2

The workforce at the start of each month:

Staff Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Pilot	62	69	61	74	80	84
Co-pilot	62	70	61	74	80	86
Attendant	176	198	173	212	230	244

Number of staff hired in each month:

Staff Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Pilot	54	7	0	13	6	4
Co-pilot	50	8	0	13	6	6
Attendant	156	22	0	39	18	14

Workforce fired:

Staff Type	Month-3
Pilot	8
Co-pilot	9
Attendant	25

Conclusion

Workforce Strategy	Objective Function Minimized Cost ($)
Demand-Driven	**30,590,800**
Stable	**31,974,300**
Hybrid – Optimization	**34,731,367.1875**

After implementing the proposed strategies, we have determined that a demand-driven strategy would be the optimal choice for SKYPAS. This approach proved to be the most cost-effective, minimizing overall expenses for the organization.

Future Scope and Potential Next Steps

During the execution of the hybrid optimization model, we observed that the cost was significantly higher than what we anticipated. Upon further inspection of the results, we identified that the additional cost we added as a penalty for idle time was the factor for the higher objective function value. This prompted us to look at the idle time of each crew type for each month. The results from GAMS are given below:

Staff Type	Month-1	Month-2	Month-3	Month-4	Month-5	Month-6
Pilot	9,636.5	10,733.5	9489.5	11,511	12,444.5	13,065
Co-pilot	9,636.5	10,889	9489.5	11,511	12,444.5	13,377
Attendant	27,363.5	30,810.5	26,919.5	32,988	35,787.5	37,967

From the idle time for each month, we observed that every crew member was only working on their initially assigned flight. This assignment led to substantial idle time and consequently higher objective function value.

Potential next steps include linking the routes for each flight by defining the origin and destination for each route. This enables flights and crew to transition to the next feasible route. Consideration would also be given to reformulating this problem as a network modeling problem to determine the optimal path each flight will take. This will help optimize scheduling crews and reduce the number of flights leased to meet demand.

References

1. Flight time limitations and rest requirements under 14 C.F.R.
2. Deep Dive: The World of Aircraft Leasing
3. Linear Programming Applications in Business, Finance, Medicine, and Social Science - Mathematics LibreTexts

Appendix

A. Data File Used for GAMS (data.gms)

```
table demand(i,j) 'Passenger demand for each route and month'
    1       2       3       4       5       6
1   2500    2750    2400    2900    3100    3200
2   2500    2600    2200    2800    3100    3300
3   2300    2100    1900    2300    2500    2700
4   2000    3000    2700    3200    3400    3600
5   2600    2900    2500    3100    3400    3600;

parameter capacity(k) 'Aircraft capacity' /
1   150
2   200
3   250/;

parameter owned_fleet(k) 'Number of owned aircraft of each type' /
1   10
2   0
3   0/;

parameter leasing_cost(k) 'Monthly leasing cost for each aircraft type' /
1   0
2   50000
3   75000 /;

table crew_req(k,e) 'Crew requirements for each aircraft type'
        pilot   copilot attendant
1       1       1       2
2       1       1       3
3       1       2       4;

parameter flight_duration(i) 'Flight duration for each route' /
1   5
2   4
3   6
4   3
5   4.5/;

parameter fuel_cost(i) 'Fuel cost for each route' /
1   5000
2   4500
3   4000
4   5500
5   5200/;

parameter salary(e) 'Monthly salary for each employee type' /
pilot       6000
copilot     5000
attendant   3000/;

parameter hiring_cost(e) 'Hiring cost for each employee type' /
pilot       4000
copilot     3500
attendant   2000/;

parameter firing_cost(e) 'Firing cost for each employee type' /
pilot       2000
copilot     1800
attendant   1000/;

parameter training_cost(e) 'Training cost for each employee type' /
pilot       1000
copilot     900
attendant   700/;

parameter initial_staff(e) 'Initial number of staff for each employee type' /
pilot       8
copilot     12
attendant   20/;
```

B. GAMS code for Demand-Driven Strategy

```
Sets
i routes /1*5/
j months /1*6/
k aircraft_types /1*3/
e employee_types /pilot, copilot, attendant/;

Parameters
demand(i,j) Passenger demand for each route and month
capacity(k) Aircraft capacity
owned_fleet(k) Number of owned aircraft of each type
leasing_cost(k) Monthly leasing cost for each aircraft type
crew_req(k,e) Crew requirements for each aircraft type
flight_duration(i) Flight duration for each route
fuel_cost(i) Fuel cost for each route
salary(e) Monthly salary for each employee type
hiring_cost(e) Hiring cost for each employee type
firing_cost(e) Firing cost for each employee type
training_cost(e) Training cost for each employee type
initial_staff(e) Initial number of staff for each employee type
max_hours Working hours limit per employee per month /160/;

$include data.gms

Variables
Z Total cost;

Integer Variables
X(i,j,k) Number of flights on route i in month j using aircraft type k
leased_fleet(k,j) Number of leased aircraft of type k in month j
staff(e,j) Number of staff of type e in month j
hire(e,j) Number of staff of type e hired in month j
fire(e,j) Number of staff of type e fired in month j;

Equations
objective  Objective Function
meet_demand(i,j) Passenger Demand
use_all_type1(j) Type-1 aircraft Utilization
additional_demand(i,j) Demand Satisfaction
aircraft_availability_other(k,j) Aircraft Balance
crew_requirements(e,j) Crew Satisfaction
working_hours_limit(e,j) Maximum Work Hours for each employee
staff_balance(e,j) Workforce Balance
initial_staff_constraint(e) Initial staff levels;

objective.. Z =E= sum((i,j,k), X(i,j,k) * fuel_cost(i))
                + sum((k,j)$(ord(k) > 1), leased_fleet(k,j) * leasing_cost(k))
                + sum((e,j), staff(e,j) * salary(e))
                + sum((e,j), hire(e,j) * (hiring_cost(e) + training_cost(e)))
                + sum((e,j), fire(e,j) * firing_cost(e));

meet_demand(i,j).. sum(k, X(i,j,k) * capacity(k)) =G= demand(i,j);

use_all_type1(j).. sum(i, X(i,j,'1')) =E= owned_fleet('1');

additional_demand(i,j)..sum(k$(ord(k) > 1), X(i,j,k) * capacity(k)) =G= max(0, demand(i,j) - owned_fleet('1') * capacity('1'));

aircraft_availability_other(k,j)$(ord(k) > 1).. sum(i, X(i,j,k)) =L= leased_fleet(k,j);

crew_requirements(e,j).. sum((i,k), X(i,j,k) * crew_req(k,e)) =L= staff(e,j);

working_hours_limit(e,j).. sum((i,k), X(i,j,k) * flight_duration(i) * crew_req(k,e)) =L= staff(e,j) * max_hours;

staff_balance(e,j)$(ord(j) > 1).. staff(e,j) =E= staff(e,j-1) + hire(e,j) - fire(e,j);

initial_staff_constraint(e).. staff(e,'1') =E= initial_staff(e) + hire(e,'1') - fire(e,'1');

Model demand_driven /all/;

Solve demand_driven using mip minimizing Z;

Display X.l, leased_fleet.l, staff.l, hire.l, fire.l, Z.l;
```

C. GAMS code for Stable Workforce Strategy

```
Sets
i 'routes' /1*5/
j 'months' /1*6/
k 'aircraft_types' /1*3/
e 'employee_types' /pilot, copilot, attendant/;

Parameters
demand(i,j) 'Passenger demand for each route and month'
capacity(k) 'Aircraft capacity'
owned_fleet(k) 'Number of owned aircraft of each type'
leasing_cost(k) 'Monthly leasing cost for each aircraft type'
crew_req(k,e) 'Crew requirements for each aircraft type'
flight_duration(i) 'Flight duration for each route'
fuel_cost(i) 'Fuel cost for each route'
salary(e) 'Monthly salary for each employee type'
hiring_cost(e) 'Hiring cost for each employee type'
firing_cost(e) 'Firing cost for each employee type'
training_cost(e) 'Training cost for each employee type'
initial_staff(e) 'Initial number of staff for each employee type'
max_hours 'Working hours limit per employee per month' /160/;

$include data.gms

Variables
Z Total cost;

Integer Variables
X(i,j,k) Number of flights on route i in month j using aircraft type k
leased_fleet(k,j) Number of leased aircraft of type k in month j
staff(e) Number of staff of type e (constant across months)
hire(e) Number of staff of type e hired at the beginning
fire(e) Number of staff of type e fired at the beginning;

Equations
objective Objective Function
meet_demand(i,j) Passenger Demand
use_all_type1(j) Type-1 aircraft Utilization
additional_demand(i,j) Demand Satisfaction
aircraft_availability_other(k,j) Aircraft Balance
crew_requirements(e,j) Crew Satisfaction
working_hours_limit(e,j) Maximum Work Hours for each employee
staff_balance(e) Workforce Balance
initial_staff_constraint(e) Initial staff levels;

objective.. Z =E= sum((i,j,k), X(i,j,k) * fuel_cost(i))
                + sum((k,j)$(ord(k) > 1), leased_fleet(k,j) * leasing_cost(k))
                + sum(e, staff(e) * salary(e) * card(j))
                + sum(e, hire(e) * (hiring_cost(e) + training_cost(e)))
                + sum(e, fire(e) * firing_cost(e));

meet_demand(i,j).. sum(k, X(i,j,k) * capacity(k)) =G= demand(i,j);

use_all_type1(j).. sum(i, X(i,j,'1')) =E= owned_fleet('1');

additional_demand(i,j).. sum(k$(ord(k) > 1), X(i,j,k) * capacity(k)) =G= max(0, demand(i,j) - owned_fleet('1') * capacity('1'));

aircraft_availability_other(k,j)$(ord(k) > 1).. sum(i, X(i,j,k)) =L= leased_fleet(k,j);

crew_requirements(e,j).. sum((i,k), X(i,j,k) * crew_req(k,e)) =L= staff(e);

working_hours_limit(e,j).. sum((i,k), X(i,j,k) * flight_duration(i) * crew_req(k,e)) =L= staff(e) * max_hours;

staff_balance(e).. staff(e) =E= initial_staff(e) + hire(e) - fire(e);

initial_staff_constraint(e).. staff(e) =G= initial_staff(e);

Model stable_workforce /all/;

Solve stable_workforce using mip minimizing Z;

Display X.l, leased_fleet.l, staff.l, hire.l, fire.l, Z.l;
```

D. GAMS code for Hybrid Optimization Strategy

```
Sets
i 'routes' /1*5/
j 'months' /1*6/
k 'aircraft_types' /1*3/
e 'employee_types' /pilot, copilot, attendant/;

Parameters
demand(i,j) 'Passenger demand for each route and month'
capacity(k) 'Aircraft capacity'
owned_fleet(k) 'Number of owned aircraft of each type'
leasing_cost(k) 'Monthly leasing cost for each aircraft type'
crew_req(k,e) 'Crew requirements for each aircraft type'
flight_duration(i) 'Flight duration for each route'
fuel_cost(i) 'Fuel cost for each route'
salary(e) 'Monthly salary for each employee type'
hiring_cost(e) 'Hiring cost for each employee type'
firing_cost(e) 'Firing cost for each employee type'
training_cost(e) 'Training cost for each employee type'
initial_staff(e) 'Initial number of staff for each employee type'
max_hours 'Working hours limit per employee per month' /160/
idle_cost_factor 'Idle time cost factor' /0.5/;

$include data.gms

Variables
Z 'Total cost';

Integer Variables
X(i,j,k) 'Number of flights on route i in month j using aircraft type k'
leased_fleet(k,j) 'Number of leased aircraft of type k in month j'
staff(e,j) 'Number of staff of type e in month j'
hire(e,j) 'Number of staff of type e hired in month j'
fire(e,j) 'Number of staff of type e fired in month j';

Positive Variables
idle_hours(e,j) 'Idle hours for each employee type in each month';

Equations
objective Objective Function
meet_demand(i,j) Passenger Demand
use_all_type1(j) Type-1 aircraft Utilization
additional_demand(i,j) Demand Satisfaction
aircraft_availability_other(k,j) Aircraft Balance
crew_requirements(e,j) Crew Satisfaction
working_hours_limit(e,j) Maximum Work Hours for each employee
staff_balance(e,j) Workforce Balance
initial_staff_constraint(e) Initial staff levels
idle_time_calculation(e,j) Idle Time Calculation;

objective.. Z =E= sum((i,j,k), X(i,j,k) * fuel_cost(i))
                + sum((k,j)$(ord(k) > 1), leased_fleet(k,j) * leasing_cost(k))
                + sum((e,j), staff(e,j) * salary(e))
                + sum((e,j), hire(e,j) * (hiring_cost(e) + training_cost(e)))
                + sum((e,j), fire(e,j) * firing_cost(e))
                + sum((e,j), idle_hours(e,j) * salary(e) / max_hours * idle_cost_factor);

meet_demand(i,j).. sum(k, X(i,j,k) * capacity(k)) =G= demand(i,j);

use_all_type1(j).. sum(i, X(i,j,'1')) =E= owned_fleet('1');

additional_demand(i,j).. sum(k$(ord(k) > 1), X(i,j,k) * capacity(k)) =G= max(0, demand(i,j) - owned_fleet('1') * capacity('1'));

aircraft_availability_other(k,j)$(ord(k) > 1).. sum(i, X(i,j,k)) =L= leased_fleet(k,j);

crew_requirements(e,j).. sum((i,k), X(i,j,k) * crew_req(k,e)) =L= staff(e,j);

working_hours_limit(e,j).. sum((i,k), X(i,j,k) * flight_duration(i) * crew_req(k,e)) + idle_hours(e,j) =E= staff(e,j) * max_hours;

staff_balance(e,j)$(ord(j) > 1).. staff(e,j) =E= staff(e,j-1) + hire(e,j) - fire(e,j);

initial_staff_constraint(e).. staff(e,'1') =E= initial_staff(e) + hire(e,'1') - fire(e,'1');

idle_time_calculation(e,j).. idle_hours(e,j) =E= staff(e,j) * max_hours - sum((i,k), X(i,j,k) * flight_duration(i) * crew_req(k,e));

Model hybrid_optimization /all/;

Solve hybrid_optimization using mip minimizing Z;

Display X.l, leased_fleet.l, staff.l, hire.l, fire.l, idle_hours.l, Z.l;
```

E. Output For Demand-Driven Strategy

```
--- Fixed MIP status (1): optimal.
--- Cplex Time: 0.00sec (det. 0.15 ticks)


Solution satisfies tolerances
MIP Solution:     30590800.000000    (102 iterations, 16 nodes)
Final Solve:      30590800.000000    (0 iterations)

Best possible:    30588850.000000
Absolute gap:         1950.000000
Relative gap:            0.000064
```

F. Output For Stable Workforce Strategy

```
Solution satisfies tolerances
MIP Solution:     31974300.000000    (735 iterations, 420 nodes)
Final Solve:      31974300.000000    (0 iterations)

Best possible:    31971416.666667
Absolute gap:         2883.333333
Relative gap:            0.000090
```

G. Output For Hybrid Optimization Strategy

```
Solution satisfies tolerances
MIP Solution:     34731367.187500    (108 iterations, 0 nodes)
Final Solve:      34731367.187500    (0 iterations)

Best possible:    34730954.687500
Absolute gap:          412.500000
Relative gap:            0.000012
```

NOTE: All files will be uploaded with the report.