MATH 111 - Calculus and Analytic Geometry I

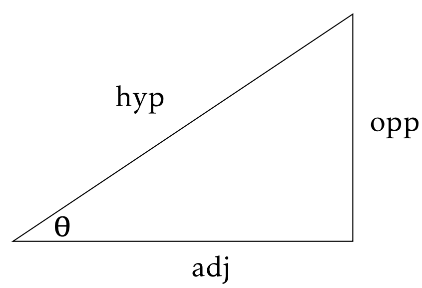
Lecture 2 Worksheet

Fall 2020 **Subhadip Chowdhury** **Aug 21**

Title: Chapter 1 - Review of Functions II

Summary: In beginning our investigations of the Calculus, we will immediately utilize transcendental functions to explore phenomenons outside of those offered by polynomial and rational functions. In particular, we will frequently use the trigonometric functions throughout Calculus. So let's review them!

§A. Trigonometry Review

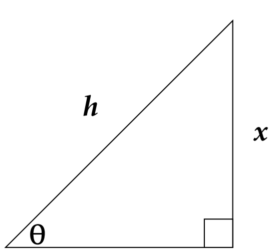


You likely learned about trig functions via right triangles. Given a right triangle and an angle inside (except for the **90** degree one), we can specify a trig value of that angle. These trig values are formed via ratios of sides of the triangle and each different ratio has a different name. Can you recall these ratios? Do you remember the names of these functions? Ask your team members for assistance if you have forgotten!

1. $\sin(\theta) = \dfrac{opp}{hyp}$
2. $\cos(\theta) =$
3. $\tan(\theta) = \dfrac{opp}{adj}$
4. $\csc(\theta) =$
5. $\sec(\theta) = \dfrac{hyp}{adj}$
6. $\cot(\theta) =$

Question 1.

Use the right triangle below to write h as a function of x and θ.



The Trig function values can also be defined as points on a circle as follows:

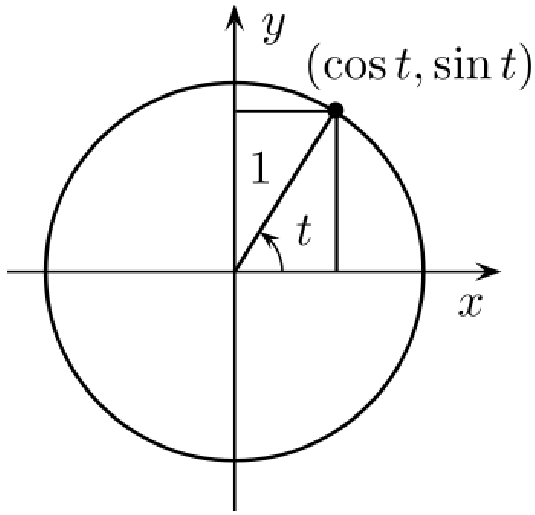


Figure 1: Sine and Cosine via a circle of radius **1**

Here's a picture I found on the internet that might help you recall the trig values of some common angles.

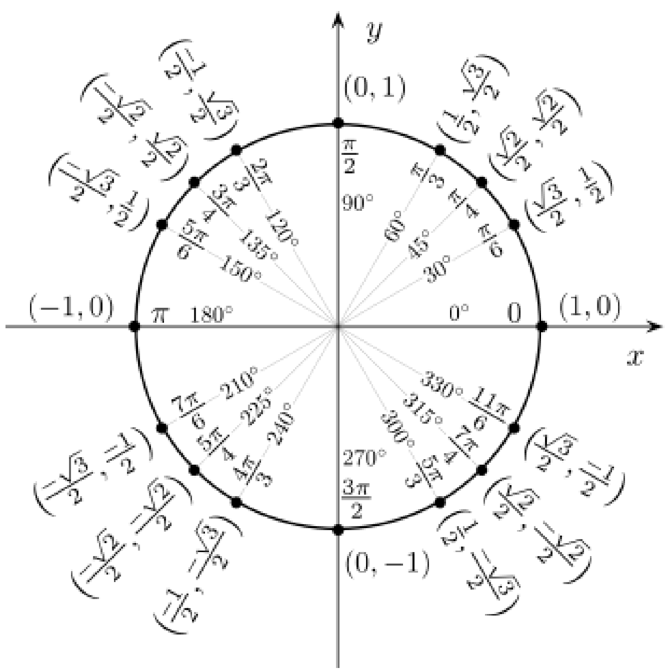


Figure 2: Common Trig Values

Using the figure 2, we can compute trig values for angles that lie in any quadrant. You just have to keep track of the sign of your x and y coordinates.

For example, $\cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$ since the angle of $\frac{3\pi}{4}$ corresponds to the point $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ on the unit circle.

■ Question 2.

Try to use the values in figure 2 above, and your knowledge of the unit circle, to compute the trig values below.

1. $\sin\left(\frac{-5\pi}{6}\right) =$
2. $\cos\left(-\frac{3\pi}{4}\right) =$
3. $\tan\left(\frac{14\pi}{3}\right) =$
4. $\sec\left(\frac{7\pi}{6}\right) =$

§B. Inverse Function

Definition B.1

We call a function f **one-to-one** if $f(a) \neq f(b)$ whenever $a \neq b$. That is, the function must send each input to a **different** output, i.e. no two outputs of the function can be the same.

Geometrically, when looking at the graph, a one-to-one function should pass a **horizontal line test** (does that sound familiar?).

■ Question 3.

Sketch a graph of a function that is **not** one-to-one.

We can determine if a function is one-to-one by solving for x in the equation $f(x) = c$ for all c in the range of f. If there is always just one unique solution, then f is one-to-one. Try this for the following functions:

■ Question 4.

(a) $f(x) = x^2$. Try to solve for x in $x^2 = c$, where c is any positive constant (or alternatively, draw a picture!). Is f a one-to-one function?

(b) $g(x) = \frac{1}{x+2}$. Try to solve $g(x) = c$ for x, where c is any constant such that $c \neq 0$.

Is g a one-to-one function?

Definition B.2

Functions that are one-to-one are precisely those that have **inverses**. For a function f, we say that f has an **inverse function**, denoted f^{-1}, if,

$$f(f^{-1}(y)) = y \text{ for all } y \text{ in the domain of } f^{-1}$$

and

$$f^{-1}(f(x)) = x \text{ for all } x \text{ in the domain of } f.$$

■ Question 5.

Verify that for $f(x) = \frac{1}{x+2}$, the inverse function is $f^{-1}(x) = \frac{1-2x}{x}$. This means you will need to check the above definition and compose these two functions together (twice!).

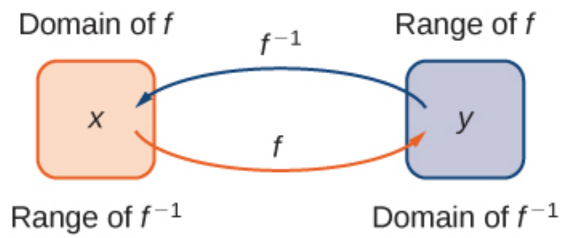


Figure 3: Domain and Range for a one-to-one function f and its inverse f^{-1}

If you have a graphing utility, sketch both f and f^{-1}. Do you notice any symmetry between the two graphs?

Question 6.

Why doesn't the function $f(x) = x^2$ have an inverse? Is there a way to **restrict the domain** of f so that the square root function, $y = \sqrt{x}$, will be its inverse?