MATH 111 - Calculus and Analytic Geometry I

Lab 5 Worksheet

Fall 2020 | **Subhadip Chowdhury** | **Sep 29**

Title: Practice Problems for Power, Product, Quotient and Chain Rule

Question 1.

Differentiate the given functions.

(a) $y = \tan(x + x^{-1})$

(b) $f(x) = \left(\frac{x-1}{x+1}\right)^3$

(c) $g(y) = (y^3 - y^2 - y - 1 - y^{-1} - y^{-2})^3$

(d) $y = \sin(x)\sec^3(x)$

(e) $y = \sin(\cos(\sin(x)))$

(f) $w = \sqrt{2 + \sqrt{4 + z^2}}$

(g) $y = \theta^2 \sin\left(\frac{4}{\theta}\right)$

Question 2.

Calculate the specified derivative.

(a) $\frac{d^2}{d\theta^2}[\sin(\theta)\cos(\theta)]$

(b) $y^{(4)}$ for $y = 11(1-x)^{-1}$.

(c) $\frac{d^{100}}{dx^{100}}[\sqrt{2}x^{100} - 88x^{99} + 87x^{64} + 17x^{36}]$

(d) $y^{(32)}$ for $y = \sin(x)$?

Question 3.

Find the equation of the tangent line at the given point.

(a) $y = \frac{\sin x - \cos x}{x}, \quad x = \frac{\pi}{6}$

(b) $y = \csc x - \cot x, \quad x = \frac{\pi}{4}$

(c) Is it possible for the graph of $y = \tan(x)$ to have a horizontal tangent? Explain.

Question 4.

Suppose f and g are functions with $g(3) = 2$, $f'(3) = -1$, and $g'(3) = 0$. What is the derivative of $h(x) = \dfrac{f(x)}{g(x)}$ at $x = 3$?

Question 5.

Let f be a function with $f(5) = 2$ and $f'(5) = -1$. Let $g(x) = x^2 f(x)$. Find $g'(5)$.

Question 6.

Suppose $h(x) = f(x)g(x)$ and $g'(x) = xg(x)$. If $g(2) = 1$, $f'(2) = 3$, and $f(2) = 4$, then find $h'(2)$.

Question 7.

Suppose f, g, and h are nonzero differentiable functions with $h(x) = f(x)g(x)$ for all real x. Suppose also that

$$h'(1) = 12h(1), \quad f'(1) = 4f(1), \quad g'(1) = \lambda g(1)$$

Then find the value of λ.

Question 8.

Consider a function $f(x)$ defined as follows:

$$f(x) = \begin{cases} b + ax - x^2 & \text{for } x < 2 \\ ax^2 + bx + 2 & \text{for } x \geq 2 \end{cases}$$

If $f(x)$ is both continuous and differentiable at $x = 2$, then find a and b.

Question 9.

If $f(x) = x^2 + x$ and $g(x) = x^3 + \lambda$, for what value of λ do we have $f(\lambda) = g(\lambda)$ and $f'(\lambda) = g'(\lambda)$?

Question 10.

Let $P(x) = ax^3 + bx^2 + cx + d$. If $P(0) = P(1) = -2$, $P'(0) = -1$, and $P''(0) = 10$, what is $P'''(0)$?

Question 11.

Suppose $f(x) = \sin x$ and $g(x) = ax^2 + bx + c$. If $f(0) = g(0), f'(0) = g'(0),$ and $f''(0) = g''(0)$, then find $a, b,$ and c.

Question 12.

Let $f(x)$ be a continuous and differentiable function defined as

$$f(x) = a\sin^2 x + b\cos x$$

where a and b are real numbers. If $f(\pi/2) = 2$ and $f'(\pi/2) = 3$, what are the values of a and b?

Question 13.

Let $f(x)$ and $g(x)$ be continuous and differentiable functions such that

$$f(x) = \sin(g(x)) + \cos x$$

If $g(0) = \pi$ and $g'(0) = \dfrac{\pi}{4}$, find the value of $f'(0)$.

Question 14.

Let a and b be real numbers such that $f(x) = ax\sin x + b\cos x$ and $f'(x) = x\cos x$ for every real number x. What are the values of a and b?
