MATH 111 - Calculus and Analytic Geometry I

Lab 3 Worksheet

Fall 2020 **Subhadip Chowdhury** **Sep 8**

§A. How Do We Measure Velocity?

Calculus can be viewed broadly as the study of change. A natural and important question to ask about any changing quantity is "how fast is the quantity changing?"

We begin with a simple problem: a grapefruit is tossed straight upward into the air at $t = 0$ seconds. It leaves the thrower's hand at high speed, slows down until it reaches its maximum height, and then speeds up in the downward direction and finally, "Splat!" (figure 1) How fast is the grapefruit moving? Questions like this one are central to our study of differential calculus.

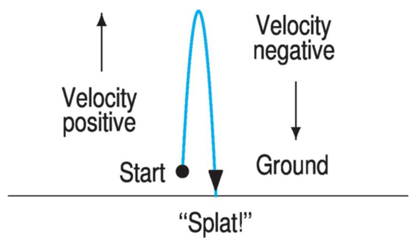

Figure 1: The grapefruit's path is straight up and down

Velocity versus Speed: Suppose an object moves along a straight line. We can designate one direction as positive and the other as negative. Speed is what you see in the speedometer of a car, it's the magnitude of the velocity and so is always positive or zero. However, if the car is moving backwards (reverse), then its velocity is negative.

§B. Position and average velocity

Definition B.1

If $s(t)$ is the position of an object at time t, then the average velocity of the object over the interval $a \leq t \leq b$ is

$$\text{Average velocity } = \frac{\text{Change in position}}{\text{Change in time}} = \frac{s(b) - s(a)}{b - a}$$

In words, the average velocity of an object over an interval is the net change in position during the interval divided by the change in time.

Suppose the following table gives the height of the grapefruit above the ground over time.

t(sec)	0	1	2	3	4	5	6
$y = s(t)$(feet)	6	90	142	162	150	106	30

■ **Question 1.**

Find the average velocity of the grapefruit over the time interval $1 \le t \le 2$.

§C. **Instantaneous Velocity**

Can you use above table to find the velocity of the grapefruit at exactly $t = 1$ second? Whether we are driving a car, riding a bike, or throwing a grapefruit, we have an intuitive sense that a moving object has a velocity at any given moment -- a number that measures how fast the object is moving right now.

How does the speedometer of a car tell the car's velocity at any given instant? The velocity on a speedometer is in reality just an average velocity that is computed over a very small time interval. If we let the time interval over which average velocity is computed become shorter and shorter, we can progress from average velocity to instantaneous velocity.

Definition C.1

Let $s(t)$ be the position at time t. Then the instantaneous velocity at $t = a$ is defined as

$$\text{Instantaneous velocity at time } a = \lim_{b \to a} \frac{s(b) - s(a)}{b - a}$$

§D. **Visualizing Velocity**

It is easy to visualize the average velocity over any time interval $a \le t \le b$ to be the slope of the line joining the points on the graph of $s(t)$ corresponding to $t = a$ and $t = b$. Since slope is rise/run, we get the definition of average velocity from above.

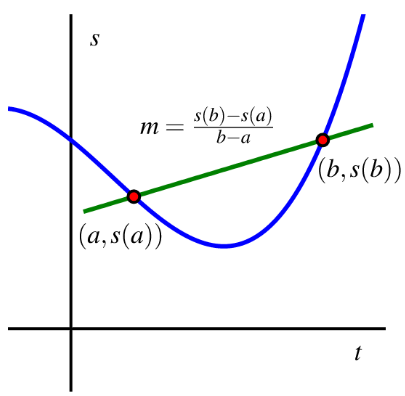

Figure 2: The graph of position function $s(t)$ together with the line through $(a, s(a))$ and $(b, s(b))$

As the point b moves closer to a, the secant joining a and b approaches towards the tangent to the graph at a. Use this DESMOS page to convince yourself of this fact. In the DESMOS page, b is written as $a + h$ so that $b \to a$ is equivalent to $h \to 0$.

We say that the slope of the tangent to the curve at $t = a$ is the slope of the curve at $t = a$. Thus the instantaneous velocity is the slope of the curve at a particular moment of time.