MATH 111 - Calculus and Analytic Geometry I

Lecture 6 Worksheet

Fall 2020 | **Subhadip Chowdhury** | **Aug 31**

Title: Limits and Continuity

Summary: We are going to postpone evaluating limits using algebra and instead introduce the formal definition of Continuity. That will help us evaluate limits for a large class of functions.

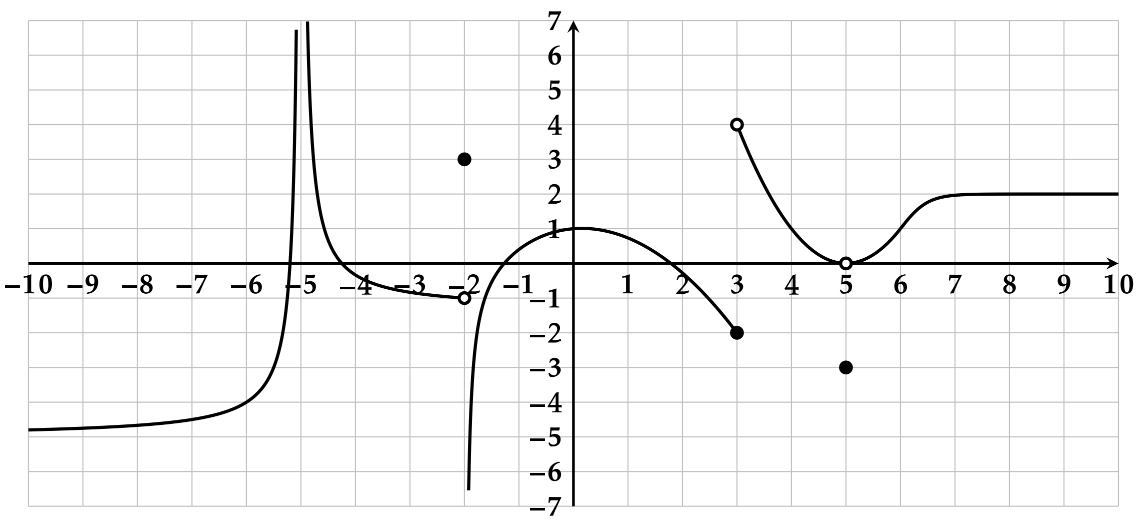
§A. Limits From a Graph

Let's review what we learned last week.

Question 1.

The graph of a function $f(x)$ is given below. Try to answer the given questions using the "**answer bank**" provided. Each **answer** goes with exactly one statement below.

Graph of $f(x)$



1. $\lim_{x\to-\infty} f(x) =$
2. $\lim_{x\to\infty} f(x) =$
3. $\lim_{x\to-5^-} f(x) =$
4. $\lim_{x\to-5^+} f(x) =$
5. $\lim_{x\to-5} f(x) =$
6. $f(-5) =$
7. $\lim_{x\to-2^-} f(x) =$
8. $\lim_{x\to-2^+} f(x) =$
9. $\lim_{x\to-2} f(x) =$
10. $f(-2) =$
11. $\lim_{x\to0^-} f(x) =$
12. $\lim_{x\to0^+} f(x) =$
13. $\lim_{x\to0} f(x) =$
14. $f(0) =$
15. $\lim_{x\to3^-} f(x) =$
16. $\lim_{x\to3^+} f(x) =$
17. $\lim_{x\to3} f(x) =$
18. $f(3) =$
19. $\lim_{x\to5^-} f(x) =$
20. $\lim_{x\to5^+} f(x) =$
21. $\lim_{x\to5} f(x) =$
22. $f(5) =$

Answer Bank for Problem 3

(a) 1
(b) -2
(c) 4
(d) DNE
(e) -2
(f) undefined
(g) ∞
(h) -5
(i) 2
(j) ∞
(k) ∞
(l) -1
(m) DNE
(n) $-\infty$
(o) 0
(p) 0
(q) -3
(r) 0
(s) 3
(t) 1
(u) 1
(v) 1

§B. Continuity

Definition B.1: Continuity at a Point

The function $f(x)$ is said to be **continuous** at a point $x = c$ if

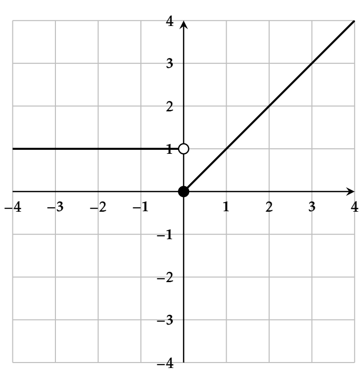
- f is defined at $x = c$, and
- if $\lim_{x\to c} f(x)$ exists, and
- if $\lim_{x\to c} f(x) = f(c)$

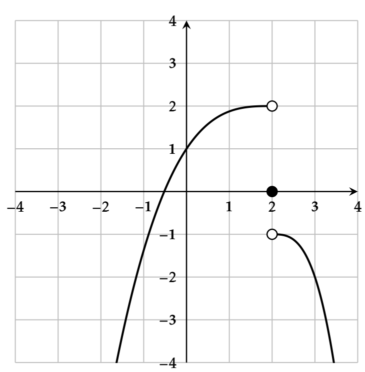
In other words, $f(x)$ can be made to remain as close as we want to $f(c)$ provided x is chosen close enough to c.

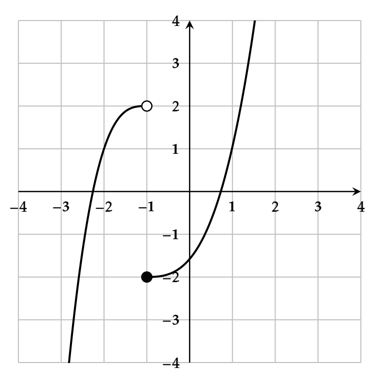
An important difference between limits and continuity: a limit is only concerned with what happens **near** a point, but continuity depends on what happens **near** a point and **at** that point.

Question 2.

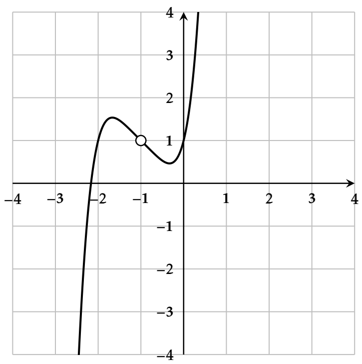
In the following pictures, if you believe the function is discontinuous at a point, discuss **why** you think it's discontinuous at that point. Which of the three parts in the definition does it fail to satisfy (if discontinuous)?

1.


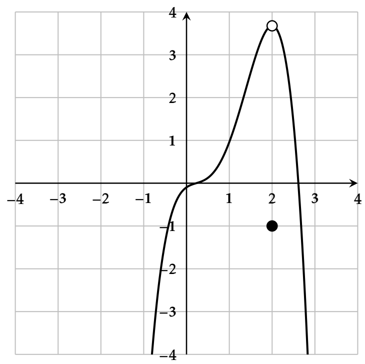
2.


3.


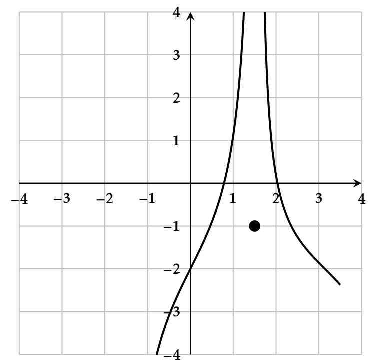
4.



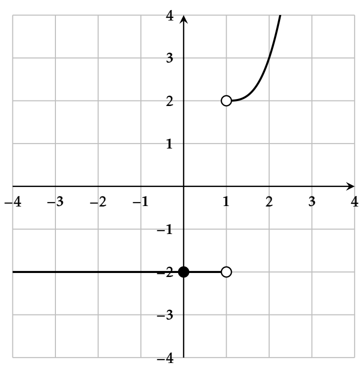
5.



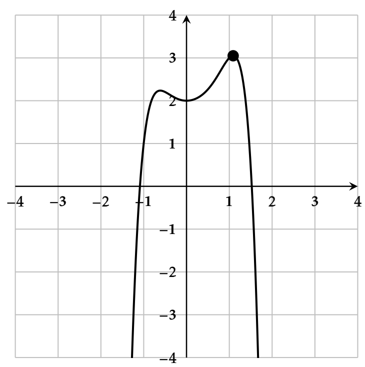
6.



7.



8.



Definition B.2: Continuity on an Interval

A function f is said to be continuous on an open interval (a,b) if it is continuous at every point in the interval.

A function f is said to be continuous from the right at c if $\lim_{x\to c^+} f(x) = f(c)$.

A function f is said to be continuous from the left at c if $\lim_{x\to c^-} f(x) = f(c)$.

A function f is said to be continuous on a closed interval $[a,b]$ if it is continuous on (a,b), and continuous from the right at a and continuous from the left at b.

Question 3.

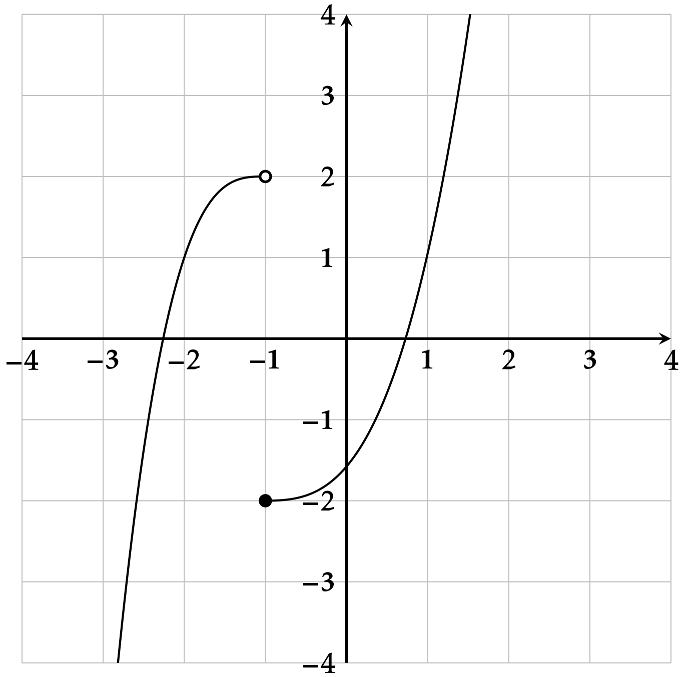
(a) Explain why $\lim_{x\to 1} \sqrt{x-1}$ does not exist.

(b) On what interval is $f(x) = \sqrt{x-1}$ continuous?

(c) On what intervals is $g(x) = \dfrac{x^2 - 10x}{x^2 - 16x + 60}$ continuous?

(d) On what intervals is $f(x) = \dfrac{\sqrt{x-4} - 1}{(x-5)(x-6)}$ continuous?

Graph of $f(x)$



Graph of $g(x)$