MATH 111 - Calculus and Analytic Geometry I

Lecture 7 Worksheet

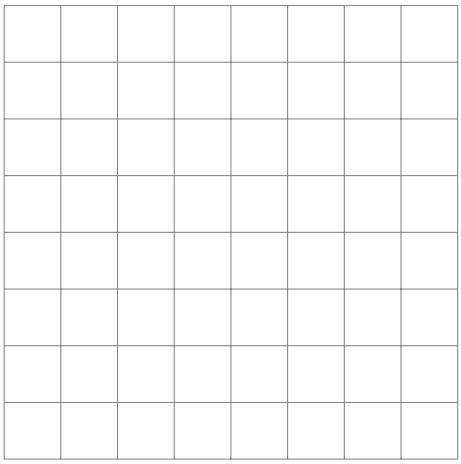
Fall 2020 **Subhadip Chowdhury** **Sep 2**

Title: IVT and other continuity theorems

Summary: We will discuss some of the most important consequences of continuity for a function.

§A. The Intermediate Value Theorem

1. What was your height at birth? _____
   The average baby born measures about 20 inches at birth.

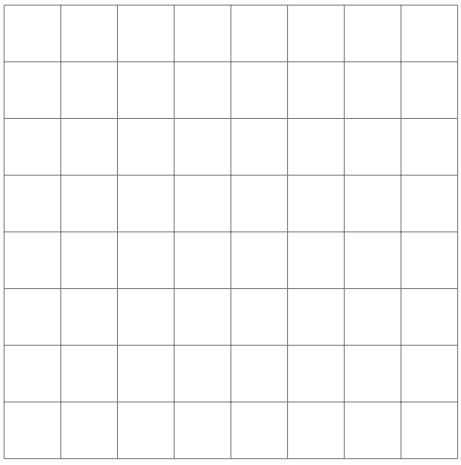


2. What is your height today? _____

3. Sketch a graph of how your height has changed over the course of your lifetime.

4. Was there a day in your life that you measured exactly 40 inches?

5. A continuous function $y = f(x)$ is known to be negative at $x = 0$ and positive at $x = 1$. Explain why the equation $f(x) = 0$ must have at least one solution between $x = 0$ and $x = 1$? Illustrate with a sketch.



6. In your own words, what do you think the Intermediate Value Theorem (IVT) means?

Theorem A.1

Suppose f is continuous on a closed interval $[a, b]$. If k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$.

7. Why do we need continuity for the Intermediate Value Theorem?

8. Prove that the function $f(x) = x^{12345} + 2x^{6789} - 1$ has at least one zero between 0 and 2 (i.e. $f(x) = 0$).

9. **True or False:** At some point since you were born your weight in pounds equaled your height in inches.

10. Can you prove (without graphing!) that the equation $\cos(\theta) = \theta^3$ has at least one real solution?

11. Suppose $g(x)$ is a continuous function with $g(0) = 3$, $g(1) = 8$, $g(2) = 4$.
True or False: $g(x)$ is an invertible function.

12. **True or False:** Along the Equator, there are two diametrically opposite sites that have exactly the same temperature at the same time.

§B. Continuity of Combinations of Functions

Theorem B.1: Continuity of Sums, Products, and Quotients of Functions

Suppose that f and g are continuous on an interval and that k is a constant. Then, on that same interval,

1. $kf(x)$ is continuous.
2. $f(x) + g(x)$ is continuous.
3. $f(x)g(x)$ is continuous.
4. $f(x)/g(x)$ is continuous, provided $g(x) \neq 0$ on the interval.

Theorem B.2: Continuity of Composite and Inverse Functions

If f and g are continuous, then

1. if the composite function $f(g(x))$ is defined on an interval, then $f(g(x))$ is continuous on that interval.
2. if f has an inverse function f^{-1}, then f^{-1} is continuous.