MATH 111 - Calculus and Analytic Geometry I

Lecture 4 Worksheet

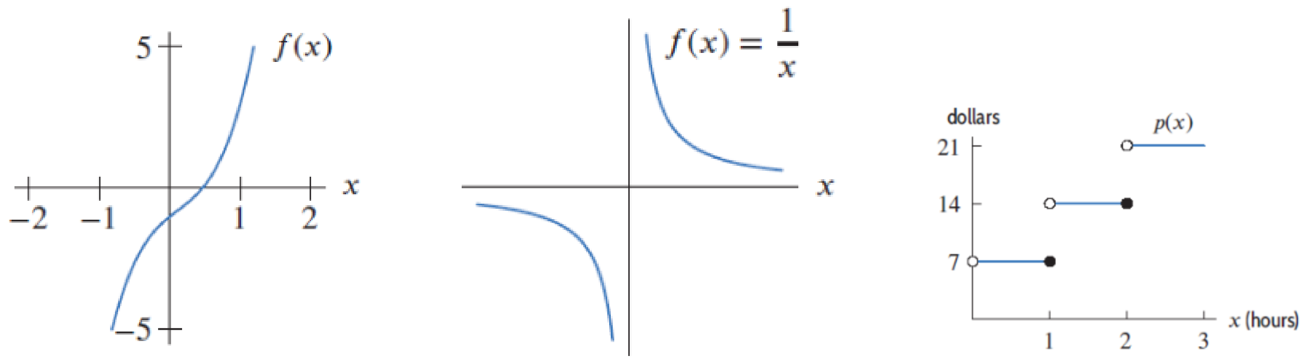
Fall 2020 | **Subhadip Chowdhury** | **Aug 26**

Title: An Informal Introduction to Limits and Continuity

Summary: Before jumping into the formal definition and calculation of Limits, we will try to motivate and showcase the necessity of the idea of limits through graphical and numerical viewpoints.

Continuity of a Function on an Interval: A Graphical Viewpoint

Consider graphs of three functions as follows:



(a) The graph of $f(x) = 3x^3 - x^2 + 2x - 1$

(b) The graph of $f(x) = 1/x$

(c) Cost of renting a car from a company

Figure 1

A function is called **continuous** if you can draw its graph without lifting the pencil from the paper. In the above three pictures, the first function is continuous everywhere; the second one is continuous on all intervals not containing 0, and the third function is continuous on intervals of the form $(n, n+1)$.

Continuity of a Function at a Point: A Numerical Viewpoint

A function $y = f(x)$ is called **continuous at a point** if nearby values of x give nearby values of y. In practical terms, this means small errors in the input leads to only small errors in output. For the same reason the idea of continuity is important in real life.

A bit more formally, we can say that if $f(x)$ is continuous at $x = c$, the values of $f(x)$ approach $f(c)$ as x approaches c. So to investigate whether or not a function f is continuous at c, we need to know what value is approached by $f(x)$ as x approaches c.

The Idea of a Limit

The notation

$$\lim_{x\to c} f(x) = \mathbf{L}$$

means the values of $f(x)$ approach $\mathbf{L}$ as x approaches c. We read it as:

"the limit of $f(x)$ as x goes to (or tends to) c is $\mathbf{L}$".

Clearly if $\mathbf{L}$ is not the same as $f(c)$ then f is not continuous at c. Here's an example of how that might happen.

Example .1

Consider the function $f(x) = \frac{x^2-4}{x-2}$. What is $f(2)$? Draw the graph of $f(x)$ vs. x. Can you guess $\lim_{x\to 2} f(x)$ from the picture?

The most important thing to note from this example is that even when $f(c)$ does not exist, the limit $\lim_{x\to c} f(x)$ might. Here's a second example where $f(c)$ exists but $\lim_{x\to c} f(x)$ doesn't.

Example .2

Consider the function from the third picture 1c above. Check that $f(1) = 7$. But $\lim_{x\to 1} f(x)$ doesn't exist. Can you explain why not?

Question 1.

Come up with an example or a scenario where neither $f(c)$ nor $\lim_{x\to c} f(x)$ exists.

Question 2.

Come up with an example or a scenario where both $f(c)$ and $\lim_{x\to c} f(x)$ exist but they are not equal to each other.