MATH 111 - Calculus and Analytic Geometry I

Lab 6 Worksheet

Fall 2020 **Subhadip Chowdhury** **Oct 6**

§A. Logarithmic Differentiation

You now know how to differentiate polynomials and exponential functions. But what about a function like the following:

$$f(x) = x^x.$$

This is **not** an exponential function, nor is it a polynomial (do you see why?). If we try to use the definition of the derivative, we get this horrible thing:

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^{x+h} - x^x}{h}$$

and, yeah - how do we compute that limit?

The method of **logarithmic differentiation** will allow us to find the derivative of x^x. This utilizes the rules of logarithms, the derivatives we have recently discovered, and implicit differentiation.

Example A.1

Let $y = f(x) = x^x$. Then

$$\ln(y) = \ln(x^x) = x \ln(x).$$

Now we differentiate:

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln(x) = 1 + \ln(x).$$

Solving for $\dfrac{dy}{dx}$,

$$\frac{dy}{dx} = y(1 + \ln(x)) = x^x(1 + \ln(x)).$$

Example A.2

Let $y = f(x) = x^{\sin(x)}$. Now take the natural log of both sides like so:

$$\ln(y) = \ln\left(x^{\sin(x)}\right) = \sin(x) \cdot \ln(x).$$

Apply differentiation (implicitly):

$$\frac{1}{y}\frac{dy}{dx} = \cos(x)\ln(x) + \frac{\sin(x)}{x}.$$

Solving for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right) = x^{\sin(x)}\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right).$$

■ **Question 1.** □

In your own words, summarize the steps involved in a logarithmic differentiation problem.

§B. Logarithmic Differentiation Practice

Now try your hand at these practice problems. Each is a function of the form $f(x)^{g(x)}$. Check your work with your team and help/ask for help if necessary!

■ **Question 2.** □

Compute $\frac{dy}{dx}$ for each function.

(a) $y = x^{x^2}$

(b) $y = x^{\tan(x)}$

(c) $y = x^{e^x}$

(d) $y = (\sin x)^x$

(e) $y = (\ln x)^{(e^x)}$

(f) $y = 2^{x^x}$