MATH 111 - Calculus and Analytic Geometry I

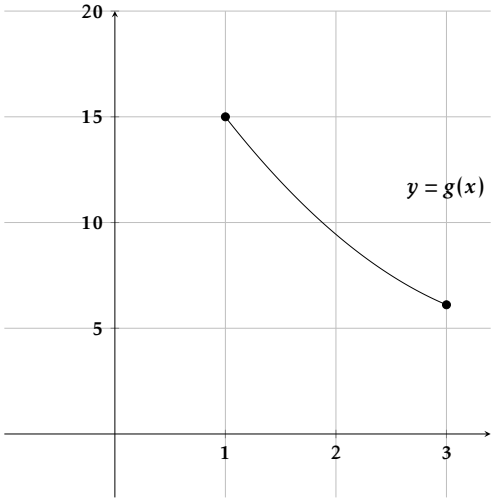
Lab 7 Worksheet

Fall 2020 | **Subhadip Chowdhury** | **Oct 13**

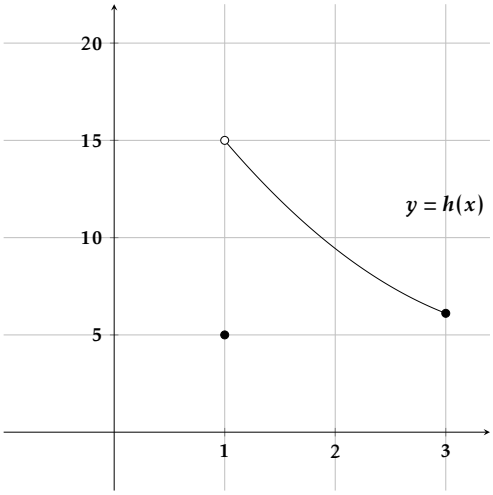
§A. Motivation

Draw the secant line between the endpoints for the given interval $[a, b]$. Can you identify a point c, with $a < c < b$, such that the slope of the tangent line to the graph at $x = c$ is equal to the slope of the secant line between a and b?

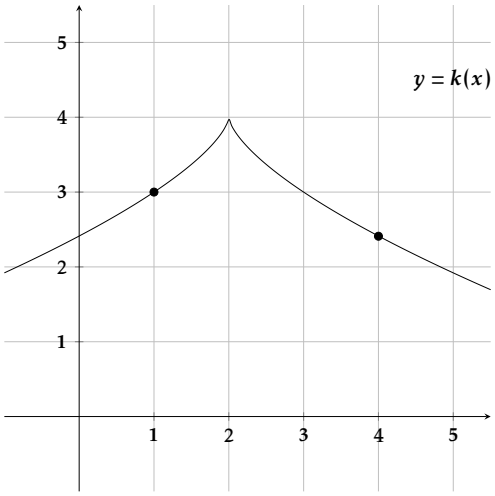
$g(x)$ on the interval $[1, 3]$



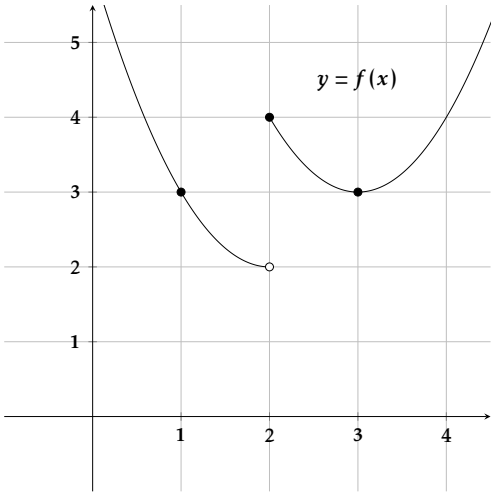
$h(x)$ on the interval $[1, 3]$



$k(x)$ on the interval $[1, 4]$



$f(x)$ on the interval $[1, 3]$

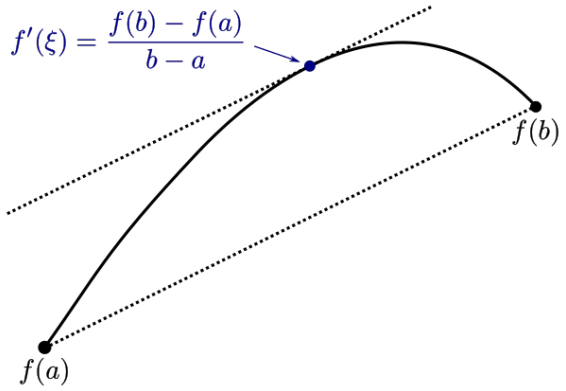


§B. The Mean Value Theorem

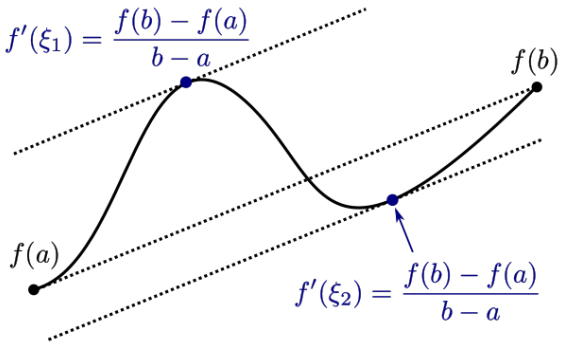
Theorem B.1

If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists at least one number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



(a) The function f attains the slope of the secant between a and b as the derivative at the point $\xi \in (a,b)$.

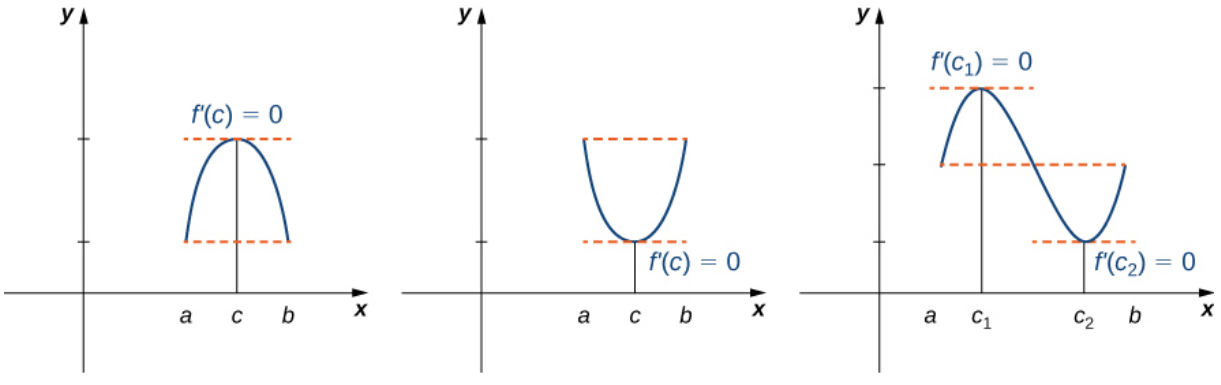


(b) It is also possible that there are multiple tangents parallel to the secant.

A special case of the Mean Value Theorem is called Rolle's Theorem.

Theorem B.2: Rolle's Theorem

Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a) = f(b)$, then there is at least one number c in (a,b) such that $f'(c) = 0$.



Question 1.

As always, the MVT is a "If-Then" statement. There are some hypotheses and there is a conclusion. Identify which part is which.

§C. Applications

Question 2.

An elevator starts at ground level at time $t = 0$ seconds. At $t = 20$ seconds, the elevator has risen 100 feet. What does the Mean Value Theorem tell you about this situation? (Be specific to this case.)

Question 3.

Let $g(x) = |x^2 - 1|$. Graph this function using Desmos and answer the questions below.

(a) Do the hypotheses of the MVT hold on $[0, 3]$? Does the conclusion hold? Explain.

(b) Do the hypotheses of the MVT hold on $[1, 3]$? Does the conclusion hold? Explain.

(c) Do the hypotheses of the MVT hold on $[-1, 3]$? Does the conclusion hold? Explain.

Question 4.

Does the MVT apply to $g(x) = x^{1/3}$ on $[0, 8]$? Why or why not? If so, find all values of c that satisfy the theorem.

Question 5.

Explain why $h(x) = x^3 + 6x + 2$ satisfies the hypotheses of the MVT on the interval $[-1, 3]$. Then find all values of c in $[-1, 3]$ guaranteed by the theorem.

Question 6.

Show a Write-up

Considering the following situation. You are driving a car on a highway, traveling at the speed limit of 55 mph. At 10:17am, you pass a police car on the side of the road, presumably checking for speeders. At 10:53am, 39 miles from the first police car, you pass another police car. You are of course obeying the speed limit and traveling exactly 55 mph. However, you are shocked when the police turn on their lights and pull you over. The officer claims you were speeding at some point in the last 39 miles. Is the officer telling the truth, or needlessly pulling you over?

Question 7.

Show a Write-up

Let $f(x) = \dfrac{1}{x^2}$. Show analytically why there cannot exist a number c in $(-1, 1)$ such that

$$f(1) - f(-1) = 2f'(c).$$

Does this contradict the MVT? Explain.

Question 8.

□

Use Rolle's theorem to show that the graph of $f(x) = x \cos x$ has a horizontal tangent in the interval $\left[0, \dfrac{\pi}{2}\right]$.

Question 9.

□

Show that $|\sin x - \sin y| \leq |x - y|$ for all x and y.

■ Question 10. □

Suppose f is a polynomial. Which of the following statements are correct? There may be more than one correct answer.

(I) Between any two consecutive roots of f, there must be at least one root of f'.

(II) Between any two consecutive roots of f', there must be at least one root of f.

(III) Between any two consecutive roots of f, there can be at most one root of f'.

(IV) Between any two consecutive roots of f', there can be at most one root of f.

We end with a wild MVT spotted on a bridge in Beijing!