Calculus & Analytical Geometry II

Lecture 1 Worksheet

Spring 2021 **Subhadip Chowdhury** **Math 112**

Our first task is to review some of the concepts from Calculus I. Share and discuss the following questions amongst your group. At the end of discussion, any one of you might be asked to give your groups answer(s), so everyone should be recording and participating.

§A. Recap on Exponential, Logarithm, and Inverse Functions

We will eventually discuss in integration techniques for exponential, logarithmic, and trigonometric functions in greater details. Let's revisit some of the basic derivative rules first.

■ Question 1. □

Write down the formula for the following derivatives.

(i) $\frac{\mathrm{d}}{\mathrm{d}x}e^x$

(ii) $\frac{\mathrm{d}}{\mathrm{d}x}b^x$

(iii) $\frac{\mathrm{d}}{\mathrm{d}x}\ln x$

(iv) $\frac{\mathrm{d}}{\mathrm{d}x}\sin x$

(v) $\frac{\mathrm{d}}{\mathrm{d}x}\cos x$

(vi) $\frac{\mathrm{d}}{\mathrm{d}x}\tan x$

■ Question 2. □

You will also need to remember helpful derivative rules such as product rule and chain rule.

(i) $\frac{\mathrm{d}}{\mathrm{d}x}[e^x \cdot \ln x]$

(ii) $\frac{\mathrm{d}}{\mathrm{d}x}[7^{7x}]$

(iii) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\ln(\ln x)]$

(iv) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\ln(\tan x)]$

(v) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\ln(2x)\cdot e^{-x/2}]$

(vi) $\dfrac{\mathrm{d}}{\mathrm{d}x}[e^{\sin x}]$

§B. Derivative of Inverse Functions

Given a one-to-one function $f(x)$, its inverse f^{-1} is defined as follows.

$$f(x) = y \implies f^{-1}(y) = x$$

So for example, if $f(x) = 2x - 1$, then we can write

$$y = 2x - 1 \implies x = \frac{y+1}{2} \implies f^{-1}(y) = \frac{y+1}{2}$$

This gives us a formula for f^{-1}. So in general, if we change the variable,

$$f^{-1}(\heartsuit) = \frac{\heartsuit + 1}{2}$$

Important thing to keep in mind: $f^{-1}(x)$ is NOT the same thing as $\dfrac{1}{f(x)}$. For example, in the above case, $f^{-1}(\star) = \dfrac{\star+1}{2}$, but $\dfrac{1}{f(\star)} = \dfrac{1}{2\star - 1}$.

The formula for the derivative of inverse function is given by

$$\frac{\mathrm{d}}{\mathrm{d}x} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

We can use this formula to find derivatives of inverse trigonometric functions.

Question 3.

(i) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\arcsin x]$

(ii) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\arctan x]$

§C. The Fundamental theorem of Calculus

Theorem C.1: The Fundamental theorem of Calculus Part I

Let f be continuous on $[a,b]$. Then for any antiderivative F of f on $[a,b]$,

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

Theorem C.2: Corollary of FTCI - Net Change Theorem

Let $F(t)$ be some quantity with a continuous rate of change $F'(t)$. Then

$$\int_a^b F'(t)\,\mathrm{d}t = F(b) - F(a)$$

In other words, the integral of a rate of change gives total change.

Example C.3

For example, suppose you are given that a particle moves in a straight line with velocity $v(t) = t^2$. We would like to find the total displacement (net change) of the particle from time $t = 0$ to time $t = 3$. Since velocity $v(t)$ is the derivative of displacement $s(t)$, we can use the net change formula to write

$$s(3) - s(0) = \int_0^3 s'(t)\,\mathrm{d}t = \int_0^3 v(t)\,\mathrm{d}t$$

Now the general antiderivative of t^2 is $\frac{t^3}{3} + C$, where C is any constant. So,

$$\begin{aligned}\int_0^3 v(t)\,\mathrm{d}t &= \int_0^3 t^2\,\mathrm{d}t \\ &= \left.\frac{t^3}{3}\right|_0^3 \\ &= 9\end{aligned}$$

Note the shorthand notation:

$$F(x)\Big|_a^b = F(b) - F(a).$$

We will often write this as part of our work when evaluating definite integrals, as it allows us to write down the antiderivative first before trying to plug values into it (and likely confusing ourselves in the process).

Theorem C.4: The Fundamental theorem of Calculus Part II

Let f be a continuous function on $[a,b]$ and define an area function $F(x)$ by:

$$F(x) = \int_a^x f(t)\,dt.$$

Then $F(x)$ is continuous on $[a,b]$, differentiable on (a,b), and $F'(x) = f(x)$. In other words,

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

Theorem C.5: Corollary of FTCII

If $F(x)$ is defined as

$$F(x) = \int_{u(x)}^{v(x)} f(t)\,dt.$$

then

$$\frac{d}{dx}F(x) = \frac{d}{dx}\int_{u(x)}^{v(x)} f(t)\,dt = f(v(x))v'(x) - f(u(x))u'(x)$$

Question 4.

□

Suppose $f(t) = t^2 - e^t$. We define a function $F(x)$ as

$$F(x) = \int_{x^2}^{e^x} f(t)\,dt$$

Find $F'(x)$.

Question 5.

□

Complete the Calc I Review assignment on Edfinity to unlock DC1 Practice Problems.

§D. A Bunch of Integrals

Here are a bunch of integrals from Calculus I. **We will be reviewing/discussing all of these in the next week.** Technically, these topics are review from Calc I, but since they came at the very end of your semester, we will review/refresh them. That being said, you might have a hard time with some of these, or have no idea what to do. That's okay! Talking about some of these will help launch us into the next part of our discussion.

(i) $\displaystyle\int_0^1 \frac{x^3}{(1+x^4)^{10}}\,\mathrm{d}x$

(ii) $\displaystyle\int \sec^2 2x\,\mathrm{d}x$

(iii) $\displaystyle\int (x+\pi)^{20}\,\mathrm{d}x$

(iv) $\displaystyle\int_0^1 x(x+1)^8\,\mathrm{d}x$

(v) $\displaystyle\int 10x\sqrt{5x^2-4}\,\mathrm{d}x$

(vi) $\displaystyle\int \sin x(\sec x)^3\,\mathrm{d}x$

(vii) $\displaystyle\int \frac{2+x}{x}\,\mathrm{d}x$

(viii) $\displaystyle\int \frac{\cos(\ln x)}{x}\,\mathrm{d}x$

(ix) $\displaystyle\int \frac{(\ln x)^{22}}{x}\,\mathrm{d}x$

(x) $\displaystyle\int \sec x\,\mathrm{d}x$

(xi) $\displaystyle\int \tan x\,\mathrm{d}x$

(xii) $\displaystyle\int_0^{999} 7\,\mathrm{d}x$

(xiii) $\displaystyle\int e^{\sin 2x}(\cos 2x)\,\mathrm{d}x$

(xiv) $\displaystyle\int \frac{12}{1+9x^2}\,\mathrm{d}x$

(xv) $\displaystyle\int_1^4 \left(x+\frac{1}{x}-\frac{5}{x^3}+111\right)\mathrm{d}x$

(xvi) $\displaystyle\int \frac{e^x}{4-e^x}\,\mathrm{d}x$