MATH 111 - Calculus and Analytic Geometry I

Lab 9 Worksheet

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§A. Sigma Notation

In calculus, we do a lot of adding. One notation is used predominantly in mathematics to help write out long formulaic sums in a concise way. This notation uses the Greek letter $\sum$ and is called *Sigma Notation*.

Example A.1

$$\sum_{i=1}^{5}(2i) = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30$$

$$\sum_{j=2}^{5} 2^j = 2^2 + 2^3 + 2^4 + 2^5 = 4 + 8 + 16 + 32 = 60$$

More generally,

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \cdots + f(n)$$

Question 1. **Your turn!**

Compute the given sum:

(a) $\sum_{i=3}^{7}(i^2 + 1)$

(b) $\sum_{i=1}^{10} i$

(c) $\sum_{j=6}^{15}(j - 5)$

(d) $\sum_{i=1}^{20} 2$

The following three formulas can be helpful when approximating area for some simple functions; they are also just cool formulas to know! (See page 510 in textbook)

Theorem A.1

The sum of n consecutive integers is given by:

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

The sum of consecutive integers square is given by:

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

The sum of consecutive integers cubed is given by

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{i=1}^{n} i\right)^2$$

Some properties involving sigma notation follow from our basic properties of arithmetic.

Theorem A.2

Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ represent two sequences of terms and let c be a constant. The following properties hold for all positive integers n and for integers m, with $1 \leq m \leq n$.

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{m} a_i + \sum_{i=m+1}^{n} a_i$$

Example A.2

$$\begin{aligned}
\sum_{i=1}^{6}(2i+3) &= 2 \cdot \sum_{i=1}^{6} i + \sum_{i=1}^{6} 3 \\
&= 2 \cdot \frac{6 \cdot 7}{2} + 3 \cdot 6 \\
&= 42 + 18 = 60
\end{aligned}$$

■ Question 2. □

Try to use above formulas, along with simple properties of sums, to compute larger sums with ease:

(a) $\sum_{i=1}^{100} i^2$

(b) $\sum_{k=0}^{20} (k^2 - 5k + 1)$

(c) $\sum_{j=11}^{20} (j^2 - 10j)$

(d) $\displaystyle\sum_{i=0}^{5} \frac{i}{n^2}$

(e) $\displaystyle\sum_{i=1}^{n} \frac{i}{n^2}$

(f) $\displaystyle\frac{1}{n}\sum_{i=1}^{n} \frac{i}{n}$

(g) $\displaystyle\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \frac{i}{n}$