MATH 221 - Differential Equations

Lecture 7 Activities

Fall 2020 | **Subhadip Chowdhury** | **Sep 2**

§A. In-class Practice Problems

■ Question 1.

For each of the following ODE, draw the phase line and identify each equilibrium point as sink, source or node. Then sketch the graph of the solution curves satisfying given initial conditions.

(a) $\dfrac{dy}{dx} = x\cos x, \qquad y(0) = 0, y(3) = 1, y(0) = 2, y(0) = -1.$

(b) $\dfrac{dP}{dt} = P\left(1 - \dfrac{P}{10}\right)^6 (P-3)^3, \qquad P(0) = 2, P(0) = 7, P(0) = 12$

■ Question 2.

Consider the autonomous equation $dy/dx = f(y)$ where $f(y)$ is continuously differentiable, and suppose we know that $f(-1) = f(2) = 0$.

(a) Describe all the possible behaviors of the solution $y(x)$ that satisfies the initial condition $y(0) = 1$.

(b) Suppose also that $f(y) > 0$ for $-1 < y < 2$. Describe all the possible behaviors of the solution $y(x)$ that satisfies the initial condition $y(0) = 1$.

■ Question 3.

Classify the $y = 0$ equilibrium point of the following ODE.

$$y' = y\left(\cos\left(y^5 + 2y\right) - 27\pi y^4\right)$$

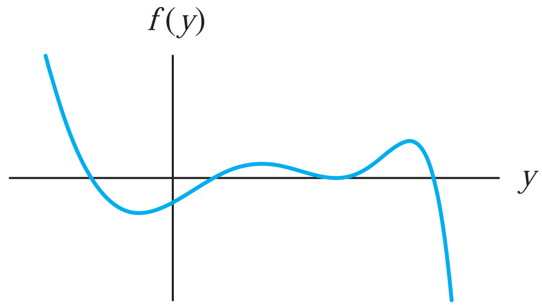
■ Question 4.

Recall the ODE $y' = \dfrac{1}{1-y}$ from the worksheet. Find the interval of definition of the solution curve that starts at $(0, 2)$.

§B. Suggested Homework Problems

■ Question 5.

The graph of a function $f(y)$ is given. Sketch the phase line for the autonomous differential equation $dy/dx = f(y)$.



■ Question 6.

Consider the following ODE with parameter μ

$$\frac{dy}{dx} = \mu y - y^3$$

Find the equilibrium point(s), possibly in terms of μ and describe the phase line.

1. For which values of μ is the phase line qualitatively the same?
2. At which value(s) of μ does the phase line undergo a qualitative change?

■ Question 7.

The Ermentrout-Kopell model[1] for the spiking of a neuron can be modeled by the ODE

$$\frac{d\theta}{dt} = 1 - \cos\theta + (1 + \cos\theta)\mathbf{I}(t)$$

where $\mathbf{I}(t)$ is the input. Often the input function $\mathbf{I}(t)$ is a constant $\mathbf{I}$. Consider the case when $\mathbf{I} = -1/3$.

1. Determine the equilibrium points for this input.
2. Classify these equilibria.
3. Draw the slope-field using DFIELD and describe the long-term behavior of the solution curves for different initial conditions.

[1] See "Parabolic bursting in an excitable system coupled with a slow oscillation" by G. B. Ermentrout and N. Kopell, in SIAM J. Applied Math, Vol. 44, 1984, pp. 1133–1149.