MATH 221 - Differential Equations

Lecture 8 Activities

Fall 2020 | **Subhadip Chowdhury** | **Sep 4**

§A. In-class Practice Problems

Question 1.

Download and run the `euler.ipynb` file.

Question 2.

The ODE model of an electronic **RC**–circuit containing a capacitor, a resistor, and a voltage source looks like

$$\frac{dv_c}{dt} = \frac{\mathbf{V}(t) - v_c}{\mathbf{RC}}$$

where $\mathbf{V}(t)$ is a variable source of input voltage and $v_c(t)$ is the voltage across the capacitor at time t. Suppose $\mathbf{V}(t) = \sin(2\pi t)$, an oscillating function. Let $\mathbf{R} = 0.5, \mathbf{C} = 1$. Use `DFIELD` to draw the slope field and draw several solution curves with different initial conditions. Can you predict about what happens to different solution curves in the long term?

Next use Euler's Method to find the approximate solutions for same initial conditions. Draw all the approximate solution curves in one single plot with appropriate axis labels and title.

[Hint: You can define **3** or four different `y_euler` vectors for different initial conditions. Then plot all on the same picture.]

Question 3.

(a) Use the Euler's method to plot the solution to the IVP

$$\frac{dy}{dt} = e^t \sin y, \quad y(0) = 5$$

[Hint: You will need `np.exp` and `np.sin`.]

(b) Check that the constant function $y(t) = n\pi$ is a solution to the ODE $y' = e^t \sin y$ for any integer n.

(c) Explain why we should not believe the numerical result from part (a).