Assignment 10 Solutions

Chapter 4.1

20. If we guess a constant function of the form $y_p(t) = k$, then substituting $y_p(t)$ into the left-hand side of the differential equation yields

$$\begin{aligned} \frac{d^2(k)}{dt^2} + p\frac{d(k)}{dt} + qk &= 0 + 0 + qk \\ &= qk. \end{aligned}$$

Since the right-hand side of the differential equation is simply the constant c, $k = c/q$ yields a constant solution.

31. **(a)** The general solution for the homogeneous equation is

$$k_1 \cos 2t + k_2 \sin 2t.$$

Suppose $y_p(t) = at^2 + bt + c$. Substituting $y_p(t)$ into the differential equation, we get

$$\frac{d^2 y_p}{dt^2} + 4y_p = -3t^2 + 2t + 3$$

$$2a + 4(at^2 + bt + c) = -3t^2 + 2t + 3$$

$$4at^2 + 4bt + (2a + 4c) = -3t^2 + 2t + 3.$$

Therefore, $y_p(t)$ is a solution if and only if

$$\begin{cases} 4a = -3 \\ 4b = 2 \\ 2a + 4c = 3. \end{cases}$$

Therefore, $a = -3/4$, $b = 1/2$, and $c = 9/8$. The general solution is

$$y(t) = k_1 \cos 2t + k_2 \sin 2t - \tfrac{3}{4}t^2 + \tfrac{1}{2}t + \tfrac{9}{8}.$$

(b) To solve the initial-value problem, we use the initial conditions $y(0) = 2$ and $y'(0) = 0$ along with the general solution to form the simultaneous equations

$$\begin{cases} k_1 + \frac{9}{8} = 2 \\ 2k_2 + \frac{1}{2} = 0. \end{cases}$$

Therefore, $k_1 = 7/8$ and $k_2 = -1/4$. The solution is

$$y(t) = \tfrac{7}{8} \cos 2t - \tfrac{1}{4} \sin 2t - \tfrac{3}{4}t^2 + \tfrac{1}{2}t + \tfrac{9}{8}.$$

Chapter 4.3

21. **(a)** The graph shows either the solution of a resonant equation or one with beats whose period is very large. The period of the beats in equation (iii) is 4π, and the period of the beats in equation (iv) is $4\pi/(4-\sqrt{14}) \approx 48.6$. Hence this graph must correspond to a solution of the resonant equation—equation (v).

(b) The graph has beats with period 4π. Therefore, this graph corresponds to equation (iii).

(c) This solution has no beats and no change in amplitude. Therefore, it corresponds to either (i), (ii), or (vi). Note that the general solution of equation (i) is

$$k_1 \cos 4t + k_2 \sin 4t + \tfrac{5}{8},$$

and the general solution of equation (ii) is

$$k_1 \cos 4t + k_2 \sin 4t - \tfrac{5}{8}.$$

Equation (iv) has a steady-state solution whose oscillations are centered about $y = 0$. Since the oscillations shown are centered around a positive constant, this function is a solution of equation (i).

(d) The graph has beats with a period that is approximately 50. Therefore, this graph corresponds to equation (iv) (see part (a)).

24. To produce the most dramatic effect, the forcing frequency due to the speed bumps must agree with the natural frequency of the suspension system of the average car. Therefore, the speed bumps should be spaced so that the amount of time between bumps is exactly the same as the natural period of the oscillator. Since the natural period of the oscillator is 2 seconds, we compute the distance that the car travels in 2 seconds. At 10 miles per hour, the car travels $1/180$ miles in 2 seconds, and $1/180$ miles is 29 feet, 4 inches.