MATH 221 - Differential Equations

Lecture 3 Activities

Fall 2020 | **Subhadip Chowdhury** | **Aug 24**

§A. In-class Practice Problems

Question 1.

Consider the IVP

$$ty' + y = 0, \quad y(1) = 1$$

Confirm that $y = 1/t$ is the solution of this IVP. What is the domain of definition of the function $y = 1/t$? What is the interval of definition of the solution of the ODE? Are these two sets identical?

Question 2.

(a) Recall that $\frac{d}{dt}\tan(t) = \sec^2 t$. Use this information to write an **autonomous** differential equation whose solution is $y = \tan(t)$.

(b) Consider the differential equation from part (a) along with the initial condition $y(0) = 1$. Clearly, $y = \tan(t)$ doesn't satisfy the IVP anymore (why?) . Can you guess another function that is the solution to the IVP? What is the interval of definition for this solution?

Question 3.

(a) Verify that $3x^2 - y^2 = c$ is a one-parameter family of solutions of the differential equation $\frac{dy}{dx} = \frac{3x}{y}$.

(b) By hand or using a graphing utility, e.g. Desmos, sketch the graph of the implicit solution $3x^2 - y^2 = 3$. Mark off segments, or pieces, on the curve that correspond to **graphs of explicit solutions**. Keep in mind that an explicit solution $y = \varphi(x)$ must be a function and differentiable.

(c) Explain how the DE can help in finding points on the graph of $3x^2 - y^2 = 3$ where the tangent line is vertical. How does knowing where a tangent line is vertical help in determining an interval I of definition of a solution $y = \varphi(x)$ of the DE? Carry out your ideas and find the interval of definition for each of the explicit solutions.

(d) The point $(-2, 3)$ is on the graph of $3x^2 - y^2 = 3$. Which of the explicit solutions in part (c) satisfies $y(-2) = 3$?

§B. Suggested Homework Problems

Question 4.

Determine a plausible value of x_0 for which the graph of the solution of the initial-value problem $y' + 2y = 3x - 6, \quad y(x_0) = 0$ is tangent to the x-axis at $(x_0, 0)$. Explain your reasoning.

Question 5.

The function $y = \sin x$ is an explicit solution of the first-order differential equation $dy/dx = \sqrt{1-y^2}$. Find an interval $\mathbf{I}$ of definition. Note that $\mathbf{I} \neq \mathbb{R}$.

Question 6.

(a) Make up a differential equation that does not possess any real solutions.

(b) Make up a differential equation that you feel confident possesses only the trivial solution $y(t) = 0$ for all t. Explain your reasoning.

Question 7.

The curve in the figure below has the implicit equation $x^3 + y^3 = 3xy$. It belongs to a family of curves called **Folia of Descartes**. It has an asymptote $x + y = -1$ which is also drawn in the picture.

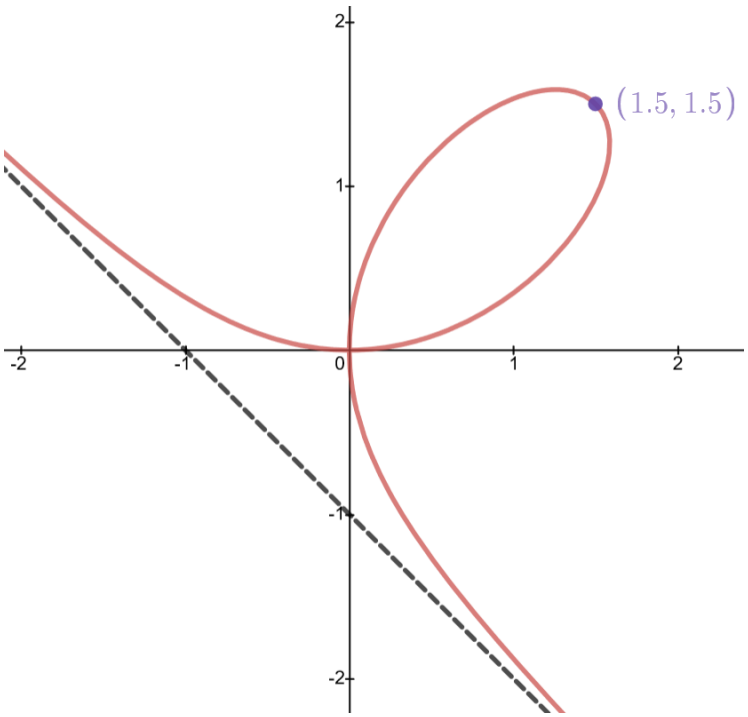

Figure 1: Folium of Descartes

(a) Show that the curve is an implicit solution to the differential equation

$$\frac{dy}{dx} = \frac{y\left(y^3 - 2x^3\right)}{x\left(2y^3 - x^3\right)}$$

(b) Mark off segments, or pieces, on the curve that correspond to **graphs of explicit solutions**.

(c) Find the interval of definition for the solution to the IVP

$$\frac{dy}{dx} = \frac{y\left(y^3 - 2x^3\right)}{x\left(2y^3 - x^3\right)}, \qquad y(1.5) = 1.5$$