Math 2208: Ordinary Differential Equations

Lecture 3 Worksheet

Spring 2020 **Subhadip Chowdhury** **Jan 29**

Title: Intro to Octave and Euler's Method

Summary: We will learn to use Octave, an open source alternative of Matlab. Then we will learn about a simple numerical technique called Eulers Method which approximates solutions to ODEs quantitatively. We will implement Euler's method using Octave.

§A. Basic Commands in Octave

- Open Octave (go to "Launchpad" → Search for "Octave").
- Now you have the "command window" open in front of you. Perform these basic calculations by entering in the appropriate text after the >> and hitting return. Write down what each command did.

a) `exp(1)`	b) `log(exp(3))`	c) `pi`
d) `clc`	e) `1/0`	f) `0/0`
g) `1:5`	h) `data = [2,3,5,7,11]`	i) `data`
j) `data.*2`	k) `data.^2`	l) `data./2`
m) `diff(data)`	n) `data(1:end)`	o) `data(1:end-1)`
p) `data(2:end)`	q) `data([1,3])`	r) `mat = [2,3,4;6,7,8]`
s) `mat(1,2)`	t) `mat(1,:)`	u) `datavector = [2;3;5;7;11]`

- Use Octave to compute $5\dfrac{\ln(12.13)e^2\pi}{\sin(3)}$.

§B. Using Scripts for more complicated computations

- Go to our blackboard page, and download the script `BasicPlotExamples.m`
- Save the script in your computer desktop (or your favourite directory of choice where you can find it).
- Inside `Octave`, go to the File Browser window on left top, navigate to the `BasicPlotExamples.m` file and open it.
- Execute the script by clicking the  button. If Octave shows a prompt asking you to add the directory to executable paths, click yes.
- What did the script do? Read the comments in the file.

- Now we will write a new script. Under the "File" tab click "New Script".
- Click "Save". Name the file `lab1_plots_lastname_firstname.m` and save it somewhere you can get access to later.

Question 1.

Plot the function $y(t) = e^t \sin(t) - t^\pi$ for $t = 0, 0.01, 0.02, \ldots, 2.99, 3.0$. Try to figure out how to make the curve a green dotted line. To see how to do this, type `help plot` in the command window.

Use the file `BasicPlotExamples.m` as a reference. But don't just blindly copy the whole file, look at what each part of the code does, and decide what to use (not all of it will be relevant). You might need to refer to the basic commands in section 1. Set the title of the plot to be your name. Make sure it is of appropriate font size to be legible. Take a screenshot of the plot. Upload the octave code (.m file) and the screenshot of the plot to the Blackboard submission link.

§C. Euler's Method

Suppose that the goal is to solve the ode

$$y' = f(t, y), \qquad y(0) = y_0$$

and to determine $y(T)$. We can find an approximation by discretizing the interval $[0, T]$ into a partition

$$0 = t_0 \le t_1 \le t_2 \le t_3 \le \ldots \le t_N = T$$

and using linear approximation to calculate $y(t_{i+1})$ from $y(t_i)$ using tangent lines.

Assume $t_{i+1} - t_i = \Delta t$. recall from your Calculus class that the equation of the tangent line to the solution at $t = t_0$ is

$$y = y_0 + y'(t_0)(t - t_0) = y_0 + f(t_0, y_0)(t - t_0)$$

Refer to figure 1.

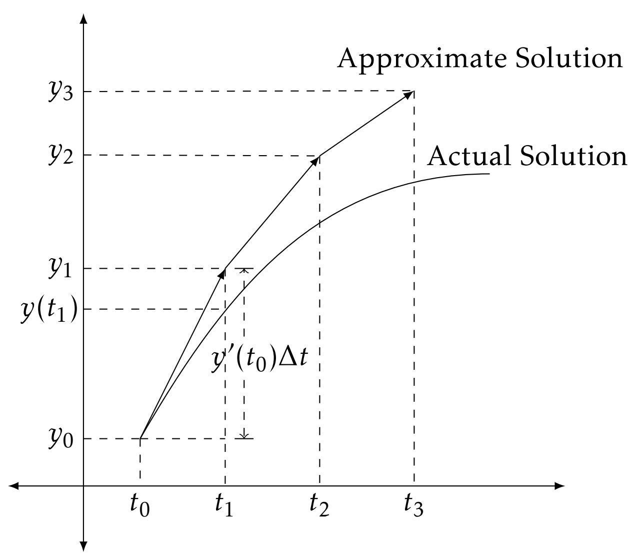
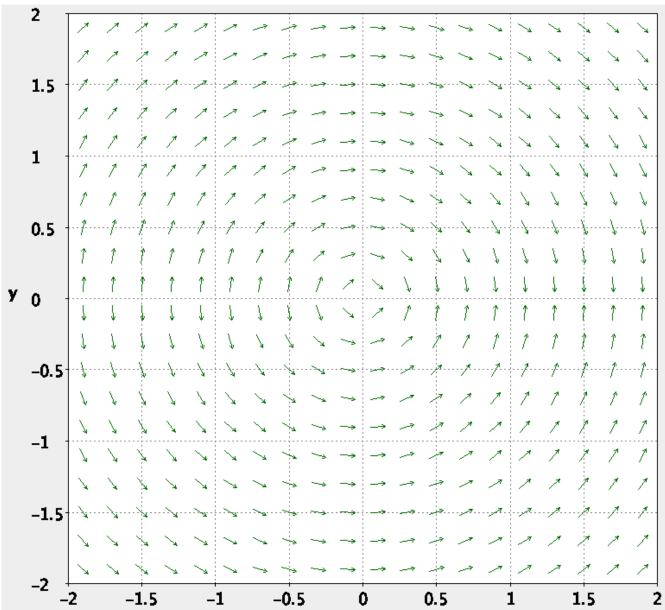


Figure 1: Euler's Method

If Δt is small enough, then $y_1 = y_0 + f(t_0, y_0)\Delta t$ is a good approximation for $y(t_1)$. Then we can continue the process with

$$y_{i+1} = y_i + f(t_i, y_i)\Delta t = y_i + \Delta y_i$$

Question 2.



t_i	y_i	$f(t_i, y_i)$	Δy_i

Consider the differential equation $y' = -t/y$ with initial condition $y(0) = 1$. You are given that the exact solution is $y(t) = \sqrt{1 - t^2}$.

(a) use the slope field to estimate $y(1/2)$ for the solution that satisfies the given initial condition.

(b) Compare your estimate with the exact value of $y(1/2)$.

(c) Use Euler's Method with $\Delta t = .25$ to estimate $y(1/2)$.

(d) Is your Euler's Method estimate and over-estimate or under-estimate? Explain why.

§D. Implementing Euler's Method in Octave

- Go to our blackboard page, and download the script `Euler.m`.
- Save the script on your computer desktop.
- Read the comments in the file `Euler.m`. It will use Euler's method to approximate a solution to the IVP from question 1. However, this time we will take $\Delta t = 0.1$.
- Execute the script. It outputs a graph, a table and the end value of y_n. Compare the resulting graph (which is the approximation solution to equation (1) for $0 \le t \le 0.5$, to the direction field above.
- How did `Euler`'s method do this time? Is this a better estimate than last question?

■ Question 3.

Complete the following sentences.

1. As the time step Δt ________________ in magnitude, the numerical error in computing $y(t_1)$ using Eulers Method decreases in magnitude.
2. As the time step Δt ________________ in magnitude, the numerical error in computing $y(t_1)$ using Eulers Method increases in magnitude.
3. When y'' is ________________ on $t_0 < t < t_1$ the function $y(t)$ is concave up and estimates of $y(t_1)$ using Eulers Method will be ________________.
4. When y'' is ________________ on $t_0 < t < t_1$ the function $y(t)$ is concave down and estimates of $y(t_1)$ using Eulers Method will be ________________.

■ Question 4.

Plot the actual solution to the IVP

$$\frac{dy}{dt} = -t/y, \qquad y(0) = 1 \tag{1}$$

and the approximate Euler's estimate in the same figure. The actual solution should be a solid blue curve. The Euler approximate should be a dotted red curve. Give your plot appropriate labels. Take a screenshot of the plot and include it with your homework due next week.