MATH 221 - Differential Equations

Lecture 23 Activity

Fall 2020 | **Subhadip Chowdhury** | **Oct 9**

§A. Suggested Homework Problems

Question 1.

Suppose that the 2×2 matrix $\mathbf{B}$ has $\lambda = -2 + 5i$ as an eigenvalue with eigenvector

$$\vec{\mathbf{R}}_0 = \begin{pmatrix} 1 \\ 4 - 3i \end{pmatrix}$$

Compute the general solution to $d\vec{\mathbf{R}}/dt = \mathbf{B}\vec{\mathbf{R}}$.

Question 2.

For each of the following systems,

(a) find the eigenvalues,

(b) determine if the origin is a spiral sink, a spiral source, or a center,

(c) determine the natural period and natural frequency of the oscillations,

(d) determine the direction of the oscillations in the phase plane (do the solutions go clockwise or counterclockwise around the origin?), and

(e) using PPLANE, sketch the xy-phase portrait and the $x(t)-$ and $y(t)-$ graphs for the solutions with the indicated initial conditions.

1. $\frac{d\vec{\mathbf{R}}}{dt} = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}\vec{\mathbf{R}}$, with initial condition $\vec{\mathbf{R}}(0) = (1, 1)$

2. $\frac{d\vec{\mathbf{R}}}{dt} = \begin{pmatrix} 0 & 2 \\ -2 & -1 \end{pmatrix}\vec{\mathbf{R}}$, with initial condition $\vec{\mathbf{R}}(0) = (-1, 1)$

Question 3.

We are continuing problem 1 from lecture 20 activity. Consider the second-order equation

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$$

where p and q are positive.

(a) Convert this equation into a linear system of two first-order ODEs.

(b) Compute the characteristic polynomial of the system.

(c) Find the eigenvalues.

(d) What conditions on p and q guarantee that the eigenvalues are complex?

(e) What relationship between $\boldsymbol{p}$ and $\boldsymbol{q}$ guarantees that the origin is a spiral sink? What relationship guarantees that the origin is a center? What relationship guarantees that the origin is a spiral source?

(f) If the eigenvalues are complex, what conditions on $\boldsymbol{p}$ and $\boldsymbol{q}$ guarantee that solutions spiral around the origin in a clockwise direction?