MATH 2208: ORDINARY DIFFERENTIAL EQUATIONS

ASSIGNMENT 4

Fall 2019 **Subhadip Chowdhury** **Due: Feb 19**

Reading

Section 1.9 from the textbook.

Exercises

Don't forget to be neat and thorough. No fringe, and please use the cover page.

Question 1.

Book problems **1.9.(4, 8)**. These problems require that you use the integrating factor method.

Additional Problems

Question 2.

Recall that in your second assignment you did a qualitative analysis of the ODE

$$\frac{dy}{dt} = \frac{\sin(2\pi t) - y}{RC}$$

with $R = 0.5$ and $C = 1$. You may recall that *no matter what the initial value*, all the solutions seemed to converge to a single solution. We would like to show that this is indeed the case by explicitly solving the ODE.

Find the general solution of the ODE above by using the integrating factor method. You may evaluate the integrals using software (like Wolfram alpha), but please show the ODE part of your work. Use your analytical formula to describe the long term behavior (what happens when $t \to \infty$?) of the solutions and compare this with your observation from the previous assignment.

Question 3.

In your first homework, you derived the following IVP to describe a population subject to a carrying capacity N,

$$\frac{dy}{dt} = k\left(1 - \frac{y}{N}\right)y, \quad y(0) = y_0$$

where k, N are positive constants and y_0 is some real number. You will now analyze this ODE using analytical, qualitative, and numerical approaches.

(a) **Analytical Approach:** Assuming that $k = 0.5$ and $N = 10$ and $y_0 = 1$, solve the IVP using the separation of variable method. Show your work.

(b) **Numerical Approach:** Assuming that $k = 0.5$ and $N = 10$ and $y_0 = 1$, solve the IVP using Octave/Matlab's `ODE45` routine. Use the time domain $[0, 30]$ and a time step size of $\Delta t = 0.5$. Make a picture showing your answer from `ODE45` (as solid circles) and the exact solution you obtained in the previous problem (shown as a solid line) in the same graph. Include a printout of your graph, which should have appropriate and legible labels.

(c) **Qualitative Approach:** Assuming that $k = 0.5$ and $N = 10$, draw the phase line. Use your phase line to describe the long term behavior for the following situations:

(i) $y_0 < 0$

(ii) $y_0 = 0$

(iii) $0 < y_0 < N$

(iv) $y_0 = N$

(v) $y_0 > N$

(d) Briefly describe the pros and cons of each of the above approaches in (a)-(c).

We have learned a number of tools to study first order ODEs. One challenge is deciding what tool is the best one for a given problem. In problems $4, 5, 6, 7$ below, you should address the following:

(a) Decide which tools we could apply to the problem.

(b) Classify each tool as analytical, numerical, or qualitative.

(c) Solve the problem with the tool of your choice (some may have multiple tools that work). You may use any computer software of your choice if necessary. If using qualitative method, use the new techniques e.g. phase lines when appropriate.

Question 4.

What is $y(5)$ if y satisfies

$$\frac{dy}{dt} = -2ty + 4t, \quad y(0) = 1$$

Question 5.

Consider the population model

$$\frac{dP}{dt} = -(\sin(P) + 2)\left(P^2 - P\right)$$

What is the population at time $t = 2$ if $P(0) = 0.5$.

Question 6.

Consider the ODE

$$\frac{dP}{dt} = (P + 1 + \cos(t))(P - 2)$$

Describe, for large values of t, all solutions that have an initial value satisfying $-3 > P(0) > -5$.

Question 7.

Consider the population model

$$\frac{dP}{dt} = -(\sin(P) + 2)\left(P^2 - P\right)$$

If $P(0) = 0.5$, does the population go toward extinction?