MATH 221 - Differential Equations

Lecture 20 Activity

Fall 2020 | **Subhadip Chowdhury** | **Oct 2**

§A. In-class Activity Problems

Question 1.

Consider the second-order equation

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$$

where p and q are positive.

(a) Convert this equation into a linear system of two first-order ODEs.

(b) Compute the characteristic polynomial of the system.

(c) Find the eigenvalues.

(d) Under what conditions on p and q are the eigenvalues two distinct real numbers?

(e) Verify that the eigenvalues are negative if they are real numbers.

§B. Suggested Homework Problems

Question 2.

In the following problems:

(a) compute the eigenvalues;

(b) for each eigenvalue, compute the associated eigenvectors;

(c) using `PPLANE` sketch the direction field for the system; and

(d) for each eigenvalue, find a corresponding straight-line solution and plot its $x(t)-$ and $y(t)-$graphs.

1. $\dfrac{d\mathbf{R}}{dt} = \begin{pmatrix} -4 & -2 \\ -1 & -3 \end{pmatrix}\mathbf{R}$

2. $\dfrac{dx}{dt} = 5x + 4y \qquad \dfrac{dy}{dt} = 9x$

3. $\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

Question 3.

A matrix of the form

$$\mathbf{A} = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$$

is called upper triangular. Suppose that $b \neq 0$ and $a \neq d$. Find the eigenvalues and eigenvectors of A.

■ Question 4.

□

A matrix of the form

$$\mathbf{B} = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{b} & \boldsymbol{d} \end{pmatrix}$$

is called symmetric. Show that $\mathbf{B}$ has real eigenvalues and that, if $\boldsymbol{b} \neq \mathbf{0}$, then $\mathbf{B}$ has two distinct eigenvalues.