Math 2208: Ordinary Differential Equations

Assignment 2

Spring 2020 **Subhadip Chowdhury** **Due: Feb 5**

Reading

Section 1.(2,3,4) from the textbook.

Exercises

Don't forget to be neat and thorough. No fringe, and please use the cover page.

Question 1.

Book problem 1.2.42.

Question 2.

Book problems 1.3.(6a, 9, 12, 14, 18, 19).
Note: For problems 6 and 9, you may use the `dfield` software instead of `HPGSolver` to check your answers, but you are NOT required to print anything out.

For problem 19, use `dfield` and attach screenshots of the three slope fields that you get (or copy them by hand). You can put all three pictures in one page, e.g. using MS Word, and save paper.

Question 3.

Book problems 1.4.(5, 6, 11, 14).
Note: You can use `Euler.m` to perform the Euler's Method in 5,6. Copy the tables to paper. The tables should at least have a t_k column and a y_k column (look in page 55 in the textbook for examples). Do not use `dfield` for 11.

Question 4.

Finish the code and attach the screenshot of the plot in question 4 from Worksheet 3.

Solution. The first picture below uses same t values for both graph. But this makes the graph of actual solution look a bit jagged. With a more densely spaced t dataset, the graph of the actual solution looks more smooth, as in the second figure. Either picture is good for now and both will get full marks. ■

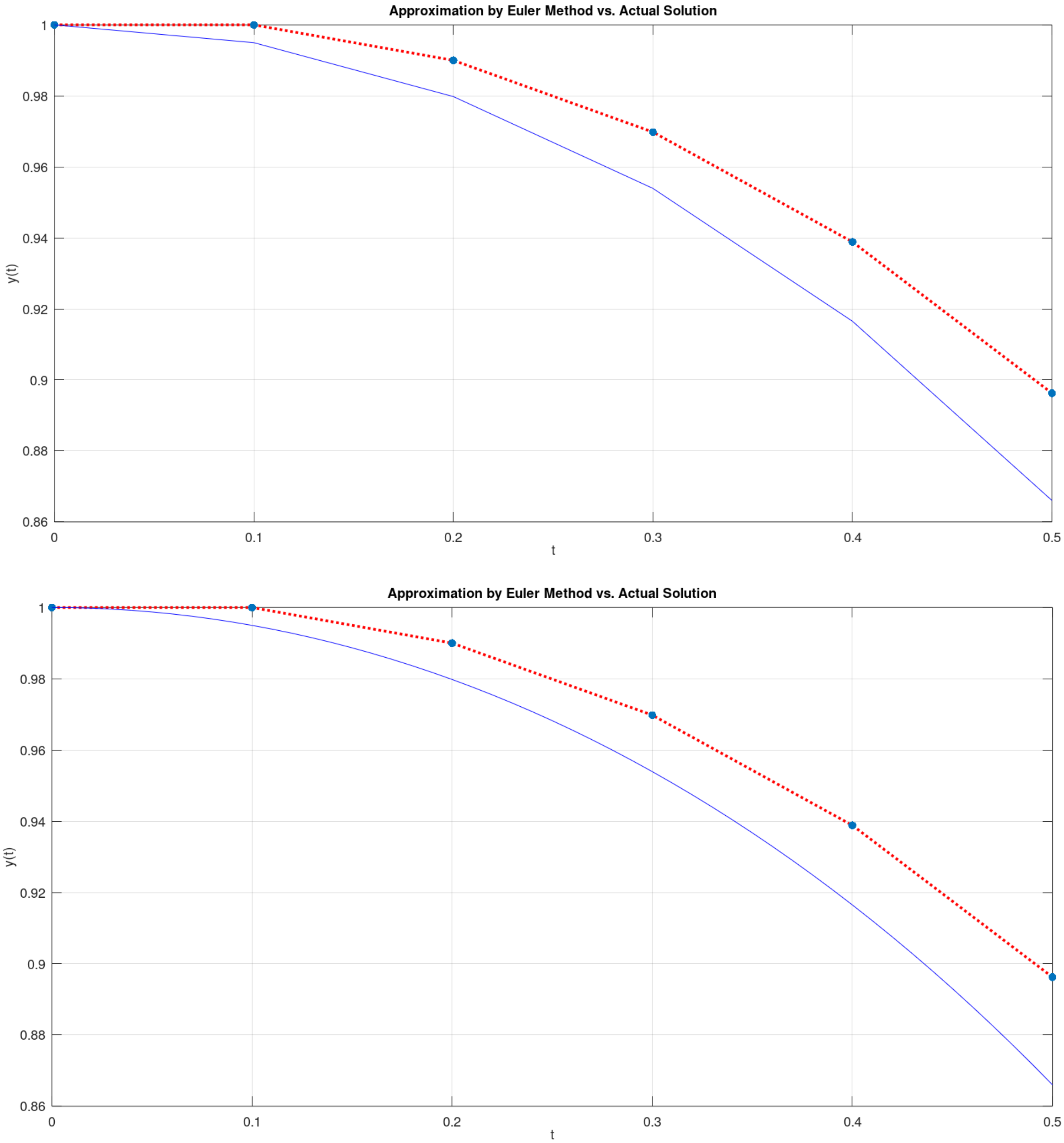

Figure 1

Additional Problems

Question 5.

The ODE model of an electronic **RC**–circuit containing a capacitor, a resistor, and a voltage source looks like

$$\frac{dv_c}{dt} = \frac{\mathbf{V}(t) - v_c}{\mathbf{RC}}$$

where $\mathbf{V}(t)$ is a variable source of input voltage and $v_c(t)$ is the voltage across the capacitor at time t. Suppose $\mathbf{V}(t) = \sin(2\pi t)$, an oscillating function. Let $\mathbf{R} = 0.5, \mathbf{C} = 1$. Use `dfield` to perform a qualitative analysis of the differential equation for different initial values. What happens to different solution curves in the long term? Include the `dfield` picture.

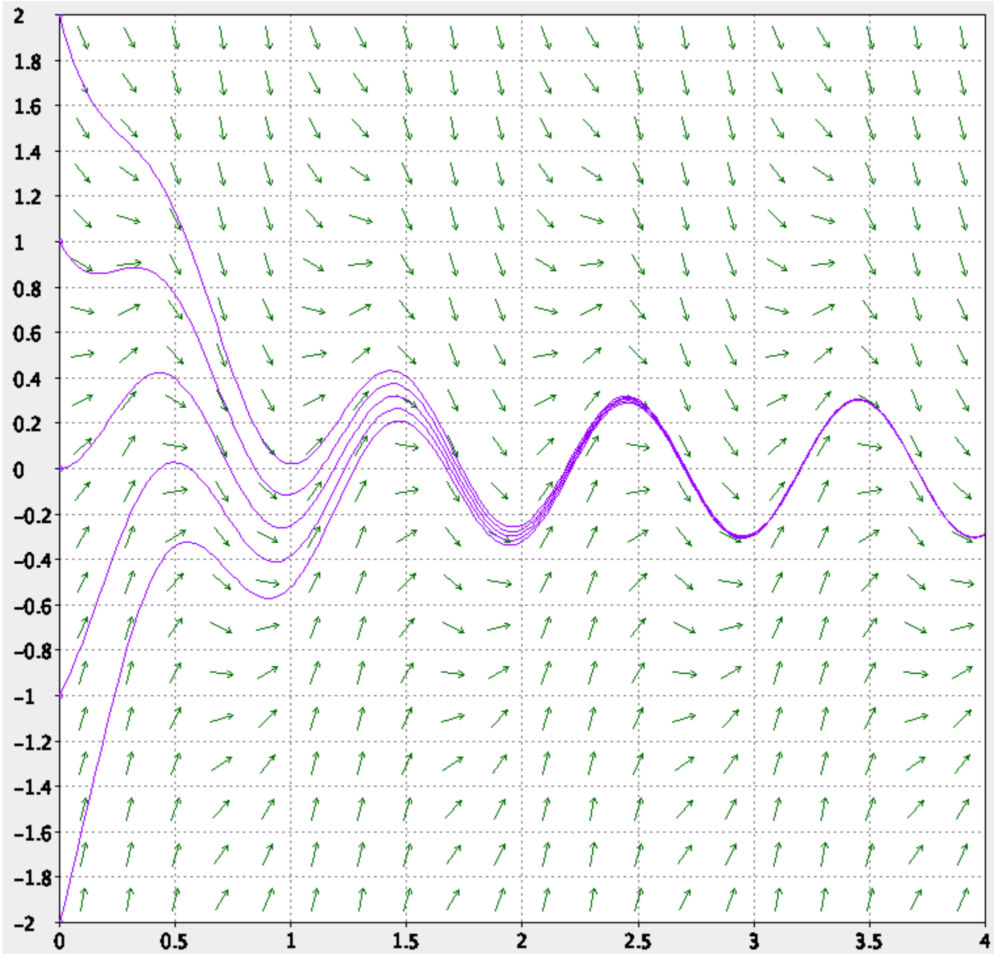

Figure 2

Solution. From the slope field for this equation, we can predict that the solutions oscillate for several different initial conditions. In addition, we see that they also approach each other and converge towards a single solution as $t \to \infty$.
Instructor's Note: This uniformity of long-term behavior is not so easily predicted from the slope field alone unless we plot some solution curve. ■

■ Question 6.

Use `Octave/Matlab` and modify the code in `Euler.m` to get an approximate solution to the ODE from question 5 for several different initial conditions (same ones you used in `dfield`). Draw all the curves in one single plot with appropriate labels and title. Attach a screenshot of the plot.

Coding instruction: If you create a figure but don't use the command `clf;`, the figure doesn't get erased in next run. This way you can plot new curves on the same figure every time you run it with a new initial condition.

Solution. Please come ask me in Office Hour if you are having trouble with the code. I have used the IC values $-2, -1, 0, 1,$ and 2. ■

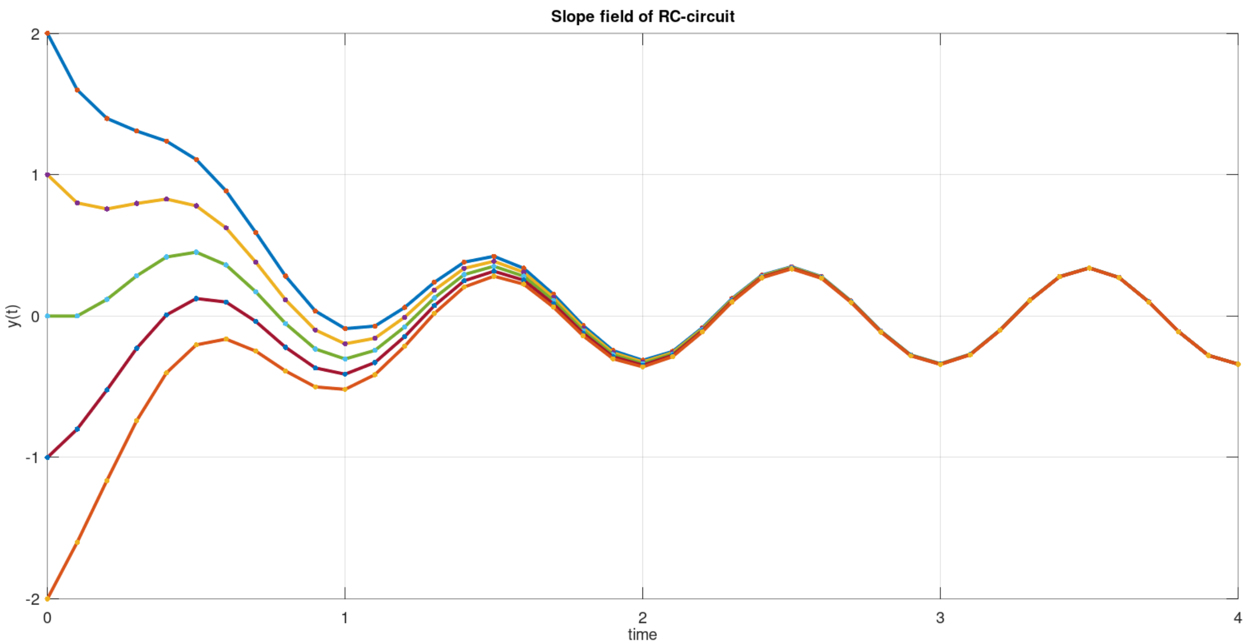

Figure 3