MATH 221 - Differential Equations

Lecture 10 Activities

Fall 2020 | **Subhadip Chowdhury** | **Sep 7**

§A. In-class Practice Problems

Question 1.

In the first week, you derived the following IVP to describe a population subject to a carrying capacity $\mathbf{N}$,

$$\frac{dy}{dt} = k\left(1 - \frac{y}{\mathrm{N}}\right)y, \quad y(0) = y_0$$

where k, N are positive constants and y_0 is some real number. You will now analyze this ODE using analytical, qualitative, and numerical approaches.

(a) **Analytical Approach:** Assuming that $k = 0.5$ and $\mathrm{N} = 10$ and $y_0 = 1$, solve the IVP using the separation of variable method.

(b) **Numerical Approach:** Assuming that $k = 0.5$ and $\mathrm{N} = 10$ and $y_0 = 1$, solve the IVP using the `ODEINT` routine. Use the time domain $[0, 30]$ and a time step size of $\Delta t = 0.5$. Make a picture showing your answer from `ODEINT` (as solid circles) and the exact solution you obtained in the previous problem (shown as a solid line) in the same graph. Include a printout of your graph, which should have appropriate and legible labels.

(c) **Qualitative Approach:** Assuming that $k = 0.5$ and $\mathrm{N} = 10$, draw the phase line. Use your phase line to describe the long term behavior for the following situations:

   (i) $y_0 < 0$

   (ii) $y_0 = 0$

   (iii) $0 < y_0 < \mathrm{N}$

   (iv) $y_0 = \mathrm{N}$

   (v) $y_0 > \mathrm{N}$

(d) Briefly describe the pros and cons of each of the above approaches in (a)-(c).

We have learned a number of tools to study first order ODEs. One challenge is deciding what tool is the best one for a given problem. In the problems below, you should address the following:

(a) Decide which tools we could apply to the problem.

(b) Classify each tool as analytical, numerical, or qualitative.

(c) Solve the problem with the tool of your choice (some may have multiple tools that work). You may use any computer software of your choice if necessary. If using qualitative method, use the new techniques e.g. phase lines when appropriate.

■ Question 2.

What is $y(5)$ if y satisfies

$$\frac{dy}{dt} = -2ty + 4t, \quad y(0) = 1$$

■ Question 3.

Consider the population model

$$\frac{dP}{dt} = -(\sin(P) + 2)\left(P^2 - P\right)$$

What is the population at time $t = 2$ if $P(0) = 0.5$.

■ Question 4.

Consider the ODE

$$\frac{dP}{dt} = (P + 1 + \cos(t))(P - 2)$$

Describe, for large values of t, all solutions that have an initial value satisfying $-3 > P(0) > -5$.

■ Question 5.

Consider the population model

$$\frac{dP}{dt} = -(\sin(P) + 2)\left(P^2 - P\right)$$

If $P(0) = 0.5$, does the population go toward extinction?