MATH 221 - Differential Equations

Lecture 1 Worksheet

Fall 2020 | **Subhadip Chowdhury** | **Aug 19**

Title: Why Differential Equations?

Summary: We will introduce the study of differential equations as mathematical modeling tools.

§A. Differential Equations as Mathematical Models

Question 1. (Group Work)

Consider the following US population data from **1790 – 1850**. Assume that the year **1790** corresponds to $t = 0$.

Year	t	Population (in millions)
1790	**0**	**3.9**
1800	**10**	**5.3**
1810	**20**	**7.2**
1820	**30**	**9.6**
1830	**40**	**12.8**
1840	**50**	**17**
1850	**60**	**23**

Can you estimate the population in **1870** based on this data?

Definition A.1: Mathematical Model

A mathematical model is a mathematical description of a system or phenomenon. Many physical systems often involve time (the variable t) so that the mathematical description of the model involves the rate of change of a variable with respect to time which can be mathematically represented using differential equations. The solution of the model produces a state of the system at certain points in time: the past, present or future.

Differential equations are used to map all sorts of physical phenomena, from chemical reactions, disease progression, motions of objects, electronic circuits, etc. Most mathematical models of real-world situations do not have analytical solutions.

Three main tools to analyze ODEs

- **Analytical**: Find an explicit solution to an ODE
- **Qualitative**: Obtain information about solutions without finding an explicit formula
- **Numerical**: Use (computer) algorithm to approximate solutions