# MATH 221 - Differential Equations

## Lecture 19 Activity

**Fall 2020** | **Subhadip Chowdhury** | **Sep 30**

## §A. In-class Activity

### Question 1.

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Consider the system $\vec{\mathbf{R}}'(t) = \mathbf{A}\vec{\mathbf{R}}(t)$. If $\mathbf{det}(\mathbf{A}) = \mathbf{0}$, show that the equilibrium solutions for the system form a straight line in the **XY** phase plane.

### Question 2.

**Housing Economy**

In this question, we consider the following model of the market for single-family housing in a community. Let $\mathbf{S}(t)$ be the number of sellers at time $t$ , and let $\mathbf{B}(t)$ be the number of buyers at time $t$. We assume that there are natural equilibrium levels of buyers and sellers (made up of people who retire, change job locations, or wish to move for family reasons). The equilibrium level of sellers is $\mathbf{S_0}$, and the equilibrium level of buyers is $\mathbf{B_0}$.

However, market forces can entice people to buy or sell under various conditions. For example, if the price of a house is very high, then house owners are tempted to sell their homes. If prices are very low, extra buyers enter the market looking for bargains. We let $b(t) = \mathbf{B}(t) - \mathbf{B_0}$ denote the deviation of the number of buyers from equilibrium at time $t$. So if $b(t) > \mathbf{0}$, then there are more buyers than usual, and we say it is a **seller's market.** Presumably the competition of the extra buyers for the same number of houses for sale will force the prices up (the law of supply and demand).

Similarly, we let $s(t) = \mathbf{S}(t) - \mathbf{S_0}$ denote the deviation of the number of sellers from the equilibrium level. If $s(t) > \mathbf{0}$, then there are more sellers on the market than usual; and if the number of buyers is low, there are too many houses on the market and prices decrease, which in turn affects decisions to buy or sell.

We can give a simple model of this situation as follows:

$$\frac{d\vec{\mathbf{R}}}{dt} = \mathbf{A}\vec{\mathbf{R}} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} b \\ s \end{pmatrix}, \quad \text{where } \vec{\mathbf{R}} = \begin{pmatrix} b \\ s \end{pmatrix}$$

The exact values of the parameters $\alpha, \beta, \gamma,$ and $\delta$ depend on the economy of a particular community. Nevertheless, if we assume that everybody wants to get a bargain when they are buying a house and to get top dollar when they are selling a house, then we can hope to predict whether the parameters are positive or negative even though we cannot predict their exact values.

Use the information given above to obtain information about the parameters $\alpha, \beta, \gamma,$ and $\delta.$ Be sure to justify your answers.

(a) If there are more than the usual number of buyers competing for houses, we would expect the price of houses to rise, and this increase would make it less likely that new potential buyers will enter the market. What does this say about the parameter $\alpha$**?** Is it positive or negative?

(b) If there are fewer than the usual number of buyers competing for the houses available for sale, then we would expect the price of houses to decrease. As a result, fewer potential sellers will place their houses on the market. What does this imply about the parameter $\gamma$ ?

(c) Consider the effect on house prices if $s > 0$ and the subsequent effect on buyers and sellers. Then determine the sign of the parameter $\beta$.

(d) Determine the most reasonable sign for the parameter $\delta$.

## §B. Suggested Homework Problems

### ■ Question 3.

**Linear Algebra Practice**

(a) Let $\mathbf{A} = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $a = -1$ and $b = 6$.

- (i) Compute $\mathbf{A}\vec{v}$.
- (ii) Compute $a\vec{v}$.
- (iii) Draw the vector $\vec{v}$ in the $(x, y)$ plane with the tail centered at the origin.
- (iv) Draw the vector $a\vec{v}$ in the $(x, y)$ plane with the tail centered at the origin. How does this compare to the drawing for $\vec{v}$?
- (v) Draw the vector $b\vec{v}$ in the $(x, y)$ plane with the tail centered at the origin. How does this compare to the drawing for $\vec{v}$?
- (vi) Draw the vector $b\vec{v}$ in the $(x, y)$ plane with the tail centered at the origin. How does this compare to the drawing for $b\vec{v}$?

(b) Compute the determinant of $\mathbf{A}$. Show your work.

### ■ Question 4.

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Write the given linear system in matrix form.

$$\frac{dx}{dt} = 3x - 5y \qquad \frac{dy}{dt} = 4x + 8y$$

### ■ Question 5.

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Verify that the vector-valued function $\vec{\mathbf{R}}(t)$ is a solution of the given autonomous linear system.

(a)

$$\frac{dx}{dt} = 3x - 4y \qquad \frac{dy}{dt} = 4x - 7y; \qquad \vec{\mathbf{R}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t}$$

(b)

$$\frac{dx}{dt} = -2x + 5y \qquad \frac{dy}{dt} = -2x + 4y; \quad \vec{\mathbf{R}} = \begin{pmatrix} 5\cos t \\ 3\cos t - \sin t \end{pmatrix} e^{t}$$

### ■ Question 6.

**Older Topic: Predator-Prey Model**

Pesticides that kill all insect species are not only bad for the environment, but they can also be inefficient at controlling pest species. Suppose a pest insect species in a particular field has population $\mathbf{R}(t)$ at time $t$, and suppose its primary predator is another insect species with population $\mathbf{F}(t)$ at time $t$. Suppose the populations of these species are accurately modeled by the system

$$\frac{d\mathbf{R}}{dt} = 2\mathbf{R} - 1.2\mathbf{RF} \qquad \frac{d\mathbf{F}}{dt} = -\mathbf{F} + 0.9\mathbf{RF}$$

studied in last week. Finally, suppose that at time $t = 0$ a pesticide is applied to the field that reduces both the pest and predator populations to very very small but nonzero numbers.

(a) Using the phase portrait from PPLANE, predict what will happen to the population of the pest species as $t$ increases.

(b) Write a couple of sentences in nontechnical language (so that a farmer might understand), warning of the possibility of the **paradoxical** effect that pesticide application can have on pest populations.