Math 2000-B Handout 3

Subhadip Chowdhury

■ Exercise 1.

For the following vectors, find the values of h for which $\vec{\mathbf{b}} \in \mathbf{Span}\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$.

$$\vec{\mathbf{u}} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{\mathbf{b}} = \begin{bmatrix} -1 \\ h \\ 1 \end{bmatrix}$$

■ Exercise 2.

Is it possible or impossible to find examples of each of the following? If possible, please provide an example. If impossible, please explain why.

(a) Two invertible matrices A and B such that $AB = 0$.

(b) A 4×3 matrix A such that $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is consistent for every choice of $\vec{\mathbf{b}}$ in $\mathbb{R}^4$.

(c) a 2×3 matrix A, not in echelon form, such that the solution of $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ is a plane in $\mathbb{R}^3$.

■ Exercise 3.

Given a $(n \times n)$ matrix A, if $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is consistent for all $\vec{\mathbf{b}} \in \mathbb{R}^n$, then show that the system has a unique solution for every $\vec{\mathbf{b}}$ (i.e. never infinitely many slutions).

■ Exercise 4.

Suppose A is a 5×3 matrix and there exists a 3×5 matrix C such that $CA = I_3$. Suppose further that for some given $\vec{\mathbf{b}}$ in $\mathbb{R}^5$, the equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has at least one solution. Show that this solution is unique.

■ Exercise 5.

True or False.

(a) A single vector by itself is linearly dependent.

(b) Elementary row reduction operations on a matrix do not change the linear dependence relation among the columns.

(c) If $AB = I$, then A is invertible.

■ Exercise 6.

Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

(a) Show that $A^3 = \mathbf{0}$.

(b) Use matrix algebra to compute the product $(I - A)(I + A + A^2)$.

(c) What is the inverse of $(I - A)$?

■ Exercise 7.

Let $A = LU$, where L is an invertible lower triangular matrix and U is upper triangular. Explain why the first column of A is a multiple of the first column of L. How is the second column of A related to the columns of L?

■ Exercise 8.

Let A be a 6×4 matrix and B a 4×6 matrix. Show that the 6×6 matrix AB cannot be invertible.