Midterm 2 Part 1

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INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.

- Answer the questions in the white space provided.

- Write legibly and clearly mark the answer.

- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are not allowed.

- If you write down the correct formula/procedure to find an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.

- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: _____

Question	Points	Score
1	15	
2	15	
Total:	30	

This exam has 2 questions, for a total of 30 points.
The maximum possible point for each problem is given on the right side of the problem.

1. For each of the following statements, find out whether it is 'always true', 'sometimes true', or 'always false'. Give a brief explanation for your answer. If you think an answer is 'sometimes true', give an example or criterion when it's false.

    (a) The map $T : \mathbb{P}_2 \to \mathbb{R}$ defined by $T(f(t)) = \displaystyle\int_0^a f(t)\,dt$ is an isomorphism, where a is some positive real number. $\boxed{3}$

    (b) If $\mathbf{Nul}(A) = \{\vec{0}\}$ for a 5×4 matrix A, if $AB = AC$ for two 4×5 matrices B and C, then B is equal to C. $\boxed{3}$

    (c) If T is a linear transformation from a vector space V to V, then $\boxed{3}$

    $$\{\vec{v} \in V \mid T(\vec{v}) = \vec{v}\}$$

    is a subspace of V.

(d) If V is a subspace of $\mathbb{R}^n$, we can find a 3×3 matrix A such that $V = \mathbf{Col}(A)$.

$\boxed{3}$

(e) Let A be an 6×4 matrix. Then the dimension of the row space and column space of A are same.

$\boxed{3}$

2. Is it possible or impossible to find examples of each of the following? If possible, please provide an example. If impossible, please explain why.

(a) A linear transformation whose kernel is the line spanned by $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ in $\mathbb{R}^3$.

$\boxed{3}$

(b) A 5×5 matrix A such that $\mathbf{Nul}(A) = \mathbf{Col}(A)$.   3

(c) A 4×3 matrix B with nonzero entries such that $\mathbf{Col}(B)$ is 2-dimensional.   3

(d) A nonhomogeneous system of seven equations in six unknowns that has a unique solution for every choice of constant on the right-hand side.   3

(e) A linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ that transforms the unit square with vertices at $(0,0), (0,1), (1,0),$ and $(1,1)$ into a triangle with vertices at $(0,0), (1,0)$ and $(1,1)$.

3