Midterm 1

FEBRUARY 22, 2019
SUBHADIP CHOWDHURY

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the white space provided. If you run out of room, use the back page.
- Write legibly and clearly mark the answer.
- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are not allowed.
- If you write down the correct formula/procedure to find an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: ______________________________

Question	Points	Score
1	6	
2	6	
3	6	
4	15	
5	7	
Total:	40	

This exam has 5 questions, for a total of 40 points.
The maximum possible point for each problem is given on the right side of the problem.

1. For each of the following statements, find out whether it is '**always true**', '**sometimes true**', or '**always false**'. Give a brief explanation for your answer. If you think an answer is 'sometimes true', give an example or criterion when it's false.

   (a) If A is a *nonzero* matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then the rank of A is 1. $\boxed{3}$

   (b) If $A^3 + I_3 = 0$ for a 3×3 matrix A, then A is invertible. $\boxed{3}$

2. Is it possible or impossible to find examples of each of the following? If possible, please provide an example. If impossible, please explain why.

   (a) A 3×4 matrix A such that $A\vec{x} = \vec{b}$ is consistent for all $\vec{b} \in \mathbb{R}^n$. $\boxed{3}$

(b) A $\mathbf{4 \times 3}$ matrix A such that $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ only has trivial solution. $\boxed{3}$

3. Suppose $A = \begin{bmatrix} \vec{\mathbf{A}}_1 & \vec{\mathbf{A}}_2 & \vec{\mathbf{A}}_3 \end{bmatrix}$ is a $\mathbf{5 \times 3}$ matrix, $\vec{\mathbf{b}}$ is some vector in $\mathbb{R}^5$, and $\boxed{6}$

$$\vec{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \text{and} \qquad \vec{\mathbf{x}}_2 = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}$$

are both solutions to $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. Find a linear dependence relation among $\vec{\mathbf{A}}_1, \vec{\mathbf{A}}_2$, and $\vec{\mathbf{A}}_3$.

4. Consider the augmented matrix

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{4} & \mathbf{-9} & \mathbf{-7} \\ \mathbf{0} & \mathbf{1} & \mathbf{-4} & \boldsymbol{k} \\ \mathbf{2} & \mathbf{1} & \boldsymbol{h} & \mathbf{0} \end{array}\right]$$

where $\boldsymbol{h}$ and $\boldsymbol{k}$ are real numbers.

(a) Row-reduce this matrix to the echelon form (not reduced echelon form). Write down the exact row operation you are doing at every step. $\boxed{6}$

(b) Give examples of real numbers $\boldsymbol{h}$ and $\boldsymbol{k}$ (one example for each case) such that the system has

i. no solution. $\boxed{3}$

ii. exactly one unique solution. $\boxed{3}$

iii. infinitely many solutions. $\boxed{3}$

5. Consider an $m \times n$ matrix A and an $n \times m$ matrix B such that $AB = I_m$. Assume $m \neq n$. Show that the columns of A are linearly dependent. $\boxed{7}$

[HINT: What can you say about m and n?]