Math 2000-B Handout 9: Diagonalizability and Similar Matrices

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■ Exercise 1.

(a) A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that A is not diagonalizable? Justify your answer.

(b) A is a 7×7 matrix with three eigenvalues. One eigenspace is two-dimensional, and one of the other eigenspaces is three-dimensional. Is it possible that A is not diagonalizable? Justify your answer.

■ Exercise 2.

(a) Construct a nonzero 2×2 matrix that is invertible but not diagonalizable.

(b) Construct a nondiagonal 2×2 matrix that is diagonalizable but not invertible.

■ Exercise 3.

Verify the following statements. The matrices are square.

(a) if A is both diagonalizable and invertible, then so is A^{-1}.

(b) If A is invertible and similar to B, then B is invertible and A^{-1} is similar to B^{-1}.

(c) If A is similar to B, then A^2 is similar to B^2.

(d) If B is similar to A and C is similar to A, then B is similar to C.

(e) If A is diagonalizable and B is similar to A, then B is also diagonalizable.

(f) If $B = P^{-1}AP$ and $\vec{x}$ is an eigenvector of A corresponding to an eigenvalue λ, then $P^{-1}\vec{x}$ is an eigenvector of B corresponding also to λ.

■ Exercise 4.

Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\vec{x}) = A\vec{x}$, where A is a 3×3 matrix with eigenvalues 5 and -2. Does there exist a basis $\mathscr{B}$ for $\mathbb{R}^3$ such that the $\mathscr{B}$-matrix for T is a diagonal matrix? Discuss.

■ Exercise 5.

Let V be a vector space with a basis $\mathscr{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n\}$ and let W be the same space as V with a basis $C = \{\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n\}$, and let I be the identity transformation $I : V \to W$. Find the matrix for I relative to $\mathscr{B}$ and $\mathscr{C}$. This is called the change-of-coordinates matrix from $\mathscr{B}$ to $\mathscr{C}$.